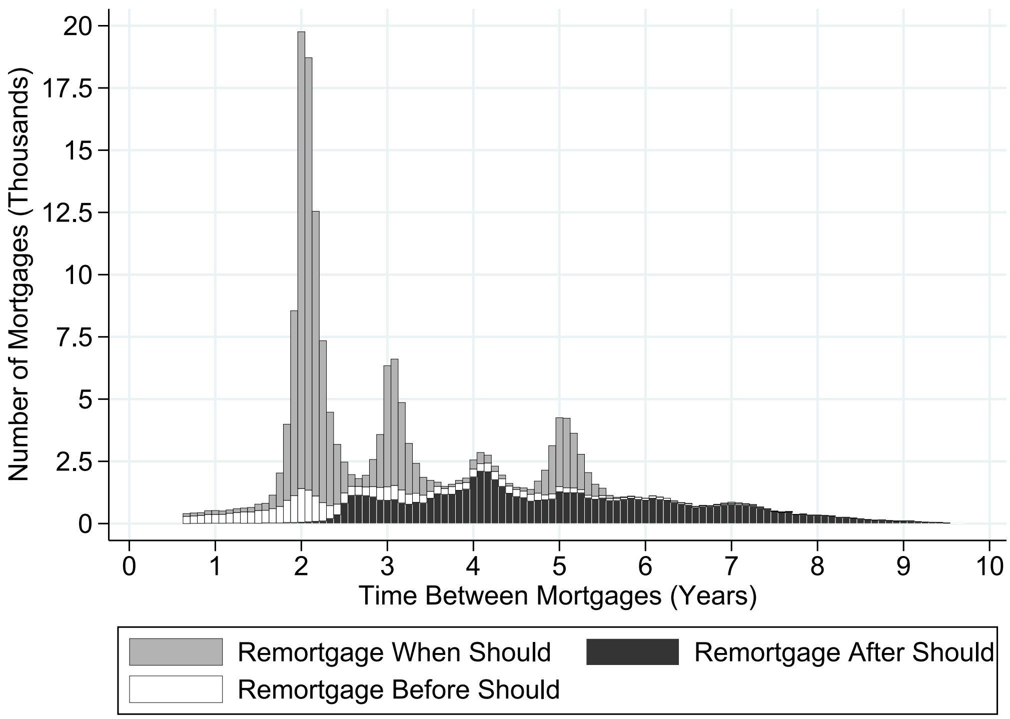
Web Appendix (Not For Publication)

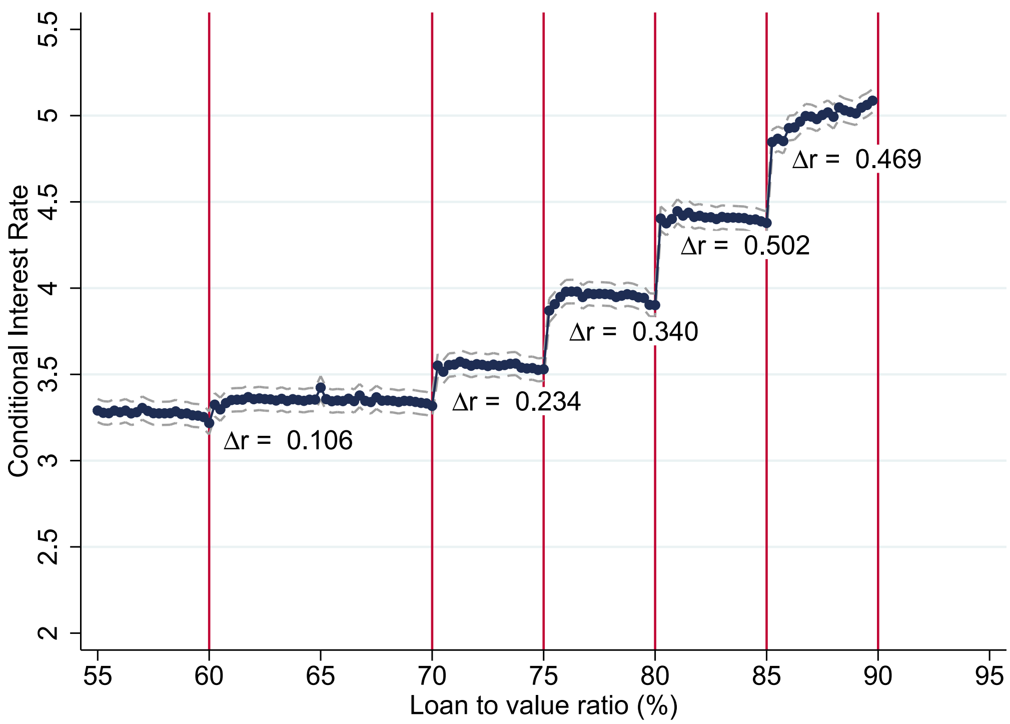
A Supplementary Figures and Tables

FIGURE A.1: REFINANCING HAPPENS WHEN THE RESET RATE KICKS IN



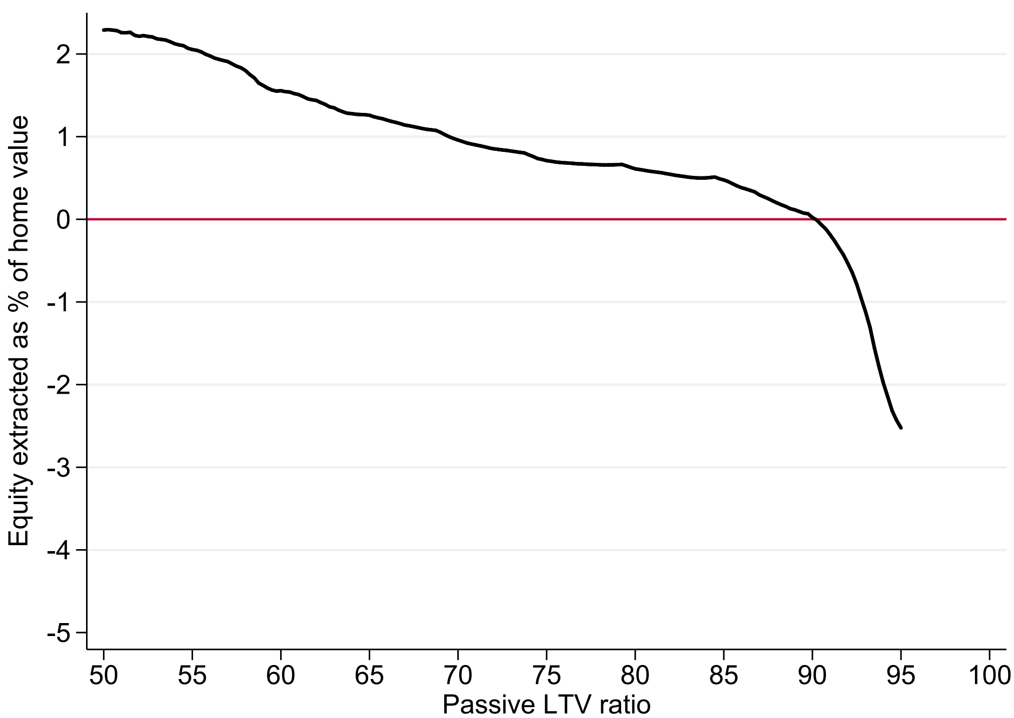
Notes: The figure shows the distribution of the time to refinance, excluding individuals where the date on which the reset rate kicks in is unobserved. The figure shows individuals individuals who refinance more than 6 months after their reset rate kicks in in black, individuals who refinance more than 2 months before their reset rate kicks in in white, and the remainder who refinance around their reset date in gray.

FIGURE A.2: ESTIMATING INTEREST RATE JUMPS WITH BORROWER DEMOGRAPHICS



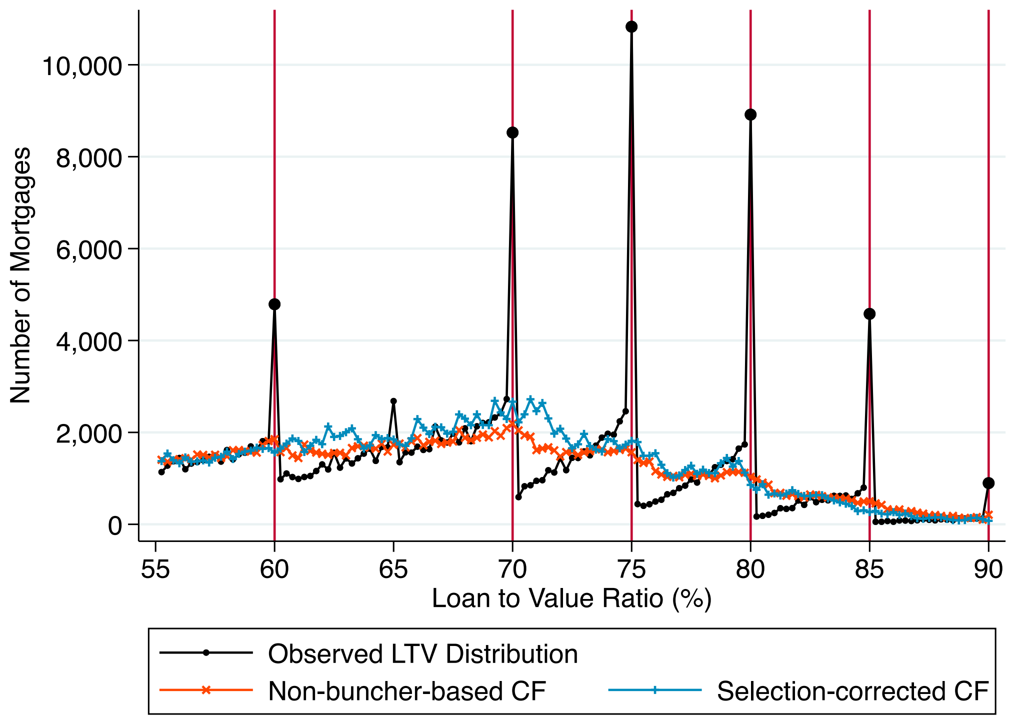
Notes: The figure shows the conditional interest rate as a function of the Loan-To-Value (LTV) ratio based on a regression like (1), but adding controls for borrower demographics. Specifically, we add controls for age, income, single/couple status, and the reason for refinancing. In each LTV bin, we plot the estimated coefficient on the LTV bin dummy plus a constant given by the mean predicted value $E[\hat{r}_i]$ from all the other covariates. The figure shows that the mortgage interest rate evolves as a step function with sharp notches at LTV ratios of 60%, 70%, 75%, 80%, and 85%. These notches are virtually unchanged compared to the specification without borrower demographics.

