Appendix

We start by showing the comparative statics for the model discussed in the text. We then add multiple shocks per period and consider the possibility that consumers have loans in later periods rather than insurance. Lastly, we allow for more flexible markets to show the "lower bound" on the value of insurance.

Model in Text

As shown in the text, in the case of a single potential shock, insurance in later periods, and limited credit markets (so one can only borrow in the case of a shock), the value of insurance is.

$$V^{UB} = (1-\pi)\left(u(y - m^I\pi^c p) - u(y)\right) - \frac{\pi}{\alpha}\Big(u\big(y - m^I\pi^c p - m^L\alpha p(1 - m^I\pi^c)\big) - u(y - m^I\pi^c p)\Big).$$

Risk. The effect of π in the case of community rating is simple:

$$\frac{\partial V}{\partial \pi} = u(y) - u(y - m^I\pi^c p) + \frac{1}{\alpha}\Big(u(y - m^I\pi^c p) - u\big(y - m^I\pi^c p - m^L\alpha p(1 - m^I\pi^c)\big)\Big) > 0.$$

In the case of experience rating it is more complicated. If we think only about the effect of risk in the first period (separating π into π_1 and π_t for later periods), then the incremental value of insurance is

$$V^{UB} = (1-\pi_1)\left(u(y - m^I\pi_1 p) - u(y)\right) + \pi_1\Bigg(\Big(u(y - m^I\pi_1 p) - u\big(y - m^I\pi_t p - m^L\alpha p(1 - m^I\pi_t)\big)\Big) + \left(\frac{1}{\alpha} - 1\right)\Big(u(y - m^I\pi_t p) - u\big(y - m^I\pi_t p - m^L\alpha p(1 - m^I\pi_t)\big)\Big)\Bigg).$$

The probability of the shock and period 1's premium are determined by π_1, but later period premiums and the amount one pays out of pocket (instead of borrowing) are determined by π_t. The effect of π_1 is

$$\frac{\partial V}{\partial \pi_1} = \Bigg(u(y) - u\big(y - m^I\pi_t - m^L\alpha p(1 - m^I\pi_t)\big) + \left(\frac{1}{\alpha} - 1\right)\Big(u(y - m^I\pi_t p) - u\big(y - m^I\pi_t p - m^L\alpha p(1 - m^I\pi_t)\big)\Big)\Bigg) - pm^I u'(y - m^I\pi_1 p).$$

The second derivative is

$$\frac{\partial^2 V}{\partial \pi_1^2} = +pm^I u''(y - m^I\pi p) < 0.$$

Also note that $[V^{UB}]_{\pi_1=0} = 0$. So if V equals zero for some $\pi > 0$ it must be that $\partial V/\partial \pi < 0$ at that π. So if someone is indifferent between insurance and loans then on the margin, raising π_1

makes them prefer loans. Obviously, risks are correlated across periods, so if we're comparing across people then we need to think about both π_1 and π_t changing

$$\frac{\partial V}{\partial \pi} = \frac{\partial V}{\partial \pi_1} - pm^I\pi\left(\left(\frac{1}{\alpha} - 1\right)u'(y - m^I\pi p) - \frac{1}{\alpha}u'\big(y - m^I\pi p - m^L\alpha p(1 - m^I\pi)\big)(1 - \alpha m^L)\right)$$

The increased premium payments in future periods raises the marginal utility of income in those periods, making it costlier to take out a loan in the first period, but this can be partially offset by borrowing less (hence the second term is multiplied by $1 - \alpha m^L$). The net effect is ambiguous.

Risk is also reflected in the price of medical care. Again, think about separating the period 1 price p_1 and later period price p_t.

$$\begin{aligned} V^{UB} = (1 - \pi)\,\big(u(y - m^I\pi^c p_1) - u(y)\big) \\ + \pi\Bigg(\Big(u(y - m^I\pi^c p_1) - u\big(y - m^I\pi^c p_t - m^L\alpha(p_1 - m^I\pi^c p_t)\big)\Big) \\ + \left(\frac{1}{\alpha} - 1\right)\Big(u(y - m^I\pi^c p_t) - u\big(y - m^I\pi^c p_t - m^L\alpha(p_1 - m^I\pi^c p_t)\big)\Big)\Bigg), \end{aligned}$$

$$\frac{\partial V}{\partial p_1} = -\pi^c m^I u'(y - m^I\pi^c p_1) + \frac{\pi}{\alpha}\alpha m^L u'\big(y - m^I\pi^c p_t - m^L\alpha(p_1 - m^I\pi^c p_t)\big)$$

The marginal utility of income is higher under loans, so if loans are "costlier" ($\pi m^L > \pi^c m^I$) then higher prices definitely push for insurance $\left(\frac{\partial V}{\partial p_1} > 0\right)$. If Insurance is costlier, there are counterveiling effects. The higher price makes the higher load on insurance costlier, but the additional dollars are more costly when the individual is making loan payments.

