Fiscal Policy in an Expectations Driven Liquidity Trap

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Abstract

We examine the impact of fiscal policy interventions in an environment where the short term nominal interest rate is at the zero bound. In the basic New Keynesian model in which the monetary authority operates a Taylor rule, globally multiple equilibria arise, some of which display all the features of a liquidity trap. A loss in confidence can set the economy on a deflationary path that eventually prevents the monetary authority from adjusting the interest rate and can lead to potentially very large output drops. Contrary to a line of recent papers, we find that demand stimulating policies become less effective in a liquidity trap than in normal circumstances. The key reason is that demand stimulus leads agents to believe that things are even worse than they thought. In contrast, supply side policies, such as cuts in labor income taxes, lead to relative optimism and become more powerful.

Keywords: Liquidity trap, fiscal policy, sunspots, confidence shocks

JEL Classification: E3, E6, E62

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1 Introduction

By what magnitude does output increase in response to temporary changes in government purchases of goods and services or to changes in various taxes? This central question in macroeconomics has received renewed attention during the global recession of 2008-2010. Many central banks responded to the recent macroeconomic events by aggressively cutting short term interest rates. This policy led to unprecedented low levels of nominal short term interest rates and forced policy makers to reach for alternative stabilization instruments, including large fiscal policy interventions. Unfortunately, there exists little convincing empirical evidence for whether fiscal policies implemented under such conditions are especially effective or not.¹ There is even less evidence on the relative attractiveness of demand and supply oriented policies. For this reason, it is pertinent to use economic theory to shed light on the issue and this is the topic of this paper.

We examine the effects of fiscal policy when the economy is in a liquidity trap, i.e. a situation of zero nominal interest rates and depressed output levels, that is caused by a sudden loss of confidence. The analysis is cast in a New Keynesian model with price setting frictions. Monetary policy follows an interest rate rule responsive to inflation that ensures local determinacy of the equilibrium when inflation is near the target. However, because of the zero lower bound, globally there exist multiple equilibria consistent with rational expectations, as discussed by Benhabib, Schmitt-Grohé and Uribe (2001a,b, 2002). The lower bound implies a non monotonic relationship between consumption growth and expected inflation and introduces a kink in the aggregate demand schedule: it is downward sloping for sufficiently high levels of inflation, but upward sloping

¹One exception is Almunia, Bénétrix, Eichengreen, O’Rourke and Rua (2009) who find large multipliers associated with defense spending in the 1930s.
when monetary policy hits the lower bound. Because of this kink, sunspot equilibria exist in which waves of pessimistic expectations can bring the economy into a temporary liquidity trap. A loss in confidence is deflationary, sends real interest rates soaring and causes large drops in output and welfare. The temporary nature of the confidence loss causes reductions in economic activity that are much larger than in a state of more permanent deflation, as in Benhabib, Schmitt-Grohé and Uribe (2001a,b, 2002), because the expectation of a future recovery fuels intertemporal substitution and makes firms reluctant to cut prices. Interestingly, Evans, Guse and Honkapohja (2008) and Evans and Honkapohja (2009) show that the possibility of self fulfilling liquidity traps is robust to alternative assumptions about the formation of expectations. We study the global properties of the New Keynesian model rather than relying on local approximations. This is important for two reasons. First, expectations driven liquidity traps can only exist when the economy is sufficiently far away from the (usual) steady state. Second, deflationary equilibria display significant price dispersion, which is a source of persistence and inefficiency that is often assumed away in local approximations.

We show that in an expectations driven liquidity trap supply side oriented fiscal policy interventions, such as cuts in labor income taxes, are more effective in stimulating the economy than during normal times. In contrast, fiscal policies that stimulate demand, such as increased government expenditures or temporary cuts in consumption taxes, become less successful in raising output than usual. This may seem counterintuitive as the main problem in a liquidity trap is the paradox of thrift and weak demand. However, in a sunspot equilibrium a policy of demand stimulus implies, for a given aggregate supply schedule, even more deflation and hence a larger increase in the real interest rate is necessary to eliminate excess savings in equilibrium. With increased demand from
the public sector, agents’ gloomy expectations become self fulfilling only for even more pessimistic views about the future outlook of the economy. In that sense, the problem of demand stimulus in a liquidity trap driven by low confidence is that it leads agents to believe that things must be even worse than they thought. The opposite is true for policies that stimulate supply as these lead to lower real interest rates and less pessimism.

Our results contrast sharply with recent contributions of Christiano, Eichenbaum and Rebelo (2009) and Eggertson (2009), who argue that government spending multipliers may be substantially larger in liquidity traps than at positive nominal interest rates. The crucial difference with our analysis is the type of exogenous shock that drives the economy into a liquidity trap. While we consider a nonfundamental shock to expectations, these authors consider a liquidity trap caused by a fundamental shock such as a large increase in households’ preference for future consumption.

In that case, government spending raises expected future inflation which lowers real interest rates and therefore boosts private spending. Eggertson (2009) and Christiano, Eichenbaum and Rebelo (2009) show that the (marginal) spending multiplier in standard New Keynesian models can be two or three times larger in a liquidity trap. Similarly, Eggertson (2009) shows that a policy of temporary cuts in consumption taxes becomes more effective. On the other hand, Eggertson (2009) finds that cuts in labor income taxes are counterproductive in a liquidity trap as they cause lower expected inflation and higher real interest rates.

The arguments in Christiano, Eichenbaum and Rebelo (2009), Eggertson (2009), and also Wood-

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2 Cogan, Cwik, Taylor and Wieland (2009) argue in the context of the Smets and Wouters (2007) model that the increase in the spending multiplier is likely to be considerably smaller. Woodford (2010) points out that the small multiplier effects found by Cogan et al. (2009) are most likely due to different assumptions about the duration of the increase in government spending.
ford (2010) lend support to governments that have engaged in, sometimes aggressive, increases in
government spending during the current recession. Our results instead favor the policy recommenda-
tions of others, such as Bils and Klenow (2008), Hall and Woodward (2008) or Feldstein (2009)
who have proposed tax cuts to counter the recession. One of the key messages of our analysis is
that the issue of the relative merits of different fiscal policy interventions cannot be separated from
the fundamental question what type of shock has driven the economy into recession in the first
place.

An important issue is whether monetary and fiscal policies can be designed to avoid the type
of expectations driven liquidity traps that we examine. Benhabib, Schmitt-Grohé and Uribe (2002)
and Atkeson, Chari and Kehoe (2010) both propose policies aimed at eliminating expectations
regimes that eliminate unintended deflationary steady state equilibria by violating transversality
conditions. Atkeson, Chari and Kehoe (2010) propose regime switching monetary rules that en-
sure implementation of the intended competitive equilibrium by making the optimal choices of
individual agents the best responses whenever the average choice of other agents deviates from
the desired outcome. Both of these proposed policies may eliminate expectations driven liquidity
traps \textit{ex ante}.\footnote{Evans and Honkapohja (2009) show in a numerical example how an economy under adaptive learning can in principle escape a self fulfilling deflation trap through increases in government spending. In their example, they show how an increase in government spending up to 60\% of GDP manages to bring the economy on a path to the desired steady state following an initial large negative shock to expectations.} However, it is not clear whether such policies are followed in practice. At least in
the current recession, actual policies have been unsuccessful in preventing short term interest rates
monetary and/or fiscal policies implemented \textit{ex post}, i.e once the economy finds itself in a liquidity
trap. They propose rules intended to manipulate agents’ inflation expectations such as temporary deviations from the inflation target once the economy exits from the liquidity trap. As discussed by Eggertson (2009), such policies may suffer from credibility issues making them hard to implement in practice. As Christiano, Eichenbaum and Rebelo (2009) and Eggertson (2009), in this paper we study an environment where monetary policy has failed to prevent zero short term rates and we focus primarily on the effects of marginal fiscal policy interventions.

The remainder of the paper is organized as follows. Section 2 describes the model environment and discusses the existence and properties of expectations driven liquidity traps. In Section 3, we examine the impact of fiscal policies implemented during the liquidity trap. Section 4 concludes and provides some directions for future research.

2 Expectations Driven Liquidity Traps in the New Keynesian framework

2.1 The Environment

There are four types of agents in the economy: A large number of identical and infinitely lived households that consume a final good and supply labor; competitive final goods producers that transform intermediate goods into a single final good; monopolistically competitive intermediate goods firms that produce differentiated varieties using labor and set prices subject to a Calvo price setting friction; and a government that is in charge of fiscal and monetary policies.

