Legal Protection in Retail Financial Markets

Bruce Ian Carlin†
Simon Gervais‡

March 10, 2010

Abstract

Given the importance of sound advice in retail financial markets and the fact that financial institutions often outsource their advice services, what legal rules maximize social welfare in the market? We address this question by posing a theoretical model of retail markets in which a firm and a broker face a bilateral hidden action problem when they service clients in the market. All participants in the market are rational, and prices are set based on consistent beliefs about equilibrium actions of the firm and the broker. We characterize the optimal law in the context of our model, and derive how the legal system splits the blame between parties to the transaction. We also analyze how complexity in assessing clients and conflicts of interest affect the law. Since these markets are large, the implications of the analysis have great welfare import.

*We would like to thank Tony Bernardo, Doug Diamond, Denis Gromb, Raghu Rajan, Adriano Rampini, Jacob Sagi, and Andrei Shleifer for helpful discussions at the inception of this project. Also providing useful comments and suggestions were Moez Bennouri, Utpal Bhattacharya, Philip Bond, Brendan Daley, Howell Jackson, Jakub Jurek, Rachel Kranton, Rich Mathews, Sébastien Michenaud, as well as seminar participants at American University, Duke University (law school and economics department), Rice University, Texas A&M University, UCLA, the University of Miami, the University of Texas at Austin, the University of Virginia, the U.S. Securities and Exchange Commission, the UBC Summer Finance Conference, the 2009 Conference of the Financial Intermediation Research Society, the 2009 Mitsui Finance Symposium held at the University of Michigan, the 2009 Conference on Recent Developments in Consumer Credit and Payments held at the Federal Reserve Bank of Philadelphia, the 2009 Conference on the Role of Government Regulation in Corporate Finance held at Vanderbilt University, the NBER Working Group on Household Finance, and the 2010 Annual Meeting of the American Finance Association. All remaining errors are of course the authors’ responsibility.

†Anderson School of Management, University of California, Los Angeles, 110 Westwood Plaza Suite C519, Los Angeles, CA 90095, bruce.carlin@anderson.ucla.edu, (310) 825-7246.

‡Fuqua School of Business, Duke University, One Towerview Drive, Durham, NC 27708-0120, sgervais@duke.edu, (919) 660-7683.
Legal Protection in Retail Financial Markets

Abstract

Given the importance of sound advice in retail financial markets and the fact that financial institutions often outsource their advice services, what legal rules maximize social welfare in the market? We address this question by posing a theoretical model of retail markets in which a firm and a broker face a bilateral hidden action problem when they service clients in the market. All participants in the market are rational, and prices are set based on consistent beliefs about equilibrium actions of the firm and the broker. We characterize the optimal law in the context of our model, and derive how the legal system splits the blame between parties to the transaction. We also analyze how complexity in assessing clients and conflicts of interest affect the law. Since these markets are large, the implications of the analysis have great welfare import.
1 Introduction

Retail financial markets are unique in that the majority of consumers who participate have an incomplete understanding of the products that are available and are generally uninformed about prices in the industry (e.g., NASD Literacy Survey, 2003). In fact, in the language of the Securities Act of 1933, public investors are described as those who are “unable to fend for themselves.” Participation at the household level, therefore, not only involves having access to good quality opportunities, but also entails being directed toward the best alternatives.

There is no clear evidence that advice increases welfare, however (e.g., Bergstresser, Chalmers and Tufano, 2009). As argued by Bolton, Freixas and Shapiro (2007), this may be due to conflicts of interest and can be a significant cause of decreased faith in the market. Given the large size of retail markets, protecting consumers who are “unable to fend for themselves” is not only an important duty of the law, but also a key driver of participation in the market, economic growth, and social welfare (e.g., Campbell, 2006; Calvet, Campbell and Sodini, 2006). The importance of regulation in such markets has been highlighted and described by Campbell, Jackson, Madrian and Tufano (2010), but there still remains a paucity of theoretical work on this subject.

In this paper, we provide a theoretical analysis of consumer protection laws and address the following questions: Who should be held accountable when consumers are wronged in financial markets? How does the difficulty in assessing a consumer’s needs affect the penalties that are imposed on the firm and its representatives? How may the law circumvent conflicts of interest when they arise?

Three stylized facts about retail financial markets make addressing these questions interesting and challenging. First, financial institutions frequently outsource their advice services to brokers. Indeed, the majority of financial products are sold through intermediaries. For example, only 40% of mutual funds are purchased directly from financial institutions (Investment Company Institute July 2003). As discussed by Jackson (2008), this trilateral arrangement between the consumer, the intermediary and the producer of the financial service complicates the legal structure that must be implemented in order to preserve efficiency. For one thing, when a household investor is wronged in the market, two parties are potentially culpable: the producer and the intermediary (e.g., an advisor or broker). Thus, any law that is implemented must take into account the potential actions of the producer of the product itself (e.g., quality and transparency choices), the actions of the advisors in placing clients into those instruments (e.g., irresponsible advice), and the contractual agreements that are present between these parties.

The second stylized fact is that financial services often require consumers to either pay up-front
(e.g., loads) or agree to fees (e.g., management fees) before their service is produced. In essence, consumers of financial products purchase a fiduciary duty from their providers. In turn, both financial institutions and advisors invest in the quality of their services after payment for their services has been made or defined. Naturally, such a situation is vulnerable to hold-up problems, which may lead to underinvestment and market breakdown.\footnote{Note that this differs from the market for durable goods. In this market, producers invest in quality during production, before consumers pay in full. Not surprisingly, there is a lower tendency for hold-up to occur.} The law, then, needs to ameliorate this problem: penalties are set to maximize participation and investment in the market and the quality of services provided there.

The third stylized fact is that assigning blame to either party is an imperfect process. Advisors can make honest mistakes when assessing the needs of clients. Indeed, despite their good intentions, it may be difficult for them to match consumers with financial products. Further, based on the ex post realization that a consumer has been wronged, it is often difficult for the law to identify where the process failed.\footnote{It may even be difficult for the law to determine whether or not a customer was wronged to begin with, as the performance risk of the product may lead to bad outcomes even for customers who are fit for it ex ante.} Therefore, the legal system not only serves to realign the incentives of producers and advisors, but must also correctly split blame across all parties in order to be effective ex ante. In doing so, the legal system must anticipate and take into account the effect that a law will have on the contractual incentives and prices that will prevail in equilibrium for the industry.

The model that we analyze proceeds as follows. A single firm produces financial products and distributes them to the public through a broker who provides advice service to potential clients. The firm has a responsibility to provide good quality opportunities for clients, and higher quality increases the chances that consumers benefit from making a purchase. The broker’s job is to sort clients and make product recommendations based on a noisy signal about their type. How the firm and the broker fulfill these responsibilities is unobservable and non-verifiable, and so market participants face the coordination problems that arise in settings with bilateral hidden action.\footnote{Similar settings of bilateral moral hazard are identified and discussed by Levmore (1993).}

The government sets a law that holds the firm and the broker responsible when a consumer is wronged. This may occur if a consumer is directed to inappropriate choices when they make a purchase or after they become a client. An example is when the consumer is unjustifiably directed towards an adjustable-rate mortgage as opposed to a fixed-rate instrument. In the model, the penalty that the law dictates is set optimally based on the incentives that the firm and the broker are anticipated to have when choosing their optimal strategies. As such, prices in the market arise from consumers’ rational expectations about the optimal actions of the firm and the broker, the law that is set to protect their interests, and their expectations about their own type.

In equilibrium, in the absence of penalties (i.e., the absence of law), neither the firm nor the
broker can commit to provide quality or advice. Since consumers are rational, prices drop, and minimal economic surplus is realized. The market suffers from an extreme underinvestment problem as a result of the dual moral hazard problem. This motivates further analysis regarding consumer protection law.

When the legal system imposes penalties, the firm improves the quality of its products and the broker provides more thorough advice. Thus, the law acts as a coordinating device to ease moral hazard problems in the market. At the same time, however, the broker and the firm have a tendency to free-ride on each other’s effort provision. Increasing penalties (blame) to each party not only increases their own effort provision, but also decreases their counterparty’s incentives to offer better services. For example, as penalties induce the firm to offer higher quality products, the marginal benefit of providing advice decreases. Likewise, the broker’s decision to offer more advice decreases the number of sales made in the market, and lowers the marginal benefit for the firm to invest in offering quality. The law must then consider not only the direct effect that penalties have on the firm’s or broker’s actions, but also the indirect effect they have because of free-riding.

In equilibrium, the law is set to maximize total welfare in the market. We show that the total penalty imposed not only makes a wronged consumer whole (i.e., expectation damages are awarded), but grants them punitive damages. The result implies that insurance alone does not maximize welfare in the market. That is, a law that makes a wronged consumer whole, but does not punish the firm or broker further, does not achieve first-best quality and advice. This finding contrasts with how the SEC typically sets public penalties, and thus provides an important normative implication. Specifically, SEC penalties usually involve either restitution for loss incurred, recision (i.e., a refund), or disgorgement of the violator’s gains. While we do not address disgorgement in our model, our analysis does suggest that restitution and recision alone are unlikely to achieve first-best in this setting.

