Capital Account Policies and the Real Exchange Rate

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Abstract
This paper presents a simple model of how a small open economy can undervalue its real exchange rate using its capital account policies. The paper presents several properties of such policies, and proposes a rule of thumb to assess their welfare cost. The model is applied to an analysis of Chinese capital account policies.

1 Introduction

There are debates about the extent to which emerging market and developing countries that have accumulated large amounts of foreign exchange reserves in the 2000s are doing so in order to undervalue their currency. However, we do not have a simple model of how a country can achieve persistent real exchange rate distortions through reserve accumulation. The main purpose of this paper is to present such a model and to use it to answer a few questions about such a policy.

Real exchange rate undervaluation is often presented, in policy debates, as the result of a monetary operation. For example, it is argued that the People’s Bank of China (PBOC) has resisted the appreciation of the renminbi by

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foreign exchange interventions. In an environment with perfect capital mobility, however, pegging the nominal exchange rate would influence the real exchange rate only to the extent that domestic price stickiness is pervasive and persistent enough to prevent internal appreciation through domestic inflation. Standard estimates suggest that nominal stickiness is not persistent enough to induce large and persistent deviations of the real exchange rate from its flexible-price equilibrium value (Rogoff (1996), Chari, Kehoe and McGrattan (2002)). In order to achieve persistent real undervaluation, thus, monetary policy must rely on something else than just nominal stickiness.

I focus in this paper on the role of imperfect capital mobility. Imperfect capital mobility can be defined, for the purpose of my analysis, as any friction inducing a deviation from Ricardian equivalence in capital flows. Imperfect capital mobility could result from “natural causes”, such as financial frictions that prevent the private sector from borrowing abroad, or deviations from rational expectations that mitigate or delay the private sector’s Ricardian response to reserve accumulation. Imperfect capital mobility could also be policy-induced and result from capital account restrictions that are imposed by the government. The fact that the country that has accumulated the most reserves in the recent period, China, also imposes tight restrictions on its capital account suggests that this is not a theoretical case. Thus, I will focus on the impact of capital account policies (defined in a broad way as the accumulation of foreign assets and liabilities by the public sector plus all the policies that affect the private sector’s access to foreign capital). However, most of my results can be extended to the case where Ricardian equivalence fails because of frictions other than capital account restrictions.

In order to simplify and streamline the analysis, I use a model that is entirely real—there is no money and no monetary policy. I consider a small open economy that consumes a tradable good and a nontradable good. The government accumulates foreign assets and imposes controls on capital flows. This combination of policies allows the government to effectively control the level of net foreign assets for the country as a whole. The other properties of the model then follow in a straightforward way. The government controls the current account balance (since it is the change in net foreign assets) and therefore the trade balance. The real exchange rate, then, has to be

Ricardian equivalence—a property of frictionless rational expectations models—stipulates that an accumulation of foreign assets by the public sector should have no impact on net capital flows because it is offset by an inflow of capital to the private sector.
consistent with the trade balance. Other things equal, accumulating more net foreign assets will depreciate the real exchange rate.

I then use the model to look at several questions related to capital account policies and real exchange rates. How can we detect in the data that these policies influence the real exchange rate? Are there limits to the impact capital account policies on real exchange rates and how are they determined? How different are capital account policies from trade policies? I also look at the recent experience of China through the lenses of the model.

The paper is related to several lines of literature. First, the literature on global imbalances, the “global savings glut”, and the fact that capital has often been flowing “upstream” (from relative poor high-growth countries to relatively rich low-growth countries). The problem in that literature is to explain high saving rates in emerging market economies. One line of explanation is precautionary savings against idiosyncratic risk: see e.g. Mendoza, Quadrini and Ríos-Rull (2009), Carroll and Jeanne (2009) or Sandri (2010). Chamon and Prasad (2010) argue that precautionary savings against idiosyncratic risk is the most likely cause of the high saving rate in China. Precautionary savings could also be against aggregate risk, in particular the risk of sudden stop: see Durdu, Mendoza and Terrones (2009) or Jeanne and Rancière (2011). Capital outflows from high-growth countries could also result from domestic financial frictions as in Caballero, Farhi and Gourinchas (2008) or (specifically about China)? A common feature of these contributions is that the saving rate is determined by the behavior of the private sector. Reserve accumulation and capital account policies play no role and it is by happenstance that a share of foreign assets ends up being accumulated as reserves.

The evidence however suggests that the upstream flow in capital is linked to public flows and in particular reserve accumulation (Aguiar and Amador (2011), Gourinchas and Jeanne (2011)). My model explains the link between reserve accumulation and net capital flows as more than a coincidence. In equilibrium, reserve accumulation must reduce net capital inflows by reducing saving (keeping investment constant). Another way of looking at the real undervaluation policy in my model, thus, is that the accumulation of foreign assets induces “forced saving” in the domestic economy. The capital controls prevent the domestic private sector from offsetting the public accumulation of foreign assets with capital inflows. The model thus provides a very simple
explanation for the high saving rate in countries such as China.\footnote{The model presented here explains the saving rate for the economy as a whole. It does not predict the breakdown of saving between the household and the corporate sectors.}

Second, the paper is also related about the literature about optimal capital account policies. One recent line of literature has studied the normative case for prudential capital controls (Bianchi (2011), Korinek (2011)). Another line of literature has studied the case for “mercantilist” real exchange rate undervaluations (Aizenman and Lee (2007), Korinek and Serven (2010)). Costinot, Lorenzoni and Werning (2011) study equilibrium capital account policies in a two-country model. By contrast with that literature, I take capital account policies as given and do not look at the reasons that real exchange rate undervaluation might be desirable from a welfare perspective.

