Official Bailouts of Sovereigns: 
Seniority, Catalytic Effects and Insolvency

DRAFT

Christophe Chamley\textsuperscript{1} and Brian Pinto\textsuperscript{2}

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Abstract

Institutional lending in crisis is evaluated from a theoretical point of view. Four points are suggested. First, the importance of seniority may be irrelevant. Second, institutional lending may be powerful in a liquidity crisis. Third, when a crisis takes place where solvency is an issue, some immediate debt reduction is efficient. Fourth, in a situation of multiple equilibria in private loan rates, institutional lending may induce a switch to a low private loan rate only if it can be done with a sufficiently high amount.

Key words: Institutional lending, liquidity crisis, solvency crisis, catalytic lending, multiple equilibria.

\textsuperscript{1}Boston University and EHESS-PSE
\textsuperscript{2}The World Bank
1 Introduction

There have been three official bailouts of countries in the euro periphery starting in 2010. The bailout of Greece, which was finalized over April-June 2010, was followed by similar bailouts of Ireland and Portugal. Financial rescue packages and fiscal programs have been jointly orchestrated by the IMF and Eurogroup, with the latter's funding channeled via the European Financial Stability Facility created in May 2010. Until the announcement of a second bailout for Greece following a euro area summit on July 21, 2011, the first-round bailouts had one feature in common: None served to quieten the markets and lower bond spreads to levels that would connote low default probabilities as one might have expected a priori given the size of the announced bailout packages and the international muscle behind them. To the contrary, bond spreads rose significantly following the implementation of the bailouts. In other words, the situation worsened.

Several reasons could potentially explain why the debt outlook for these countries worsened in the wake of the initial bailout announcements and the start of their implementation in spite of the announcement of progressively bigger official sums being put on the table. These include news about hidden deficits and debt, the lack of a credible fiscal adjustment program and political impediments to the needed austerity or to design flaws in the bailout packages. A more cynical view is that the private creditors of the sovereigns have already factored an official bailout into their calculations, in which case the hype about a large enough bailout package to calm the market is very much in their self-interest as it could enable exit at 100 cents on the euro.

An examination of the case of Greece (Chamley and Pinto 2011) isolates the seniority of the official debt involved in the bailout in conjunction with solvency problems in the affected countries as the key reason official bailouts of sovereigns tend not to work. The argument is simple: official loans to the troubled government provide liquidity support without any guarantee of an increase in the present value of primary fiscal surpluses net of the official loans. In this case, if the official loans are perceived as senior, short-term creditors who are first in
the queue may seize the opportunity to exit without a loss while those further back in the queue will perceive their claims as devalued, prompting a sell-off. In a related paper, Gros (2010) argued that the formal announcement of seniority could cause the sort of problems described in Chamley and Pinto (2011). He attributes the problems with the Irish bailout in part to the November 2010 announcement that official financing from the Eurogroup would be considered senior to private loans; but the case of Greece itself as well as that of Russia in 1998 convincingly shows that all that matters is a perception of seniority based on precedents.

In this paper, we revisit the analytics of the official bailouts of countries. We start by reviewing the existing literature. This focuses on the catalytic role of official lending (e.g., IMF) in two respects: persuading the country to make a bigger effort to pay off its loans (which can be thought of as the accompanying fiscal program conditions aimed at raising primary fiscal surpluses); and persuading private creditors to rollover their loans or make new loans thereby preventing a default. The role of the IMF is to solve a coordination problem and increase the chances of a good equilibrium materializing. This literature treats the IMF pari passu with private creditors and concerns itself with liquidity problems typically in the context of a single-period model. Following the review, we introduce the notion of seniority of official lenders in a multi-period context. This approach replicates well the behavior of market investors following the first-round bailouts of Greece, Ireland and Portugal. In addition to seniority, the pricing of official loans relative to the risk-free rate is of critical importance in replicating the behavior observed because it affects the level of fiscal effort required and hence the credibility of the program. An implication of our approach is that pricing official loans at the risk-free rate (in line with standard corporate finance principles since they are more-or-less zero risk relative to private creditors) and insisting on an orderly upfront haircut for private creditors would greatly increase the chances of a successful bailout (defined as lowering the bond spreads of the bailed out countries to non-default levels).

According to the official policy of the IMF, “the IMF, when it lends, provides
only a small portion of a country’s external financing requirements,” and “IMF financing can act as an important lever, or catalyst, for attracting other funds” (IMF web site). That effect operates through two channels. In the first, the IMF lending induces the country to undertake policies reforms that increase future growth and the profitability of investment. These prospects enhance the incentive of private agents for more lending. In this paper we ignore that effect, not because we think it is not important. On the contrary, the coordination and the monitoring by the IMF, or another institution, may be critical for a recovery, but that channel works through a mechanism that is different from the interaction between IMF lending and private lending and we focus on the latter.

All loans by the IMF have a status of seniority when a country faces a repayment problem. That seniority clause has been criticized (Gros, 2011) because it could reduce the incentive of private lenders. We think that this is a misconception and we address the issue in the next section. In the first result, the seniority of some loans have no impact on the incentive of private loans and on the default incentives of a country.

In Section 3, we focus on liquidity crises. We build on previous work by Morris and Shin (2006). In the context of a pure liquidity crisis, IMF lending may have a beneficial impact. That is not surprising, but we analyze some specific properties about the catalytic impact of IMF lending and on the rate of private loans. We also propose a criterion to analyze cases of liquidity short-falls that may generate multiple equilibria under common knowledge.

Section 4 is devoted to the case of a solvency problem that follows an adverse shock. The main result is that an immediate reduction of the existing debt is efficient in the sense that it reduces the expected total cost of default.

In Section 5, we turn to the issue of multiple equilibrium lending rates. For example, a high rate may be self-fulfilling because it increases the probability of default. It is shown that small intervention by an institution have no impact. Only sufficiently large interventions may trigger a switch to an equilibrium with
a lower rate on private loans.