Households

The preferences of the representative household are given by

\[ U = E_0 \sum_{t=0}^{\infty} (\omega_t \beta)^t u(c_t, l_t, m_t) \]  \hspace{1cm} (1)
where $E_t$ denotes the mathematical expectations operator conditional on all information available at date $t$, and $\beta \in (0, 1)$ is the subjective discount factor. Households derive utility from consumption of final goods, $c_t \geq 0$, leisure, $0 \leq l_t \leq 1$, and liquidity services derived from real money balances, $m_t \geq 0$. Households have a time endowment of one unit each period. $\omega_t > 0$ is a taste shock that gives rise to variations in the rate of time preference. For much of the analysis we will assume that $\omega_t = 1$ for all $t$. The instantaneous utility function is increasing and concave in all arguments. For simplicity, we assume that the utility function is additively separable between real balances and consumption and leisure,

$$u(c_t, l_t, m_t) = U(c_t, l_t) + V(m_t)$$

We also impose the following regularity conditions on preferences

$$\lim_{c \to 0^+} U_c(c, l) = \infty$$
$$\lim_{c \to \infty} U_c(c, l) = 0$$
$$\lim_{l \to 0^+} U_l(c, l) = \infty$$
$$\lim_{l \to 1^-} U_l(c, l) = 0$$
$$\lim_{m \to \infty} V_m(m) < 0$$

where we use $U_i$ for $\partial U(x)/\partial x_i$ for $x_i = c, l$ and $V_m = \partial V(m)/\partial m$. The first four conditions guarantee an interior solution for consumption and leisure choices. The last condition implies that real money demand remains finite even if short term nominal interest rates reach zero. Alternatively, one can assume that preferences display a satiation point in real money balances, see Benhabib, Schmitt-Grohé and Uribe (2002).
Households face a sequence of budget constraints,

\[
(1 + \tau_{ct}) P_t c_t + M_t + \frac{B_t}{1 + i_t} \leq (1 - \tau_{nt}) W_t (1 - l_t) + B_{t-1} + M_{t-1} + \Upsilon_t + T_t \tag{6}
\]

\[
M_{-1} \geq 0, \quad B_{-1} \geq 0 \text{ given}
\]

\(P_t\) denotes the nominal price level of the final good and \(\tau_{ct}\) is a sales tax imposed by the government on final goods purchases. Thus, \((1 + \tau_{ct}) P_t c_t\) is the nominal consumption expenditure required to purchase \(c_t\) consumption goods in period \(t\). Households can purchase a one period nominal discount bond in period \(t\) at the price \(1/(1 + i_t)\). The nominal interest rate on the bond is \(i_t\). Households earn after tax labor income \((1 - \tau_{nt}) W_t (1 - l_t)\) where \(\tau_{nt}\) is a proportional labor income tax, and \(W_t\) is the nominal wage rate. The households’ asset income is the sum of the payout on its bond portfolio, \(B_{t-1}\), its nominal cash balances, \(M_{t-1}\), and dividend income received from firm ownership, \(\Upsilon_t\). Finally, \(T_t\) are transfers received from the government.

In equilibrium, the short-term nominal interest rate needs to be nonnegative in order to guarantee the agents’ budget sets are bounded, i.e. \(i_t \geq 0, \forall t \geq 0\). Else, agents can make arbitrarily large profits by choosing arbitrarily large money holdings financed by issuing bonds.

The households face the no Ponzi constraints

\[
\lim_{s \to +\infty} E_t \frac{B_{t+s}}{(1 + i_t) \cdots (1 + i_{t+s})} \geq 0 \tag{7}
\]

The household’s problem is to maximize utility in (1) subject to the nonnegativity and time endowment constraints, the budget constraints, initial asset positions in (6), and the condition in (7). The
The optimality conditions include

\[
\frac{U_t(c_t, l_t)}{U_c(c_t, l_t)} = \frac{(1 - \tau_{n,t}) W_t}{(1 + \tau_{c,t}) P_t} \tag{8}
\]

\[
U_c(c_t, l_t) = \beta(1 + i_t) E_t \left[ \frac{\omega_{t+1}}{\omega_t} \frac{(1 + \tau_{c,t}) P_t}{(1 + \tau_{c,t+1}) P_{t+1}} U_c(c_{t+1}, l_{t+1}) \right] \tag{9}
\]

\[
\frac{V_{m}(m_t)}{U_c(c_t, l_t)} = \frac{i_t}{1 + i_t (1 + \tau_{c,t})} \tag{10}
\]

Equation (8) equates the marginal rate of substitution between consumption and leisure to the after-tax consumption real wage (the nominal wage relative to the price of consumption goods including sales taxes). Equation (9) implies that the expected intertemporal marginal rate of substitution of consumption equals the expected consumption based real interest rate. Equation (10) defines money demand and sets the marginal rate of substitution between consumption and real balances equal to the user cost of money corrected for the wedge introduced by the sales tax. Besides satisfying all constraints and conditions (8)-(10), optimal decisions must also obey the transversality condition

\[
\lim_{s \to \infty} E_t \left[ \frac{B_{t+s} + M_{t+s}}{(1 + i_t) \cdots (1 + i_{t+s})} \right] = 0 \tag{11}
\]

**Final Goods Sector** There is a competitive sector of final goods producers. Final goods are produced by aggregating a continuum of intermediate goods through a constant elasticity of substitution (CES) technology. The production function of the representative final goods producer is given by

\[
y_t = \left( \int_0^1 y_u^{1-1/\eta} du \right)^{1/(1-1/\eta)} \tag{12}
\]
where $\eta > 1$ is the elasticity of substitution between the intermediate goods and $y_{it}$ is the input of intermediate good of variety $i$. Cost minimization yields demand functions for variety $i$:

$$y_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\eta} y_t$$  \hspace{1cm} (13)

where $P_{it}$ is the date $t$ price of intermediate good of variety $i$. $P_t$ is the price of the final good and is given by

$$P_t = \left( \int_0^1 P_{it}^{1-\eta} di \right)^{1/(1-\eta)}$$  \hspace{1cm} (14)

The final goods are used either for private or government consumption. Thus, the economy wide resource constraint is

$$y_t \geq c_t + g_t$$

where $g_t$ denotes government purchases of the final good.

**Intermediate Goods Sector** There is a continuum of monopolistically competitive intermediate goods producers. Intermediate goods are produced using labor through the linear technology

$$y_{it} = n_{it}$$  \hspace{1cm} (15)

where $n_{it}$ denotes intermediate producer $i$'s use of labor services. Intermediate producers set the price of their good, $P_{it}$, and satisfy all demand at this price. They set prices subject to the Calvo friction of staggered price setting: Each period, whether the firm can reset the price is determined by a Poisson process with arrival rate $(1 - \xi) \in (0, 1]$. The problem of firm $i$ after receiving the
opportunity to reset the price in period $t$ is to choose a new price $P_{it}^*$ to maximize

$$E_t \sum_{s=t}^{\infty} \xi^{s-t} Q_{t,s} \Upsilon_{is}(P_{it}^*)$$ (16)

subject to the demand functions in equation (13). $Q_{t,s} = \beta^{t-s} (U_c(c_s,l_s)/U_c(c_t,l_t))(P_t/P_s)$ is the discount factor between period $t$ and $s$ and $\Upsilon_{is}(P_{it}^*)$ are period $s$ profits of firm $i$ when charging $P_{it}^*$, which are given by

$$\Upsilon_{is}(P_{it}^*) = (P_{it}^* - (1 - \tau_r)W_s) \left( \frac{P_{it}^*}{P_s} \right)^{-\eta} y_s$$ (17)

$\tau_r$ is a proportional employment cost subsidy that firms receive from the government. We include the subsidy to eliminate the distortion stemming from monopoly pricing and henceforth we assume $\tau_r = 1/\eta$.

The first-order condition for $P_{it}^*$ can be expressed as:

$$E_t \sum_{s=t}^{\infty} \xi^{s-t} Q_{t,s} [(P_{it}^* - W_s)\Upsilon_{is}] = 0$$ (18)

Since all firms that are given the chance to reset the price of their good have the same marginal costs, face the same demand functions, and have the same future prospects of being able to reoptimize, they all set the same price, $P_{it}^*$, which is the solution to equation (18). Consequently, using the law of large numbers we can express the aggregate price index as

$$P_t^{1-\eta} = \xi P_{t-1}^{1-\eta} + (1 - \xi) P_t^{*1-\eta}$$ (19)
Equalizing supply and demand for intermediate good \( i \) and aggregating across firms implies that

\[
\int_0^1 n_i \, di = \int_0^1 \left( \frac{P_i}{P_t} \right)^{-\eta} \, y_i \, di
\]

It follows that aggregate output can be expressed as:

\[
y_t = v_t^{-1} n_t \tag{20}
\]

where \( n_t = \int_0^1 n_i \, di \). In this equation the variable \( v_t = \int_0^1 (P_i/P_t)^{-\eta} \, di \) is a price dispersion term that is determined recursively as

\[
v_t = \xi \pi_t \, v_{t-1} + (1 - \xi) \, p_t^{\ast -\eta} \tag{21}
\]

where \( \pi_t = P_t/P_{t-1} \) is inflation and \( p_t^{\ast} = P_t^{\ast}/P_t \) is the optimal reset price relative to the general price level. From equation (20) it is clear that price dispersion acts like an inefficiency wedge that arises because firms charge different prices in equilibrium due to the price setting friction. One property of the price dispersion term is that its minimum value is one, \( v_t \geq 1 \). The minimum is reached when either prices are fully flexible or in equilibria in which the price level is constant.