The difficulty that a broker experiences in assessing his clients’ needs not only impacts the optimal actions of the firm and the broker, but also affects the law. We model this difficulty as a tendency for the broker to make advising errors. As the probability of making such errors increases, the broker has a lower incentive to give advice. This arises because the marginal benefit of doing so drops and the broker is more willing to take his chances by selling products to all-comers without sorting them. In contrast, as the probability of errors rises, the firm has a greater incentive to provide quality because higher quality increases the chances that consumers are properly served. The effect that such errors has on the law is to penalize the broker more when assessment is more precise.

For most of the analysis, we assume that consumers cannot circumvent the broker and ignore
their advice. We extend our analysis to relax this assumption and consider the law when the broker does not act as a gate-keeper. In that case, the law cannot achieve the same first-best outcome by including punitive damages, since such payoffs would cause the value of advice to deteriorate. That is, the presence of punitive damages creates an incentive for consumers to ignore advice and take chances with products that are not as well suited to their profile. The legal rules that maximize welfare involve an insurance-type remuneration in which a wronged consumer is made whole, but is not entitled to other damages. The law splits this obligation between the firm and the broker, based on the other parameters in the market.

Finally, we extend our analysis to consider the presence of conflicts of interest in the market. Specifically, we analyze how sales commissions affect the law that is set and the optimal actions of the firm and the broker. We show that sales commissions cause advice to drop, but induce the firm to produce more quality. The time and effort spent by the broker in his advising function has the negative effect of excluding some consumers from buying the firm’s products. In the presence of commissions, the agent has an incentive to sell more and thus to be negligent in his advising role. Anticipating this, the firm chooses higher quality to avoid the penalties that are associated with such wrongdoing. The legal rules that maximize welfare in this setting involve higher penalties for the broker and lower ones for the firm, which helps to circumvent this conflict of interest. This result is consistent with the case law that deals with conflicts of interest and financial intermediaries (e.g., Kumpan and Leyens, 2008).

As suggested by Campbell et al. (2010), the law plays a crucial role in retail financial markets because other commitment devices and market discipline devices are inadequate in this setting. For example, because financial products are inherently risky and it is difficult to measure their ex ante suitability based on ex post outcomes, it is generally implausible to offer warranties on such products. Warranties that protect against performance create easy ex post arbitrage opportunities for buyers. For example, granting a free option to return a portfolio would clearly create insurmountable adverse selection problems, and would make the firm vulnerable to opportunistic behavior by buyers disappointed by the portfolio’s performance. Moreover, perfect protection would essentially transform the portfolio into a risk-free security, making it a redundant investment vehicle. Thus, whereas Spence (1977), Grossman (1981), and Mann and Wissink (1990) suggest that warranties and refunds can increase the surplus generated by transactions, commitment to quality via such mechanisms is next to impossible in retail financial markets. Instead we expect the legal system to play a more significant role in these markets, as is the case in Palfrey and Romer’s (1983) analysis of disputes over product performance between buyers and sellers.

Reputation is also insufficient to induce full commitment to quality or advice, as proposed by
Klein and Leffler (1981), Shapiro (1982, 1983) and Allen (1984). The reason is that products and prices in these markets are inherently difficult for consumers to decipher. As a result, consumers often settle on a suboptimal product, as documented by Capon, Fitzsimmons and Prince (1996), Agnew and Szykman (2005), Choi, Laibson and Madrian (2009a), and Agarwal et al. (2009), among many others. Moreover, as shown by Ausubel (1991), Jain and Wu (2000), Jones and Smythe (2003), and Choi, Laibson and Madrian (2004) in different contexts, these consumers are frequently unable to discriminate among brokers and providers of services, due to various constraints on their discovery processes (e.g., ability or cost to learn). Finally, the low frequency with which the average consumer interacts with a financial product provider seriously limits the efficiency of reputation-building as a disciplining device, especially when the transactions and experiences of other market participants are not publicly observable.

As such, our paper contributes to a growing theoretical literature on household finance, law and finance, and the legal foundations of agency law. We highlight this contribution in the next section. Following that, we set up our benchmark model in Section 3, and consider the legal system when brokers advise clients as to what product to buy in the market. We start by analyzing the strategic choices of the firms and the brokers, and then derive and characterize the legal rules that the government sets in order to maximize welfare. We finish the section by analyzing an extension in which the broker is compensated with sales commissions. In Section 4, we relax the assumption that the broker acts as a gate-keeper in the market and compare our results to those derived in previous sections. Section 5 offers some concluding remarks. The Appendix contains all the proofs.

2 Related Literature

Our paper contributes to the theoretical literature on household finance, in which rational financial institutions interact with heterogeneous consumers who rationally participate in the market, but must make decisions based on a constrained learning process. Whereas consumers are assumed to have incomplete knowledge about prices in the market in Carlin (2009) and about the quality of

---

4 Other relevant papers include Alexander, Jones and Nigro (1998), Sirri and Tufano (1998), Wilcox (2003), Barber, Odean and Zheng (2005), and Choi, Laibson and Madrian (2009b). Also note that there exists extensive evidence of significant pricing effects in the market (Ausubel, 1991; Mitchell et al., 1999; Baye and Morgan, 2001; Brown and Goolsbee, 2002; Christoffersen and Musto, 2002; Hortaçsu and Syverson, 2004; Green, 2007; Green, Hollifield and Schürhoff, 2007), and that this has a substantial welfare impact (e.g., Campbell, 2006; Calvet, Campbell and Sodini, 2006).

5 In fact, it could be argued that a legal system is necessary for reputation to form in these markets. That is, given poor access to information, the presence of lawsuits acts as a device for reputation to form and get disseminated in the population. We leave this additional role for the legal system to future research.

6 This literature has evolved from the initial insight of Stigler (1961) about price dispersion and the subsequent consumer models of Shilony (1977), Varian (1980), and Burdett and Judd (1983).
products in Carlin and Manso (2009), we assume instead that consumers have limited information about the appropriateness of a specific financial product for their own situation. In this context, consumers not only benefit from a higher commitment to quality by the firm, but also from the advice of the agent hired by the firm to match products and customers. As in Kronman’s (1978) discussion of voluntary disclosures and in Shavell’s (1994) model of the same problem, the presence of legal obligations changes the agent’s incentives to gather and communicate information that is socially useful. Our analysis adds the aforementioned tensions between the firm and the agent to this problem, characterizes their contractual relationship, and derives the regulation that maximizes economic welfare.

Our paper also adds to the literature on law and finance, which highlights the link between strong legal and financial institutions and economic growth. For example, Shleifer and Wolfenzon (2002) analyze the effects that legal protection has on the type and quality of investments that occur in the market. Similarly, Stulz (2009) shows how strong securities laws that mandate disclosures can significantly impact firms’ access to capital and their value, as suggested by La Porta, Lopez-de-Silanes and Shleifer (2006). This work underscores several empirical observations that there is a strong relationship between legal institutions and economic progress. Indeed, La Porta et al. (1997, 1998) document substantial cross-sectional variation in the legal protection that investors receive in different countries, and posit that there exists a positive correlation between government regulation and economic growth. Following them, Levine (1999), Glaeser, Johnson and Shleifer (2001), and Beck, Demirgüç-Kunt and Levine (2005) also argue for this positive relationship. In a similar vein, Levine (1998), Levine, Loayza and Beck (2000), and Haselmann, Pistor and Vig (2010) provide evidence that financial intermediation and the provision of credit are greatly affected by the legal system, while Nunn (2007) shows that the ability of a legal system to enforce contracts is a significant driver of economic activity. Consistent with these empirical observations, the analysis in this paper demonstrates that consumer protection law is necessary for both the preservation and the prosperity of retail financial markets. In this sense, our work complements that of Acemoglu, Antràs and Helpman (2007) who show that strong contract enforcement facilitates the adoption of more advanced technology.

The paper may also be viewed as an economic analysis of agency law (e.g., Rasmusen, 2004). Indeed, following Ross (1973), Jensen and Meckling (1976), and Holmström (1979), economists have focused their study of agency theory on the search for contractual arrangements that realign the incentives of agents with those of the principal, thereby maximizing the production potential

\[^{7}\text{For a comprehensive overview of the literature on law and economic growth, see La Porta, Lopez-de-Silanes and Shleifer (2008).}\]
and value of the firm. In contrast, and as laid out by Sykes (1984, 1988), the primary objective of legal scholars studying agency law is to determine who is to blame (principal or agent, or both) when an outsider is wronged. As Rasmusen (2004) writes, “for the economist, the agency problem is how to give the agent incentives for the right action; for the lawyer, it is how to ‘mop up’ the damage once the agent has taken the wrong action” (page 370). In this paper, we analyze the interaction of these forces, that is, where the law must take into account incentives within the firm when it assigns blame. To our knowledge, this interaction has not been analyzed or modeled before.8

Finally, our paper is closest in spirit to recent discussion papers by Barr, Mullainathan and Shafir (2008) and by Lipner and Catalano (2009) about the regulation of home mortgage credit and negligent investment advice respectively. Like us, Barr, Mullainathan and Shafir argue that the complexity of the decisions that consumers are asked to make about mortgages requires a legal system that properly internalizes the incentives, motives and biases of market participants. Similarly, Lipner and Catalano advocate a system of legal responsibility that fills the gaps in existing securities statute and holds brokers and advisors accountable for “negligent misrepresentation.” Our paper complements these papers by providing an economic analysis that formalizes the main ideas, explicitly characterizes the economic forces, and extends their applicability to the entirety of retail financial markets.