FIGURE A.3: EQUITY EXTRACTION BY PASSIVE LTV FOR NON-BUNCHERS



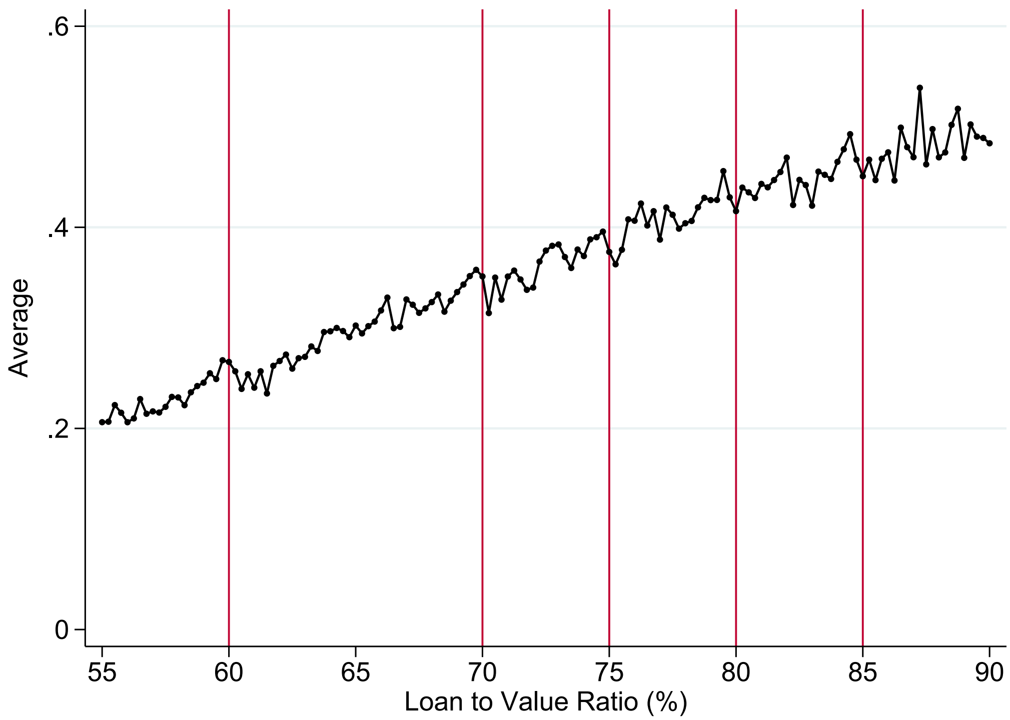
Notes: The figure shows the moving average of equity extracted on the y-axis, calculated among households that do not bunch in the actual LTV distribution. The x-axis is the passive LTV, i.e. the LTV that results from applying the amortization to the previous mortgage and using the new lender-assessed property valuation. This moving average is used to adjust the passive LTV distribution to obtain the counterfactual LTV distribution.

FIGURE A.4: COUNTERFACTUAL LTV DISTRIBUTION CORRECTING FOR SELECTION INTO BUNCHING



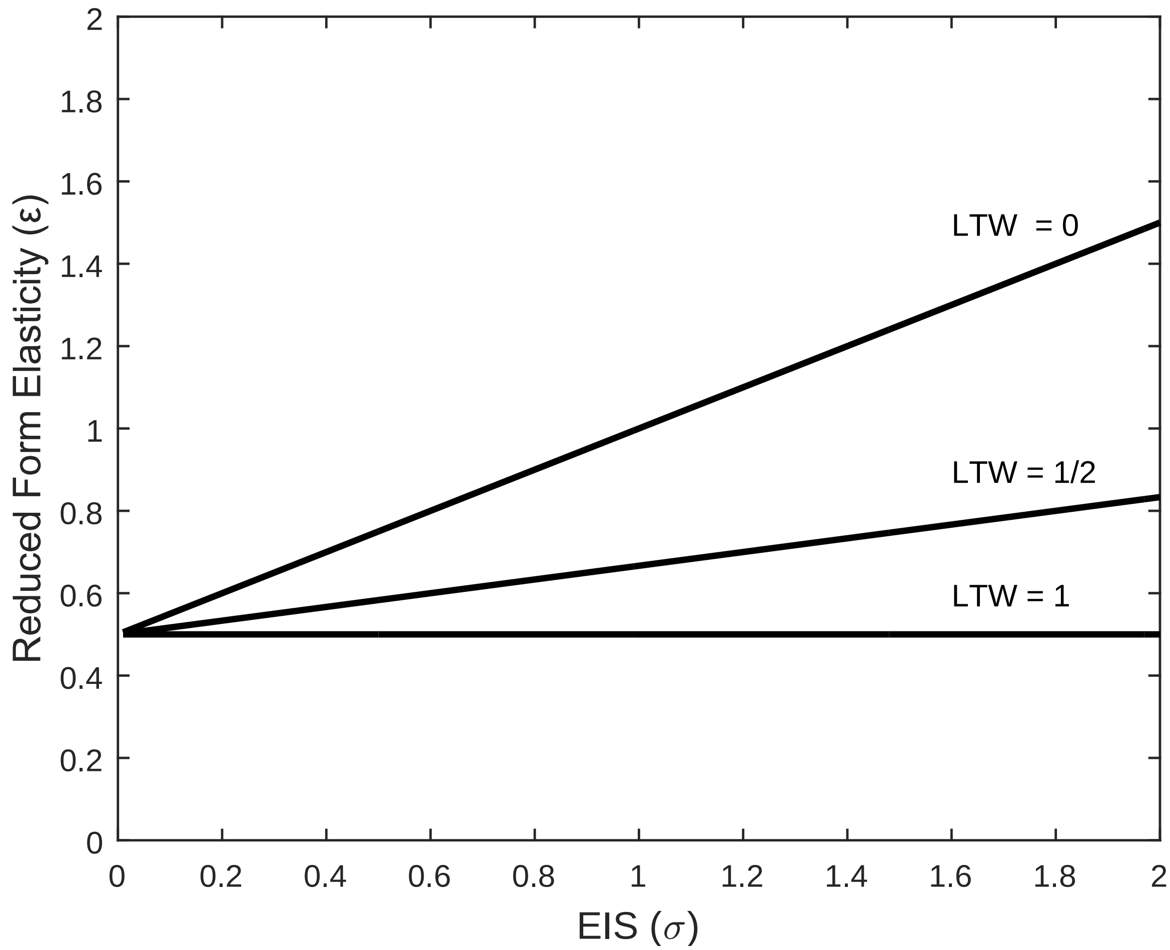
Notes: The figure shows the observed distribution of Loan to Value Ratios in black circles. The figure also shows two counterfactual distributions. First, in orange exes, the counterfactual created by adjusting the passive LTV distribution (Figure 4A) for equity extraction using non-bunchers to predict equity extraction by both bunchers and non-bunchers. Second, in blue crosses, the counterfactual created by estimating equity extraction correcting for sample selection created by selection into bunching. Both methods are described in greater detail in section 2.4.

FIGURE A.5: NUMBER OF PAST AND FUTURE BUNCHING EVENTS BY CURRENT LTV



Notes: The figure shows the average number of past and future bunching events as a function of current LTV choice.