**Government**  The government is in charge of monetary and fiscal policies. We specify monetary policy by an interest rate rule

\[
1 + i_t = \phi \left( \frac{\pi_t}{\bar{\pi}} \right) \tag{22}
\]

where \( \bar{\pi} \geq 1 \) is the inflation target. We assume that \( \phi(1) = \beta^{-1} \bar{\pi} \) and that \( \phi(\cdot) \geq 1 \) for all \( \pi_t \) such that the nominal interest rate always satisfies the zero bound on interest rates, and that \( \phi'(\cdot) \)
is sufficiently large when \( i_t > 0 \) to satisfy the Taylor principle, ensuring local determinacy in the neighborhood of the intended level of inflation.\(^4\) The monetary authority sets the nominal money stock to implement the inflation feedback rule for the nominal interest rate. Below a critical value of \( \pi_t \), the monetary authority implements a zero nominal interest rate. Although zero is a lower bound on interest rates, in practice central banks may also abandon the interest rate rule at strictly positive interest rates, but this is not important for our analysis. The fact that (22) does not include the output gap, expected inflation or other observable macroeconomic variables is also not important. What is important is that we ignore possible unconventional monetary policy measures at the lower bound in order to focus on fiscal policy.

Fiscal policy involves a choice of taxes, government spending, and debt. The government’s budget constraint is given as

\[
\frac{B_t}{1 + i_t} = B_{t-1} - M_t + M_{t-1} + D_t
\]

(23)

where \( D_t \) is the deficit in period \( t \)

\[
D_t = P_t g_t + T_t + \frac{1}{\eta} W_t n_t - (\tau_{c,t} P_t c_t + \tau_{n,t} W_t (1 - l_t))
\]

(24)

Below we examine fiscal policies that are consistent with the government budget constraint. Unless mentioned otherwise, we assume that policies are Ricardian, in the sense that they always satisfy equation (11).

\(^4\)As shown by Coibion and Gordonichenko (2009), at positive trend inflation levels \( \pi > 1 \) local determinacy may require that the central bank raises interest rates more than one for one with inflation.
2.2 Equilibrium Analysis

Let \( w_t = W_t/P_t, b_t = B_t/P_t, t_t = T_t/P_t, d_t = D_t/P_t. \)

**Equilibrium Definition**  A competitive rational expectations equilibrium is a sequence of allocations \((c_t, n_t, l_t, y_t)_{t=0}^{\infty}\), a price system \((\pi_t, w_t, p_t, v_t)_{t=0}^{\infty}\), monetary policies \((i_t, m_t)_{t=0}^{\infty}\), and fiscal policies \((b_t, d_t, g_t, \tau_{c,t}, \tau_{n,t}, t_t)_{t=0}^{\infty}\) such that (i) households maximize utility subject to all constraint, (ii) final goods producers maximize profits, (iii) intermediate goods producers maximize profits, (iv) monetary policy is guided by the interest rate rule, (v) fiscal policies are consistent with the government budget constraint, and (vi) goods, asset and labor markets clear, for given initial conditions \( b_{-1}, m_{-1} \geq 0 \) and \( v_{-1} \geq 1 \), a law of motion for \( \omega_t \) and specifications of fiscal policies.

Market clearing requires

\[
\begin{align*}
    n_t &= 1 - l_t \\
    y_t &= c_t + g_t
\end{align*}
\]

(25)  (26)

For a given specification of fiscal policies and a law of motion of \( \omega_t \), equilibrium sequences for output, inflation and price dispersion \((y_t, \pi_t, v_t)_{t=0}^{\infty}\) are solutions to the following system of nonlinear
stochastic difference equations:

\[ 1 = \beta \phi \left( \frac{\pi_t}{\pi} \right) E_t \left[ \frac{\omega_t + 1}{\omega_t} \frac{(1 + \tau_{e,t})}{\pi_{t+1}} \frac{U_c(y_{t+1} - g_{t+1}, 1 - v_{t+1}y_{t+1})}{U_c(y_t - g_t, 1 - v_ty_t)} \right] \quad (27) \]

\[ p_t^* \pi_t = \frac{E_t \sum_{s=t}^{\infty} (\beta \xi)^{s-t} \omega_s U_{t}(y_s - g_s, 1 - v_s y_s) \left( \prod_{j=0}^{s-t} \pi_{t+j} \right)^{\eta - 1} y_s}{E_t \sum_{s=t}^{\infty} (\beta \xi)^{s-t} \omega_s U_{t}(y_s - g_s, 1 - v_s y_s) \left( \prod_{j=0}^{s-t} \pi_{t+j} \right)^{\eta - 1} y_s} \quad (28) \]

\[ v_t = \xi \pi_t^{\eta n} v_{t-1} + (1 - \xi) p_t^* n \quad (29) \]

for a given initial condition \( v_{-1} \), where \( p_t^* \) is implicitly determined by

\[ 1 = \xi \pi_t^{\eta n - 1} + (1 - \xi) p_t^{* n - 1} \quad (30) \]

Equation (27) is the equilibrium version of the agent's intertemporal Euler equation combined with the interest rate rule and with the equilibrium condition in equation (20). Equation (28) is the equilibrium version of the optimality condition for the optimal reset price. Equation (29) is the law of motion for the degree of price dispersion.

In order to facilitate the global analysis we focus exclusively on Markovian equilibria which can be generated from recursion of a state space system of the form

\[ u_t = f(s_t) \quad (31) \]

\[ s_t = h(s_{t-1}) + \mu \varepsilon_t, \; s_0 \text{ given} \quad (32) \]

where \( s_t \) denotes the vector of state variables of the economy, \( u_t \) is the inflation/output vector, \( \varepsilon_t \) contains random innovations to any exogenous stochastic processes in the state vector and \( \mu \) is an
appropriate selection vector. In terms of the model, the state variables include price dispersion and the exogenous forcing process for \( \omega_t \) when it is active. Throughout this paper, we preserve the nonlinear nature of the functions \( f(\cdot) \) and \( h(\cdot) \) that solve the system of equations in (27)-(29). The focus on the global dynamics of output and inflation contrasts with most of the literature that typically studies local approximations based on perturbations of the equilibrium conditions around a deterministic steady state (see Wolman (2005), Evans, Guse and Honkapohja (2008) for exceptions). The global analysis allows us to analyze equilibrium behavior for which the dynamics of the economy is very different from that in the neighborhood of the usual point of approximation.

It is well known at least since Sargent and Wallace (1975) that under an interest rate rules rational expectations monetary models can display equilibrium indeterminacy. More recently, Atkeson, Chari and Kehoe (2010) show that the Taylor principle is neither necessary nor sufficient for uniqueness. The equilibria we study in this paper are closely related to Benhabib, Schmitt-Grohe and Uribe (2002). There are however three main differences: First, we analyze liquidity traps with stochastic duration instead of permanent liquidity traps in a perfect foresight context. Second, we look at a production economy instead of an endowment economy. Finally, we assume nominal rigidities instead of perfect price flexibility.

**Steady States** We begin by studying the steady state properties of a deterministic version of the model. We set \( \omega_t = 1 \) for all \( t \) and, for simplicity, ignore fiscal policy for now and set \( g_t = \tau_{c,t} = \tau_{n,t} = 0 \) for all \( t \geq 0 \). By steady state \((s, u)\) here we mean a fixed point such that \( s = h(s) \) and \( u = f(s) \). As discussed by Benhabib, Schmitt-Grohé and Uribe (2001a,b, 2002), when fiscal policy is Ricardian and monetary policy follows an interest rate rule subject to a lower
bound, there generally exist two different steady states.\textsuperscript{5} This can be seen from the Euler equation in (27) which requires that the steady state real interest rate equals $1/\beta$ in order for consumption to be constant. The lower bound on the interest rate implies that this condition can hold for two different combinations of nominal interest rates and inflation.