Some qualifying remarks are added in the concluding section.

*Short Literature review*

Previous studies that consider the effort effect include Penalver (2004) and the first part of Morris and Shin (2004). A general result is that IMF lending may be effective when the fundamental of the countries are neither too good (in which case no lending is necessary), nor too bad (in which case, the incentive effect is small and a waste).

to be completed: The papers that will be (briefly) discussed are (see the reference list) Arellano, Bird and Rowlands, Bolton and Jeanne, Chamley and Pinto, Cole and Kehoe, Conesa and Kehoe, Corsetti, Gros, Roch and Uhlig.

2 One-period debt contracts and sovereign loans

Sovereign loans are in the form of debt contracts with a repayment that is not contingent. Failure to meet the scheduled triggers default. The efficiency of the standard debt contract between two private parties has been demonstrated by Townsend (1979) and Gale and Hellwig (1985) in a one period framework when lenders do not have costless information on the *ability* of the borrower to pay back\(^3\).

In the standard debt contract, default takes place when the borrower cannot pay the fixed amount that is due when there is no monitoring. The creditor is able to recover all resources that are found after monitoring, net of the monitoring cost.

\(^3\)Either there is no monitoring and the payment by the debtor is fixed, or there is monitoring. In order to save on the cost of monitoring, if the debtor can pay he pays, and if he cannot pay, the creditor gets all the available resources net of the monitoring cost in order lower the *ex ante* cost of the contract.
Such ability may not be relevant in sovereign debt contracts. The standard enforcement mechanism is the penalty that follows default. Assume a debt contract of one period (between loan and payoff) and focus on the payoff period when there payment that is due by the borrower is equal to some value $B$ and that the penalty is an increasing function $\tau(y)$ of the income $y$ of the borrower: a penalty in form of credit restrictions is more costly when the economy is in a boom. An equivalent formulation is obtained when the cost of avoiding the penalty increases if the economy is in a recession. In this setting, default takes place if the income $y$ is smaller than some threshold value $\bar{y}$ such that $B = \tau(\bar{y})$: the default rule has the same form for a sovereign debt contract as for a private contract.

Set the loan to be normalized to a principal of 1 and that the income that can service the loan is the realization of a random variable $y$ with cumulative distribution function $F$. The cost of default is assumed to be a function $\tau(y)$ that may be constant. If the opportunity cost of lenders (principal and return) is $R^*$ and the contract rate is $R$, we have

$$R^* = \int_R dF(y) + \int_0^R (y - \tau(y))dF(y).$$

(1)

In general this equation may have more than one solution in $R$. When there are two parties, as in private contracts, the two parties may agree on the lowest value of $R$, which is the most efficient since it minimizes the probability of default and the expected cost. When the number of lenders is large, as that may be the case in sovereign loans, the coordination on the lowest interest rate may not take place. This issue will be discussed in the next Section.

In sovereign lending, it is common to have different contract with different lenders. For example, institutional lending by the IMF benefits from seniority. To focus on the main argument, assume that the IMF and private investors have the same opportunity cost that is the risk-free return. The seniority enables the IMF to lend at a lower rate but it increases the lending rate by private investors. However, the seniority has per se no impact on the default decision of the borrower if that decision is determined a threshold rule of income as in...
equation (1). The independence of the default on the composition of the debt is similar to a Modigliani-Miller theorem. This simple point needs to be recalled before any discussion on the impact of institutional lending.

Lemma 1 (Modigliani-Miller)

Assume that a borrower has a given one period debt, normalized to one, in the form of a standard contract, and a random income \( y \). The return of the debt contract is \( R \) and the risk-free return is \( R^* \). The borrower defaults if \( y < y^* = R \), in which case lenders get \( y - \tau(y) \). Suppose that a fraction of the debt \( m \) is replaced by a contract with seniority repayment and ex ante rate of return equal to \( R^* \) with \( R^* m \leq y \) for all \( y \). The default threshold \( y^* \) is independent of \( m \).

We assumed for simplicity that \( y \geq R^* m \) in order to fit with a risk less loan and the IMF insistence on being repaid back. But Lemma 1 obviously holds even when the senior claim is risky. As any Modigliani-Miller type result, the property is simple, but the property should be emphasized in order to dispel some notions that seniority loans reduce the incentive of private lenders.

The property can be restated in different terms: an institution such as the IMF can charge a lower rate because of the senior claim. That lower rate reduces the debt service of the country. The remaining part of the debt is more risky, obviously. But that higher risk is associated with a higher loan rate on a debt that is reduced by the IMF loan. The total service is not changed, and if the default decision is triggered by the relation between the debt service and some other variable (e.g., future income or penalty for default), then the default decision is not affected by the seniority clause.

3 Liquidity crisis

Throughout the paper we will consider a model with three periods, 0, 1 and 2, and a country that has income in periods 1 and 2, \( y_1 \) and \( y_2 \), that are random.
We will focus essentially on the periods 1 and 2. The country borrows an amount 1 in period 0 to be serviced by interest \( R_{01} \) and \( R_{02} \). Initially \( y_i > R_{0i} \).

Assume that in period 1, there is a shock and the country has to borrow. With \( z < 0 \),

\[
y_1 + z < R_{01}.
\]

We can assume that the amount due is normalized to 1. The available resources are \( \theta \) and the short-fall that has to be financed is therefore \( 1 - \theta \). The country borrows on the market at the rate (principal plus interest) \( R \). The proceeds of the borrowings are used to make the payment that is due in period 1. If the country is unable to make all the payment due it defaults. We assume that in this case, the loans that are made in period 1 by private lenders are lost with no return in period 2.