3 A Market for Financial Services and Advice

3.1 Model Setup

Consider a risk-neutral financial institution (i.e., a firm or a principal) that markets a continuum of financial products, \( i \in [0, 1] \), to a unit mass of consumers. The products could be a group of instruments used to finance the purchase of consumption goods (for example, a line of credit cards) or a set of investment vehicles that are available to maximize lifetime utility (for example, a family of mutual funds).

Each consumer is naturally well-suited for a subset of measure \( \phi \in (0, 1) \) of these products. Specifically, each consumer’s type \( \tilde{\tau} \) is uniformly distributed on the continuum \([0, 1]\) and a consumer of type \( \tilde{\tau} = \tau \) is a match for the class of products \( I_{\tau} = \left[ \tau - \frac{\phi}{2}, \tau + \frac{\phi}{2} \right] \), where the subinterval below zero (above one) in \( I_{\tau} \) is remapped to the upper (lower) portion of the unit interval; the same

---

8Hiriart and Martimort (2006) study a regulation problem in which the legal system must anticipate the firm’s incentives to undertake environmentally risky activities. The similarity between our problem and theirs is limited to this anticipatory component and the fact that blame is ultimately shared by the firm and its agent. Indeed, the market, the product, the game between the firm and its agent, and the presence of utility-maximizing consumers are all specific to our setting.
\[ \tilde{\tau} = \tau \]

\[ \tau + \frac{\phi}{2} \]

\[ \tau - \frac{\phi}{2} \]

\[ I_{\tau} \]

\[ 0 \]

\[ 1 \]

Figure 1: The set of all the firm’s products $[0, 1]$ are set in a circle. Products are labeled between zero and one as we move counter-clockwise along the circle’s circumference. The customer’s type $\tilde{\tau} = \tau$ determines an interval of products $I_{\tau}$ for which this customer is a natural match.

customer is a mismatch for all the other products. To visualize this, we can think of the unit interval of products as the circumference of a perfect circle, as in Figure 1. Each consumer’s type is a point $\tilde{\tau} = \tau$ on the circle’s circumference, with all of the points within a distance of $\frac{\phi}{2}$ from $\tau$ representing products that are a good match for the customer. For example, a consumer with a long run objective could be a good match for a set of riskier equity funds offered by a fund family.

When a customer of type $\tau$ is matched with a product in $I_{\tau}$, he is said to be satisfied and as such derives a positive money-equivalent value of $m > 0$ from owning the product. When mismatched (i.e., if the product is from $[0, 1] \setminus I_{\tau}$), the same customer is said to be dissatisfied and suffers a money-equivalent loss of $-m < 0$. Dissatisfied customers would be willing to pay as much as $m$ to avoid or get rid of the product, and are likely to seek damages based on the law that is present in the economy. Note that the circular representation that we use differs from standard circular city models of spatial differentiation in which utility is continuous and a function of the distance between a customer’s type and the product that he purchases (e.g., Lerner and Singer, 1939; Vickrey, 1964; Salop 1979). Here, utility takes on a binary value depending on whether a customer is inclined to seek legal remedies (to be introduced shortly). We choose this approach in order to maintain tractability, but our analysis extends to standard circular city models when customers must incur a fixed cost to enter lawsuits.\(^9\)

As such, the parameter $\phi$ is an ex ante measure of the specificity of the products offered by the firm. When $\phi$ is low, products are more specialized and greater care is needed during product selection. When $\phi$ is high, products have more widespread use. Ex ante, consumers are unaware of

\(^9\)Our model also has the interpretation that customers cannot perfectly discern the quality of the match. This is particularly well-suited to retail financial markets where, because products are challenging to understand, customers are less precise when calculating how close they are to their optimal product.
their own type, but do know the distribution of products and types in the population. Consumers have consistent beliefs about the market, and we set $\phi m - (1 - \phi)m = 0$ so that without any other information (e.g., advice), consumers are not willing to pay anything for the product.\footnote{We could normalize $\phi m - (1 - \phi)m$ to any constant without affecting our results. This particular parametrization eliminates corner solutions that come with uninteresting properties (e.g., negative firm profits).}

Sales in the market are intermediated by a risk-neutral broker (i.e., an agent). The broker’s role is to distribute the products and to direct customers to specific products offered by the firm. The broker receives a wage $w$ from the firm for providing this service and the firm does not interact with consumers directly.\footnote{We assume that the broker attracts clients costlessly through referrals from the firm. Thus, we do not model the moral hazard problem associated with the effort required to attract consumers. This problem is analyzed by Inderst and Ottaviani (2009), and Inderst (2008).} As such, there is a division of labor in which the firm is responsible for producing the good, while the broker is responsible for providing potential clients with financial advice. For example, a mutual fund family that offers multiple funds with various risk characteristics will rely on brokers to guide customers towards the fund that is appropriate for each of them. Similarly, a lender will rely on a mortgage broker to advise customers in terms of the appropriate instrument to finance a house purchase (e.g., an adjustable-rate mortgage versus a fixed-rate mortgage).

The broker chooses an unobservable level of advice $a \in [0, 1]$ and incurs a private effort cost of $\frac{w}{2}a^2$ in doing so. If $a = 0$, the broker does not gather any information about any customer, and so cannot provide them with any useful advice. If $a = 1$, the broker responsibly sorts customers based on his (possibly imperfect) information about their needs for different products. For every customer, the agent receives a signal $\tilde{s} = \tilde{\epsilon} \tilde{\tau} + (1 - \tilde{\epsilon}) \tilde{\eta}$, where

$$\tilde{\epsilon} = \begin{cases} 1, & \text{prob. } a \gamma \\ 0, & \text{prob. } 1 - a \gamma, \end{cases} \quad (1)$$

and the variable $\tilde{\eta}$ is noise that, like $\tilde{\tau}$, is uniformly distributed on $[0, 1]$, but whose realization is independent from $\tilde{\tau}$. A larger $a \in [0, 1]$ allows the agent to observe a customer’s true type more frequently but, when $\gamma$ is smaller than one, his information can never be perfect.\footnote{In Section 3.4, we consider an alternative contract in which the firm compensates the broker with sales commissions. As will become apparent there, even though such incentives are necessary for the broker to distribute the product, sales commissions induce a conflict of interest, which we characterize in detail. For now, we ignore this source of moral hazard and focus on the broker’s tendency to give advice.} Of course, since $\tilde{s}$ is more likely to be in $I_\tau$ than a random draw from a uniform distribution, the agent’s advice is always to recommend product $\tilde{s}$ to the consumer.\footnote{To highlight the fact that a fraction $a$ of consumers receive information that increases their posterior probability of being correctly matched with a financial product, one can write the agent’s signal as $\tilde{s} = \psi \tilde{\epsilon} \tilde{\tau} + (1 - \tilde{\epsilon}) \tilde{\eta} + (1 - \psi) \tilde{\eta}$, where $\psi$ is equal to 1 with probability $a$ and equal to zero otherwise. The analysis is completely unaffected by this alternative representation of $\tilde{s}$.}
The parameter $\gamma$ captures the idea that some financial decisions are complicated and, despite the agent’s goodwill, errors do occur. When $\gamma = 1$ and $a = 1$, the agent always observes the consumers’ types with perfect precision. However, when $\gamma = 0$, effort in giving advice does not improve the chances that consumers are sorted appropriately. We can also think of $1 - \gamma$ as the difficulty of the agent’s task. For example, financial products that are especially difficult to match with consumers are characterized by a low $\gamma$. In what follows, we will see that $\gamma$ not only plays an important role when we analyze the optimal actions of the firm and the broker, but also when we derive the optimal law.

Difficulty in advising consumers not only affects the signal that the broker receives about consumers, but also affects the time that he needs to spend with his clients. Although we do not model the agent’s role in attracting customers, we do capture this idea by assuming that the advising function displaces some of the agent’s attention and reduces the flow of potential customers. More precisely, we assume that the agent’s effort $a$ that is directed towards advising customers results in a loss of $\delta a$ in customer flow, where $\delta \in [0, 1 - \gamma)$ can be interpreted as the time required for the advice process. That is, of the initial mass of customers, only $1 - \delta a$ sales are made by the firm. The other $\delta a$ customers are assumed to receive a payoff of zero.\(^{15}\)

When producing the good, the firm chooses an unobservable quality level for the services that it provides to its customers. This quality improves the chances that mismatched customers experience a positive outcome even though they have been placed into an inappropriate product. Specifically, the firm chooses $q \in [0, 1]$, the probability that a mismatched customer receives a satisfactory outcome, and incurs a cost of $k_F q^2$ in doing so. Therefore, a high (low) choice of $q$ lowers (raises) the probability that a customer seeks damages ex post. This captures the idea that the firm may avoid legal liability by improving the performance of its products. For example, consider a situation in which an elderly investor is advised to place an inappropriate fraction of his wealth in a high growth equity fund. If the efficient management of the fund enables it to outperform other investment vehicles, the investor may end up realizing high returns despite having taken inappropriate risks. Ex post, such an investor is likely satisfied and unlikely to seek legal remedies.