Third, the paper is related to the literature on the impact of reserves on exchange rates. Most of this literature is empirical and looks at high-frequency data [to develop].

The paper is structured as follows. Section 2 motivates the model by looking at the capital account policies of China. Section 3 presents the model. Sections 4 to 7 present various properties of the model and section 8 goes back to the Chinese experience, this time examining it through the lens of the model.

\section{Capital Account Policies of China}

The capital account is very restricted in China. On the side of inflows, FDI is encouraged but other inflows are constrained. The financial assets that foreigners might want to invest in are equity, debt securities and bank deposits. Those assets are not scarce. Figure 1 reports the outstanding stocks of the different types of financial assets as shares of GDP. At $3,408bn at the end of 2011, the Chinese stock market capitalization is not large relative to that in advanced economies, but it started from a very low level and was multiplied by more than 80 since the end of 1995. The market for debt securities is less developed. The main vehicle for households’ and firms’ financial saving is bank deposits, which has amounted to more than 100 percent of GDP since 2000. Most of those deposits are time and saving deposits that bear an interest rate.

The access of foreign investors to Chinese financial assets is severely limited. For equity, two types of shares are traded in the Shanghai and
Figure 1: Outstanding stocks of Chinese financial assets: bank deposits, bonds and stock market (percent of GDP, 2000-2010). Source: People’s Bank of China, China Securities Regulatory Commission, Shanghai and Shenzhen Stock Exchanges.
Figure 2: Composition of Chinese foreign assets and liabilities (percent, 2010). Source: SAFE.

Shenzhen stock markets, “A shares” that can be owned only by domestic investors and “B shares” that can be purchased by foreigners. The value of B shares have never exceeded 3 percent of total stock market capitalization since 2000. Foreign investors can invest in financial assets other than B shares through the “qualified investor program”, but the quota allocated to this program is small and the range of investable assets limited (Lardy and Douglass (2011), Cappiello and Ferrucci (2008)). Foreign investors cannot otherwise invest in domestic debt securities or hold bank deposits. Capital outflows are severely limited too.

As a result of these restrictions, most of the capital inflows and foreign liabilities are accounted for by FDI and most of the capital outflows and foreign assets take the form of foreign exchange reserves. This is visible in Figure 2, which reports the breakdown of Chinese foreign assets and liabilities at the end of 2010.

It should be noted that although FDI is encouraged (in particular through tax incentives) it is also controlled as it is subject to authorizations from the Chinese authorities. In principle, thus, the Chinese authorities can influence the level of FDI inflows. Even they could not, FDI is unlikely to respond one-for-one to reserve accumulation so as to neutralize its impact on net capital inflows. This suggests that the Chinese authorities can indirectly control net
capital flows with their reserve accumulation policy—the central assumption in the model presented in the next section.

3 Model

The model aims at capturing the essential features of the Chinese capital account policies documented in the previous section. It is deterministic and in continuous time. I consider a small open economy populated by an infinitely-lived representative consumer who consumes a tradable good and a nontradable good. The utility of the representative consumer is given by

$$U_t = \int_0^{+\infty} e^{-\rho s} u(c_{t+s}) ds,$$

(1)

where $c_t = c(c_{Tt}, c_{Nt})$ is a function of the consumption of the tradable good, $c_T$, and the consumption of the nontradable good, $c_N$, which is homogeneous of degree 1. I denote by $p_t$ the price of the nontradable good in terms of the tradable good, and by $q_t$ the price of the tradable good in terms of domestic consumption. I will call $q_t$ the real exchange rate (an increase in $q$ is a real depreciation). By an abuse of language that is common in the literature I will sometimes call the tradable good “dollar”.

The domestic consumer receives exogenous flows of nontradable and tradable goods. The budget constraint of the domestic consumer is

$$c_t + \dot{a}_t + q_t \dot{a}_t^* = q_t(y_{Tt} + p_t y_{Nt}) + r_t a_t + q_t r^* a_t^* + z_t,$$

where $a_t$ and $a_t^*$ are the consumer’s holdings of bonds respectively denominated in consumption good and tradable good; $y_{Tt}$ and $y_{Nt}$ are the country’s endowments of the tradable good and nontradable good; and $z_t$ is a lump-sum transfer from the government. Although the difference between $a_t$ and $a_t^*$ is the denomination of the bonds, by a slight abuse of language I will sometimes call $a_t$ and $a_t^*$ the private sector’s holdings of “domestic assets” and “foreign assets”, respectively. The assumption that the output of tradable good and nontradable good are endowments can be interpreted as the fact that labor is not mobile between the two sectors. I will assume, to simplify the analysis, that the consumer’s psychological discount rate is equal to the interest rate, $\rho = r^*$, but this assumption can easily be relaxed.
The budget constraint of the government is
\[ q_t \dot{b}_t^* + \dot{b}_t + z_t = q_t r^* b_t^* + r_t b_t, \]
(2)
where \( b_t^* \) and \( b_t \) are the government’s holdings of bonds denominated in dollars and in consumption good respectively. I call \( b_t^* \) “international reserves.” If the government accumulates foreign assets by issuing domestic liabilities, \( b_t \) is negative and \(-b_t\) is the government’s domestic debt.