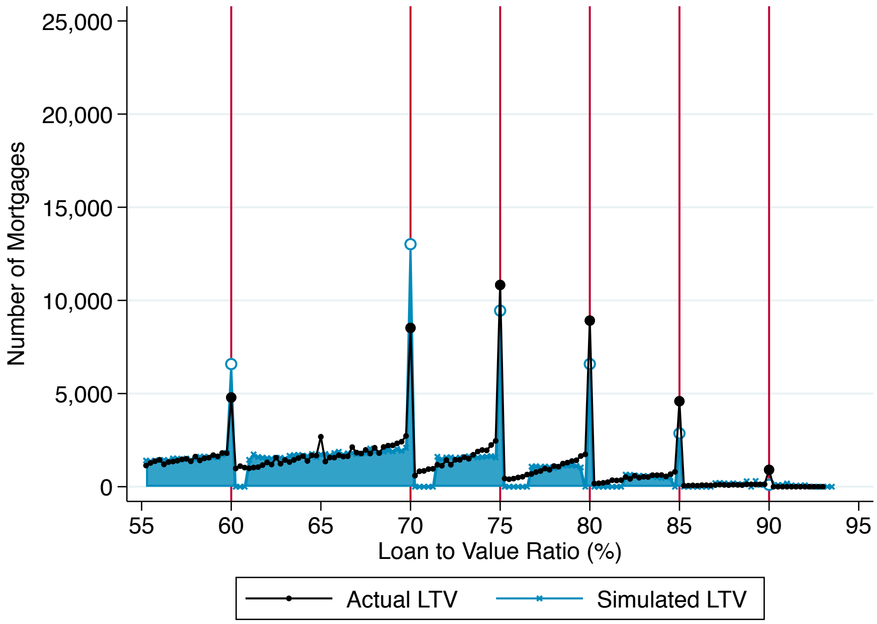
FIGURE A.6: STRUCTURAL EIS VS REDUCED-FORM ELASTICITY



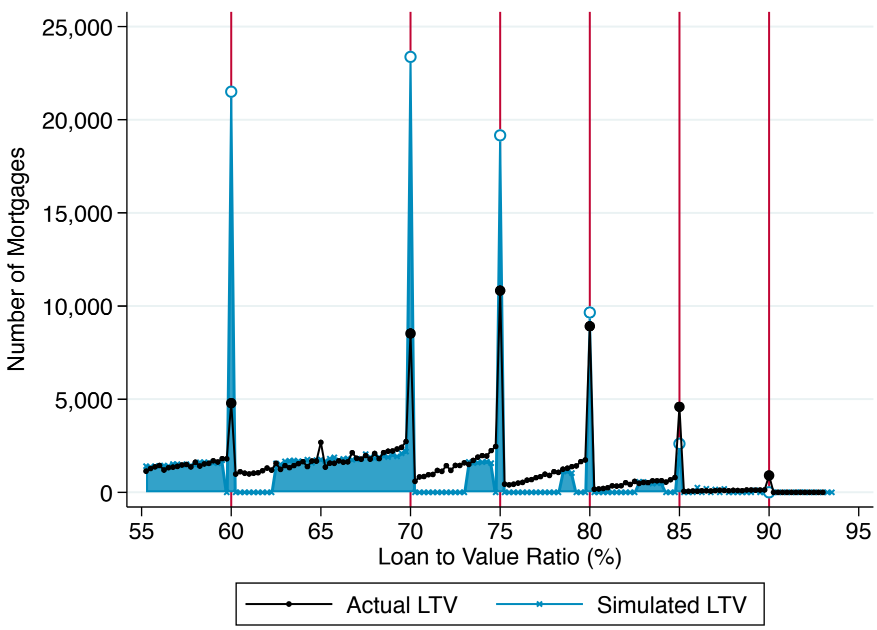
Notes: The figure shows the reduced-form borrowing elasticity ε as a function of the structural EIS σ, assuming that $\delta = R = 1$. The correspondence between the two follows from equation (10). The three curves correspond to three values of the loan-to-wealth (LTW) ratio, which is the ratio of the mortgage loan to total future housing and human wealth. The reduced-form elasticity is increasing in σ, but is also affected by LTW. A given reduced-form estimate is thus consistent with a wide range of structural estimates of the EIS.

FIGURE A.7: OBSERVED VS SIMULATED LTV DISTRIBUTIONS WHEN CALIBRATING NON-EIS PARAMETERS

Panel A: $\sigma = 0.06$**; Realistic** δ, y, P

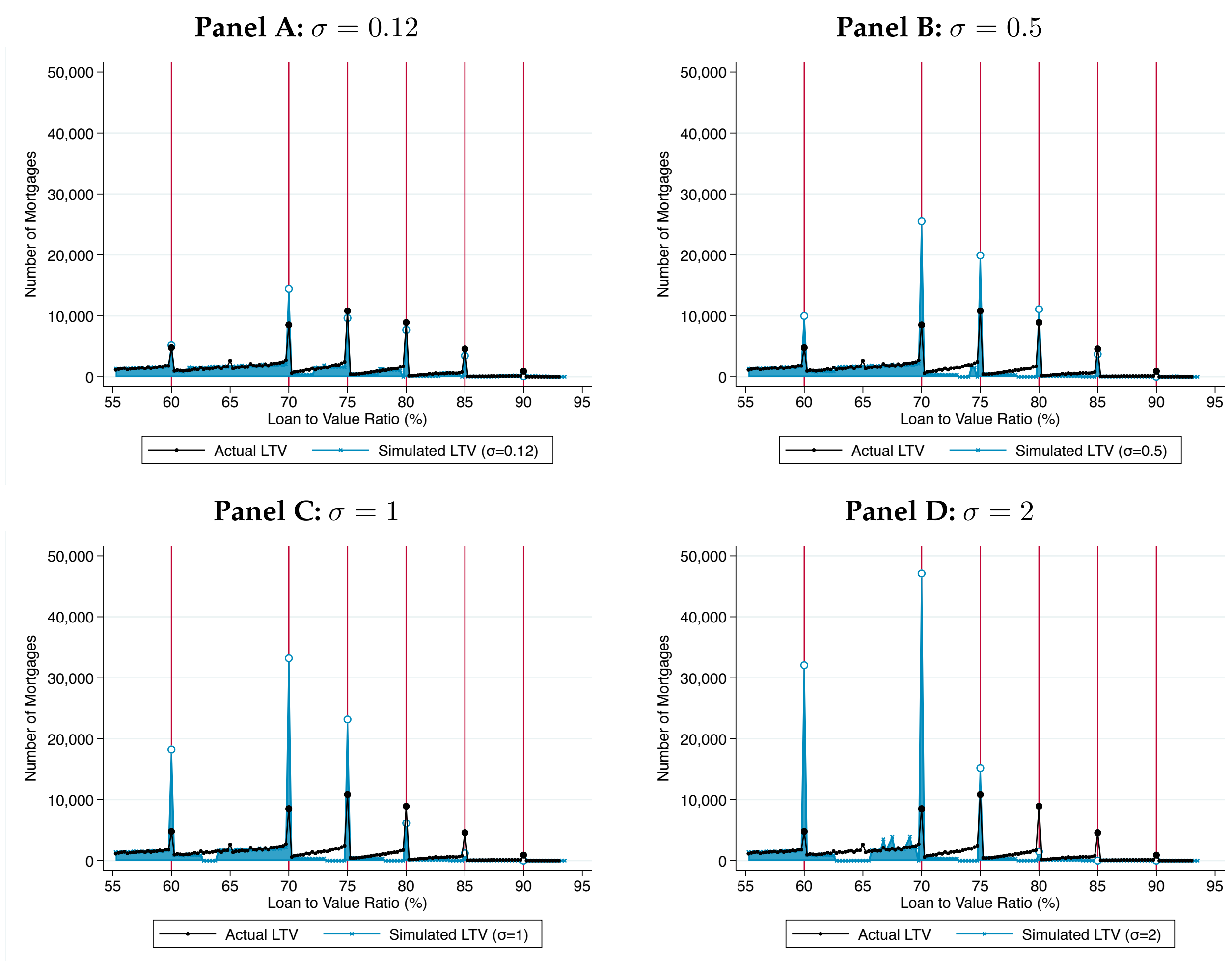


Panel B: $\sigma = 1$**; Calibrated** δ, y, P



Notes: The figure shows two simulations of a model introduced in Section 3. In the upper panel, the EIS is calibrated (to $\sigma = 0.06$) to minimize the MSE of the bunching moments, while other parameters are externally calibrated to realistic values. In the lower panel, the EIS is set to $\sigma = 1$ and remaining parameters are calibrated to minimize the MSE of the bunching moments. The blue lines show the predicted LTV distribution if households choose leverage optimally according to the model. The black lines show the empirical LTV distribution. The model can match the LTV distribution when calibrating the EIS alone, but has difficulty in doing so when $\sigma = 1$, even if all other parameters are set for this purpose. Further, the parameter values arising from this latter calibration are unrealistic, with a discount factor of $\delta = 0.24$, house price expectations of -12% annually and income growth expectations of -42% annually.