Lenders in period 1 face a risk and their lending rate is higher than the risk-free rate \( R^* \) to compensate for the probability of default. A period in a discrete model takes some real time. During that real time, the process of lending by private agents actually involves a number of decisions in a sequence and agents have opportunities to observe the unfolding of events and adjust their decision accordingly. This issue is complex and clearly a subject of further research. We sidestep the problems here by assuming that all lenders make their decision simultaneously and that each lender cannot observe the actions of other lenders.

Adapting Morris and Shin (2006), we assume that there is a continuum of lenders of mass \( A \) each risk-neutral and able to lend a fixed amount that is normalized to 1 (like 1 million and 1000 lenders). Each lender cannot observe exactly the amount that is available \( \theta \) in the country and has a noisy signal \( s = \theta + \epsilon \) where \( \epsilon \) has a variance \( \sigma^2 \). Each lender is willing to make a loan if his signal is greater than some threshold level \( s^* \), that is when \( s = \theta + \epsilon \geq s^* \) which is equivalent to \( \epsilon \geq s^* - \theta \). Let \( F \) be the cumulative distribution function of \( \epsilon \). The amount of loanable funds is therefore \( A(1 - F(s^* - \theta)) \). When that amount is shorter than the financing need, the country defaults. When the amount is greater, the
country uses only the need amount.

We assume that an institution, \( e.g. \), the IMF makes a loan \( m \) in period 1 at the risk-free rate \( R^* \) due in period 2. The loan is observable to all agents before making a lending decision. This loan could have seniority over other payments that are due in period 2. However, because we focus on the issue of liquidity we assume that there is no solvency problem in period 2 and the issue of seniority is irrelevant. Moreover, if the country defaults in period 1, by assumption here, the IMF can extract the risk-free rate in period 2. The total amount of resources that are available in period 1 is therefore \( \theta + m \).

It follows that the country is able to make the payment due when the level of its resources, \( \theta \), is greater than some value \( \theta^* \) that is solution of the equation

\[
1 - \theta^* - m = A(1 - F(s^* - \theta^*)).
\]

That cutoff point depends on the strategy \( s^* \) of the lenders.

The minimum level of the private signal for agents willing to lend, \( s^* \), is determined by the arbitrage of an agent with signal \( s^* \) who is indifferent between the risk-free loan at the rate \( R^* \) and the risky loan to the country at the rate \( R \).

One can show (Appendix) that if the noise of the private signals is vanishingly small, we have

\[
F(s^* - \theta^*) = \frac{R^*}{R}.
\]

Combining with equation (2), there is a default if \( \theta < \theta^* \) with

\[
\theta^* = 1 - m - A(1 - \frac{R^*}{R}).
\]

This equation shows two properties. First, the IMF loan obviously lowers the threshold level of resources that that are required to prevent default. It is remarkable here that it lowers that threshold one for one: a 1 billion loan by the IMF lowers \( \theta^* \) by 1 billion. There is no catalytic effect that would induce more
private loans and lower $\theta^*$ by more than 1 through a multiplier effect\(^4\).

**Proposition 1**

*When lenders have nearly perfect (but heterogeneous) information on the financing need of the country, for a fixed loan rate on private loans, the level of the country’s reserves that prevent a default is reduced by the exact amount of the IMF loan. Private loans have no impact on the default outcome and there is no catalytic effect.*

We show in the Appendix that the heterogeneity of lenders’ beliefs is not negligible, then a loan by the IMF has a catalytic effect: the contribution of private lenders reduces the level of reserves that prevent a default.

Equation (4) shows a second property. A higher value of the loan rate $R$ attracts more loans and lowers the threshold value of the resources. In a full fledged model, the rate $R$ should be determined by the country or by the market. In the first case, that rate becomes an additional signal either on the information by the country about its reserves $\theta$. In the second case, it is a signal that aggregates the private information of lenders. That issue is beyond the scope of the present study and it is ignored here.

Equation (4) can be rewritten

$$R = \frac{R^*}{1 - \lambda(1 - \theta^* - m)} \quad \text{with} \quad \lambda = \frac{1}{A}. \quad (5)$$

We now provide a different interpretation of this equation. Define $g = 1 - \theta$ where $\theta$ is the actual level of resources of the country. The shortfall that is to be financed in financial market and is subject to the risk of coordination failure is $g - m$. It is plausible to assume that the probability of a coordination failure is proportional to the short-fall that is normalized in the interval $[0, 1]$ and with a

\(^4\)That property is omitted by Morris and Shin (2006).
coefficient of proportionality $\lambda < 1$. In this case, the lending rate $R$ should be given by the equation (5) which is rewritten as

$$R = \frac{R^*}{1 - \lambda(g - m)} \quad \text{with} \quad \lambda = \frac{1}{A}. \quad (6)$$

We will keep both interpretation in the analysis of the institutional lending. In the interpretation of the coordination game, $g = 1 - \theta^*$ is the funding need of the country when it is about to default (given the loan rate $R$). In the interpretation of the probability of the coordination failure, $g = 1 - \theta$ is simply the funding need of the country.

**The impact of institutional lending on the debt service**

Define the amount of private lending $L = g - m$. The debt service in period 2 is equal to $B$ with

$$B = LR + mR^* = (g - m)R + mR^*. \quad (7)$$

Using the expression of $R$ in (5),

$$B = R^*\phi(m), \quad \text{with} \quad \phi(m) = \frac{g - m}{1 - \lambda(g - m)} + m = \frac{1}{g - m - \lambda} + m.$$

One verifies that

$$\phi'(m) = -\frac{1}{(1 - \lambda(g - m))^2} + 1 < 0.$$

An increase of the IMF lending lowers the debt service. In the interpretation of the cost of coordination, interpretation is intuitive: the IMF lending lowers the cost of coordination.

Assume that the country has issued some “long-term” bonds with payments in periods 1 and 2. A loan by the IMF lowers the probability of default. It has a
positive impact on the price of the long-term bonds. We think that a positive impact of IMF lending on long-term bonds is clear evidence that the intervention addresses a liquidity problem and not a solvency problem.