Obviously, a higher price this period makes us think that prices will be higher in the future.

$$\begin{aligned} \frac{\partial V}{\partial p} = \frac{\partial V}{\partial p_1} - \pi\pi^c m^I\Bigg(u'(y - m^I\pi^c p)\left(\frac{1}{\alpha} - 1\right) \\ - \frac{1}{\alpha}(1 - m^L\alpha)u'\big(y - m^I\pi^c p_t - m^L\alpha(p_1 - m^I\pi p_t)\big)\Bigg) \end{aligned}$$

Again, the increased premium payments in future periods raises the marginal utility of income in those periods, making it costlier to take out a loan in the first period, but this can be partially offset by borrowing less (hence the second term is multiplied by $1 - \alpha m^L$). The net effect is ambiguous.

Effect of credit market access. Changing n (or r) affects α, which increases V. Again using $\tilde{y} = y - m^I\pi^c p$, we have

$$\begin{aligned} \frac{\partial V}{\partial \alpha} = \frac{\pi}{\alpha^2}\Big(-u(\tilde{y}) + u\big(\tilde{y} - \alpha m^L p(1 - m^I\pi^c)\big) + \alpha m^L p(1 - m^I\pi^c)u'\big(\tilde{y} - \alpha m^L p(1 - m^I\pi^c)\big)\Big) \\ > 0, \end{aligned}$$

which is positive since u is concave.

Effect of administrative costs and price elasticity. The effect on the load on loans is simple.

$$\frac{\partial V}{\partial m^L} = \frac{\pi}{\alpha}\Big(\alpha p(1 - m^I\pi^c)u'\big(\tilde{y} - \alpha m^L p(1 - m^I\pi^c)\big)\Big) > 0$$

When the load on loans is higher, loans are worse and the incremental value of insurance is higher. Insurance load is more complicated because it also affects future premium payments.

$$\frac{\partial V}{\partial m^I} = -\pi^c p\ \left(\left(1 - \pi + \frac{\pi}{\alpha}\right)u'(\tilde{y}) - \frac{\pi}{\alpha}u'\big(\tilde{y} - m^L\alpha p(1 - m^I\pi^c)\big)(1 - \alpha m^L)\right)$$

The raising current period m^I decreases V^{UB} by $\pi^c p\ (1 - \pi)u'(\tilde{y})$, but raising the future m^I decreases the value of loans (increases V^{UB}) by $\pi^c p\ \frac{\pi}{\alpha}\Big(u'\big(\tilde{y} - m^L\alpha p(1 - m^I\pi^c)\big)(1 - \alpha m^L) - u'(\tilde{y})\Big)$.

More complicated model

In reality, people face multiple potential health risks in multiple periods. This more complicated model has the same basic phenomenon as in simpler one used in the text, but what it means to smooth consumption becomes a bit more complicated.

In each period, an individual faces a variety of potential health shocks, indexed by i, for which the price of treatment is p_i and the probability of occurrence is π_i. Let $p_0 = 0$ so π_0 is the probability of no health shock. If insurance is community rated, then the premium depends on $E_c[p^t]$, the expected cost for the average person in the pool. The utility of buying insurance every period is

$$\sum_{t=1}^{T} \beta^{t-1} u(y - E_c[p]m^I).$$

The value of switching to loans for just period 0 is

$$\sum_{i:\, p_i < m^I E_c[p]} \pi_i \left(u(y - p_i) + \sum_{t=2}^{T} \beta^{t-1} u(y - E_c[p]m^I)\right) + \sum_{i:\, p_i > m^I E_c[p]} \pi_i \left(\sum_{t=1}^{n} \beta^{t-1} u(y - E_c[p]m^I - \alpha m^L(p_i - E_c[p]m^I)) + \sum_{t=n+1}^{T} \beta^{t-1} u(y - E_c[p^t]m^I)\right).$$

The difference is

$$\sum_{i:\,p_i<m^I E_c[p]} \pi_i \left(u(y - m^I E_c[p]) - u(y - p_i)\right) + \sum_{i:\,p_i>m^I E_c[p]} \frac{\pi_i}{\alpha}\left(u(y - E_c[p]m^I) - u(y - E_c[p]m^I - \alpha m^L(p_i - E_c[p]m^I))\right).$$