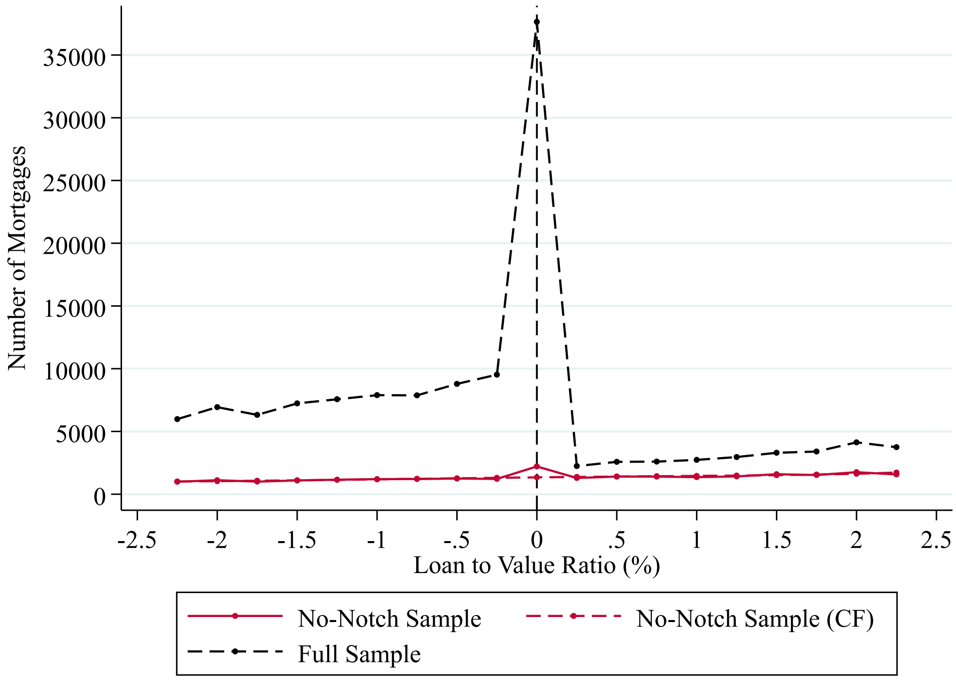
FIGURE A.8: OBSERVED VS SIMULATED LTV DISTRIBUTIONS WITH FRICTION ADJUSTMENT



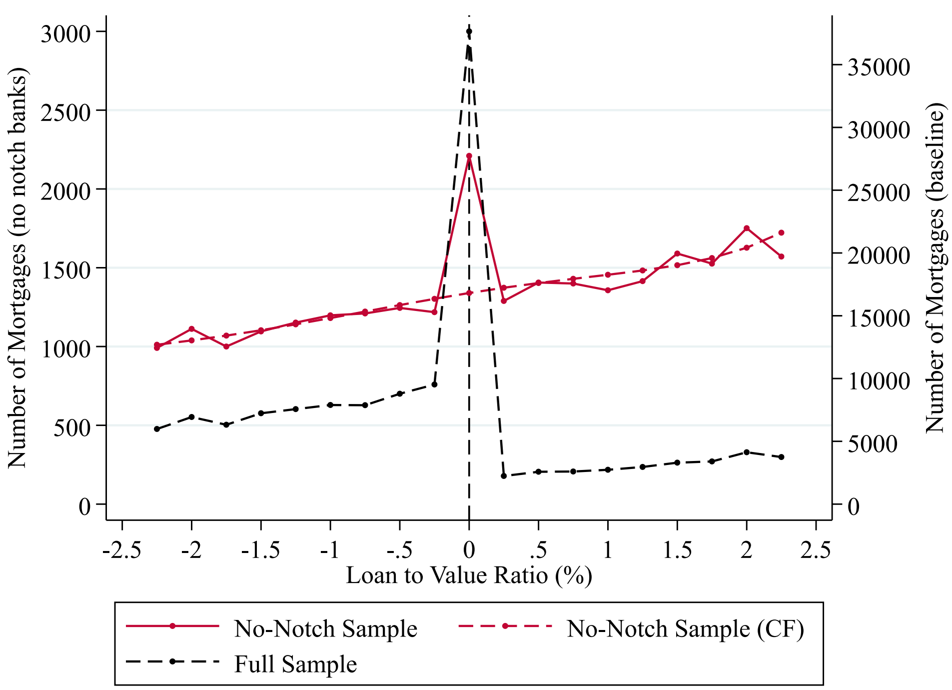
Notes: The figure shows simulations of a model introduced in Section 3 for a range of EIS values. The simulations include a friction adjustment so that a fraction $a^*$of non-bunching households are assumed to be "non-optimizers", who behave as though they face the counterfactual interest rate schedule (and thus choose the corresponding counterfactual LTV). The blue lines show the predicted LTV distribution from the model. The black lines show the empirical LTV distribution. The upper left hand corner has $\sigma = 0.12$, which is the EIS that minimizes the MSE of the predicted bunching masses. Higher EIS values predict far greater bunching masses than found in the data, with a large share of households jumping more than one notch in the LTV distribution to exploit lower interest charges. The distribution largely hollows out between notches, in contrast to the data.

FIGURE A.9: LTV DISTRIBUTION IN THE NO-NOTCH SAMPLE

Panel A: No-Notch Sample vs Full Sample

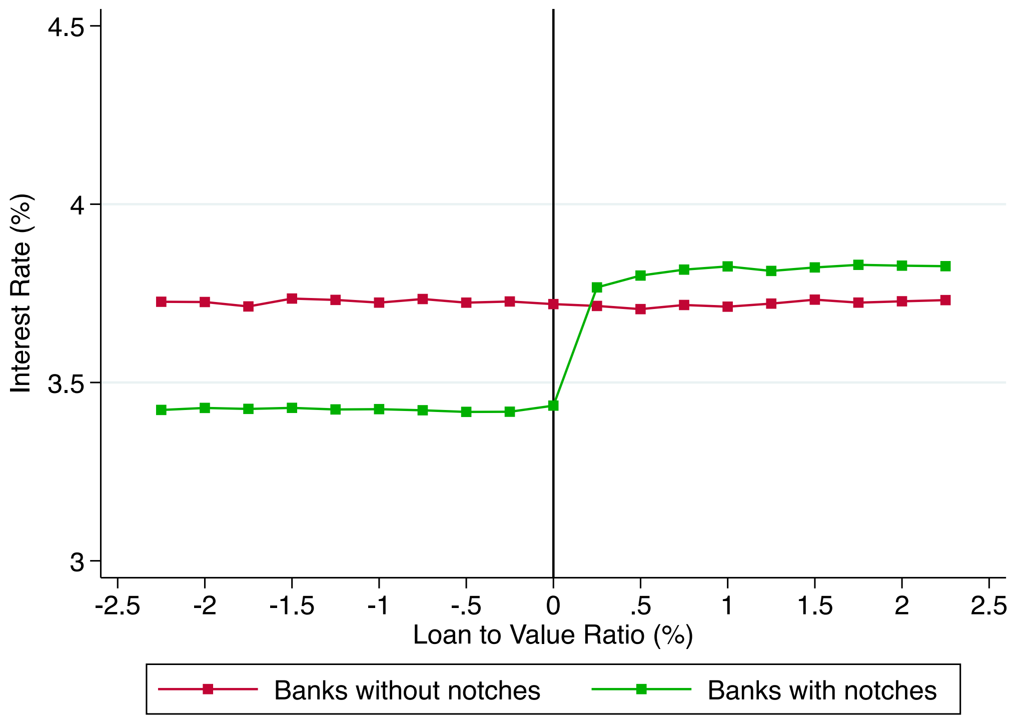


Panel B: Round-Number Bunching in the No-Notch Sample



Notes: This figure shows frequency of refinancers in the neighborhood of notches, in bank-month observations where the bank didn't feature a interest rate jump at the notch. Panel A show this distribution alongside the frequency of refinancers in the neighborhood of notches, in bank-month observations where a notch was present. It demonstrates that "no-notch banks" are a relatively small portion of our sample. Panel B zooms in on the distribution of mortgages at "no-notch banks", together with the counterfactual distribution. It shows a small amount of round number bunching. We correct our estimates of bunching in response to interest rates with the magnitude of round number bunching.

FIGURE A.10: INTEREST RATE SCHEDULES IN BANKS WITH AND WITHOUT NOTCHES



Notes: This figure shows the average interest rate in the neighborhood of the pooled notch for bank-months that featured a notch and those that did not. The interest rate is estimated using equation (1) for these subsamples. Relative to "no-notch banks", banks with notches offer a discount at LTVs below the notch."notched

TABLE A.1: BUNCHING AND EIS ESTIMATES WITH SELECTION-CORRECTED COUNTERFACTUAL DISTRIBUTION

Statistic	Notch 60	70	75	80	85	Pooled
Panel A: Bunching Evidence						
b	1.54	1.61	5.57	8.05	18.32	4.22
	(0.27)	(0.14)	(0.32)	(0.93)	(2.45)	(0.19)
a	0.42	0.27	0.20	0.20	0.12	0.08
	(0.04)	(0.02)	(0.02)	(0.03)	(0.05)	(0.00)
b_{Adj}	2.14	1.70	5.98	8.29	18.49	6.12
	(0.50)	(0.21)	(0.39)	(1.19)	(3.18)	(0.29)
$\Delta\lambda_{Adj}$	0.42	0.42	1.93	2.57	5.45	1.59
	(0.09)	(0.05)	(0.11)	(0.38)	(1.22)	(0.08)
Panel B: Elasticities						
EIS σ	0.02	0.02	0.06	0.07	0.28	0.05
	(0.00)	(0.00)	(0.01)	(0.03)	(0.10)	(0.01)
Reduced-form ε	0.52	0.52	0.54	0.55	0.65	0.54
	(0.00)	(0.00)	(0.00)	(0.01)	(0.04)	(0.00)

Notes: This table shows estimates of bunching, LTV responses, and the implied Elasticity of Intertemporal Substitution and reduced-form LTV elasticity when using the selection-corrected equity extraction predictions described in section 2.4 to form the counterfactual LTV distribution shown in figure A.4. The results are extremely similar to our baseline results in Table 2 using non-bunchers to predict equity extraction.

TABLE A.2: PARAMETER VALUES IN THE FULL STRUCTURAL MODEL

Parameter		**Value**	**Source**
Refinancing Cost	Ω	£1,000	Moneyfacts
House Price Process	Autocorrelation ρ_h Trend p_1 Variance σ_p^2	0.875 0.006 0.006	Nationwide mortgage data 1974–2016
Quadratic lifecycle income profile coefficients	linear quadratic	1,360 14	Her Majesty's Revenue & Customs
Unemployment probability		5%	Historical average
Replacement Rate		60%	Benefit formulas
Future Bank of England policy rate		Calibrated to yield curve	
Inflation expectations		2%	Bank of England target
Bequest motive	Γ	0.1	Internally calibrated
Mortgage amortization rate	μ_t	1 / (70 - Age + 1)	Moneyfacts
Risk aversion	γ	2	Literature
Housing depreciation	d	0.025/annum	Harding *et al.* (2007)
Discount factor	δ	0.96	Literature

Notes: This table shows calibrated parameters, their values, and source. A detailed description is found in Section D.2.