By construction in this model, the broker and the firm cannot directly observe each other’s actions when making their choices of $a$ and $q$. The resulting model is one of bilateral hidden consumers have no other choice but to heed their recommendation. They cannot bypass the advisor. We relax this assumption in Section 4.

\(^{15}\)The same setup endogenously arises if we assume that the agent must also exert effort to attract customers and that the agent has a limited effort capital. More specifically, assume that $1 - \delta$ customers come to the firm if the agent exerts no effort to attract customers, and that an effort of $\alpha \in [0, 1]$ increases the flow of customers by $\delta \alpha$. The above setup results if $\alpha$ can be exerted without any effort cost being incurred by the agent and if his total effort supply, $a + \alpha$, cannot exceed one. Indeed, the agent then allocates all of his non-advising effort, $1 - a$, to attracting customers for a total customer flow of $1 - \delta + \delta(1 - a) = 1 - \delta a$. 

10
action in which both parties are rational and have consistent beliefs about each other’s equilibrium behavior. The agent’s advice helps to match the investors’ needs with the correct instrument, but the agent is unable to advise consumers about how resources and products are managed internally within the firm. Likewise, the firm chooses how much capital it expends to add quality to its investment products or services, but cannot oversee the advice that consumers receive when they purchase the product.

Prices in the market arise from the fully rational behavior of all parties to the transaction. In equilibrium, this price depends on the consumers’ consistent beliefs about the equilibrium actions of the firm and the broker, as well as their bargaining power.\textsuperscript{16} Ex ante, all of the consumers are symmetric and every product offered by the firm is identical. Therefore, all products in the market sell for the same price, which we denote by $p$ and characterize in Section 3.2.

The legal environment is set as follows. The government chooses a law that protects consumers if they purchase a product they should not have purchased and suffer a loss.\textsuperscript{17} We assume that the legal system is set to maximize total welfare in the market. As such, the law $\mathcal{L} = \{\rho_A, \rho_F\}$ is the financial burden imposed on the two parties who are responsible for marketing and selling the financial product or service to a consumer: $\rho_A \geq 0$ is paid by the broker and $\rho_F \geq 0$ is paid by the firm.\textsuperscript{18} Therefore, for each consumer who suffers a loss $\overline{m}$ after purchasing the product, $\rho_T \equiv \rho_A + \rho_F$ is the total value recovered via the legal system. If $\rho_T = \overline{m} + \overline{m}$, the consumer is said to be made “whole” by the law. If $\rho_T > \overline{m} + \overline{m}$, the consumer is not only made whole, but is also entitled to additional punitive damages.

Note that we deliberately assume that penalties are not affected by the number of consumers who end up suing. Two considerations drive this assumption. First, by making each customer’s penalty independent from the penalties of others, we capture the idea that in reality transactions for financial products and the lawsuits that results from them do not all occur at the same time, as they do in the model. Instead, each consumer can appeal to the court system if and when they are wronged, regardless of what is likely to happen to others down the road. In fact, in this light, our assumption is consistent with that of Inderst and Ottaviani (2009) who consider only one consumer. Indeed, given that the firm and agent are risk-neutral, $a$ and $q$ then jointly determine the probability that this one consumer buys the product and the probability that he sues, without

\textsuperscript{16}For some of our analysis, it will be convenient to assume that the firm is a monopolist that extracts all consumer surplus, but our results do not depend on this particular split of bargaining power.

\textsuperscript{17}We assume that customers’ types can be verified perfectly in a court of law. This is without loss of generality as, in our setting, imperfect verification could be easily overcome by appropriately scaled penalties.

\textsuperscript{18}Alternatively, the parameters $\rho_A$ and $\rho_F$ could also be interpreted as expected penalties given the probability that a lawsuit is successful and the damages awarded by the law. For example, it might be that $\rho_A = zp_A$ and $\rho_F = zp_F$ where $z$ is the probability that any lawsuit succeeds, and $p_A$ and $p_F$ are the payments to the consumer when it does succeed.
Figure 2: Consumer protection game. At $t = 1$, the law is set. At $t = 2$, the price $p$ for the financial service is posted. If consumers are willing to pay $p$, they present to the broker to be placed into an appropriate financial product. At $t = 3$, the broker chooses $a$ and the firm chooses $q$. A fraction $1 - a\delta$ of the consumers are served and each pay the firm $p$. Finally, at $t = 4$, consumers realize their utility payoffs and those who have been wronged sue for damages based on the law.

affecting the analysis. Second, as we show in Section 3.3, treating each customer independently is sufficient for the legal system to recover first-best when customers are obliged to follow the broker’s recommendations.

The timing of the game is outlined in Figure 2. At $t = 1$, the law is set. At $t = 2$, the price $p$ for the financial service is set in the market. If $p$ is less than or equal to consumers’ willingness to pay for the product (based on rational expectations), then they present to the broker for advice. At $t = 3$, the broker chooses $a$ and the firm chooses $q$. Based on this, $1 - a\delta$ consumers are served and each pay the firm $p$. Finally, at $t = 4$, consumers realize their utility and those who have been wronged sue for damages based on the law.

A key feature of this timing structure is that prices are set before $a$ and $q$ are chosen by the broker and the firm. As we noted in the introduction, this is frequently the economic interaction that arises in retail financial markets where consumers are in essence buying the “fiduciary duty” of either the firm that produces the financial service or the broker who provides advice. For example, when consumers initially invest in a mutual fund, the loads or management fees are posted before they present for service. If these fees are agreeable, consumers will proceed with their broker and receive advice and financial service.

The natural question that arises is whether the broker and the firm will actually uphold their fiduciary duty to their customers. Indeed, a natural hold-up problem arises because prices have already been negotiated up front. Not surprisingly, this affects the price that consumers are willing to pay, and the investment they are willing to make. We will characterize the inefficiencies that arise from this problem in the next section.
3.2 Equilibrium Behavior

We begin by calculating the number of sales that are made by the broker and the fraction of consumers who seek remedies because they were wronged in the transaction. We then characterize the equilibrium actions of the firm and the broker. An analysis of the law is contained in Section 3.3. Throughout, we restrict $k_F > \gamma + \delta m$ and $k_A > (\gamma + \delta)m$ in order to guarantee interior solutions.\footnote{This assumption is made purely for technical convenience. It is sufficient, but not necessary, for interior solutions to exist. The avoidance of corner solutions is for expositional clarity, and does not qualitatively change our results.}

3.2.1 The Number of Sales and Lawsuits

Figure 3 shows the repartition of consumers when the broker chooses an advice level of $a$ and the firm chooses a product quality level of $q$. Of the initial unit mass, only $1 - \delta a$ show up to buy a product. The $a$ customers who are properly advised by the broker experience an increment of $(1 - \phi)\gamma$ in the probability of being matched with a product that is right for them. Product quality further improves the possibility of a match for those customers who do not benefit from the advice process. Importantly, customers do not know $a, q,$ or where they will eventually end up in this tree when they purchase the product. Instead, they must rely on the equilibrium values of $a$ and $q$ that result from the publicly observable incentives faced by the broker and the firm.

---

Figure 3: This figure shows how the initial unit mass of consumers gets split depending on the advising choice $a$ of the broker and the quality choice $q$ of the firm.
Let us define \( n_S \) as the total number of sales made by the firm. By construction,

\[
    n_S = 1 - a\delta,
\]

which is decreasing in the quality \( a \) of advice offered to consumers and the difficulty \( \delta \) of the task. Let us also define \( n_H \) (\( n_L \)) as the number of matched (mismatched) consumers who purchase the product and ultimately experience a utility of \(+m\) (\(-m\)). Clearly, \( n_H \) is the source of positive welfare in the market since these are the consumers who gain value from the product. In contrast, \( n_L \) not only represents the fraction of consumers who suffer losses, but also measures the fraction who seek remedies through the law. The quantity \( \rho_T n_L \), which represents the total amount of penalties paid to wronged customers, thus provides a good measure of the size of the legal system. Both \( n_H \) and \( n_L \) can be easily calculated from Figure 3. The following lemma computes and characterizes these quantities for given values of \( a \) and \( q \).

**Lemma 1** (Sales and Lawsuits). The number of consumers who are matches and mismatches are

\[
    n_H = \phi + q(1-\phi) + a[(1-q)(1-\phi)(\gamma + \delta) - \delta],
\]

\[
    n_L = (1-q)(1-\phi)[1 - a(\gamma + \delta)].
\]

The number \( n_L \) of mismatches is decreasing in \( \delta \), \( q \) and \( a \). The number \( n_H \) of matches is decreasing in \( \delta \) and increasing in \( q \). An increase in the advice level \( a \) increases (decreases) \( n_H \) when \( \delta \) is sufficiently small (large).