In the present analysis, the country is assumed to be solvent in that it has sufficient resources in period 2 such that the present value of resources for all periods, discounted to period 0 is equal to the value of the debt in period 0. However, the liquidity crisis in period 1 increases the loan rate to period 2. If that impact is sufficiently high, the country may be insolvent. More precisely, if the lending rate $R$ from period 1 to period 2 increases above some threshold that is determined by solvency, then lenders can determine in period 1 that the country is insolvent and default occurs in period 1. A liquidity problem can degenerate into a solvency problem.

Another sequence of events could also take place. Assume that the country defaults if the debt service in period 2 is greater than some penalty and that the penalty is the realization of a random variable. For example the penalty could be a fixed fraction of the random income in period 2 and therefore be lower if the country has a low income. For some values of the return $R$, a default would take place in period 2. The expectation of this default would raise the rate $R$ even more. For some values of $R$, the default could occur in period 1 rather than period 2. One could also observe multiple equilibria with a model structure as in Cole and Kehoe (2000).

In both the previous contexts, the institutional lending by lowering the lending rate could eliminate the possibility of a default or lower its probability. Such a policy would have obvious benefits.

**The impact of debt reduction**

Assume that in period 1, the debt is reduced in the form of a forgiveness of part of the due payment. Let $m$ be this reduction. In the present framework, it is equivalent to an institutional loan with no payment due in period 1. The
reduction of the due payment lowers the refinancing requirement in the same way as the institutional loan. That effect generates the same reduction of the loan rate from period 1 to period 2. That is the first term in the expression (7) of the debt service in period 2. The reduction of that debt service is now stronger because there is no more second term for the service of the institutional loan.

4 Solvency crisis

In the previous section, we focused on the liquidity issue. The country’s present value of resources was greater than its liabilities even if some loans had to be refinanced at a rate slightly above the risk-free rate. The country could become insolvent only if the refinancing rate would be sufficiently higher than the risk-free rate. But such a failure would not be a failure of the country. It would be a failure of coordination between lenders and a failure of the capital market.

For the problem of closing a financing gap in the previous section, we focused on the periods 1 and 2. What may have happened before period 1 (here period 0), could be taken as given. That assumption sidesteps the critical element in the context of an institutional loan. All institutional interventions take place only after a shock puts the economy in a state that was different from the expectation of private agents. In this section, we...

4.1 Common knowledge

Assume first that because of a shock, the country needs to raise the amount \( z \) in period 1. The country has a random about of resources \( y \) in period 2, net of the payments for previous debt commitments. Assume that the financing \( z \) is done through institutional and private loans. The institutional loan is \( m \) and is charged at the risk-free rate \( R^* \) with a seniority clause. Private lending is done at the rate \( R \). For simplicity assume that the random income \( y \) is always greater than \( mR^* \) and that the country defaults in period 2 only if it does not have
the resources to meet the due payment. From Lemma 1, the default decision is
independent of the institutional loan \( m \). A higher value of \( m \) obviously raises the
private rate \( R \) but it does not affect the default decision. The seniority clause
of institutional lending has no adverse impact.

Assume the country borrows an amount normalized to 1 in period 0. At this
first state, all loans in period 0 are one period loans with a rate \( R_{01} \) in period
1. The economy has a fixed income in period 1 that is equal to the payment
\( R_{01} \). In period 2, the economy has a random income \( y \), net of its debt obligation
in that period. If the economy has no shock, there is no need for finance in
period 1. If a shock takes place the economy’s income in period 1 is reduced
by \( z \). If a default takes place, there are in general two costs, borne by the
country and the lenders, respectively. We ignored the cost borne by the country
and assume that the lenders can claim what is available in the country minus a
cost that increases with the level of the default, that is the due payment minus
the available resources. That increasing property is plausible. Note also non
linearity, probability of terminating a bank, etc...). The cost of debt reduction
is assumed here to be an increasing function \( \lambda(\cdot) \) of the amount of the debt
reduction. We assume that

\[
\lambda(0) = 0, \quad \lambda'(x) > 0, \quad \lambda''(x) \geq 0.
\]  

Note that the function \( \lambda \) does not need to be strictly convex. We will see that
the second derivative \( \lambda''(x) \) may even be negative provided that it is not too
small.

In order to simplify the presentation, we assume that if a shock does not occur,
the country can meet its debt obligations in period 2. The shock in period 1 has
no impact on the random income in period 2, it is therefore an adverse shock
to the country. We assume that when the shock occurs, the country receives a
reduction \( x \) of the payment due in period 1. The difference between the funding
requirement and the debt relief, \( z - x \), is financed by a one-period loan at the
rate \( R \).
In period 2, the country defaults if the income \( y \) is less than the payment due \( R(z - x) \). The value of \( R \) is determined by arbitrage with risk-free investment:

\[
R^*(z - x) = (z - x)R \int_{(z-x)R} f(y)dy + \int_{(z-x)R}^{(z-x)R} (y - \lambda((z - x)R - y)) f(y)dy. \tag{9}
\]

On the right-hand side of the previous equation, the first term is the payment in period 2 with no default. The second term is the payment under default. The solution in \( R \) is a function of the debt reduction in period 1, \( x \).

We consider the total cost of all debt reductions in periods 1 and 2. These costs may be burned by different agents but from a social point of view, we just add them up. (We assume no interaction between the costs in periods 1 and 2). The total cost of debt reduction is therefore a function of the debt reduction in period 1.

\[
C(x) = \lambda(x) + \frac{1}{R^*} \int_b^b \lambda(b - y)f(y)dy, \quad \text{with} \quad b = (z - x)R, \tag{10}
\]

where \( R \) is a function of \( x \) in equation (9). Note that the cost of default in period 2 is discounted to period 1. The most appropriate discount rate is probably the risk-free rate.