B Proofs of Propositions in the Simple Model

B.1 Proposition 1

The Euler equation (4) and the budget constraints (2) and (3) imply:

$$y_1 - R\lambda P_0 H + (1-d)\,P_1 H = (\delta R)^{\sigma}\left[W_0 + y_0 - (1-\lambda)\,P_0 H\right]. \tag{B.1}$$

Applied to the marginal buncher at the counterfactual, this gives (5). Applied at the optimal interior LTV it gives

$$\lambda^I P_0 H = \frac{y_1 + (1-d)\,P_1 H - (\delta\,(R+\Delta R))^{\sigma}\,(W_0 + y_0 - P_0 H)}{(\delta\,(R+\Delta R))^{\sigma} + (R+\Delta R)}$$

Then consumption in period zero at the optimal interior LTV is given by

$$\begin{aligned} c_0^I &= W_0 + y_0 - \left(1-\lambda^I\right) P_0 H \\ &= \frac{1}{(\delta R)^{\sigma}}\frac{((\delta R)^{\sigma} + R + \Delta R)\left(\frac{y_1}{P_0 H} + (1-d)\,\Pi_1\right) - (R+\Delta R)\,((\delta R)^{\sigma} + R)\,(\lambda^* + \Delta\lambda)}{(\delta\,(R+\Delta R))^{\sigma} + (R+\Delta R)} P_0 H. \end{aligned} \tag{B.2}$$

Using the Euler equation, the value of bunching at the interior is given by

$$V^I = \frac{\sigma}{\sigma-1}\left(1 + \delta^{\sigma}\,(R+\Delta R)^{\sigma-1}\right)\left(c_0^I\right)^{\frac{\sigma-1}{\sigma}}.$$

Plugging (B.2) into this last equation gives the value of the best interior LTV in (6).

Using the budget constraints (2) and (3), with LTV at the notch, λ^*, gives consumption of

$$\begin{aligned} c_0^N &= W_0 + y_0 - P_0 H + \lambda^* P_0 H \\ &= \frac{\frac{y_1}{P_0 H} + (1-d)\,\Pi_1 - R\lambda^* - ((\delta R)^{\sigma} + R)\,\Delta\lambda}{(\delta R)^{\sigma}} P_0 H \end{aligned}$$

and

$$\begin{aligned} c_1^N &= y_1 - R\lambda^* P_0 H + (1-d)\,P_1 H \\ &= \left(\frac{y_1}{P_0 H} + (1-d)\,\Pi_1 - R\lambda^*\right) P_0 H. \end{aligned}$$

Together, these give lifetime utility as in (7). The marginal buncher is defined as one who is indifferent between the optimal interior c_{0}=c_{1}LTV and bunching at the notch, so that $V^N = V^I$, giving the statement in Proposition 1.

B.2 Proposition 2

At an optimal interior LTV choice, (2) to (4) give

$$\lambda = \frac{y_1 + (1-d)\, P_1 H + (\delta R)^{\sigma}\, (P_0 H - W_0 - y_0)}{\left((\delta R)^{\sigma} + R\right) P_0 H}.$$

Differentiating this equation with respect to the interest rate R gives (10).

B.3 Proposition 3

As $\sigma \to 0$, the Euler equation gives $c_1 = c_0$. Lifetime utility converges to Leontief preferences and utility is equal to the smaller of c_0 and c_1. The Euler equation holds at the best interior LTV so that lifetime utility is given by period zero consumption c_0^I. Bunching at the notch requires forgoing current consumption for future consumption, so that $c_0^N < c_1^N$ and lifetime utility at the notch is given by c_0^N. Thus households are better off bunching at the notch even with a zero EIS for counterfactual LTVs that give $c_0^I < c_0^N$.

Applying the Euler equation $c_1 = c_0$ and the budget constraints (2) and (3) at the counterfactual with $\sigma = 0$ imply that initial wealth satisfies:

$$W_0 + y_0 - P_0 H = y_1 + (1-d)\, P_1 H - (R+1)\,(\lambda^* + \Delta\lambda)\, P_0 H.$$

At this level of initial wealth $c_1^I = c_0^I$ and the budget constraints imply that the best interior LTV is

$$\lambda^I = \frac{R+1}{R+\Delta R+1}\,(\lambda^* + \Delta\lambda)\,.$$

Applying this to (2) gives period zero consumption of

$$c_0^I = y_1 + (1-d)\, P_1 H - \frac{(R+1)\,(R+\Delta R)}{R+\Delta R+1}\,(\lambda^* + \Delta\lambda)\, P_0 H.$$

Applying (2) at the notch, where $\lambda = \lambda^*$ and the interest rate is R gives

$$c_0^N = y_1 + (1-d)\, P_1 H + \left(\lambda^* - (R+1)\,(\lambda^* + \Delta\lambda)\right) P_0 H.$$

As noted above, a region of the counterfactual distribution is strictly dominated by bunching if $c_0^N > c_0^I$ even as $\sigma \to 0$. Applying the last two equation to this inequality gives

$$\frac{\lambda^* + \Delta\lambda}{\lambda^*} \leq \frac{R + \Delta R + 1}{R + 1},$$

or

$$\lambda^* + \Delta\lambda < \left(1 + \frac{\Delta R}{R + 1}\right)\lambda^*,$$

giving the dominated range in (11).

C Multi-Period Version of the Simple Model

The two-period model in section 3 can easily be extended to have many periods, $t = 0, 1, ..., T$. In the multi-period version of the model, we assume that households face a notched interest rate schedule in period 0, but do not face notches after this time. We also assume that house price growth net of depreciation is constant. Households maximize their lifetime utility from non-housing consumption $\frac{\sigma}{\sigma-1}\sum_{t=0}^{T}\delta^t\left(c_t^i\right)^{\frac{\sigma-1}{\sigma}}$ and face a sequence of budget constraints given by

$$c_t = \begin{cases} y_0 + W_0 - (1 - \lambda_1) P_0 H_0 & \text{if } t = 0 \\ y_t - R_t \lambda_t P_{t-1} H_{t-1} + \lambda_{t+1} P_t H_t & \text{if } 1 \leq t < T \\ y_T - R_T \lambda_T P_{T-1} H_{T-1} + P_T H_T & \text{if } t = T \end{cases} \tag{C.3}$$

In the absence of notches, household maximization yields the Euler equation

$$c_t = (\delta R_t)^\sigma c_{t-1} \quad 1 \leq t \leq T - 1. \tag{C.4}$$

Combining this with the budget constraints from period 1 onward, period 1 consumption satisfies

$$c_1 = \frac{Y + (R_H - R_1\lambda_1) P_0 H_0}{\tilde{R}} \tag{C.5}$$

where $\tilde{R} \equiv \sum_{t=1}^{T}(\delta^\sigma)^{t-1}\prod_{s=1}^{t-1}(R_{s+1})^{\sigma-1}$ is a sufficient statistic for the future path of interest rates, $R_H \equiv \left(\prod_{s=2}^{T} R_s^{-1}\Pi^T\right)$ gives the value of house price appreciation to period T, and $Y \equiv \sum_{t=1}^{T} y_t \prod_{s=1}^{t-1} R_{s+1}^{-1}$ is the net present value of the household's income from period 1 inwards. Note that if interest rates are constant at R these become $\tilde{R} = \left[1 - \left(\delta^\sigma R^{\sigma-1}\right)^T\right] / \left[1 - \left(\delta^\sigma R^{\sigma-1}\right)\right]$, $R_H = R^{T-1}\Pi^T$, and

$Y = \left(1 - R^{-T}\right) / \left(1 - R^{-1}\right)$.

To derive the indifference condition of the marginal buncher in the multi-period model, we start by analyzing the marginal bunching household's counterfactual LTV choice at a constant interest rate R_1, $\lambda_1^* + \Delta\lambda_1$. This choice satisfies the Euler equation (C.4) in period 1 and using (C.5) allows us to express the marginal bunching household's wealth as a function of the other parameters of the model through

$$W_0 = P_0 H_0 - y_0 + \frac{\frac{Y}{\tilde{R}} + \left[\frac{R_H}{\tilde{R}} - \left(\frac{R_1}{\tilde{R}} + (\delta R_1)^\sigma\right)(\lambda_1^* + \Delta\lambda_1)\right] P_0 H_0}{(\delta R_1)^\sigma} \tag{C.6}$$

The marginal buncher's optimal interior choice of LTV λ^I at the higher interest rate $R_1 + \Delta R$ also satisfies the Euler equation in period 1. Inserting the period-0 budget constraint (C.3), the period-1 budget constraint (C.5) and the expression for wealth (C.6) yields

$$\lambda_1^I P_0 H = \frac{\frac{Y}{\tilde{R}} + \frac{R_H}{\tilde{R}} P_0 H_0 - \left[\delta\left(R_1 + \Delta R\right)\right]^\sigma \left(y_0 + W_0 - P_0 H\right)}{\frac{R_1 + \Delta R}{\tilde{R}} + \left[\delta\left(R_1 + \Delta R\right)\right]^\sigma} \tag{C.7}$$