According to Lemma 1, as the firm increases scope for its products (increases quality) while the broker keeps his advising constant, fewer consumers are wronged since each product becomes a match for a larger fraction of consumers. However, although an increase in \( a \) still lowers the numbers of sales to low types, it is possible for advice to have a negative side effect and reduce the number of sales to high types. Indeed, when \( \delta > \frac{(1-q)(1-\phi)\gamma}{\phi + q(1-\phi)} \), the broker’s effort to sort consumers ends up costing the firm many sales. So, although the consumers who buy are better matched, the fact that many of them no longer buy products at all reduces the total number of consumers who benefit from the firm’s product offering. Finally, \( \delta \) has the same negative effect on \( n_H \) and \( n_L \): for a given \( a \) and \( q \), an advising function that is more attention-consuming reduces the total number of customers, some of whom would end up matched and some mismatched.

### 3.2.2 Optimal Broker and Firm Behavior

We begin our study of the model’s equilibrium by characterizing the behavior of the broker. The broker is paid a wage \( w \) for distributing the product to clients and incurs a cost of \( \frac{4}{2}a^2 \) for giving
advice $a$. He therefore solves
\[
\max_{a \in [0,1]} w - n_t \rho_A - \frac{k_A}{2} a^2,
\]
or equivalently using (4),
\[
\max_{a \in [0,1]} w - (1 - q)(1 - \phi)[1 - a(\gamma + \delta)] \rho_A - \frac{k_A}{2} a^2.
\]
(5)

The first-order condition yields
\[
a = \frac{(1 - q)(1 - \phi)(\gamma + \delta) \rho_A}{k_A}.
\]
(6)

By inspection of (6), the higher the penalties imposed by the law, the higher the advice that is given in the market. Interestingly, though, the higher the quality of the product, the less advising the broker chooses to do. This occurs because the marginal benefit to advice decreases due to the fact that clients are more likely to gain positive value from their purchase. This implies a natural tendency for the broker to free-ride on the quality provided by the firm. For lower levels of $\gamma$, the broker also tends to advise less. When consumers are difficult to sort, the broker will “take his chances” and allocate them to products almost randomly in order to save on the effort cost of advising clients.

Now, we consider the optimal behavior of the firm, given the price $p$ that forms in the market. The firm solves
\[
\max_{q \in [0,1]} n_sp - n_t \rho_F - \frac{k_F}{2} q^2 - w,
\]
or equivalently using (2) and (4),
\[
\max_{q \in [0,1]} (1 - a\delta) p - (1 - q)(1 - \phi)[1 - a(\gamma + \delta)] \rho_F - \frac{k_F}{2} q^2 - w.
\]
(7)

Note that, in this maximization problem, $p$ is treated as an exogenous constant by the firm. This arises because customers do not know $a$ or $q$ when $p$ is set; that is, neither the broker nor the firm can directly affect the customers’ willingness to pay more for financial products through their choice of $a$ and $q$. The first-order condition for the firm’s problem yields
\[
q = \frac{(1 - \phi)[1 - a(\gamma + \delta)] \rho_F}{k_F}.
\]
(8)

As such, the optimal quality choice of the firm is increasing in $\rho_F$ and is decreasing in $a$ and $\gamma$. As the amount of advice rises, there is a natural tendency for the firm to free-ride on the effort provision of the agent. Likewise, as the tendency for the broker to make errors decreases (i.e., as $\gamma$

\[\text{As mentioned above, we analyze sales commissions and the incentives they create in section 3.4.}\]
\[\text{This price will be further discussed later.}\]
increases), the marginal benefit of quality for the firm decreases, as it can rely more heavily on the agent to match customers and products and thereby to reduce the firm’s expected liabilities. Thus, as in the moral-hazard-in-teams problem of Groves (1973) and Holmström (1982), the firm and the agent free-ride on each other when they make their unobservable choices of quality and advice. Because this free-rider problem is affected by the difficulty of the agent’s task, the equilibrium degree of moral hazard that customers can expect also depends on $\gamma$.

Of course, in (6) and (8), $a$ and $q$ are expressed in terms of the other party’s optimal choice. In the following proposition, we solve for $a^*$ and $q^*$, the broker’s and firm’s optimal choices in terms of the primitives of the model.

**Proposition 1.** In equilibrium, the optimal amount of advice is given by

$$a^* = \frac{(1 - \phi)(\gamma + \delta)[k_F - (1 - \phi)\rho_F]\rho_A}{k_A k_F - (1 - \phi)^2(\gamma + \delta)^2\rho_A \rho_F},$$

whereas the optimal choice of quality is

$$q^* = \frac{(1 - \phi)[k_A - (1 - \phi)(\gamma + \delta)^2\rho_A]\rho_F}{k_A k_F - (1 - \phi)^2(\gamma + \delta)^2\rho_A \rho_F}.$$  

The broker’s choice of advice $a^*$ is increasing in $\rho_A$ and $\gamma$, and is decreasing in $\rho_F$. The firm’s optimal choice of quality $q^*$ is increasing in $\rho_F$ and is decreasing in $\rho_A$ and $\gamma$.

According to Proposition 1, the more the law holds the firm liable for the consumers’ misfortune, the higher the tendency for the firm to add more quality to their product. Likewise, the more the law penalizes the broker when a consumer is wronged, the higher effort the broker employs in giving sound advice. This has an important effect given the tendency for free-riding among the parties. That is, higher penalties for the firm will cause $q^*$ to rise, which will make advice less likely. In the same way, raising $\rho_A$ causes advice to increase, but leads to a lower quality in the market. Each party takes into account the penalties imposed on the other when they make their optimal choices. This will have important implications for the optimal law, as we show in Section 3.3.

Proposition 1 also shows that a more accurate advice channel leads to more advice and less quality. When $\gamma$ is large, the broker’s marginal benefit of effort is higher, making his investment in advising customers more appealing. Of course, this higher effort provision induces the firm to free-ride on better advice, thereby causing quality to decrease. Therefore, when $\gamma$ is high, the low quality of products makes each of them a more specialized match, but greater care is used when sorting consumers. When $\gamma$ is low, each product comes with more quality and more widespread appeal, but less care is used when sorting consumers.
3.2.3 Prices

Of the $n_S = n_H + n_L$ customers who show up to buy a product from the firm for a price $p$, $n_H$ will experience a utility of $+m$, while $n_L$ will experience a utility of $-m$ and receive $\rho_T = \rho_A + \rho_V$ through the legal system. Thus, the expected utility of any one customer is

$$U \equiv -p + \frac{n_H}{n_S}m + \frac{n_L}{n_S}(-m + \rho_A + \rho_V).$$

(11)

Since investors who do not show up experience a utility of zero, the customers’ reservation price $\hat{p}$ for the firm’s products is the price $p$ that makes (11) equal to zero:

$$\hat{p} = \frac{n_H}{n_S}m + \frac{n_L}{n_S}(-m + \rho_A + \rho_V).$$

(12)

Given a law $\mathcal{L} = \{\rho_A, \rho_V\}$, customers can anticipate $a^*$ and $q^*$, as chosen by the broker and the firm in (9) and (10). This means that they can also anticipate, through (3) and (4), the odds of eventually experiencing a match from their purchase.

The price $p$ that is charged by the firm lies in $[0, \hat{p}]$ and depends on who has more bargaining power. If the product market is competitive, then $p = 0$. If the firm is a monopolist, $p = \hat{p}$. However, as we will show in the next section, the optimal law that is set does not depend on which party has bargaining power. Since prices are merely transfers between market participants, they do not directly impact aggregate welfare.

By inspection of (12), when $\rho_A = \rho_V = 0$, $\hat{p} = 0$. That is, without advice or quality, consumers are not made better off from purchasing the firm’s products, and so they are not willing to pay positive prices for them. This market breakdown equilibrium can only be perturbed via positive values for $\rho_A$ and $\rho_V$, which make $a^*$ and $q^*$ positive and in turn $\hat{p}$ also positive. The law that is set becomes a major driver of positive welfare creation in the market.

3.3 Welfare and Legal Protection

We now analyze consumer protection law in this market. We begin by showing that without the law, the sale of financial products does not enhance welfare in the market: quality and advice are zero. Following this discussion, we derive and characterize the legal rules that maximize welfare in the market.

Since prices, wages, and penalties are transfers among market participants, total welfare reduces to the value that consumers gain minus the losses that wronged consumers suffer minus the costs of quality and advice. As such, maximizing total welfare in the market does not depend on which party has market power, that is whether the firm is a monopolist or there is perfect competition.
Welfare can be computed as
\[
W \equiv n_H \overline{m} - n_L \overline{m} - \frac{k_F}{2} q^2 - \frac{k_A}{2} a^2.
\] (13)

This is the quantity that the government seeks to maximize by setting the law \( L = \{\rho_A, \rho_F\} \), which affects the broker’s choice of \( a \) and the firm’s choice of \( q \), and therefore impacts the quantities \( n_H \) and \( n_L \).