The first steady state, which we will refer to as the intended steady state $(\pi^I, y^I, v^I)$, has inflation at the target level $\pi^I = \tilde{\pi}$ and a positive nominal interest rate. Output and inflation are implicitly determined by

$$\frac{U_l(y^I, 1 - v^I y^I)}{U_c(y^I, 1 - v^I y^I)} = \frac{1 - \xi \beta \tilde{\pi} \eta}{1 - \xi \beta \tilde{\pi} \eta - 1} \left( \frac{1 - \xi}{1 - \xi \tilde{\pi} \eta - 1} \right)^{\frac{1}{\eta}}, \quad \nu^I = \frac{1 - \xi}{1 - \xi \tilde{\pi} \eta - 1} \left( \frac{1 - \xi \tilde{\pi} \eta - 1}{1 - \xi \tilde{\pi} \eta - 1} \right)^{\frac{1}{\eta}} \quad (33)$$

For most of the analysis, we set $\tilde{\pi} = 1$ so that the government has a zero inflation target. In that case, there is no price dispersion in the intended steady state, $v^I = 1$. Intuitively, since the price level is constant, all firms that get the opportunity to reset the price of their commodity set a relative price of one and therefore the price distribution is degenerate. This also implies that $y^I = y^E$ where $y^E$ equals the efficient (flexible price) level of output determined implicitly by the condition $U_l(y^E, 1 - y^E) / U_c(y^E, 1 - y^E) = 1$. The intended steady state with a zero inflation target therefore acts as a useful welfare benchmark when welfare effects of real money holdings are ignored.

There exists a second, unintended, steady state $(\pi^U, y^U, v^U)$, in which the nominal interest rate is at the lower bound, i.e. $\phi(\pi / \tilde{\pi}) = 1$, and there is deflation $\pi^U = \beta$. As in the intended steady

\textsuperscript{5}In our model with endogenous labor supply, more than two steady states can exist depending on the properties of labor supply. Because this is not the mechanism that generates multiple equilibria in this paper, we ignore this possibility. To rule out more than two steady states, it suffices to assume preferences displaying either additive separability or Edgeworth complementarity of consumption and leisure.
state, the real interest rate equals $1/\beta$. With declining price levels, intermediate goods producers that can reset the price of their good set a relative price below unity, $p^{*U} < 1$. The unintended steady state output and price dispersion is implicitly given by

$$\frac{U_l(y^U, 1 - v^U y^U)}{U_c(y^U, 1 - v^U y^U)} = \frac{1 - \xi \beta^{1+\eta}}{1 - \xi \beta^{\eta}} \left( \frac{1 - \xi}{1 - \xi \beta^{\eta-1}} \right)^{\frac{1}{1 - \xi}} \ , \ v^U = \frac{1 - \xi}{1 - \xi \beta^{\eta}} \left( \frac{1 - \xi \beta^{\eta-1}}{1 - \xi} \right)^{\frac{\eta}{1 - \xi}}$$

(34)

One property of the unintended steady state is that output differs from the efficient output level, $y^U \neq y^E$. Price dispersion drives a wedge between the marginal utility of consumption and leisure and $U_l(y^U, 1 - v^U y^U) < U_c(y^U, 1 - v^U y^U)$. If labor and leisure are complements, this translates to labor supply and output levels below the efficient level. However, as we will show below, the discrepancy $y^U - y^E$ tends to be small relative to how far output can be below the efficient level in a temporary liquidity trap.

Intuitively, under the assumptions of Ricardian fiscal policies and the simple interest rate rule, there is no unique rational expectation that is consistent with a single steady state outcome. There is an optimistic expectation, according to which the level of inflation is as intended by the central bank and expected lifetime wealth is high. The optimistic expectation leads households to choose a high level of consumption and labor supply. But there is also a pessimistic expectation, according to which there is deflation and lifetime wealth is lower. The pessimistic expectation leads households to choose a lower level of consumption. The lower consumption choice based on pessimistic expectations is rational because deflation generates price dispersion, lowering labor supply and lifetime wealth. In both steady states, real interest rates are identical, but there are two levels of inflation that can ensure that national savings is zero in equilibrium.
The existence of the unintended steady state is due to the zero lower bound on interest rates, as pointed by Benhabib, Schmitt-Grohé and Uribe (2002): the central bank cannot prevent market clearing at deflationary levels, because it lacks the ability to generate a real interest rate that would be inconsistent with zero national savings at deflationary levels. The reason output and consumption are at inefficient levels in the unintended steady state is because of the nominal price setting friction and price dispersion. With flexible prices, or automatic indexation to lagged inflation, the unintended steady state still exists but output and consumption are at efficient levels. Abstracting from utility derived from real money balances, the only potential source of welfare loss from being in a perpetual liquidity trap comes from the effect of price dispersion.

**Sunspot Equilibria** The multiplicity of steady states is a strong indicator for the existence of sunspot equilibria. In a sunspot equilibrium, rational agents condition their expectations on an information set that contains a random variable that otherwise has no impact on fundamentals. This random variable is often referred to as a sunspot (see Shell (1977) and Cass and Shell (1983)) and we interpret it as a variable formally measuring exogenous variation in confidence or sentiment. We denote this confidence variable by $\psi_t$ and assume it evolves according to a $n$-state discrete Markov chain, $\psi_t \in [\psi_1, \ldots, \psi_n]$. Equilibrium dynamics are still described by the system of the form (31)-(32), but the vector of state variables contains the confidence variable $\psi_t$, i.e. $s_t = [v_{t-1}, \omega_t, \psi_t]$.

---

6 This is because sunspot equilibria usually exist near distinct steady states. Eliminating the second steady state, for instance by targeting deflation $\pi = \beta$, however is not sufficient to rule out sunspot fluctuations. We analyze sunspot equilibria that are generally far away from the steady states.

7 See Benhabib and Farmer (2000) for an excellent survey of macroeconomic models with indeterminacy and sunspot equilibria.
Formally, a Markov sunspot equilibrium is a Markov competitive equilibrium defined by a pair of functions $f(s_t)$ and $h(s_t)$ for which $f([v_{t-1}, \omega_t, \psi_t = \psi_i]) \neq f([v_{t-1}, \omega_t, \psi_t = \psi_j])$ and $h([v_{t-1}, \omega_t, \psi_t = \psi_i]) \neq h([v_{t-1}, \omega_t, \psi_t = \psi_j])$ for $i \neq j$, where $i, j = 1, \ldots, n$. Therefore, output and inflation are stochastic processes whose values depend on the realization of the variable $\psi_t$.

In the New Keynesian model, stochastic fluctuations in confidence allow for temporary liquidity traps that may involve output drops that far exceed the difference between intended and unintended steady state levels of output. The main reason is that the real interest rate must always adjust to ensure market clearing. Suppose agents grow pessimistic and expect a temporary but persistent drop in income leading to lower desired consumption. The nominal friction and market clearing require a reduction in output and a fall in prices. The fact that the wave of pessimism is temporary has two important implications: it makes the price setters more reluctant to cut prices, as they are considering the profit impact of their decision in all states of the world including a recovery. Furthermore, with constant short term nominal interest rates a fall in prices leads to a temporary increase in the real rate, triggering intertemporal substitution effects. A temporary rise in interest rates makes current consumption more expensive relative to future consumption and leads to an increased desire to save. Because saving must be zero in equilibrium, this requires a further drop in output and stronger price declines, which again increase real interest rates and lower consumption, etc. This downward spiral ends when output and wealth have fallen sufficiently to discourage saving and the real interest rate equates consumption to output. Because of the lower output level, the initial loss of confidence becomes a self fulfilling prophecy consistent with rational expectations. Locally, the monetary authority can prevent this downwards spiralling savings glut by lowering nominal interest rates sufficiently to offset the real rate increases. Globally, however, it is unable
to do so because of the zero lower bound.

**A Two State Example**  Suppose the sunspot variable $\psi_t$ follows a two state Markov chain with transition matrix $R$,

$$\psi_t \in [\psi_O, \psi_P], \quad R = \begin{bmatrix} 1 & 0 \\ 1 - q & q \end{bmatrix}, \quad 0 < q < 1$$  \hspace{1cm} (35)

The first state, $\psi_O$, is the intended state where (relative) optimism prevails. The second state, $\psi_P$, is characterized by pessimism. Once in a wave of pessimism, the probability of continued pessimism in the next period is given by the parameter $q$. If pessimism switches to optimism, a return to pessimism cannot occur. We focus on this two state case with one absorbing state to simplify the intuition as much as possible and to keep the number of new parameters to a minimum. It also facilitates comparison with Christiano, Eichenbaum and Rebelo (2009), Eggertson (2009) and Woodford (2010), whose liquidity trap inducing shock has the exact same stochastic properties. The transition matrix $R$ implies that, once optimism arrives, the economy converges to the intended steady state values of output and inflation, $y^I$ and $\pi^I$, which are also the efficient levels if the inflation target is zero. It is straightforward to allow for transitions from the optimistic state to the pessimistic state, in which case output and inflation in the optimistic state no longer converge to the intended steady state.

