ONLINE APPENDIX

"Asset Liquidity and International Portfolio Choice"

The case of International Asset Equilibria

In this appendix we state and prove the analogue of Propositions 2-4 for the case in which international asset equilibria arise. Recall that in this case $t_{FF} = 0$ and so $t_F = 0$. However, this does not mean that we have 100% bias towards the domestic asset. Sellers who match with foreign buyers will hold some foreign assets at the end of the first subperiod (DM trade). Another interesting feature of this type of equilibrium concerns the nature of consumption and has important implications for accounting. Buyers from country i hold home assets in order to trade in DM_j. If they match in the foreign DM, country i has some imports from country j (the special good) and country j has some imports from country i (the fruit consumed by the seller from country j who received asset i). However, if the buyers from i do not match, they return home with asset i, and no imports are generated. This is in contrast with the local asset dominance equilibrium, where regardless of the success of the match in the foreign DM, imports from country j are always generated.

We now state the three main Propositions regarding asset home bias, consumption home bias, and asset turnover rates in the case of an international asset equilibrium. For the proofs, the classification of agents into seven groups, as discussed in Appendix B of the main text, is extremely helpful. For the reader's convenience, we repeat these classifications: 1) sellers who got matched with a local buyer (group 1), 2) sellers who got matched with a foreign buyer (group 2), 3) sellers who did not get matched (group 3), 4) buyers who got matched in both DM's (group 4), 5) buyers who got matched only in the home DM (group 5), 6) buyers who got matched only in the foreign DM (group 6), and 7) buyers who did not get matched (group 7).

Proposition 2. Assume that parameter values are such that the international asset regime arises. Then, for all T, agents' portfolios exhibit home bias, in the sense that the home asset's share in the entire portfolio is greater than fifty percent.

Proof. As in the main text, we define asset home bias as the ratio of the weighted sum of domestic asset holdings over the weighted sum of all asset holdings for all citizens of a certain country. We count asset holdings in both the CM and the DM and we assume equal weights for the two markets. It is understood that we count asset holdings at the end of the subperiods.

a) First consider the case $T < T_2^*$. The weighted sum of domestic asset holdings is given by $2T - p_F t_{HF}$. Intuititvely, local agents hold all the supply of the home asset (multiplied by 2, because we count both subperiods), except from the volume of assets held by foreign sellers who received home assets, $p_F t_{HF}$. Moreover, the weighted sum of all asset holdings for local agents is $2T$. Hence,

$$HA = \frac{2T - p_F t_{HF}}{2T} = 1 - \frac{1}{2}\frac{p_F t_{HF}}{T} > \frac{1}{2}.$$

The inequality follows from the facts that $p_F < 1$ and $t_{HF} < T$.

b) Now let $T \geq T_2^*$. The weighted sum of domestic asset holdings is given by $2T - p_F(1-\beta)q_{F,2}^*/(d-\kappa(1-\beta))$, and the weighted sum of all asset holdings is unchanged. Hence,

$$HA = \frac{2T - p_F \frac{1-\beta}{d-\kappa(1-\beta)} q_{F,2}^*}{2T} = 1 - \frac{1}{2}\frac{p_F \frac{1-\beta}{d-\kappa(1-\beta)} q_{F,2}^*}{T}.$$

We claim that this expression is greater than 0.5. Our claim will be true if $p_F(1-\beta)q_{F,2}^*/(d-\kappa(1-\beta)) \leq T$. But here $T \geq T_2^*$ and

$$T_2^* > p_F \frac{1-\beta}{d-\kappa(1-\beta)} q_{F,2}^* \Leftrightarrow \frac{q^*}{d} > (p_F - 1)\frac{q_{F,2}^*}{d-\kappa(1-\beta)},$$

which is true since the last expression is negative. Hence,

$$T \geq T_2^* > p_F \frac{1-\beta}{d-\kappa(1-\beta)} q_{F,2}^*,$$

verifying our claim that $HA > 1/2$. $\square$

Recall that in the original Proposition 2, we required that trading opportunities at home are not significantly less than trading opportunities abroad. Hence, the "international asset" version of Proposition 2 is stronger than the "local asset dominance" version. This is because the asset bias in the international asset regime is stronger. In fact, under the international asset regime, agents never buy any foreign assets in the CM, and the only agents that ever hold some foreign assets are sellers from group 2 (matched with foreign buyers). It turns out that these holdings are never large enough in order to generate $HA < 1/2$, so the asset home bias result holds universally.