The effect of a change in price includes its effect on the premium, where $\frac{\partial E_c[p]}{\partial p_i} = \pi_i^c$. If $p_j < m^I E_c[p]$, the effect of the price in period one is

$$\frac{\partial\, V}{\partial p_j} = \left(-m^I \pi_j^c u'(y - E_c[p^1]m^I) + \pi_j u'(y - p_j)\right)$$

which is negative (since $m^I \pi_j^c > \pi_j$ and $u'(y - E_c[p^1]m^I) > u'(y - p_j)$). If $p_j > m^I E_c[p]$, then the effect of the price in period one is

$$\frac{\partial\, V}{\partial p_j} = \left(-m^I \pi_j^c u'(y - E_c[p^1]m^I) + \frac{\pi_j}{\alpha}\alpha m^L u'(y - E_c[p]m^I - \alpha m^L(p_i - E_c[p]m^I))\right)$$

Which, as in the simpler case, is positive if $m^L \pi_j > m^I \pi_j^c$. The effect of changing later prices is

$$\frac{\partial\, V}{\partial p_j} = \sum_{i:\,p_i>m^I E_c[p]} \pi_i m^I \pi_j^c \left(\frac{1}{\alpha} u'(y - E_c[p]m^I - \alpha m^L(p_i - E_c[p]m^I))(1 - \alpha m^L) - \left(\frac{1}{\alpha} - 1\right) u'(y - E_c[p]m^I)\right) - \frac{\pi_j}{\alpha}\alpha\, m^L u'(y - E_c[p]m^I - \alpha m^L(p_i - E_c[p]m^I)).$$

If we approximate

$$V \approx u'(\tilde{y})\left(\sum_{i:\,p_i<m^I E_c[p]} \pi_i\,(p_i - m^I E_c[p]) + \sum_{i:\,p_i>m^I E_c[p]} \frac{\pi_i}{\alpha}\alpha m^L(p_i - E_c[p]m^I)\right) + u''(\tilde{y})\left(\sum_{i:\,p_i<m^I E_c[p]} \pi_i\,(p_i - m^I E_c[p])^2 + \sum_{i:\,p_i>m^I E_c[p]} \frac{\pi_i}{\alpha}\,(\alpha m^L(p_i - E_c[p]m^I))^2\right).$$

$$V \approx u'(\tilde{y})\left(E[p] - E_c[p]m^I + (m^L - 1)\left(\tilde{E}[p] - m^I E_c[p]\right)\Pr\left[p_i > m^I E_c[p]\right]\right) + u''(\tilde{y})\left(E[p^2] - 2m^L E[p]E_c[p] + {m^I}^2 E_c^2[p] + \left(\alpha m^{L^2} - 1\right)\Pr[p_i > m^I E_c[p]](\tilde{E}[p^2] - 2m^L \tilde{E}[p]E_c[p] + {m^I}^2 E_c^2[p])\right).$$

The two differences from the baseline case in the text are that we have $E_c[p^1]$ instead of $\pi^c p$ and there are the future insurance payments affecting the marginal utility. We still have that the total weight on the marginal utilities with loans equals $\frac{1}{\alpha}$ and that $\pi_i < \pi_{i,c}$, but there are two potentially countervailing effects. The first is that the marginal utility of future consumption is higher because of the need to pay for insurance in the future. This works against loans, particularly when a higher price raises the loan payment. The second is that it may be the case that $\alpha p_i > E_c[p]$, in this case the increased price matters in a state of the world where the marginal utility of income is particularly high, making loans worse relative to insurance where the price increase is spread across all states of the world.

The effect of income on the value function is

$$\begin{aligned}\frac{\partial V}{\partial y} = u'(y - E_c[p^1]m^I)& \\ &- \sum_i \pi_i \Bigg(u'(y - \alpha m^L p_i^1) \\ &+ \sum_{t=2}^{n} \beta^{t-1} \Big(u'(y - E[p^t]m^I) - u'(y - E[p^t]m^I - \alpha m^L p_i^1) \Big) \Bigg).\end{aligned}$$

The effect of later period income is unambiguously negative. The wealthier one is in the future, the smaller the utility loss from future loan payments so the less valuable insurance is relative to loans.