It is useful to provide a graphical representation of the equilibrium dynamics. For now, we maintain the assumptions that $\omega_t = 1$ for all $t$ and that there is no fiscal policy besides the constant employment subsidy that corrects for the monopolistic distortion, i.e. $g_t = \tau_{n,t} = \tau_{c,t} = 0$ for all
Let \( \pi, y \) and \( v \) denote the fixed points of the system defined by \( f([v_{t-1}, \psi_t = \psi_P]) \) and \( h([v_{t-1}, \psi_t = \psi_P]) \). These are the values of inflation, output and price dispersion to which the economy converges conditional on the realization \( \psi_P \) and \( \psi_O \). Furthermore, let \( \pi^O, y^O \) and \( v^O \) denote the values obtained from evaluating \( f([v_P, \psi_t = \psi_O]) \) and \( h([v_P, \psi_t = \psi_O]) \). These are the values of inflation, output and price dispersion immediately after returning to optimism from the pessimistic state \([v_P, \psi_P] \).

Evaluating equations (27) and (28) at the point \( \pi_P, y_P \) and \( v_P \) yields:

\[
U_c(y_P, 1 - v_P y_P) = \beta \phi \left( \frac{\pi_P}{\pi} \right) \left[ \frac{q}{\pi_P} U_c(y_P, 1 - v_P y_P) + \frac{1 - q}{\pi_O} U_c(y^O, 1 - v^O y^O) \right]
\]

where \( 0 < \Lambda_P < 1 \) is still a complicated function of expectations of future inflation and output levels and \( p^O_P, \pi^O, y^O, v^O \) as well as \( \Lambda_P \) are functions of \( v_P \) and that \( v_P \) is related to \( \pi_P \) through \((1 - \xi \pi_P \eta) v_P = (1 - \xi) p^O P \pi_P\), the above equations describe more generally two relationships between inflation and output that can be graphed in the two dimensional plane. We will refer to these relationships as \((\pi, y)^{EE}\) and \((\pi, y)^{AS}\), respectively, where \((\pi, y)^{EE}\) are the combinations of \( \pi \) and \( y \) that are consistent with the equilibrium Euler condition (36) and \((\pi, y)^{AS}\) are those consistent with the supply relationship (37). Any intersection describes possible limit points to which the economy may converge as it evolves while in the pessimistic state \( \psi_P \).

Figure 1 depicts two possible cases that can arise for identical preference and policy parameters, but different values of the parameter \( q \). There is always an intersection that corresponds to \( y^f \) and \( \pi^f \), i.e. the output and inflation levels at the intended steady state. This cannot be a limit point of
a sunspot equilibrium, since the equilibrium outcomes in that case are identical across realizations of $\psi_t$. For a wide range of values of the parameter $q$, there exists a second intersection at $\pi_P$ and $y_P$ that is characterized by deflation and zero nominal interest rates. The reason is that the $(\pi, y)^{EE}$ schedule implies a downward sloping relationship between output and inflation for sufficiently high levels of inflation, but becomes upward sloping for levels of inflation for which the constraint on monetary policy is binding.

The left panel of Figure 1 depicts a situation for a specific value of $q$ for which the intended steady state is the only limit point. There is no sunspot equilibrium for a confidence variable $\psi_t$ with persistence equal to that particular value of $q$. The right panel of Figure 1 shows a case with a different value of $q$ for which a second limit point does exist. As $q$ tends to zero and the wave of pessimism is expected to be shorter lived, the upward sloping part of the $(\pi, y)^{EE}$ becomes steeper, while the $(\pi, y)^{AS}$ curve becomes flatter. For low enough values of $q$, a second intersection does not exist. Intuitively, if the duration of the pessimistic state is short, agents cannot rationally expect a sufficient amount of deflation that would lead to a binding lower bound on the short term nominal interest rate. For $q$ larger than a certain critical value, i.e. if pessimism is expected to persist long enough, the equilibrium conditions support a sunspot limit point, which will be characterized by depressed output levels, deflation and zero nominal interest rates. For $q \to 1$, the second intersection converges to the unintended steady state $\pi^U$ and $y^U$.

At any second intersection, the $(\pi, y)^{AS}$ curve is steeper than the $(\pi, y)^{EE}$ curve, which is a necessary condition for the existence of the sunspot equilibria we study. Interestingly, this condition on the relative slopes of $(\pi, y)^{AS}$ and $(\pi, y)^{EE}$ is the exact opposite of the parameter restrictions of
Christiano, Eichenbaum and Rebelo (2009), Eggertson (2009) and Woodford (2010) required to
generate a liquidity trap outcome after a discount factor or interest rate spread shock.\(^8\) Suppose
that there is a shock to preferences \(\omega_t\) that evolves according to a Markov process with transition
matrix \(R\) in (35). The main effect of such a shock is to shift the \((\pi, y)^{EE}\) schedule to the left. If
the shock is large enough and its persistence \(q\) is sufficiently low, it can generate a liquidity trap in
equilibrium, as illustrated in the left panel of Figure 2. In such a liquidity trap, the \((\pi, y)^{EE}\) curve
must be steeper then the \((\pi, y)^{AS}\) curve. However, if the expected duration of the regime with the
high value of \(\omega_t\) is too long, as in the right panel of Figure 2, a liquidity trap cannot arise. The
difference in slopes of the two schedules between the right panel of Figure 1 and the left panel of
Figure 2 is the reason why, as we discuss below, policy interventions lead to different outcomes
depending on the type of shock, fundamental of nonfundamental, generating the liquidity trap.

**Sunspots in a Calibrated Model** The analysis of limit points in the pessimistic state of a sunspot
equilibrium ignores the transitional dynamics which occur because of price dispersion. In order to
present the full dynamics surrounding expectations driven liquidity traps, consider the following
functional forms for preferences and the monetary policy rule:

\[
U(c_t, l_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \theta \frac{l_t^{1-\kappa} - 1}{1-\kappa}, \quad \sigma, \theta, \kappa > 0
\]

\[
\phi\left(\frac{\pi_t}{\bar{\pi}}\right) = \max\left(\frac{\pi_t^{\phi_\pi}}{\beta}, 1\right), \quad \phi_\pi > 1
\]

Preferences over consumption and leisure are additively separable. The policy rule in (39) implies
a zero inflation target and the restriction \(\phi_\pi > 1\) guarantees local determinacy in the neighborhood

\(^8\)Note that both types of liquidity traps can always emerge for identical preference, policy and technology parame-
ters when the sunspot and discount factor shocks have different persistence.
of the intended steady state. The max operator ensures that the short term interest rate implied by the rule satisfies the zero lower bound. We continue to assume that there are no fundamental sources of uncertainty, i.e. \( \omega_t = 1 \) for all \( t \). We set the parameter values to \( \beta = 0.99, \kappa = 2.65, \sigma = 1, \eta = 10, \phi = 1.5 \) and \( \xi = 0.65 \). The value for \( \beta \) implies an annual real interest rate in the intended steady state of 4 percent. The value for \( \xi \) is such that firms are able to adjust prices approximately once every three quarters, which is in line with the micro evidence of for instance Nakamura and Steinsson (2008). The value of \( \eta = 10 \) translates to a markup of 11 percent in the intended steady state. We set \( \theta \) such that in the intended steady state households spend 30 percent of their time endowment working. The value of \( \kappa \) implies a Frisch elasticity of around 0.75, which is in upper end of the range deemed realistic by labor economists. The value of \( \sigma \) implies an elasticity of intertemporal substitution of unity.

Figure 3 depicts the equilibrium path in a two state sunspot equilibrium with transition matrix given in (35) with \( q = 0.80 \). At date 0, the economy is in the pessimistic state displaying a liquidity trap. The economy regains confidence and moves to positive interest rates at (the stochastic) date \( T \). The initial distribution of prices is characterized by \( v_{-1} = 1 \). The black horizontal lines denote the equilibrium values at the intended steady state (full lines) and the unintended deflationary steady state (broken lines). Output in the upper left panel of Figure 3 is plotted in percentage deviation of the intended steady state level, which also corresponds to the efficient level because of the zero inflation target. The other panels display the annual inflation rate, the level of price dispersion, and the annual level of short term nominal interest rate. Starting from the initial state

\footnote{The functions \( h(\cdot) \) and \( f(\cdot) \) are approximated numerically by piecewise linear functions obtained from time iteration of a recursive version of the system in (27)-(29). Different equilibria (for a fixed value of \( q \)) can be found by varying the starting point in the iterations appropriately. Matlab programs are available on the authors’ website.}
\((v_{-1}, \psi_P)\), output and inflation converge to the sunspot limit point. As long as deflation persists, the distribution of prices becomes more dispersed, increasing the inefficiency wedge in the labor market. However, the transitional dynamics that arise because of gradual changes in price dispersion turn out to be relatively unimportant. Until pessimism turns to optimism at date \(T\), output and consumption are about 0.9% below the efficient level, whereas the annual rate of inflation is 6% below the (zero) target.