In order to evaluate the impact of the debt reduction \( x \) on the cost \( C \), we differentiate equation (9):

\[
-R^* dx = \left( -R dx + (z - x)dR \right) \int_b^b f(y)dy + \left( -R dx + (z - x)dR \right) \int_b^b \lambda'(b - y)f(y)dy.
\]

Using \( \lambda(0) = 0 \),

\[
-R^* dx = \left( -R dx + (z - x)dR \right) \int_b^b f(y)dy + \int_b^b \lambda'(b - y)f(y)dy. \tag{11}
\]

Differentiating (10), and using \( \lambda(0) = 0 \),

\[
dC = \lambda'(x)dx + \frac{1}{R^*} \left( (z - x)dR - R dx \right) \int_b^b \lambda'(b - y)f(y)dy.
\]
Substituting in this expression for \((z - x)dR - Rdx\) in (11),

\[
C'(x) = \lambda'(x) - \left(1 + \frac{R}{R^*} \int_{b}^{b} \lambda'(b - y)f(y)dy\right) \frac{\int_{b}^{b} \lambda'(b - y)f(y)dy}{1 - \pi},
\]

where \(\pi\) is the probability of default in period 2, \(\pi = \int f(y)dy\).

We evaluate this derivative at the point of no debt relief in the period 1, when \(x = 0\). Let \(\alpha = \lambda'(0)\). Using \(\lambda'' > 0\), we have

\[
C'(0) \alpha \leq 1 - \left(1 + \frac{R}{R^*} \int_{b}^{b} \lambda'(b - y)f(y)dy\right) \frac{1}{1 - \pi}. \tag{12}
\]

It follows that

\[
C'(0) < 0. \tag{13}
\]

Conditional on the occurrence of a shock, at the position of no debt relief, \((x = 0)\), an increase of \(x\) lowers the total cost of default.

**Proposition 2**

*Under the assumptions in (8), if a shock takes place in period 1 with no change of the distribution of income in period 2, some debt relief in period 1 is efficient in the sense that it reduces the total expected cost of default.*

### 4.2 Heterogeneous beliefs

*To be done*

### 5 Multiple equilibria with finite rates in debt contracts

In this section, we address an issue that is different from the liquidity and the solvency crises of the previous sections. Can a country (e.g., Italy, Ireland), face
a high cost of debt because the probability of default is higher and that higher probability is induced by a self-fulfilling high rate, *ex ante*. We examine here the issue of multiple equilibrium loan rates.

Consider a two-period setting where the country borrows a quantity normalized to 1 in the first period and pays in the second period. In that period, there are three possible states \( j = 0, 1, 2 \), where the country’s income is \( y_j \) with \( y_0 < y_1 < y_2 \). State \( j \) has probability \( \pi_j \) for state \( i \) and is known by the lenders. Let \( R_j \) the loan rate if the country defaults in state \( k \leq j \). Since the country does not default in the best state, \( j = 0 \) or 1. If the income of the country is greater than the loan rate \( R_j \), it pays the loan rate. If the income is smaller than the loan rate, it defaults and the payment on the loan is \( \lambda R_j \) with \( \lambda \in [0, 1) \). The default can be triggered either because the income is short of the debt payment or because the cost of default is smaller than the benefit from not paying the debt\(^5\).

By arbitrage, the expected return on a loan is equal to the risk-free rate \( R^* \). The loan rates with default in state 0, \( R_0 \), and default in states 0 and 1, \( R_1 \), are therefore determined by

\[
\begin{aligned}
R^* &= (1 - \pi_0)R_0 + \pi_0(1 - \lambda)y_0, \\
R^* &= (1 - \pi_0 - \pi_1)R_1 + \pi_0(1 - \lambda)y_0 + \pi_1(1 - \lambda)y_1,
\end{aligned}
\]

Hence,

\[
\begin{aligned}
R_0 &= \frac{R^* - \pi_0(1 - \lambda)y_0}{1 - \pi_0}, \\
R_1 &= \frac{R^* - \pi_0(1 - \lambda)y_0 - \pi_1(1 - \lambda)y_1}{1 - \pi_0 - \pi_1}.
\end{aligned}
\]

By definition, \( R_0 < y_1 < R_1 \), and using the expressions of \( R_0 \) and \( R_1 \),

\[
\pi_0(1 - \lambda)y_0 + (1 - \pi_0)y_1 - \lambda \pi_1 y_1 < R^* < \pi_0(1 - \lambda)y_0 + (1 - \pi_0)y_1.
\]

\(^5\)For example if the default penalty is a fraction \( \mu \) of income, the country defaults if \( (\lambda + \mu)y < R \).
The right-hand inequality (for the equilibrium with low interest rate), holds when \( RR^* < y_1 \) and the probability of a very bad state, \( \pi_0 \), is sufficiently small. The left-hand inequality, (for the high interest rate equilibrium), holds when both the probability of the middle state and the cost of default in that state are sufficiently high.

**Proposition 3**

Consider a country that borrows one unit to be paid in the next period according to the model of this section with three states for the level of income in the next period. There are two equilibria under inequality (16). In one equilibrium, there is default only in the state with the lowest income. In the other equilibrium, the loan rate is higher and the country does not default only in the state with the highest income.

The critical assumption in the inequality (16) is that in the middle state, there is a sufficient loss of resources to be claimed by lenders.

**The impact of senior institutional lending**

Suppose all agents coordinate on the equilibrium with higher interest rate and default in the second period. Assume that an institution such as the IMF lends \( m \) at the risk-free rate \( r^* \), with senior claim on repayment, and that in the middle state, with income \( y_1 \), the country has sufficient resources to pay off the IMF loan (which can therefore be granted at the risk-free rate): \( y_1 > m(1 + r^*) \).