Government policy consists in the announcement of paths for public assets, \((b_t^*, b_t)\), that satisfy the transversality condition,
\[ \lim_{t \to +\infty} \frac{(b_t^* + b_t) e^{-r^* t}}{q_t} \geq 0. \]
(3)

The impact of government policy crucially depends on the extent of capital mobility between the country and the rest of the world. With perfect capital mobility, government policy has no effect on the domestic economy and the real exchange rate (Ricardian equivalence). Using the first-order condition, \( q_t = \partial c_t / \partial c_T \), and the fact that the consumption of nontradable good is equal to its supply in each period, \( c_{Nt} = y_{Nt} \), the real exchange rate can be written in reduced form as a function of \( c_T \) and \( y_{Nt} \),
\[ q_t = q(c_T, y_{Nt}). \]
The equilibrium under perfect capital mobility is then characterized by the following conditions,
\[ u'(c(c_T, y_{Nt})) q(c_T, y_{Nt}) = \mu, \]
\[ \int_0^{+\infty} c_T e^{-r^* t} dt = \int_0^{+\infty} y_T e^{-r^* t} dt + b^*_0 + a^*_t + \frac{b_0 + a_0}{q(c_T, y_{Nt})}. \]
The first equation says that the marginal utility of consuming the tradable good must be constant over time (since the dollar interest rate is equal to the consumer’s psychological discount rate). The second equation is the country’s intertemporal budget constraint. Together, these conditions pin down the path for the consumption of tradable good, \( c_T \), and the path for the country’s total net foreign assets, \( b_t + a_t + q_t(b_t^* + a_t^*) \), but they do not determine the individual components of foreign assets. In particular, an open market operation in which the government purchases reserves by issuing domestic debt has no impact on the domestic economy. This is clear if the government makes the transaction with foreign investors, since in this case nothing
changes for the domestic private sector. This is also true if the government’s
debt is not traded internationally and must be sold to the domestic private
sector. In this case, the domestic private sector simply finances the purchase
of domestic government debt by selling foreign assets (or issuing foreign lia-
bilities) to foreign investors. Government policy is irrelevant if the domestic
private sector is connected to the international financial market through the
frictionless trade of one asset or liability.\footnote{The various classes of assets and liabilities are perfectly substitutable, in our model, because of the absence of risk.}

The situation is quite different if the access of the domestic private sector
to foreign borrowing and lending is restricted. To simplify, let us consider
the extreme case where the government is the only agent in the economy
that can enter into financial relationships with the rest of the world (a closed
capital account).\footnote{We will consider later the case where the government can impose a tax on capital flows (see section 5).} Let us assume that government domestic debt can be held
only by the domestic private sector \((a_t = d_t)\) and that foreign assets can be
held only by the government \((a^*_t = 0)\). This assumption is meant to capture
Chinese-style capital account policies in which the access of foreign investors
to domestic financial assets and the access of domestic private investors to
foreign assets are restricted. The country’s consolidated budget constraint
can then be written,

\[ c_{Tt} = y_{Tt} + r^* b^*_t - \dot{b}^*_t. \]

By setting the path for reserves, \((b^*_t)_{t \geq 0}\), the government completely deter-
mines the paths for the consumption of the tradable good, \((c_{Tt})_{t \geq 0}\), and for
the trade balance, \((y_{Tt} - c_{Tt})_{t \geq 0}\). It also determines the path for the real
exchange rate, \(q_t = q(c_{Tt}, y_{Nt})\).

This result is, as a matter of accounting, obvious. If the government can
determine the country’s total net foreign assets, then it can also determine
the current account balance (the change in the country’s net foreign assets)
and the trade balance (the change in the country’s net foreign assets minus
the return on those assets). In particular, the government can induce “forced
saving” in the domestic economy by forcing the private sector to buy domestic
debt and by using the proceeds to buy foreign assets. With a closed capital
account, the domestic private sector cannot undo this operation by selling
assets to—or borrowing from—the rest of the world.

Denoting by \(c_T(q, y_N)\) the level of tradable good consumption when the
Proposition 1 With a closed capital account, the government can implement any real exchange rate path, \((q_t)_{t \geq 0}\), satisfying the country's external budget constraint

\[
\int_0^{+\infty} c_T(q_t, y_N t) e^{-r^* t} dt \leq \int_0^{+\infty} y_T t e^{-r^* t} dt + b^*_0.
\]  

Proof. See discussion above. ■

Inequality (4) is binding if the reserves satisfy the transversality condition as an equality,

\[
\lim_{t \to +\infty} b^*_t e^{-r^* t} = 0.
\]

But in general the left-hand side of this equation could be strictly positive, i.e., the government could never allow residents to consume the return on the reserves (which is equivalent to “throwing away” a fraction of the reserves, as in Korinek and Serven (2010)).

A realistic case in which the government might want to use capital account policies to control the real exchange rate is to resist a real exchange rate appreciation resulting from a take-off in the tradable good sector. One can capture this case in the model by assuming that the endowment of tradable good, \(y_T t\), increases over time. There is a trade deficit in the early times if the consumption of tradable good exceeds the endowment, because it reflects the anticipation of the higher future tradable endowments.

I will assume that (for a reason outside of the model), the government tries to smooth the trade balance by limiting the initial trade deficit, or even maintaining a surplus. This is possible if the government closes the capital account and accumulates reserves. More formally, I will define an episode of “resistance to real exchange rate appreciation” as follows.