Figure 4 displays equilibrium paths for a different calibration where \(\sigma = 0.7\) and \(\xi = 0.82\) (the other parameters remain the same). Hence, consumption is more interest rate elastic, which flattens the \((\pi, y)^{EE}\) schedule. The lower value of \(\sigma\) also makes marginal cost less elastic with respect to output and the higher value of \(\xi\) implies more rigid prices, both of which flatten the \((\pi, y)^{AS}\) schedule. In this case, there is a 6% gap with the efficient level of output, whereas the annual rate of inflation is 8.5% below the target. For other parameter values the output loss in a liquidity trap can be even larger. The largest deviations from the intended steady state are obtained when the \((\pi, y)^{AS}\) and \((\pi, y)^{EE}\) curves are similar in slope.\(^{10}\) If that is the case, large adjustments are required before the downward spiral triggered by the loss in confidence ends.

**Sensitivity Analysis** Figure 5 plots the levels of output and inflation in the pessimistic state of the sunspot equilibrium for different parameter values. We center the sensitivity analysis around the parameter values of the first calibrated example, i.e. \(\sigma = 1\), and \(\xi = 0.65\). We choose these values to convince the reader that sunspot equilibria exist for a wide range of parameters that are in line with standard calibrations of the New Keynesian model. The other parameters are the same as

\(^{10}\)This also true when the liquidity trap is generated by a fundamental shock, as in Christiano, et al. (2009) and Eggertson (2009). The key difference is that here the slope of the AS curve needs to approach the slope of the EE curve from below, and not vice versa.
before. In the figure, the squares denote the benchmark parameter values. Given that transitional
dynamics are relatively unimportant, the graphs display output and inflation levels in the sunspot
limit points obtained from the system in (36)-(37).

The first row of Figure 5 shows how the output drop becomes larger for lower values of $\sigma$. A
lower value of $\sigma$ flattens the $(\pi, y)^{EE}$ curve and strengthens intertemporal substitution in response
to price declines. It also makes marginal cost less elastic with respect to output and hence flattens
the $(\pi, y)^{AS}$ curve. Therefore, a lower value of $\sigma$ implies a larger fall in income and a more modest increase in the real interest rate. The second row of Figure 5 illustrates how output losses are
larger when the degree of price stickiness increases and the $(\pi, y)^{AS}$ curve becomes flatter. When
firms expect to be able to reset prices in the future with lower probability, they are less willing to accommodate demand through reductions in their current price and instead reduce production. But this requires stronger deflation and real interest increases to equate consumption with output. As the value of $\xi$ increases, the slope of the $(\pi, y)^{AS}$ curve approaches the slope of the $(\pi, y)^{EE}$ curve and output drops grow very large. Above a critical value of $\xi$ (in this case approximately 0.82), there does not exist a sunspot limit point. As $\xi$ approaches zero, output converges to the flexible price efficient output level, $y_P \to y^E$, while inflation approaches $\pi_p \to \beta q/(1 - \beta(1 - q))$.

The third row of Figure 5 shows the range of the persistence parameter $q$ for which a sunspot
equilibrium exists. In this case the critical value of $q$ is approximately 0.56. The longer pessimism
is expected to prevail, the smaller are the output losses and levels of deflation. As explained before, the temporary nature of the confidence crisis creates the motive for intertemporal substitution and introduces a reluctance of firms to cut prices. Higher values of $q$ flatten the $(\pi, y)^{EE}$ curve and
steepen the \((\pi, y)^{AS}\) curve. For \(q \to 1\), inflation and output levels in a liquidity trap approach the levels of the unintended steady state, \(y^U\) and \(\pi^U\). Finally, the last row in Figure 5 shows the effect of different elasticities of labor supply. A higher Frisch elasticity (lower \(\kappa\)) flattens the \((\pi, y)^{AS}\) curve as marginal cost becomes less elastic with respect to output and therefore leads to stronger declines in output and inflation.

These numerical examples illustrate that the conditions for the existence of expectations driven liquidity traps are realistic. Roughly speaking, the requirement is that expectations are such that the \((\pi, y)^{EE}\) curve in (36) is flatter than the \((\pi, y)^{AS}\) curve in (37), which is the case for a wide range of plausible parameter values. Temporary expectations driven liquidity traps are also very likely to exist in more complicated monetary models. The fact that the monetary authority operates an interest rate target subject to the zero bound is why self fulfilling liquidity traps can occur. If there exists an inflation-output trade off, a liquidity trap induced by a loss in confidence will automatically be associated with potentially very large drops in output and welfare.

We conduct one last experiment before turning to the effects of fiscal policy interventions and examine how liquidity trap outcomes depend on the numerical inflation target adopted by the monetary authority. The possibility of reaching the zero bound is a widely used argument for adopting strictly positive numerical targets for inflation. Some economists, such as Blanchard, Dell’Ariccia and Mauro (2010), have suggested raising targets to alleviate constraints on monetary policy in the future. In the context of our model, a permanently higher inflation target has several effects. Figure 6 depicts inflation and output (in percent deviation of the efficient level) in the intended steady state as well as in a liquidity trap for different values of the inflation target \(\tilde{\pi}\). Larger deviations
from a nonzero target generate more price dispersion, which increases the inefficiency wedge in
the intended steady state and lowers output relative to the efficient level. In a liquidity trap, a higher
inflation target makes price setters more reluctant to cut prices and therefore flattens the \((\pi, y)^{AS}\)
schedule. As long as the \((\pi, y)^{AS}\) remains steeper than the \((\pi, y)^{EE}\) curve, for a sunspot of given
persistence \(q\), this results in larger output drops and more deflation in a liquidity trap.

Another important effect, however, is that a different inflation target alters the range of values
of \(q\) for which expectations driven liquidity traps can exist. Figure 7 shows the combinations of
the probability \(q\) and the inflation target \(\tilde{\pi}\) for which there exist two intersections as in panel \(b\) of
Figure 1. When the inflation target is exactly \(\tilde{\pi} = \beta\), the \((\pi, y)^{AS}\) and \((\pi, y)^{EE}\) schedules intersect
once and exactly at the kink. For higher inflation targets \(\tilde{\pi} > \beta\) there may be two intersections de-
dpending on the value of \(q\). The figure shows that higher inflation targets raise the critical value of
\(q\). In general, however, higher inflation targets alone will not succeed in ruling out the possibility
of expectations driven liquidity traps, while output losses and deflation become more pronounced
when they occur. In our calibrated example, an inflation target as high as 7 percent still permits
expectations driven liquidity traps for values of \(q\) at least 0.70.

3 Fiscal Policy in a Liquidity Trap

Given the existence of equilibria in which pessimism brings the economy into a (deep) recession
and forces the monetary authority to lower interest rates to their lowest possible levels, it is inter-
esting to examine how changes in fiscal policy affect equilibrium outcomes. Since the key problem
in a deflationary liquidity trap is weak demand, either because of a fundamental demand shock or
a loss of confidence, a natural policy response is to increase public sector demand. Christiano,
Eichenbaum and Rebelo (2009), Eggertson (2009) and Woodford (2010) examine fiscal stimulus in a liquidity trap generated by a fundamental demand shock. They show how demand stimulating policies can have much stronger effects on output when nominal rates are constant. Eggertson (2009), for example, finds a multiplier after a marginal increase in government spending of 2.3 in a liquidity trap compared to 0.3 when the short term interest rate is positive. The intuition is that temporary but persistent expansionary fiscal policy raises inflation expectations. At zero nominal rates, this lowers real interest rates and crowds in private consumption. On the other hand, interventions intended to stimulate the supply side, such as cuts in the marginal labor income tax rate, are counterproductive in a liquidity trap. In Eggertson (2009), the output multiplier of a labor income tax rate cut is mildly positive when the interest rate is positive, but negative in a liquidity trap. This is because in a liquidity trap supply stimulus lowers inflation expectations, increases real rates and crowds out private consumption.