It is also natural to assume that if the country defaults, the IMF has special means to enforce the payment of its loan\(^6\). Assume that the country defaults in states 0 and 1. The loan rate, \( R_1 \), is such that

\[
R^*(1 - m) = (1 - \pi_0 - \pi_1)R_1(1 - m) + \pi_0((1 - \lambda)y_0 - mR^*) + \pi_1((1 - \lambda)y_1 - mR^*).
\]

---

\(^6\)This point is discussed later ***.
Note that on the right-hand side, the payment to the IMF has seniority and is deducted from the available resources \( y_i, (i = 0, 1) \) before any resource is transferred to private creditors. The cost of default applies to this transfer but not to the transfer to the IMF. We assume that in the lowest state, there are sufficient resources to pay the IMF: \( y_0 > mR^* \).

A straightforward algebra shows that the rate \( R_1 \) on the private loans that is required for an *ex ante* return \( R^* \) is given by:

\[
R_1 = \frac{R^*}{1 - \pi_0 - \pi_1} - \frac{1}{1 - \pi_0 - \pi_1} \left( \pi_0 \frac{(1 - \lambda)y_0 - mR^*}{1 - m} + \pi_1 \frac{(1 - \lambda)y_1 - mR^*}{1 - m} \right).
\]

It increases with the amount of the IMF loan. The statutory debt service, \( B \) is equal to

\[
B = R_1(1 - m) + R^*m = \frac{R^*}{1 - \pi_0 - \pi_1} + \frac{-1}{1 - \pi_0 - \pi_1} \left( \pi_0((1 - \lambda)y_0 + \pi_1(1 - \lambda)y_1) \right).
\]

That service is independent on the IMF loan. The property is not surprising and is just an application of Lemma 1. IMF lending at the risk-free rate is neutral, *per se*. The intervention of the IMF may trigger a switch to the equilibrium with a lower rate (and default in state 0 only), but this effect is independent of the amount of the loan. Note that if the IMF would make a loan at a rate lower than the risk-free rate, it would lower the statutory payment and that subsidy, if sufficiently large, may trigger the switch to the better equilibrium.

Assume now that the country is initially in the bad equilibrium with rate \( R_1 \) and renegotiates a fraction \( m \) of its debt to be serviced at the rate \( \tilde{R} = R_0 \), that is the lower rate of the good equilibrium.

The statutory debt service is now

\[
B(m) = (1 - m)R_1 + mR_0.
\]

Under the previous assumptions about the parameters of the model (for which there are two equilibria), we have

\[
B(0) = R_1 > y_1, \quad B(1) = R_0 < y_1.
\]
For some value $m^* \in (0, 1)$, if $m > m^*$, the country does not need to default in state 1 on the debt that earns a return $R_1$. In other terms, the market value of that debt is greater than 1. The country could buy that debt back by issuing a new debt with a return $R_0$. Note that the IMF’s loan at the rate $R^*$ that is even lower than $R_0$, cannot achieve the switch from bad to good equilibrium because of the seniority clause and because it shifts the entire cost of the default to the private creditor. This discussion is summarized by the following proposition.

**Proposition 4**

*If there are two equilibria in the model of Proposition 6, an institutional loan with seniority at the risk-free rate does not trigger a shift from the bad to the good equilibrium. If the country refinances a fraction of its private debt $m \geq m^*$ for some value $m^* \in (0, 1)$, it is able to refinance the whole debt at the rate $R_0$ and switch to the good equilibrium at the rate $R_0$.***

This property mitigates the benefit of the long maturity of the debt that were presented by Giavazzi and Pagano (1989). In their argument a long maturity enables the country to reduce the amount that is due in each period and that may be subject to a temporary crisis of liquidity or confidence. Consider a country that faces a high rate on its debt because it has gone through difficult times in the past and suppose that current conditions are much improved. From the previous result, if it refinances its debt gradually as the old bonds come to maturity, the high rate may be the only equilibrium rate. The switch to the better equilibrium requires a quantum refinancing of the debt at a lower rate. Of course, an institution could provide a bridge loan to facilitate that transition.

As usual for all cases of multiple equilibria, the issue is the “choice” by agents of the equilibrium. We may assume here that the choice of equilibrium is common knowledge before any loan contract is made, for example through a futures market. If the good equilibrium takes place, the country borrows at the low risk-free rate and there is obviously no need to call the IMF. The IMF will be called here only if lenders coordinate on the bad equilibrium with high interest
rate. When the high rate is caused by the expectation of default, the previous analysis shows that the IMF senior lending has clearly a negative impact on the welfare of the country: when its state of high income, it has to pay more to the lenders than if the IMF had not intervened; in the low income state, the country has no net revenues as without the IMF intervention. The lenders get less, but get more in the high state. Ex ante, they are indifferent since they adjust their lending rate, r, to get an expected rate that is equal to the risk-free rate.

To be continued

6 Conclusion

to be added
APPENDIX A

Greek Debt Crisis Timeline

The timeline below is built around the two official bailouts orchestrated for Greece. The first bailout was finalized between March and June 2010. Talk of a second bailout began in May 2011 with a formal announcement during a July 21 2011 EU Summit. It was eventually approved when euro area governments completed the ratification of a related expansion plan for the European Financial Stability Facility–EFSF, the region’s rescue fund on October 13, 2011. An EU Summit concluded on October 26 2011 agreed to further leverage the EFSF to backstop the debt of vulnerable sovereigns like Italy and Spain, to which contagion had spread in August 2011, with the European Central Bank (ECB) stepping in to buy their sovereign bonds. The timeline ends with a call for a national referendum on the bailout by the Greek Prime Minister on October 31 which was subsequently withdrawn on November 4, 2011.