First, I denote with tilde the values of the variables in the undistorted equilibrium (with free capital mobility). For example, \((\tilde{c}_T t)_{t \geq 0}\) is the path for the consumption of tradable good when the domestic consumer has unrestricted access to foreign borrowing and lending, and \((\tilde{b}^*_t)_{t \geq 0}\) is the path for foreign exchange reserves that is consistent with the undistorted equilibrium (when reserves are the only foreign assets). An episode of resistance to appreciation is when the government depreciates the real exchange relative to the undistorted level by purchasing reserves.
Definition 2 There is resistance to real exchange rate appreciation between time 0 and time t if:

- the government closes the capital account between time 0 and time t;

- the government accumulates more reserves than in the undistorted equilibrium while the capital account is closed: \( b^*_s > \tilde{b}^*_s \) for \( 0 < s \leq t \);

- the initial real exchange rate is depreciated relative to its undistorted value: \( q_0 > \tilde{q}_0 \).

The difference \( q_0/\tilde{q}_0 - 1 \) is a measure of the initial real exchange rate undervaluation. Note that the resistance to appreciation is assumed to last a finite time \( t \), after which there is free capital mobility and Ricardian equivalence applies. This assumption is analytically convenient and is not very restrictive since \( t \) can be taken to be arbitrarily large. After time \( t \), the economy follows its undistorted path conditional on the initial level of foreign assets \( b^*_t \). Also note that in the long run (after time \( t \)), the effects of the resistance to appreciation are reversed. If the government does not “throw away” the reserves, the real exchange rate is appreciated in the long run as the country consumes the return on the foreign assets that have been accumulated during the episode of resistance to appreciation.

4 Capital Account Policies and Trade Protectionism

Because capital account policies affect the real exchange rate and the trade balance, it is natural to ask how these policies compare with standard protectionist measures, such as a tariff on imports and a subsidy on exports. Let us assume that the government can impose a time-varying tariff (subsidy) \( \tau_t \) on imports (exports). In this model with only one tradable good, this is equivalent to imposing a tax \( \tau \) on the consumption of tradable good. The tax receipts are rebated to the consumer in a lump-sum way. The combination of tariff and subsidy raises the relative price of the tradable good and induces a real depreciation.\(^5\) It is then possible to show that capital account policies can achieve the same outcomes as trade protectionism.

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\(^5\)It is well understood since at least Johnson (1953) that it is possible the same effects as a real depreciation with a tariff on imports and a subsidy on exports.
Proposition 3. Any real allocation that can be achieved with a time-varying tariff (subsidy) on imports (exports) under perfect capital mobility can also be implemented with capital account policies.

Proof. The real exchange rate $q_t$ is the price of the tradable good in terms of the consumption good before tax. Then using $c_{Nt} = y_{Nt}$ and assuming (without loss of generality) that $a_t = 0$, the consumer’s budget constraint is,

$$\left(1 + \tau_t\right)c_{Tt} + a^*_t = \left(1 + \tau_t\right)y_{Tt} + r^*a^*_t + \frac{z_t}{q_t}.$$

Maximizing welfare (1) under this constraint gives the first-order condition,

$$u'(c(t_{Tt}, y_{Nt})) q(c(t_{Tt}, y_{Nt})) = \mu(1 + \tau_t),$$

where $\mu$ is the (constant) shadow cost of the budget constraint. This equation and the country’s intertemporal external budget constraint (which remains the same as before since the tax receipts are rebated and consumed domestically) pin down the paths $(c_{Tt})_{t \geq 0}$ and $(q_t)_{t \geq 0}$. Since the path $(q_t)_{t \geq 0}$ satisfies condition (4) it can be implemented with capital account policies.

This result is interesting in light of the recent debates on whether there is a need for international rules for capital account policies in the same way as there are rules for international trade. The status quo is characterized by a strong international regime to discourage the use of policies distorting international trade (with the World Trade Organization) whereas capital account policies are largely left to the discretion of country authorities.\textsuperscript{6} The status quo is often justified by the fact that the welfare gains from international integration seem larger for free trade in goods than for free trade in assets, so that the international community has a stronger stake in maintaining the former than the latter (Bhagwati, 1998).

The insight that Proposition 3 brings to this debate is that capital account policies may (under certain conditions) be used to achieve exactly the same outcomes as trade protectionism.\textsuperscript{7} From this point of view, the case for international rules would be no less strong—and not fundamentally different—for

\textsuperscript{6}For example, the IMF does not have jurisdiction over its members’ capital accounts. There are regional exceptions: for example, members of the European Union must maintain an open capital account, and bilateral free trade agreements with the US often limit the use of capital account policies.

\textsuperscript{7}The equivalence between trade policies and capital account policies does not hold in any environment, and in particular it does not hold in general in models with several...
capital account policies than for trade policies (Jeanne, Subramanian and Williamson, 2012).

5 Excess Returns

Restrictions on free capital mobility introduce a wedge between domestic and foreign returns. I study in this section the behavior of this wedge, because it can be measured (as I will do for China in section 8) and thus offers an interesting point of comparison between the model and the data. Moreover, this wedge plays a role in measuring the welfare consequences of capital account policies (as discussed in section 6).

Based on an exogenous path for reserve accumulation, \((b_t^*)_{t \geq 0}\), one can compute (if the capital account is closed) the paths for tradable consumption, \(c_{Tt}\), total consumption, \(c_t\), the marginal utility of consumption \(\lambda_t = u'(c_t)\), and the real exchange rate \(q_t\). The interest rate on domestic bonds is given by

\[
r_t = r^* - \frac{\dot{\lambda}_t}{\lambda_t}.
\]

(6)

The dollar return on domestic bonds is then equal to this interest rate plus the rate of real exchange rate appreciation, \(r_t - \dot{q}/q\). The excess dollar return on domestic bonds is the difference with the dollar interest rate,

\[
\epsilon_t = r_t - \frac{\dot{q}_t}{q_t} - r^*.
\]

(7)

If this excess return is positive, foreign capital “wants” to flow into the country and must be kept at bay with controls on inflows. Conversely, if the excess return is negative, domestic capital wants to flow out and can be retained in the country using control on outflows.