Consider now the analogue of Proposition 3.

Proposition 3. Assume that parameter values are such that the international asset regime arises. Define C_F, C_T, and C_H as the value of foreign (or imported) consumption, total consumption, and consumption produced at home, respectively. If $p_H \geq p_F$, then $C_H > C_F$, implying $\frac{C_H}{C_T} > 0.5$, for any T.

Proof. We only prove the result for $T < T_2^*$, since the methodology is identical. Let C_F^i, C_H^i be the value of foreign and domestic consumption for the typical agent in group i, $i = 1, ..., 7$, respectively. All consumption is denominated in terms of the general good. Moreover, let H^i

denote the hours worked by the typical agent in group i, $i = 1, ..., 7$.

Groups 1, 3, and 7 consume X^*, and this consumption is entirely produced by local resources (trees or labor). Group 2 also consumes X^*, but part of that is imported. The imported consumption, per agent in this group, is given by $(\psi + d - \kappa)t_{HF}$. Group 4 consumes X^* in the CM, which is produced locally. In the DM, this group also consumes $(\psi + d)t_{HH} = q_H$ of special good produced locally and $(\psi + d - \kappa)t_{HF} = q_F$ of special good produced abroad. Group 5 consumes X^* in the CM and $(\psi + d)t_{HH} = q_H$ in the DM, all of which is locally produced. Group 6 consumes X^* in the CM, which is produced locally, and $(\psi + d)t_{HH} = q_H$ in the DM, which is imported.

Summarizing the findings in the previous paragraph, the only groups that have some imported consumption are 2, 4, and 6. We prove the proposition in two steps. First, we show that $C^4_H > C^4_F$. Then, following a similar strategy as in the proof of the original Proposition 3, we couple the groups 2 and 6 with groups 1 and 5, respectively, and we show that $C^5_H + C^6_H > C^5_F + C^6_F$ and $C^1_H + C^2_H \geq C^1_F + C^2_F$.

Step 1: We claim that $p_H \geq p_F$ implies $q_H > q_F$. This necessarily implies $C^4_H > C^4_F$. From the definition of an international asset equilibrium we know that

$$\begin{aligned} q_H = q_H(p_H) &\equiv \left\{ q : u'(q) = 1 + \frac{\psi - \beta(\psi + d)}{\beta p_H(\psi + d)} \right\}, \\ q_F = q_F(p_F) &\equiv \left\{ q : u'(q) = \frac{\psi - \beta(\psi + d)(1 - p_F)}{\beta p_F(\psi + d - \kappa)} \right\}. \end{aligned}$$

Clearly, q_H is increasing in p_H and q_F is increasing in p_F. Thus, our claim will be true as long as $q_H > q_F$ for any $p_H = p_F = p$. Fixing $p_H = p_F = p$, we have

$$q_H > q_F \Leftrightarrow u'(q_H) < u'(q_F) \Leftrightarrow 1 + \frac{\psi - \beta(\psi + d)}{\beta p(\psi + d)} < \frac{\psi - \beta(\psi + d)(1 - p)}{\beta p(\psi + d - \kappa)}.$$

It is easy to verify that the last statement is true, as long as $\kappa > 0$. Hence, we have proved our claim, and we can conclude that $C^4_H > C^4_F$.

Step 2: Consider the joint consumption of groups 1 and 2. $C^1_H + C^2_H \geq C^1_F + C^2_F$ will hold as long as $p_H X^* + p_F X^* \geq 2p_F(\psi + d - \kappa)t_{HF}$. But $p_H X^* + p_F X^* \geq 2p_F X^* \geq 2p_F(\psi + d - \kappa)t_{HF}$, where the last inequality follows from the fact that $H_2 \geq 0$. Hence, we have established that $C^1_H + C^2_H \geq C^1_F + C^2_F$.