1. First Bailout

In March 2010, the Greek government announced fiscal austerity measures and began discussions with the EU and IMF. On March 26, Eurozone leaders endorsed a plan to help Greece avoid default, which would include IMF assistance. At the end of the month, the Greek government warned that it might not be able to rein in the deficit unless interest rates came down. There were then three successive announcements of assistance, each involving a substantial increase in the sum of money forthcoming:

- The Eurozone announced a 45 billion package on April 12 (of which 15 bn from the IMF)
- On Monday May 3, the size of the EU-IMF package for Greece was upped to 110 bn
- Exactly a week later, a 750 billion facility, eventually named the European
Financial Stability Facility (EFSF) was announced to help Greece as well as countries vulnerable to contagion from Greece (440 billion in EU loans and guarantees; 60 bn in EU balance-of-payments support; and 250 bn from the IMF)

- The EU finalized this first bailout on June 8, 2010. Greece’s credit rating was cut to junk a week later and by the end of the year, there was little sign of improvement or sustainable debt dynamics; if anything, matters were worse as indicated by a relentless increase in bond and credit default swap (CDS) spreads even as the size of the bailout package kept increasing (see table).

2. Second Bailout

Talk of a second bailout began around May 10 2011 to keep Greece out of the debt markets at least until 2013 in the hope that fiscal credibility would be secured by then but ruling out any haircuts.

- Eventually, a July 21 2011 euro area Summit announced a second bailout for Greece with a 21 percent net present value haircut on Greek debt as well as longer maturities and lower interest rates on EFSF loans to the three program countries, Greece, Ireland and Portugal; the latter two had joined Greece as countries being bailed out. The summit also noted the need to support Italy and Spain by buying their bonds in the event of contagion.

- It took until October 13 2011 to ratify this plan.

- Concrete agreements were announced after an EU summit concluded on October 26, 2011. These included a 50 percent haircut for private Greek bondholders, a decision to leverage the EFSF to 1 trillion to support vulnerable sovereigns like Italy and Spain and to have banks attain a 9 percent capital adequacy ratio by end-June 2012.
Stock markets reacted euphorically but Greece announced and then withdrew a referendum on the bailout, while Italian 10-year bond yields approached the 7 percent threshold at which other countries had been bailed out as its political problems intensified.

Table 1 contains a timeline of key events with the accompanying bond and credit default swap (CDS) spreads for Greece; the Annex contains a more detailed timeline. From an analytical perspective, three features are curious: first, the debt situation got progressively worse based on the bond and CDS spreads in spite of growing official support; an explanation for this counterintuitive point is provided in Chamley and Pinto (2011). Second, two-year bond spreads were higher than 10-year bond spreads in spite of the bailout efforts to take Greece out of the markets until 2013, which effectively meant a guarantee of these bonds. The fact that spreads on two-year bonds did not then fall to zero indicated credibility problems with the bailout package (for example, official disbursements might be held up because Greece does not meet its fiscal targets) or concern that Greece might pull the plug on the bailout (the government might decide that that taking on all the official debt under the program, which is hard to restructure, is not in the interest of the country). Third, would a more decisive approach to the Greek problem when it was first uncovered in early 2010 have prevented the larger debt crisis which emerged after July 2011?

From the perspective of a narrative on a bailout which is clearly not working, some parallels with Russia 1998 are worth mentioning7: (a) the notion that it is worth leaning against the market because of some higher objective in Russias case, saving what was regarded as a hard-won stabilization with the 1998 default and devaluation occurring less than six months after single-digit inflation was attained; in Greeces case, saving the euro and foreign banks; (b) official refusal to accept that a haircut and debt restructuring were needed until it was too late in spite of unambiguous market signals of a high-probability and large default for Greece, a haircut was ruled out categorically until June 2011; (c) official

7For details on Russia 1998, see Kharas, Pinto and Ulatov (2001).
expressions of support and attempts to talk down the market. In the case of Russia, it was common to have G-7 leaders make periodic announcements of how Russia must not fail prior to the 1998 crisis, following it up with the assembling of a large official rescue package in June and July 1998ironically, the default and devaluation occurred just a few weeks after the package was approved with the Russians electing to pull the plug on it. Similarly, for Greece, there were various expressions of solidarity and statements ruling out debt restructuring even as news stories quoted private investors as anticipating huge losses and pricing these in; and (d) market investors quite naturally sought to avoid losses by getting the official sector either to come up with a large financing package or seeking guarantees against losses for rolling over their holdings of Greek debt.