The effects of fiscal policy are much different when a liquidity trap is generated by a confidence shock, because higher spending means agents become even more pessimistic. When a confidence shock brings about zero nominal interest rates, the real interest rate increases and the output drop must be larger for higher levels of spending to ensure zero national savings. Fiscal stimulus in the form of government spending may therefore not be very successful in raising demand. This can arise in equilibrium since price setters meet the low demand with further price cuts and lower production. Given the price declines, real interest rates increases crowd out household consumption, lowering demand. Ultimately, whether fiscal stimulus increases or lowers real interest rates

11 Christiano, et al. (2009) specify this shock as a taste shock that increases consumers’ patience. Woodford (2010) instead consider an increase in the spread between lending and deposit rates due to e.g. an increase in the risk of default of borrowers. Eggertson (2009) show that the two formulations under some conditions are equivalent.
depends on the relative slope of the \((\pi, y)^{AS}\) and \((\pi, y)^{EE}\) schedules. Because the relative slopes must be different depending on whether a liquidity trap arises from a fundamental demand shock or from a loss of confidence, the effects of demand and supply side policies in both cases have essentially the opposite sign.

To see this graphically, consider Figure 8. The first row depicts the effects of policy induced shifts in aggregate demand or supply in a liquidity trap that is generated by an initial large leftward shift of the \((\pi, y)^{EE}\) schedule (from 1 to 2). In this case expansionary demand policies (from 2 to 3, left panel) are inflationary and the output effect is positive and can be very large. At the same time, an increase in \((\pi, y)^{AS}\) (from 2 to 3, right panel) only leads to more deflation and lower output. The second row in Figure 8 depicts the case where the economy is in an expectations driven liquidity trap (point 2). In this case, expansionary demand policies (from 2 to 3, left panel) further depress output and lead to more deflation. In contrast, expansionary supply policies increase output and lead to more moderate deflation (from 2 to 3, right panel).

The graphical representation of the effects of policy changes is incomplete because it ignores the fact that fiscal interventions such as increased government spending have simultaneous demand and supply effects. We therefore quantify the multipliers associated with the various fiscal policy instruments in numerical solutions of the model. We do this by looking at small perturbations of the equilibrium paths. Our approach to computing the fiscal multipliers is conceptually identical to Eggertson (2009) and Christiano, et al. (2009). The standard multipliers are found by analyzing the effect of small changes in fiscal policy in the neighborhood of the intended steady state. The multipliers in a liquidity trap are found by considering small changes in fiscal policy in the liquid-
ity trap state of the sunspot equilibrium. One difference is that we compute the multipliers based on the nonlinear solution of the functions \( f(\cdot) \) and \( h(\cdot) \), whereas previous studies rely on linear approximations. In the neighborhood of the intended steady state, both approaches yield the exact same numbers for the fiscal multipliers. For larger deviations from the intended steady state, such as required to generate zero interest rates, the multipliers will be quantitatively different as a result of nonlinearities. We stress however that it is not the nonlinear approximation, but the nature of the shock that triggers a liquidity trap that is the reason why our results are qualitatively different from Eggertson (2009) and Christiano, et al. (2009).

**Spending multiplier**  Let \((y_t)_{t=0}^\infty\) be an equilibrium path for output in the model where government spending is constant, i.e. \(g_t = g\). Next, let \((y_t(\delta))_{t=0}^\infty\) be an equilibrium path where government spending starts at \(g + \delta\) where \(\delta > 0\) and in subsequent periods, spending remains at \(g + \delta\) with probability \(p_g\) and returns to \(g\) with probability \(1 - p_g\). Once spending has returned to \(g\), it remains at that level forever. The marginal spending multiplier in period \(t\) is computed as

\[
m^g_t = \lim_{\delta \to 0} \frac{y_t(\delta) - y_t}{\delta}
\]

In the case of the sunspot equilibrium, we impose that spending is \(g + \delta\) in the liquidity trap state and \(g\) when interest rates are positive. This means that the spending process is perfectly correlated with the sunspot, or alternatively that spending is automatically adjusted in response to the liquidity trap state. Again, this approach corresponds exactly to the setup of Eggertson (2009) and Christiano, Eichenbaum and Rebelo (2009). The level of government spending \(g\) is set to be 20% of output in the intended steady state. Figure 9 depicts the spending multipliers for different values of \(\xi\), \(p_g = q\), \(\sigma\) and \(\kappa\). The parametrization is centered around the same values as before, including
\[ \xi = 0.65 \text{ and } \sigma = 1. \] The multipliers are very similar for different levels of price dispersion, so the figures only plot their values at the limit points (the intended steady state and the sunspot limit point).

The standard multiplier, i.e. in the neighborhood of the intended steady state, is about 0.55 for the benchmark parameter values. It always positive and, for the range of parameters we consider, smaller than one because of crowding out. The multiplier becomes smaller as consumption becomes more interest rate sensitive (lower \( \sigma \)), the spending increase more persistent (higher \( q \)), prices less sticky (lower \( \xi \)) or labor supply less elastic (higher \( \kappa \)). For all parameter values, the spending multiplier at the zero bound is smaller than the multiplier at the intended steady state with a positive nominal interest rate. For the benchmark case it is roughly 0.35. The multiplier usually remains positive in a liquidity trap despite a shift in \((\pi_P, y_P)^{EE}\) (as in the left panel of the second row in Figure 8) because there is also an outward shift of the aggregate supply schedule due to the wealth effect on labor supply. The liquidity trap multiplier is always smaller than under positive short term nominal interest rates, because the increase in spending leads to higher real interest rates and crowding out, as explained above. For parameter values where the output drop in a liquidity trap is the largest, the spending multiplier declines the most. This happens when the slopes of the \((\pi, y)^{AS}\) and \((\pi_P, y_P)^{EE}\) curves become similar, which occurs for low values of the persistence \( q \) or high degrees of price stickiness. For the lowest values of \( q \), the liquidity trap multiplier becomes mildly negative.
**Tax multipliers** We compute the multipliers associated with temporary changes in sales and labor income taxes as

\[ m^*_t = -\lim_{\delta \to 0} \frac{y_t(\delta) - y_t}{y_t \delta} \]

where \( \{y_t\}_{t=0}^{\infty} \) is the equilibrium path with a constant tax rate \( \tau \) and \( \{y_t(\delta)\}_{t=0}^{\infty} \) is an equilibrium path where the tax rate starts at \( \tau + \delta \) where \( \delta > 0 \) and in subsequent periods, the tax rate remains at \( \tau + \delta \) with probability \( p_\tau \) and returns to \( \tau \) with probability \( 1 - p_\tau \). To be precise, \( m^*_t \) is the tax semi-elasticity of output, i.e. it is the percent change in output associated with a marginal decrease in the tax rate. We use the same parameters as before but set \( \tau_c = 0.10 \) and \( \tau_n = 0 \) when computing the sales tax multipliers, and \( \tau_c = 0, \tau_n = 0.25 \) when computing the labor income tax multipliers.

Figure 10 depicts the sales tax multipliers, whereas Figure 11 plots the labor income tax multipliers for different values of \( \xi, p_\tau, q, \sigma \) and \( \kappa \). The effects of a temporary decrease in sales taxes are qualitatively very similar to an increase in government spending. There is a rightward shift in the \((\pi, y)^{EE}\) schedule because current consumption becomes cheaper relative to future consumption, while the \((\pi, y)^{AS}\) curve shifts to the right because of intertemporal substitution of labor supply. For the benchmark parameters, a one percent decrease in the sales tax rate increases output by 0.4 percent. The effect is larger for higher elasticities of intertemporal substitution of consumption and labor supply (low \( \sigma \) and low \( \kappa \)), more temporary tax cuts (lower \( q \)) and a larger degree of price stickiness (higher \( \xi \)). For all parameter values, the effects of sales tax cuts are reduced in a liquidity trap driven by confidence loss. As was the case for spending increases, the rightward shift in \((\pi, y)^{EE}\) leads to more deflation crowding out consumption. In the benchmark case, the sales tax multiplier drops to 0.27. The impact on the multiplier can be much larger when \((\pi, y)^{AS}\) and \((\pi, y)^{EE}\) slopes are close, which is the case when \( q \) declines and \( \xi \) grows large.
Whereas the demand stimulating fiscal measures lose effectiveness in a confidence driven liquidity trap, a labor income tax cut becomes more powerful. In the model, a labor income tax cut raises aggregate supply through an increase in labor supply, but leaves the $(\pi, y)^{EE}$ relationship unchanged. With positive nominal interest rates, this leads to lower prices, which the monetary authority accommodates by a nominal rate cut. For the benchmark case, a one percent decrease in the labor tax increases output by approximately 0.45 percent. The output effect becomes larger when intertemporal substitution effects are stronger (low $\sigma$ and low $\kappa$), the tax cut is more temporary (lower $q$) and prices are more flexible (lower $\xi$). In a liquidity trap, the tax multiplier is always larger than in a normal environment. For the benchmark case, a one percent cut in the labor tax increases output by more than 0.65 per cent. Again, the difference in multipliers grows larger for parameter values that lead to the largest drops in output, which occurs when $(\pi, y)^{AS}$ and $(\pi, y)^{EE}$ have similar slopes.