Inserting equations (C.6) and (C.7) into the period-0 budget constraint, this choice of LTV yields consumption of

$$\begin{aligned} c_0^I &= y_0 + W_0 - P_0 H + \lambda_1^I P_0 H \\ &= \frac{\left(\frac{Y}{\tilde{R}} + \frac{R_H}{\tilde{R}} P_0 H_0\right)\left[(\delta R_1)^\sigma + \frac{R_1 + \Delta R}{\tilde{R}}\right] - (\lambda_1^* + \Delta\lambda_1)\frac{R_1 + \Delta R}{\tilde{R}}\left[\frac{R_1}{\tilde{R}} + (\delta R_1)^\sigma\right] P_0 H_0}{(\delta R_1)^\sigma \left[\frac{R_1 + \Delta R}{\tilde{R}} + \left[\delta\left(R_1 + \Delta R\right)\right]^\sigma\right]} \end{aligned} \tag{C.8}$$

and so the lifetime non-housing consumption utility of the marginal buncher at the interior choice λ^I is given by

$$\begin{aligned} V^I &= \frac{\sigma}{\sigma - 1}\left[\left(c_0^I\right)^{\frac{\sigma-1}{\sigma}} + \delta\tilde{R}\left(c_1^I\right)^{\frac{\sigma-1}{\sigma}}\right] = \frac{\sigma}{\sigma - 1}\left[\left(c_0^I\right)^{\frac{\sigma-1}{\sigma}} + \delta\tilde{R}\left(\left[\delta\left(R_1 + \Delta R\right)\right]^\sigma c_0^I\right)^{\frac{\sigma-1}{\sigma}}\right] \\ &= \frac{\sigma}{\sigma - 1}\frac{\tilde{R}}{R_1 + \Delta R}\left[\frac{R_1 + \Delta R}{\tilde{R}} + \left[\delta\left(R_1 + \Delta R\right)\right]^\sigma\right]^{\frac{1}{\sigma}} (\delta R_1)^{1-\sigma} \\ &\times \left(\left[\frac{Y}{\tilde{R}} + \frac{R_H}{\tilde{R}} P_0 H_0\right]\left[(\delta R_1)^\sigma + \frac{R_1 + \Delta R}{\tilde{R}}\right] - (\lambda_1^* + \Delta\lambda) P_0 H_0 \frac{R_1 + \Delta R}{\tilde{R}}\left[\frac{R_1}{\tilde{R}} + (\delta R_1)^\sigma\right]\right)^{\frac{\sigma-1}{\sigma}} \end{aligned} \tag{C.9}$$

If instead the marginal buncher chooses to be at the notch, the household's period-0 consumption is gets

$$
\begin{aligned}
c_0^* &= y_0 + W_0 - P_0 H + \lambda_1^* P_0 H \\
&= \frac{\frac{Y}{\tilde{R}} + \left[\frac{R_H}{\tilde{R}} - \left(\frac{R_1}{\tilde{R}} + (\delta R_1)^\sigma\right)(\lambda_1^* + \Delta\lambda_1)\right] P_0 H_0 + \lambda^* P_0 H_0 (\delta R_1)^\sigma}{(\delta R_1)^\sigma}
\end{aligned} \tag{C.10}
$$

where the second equality follows by substituting wealth using equation (C.6). Equation (C.5) implies that their period-1 consumption is

$$
c_1^* = \frac{Y + (R_H - R_1 \lambda_1^*) P_0 H_0}{\tilde{R}} \tag{C.11}
$$

giving lifetime consumption utility of

$$
\begin{aligned}
V^N = \frac{\sigma}{\sigma - 1} \Bigg[& \left(\frac{\frac{Y}{\tilde{R}} + + \left[\frac{R_H}{\tilde{R}} - \left(\frac{R_1}{\tilde{R}} + (\delta R_1)^\sigma\right)(\lambda_1^* + \Delta\lambda_1)\right] P_0 H_0 + \lambda^* P_0 H_0 (\delta R_1)^\sigma}{(\delta R_1)^\sigma} \right)^{\frac{\sigma-1}{\sigma}} \\
& + \delta \tilde{R} \left(\frac{Y + (R_H - R_1 \lambda_1^*) P_0 H_0}{\tilde{R}} \right)^{\frac{\sigma-1}{\sigma}} \Bigg]
\end{aligned} \tag{C.12}
$$

The EIS in the extended model is therefore the solution to $V^N = V^I$.

We can also derive the reduced-form elasticity ε in the multi-period model by differentiating the period-1 Euler equation with respect to the period-1 interest rate (holding all future interest rates constant), yielding

$$
\varepsilon \equiv -\frac{d \log \lambda_1}{d \log R_1} = \frac{\left(R_1/\tilde{R}\right) + \sigma (\delta R_1)^\sigma}{\left(R_1/\tilde{R}\right) + (\delta R_1)^\sigma} - \frac{\sigma (\delta R_1)^\sigma \frac{P_0 H_0 - y_0 - W_0}{(Y + R_H P_0 H_0)/\tilde{R}}}{1 + (\delta R_1)^\sigma \frac{P_0 H_0 - y_0 - W_0}{(Y + R_H P_0 H_0)/\tilde{R}}} \tag{C.13}
$$

As $\sigma \to 0$, $\varepsilon \to \frac{R_1/\tilde{R}}{\left(R_1/\tilde{R}\right)+1} \simeq \frac{1}{1+T}$ bounding ε from below in the generalized model.

D Solving the Full Structural Model

In each period, households face a choice between the liquid asset and consumption. At the end of an existing mortgage (every m periods), or when moving, they refinance and also face a choice of debt (or LTV). Finally, households face a discrete choice of housing quality (moving choice). We analyze these three margins in turn.

LIQUIDITY CHOICE: A household that neither moves ($H_{t+1} = H_t$) nor refinances ($D_{t+1} = (1 - \mu_t) D_t$)

chooses consumption c_t and liquidity L_{t+1} to maximize lifetime utility, i.e.

$$V_t^L(L_t, H_t, D_t) = \max_{L_{t+1}, c_t} \frac{\sigma}{\sigma - 1} \left(c_t^{\alpha} H_{t+1}^{1-\alpha} \right)^{\frac{\sigma-1}{\sigma}} + \delta E_t \left\{ V_{t+1}(L_{t+1}, H_{t+1}, D_{t+1}) \right\}$$

subject to the budget constraint

$$c_t = y_t + (1 - \pi_t) L_t - L_{t+1} - (r_t + \mu_t) D_t - dP_t H_t.$$

$V_t^L(.)$ gives the value to a borrower entering period t, if she chooses to remain in the same house and with the same mortgage. $V_{t+1}(.)$ gives the value to a borrower entering period $t+1$. This maximization problem gives the following *short-run Euler equation:*

$$\psi_t = \delta E_t \left\{ (1 - \pi_{t+1}) \psi_{t+1} \right\} + \zeta_t, \tag{D.14}$$

where ζ_t is the shadow value of relaxing the liquidity constraint, and ψ_t is the marginal utility of non-durable consumption given by

$$\psi_t \equiv \alpha \left(\frac{H_{t+1}}{c_t} \right)^{1-\alpha} \left(c_t^{\alpha} H_{t+1}^{1-\alpha} \right)^{-\frac{1}{\sigma}}. \tag{D.15}$$

Equation (D.14) is a standard Euler equation that governs how a household draws down or accumulates liquidity in order to smooth non-housing consumption. The non-negativity constraint on liquidity creates a precautionary motive to hold liquid assets. In effect, a household that neither moves nor refinances faces a cake-eating problem as it runs-down liquidity until the next time it refinances.

MORTGAGE DEBT CHOICE: When refinancing an existing house, the household faces the following decision problem

$$V_t^R(L_t, H_t, D_t) = \max_{L_{t+1}, D_{t+1}, c_t} \frac{\sigma}{\sigma - 1} \left(c_t^{\alpha} H_{t+1}^{1-\alpha} \right)^{\frac{\sigma-1}{\sigma}} + \delta E_t \left\{ V_{t+1}(L_{t+1}, H_{t+1}, D_{t+1}) \right\} \tag{D.16}$$

subject to the budget constraint

$$c_t = y_t + (1 - \pi_t) L_t - L_{t+1} - dP_t H_t + D_{t+1} - R_t D_t - \Omega. \tag{D.17}$$

Here $V_t^R(.)$ is the value to a borrower entering period t, conditional on refinancing. Recall that the interest rate in the following period(s) is a function of the choice of current LTV and therefore of current debt. Specifically, it is a flat function of LTV between notches and features discrete jumps at notches. Hence, the continuation value $V_{t+1}(L_{t+1}, H_{t+1}, D_{t+1})$ is discontinuous at the critical LTV ratios and therefore at critical values of debt D_{t+1}.[46] The choice of debt D_{t+1} can therefore be separated into a discrete and continuous component. We define $V_t^I(L_t, H_t, D_t)$ as the value of choosing the best interior value of debt, i.e. the value of maximizing (D.16) s.t. (D.17) while ignoring the presence of notches. Moreover, we define $V_t^N(L_t, H_t, D_t)$ as the value of borrowing to the notch, i.e. the value of maximizing (D.16) s.t. (D.17) when restricting to $D_{t+1} = \lambda^* P_t H_{t+1}$. A household chooses to bunch at the notch iff $V_t^N(L_t, H_t, D_t) \geq V^I(L_t, H_t, D_t)$. This is equivalent to the bunching decision in the 2-period model of Section 3 that led to the indifference equation (8). Hence $V_t^R(L_t, H_t, D_t) \equiv \max\left\{V_t^N(L_t, H_t, D_t), V^I(L_t, H_t, D_t)\right\}$ gives the value of refinancing.[47]

Whether borrowing at the interior optimum or at the notch, liquidity choice is given again by (D.14). When refinancing, a household chooses the liquid buffer stock it wishes to store in anticipation of the cake-eating it will face while locked in to the current mortgage. The interior choice of debt is given by

$$\psi_t = -\delta E_t \left\{ \frac{\partial V_{t+1}(L_{t+1}, H_{t+1}, D_{t+1})}{\partial D_{t+1}} \right\},$$

where $H_{t+1} = H_t$ (not moving). The envelope theorem cannot generally be used to evaluate the marginal cost of debt (the right hand side of the equation), because of the fixed cost to refinancing and the discontinuities in the value function due to the notched mortgage schedule. But conceptually, the marginal cost of debt is driven by the discounted marginal utility of non-durable consumption at the next refinancing event. Specifically, if the time of next refinancing were known with certainty and the household never ran of out liquidity between mortgages, the first order condition would be rewritten as

$$\psi_t = \delta^m E_t \left\{ R_{t,t+m} \psi_{t+m} \right\}, \tag{D.18}$$

where $R_{t,t+m}$ is the cumulative marginal cost of a unit of debt carried until the next refinancing year.[48] This is a *long-run Euler equation* governing the choice of debt over the lifecycle. The long- and short-run Euler equations echo those studied in Kaplan & Violante (2014). Using their terminology, households in this model are wealthy hand-to-mouth: They have positive net worth, but

[46] This was also the case in the liquidity choice problem discussed above, but didn't affect the analysis of liquidity choice.