Table 1: Key Events and Accompanying Market Signals for Greece
<table>
<thead>
<tr>
<th>Date/Event</th>
<th>2-year gond spread</th>
<th>10-year bond spread</th>
<th>CDS spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan 4 2010: Reference point</td>
<td>183</td>
<td>233</td>
<td>281</td>
</tr>
<tr>
<td><strong>First Bailout</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>March 26: Eurozone reaches accord on Greek contingency plan but at market interest rates</td>
<td>346</td>
<td>305</td>
<td>296</td>
</tr>
<tr>
<td>March 31: Greek government warns it may not be able to cut fiscal deficit if bond yields stay high</td>
<td>416</td>
<td>344</td>
<td>343</td>
</tr>
<tr>
<td>April 9: (Friday) bailout package announcement</td>
<td>593</td>
<td>399</td>
<td>426</td>
</tr>
<tr>
<td>April 12: (Monday), Eurozone announces 30 bn plus 15 bn from IMF for Greece</td>
<td>502</td>
<td>350</td>
<td>364</td>
</tr>
<tr>
<td>May 3: Monday, EU-IMF 110 bn package announced for Greece</td>
<td>947</td>
<td>544</td>
<td>NA</td>
</tr>
<tr>
<td>May 7: Friday: Contagion fears hit global markets</td>
<td>1773</td>
<td>965</td>
<td>941</td>
</tr>
<tr>
<td>May 10 2010: Monday, EU-IMF 750 billion funding facility announced to help Greece and curb contagion</td>
<td>693</td>
<td>481</td>
<td>586</td>
</tr>
<tr>
<td>June 8: EU finalizes its 440 bn rescue package</td>
<td>708</td>
<td>561</td>
<td>795</td>
</tr>
<tr>
<td>June 15: Moodys cuts Greek rating to junk</td>
<td>811</td>
<td>641</td>
<td>815</td>
</tr>
<tr>
<td>December 31 2010: Markets seem unconvinced</td>
<td>1137</td>
<td>951</td>
<td>977</td>
</tr>
<tr>
<td><strong>Second Bailout</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>May 10 2011: Fateful day. Talk of second bailout aimed at keeping Greece out of the debt markets until 2013 as well as risk of contagion spreading from Greece, Ireland and Portugal to core of euro area. Greek debt restructuring strictly off the table, focus on fiscal and privatization</td>
<td>2347</td>
<td>1231</td>
<td></td>
</tr>
<tr>
<td>May 23: signs of contagion spreading to Italy, Spain. S&amp;P moves Italian credit rating from stable to negative</td>
<td>2456</td>
<td>1402</td>
<td></td>
</tr>
<tr>
<td>June 7: Concerns expressed about ECB 444 bn exposure to periphery sovereigns and banks</td>
<td>2099</td>
<td>1280</td>
<td></td>
</tr>
<tr>
<td>June 10: ECBs Trichet rules out Greek rollover that would impose losses, German FM Schaeuble says: Participation of private creditors in cases of insolvency is indispensable.</td>
<td>2441</td>
<td>1376</td>
<td></td>
</tr>
<tr>
<td>Date</td>
<td>Event Description</td>
<td>Word Count</td>
<td>Character Count</td>
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<td>------------</td>
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</tr>
<tr>
<td>June 22</td>
<td>Greek government wins confidence vote</td>
<td>2640</td>
<td>1387</td>
</tr>
<tr>
<td>June 23</td>
<td>Chancellor Merkel tells German parliament that new program for Greece should include costs borne by private bondholders, German and Dutch banks ask for guarantees in case of rollover</td>
<td>2726</td>
<td>1401</td>
</tr>
<tr>
<td>June 30</td>
<td>Greek parliament approves austerity package plus privatization previous day</td>
<td>2512</td>
<td>1332</td>
</tr>
<tr>
<td>July 6</td>
<td>Portugal cut to junk by Moodys previous day</td>
<td>2686</td>
<td>1389</td>
</tr>
<tr>
<td>July 12</td>
<td>European FMs warned by big European banks that if buy-backs of Greek debt in the secondary market not financed, contagion would definitely spread to Spain and Italy. Finland insists on collateral for new Greek loans. Italian bonds trade at deepest discount to par since Sep 1992 ERM crisis</td>
<td>2900</td>
<td>1407</td>
</tr>
<tr>
<td>July 18</td>
<td>Italian 10-year yields reach 6% in spite of fiscal tightening plans</td>
<td>3478</td>
<td>1556</td>
</tr>
<tr>
<td>July 22</td>
<td>Day after July 21 euro area Summit announcement of second bailout for Greece extending to mid-2014 plus help for Greek banks and 21% NPV haircut for private bondholders. EFSF loans to have longer maturities and much lower interest rates. Yields fall sharply</td>
<td>2624</td>
<td>1186</td>
</tr>
<tr>
<td>Aug 8</td>
<td>Monday following Aug 5 Friday downgrade of US by S&amp;P. US yields fall!</td>
<td>3310</td>
<td>1283</td>
</tr>
<tr>
<td>Aug 24</td>
<td>Greek 2-year spreads reach historic highs</td>
<td>4331</td>
<td>1568</td>
</tr>
<tr>
<td>Sep 2</td>
<td>Greek FM says output contraction in 2011 will be 5% rather than 3.9% and fiscal deficit targets will not be met but does not accept that debt dynamics are explosive</td>
<td>4668</td>
<td>1627</td>
</tr>
<tr>
<td>Sep 19</td>
<td>Monday following indecisive meeting of Euro group FMs re implementation of July 21 summit announcements. Greek cabinet holds emergency meeting on austerity as doubts grows about next tranche disbursement under bailout. Italy downgraded by S&amp;P</td>
<td>6092</td>
<td>2122</td>
</tr>
<tr>
<td>Sep 23</td>
<td>Dutch CB head says he no longer rules out Greek default</td>
<td>6930</td>
<td>2188</td>
</tr>
<tr>
<td>Oct 7</td>
<td>German-French rift on Greece, with Germans seeing default as unavoidable and French unwilling to accept this</td>
<td>6662</td>
<td>2153</td>
</tr>
<tr>
<td>Oct 11</td>
<td>EC President Van Rompuy announces postponement of EU summit to Oct 23 to allow for comprehensive debt resolution strategy</td>
<td>7255</td>
<td>2197</td>
</tr>
</tbody>
</table>
Oct 17: Monday following G20 finance ministers and central bank heads endorsement of plan to avoid Greek default, support banks and limit contagion with Oct 23 European summit set as deadline for final plan but German FM dampens expectations. EU policymakers target at least 50% write down on Greek debt

| Oct 27: Day after EU summit (stretched to Oct 26 from Oct 23) announces comprehensive plan with 50% principal haircut on Greek debt, plan to bolster banks to achieve 9% capital adequacy and leveraging EFSF to 1 trillion to backstop vulnerable sovereigns. Markets rally hugely. |
|---|---|
| Oct 31: Greek PM announces he will seek referendum on bailout |
| Nov 4: Greek referendum idea withdrawn |
| Nov 7: Italian political problems intensify, 10-year bond yields begin to approach 7 percent threshold |

Source: Bloomberg. Bond spreads relative to German bunds. CDS spread is the five-year credit default swap spread based on New York data.

### APPENDIX B

Technical remarks following Proposition 2.
REFERENCES