These results confirm a general consensus among economists that the effect of fiscal policy may change when monetary policy is constrained by the lower bound. However, many economists have argued on various grounds that increased spending is likely to generate larger effects in a recessionary liquidity trap. The analysis above provides a counterexample to this assertion. It highlights the importance of knowing the cause of the downturn for determining the relative merits of different fiscal policy interventions. Finally, we want to point out that the fiscal multipliers in this section are for marginal policy changes only. The average effect of any non infinitesimal policy intervention may be much different from its marginal effect. This has been highlighted recently in the context of liquidity traps in fully dynamic models by Erceg and Lindé (2010). For instance, a
fiscal measure may succeed in avoiding or shortening the liquidity trap in which case it becomes all the more desirable.

4 Conclusion and Directions for Further Research

In monetary models where the central bank operates an interest rate rule, there are equilibria in which the zero bound on short term nominal interest rates is occasionally binding. If there exists an inflation-output trade off, as in the New Keynesian model, temporary liquidity traps may occur in equilibrium during which economic activity is severely depressed. Losses in confidence lead to a downward spiral of increased savings, deflation and output drops that is aggravated by intertemporal substitution effects and forward looking price setting behavior when the crisis is expected to be temporary. We have shown that attempts to raise demand through fiscal policy become less effective in an expectations driven liquidity trap, whereas supply side stimulus becomes more potent. These findings provide a counterexample to existing results on the effects of fiscal policy in a zero interest rate environment recently derived in the context of the New Keynesian model. In this paper, we do not take a stance on whether current and past experiences of (near) zero interest rates are best described by a fundamental shocks or self fulfilling changes in confidence. We simply point out that whereas both scenarios can lead to large recessions and deflation, they have different implications for ex post policy responses.

There are several interesting avenues for future research. To the extent that empirical research can uncover differences in the effects of fiscal policy in and outside of a liquidity trap, it is possible to discriminate between the two liquidity trap scenarios empirically. The possibility of expectations driven liquidity traps also introduces new considerations relevant for the choice of a
numerical inflation target. Another important policy question is how to eliminate expectations driven liquidity traps through policy ex ante. Benhabib, Schmitt-Grohé and Uribe (2002) propose monetary and fiscal policies that violate the households’ transversality conditions along candidate equilibrium paths with strong deflation. Under their strategy, the government manages to prevent the unintended deflationary steady state equilibrium by threatening to implement a fiscal stimulus package consisting of a severe increase in the deficit should the inflation rate become sufficiently low. In appendix A, we show how their proposed rule can be extended to the case of temporary liquidity traps as long as the government threatens to increase the deficit at a sufficiently high rate. In addition to possible practical objections to a commitment to unsustainable deficits, a potential complication arises when liquidity traps may be triggered not only by a loss in confidence, but also by a fundamental shock. Unless the deficit rule can be made contingent on the type of shock, this fiscal strategy becomes inconsistent with the existence of an equilibrium. Correia, Fahri, Nicolini and Teles (2010) instead show how locally the appropriate choice of consumption and labor income taxes can implement the same allocation that would be achieved if nominal interest rates could be reduced following a negative fundamental shock. A similar systematic tax policy may well prove to be successful in ruling out expectations driven liquidity traps as well. Alternatively, Atkeson, Chari and Kehoe (2010) describe sophisticated monetary policies that implement the intended competitive equilibrium uniquely in a linear version of our model (in which there are no endogenous state variables) by switching to an appropriate monetary growth rule. We leave it for future research to construct such policies in nonlinear settings with endogenous state variables.
References


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A Using Fiscal Policy To Avoid Liquidity Traps

This appendix shows an extension of the proposal in Benhabib, Schmitt-Grohé and Uribe (2002) to avoid expectations driven liquidity traps which relies on violating the transversality condition on the end of time stock of household wealth whenever deflationary expectations arise for nonfundamental reasons.

Defining $a_t = A_t / P_t$ as total real government liabilities, $A_t = B_t + M_t$, the government budget con-
straint in (23)–(24) can be reformulated as

\[ a_t = \frac{1 + i_t}{\pi_t} a_{t-1} + \tilde{d}_t \]  \hspace{1cm} (40)

where \( \tilde{d}_t = (1 + i_t) d_t - i_t m_t \) is the real primary deficit including seigniorage. The household optimality condition in (11) requires intertemporal fiscal solvency, and government policies in equilibrium must be such that

\[
\lim_{s \to \infty} E_t \left[ a_{t+s} \frac{\pi_t}{1 + i_t} \cdots \frac{\pi_{t+s}}{1 + i_{t+s}} \right] = 0
\]  \hspace{1cm} (41)

When this transversality condition holds, the net present value of current and all future tax and seigniorage revenues equals current outstanding debt and the net present value of all current and future expenditures. In order to rule out the unintended steady state, Benhabib, Schmitt-Grohé and Uribe (2002) propose fiscal rules of the type:

\[ \tilde{d}_t = \kappa(\pi_t) a_{t-1} \]  \hspace{1cm} (42)

Analogous to Benhabib, Schmitt-Grohé and Uribe (2002), consider the following policy:

\[ \varpi(\bar{\pi}) < \frac{1}{\beta} \ , \ \varpi(\pi^U) > \frac{1}{\beta} \]  \hspace{1cm} (43)

where \( \bar{\pi} \) is the inflation target and \( \pi^U \) denotes the inflation rate in the unintended steady state. It follows from the government budget constraint in (40) that:

\[ a_{t+s} = \Pi_{j=0}^s \left( \frac{1 + i_{t+j}}{\pi_{t+j}} + \varpi(\pi_{t+j}) \right) a_{t-1} \]  \hspace{1cm} (44)
such that the transversality condition can be expressed as

\[
\lim_{s \to \infty} E_t \left[ \Pi^s_{j=0} \left( \kappa \left( \pi_t + j + i_t + j \right) \right) a_{t-1} = 0 \right]
\]

(45)

In candidate equilibrium paths that converge to the intended steady state, \( \pi_t / (1 + i_t) \to \beta \) and \( \pi_t \to \bar{\pi} \) and since \( \kappa (\bar{\pi}) < 1/\beta \), the transversality condition is satisfied. For candidate equilibrium paths that converge to the unintended steady state, \( \pi_t / (1 + i_t) \to \beta \) and \( \pi_t \to \pi^U = \beta \) and since \( \kappa (\pi^U) > 1/\beta \), the transversality condition does not hold, unless in the very specific case where \( a_{t-1} = 0 \). Therefore, the fiscal policy in (43) can rule out equilibria that converge to the unintended steady state. Under this fiscal strategy, the government manages to prevent the unintended deflationary equilibrium by threatening to implement a fiscal stimulus package consisting of a severe increase in the deficit should the inflation rate become sufficiently low.

A similar fiscal strategy can be devised in order to rule out temporary liquidity traps driven by a sunspot with stochastic properties given in (35). In this case, the threat to the deficit must be such that

\[
\lim_{s \to \infty} E_t \left[ \Pi^s_{j=0} \left( q \kappa \left( \pi_t + j + i_t + j \right) \right) a_{t-1} \neq 0 \right]
\]

(46)

If \( \pi_P \) is the inflation rate in the sunspot limit point for a given persistence \( 0 < q < 1 \), the requirement on fiscal policy to rule out the sunspot equilibrium is modified:

\[
\kappa (\pi_P) > \frac{1}{q\pi_L} > 1/\beta
\]

(47)
Recall that sunspot equilibria exists for all $q$ greater than a certain critical value. To rule out all these sunspot equilibria, the condition in (47) must hold for $q$ approaching this critical value. Therefore, the government could also avoid temporary liquidity traps as long as it commits to sufficiently large increases in the deficit in response to deflationary pressures.
Figure 1: An Expectations Driven Liquidity Trap

Figure 2: Liquidity Trap After a Fundamental EE Shock
Figure 3: Dynamics in a Expectations Driven Liquidity Trap ($\xi = 0.65, \sigma = 1$)
Figure 4: Dynamics in a Expectations Driven Liquidity Trap ($\xi = 0.82, \sigma = 0.7$)
Figure 5: Deflation and Output Loss in a Expectations Driven Liquidity Trap: Sensitivity Analysis
Figure 6: Inflation and Output in the Intended Steady State and in an Expectations Driven Liquidity Trap: Different Inflation Targets
Figure 7: Expectations Driven Liquidity Traps and the Inflation Target

- Single EE–AS Intersection
- Two EE–AS Intersections
Figure 8: The Effect of Demand and Supply Policies in Liquidity Traps
Figure 9: Marginal Spending Multipliers

- Red line: Standard Multiplier
- Blue line: Zero Bound Multiplier
Figure 10: Marginal Sales Tax Multipliers
Figure 11: Marginal Labor Tax Multipliers