The path for the excess return depends on the path of reserve accumulation in a way that is not easy to characterize in general. However, economic intuition suggests that foreign capital should try to flow into the countries that resist the appreciation of their currency in order to take advantage of tradable goods. Capital account policies do not affect the relative prices of tradable goods (and the terms of trade) in the same way as taxes on trade flows. See Costinot, Lorenzoni and Werning (2011) for an analysis of the impact of taxes on capital flows in a two-country model with several tradable goods.
the real exchange rate appreciation. The following proposition spells out a rigorous formulation of this intuition.

**Proposition 4** Assume that the government resists to appreciation between time 0 and time $t$ (according to Definition 2). Then the cumulated excess return on domestic bonds between time 0 and time $t$ is positive and larger than the amount of undervaluation at time 0,

$$\int_0^t \epsilon_s ds > \log \left( \frac{q_0}{\tilde{q}_0} \right).$$  \hspace{1cm} (8)

**Proof.** Using $\epsilon_s = -\frac{\dot{\lambda}_s}{\lambda_s} - \frac{\dot{q}_s}{q_s}$, the cumulated excess return can be written

$$\int_0^t \epsilon_s ds = \log \left( \frac{u'(c_0)q_0}{u'(\tilde{c}_t)\tilde{q}_t} \right).$$

In the undistorted equilibrium, there is no excess return and one has,

$$\frac{u'(\tilde{c}_0)\tilde{q}_0}{u'(\tilde{c}_t)\tilde{q}_t} = 1.$$

At time $t$, the real exchange rate is lower and consumption is higher than in the undistorted equilibrium because of the larger level of foreign assets ($b^*_t > \tilde{b}^*_t$). This implies $u'(\tilde{c}_t)\tilde{q}_t > u'(c_t)q_t$. Hence,

$$\frac{u'(c_0)q_0}{u'(c_t)q_t} > \frac{u'(c_0)q_0}{u'(\tilde{c}_t)\tilde{q}_t} = \frac{u'(c_0)q_0}{u'(\tilde{c}_0)\tilde{q}_0} > \frac{q_0}{\tilde{q}_0}.$$

The last inequality results from $c_0 < \tilde{c}_0$ because $q_0 > \tilde{q}_0$. ■

To illustrate, a country that undervalues its currency by 20 percent by closing its capital account for ten years will have an excess return on its domestic bonds of at least 2 percent per year on average while the capital account is closed. The average annual excess return can be reduced by lengthening the time $t$ during which the capital account is closed.

There is an excess return on domestic bonds for two reasons. First, there is a valuation gain coming from the real appreciation of the domestic currency relative to the dollar. In addition, the domestic real interest between time 0 and time $t$ is higher than the dollar interest rate because resistance to
appreciation works by postponing domestic consumption. The cumulated excess return, thus, is larger than the undervaluation at time 0.

Note that Proposition 4 says that the excess return on domestic bonds is positive on average during the episode of resistance to appreciation, not that it should be positive at every point in time during that episode. Thus, one should not conclude that a country is not undervaluing its exchange rate from the observation that the excess return is zero or even negative at a given point in time. It is not difficult to construct examples where the excess return is negative at some times during the episode of resistance to currency appreciation.

Proposition 4 is also useful to derive an upper abound on the amount of undervaluation that can be achieved with price-based capital controls. Many real world controls are price-based, i.e., they take the form of a tax on capital inflows (like for example in Brazil since 2009). It is interesting to know how much these controls can “buy” in terms of real exchange rate undervaluation.

I now assume that the government taxes the purchase of domestic bonds by foreigners at a constant rate $\tau$ between time 0 and time $t$ (instead of completely closing the capital account). The tax is removed after time $t$. Like for the controls on capital inflows that Brazil has been using since 2009 (or the Chilean capital controls of the 1990s), the tax is paid on the purchase of the assets and does not depend on how long the foreign investor holds the asset. Namely, a foreign investor can buy $(1 - \tau)q_s$ units of domestic bonds in exchange of one unit of the tradable good at any time $s \leq t$. He can sell the bonds and repatriate the proceeds at any time (there are no controls on outflows).

The government still accumulates reserves to depreciate the real exchange rate but there is now a limit to the extent that it can do so effectively. If pushed too far, this policy will raise the excess return on domestic bonds above the level where it is worth for foreign investors to buy domestic bonds in spite of the tax, thus undoing the impact of the reserve accumulation on the real exchange rate. An implication of Proposition 4 is that because of this constraint, it is not possible for the government to undervalue the real exchange rate by more than the amount of the tax.

**Corollary 5** Assume that the government resists appreciation by taxing the purchase of domestic assets by foreigners at rate $\tau$ between time 0 and time $t$. Then the extent of the undervaluation that can be achieved by reserve
accumulation is bounded by the tax rate,

\[(1 - \tau) \frac{q_0}{\tilde{q}_0} \leq 1. \tag{9} \]

**Proof.** Let us consider the problem of a foreign investor who considers buying domestic bonds at time 0 and keep them until period \(t\). For reserve accumulation to work (not be offset by capital inflows) this must yield a lower return than investing in dollar bonds, i.e.

\[(1 - \tau) \exp \left( \int_0^t \epsilon_s ds \right) \leq 1. \]

Using (8) this gives (9). \(\blacksquare\)

To illustrate, Corollary 5 says that a country that imposes a 6 percent tax on its capital inflows (as Brazil did in 2010) cannot undervalue its real exchange rate by more than 6 percent. One advantage of tax-based capital account policies, thus, is that they imply a transparent upper bound on the level of undervaluation that a country can achieve by accumulating reserves.