Suppose indeed that no law exists, so that \( \rho_A = 0 \) and \( \rho_F = 0 \). From (9) and (10), we have \( a^* = 0 \) and \( q^* = 0 \). This implies that the firm and the agent cannot commit to provide quality service to clients in the absence of external incentives to do so. In other words, if customers expect any \( q \) or \( a \) above zero, it is always optimal for the firm and agent to provide them with less than that, as their choices are unobservable. Anticipating this problem, clients are unwilling to pay positive prices for the product (i.e., \( \hat{p} = 0 \)), as the ex ante surplus they derive from it is \( \phi \overline{m} - (1 - \phi) \overline{m} = 0 \). Thus the market is fully affected by the the moral hazard problem that consumers face and, as a result, the firm’s value and total welfare are both zero. The following proposition summarizes this finding.

**Proposition 2 (Absence of Law).** **Without a legal system (i.e., when \( L = \{\rho_A, \rho_F\} = \{0, 0\} \)), advice, quality and welfare are all zero (i.e., \( a^* = q^* = W = 0 \)).**

There are two reasons why retail financial markets depend so critically on the law for both preservation and prosperity. The first reason rests on the fact that financial products and services cannot be sold with warranties. For example, it is implausible for a firm to commit to a return policy on a portfolio, as such a guarantee would clearly create arbitrage opportunities and make the firm vulnerable to opportunistic behavior by consumers. Therefore, in the absence of such warranties, the law becomes necessary to prevent market breakdown.\textsuperscript{22, 23}

The second reason why the law may be necessary depends on whether the social structure allows for the natural formation of public trust in the market without formal legal incentives. Indeed, as Carlin, Dorobantu and Viswanathan (2009) show, if the value of social capital and the potential for productivity are sufficiently high, the law may be superfluous and even value destroying. However, in most cases when the market cannot depend on these other forces, some investor protection through the law enhances welfare. Economic growth and prosperity may require legal institutions that allow firms to credibly signal the quality of their products (e.g., Glaeser, Johnson and Shleifer, 2001).

\textsuperscript{22} As we will show shortly, providing insurance policies that make consumers whole if they are wronged does not achieve first-best anyway. Rather, the optimal law must include punitive damages as well.

\textsuperscript{23} In this one-period model, we implicitly ignore reputation effects that refunds and other warranties could have on subsequent buyers. Such effects would clearly complement the legal issues discussed here. Note however that reputation forces depend heavily on the public observability of wrong-doing and refunds, which may themselves require the presence of a legal system.
Without government, the presence of the market leaves total welfare unaffected, motivating an analysis of the law. The government’s objective is to maximize welfare by choosing the optimal law:

$$\max_{\rho_A, \rho_F} \quad W = n_H m - n_L m - \frac{k_F}{2} q^2 - \frac{k_A}{2} a^2.$$  \hspace{1cm} (14)

The following proposition solves this problem and characterizes the optimal law.

**Proposition 3.** The optimal law $L^* = \{\rho_A^*, \rho_F^*\}$ is given by

$$\rho_F^* = m + \bar{m} > 0,$$  \hspace{1cm} (15)

$$\rho_A^* = \frac{k_A \left[ \gamma k_F - (\gamma + \delta) \bar{m} \right] \bar{m}}{(1 - \phi)(\gamma + \delta) \left[ k_A (k_F - \bar{m}) - \delta (\gamma + \delta) \bar{m}^2 \right]} > 0.$$  \hspace{1cm} (16)

The penalty $\rho_A^*$ is strictly increasing in $\gamma$ and decreasing in $\delta$. The equilibrium level of advice and quality induced by $L^* = \{\rho_A^*, \rho_F^*\}$ is

$$a^* = \frac{[\gamma k_F - (\gamma + \delta) \bar{m}] \bar{m}}{k_A k_F - (\gamma + \delta) \bar{m}^2},$$  \hspace{1cm} (17)

$$q^* = \frac{[k_A - \gamma (\gamma + \delta) \bar{m}] \bar{m}}{k_A k_F - (\gamma + \delta) \bar{m}^2}.$$  \hspace{1cm} (18)

The equilibrium advice level $a^*$ is decreasing in $k_A$, increasing in $k_F$ and $\gamma$, and decreasing in $\delta$ for $\gamma$ sufficiently close to zero. The equilibrium quality level $q^*$ is increasing in $k_A$, decreasing in $k_F$ and $\gamma$, and increasing in $\delta$ for $\gamma$ sufficiently close to zero.

The fact that $\rho_F^* = m + \bar{m}$ implies that when consumers sue for damages, they capture $\rho_A^* + \rho_F^* > m + \bar{m}$. So, not only are they made whole through the suit, they are also awarded punitive damages for their troubles. Thus, punitive damages are inherent to a legal system that seeks to optimize welfare in a market for retail financial products, as insurance against bad outcomes, full or partial (i.e., $\rho_T \leq m + \bar{m}$), fails to maximize consumer welfare. Two key aspects of this market lead to this result: the fact that both the producer and the intermediary can improve the customer’s experience, and the fact that it is close to impossible for the legal system to assign blame with any kind of precision when a customer is wronged. The former implies that either party, or even both parties, can be responsible for mismatches between customers and products, while the latter implies that free-riding behavior will come with any law designed to protect consumers. This combination of factors directly causes the penalty escalation that is required in order to restore confidence in the market. In fact, consumers who are wronged achieve a better outcome than consumers who are properly served. We revisit this issue in Section 4.

As shown in the proof of Proposition 3, the optimal law induces the broker and the firm to choose the first-best levels of advice and quality respectively. As such, the comparative statics about $a^*$...
and $q^*$ characterize not only the model’s equilibrium but also the societal tradeoff between advice and quality. For example, when advice is relatively cheaper than quality ($k_\lambda$ small, $k_F$ large) and of greater quality ($\gamma$ large), welfare is improved with more advice and less quality. The effect of an increase in the importance of the marketing component of the broker’s function (i.e., an increase in $\delta$) depends on the size of $\gamma$. When $\gamma$ is small, it is socially optimal to rely less on the broker and more on the firm, as the marginal benefit of advice is then smaller than the marginal cost of losing customers. When $\gamma$ is large on the other hand, an increase in $\delta$ can lead to a decrease in both $q^*$ and $a^*$ and, in some cases, to a decrease in $q^*$ and an increase in $a^*$. Indeed, when $k_F$ is large, the marginal cost of increasing quality is so high that it is optimal to reduce it when fewer customers show up. If in addition $k_\lambda$ is small relative to $k_F$, it may even be optimal to serve this smaller set of customers with more broker advice.

Finally, according to Proposition 3, as the precision in evaluating consumers (i.e., $\gamma$) rises, the law makes penalties more severe for brokers when a consumer is wronged. This makes intuitive sense as the higher the precision is, the more likely the broker is at fault, given that a consumer is wronged. Less intuitive is the fact that the number of lawsuits and the size of the law, as measured by $(\rho_A + \rho_F)n_L$, are in general non-monotonic in $\gamma$. For example, when $\delta = 0$, it can be shown that the number of lawsuits and the legal system both get larger with $\gamma$ if and only if $k_F < \bar{m} \left( 2 - \frac{\bar{m}^2}{k_\lambda} \right)$, that is, when $k_\lambda$ is large, and $k_f$ and $\gamma$ are small. So although an increase in $\gamma$ facilitates customer sorting by the broker, it is suboptimal for the law to fully correct the simultaneous effect on lawsuits that results from the firm’s free-riding behavior.

### 3.4 Incentives and Conflicts of Interest

So far, we have assumed that the advisor is paid a fixed wage $w$ for distributing the product to consumers. We now consider the possibility for the firm to compensate the agent based on the number of sales. More specifically, suppose that the advisor receives a sales commission of $b$ for each consumer who makes a purchase. Total compensation in this case is $n_s b$, and the problem that the broker faces is

$$\max_{a \in [0,1]} n_s b - n_L \rho_A - \frac{k_\lambda}{2} a^2,$$

or equivalently,

$$\max_{a \in [0,1]} (1 - a \delta)b - (1 - q)(1 - \phi)[1 - a(\gamma + \delta)]\rho_A - \frac{k_\lambda}{2} a^2. \quad (19)$$

The following proposition characterizes the optimal choices of the firm and the advisor. It also characterizes the law $\mathcal{L}^b = \{\rho_A^b, \rho_F^b\}$ that is set by the government, taking into account the contractual

\[\text{Numerical solutions indicate that similar comparative statics hold when } \delta > 0, \text{ but closed-form solutions were unobtainable.}\]
arrangement between the firm and the broker.

**Proposition 4 (Incentives and the Law).** The optimal advice and quality choices by the broker and the firm are given by

\[ a^* = \frac{(1 - \phi)(\gamma + \delta)(k_F - (1 - \phi)\rho_F)\rho_A - k_F\delta b}{k_A k_F - (1 - \phi)^2(\gamma + \delta)^2 \rho_A \rho_F}, \]  

whereas the optimal amount of advice is

\[ q^* = \frac{(1 - \phi)[k_A - (1 - \phi)(\gamma + \delta)^2 \rho_A + (\gamma + \delta)\rho_F]b}{k_A k_F - (1 - \phi)^2(\gamma + \delta)^2 \rho_A \rho_F}. \]  

The law in the presence of sales commissions \( L^b = \{\rho^b_A, \rho^b_F\} \) is such that \( \rho^b_A > \rho^*_A \) and \( \rho^b_F < \rho^*_F \).