Lower bound

Returning to the case of a single risk, we consider the case where consumers can save and borrow more generally, so loan markets are more valuable and the incremental value of insurance is smaller.

$$\begin{aligned}V^{LB} = \frac{1}{\alpha}\Big(u(y - m^I\pi^c p) - \pi u\big(y - m^I\pi^c p - m^L \alpha p(1 - m^I\pi^c)\big) \\ - (1-\pi)u(y - m^I\pi^c p + \alpha p m^I \pi^c)\Big),\end{aligned}$$

The value is increasing in α

$$\begin{aligned}\frac{\partial V}{\partial \alpha} &= \frac{1}{\alpha^2}\Big(-u(\tilde{y}) + \pi u\big(\tilde{y} - \alpha m^L p(1 - m^I\pi^c)\big) + (1-\pi)u(\tilde{y} + \alpha p\, m^I\pi^c) \\ &\qquad + \alpha\big(\pi p m^L(1 - m^I\pi^c)u'\big(\tilde{y} - \alpha m^L p(1 - m^I\pi^c)\big) \\ &\qquad - (1-\pi)u'(\tilde{y} + \alpha p m^I\pi^c)p m^I\pi^c m^L\big)\Big) \\ &= \frac{1}{\alpha^2}\Big(\pi\Big(u\big(\tilde{y} - \alpha m^L p(1 - m^I\pi^c)\big) - u(\tilde{y}) \\ &\qquad + \alpha p m^L(1 - m^I\pi^c)u'\big(\tilde{y} - \alpha m^L p(1 - m^I\pi^c)\big)\Big) \\ &\qquad + (1-\pi)(u(\tilde{y} + \alpha p\, m^I\pi^c) - u(\tilde{y}) - u'(\widetilde{y +} \alpha p m^I\pi^c)p m^I\pi^c m^L)\Big) > 0,\end{aligned}$$

and increasing in the load on loans

$$\frac{\partial V}{\partial m^L} = \pi \left(p(1 - m^I \pi^c) u'\left(\tilde{y} - \alpha m^L p(1 - m^I \pi^c)\right) \right) > 0.$$

Like the upper bound, there is both a direct effect of the insurance markup in the first period of making insurance less valuable and an indirect effect of a more expensive insruance in subsequent periods making loans costlier

Income Shocks

Income shocks reduce the relative value of loans. If the shock is only in one period, the value of insurance is

$$\begin{aligned} V^{UB} = (1 - \pi) &\left(u(y - m^I \pi^c p_1) - u(y) \right) \\ &+ \pi \Bigg(\left(u(y - m^I \pi^c p_1 - \Delta y) - u\left(y - m^I \pi^c p_t - m^L \alpha (p_1 + \Delta y - m^I \pi^c p_t)\right) \right) \\ &+ \left(\frac{1}{\alpha} - 1 \right) \left(u(y - m^I \pi^c p_t) - u\left(y - m^I \pi^c p_t - m^L \alpha (p_1 + \Delta y - m^I \pi^c p_t)\right) \right) \Bigg). \end{aligned}$$

So income shocks make insurance more valuable:

$$\begin{aligned} \frac{\partial V^{UB}}{\partial \Delta y} = \pi &\Bigg(-u'(y - m^I \pi^c p_1 - \Delta y) + m^L \alpha u'\left(y - m^I \pi^c p_t - m^L \alpha (p_1 + \Delta y - m^I \pi^c p_t)\right) \\ &+ m^L \alpha \left(\frac{1}{\alpha} - 1 \right) u'\left(y - m^I \pi^c p_t - m^L \alpha (p_1 + \Delta y - m^I \pi^c p_t)\right) \Bigg) \end{aligned}$$

$$\frac{\partial V^{UB}}{\partial \Delta y} = \pi \left(-u'(y - m^I \pi^c p_1 - \Delta y) + u'\left(y - m^I \pi^c p_t - m^L \alpha (p_1 + \Delta y - m^I \pi^c p_t)\right) \right) > 0.$$

If the shock occurs in later periods (for convenience same number as loan length, but general idea holds)

$$\begin{aligned} V^{UB} = (1 - \pi) &\left(u(y - m^I \pi^c p_1) - u(y) \right) \\ &+ \frac{\pi}{\alpha} \left(\left(u(y - m^I \pi^c p_1 - \Delta y) - u\left(y - m^I \pi^c p_t - \Delta y - m^L \alpha (p_1 - m^I \pi^c p_t)\right) \right) \right). \end{aligned}$$

Again, the shocks increase the incremental value of insurance relative to loans:

$$\frac{\partial V^{UB}}{\partial \Delta y} = \pi \left(-u'(y - m^I \pi^c p_1 - \Delta y) + u'\left(y - m^I \pi^c p_t - \Delta y - m^L \alpha (p_1 - m^I \pi^c p_t)\right) \right) > 0.$$