### 6 Welfare Cost of Resistance to Appreciation

In this model, resisting the real appreciation of the currency reduces domestic welfare (it might increase welfare if, for example, growth were endogenous to the real exchange rate—see Korinek and Serven (2010)—but under our assumptions, the first welfare theorem applies and an undervaluation policy unambiguously reduces welfare). What is the welfare cost of the undervaluation? In policy discussions, this cost is sometimes identified with the valuation loss to the central bank when the currency finally appreciates. But it has also been argued that the intervention might in fact be costless since the revaluation does not reduce the purchasing power of the reserves in terms of the tradable good.

We study the welfare impact of resistance to appreciation by looking first at a small change in the accumulation of foreign assets. Let us assume that the government changes the path \((b_t^*)_{t \geq 0}\) by an infinitesimal (first order) amount \((\delta b_t^*)_{t \geq 0}\). If \(\delta b_t^*\) is continuously differentiable (so that \(c_{Tt}\) and \(q_t\) are continuous), then the impact on welfare can be written

\[
\delta U_0 = \int_0^{+\infty} u'(c_t) \frac{\partial c_t}{\partial c_{Tt}} \delta c_{Tt} e^{-r^* t} dt,
\]
where $\delta c_{T_t} = r^* \delta b_t^* - \dot{\delta} b_t^*$. Then using $\partial c_t / \partial c_{T_t} = q_t$, $\dot{\lambda}_t / \lambda_t = r^* - r_t$ (where $\lambda_t = u'(c_t)$) and integrating by parts gives,

$$\delta U_0 = - \int_0^{+\infty} u'(c_t) q_t \epsilon_t \delta b_t^* e^{-r^* t} dt. \quad (10)$$

The term $\epsilon_t \delta b_t^*$ is the government’s flow opportunity cost of financing the marginal increase of reserves by issuing domestic debt. The impact of the marginal increase in reserves on welfare is the present discounted value of this flow cost weighted by the marginal utility of consumption.

This formula can be extended to the case where the real exchange rate is discontinuous. Let us assume that $c_t$, $c_{T_t}$ and $q_t$ are discontinuous at time $t^*$, and denote with a superscript “−” (“+”) the levels of the variables just before (after) that time. The variation of the welfare is then given by

$$\delta U_0 = - \int_{t \geq 0, t \neq t^*} u'(c_t) q_t \left( r_t - r^* - \frac{\dot{q}_t}{q_t} \right) \delta b_t^* e^{-r^* t} dt - \left[ u'(c_{t^*}) q_{t^*} - u'(c_{t^*}) q_{t^*}^+ \right] \delta b_{t^*}^+. \quad (11)$$

If consumption were continuous, $u'(c_{t^*}) = u'(c_{t^*}^+)$, the term that is added by the revaluation would reduce to

$$- u'(c_{t^*}) \left( q_{t^*} - q_{t^*}^+ \right) \delta b_{t^*}^+,$$

which is the valuation loss on the foreign assets held at time $t^*$, $\delta b_{t^*}^+$, weighted by the marginal utility of consumption. The fact that consumption jumps magnifies the welfare loss, which can be written,

$$- u'(c_{t^*}) \left( q_{t^*} - q_{t^*}^+ \right) \delta b_{t^*}^+ - \left[ u'(c_{t^*}) - u'(c_{t^*}^+) \right] q_{t^*}^+ \delta b_{t^*}^+.$$

The results so far were local. To derive global results, define $U_0(z)$ as time-0 welfare when the reserve path is given by

$$b_s^* = \tilde{b}_s^* + z (b_s^* - \tilde{b}_s^*),$$

for $s \leq t$, where $z$ is between 0 and 1. For $z = 0$, this path corresponds to the undistorted equilibrium. For $z = 1$, it corresponds to the equilibrium with resistance to appreciation. By increasing $z$ from 0 to 1 we continuously move from the undistorted equilibrium to the equilibrium with resistance to appreciation. Using equation (10 we have

$$U_0'(z) = - \int_0^t u'(c_s) q_s \epsilon_s (b_s^* - \tilde{b}_s^*) e^{-r^* s} ds, \quad (11)$$
where \((\epsilon_s)_{0 \leq s \leq t}\) is the excess return path corresponding to the \(z\)-path for reserves.

We have \(U_0'(0) = 0\) and, using the quadratic approximation \(U_0(z) = U'(1)/2z^2\), domestic welfare under resistance to appreciation is given by \(U_0 = U_0'(1) = U_0'(1)/2\).

**Proposition 6** A quadratic approximation to domestic welfare under resistance to currency appreciation is

\[
U_0 = U^* - \frac{1}{2} \int_0^t u'(c_s)q_s\epsilon_s(b_s^* - \tilde{b}_s^*)e^{-r^*s}ds,
\]

(12)

where \(U^*\) is welfare in the undistorted equilibrium.

**Proof.** See discussion above. 

This proposition gives a “rule of thumb” for assessing the welfare cost of resisting to appreciation with capital account policies. First, one needs to assess by how much the observed level of net foreign assets exceeds the counterfactual level of net foreign assets that would be observed in the absence of capital account distortion. The welfare cost of resisting to appreciation, then, is approximately equal to the excess dollar return on domestic assets times one half the excess net foreign assets. (The factor one-half comes from the fact that the welfare loss is a triangle, not a rectangle.)