According to Proposition 4, incentives cause the optimal amount of advice to decrease and the optimal choice of quality to increase. Comparing (20) and (21) to (10) and (9) in Section 3.2, we can see that

\[ a^b = a^w - \frac{\delta k_F b}{k_A k_F - (1 - \phi)^2(\gamma + \delta)^2 \rho_A \rho_F}, \]  

and

\[ q^b = q^w + \frac{(1 - \phi)(\gamma + \delta)\rho_F b}{k_A k_F - (1 - \phi)^2(\gamma + \delta)^2 \rho_A \rho_F}, \]  

where \( q_j \) and \( a_j \) are the quality and advice for each form of compensation \( j \in \{b, w\} \). This implies that there exists a conflict of interest in which the advisor will look the other way when consumers place purchase orders. So while incentives are clearly required for advisors to distribute the product, such commissions may indeed make advisors less likely to do so responsibly. Of course, the firm takes this into account when it chooses \( q \). When there are conflicts of interest, the firm provides more quality to protect itself and avoid the penalties that would ensue due to the broker’s careless recommendations. The government also takes this into account when devising the legal system, relieving the firm a bit from responsibility and placing more of the blame on the advisor when consumers are wronged. This is consistent with the case law that deals with conflicts of interest and financial intermediaries (e.g., Kumpan and Leyens, 2008).

### 4 Heeding Advice and the Law

As pointed out in the previous section, the law is only able to impose severe punitive damages on the firm and advisor as long as consumers are not able to circumvent the advice that they receive from brokers. In this section, we investigate the role of the legal system when brokers cannot act as gate-keepers. Specifically, damages cannot be too great, and the expected payoff to heeding
advice must be superior to seeking the lottery-type payoffs that we derived previously. To make the analysis more tractable and our results more intuitive, we assume that the agent’s effort to advise does not reduce the flow of customers; that is, we assume that $\delta = 0$.

For a consumer to heed advice, it must be that the payoffs to ignoring advice are lower than the payoffs of following it. Given the information structure of Section 3, a customer following the broker’s advice to buy product $\tau$ can expect a payoff of

$$m \text{ with probability } \Pr\{\tilde{\tau} \in I_\tau | \tilde{s} = \tau\} = \gamma a + (1 - \gamma a)[\phi + (1 - \phi)q] = [\phi + (1 - \phi)q] + \gamma a(1 - \phi)(1 - q) \equiv \mu_1.$$  

This customer’s expected utility from buying product $\tau$ for a price $p$ is

$$E[\tilde{u} | \tilde{s} = \tau] = \mu_1 \bar{m} + (1 - \mu_1)(-\bar{m} + \rho_\lambda + \rho_F) - p. \tag{24}$$

If on the other hand the consumer decides not to follow the broker’s advice and to buy a random product $\tilde{t} \in [0, 1]$, the probability that the product is a match is only

$$\Pr\{\tilde{\tau} \in I_{\tilde{t}}\} = \phi + (1 - \phi)q \equiv \mu_0,$$

and so his expected utility from the transaction is

$$E[\tilde{u}] = \mu_0 \bar{m} + (1 - \mu_0)(-\bar{m} + \rho_\lambda + \rho_F) - p. \tag{25}$$

A simple comparison of (24) and (25) establishes that the customer follows the broker’s advice if and only if

$$(\mu_1 - \mu_0)\bar{m} \geq (\mu_1 - \mu_0)(-\bar{m} + \rho_\lambda + \rho_F),$$

or equivalently,

$$\rho_\lambda + \rho_F \leq \bar{m} + \bar{m}. \tag{26}$$

Thus the penalties set by the government cannot exceed the value that is on the line during the purchase. The higher the potential benefit to owning the right product or the loss that may be suffered in a mismatch, the higher the penalties that may be assessed without causing the advice market to break down. Because $\rho_\lambda + \rho_F > \bar{m} + \bar{m}$ in Proposition 3, we already know that this condition constrains the government’s welfare maximization problem (in (14)). The following proposition characterizes the optimal law under this constraint.

**Proposition 5** (Heeding Advice and Optimal Law). When the broker cannot impose his recommendation on the customers’ choice of product, the law set by the government is given by

$$\rho^*_\lambda = \frac{k_\lambda k_F[(k_F - \gamma \bar{m})(1 + \gamma) - 2\gamma \bar{m}(1 - \gamma)] - 2\gamma^2 \bar{m}^2(k_F - \bar{m}) - Q}{\gamma(1 - \phi)\{k_\lambda[2k_F\gamma(k_F - \gamma \bar{m}) + 2\gamma^2 \bar{m}^2 - k_F\gamma \bar{m}(1 + \gamma) - k_\lambda k_F(1 - \gamma)] + (k_\lambda - \gamma \bar{m})Q\}}, \tag{27}$$

$$\rho^*_F = \bar{m} + \bar{m} - \rho^*_\lambda. \tag{28}$$

22
where

\[
Q = \sqrt{k_A k_F \left\{ k_A [k_F (1 + \gamma)^2 - 4 \gamma \bar{m}] - 4 \gamma^2 \bar{m} (k_F - \bar{m}) \right\}}.
\]

(29)

The agent’s penalty \((\rho^*_A)\) is increasing in \(\gamma\), while the firm’s penalty \((\rho^*_F)\) is decreasing in \(\gamma\).

An important implication of Proposition 5 is that insurance is the only way to protect consumers in this market; that is, it is optimal to have \(\rho_A + \rho_F = \bar{m} + \bar{m}\). Penalties that make consumers whole can be assessed, but no punitive damages may be added. Given our discussion in Section 3, this means that the first-best scenario is not achievable in these markets. In other words, when the advisor cannot act as a gate-keeper and customers must effectively be persuaded to use the agent’s advice, optimal quality and advice cannot be reached. This means that freedom in the market leads to lower value creation, a striking result.

Figures 4-6 provide us with more insight into the equilibrium of Proposition 5. In these figures, we plot various equilibrium quantities as functions of \(\gamma\) and \(\bar{m}\). In each graph, there are two plots: the dashed curve represents the relationship that arises when the law is unconstrained (as in Section 3) and the solid curve represents what arises when the law needs to make sure that the advice market does not breakdown. This allows us to characterize the effects of such constraints.

Consistent with Proposition 5, we can see from Figures 4(a) and 4(b) that \(\rho_A\) is decreasing in \(\gamma\), while \(\rho_F\) is increasing in \(\gamma\): more blame is put on the agent when his task of sorting customer becomes more precise and thus easier. At the same time, more of the consumer surplus that is generated in the market depends on advising as opposed to quality products. Indeed, as we can see from Figures 4(c) and 4(d), \(a\) increases with \(\gamma\), while \(q\) decreases with \(\gamma\).

Interestingly, as we can see from Figures 4(e) and 4(f), this does not translate into monotonic relationships of \(n_H\) and \(n_L\) with \(\gamma\). This is where the nature of the relationship differs in the constrained and unconstrained settings. When heeding advice is important, the number of matched customers hits a minimum at \(\gamma \approx 0.47\). More consumers are matched with a product that is appropriate for them when \(\gamma\) is small or when it is large. In the former case, this is because the firm produces more quality; in the latter case, this is because the advice channel is reliable. As shown in Figure 4(g), this translates into a larger legal system for intermediate values of \(\gamma\). This directly contrasts with the relationship when the law is unconstrained. In fact, for high values of \(\gamma\), the size of the legal system under the constrained setting is higher than when the law is unencumbered in setting penalties. The two curves in Figure 4(g) cross when \(\gamma\) is high. The intuition for this finding is that in the unconstrained case, penalties can be sufficiently high so that lawsuits become less likely. The quantity \(n_L \rho_T\) drops because \(n_L\) decreases. In the constrained case, the law is less effective because there is a limit on the penalties that can be set. This allows \(n_L \rho_T\) to remain higher because \(n_L\) is higher.
Finally, let us define $B$ to be the fraction of consumers who are made better off through the advice channel. Because the firm's choice of $q$ implies that $\phi + (1 - \phi)q$ customers would be appropriately matched with a product if they picked one randomly, we have

$$B \equiv \frac{n_H}{\phi + (1 - \phi)q} - 1.$$  

Figure 4(h) plots this quantity as a function of $\gamma$. Clearly, because the broker advises more and because he does so with more accuracy as $\gamma$ goes up, the fraction of people that benefit from the advice channel increases at a faster rate than $a$ and $\gamma$.

Let us now turn to the effects of $m$ on the equilibrium quantities. Figures 5 and 6 explore these relationships. In these figures, we set $\phi = 0.5$, so that the restriction that $\phi m + (1 - \phi)w = 0$ implies that $\bar{m} = w$. That is, as we increase $\bar{m}$, it must be the case that $w$ increases along with it. In essence therefore, the horizontal axis in all these graphs measures the utility spread between a customer who is matched and one who is mismatched. When $\bar{m}$ and $w$ are both small, customers cannot benefit much from a match, nor can they be hurt much from a mismatch. As $\bar{m}$ and $w$ increase, the gain to a match and the loss to a mismatch both increase at the same rate. In Figure 5, $\kappa_A = 1$, whereas in Figure 6, $\kappa_A = 2$.