[47] The household may also choose to jump several notches, so formally this comparison must be done against all interest rate notches.

[48] $R_{t,t+m}$ is a function of the mortgage interest rate, the inflation rate and the amortization rate in the years of the existing mortgage's duration.

can liquidate their wealth between refinancing episodes only at a cost. When they do not refinance, households can only use their liquid wealth for intertemporal substitution. In contrast, in a refinancing period, housing wealth becomes liquid again. Hence two separate Euler equations govern household behavior in these two instances. The short-term Euler equation governs the household's liquidity management between mortgages and–when the liquidity constraint binds–their quasi-hand-to-mouth behavior. The long-run Euler equation determines the household's longer-term lifecycle debt management choices.

How do the two Euler equations relate to each other? Assuming zero consumer good inflation (to sharpen the intuition) and using the law of iterated expectations, the two combine to give

$$E_t\left\{\sum_{s=0}^{m}\delta^s\zeta_{t+s}\right\} = E_t\left\{R_{t,t+m}\psi_{t+m}\right\} - E_t\left\{\psi_{t+m}\right\}. \tag{D.19}$$

This equation equates the marginal benefit of paying down debt to that of holding liquidity. The left hand side of the equation gives the marginal value of holding liquidity, given by the expected net present value of the shadow cost of the liquidity constraint. The right hand side gives the marginal benefit of paying down debt. It gives the excess return on (paying down) mortgage debt relative to the (zero) return on liquid assets: The liquidity premium.

HOUSING CHOICE: A moving household faces the following decision:

$$V_t^M\left(L_t, H_t, D_t\right) = \max_{L_{t+1},H_{t+1},D_{t+1},c_{t+1}} \frac{\sigma}{\sigma-1}\left(c_t^{\alpha}H_{t+1}^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}} + \delta E_t\left\{V_{t+1}\left(L_{t+1}, H_{t+1}, D_{t+1}\right)\right\}$$

subject to

$$\begin{aligned} c_t &= y_t + \left(1-\pi_t\right)L_t - L_{t+1} \\ &\quad + P_t\left(\left(1-d\right)H_t - H_{t+1}\right) \\ &\quad + D_{t+1} - R_tD_t - \Omega. \end{aligned}$$

The first-order conditions (D.14) and (D.18) still hold: The household is on its short-run and long-run Euler equations. In choosing a new mortgage, households face a similar bunching decision as in the refinancing decision described above. Housing choice is given by

$$\left[1-\delta\left(1-d\right)E_t\left\{\frac{P_{t+1}}{P_t}\frac{\psi_{t+1}}{\psi_t}\right\}\right]\frac{P_tH_{t+1}}{c_t} = \frac{\alpha}{1-\alpha}, \tag{D.20}$$

This first-order condition gives the relative expenditure on consumption c_t and housing H_{t+1}. With Cobb-Douglas preferences, relative expenditure on commodities is equal to the ratio of their loadings in the Cobb-Douglass function (in this case α and $1-\alpha$). However, in evaluating housing expenditure, the price of housing isn't evaluated at its spot price P_t, but also includes an additional term (given in in square brackets) that considers the asset value of housing.

MOVING CHOICE: The household moves if $V_t^M(L_t, H_t, D_t)$ exceeds $V_t^R(L_t, H_t, D_t)$ (when refinancing) or $V_t^L(L_t, H_t, D_t)$ (when not refinancing). Conceptually, households will choose to move when housing expenditure is sufficiently far from optimal, as per (D.20). When refinancing, households extract or inject equity when they are sufficiently far off of their long-run Euler equations. This occurs when interest rates are low relative to the value of liquidity (equity extraction decision) or interest rates are high and the household has sufficient liquidity (equity injection).

BEQUESTS: Finally, in period T, the households may no longer borrow and choose housing and liquidity as follows:

$$V_T(L_T, H_T, D_T) = \max_{L_{T+1}, H_{T+1}} \frac{\sigma}{\sigma-1}\left[\left(c_T^{\alpha} H_{T+1}^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}} + \delta\left(\Gamma V_{T+1}\right)^{\frac{\sigma-1}{\sigma}}\right].$$

The overall magnitude of bequests is largely driven by the bequest parameter Γ. We evaluate terminal wealth at period T prices. Hence there is no reason to bequeath any amount of the liquid asset unless house prices are expected to decline. Evaluating bequests at expected prices adds a portfolio motivation to bequeath some quantity of the liquid asset as a hedge against declining house prices, but doesn't impact estimates of the EIS that are based on bunching decisions taken more than 30 years earlier.

D.1 Bunching and Solving for the EIS computationally

We now consider the bunching decision in more detail and how we confront it with the bunching moments to estimate the EIS. The model is solved computationally via backward induction starting from age 70 (bequest decision) and solving back to the age τ at which we observe households in the data (age 38 on average in the full sample, but this varies across cuts of the data). For each guess of σ, we iterate on the model to solve for the value function $V_{\tau+1}(L_{\tau+1}, H_{\tau+1}, D_{\tau+1}|\sigma)$. We use this value function to evaluate households' continuation value as they make their refinancing choice. Households observed in the data are non-moving refinancers. In our model, they therefore face a choice of debt, liquidity, and consumption at time τ. Given their debt choice $D_{\tau+1}$ and using initial wealth W_τ, we can solve for optimal consumption and liquidity as the maximands of

$$V_\tau\left(W_\tau|\sigma\right) = \frac{\sigma}{\sigma-1}\left(c_\tau^\alpha H_{\tau+1}^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}} + \delta V_{\tau+1}\left(L_{\tau+1}, H_{\tau+1}, D_{\tau+1}|\sigma\right),$$

subject to the budget constraint

$$c_\tau = W_\tau - L_{\tau+1} - \left(1-\lambda_{\tau+1}\right) P_\tau H_{\tau+1}, \tag{D.21}$$

where $\lambda_{\tau+1} = \lambda^*$ when bunching and $\lambda_{\tau+1} = \lambda^I$ is solved as the optimal interior LTV choice. In either case, debt is given by $D_{\tau+1} = \lambda_{\tau+1} P_\tau H_{\tau+1}$. The solution of the liquidity-choice problem for the two cases gives value functions $V_\tau^N\left(W_\tau|\sigma\right)$ (bunching) and $V_\tau^I\left(W_\tau|\sigma\right)$ (interior). The marginal buncher is indifferent between bunching at locating at the optimal interior LTV. For this borrower, the indifference equation

$$V_\tau^B\left(W_\tau|\sigma\right) = V_\tau^I\left(W_\tau|\sigma\right) \tag{D.22}$$

holds and can be solved for σ. This is done by repeating the entire process for a range of σ values and searching for the EIS that solves the indifference equation.

Of course, (D.22) contains parameters other than σ and a number of state variables. How, then, is σ identified from this equation? The discount factor δ is an important determinant of the level of borrowing, but has only second order implications for the marginal response to interest rates, as discussed in Section 3. Accordingly, we find that our results are robust to a wide range of δ and to hyperbolic discounting. Risk aversion γ could potentially play a role in bunching responses as it governs the elasticity of demand for liquidity. We experiment with a wide range of values for this parameter and show that for any degree of risk aversion, a low EIS is nevertheless necessary to explain the magnitude of bunching moments. Expectations are affected by the stochastic processes of house prices and income and the future path of interest rates (as well as the depreciation rate and bequest motives). We discuss their calibration below. However, as we show in our robustness analysis, our empirical methodology isn't sensitive to the calibration of these processes. This is because expectations shift both sides of (D.22) by similar amounts and roughly cancel out from the estimating equation.

Finally, we do not observe initial wealth directly in our data, but use the method outlined in Section 3 to estimate it. That is, we back out initial wealth W_τ from the optimality condition of the marginal buncher at the counterfactual. In the context of the full model, we define initial wealth as the sum of housing net worth and the liquid asset, net of the refinancing fee:

$$W_\tau \equiv (1-\pi_\tau)\, L_\tau + (1-d)\, P_\tau H_\tau - R_\tau D_\tau - \Omega.$$

In the extended model studied here, a closed-form solution for initial wealth is unavailable, but we can solve computationally for initial wealth with the following steps.

1. Invert the Euler equations (D.14) and (D.18) and use the counterfactual LTV $\lambda + \Delta\lambda$ from the bunching moment to back out optimal consumption c_τ and liquidity $L_{\tau+1}$.