### 7 Numerical Illustration

This section illustrates the properties of the model in a calibrated version of the model. I assume that domestic consumption is a CES index of the consumption of the tradable good and that of the nontradable good,

\[
c(c_T, c_N) = \left[ \eta^{1/\theta} c_T^{(\theta-1)/\theta} + (1 - \eta)^{1/\theta} c_N^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)},
\]

and that the consumer has a constant relative risk aversion,

\[
u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}.
\]
The economy is in a steady state and is subject, at time 0, to an unexpected increase in the endowment of tradable good. Namely, the endowment of tradable good unexpectedly increases from \( y_T \) to a higher level \( y'_T = y_T + \Delta y_T \) and then remains constant. For simplicity, the supply of the nontradable good, \( y_N \), remains constant. The economy has no foreign assets in the initial steady state. This experiment is meant to capture, in a very stylized way, the case of a country having a positive shock in the tradable good sector.

Under free capital mobility, the real exchange rate would appreciate at the time of the shock, i.e., \( q \) would jump downward from \( q_0 = q(y_T, y_N) \) to \( \tilde{q}_0 = q(y'_T, y_N) \) at time 0. But we assume that the government wants to smooth the impact of the shock on the real exchange rate and let the currency appreciate gradually over the time interval \([0, t]\), according to

\[
q_s = q_0 + \frac{s}{t} \Delta q,
\]

(the real exchange rate is constant and equal to \( q_0 + \Delta q \) from time \( t \) onwards).

In order to achieve this objective, the government closes the capital account and accumulates reserves during the time interval \([0, t]\).

<table>
<thead>
<tr>
<th>Table 1. Calibration</th>
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<tr>
<td>( y_N )</td>
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<td>1</td>
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The values of the parameters used in the numerical exercise are given in Table 1. The pre-shock endowments of goods are normalized to 1. The endowment of tradable good increases by 10 percent at the time of the shock. Utility is logarithmic. The share of the tradable good in the consumption index is 0.3, and the elasticity of substitution between the tradable good and the nontradable good is set to 1 (this is the value assumed by Obstfeld and Rogoff (2005) in their three-country model of the world). Finally, the government resists appreciation for a period of 10 years. The equilibrium level of appreciation, \( \Delta q \), is the solution to a fixed-point problem that must be solved numerically.

For these parameter values, the real exchange rate falls from \( q_0 = 0.3 \) to \( \tilde{q}_0 = 0.281 \) at the time of the shock, an appreciation of 6.7 percent. This is also the amount of the real undervaluation at time 0 if the government resists appreciation. Figure 3 shows the impulse responses of the real exchange rate, the ratio of the trade balance to GDP, the ratio of foreign assets (reserves)
to GDP, and the excess dollar return on domestic assets. The real exchange rate is equal to $q_t = 0.276$ after ten years. The real exchange rate eventually appreciates by 8.4 percent, more than if it had appreciated at the time of the shock because of the accumulated foreign assets. The trade balances increases to 2.9 percent of GDP at the time of the shock and then decreases smoothly over the following ten years. Foreign assets increase to 15.1 percent of GDP after ten years and then remain equal that level in the new steady state. The excess return on domestic assets remains close to 1.2 percent over the whole episode. The cumulated excess return over the ten-year period, equal to 12.1 percent, is larger than the initial amount of undervaluation, as predicted by Proposition 4.

Although resistance to appreciation has a nonnegligible impact on trade
flows and foreign assets, its impact on welfare is small. Welfare falls by 0.0044 below the undistorted level, which is equivalent (with a logarithmic utility) to a one-time wealth loss of 0.44 percent of GDP or a permanent consumption loss of about 0.022 percent. However the welfare loss increases more than proportionately with the size of the initial undervaluation. To see this, I computed the sizes of the undervaluation and of the welfare loss when the shock $\Delta y_T$ varies between 0 and 0.5. Figure 4 reports how the welfare loss (on the vertical axis) varies with the size of the initial undervaluation (on the horizontal axis). The welfare loss from a 25 percent undervaluation amounts to a one-time wealth loss of about 6 percent of GDP. The figure also shows that the rule of thumb given in Proposition 6 (equation (12)) provides a good approximation to the true welfare loss, although it tends to slightly overestimate it for high levels of undervaluation.

Figure 4: Welfare loss from undervaluation (Source: author’s computations)
8 China Again

This section comes back to the case of China, but now taking advantage of the framework developed in the previous sections.

The real-world analog of the "government" in the model is the Chinese banking sector, including the central bank. The domestic banking sector issues most of the domestic liabilities that are the counterpart of reserve accumulation, and it is tightly controlled by the domestic authorities. Leaving the central bank aside, most of the banking sector is composed of four large banks that are owned or controlled by the government. In addition, the interest rate on deposits and loans is set by the authorities, which also have considerable influence over the lending policies, the credit flows and the sectors to which they are directed. From this point of view, it makes sense to consider the banking sector as a branch of the government.

Figure 5 gives a stylized representation of how the balance sheet of the

Figure 5: Banking sector and real economy

Figure 5 gives a stylized representation of how the balance sheet of the
banking sector fits into the Chinese economy. The banking sector issues deposits that are held by the domestic real sector and uses them to buy foreign assets and finance loans to the real sector. The banking sector’s foreign assets can be interpreted as variable $b^\ast$ in the model. Variable $b$ (the government’s domestic debt in the model) is the banking sector net debt vis-a-vis the real sector, i.e., bank deposits minus the banking sector’s claim on the domestic real sector.