Now consider the joint consumption of groups 5 and 6. Since $p_H \geq p_F$, we also have $1 - p_F \geq 1 - p_H$, and so $\mu_5 = p_H(1 - p_F) \geq \mu_6 = p_F(1 - p_H)$. We can easily calculate

$$C^5_H + C^6_H = (\mu_5 + \mu_6)X^* + \mu_5 q_H$$

and, similarly,

$$C_F^5 + C_F^6 = \mu_6 q_F.$$

Therefore, $C_H^5 + C_H^6 > C_F^5 + C_F^6$ holds true, since $\mu_5 \geq \mu_6$, $q_H \geq q_F$, and $X^* > 0$. This concludes the proof of Proposition 3. $\square$

Numerically, one can show that $C_H > C_F$ can be true even if $p_H < p_F$. However, showing that $C_H > C_F$ is true theoretically requires the stronger assumption that $p_H \geq p_F$. In general, agents carry out transactions in both DM's using the home asset. If p_F is large, many buyers consume imported special goods. Hence, if p_F is very large and, at the same time, p_H is very small and X^* is small, it is not impossible to end up with a situation where $C_H < C_F$. For example, a very small and very open economy may be characterized by such parameters. However, this scenario is unlikely to hold for the average developed economy.

We conclude this section with the statement and proof of the analogue of Proposition 4 in the international asset regime.

Proposition 4. Assume that parameter values are such that the international asset regime arises. Define the turnover rates of home and foreign assets as TR_H and TR_F, respectively. There exists a level of asset supply $\tilde{T}$, with $T_2^* \leq \tilde{T} < \infty$, such that $T \geq \tilde{T}$ implies $TR_F > TR_H$.

Proof. We define the turnover rate of an asset as the ratio of the total volume of the asset *traded* by citizens of a certain country (numerator) over the total volume of the asset *held* by the citizens of the same country (denominator). Like above, we count asset holdings in both the CM and the DM and assume equal weights for the two markets. It is understood that we count asset holdings at the end of the subperiods.

a) First, focus on the case $T < T_2^*$. Consider TR_F. The numerator of this term consists of all the trades of foreign assets carried out by the citizens of a certain country.[1] The only agents who participate in transactions that involve the foreign asset are members of group 2. These agents receive t_{HF} units as a means of payment in the DM and also sell these assets in the second subperiod (in the CM). The denominator of TR_F corresponds to the asset holdings by group 2 at the end of the first subperiod. Summing up,

$$TR_F = \frac{2p_F t_{HF}}{p_F t_{HF}} = 2. \tag{1}$$

[1] As in the main text, we adopt the following accounting procedure: (i) In the DM, we count each transaction only once, since, by definition, the meeting is bilateral in the sense that the buyer and the seller trade with each other; (ii) In the CM, we count the amount of assets bought by the buyers and sold by the sellers because the market is Walrasian and, therefore, agents trade against the market and not with each other. In fact, the latter procedure is applied when accounting for stock-market transactions in the data.

We now turn to TR_H. Consider the numerator of this term. During the first subperiod, the following transactions involve the use of local assets: i) buyers who got matched in the local DM give away t_{HH} units of the asset, and ii) buyers who got matched in the foreign DM give away t_{HF} units of the asset. During the second subperiod, the following transactions involve the use of local assets: i) agents in group 1 sell t_{HH} units of home assets in the CM. ii) Members of group 4 have zero asset holdings as they enter the CM, because they got matched in both DM's. Hence, they need to re-balance their portfolios by purchasing $t_{HH}+t_{HF}$ units. iii) Agents in group 5 purchase t_{HH} units. iv) Agents in group 6 purchase t_{HF} units. v) Finally, agents in groups 2, 3, and 7 do not need to re-balance their home asset holdings, and so they do not participate in any transactions involving this asset. This concludes the calculation of the numerator of TR_H.