2. Use the budget constraint (D.21), the counterfactual LTV, c_τ, and $L_{\tau+1}$ to back out initial wealth W_τ.

To see how this is applied in practice, let $\lambda^* + \Delta\lambda$ be the counterfactual LTV estimated for the average marginal buncher. We observe house value $P_\tau H_{\tau+1}$ in the data and can translate this into debt $D_{\tau+1} = (\lambda^* + \Delta\lambda)\, P_\tau H_{\tau+1}$. The solution of the lifecycle model gives us the value function $V_{\tau+1}(L_{\tau+1}, H_{\tau+1}, D_{\tau+1}|\sigma)$ and the long- and short-run Euler equations give

$$\psi_\tau = -\delta E_\tau \left\{ \frac{\partial}{\partial D_{\tau+1}} V_{\tau+1}(L_{\tau+1}, H_{\tau+1}, D_{\tau+1}|\sigma) \right\}$$

and

$$\psi_\tau = \delta E_\tau \left\{ \frac{\partial}{\partial L_{\tau+1}} V_{\tau+1}(L_{\tau+1}, H_{\tau+1}, D_{\tau+1}|\sigma) \right\}.$$

The marginal utility of consumption ψ_τ is a function of consumption c_τ and housing $H_{\tau+1}$ as in (D.15). Given housing $H_{\tau+1}$, the two Euler equations can be solved (computationally) for consumption c_τ and liquidity choice $L_{\tau+1}$.[49] We can then use the budget constraint to back out initial wealth:

$$W_\tau = c_\tau - y_\tau + L_{\tau+1} - (1 - (\lambda^* + \Delta\lambda))\, P_{\tau+1} H_{\tau+1}. \tag{D.23}$$

Initial wealth W_τ can then be applied to the budget constraint (D.21) when evaluating the indifference equation (D.22).

[49] We observe the nominal value of housing $P_\tau H_{\tau+1}$, but housing quality $H_{\tau+1}$is unobservable. In our baseline estimates, we assume households have the lowest house quality at the bunching choice, consistent with the lifecycle pattern of housing choices. Results were robust to allowing any value of initial housing quality. This is partially due to the unit elasticity between housing and non-housing consumption in our assumed preferences. Strong complementarities between housing and non-housing consumption would lead to behavior that is observationally equivalent to a low EIS in our model. See Flavin (2012) for a discussion. As we discuss below, strong complementarity between housing and consumption are a potential alternative explanation for the low EIS estimated in our model.

D.2 Calibration

Calibrated parameter values are summarized in Table A.2. We now detail how these parameters were calibrated.

General Assumptions: We assume that households always refinance when the reset rate kicks in and set $m = 3$, based on the average time to refinance in our data. The household faces a liquidity choice in all periods, as summarized by the short term Euler equation (D.14). In addition, the household faces a refinancing (and potentially housing) choice every third period. These variables are chosen in accordance with the the short term and long term Euler equations (D.14) and (D.18) and housing choice (D.20). We set the fixed refinancing cost to $\Omega = £1,000$, which is the origination fee on the typical mortgage product in the UK.

House Prices: We assume house prices follow a log linear AR(1) process around a deterministic growth rate. Accordingly:

$$\ln P_t = p_0 + p_1 t + \rho_h \ln P_{t-1} + \varepsilon_t^p$$

$$\varepsilon_t^p \sim N\left(0, \sigma_p^2\right)$$

Using data from the mortgage lender Nationwide from 1974 to 2016 we calibrated the parameters of this process to $\rho_h = 0.875$, $p_1 = 0.006$, and $\sigma_p^2 = 0.006$. We set p_0 so as to match the house price at the time of refinancing in our own data, i.e. we treat individual house prices as having a constant level shift relative to the national house price process. We will show that our results are robust to different assumptions about house-price growth and uncertainty.

Income: We assume that households face i.i.d. unemployment shocks around a deterministic age profile y_t^{LC}. The i.i.d assumption reduces the state space and eases computation. We will show that our results are robust to different degrees of income uncertainty and different lifecycle income patters. Using HMRC data, the average lifecycle profile y_t^{LC} is roughly quadratic with

$$y_t^{LC} = 1,360 * \text{Age} - 14 * \text{Age}^2 - y_0^i. \tag{D.24}$$

In the data the average intercept is $y_0^i = 6,830$. However, we observe households' income and age at time $t = \tau$: The bunching decision. We can therefore match individual's y_0^i based on their age in the data. In other words, we treat the household's cross-sectional deviation from the average age-income profile as a permanent level shift.

We set the probability of unemployment to 5%, roughly the historical average, although results are robust to different probabilities as we show in our robustness analysis. Applying formulae for unemployment benefits to the typical household in our sample gave a replacement rate of approximately 60% in the first year of unemployment when considering all available benefits, including the universal credit and the job seeker's allowance. Given our i.i.d. assumption, households rarely face an unemployment spell exceeding a year.[50]

INTEREST RATES: We assume households face a fixed interest rate for the $m = 3$ years of the mortgage. Mortgage interest rates have a risk premium $\rho(\lambda)$ over the Bank of England Policy (real) rate r_t^0. We assume that the risk premium is a constant function of LTV as represented in the notched LTV schedule shown in Figure 3, but that the reference policy rate varies over time. We assume that the policy rate follows a deterministic time path to reduce the dimensionality of the problem and ease computation. We forecast the (real) Bank of England policy rate with forward rates implied by the UK yield curve. This implies a slowly increasing path of interest rates over time.

INFLATION (EXPECTATIONS): We assume inflation is 2% a year each year, as per the Bank of England's target. Higher or stochastic inflation has some implications for portfolio choice (high inflation expectations make nominal liquid assets relatively less attractive), but little implication for the estimated EIS.

BEQUEST MOTIVE: We experimented with a range of parameters for the bequest motive Γ. Bequests are 30 years removed from the bunching decision for the average household in our sample and thus have little impact on our estimates of the EIS. The median British household leaves no bequests and the median British homeowner leaves only housing wealth as a bequest. The assumption $\Gamma = 0.1$ leads to bequests that are of similar magnitude to those observed in the data and we use this in our baseline simulations.

RISK AVERSION: We estimate the model with Epstein-Zin-Weil preferences. In our baseline estimates, we set risk aversion to $\gamma = 2$, as is common in the macro literature. We conduct robustness analysis with respect to risk aversion, including the possibility of $\gamma = \frac{1}{\sigma}$, e.g. CRRA preferences.

DEPRECIATION: Harding *et al.* (2007) estimate an annual depreciation rate of $d = 0.025$ per annum. This rate is close to UK estimates of the office of the deputy prime minister, as reported by Attanasio *et al.* (2012).

DISCOUNTING: We set $\delta = 0.96$, as is common in the literature and conduct robustness checks with

[50] One might expect the probability of unemployment to be lower for homeowners than the general population. Moreover, couples comprise half our sample and their replacement rate is higher if only one breadwinner is unemployed. Our results are robust to a wide range of unemployment probabilities and replacement rates. Generally, unemployment affects liquidity choice, but not the estimated EIS.

respect to this parameter. We also allow for hyperbolic discounting in additional robustness checks.

D.3 Model Simulation

As we turn to the results, it is useful first to illustrate the workings of the extended model. We do so by showing the output from a single (illustrative) simulation round of the model. The output is shown in Figure **??**. This simulation assumes $\sigma = 0.1$, which is within our range of estimates. We consider a household that begins in the neighborhood of the 75% interest notch at age 38 (a typical household in our sample). The top panels show the evolution of the exogenous state variables: house prices and income. House prices grow over time, but feature two slumps in this simulation round: one beginning when the borrower is 41 years old and the other in her late 50s. Income follows a lifecycle pattern, peaking in the early 50s. This pattern is punctuated by a brief spell of unemployment at age 49.

The middle panels show the borrower's housing and mortgage choices. Housing follows a lifecycle pattern: the borrower begins with the lowest housing quality (by assumption) and increases her house quality in her early 40s and again in her late 40s. She then downsizes late in life. Note that this occurs without any exogenously imposed lifecycle needs (e.g. children), but rather through an endogenous accumulation of wealth over the lifecycle. The borrower times her housing upgrades to take advantage of low prices during the first housing slump.

Turning to leverage, the borrower repays her mortgage over the lifecycle at the assumed amortization rate between mortgages. Early in life, the borrower tends to extract equity when refinancing and she then repays more towards the end of life. The household borrows to finance the two housing upgrades and notice that that household bunches at the 70% notch at the second move.

Turning to the bottom panels, the household chooses its lifetime amortization profile to smooth consumption over the lifecycle, giving a flat lifetime consumption profile.[51] There is a gradual increase in consumption at the end of the life, because the precautionary savings motive represses consumption at earlier ages. The net present value of lifetime (non-housing) consumption is smaller than the net present value of income because of housing purchases, maintenance, and debt servicing. House prices follow a very persistent process, so that the household views house price increases as reflecting substantial increases in net worth and she increases her consumption accordingly. This is particularly true late in life, when the increase in net worth allows a large increase in annual con-

[51] A hump-shaped consumption profile would be more empirically accurate and could be easily accommodated by adding an exogenous demand shifter. This addition has essentially no implications for the estimated EIS. We therefore abstract from lifecycle consumption preferences in the current specification.

sumption. At each refinancing episode, the household takes out a substantial amount of equity in the liquid asset (around £30,000) and runs down this liquidity over the duration of the mortgage.[52] Recalling that the borrower faces an unemployment spell at age 49, note that she rapidly draws down her liquid holdings to smooth the shock. This shock unfortunately hits shortly after she upgraded her property. Hefty precommitted mortgage repayments combined with the income loss due to unemployment rapidly exhausts the borrower's liquid holdings and requires a reduction in consumption. This is precisely the "wealthy hand-to-mouth" behavior that Kaplan & Violante (2014) postulate.

[52] The increased demand for liquid holdings later in life is because the liquid asset is the only financial asset available to households in the model and the household has already the maximal house quality allowed in our model.