Figure 6 reports the main items in the balance sheet of the Chinese consolidated banking sector between the end of 1999 and the end of 2011. Like in Figure 5, the two components on the asset side are foreign assets and credit to the domestic real sector. Lending to the real sector has gone primarily to firms in the state-owned sector and to a lesser extent to households. The liability side is composed mainly of bank deposits held by households and firms. Deposits are lower than the sum of foreign assets and credit to the domestic real sector because the banking sector also finances itself by issuing bonds, equity and the currency in circulation. The variables are expressed as shares of GDP.

One can distinguish two different epochs in the Chinese capital account and banking policies. From the end of 2002 to the end of 2008, the banking sector accumulated foreign assets at a high pace, mostly in the form of foreign exchange reserves at the central bank. Bank deposits did not increase during this period (as a share of GDP) because credit to the domestic real sector was reduced to offset the increase in foreign exchange reserves. In terms of the model, the Chinese authorities induced forced saving by allocating the Chinese loanable funds to the accumulation of foreign reserves rather than credit to the domestic sector.

The monetary authorities have used different policies to mitigate the impact of reserve accumulation on deposits. The central bank issued an increasing amount of sterilization bonds and steadily increased the regulatory cash reserves of banks. As a result the share of claims on the central bank doubled in the assets of banks in the 2000s (from about 11 percent in 2000 to about 22 percent in 2010). These policies reduced the impact of reserve accumulation on deposits.
accumulation on deposit creation, at the cost of reducing the amount of funds that the banks could lend to the real sector.

As shown in Figure 6, this policy mix was reversed in 2009 when the global financial crisis started. The Chinese authorities stopped increasing reserves (relative to GDP) and instead started to increase lending to the real sector in an attempt to stimulate the economy. The increase in lending to the real sector was matched by an equivalent increase in deposits, and so would not have resulted in higher total domestic spending if reserve accumulation had stayed on the same course as before the crisis (although it might have changed the composition of spending). Domestic spending was increased, though, because less loanable funds were used to finance the accumulation of foreign exchange reserves after 2008.

This policy course can be interpreted as follows in light of the model. Throughout the 2000s, the Chinese growth rate was very high and growth was pulled primarily by the development of the Chinese tradable sector. This should have led to an appreciation of the Renminbi because of the Balassa-Samuelson effect. However, between 2002 and 2008, the Chinese authorities were resisting currency appreciation by accumulating reserves and repressing

Figure 6: Foreign Exchange Reserves, Loans and Deposits in China (% of GDP). Source: PBOC.

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domestic demand. They stopped doing that once the global financial crisis started. Figure 7 shows the macroeconomic correlates of this interpretation in the data. Between 2002 and 2008, the Renminbi did not appreciate relative to the US dollar in real terms, whereas the underlying pressure was reflected instead in a booming trade surplus. After 2008, when reserve accumulation slowed down, the trade surplus was reduced at the same time as the Renminbi appreciated.

Figure 7: Foreign Exchange Reserves, Trade Surplus and Real Exchange Rate with the US Dollar. Source: PBOC.

The evidence so far is consistent with the view (analyzed in the model) that the Chinese authorities were constraining domestic demand by accumulating reserves and restricting capital inflows. An important implication of the model, if this was going on, is that Chinese financial assets should have delivered an excess return relative to foreign assets. Was this the case in the data?

Estimating returns on Chinese fixed-income assets is difficult because the observed interest rate levels do not reflect market forces in a repressed financial system. Presumably, foreign investors would invest in the fixed-income assets with the highest interest rates. In China this would mean lending in

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9The real exchange rate is CPI-based and normalized to 100 in .
the interbank market. Figure 8 reports the dollar return on investing in the Chinese one-month interbank market, which was computed as the one-month interest rate in the Chinese interbank market plus the one-month rate of appreciation of the Renminbi relative to the dollar, and compares it to the US Federal Funds rate. The data are monthly from 2000 to 2011, and for both returns, we show the cumulated return over a rolling twelve-month window ending in the month under consideration. We observe that after 2002 the dollar return on the Chinese interbank market is almost always larger than the Fed Funds Rate. The difference starts to be large in 2007, and it is mostly due to the appreciation of the Renminbi, which started in 2006, and resumed in 2010 after an interruption in 2008.

The behavior of households and firms, however, should be determined by the interest rate to which they have access, which is lower than the interbank market rate. A large fraction of the households’ and firms’ financial wealth is held in the form of time and savings deposits, on which banks pay a lower

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Footnote 10: The Chinese interbank market rate should not be affected by a default risk since banks are public.
return than the interbank interest rate. The excess dollar return on the 6-month deposits in Chinese banks is reported in Figure 9. It is lower than in Figure 8 but is nevertheless positive and significant after 2007.

The model also suggests a measure of the welfare cost of the Chinese policies. Proposition 6 tells us that the flow welfare cost is approximately equal to the excess dollar return on domestic assets times one half the “excess reserves” (i.e., the reserves in excess of what China would hold in the absence of distortion). If, for example, one assumes that the undistorted level of reserves was 20 percent of GDP (about the level observed at the beginning of the 2000s), excess reserves amounted to about 30 percent of GDP at the end of the decade. The average excess dollar return on Chinese deposits was 4.7 percent in 2010-11, implying a flow welfare cost of $\frac{1}{2} \times 0.3 \times 4.7 = 0.7$ percent of GDP in 2010-11.

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11Again, those policies might have welfare benefits which are not taken into account by the model.
9 Conclusions

There are several directions in which the analysis could be extended. First, it would be interesting to provide a welfare analysis of an undervaluation policy that has some benefits in terms of growth. Second, one could also look at a two-country model in order to assess the general equilibrium effects of capital controls. This is left for further research.
References