The denominator of TR_H corresponds to domestic asset holdings. They are given by $2T$ minus the amount of home assets held by foreign sellers of group 2. Using all these pieces of information, together with the appropriate measures of the various agents' groups, implies that

$$TR_H = \frac{3p_H t_{HH} + 2p_F t_{HF}}{2T - p_F t_{HF}}. \tag{2}$$

b) Next, assume $T \geq T_2^*$. Once again $TR_F = 2$. The calculation of TR_H is based on the same logic as above. Substituting out the asset holdings in (2) using the appropriate formulas for the liquidity-unconstrained case yields

$$TR_H = \frac{3p_H \frac{1-\beta}{d} q^* + 2p_F \frac{1-\beta}{d-\kappa(1-\beta)} q_{F,2}^*}{2T - p_F \frac{1-\beta}{d-\kappa(1-\beta)} q_{F,2}^*}.$$

The term TR_F is a constant, so it is unaffected by the asset supply. On the other hand, TR_H is decreasing in T, for all T, and $TR_H \to 0$ as $T \to \infty$. Therefore, there exists $\tilde{T}$, with $T_2^* \leq \tilde{T} < \infty$, such that $T \geq \tilde{T}$ implies $TR_F > TR_H$. $\square$

The interpretation of this result is no different than the local asset dominance equilibrium. Buyers use the local asset to trade in both DM's. Nevertheless, when $T \geq T_2^*$, the liquidity properties of the asset have been exhausted, and increasing T even more does not increase the volume of transactions involving local assets, but it does increase the home asset holdings, since the holding cost of home assets is zero for large T. In other words, increasing T, increases the denominator of TR_H without bound, leaving the numerator unaffected. On the other hand, TR_F is a constant. Thus, one can always find a T, which guarantees that $TR_F > TR_H$.

In the theoretical analysis above, we adopted a definition of the foreign turnover rate that is standard in the empirical literature. Given this definition, we constructed a theoretically-consistent definition of the domestic turnover rate. However, since the existing empirical literature has not considered the liquidity mechanism developed in the present paper, it does not report measures of the theoretically-consistent domestic turnover rate. Instead, the literature defines domestic turnover as the ratio of annual transactions on a market to its capitalization. The market for which estimates are reported is the stock exchange. Consequently, we repeat the turnover exercise using the empirically-relevant definition of a domestic turnover rate.

In the model, the CM represents the stock exchange. Its market capitalization is the total asset supply T.[2] The transactions that constitute the numerator are all the trades (purchases and sales) of domestic claims by both domestic and foreign agents. The following transactions are undertaken by domestic agents: i) agents in group 1 sell t_{HH} units of home assets in the CM. ii) Members of group 4 have zero asset holdings as they enter the CM because they got matched in both DM's. Hence, they need to re-balance their portfolios by purchasing $t_{HH} + t_{HF}$ units. iii) Agents in group 5 purchase t_{HH} units. iv) Agents in group 6 purchase t_{HF} units. v) Finally, agents in groups 2, 3, and 7 do not need to re-balance their home asset holdings, and so they do not participate in any transactions involving this asset. The only foreign agents who participate in transactions on the domestic CM are members of group 2—foreign sellers who matched with domestic buyers abroad. These agents sell t_{HF} units of the domestic asset.

Using all these pieces of information, together with the appropriate measures of the various agents' groups, implies that

$$TR_H = \frac{2p_H t_{HH} + 2p_F t_{HF}}{T}.$$

If $T \geq T_2^*$, one can substituting out the asset holdings using the appropriate formulas for the liquidity-unconstrained case and obtain

$$TR_H = \frac{2p_H \frac{1-\beta}{d} q^* + 2p_F \frac{1-\beta}{d-\kappa(1-\beta)} q^*_{F,2}}{T}. \tag{3}$$

Once again, the foreign turnover rate in (1) is constant, so it is unaffected by the asset supply. On the other hand, TR_H in expression (3) is decreasing in T, for all T, and $TR_H \to 0$ as $T \to \infty$. Therefore, there exists $\tilde{\tilde{T}}$, with $T_2^* \leq \tilde{\tilde{T}} < \infty$, such that $T \geq \tilde{\tilde{T}}$ implies $TR_F > TR_H$.

In sum, when T is large and $p_H \geq p_F$, economies in an international asset equilibrium exhibit asset and consumption home bias as well as higher foreign over domestic asset turnover rates.

[2] As in the main text, we use volumes rather than values, but the two are identical in the steady state.