First, let us consider the case when $\kappa_A = 1$ in Figure 5. The quantities $\rho_A$, $\rho_F$, $a$, $q$, $n_H$, $n_L$, and $n_L\rho_T$ are all monotonically increasing in $m$. This generally implies that the strictness of the law, the effort provisions of market participants, and the size of the law grow as there is more value at risk in the market. In this case, identical costs of providing quality and advice cause the law to split the responsibility evenly, thereby driving these monotonic relationships.

Now, let us consider the case in which costs are highly asymmetric, that is, $\kappa_A = 2$. Although the firm’s penalty (Figure 6(b)) and provision of quality (Figure 6(d)) are monotonically increasing in $\bar{m}$, this is not the case for the agent’s penalty (Figure 6(a)) and advising intensity (Figure 6(c)). Instead, both of these quantities peak for intermediate values of $\bar{m}$. Because it is relatively more costly for the agent to provide advice, the agent is not relied upon to create consumer surplus when the stakes are small or when they are large. In fact, although we do not plot this in Figure 6, it is the case that the fraction of the total payment received by mismatched customers, $\frac{n_L\rho_T}{\bar{m} + w}$, is monotonically decreasing in $\bar{m}$. Thus, although the agent’s advice would be welcome by consumers who have a lot to gain or lose, it is optimal for the legal system to ensure a good matching process directly via the firm’s choice of quality.

As we see from Figures 6(e) and 6(f), this reliance on the firm is quite strong, as every consumer gets matched when $\bar{m} = 1$. As shown in Figure 6(g), the result is a legal system that is small with a small $\bar{m}$ (no point in punishing since there is not much to lose) or a large $\bar{m}$ (the quality is such
Figure 4: These figures show equilibrium quantities as functions of $\gamma$, when the law is unconstrained (dashed lines) and when the law is constrained to satisfy $\rho_A + \rho_F \leq \bar{m} + \bar{m}$ (solid lines). The parameters used for all figures are $\phi = 0.5$, $\bar{m} = \bar{m} = 1$, $k_A = 1$, and $k_F = 3$. 

(a) $\rho_A$ as a function of $\gamma$. 
(b) $\rho_F$ as a function of $\gamma$. 
(c) $a$ as a function of $\gamma$. 
(d) $q$ as a function of $\gamma$. 
(e) $n_H$ as a function of $\gamma$. 
(f) $n_L$ as a function of $\gamma$. 
(g) $n_L \rho_T$ as a function of $\gamma$. 
(h) $B$ as a function of $\gamma$. 


that every consumer is matched, and so no lawsuits take place). Finally, Figure 6(h) shows that the biggest gains from the advising process occur when the broker’s choice of $a$ is large.

5 Concluding Remarks

Protecting consumers in financial markets who are “unable to fend for themselves” is not only an important duty of the law, but also an important driver of participation in the market and economic growth. In this paper, we characterize the legal rules that maximize welfare in markets in which producers of financial markets outsource their advice services.

The model that we analyze is one of bilateral hidden action: firms choose the quality of the goods they produce and brokers advise consumers when they make their purchases. Without the law, neither party can commit to acting in the best interest of consumers, and little of the economic surplus that markets can potentially generate is actually realized. With the law, the two parties tend to free-ride on each other’s effort provision: as the firm commits to higher quality, the broker has a lower incentive to give advice, and vice versa. When financial decisions are more complex and matching consumers with products becomes more difficult, the broker adds less value in the market.

We show that the law not only makes wronged consumers whole, but provides them with punitive damages. In fact, such a welfare-maximizing legal system achieves first-best quality and advice. In addition, we show that the use of sales commissions in compensation contracts causes a conflict of interest in which brokers tend to give less advice. The law circumvents this problem by increasing penalties to brokers and decreasing penalties to firms when customers are wronged in the market.

Given the large size of retail financial markets and the recent economic impact of the subprime mortgage crisis in the U.S., we feel that the analysis in this paper has significant welfare import.
Figure 5: These figures show equilibrium quantities as functions of $\bar{m}$, when the law is unconstrained (dashed lines) and when the law is constrained to satisfy $\rho_A + \rho_F \leq \bar{m} + \bar{m}$ (solid lines). The parameters used for all figures are $\phi = 0.5$ (so that $\bar{m} = \bar{m}$), $\gamma = 0.5$, $k_A = 2$, and $k_F = 2$. 
Figure 6: These figures show equilibrium quantities as functions of $\overline{m}$, when the law is unconstrained (dashed lines) and when the law is constrained to satisfy $\rho_A + \rho_F \leq \overline{m} + \overline{m}$ (solid lines). The parameters used for all figures are $\phi = 0.5$ (so that $\overline{m} = \overline{m}$), $\gamma = 0.5$, $k_A = 2$, and $k_F = 1$. 

(a) $\rho_A$ as a function of $\overline{m}$.
(b) $\rho_F$ as a function of $\overline{m}$.
(c) $\alpha$ as a function of $\overline{m}$.
(d) $\eta$ as a function of $\overline{m}$.
(e) $n_H$ as a function of $\overline{m}$.
(f) $n_L$ as a function of $\overline{m}$.
(g) $n_L \rho_T$ as a function of $\overline{m}$.
(h) $B$ as a function of $\overline{m}$. 

28
Appendix

Proof of Lemma 1

We can use Figure 3 to calculate \( n_H \) (\( n_L \)), by adding up the number of customers who experience a utility of \( +\mathfrak{m} \) (\(-\mathfrak{m}\)). Thus we have

\[
n_H = (1 - a - \delta a)[\phi + (1 - \phi)q] + a[\phi + (1 - \phi)\gamma + (1 - \phi)(1 - \gamma)q]
\]

and

\[
n_L = (1 - a - \delta a)(1 - \phi)(1 - q) + a(1 - \phi)(1 - \gamma)(1 - q),
\]

which simplify to (3) and (4) respectively. The comparative statics follow from straightforward differentiation. ■

Proof of Proposition 1

In a Nash equilibrium, the agent (the firm) correctly anticipates the firm’s (agent’s) choice of \( q \) (\( a \)). Thus their equilibrium choice of \( a \) and \( q \) must solve (6) and (8). This leads to (9) and (10). Simple but tedious differentiation of these two expressions with respect to \( \rho_A \), \( \rho_F \), \( \gamma \) and \( \delta \) completes the proof. ■

Proof of Proposition 3

The problem in (14) is equivalent to

\[
\max_{a,q} \quad W = n_H\mathfrak{m} - n_L\mathfrak{m} - k_F q^2 - k_A a^2,
\]

subject to (9) and (10). In fact, when first-best is attainable, we can simply maximize (A1) with respect to \( a \) and \( q \), and find the penalties \( \rho_A \) and \( \rho_F \) that make (9) and (10) equal to the first-best values of \( a \) of \( q \). After replacing \( n_H \) and \( n_L \) by (3) and (4) respectively, the first-order condition for this maximization problem is found to be

\[
0 = [(1 - q)(1 - \phi)(\gamma + \delta) - \delta]\mathfrak{m} + (1 - q)(1 - \phi)(\gamma + \delta)\mathfrak{m} - k_A a
\]

and

\[
0 = [(1 - \phi) - a(1 - \phi)(\gamma + \delta)]\mathfrak{m} + (1 - \phi)[1 - a(\gamma + \delta)]\mathfrak{m} - k_F q.
\]

Solving for \( a \) and \( q \) in these two equations, using the restriction that \( \phi\mathfrak{m} - (1 - \phi)\mathfrak{m} = 0 \) to simplify, yields (17) and (18). The expressions in (15) and (16) result from equating (9) and (10) with (17) and (18), and solving for \( \rho_F \) and \( \rho_A \). Given the condition that \( k_F \) > \( \frac{k_A}{\gamma} \mathfrak{m} \), the numerator in (16) is
clearly strictly greater than zero. The same condition, along with the condition that $k_A > (\gamma + \delta)\bar{m}$, also implies that the denominator of (16) is strictly positive, as

$$k_A(k_F - \bar{m}) - \delta(\gamma + \delta)\bar{m} > k_A \frac{\delta}{\gamma} \bar{m} - \delta(\gamma + \delta)\bar{m}^2 = \frac{\delta}{\gamma} \bar{m}[k_A - \gamma(\gamma + \delta)\bar{m}] > 0.$$ 

It is clear from (17) that $a^*$ is decreasing in $k_A$. The derivative of $a^*$ with respect to $k_F$ is

$$\frac{\partial a^*}{\partial k_F} = \frac{m^2(\gamma + \delta)\{k_A - \bar{m}(\gamma + \delta)\}}{[k_A k_F - m^2(\gamma + \delta)^2]^2}.$$ 

This quantity is clearly positive given the assumption that $k_A > (\gamma + \delta)\bar{m}$ and the fact that $\gamma < 1$. The derivative of $a^*$ with respect to $\gamma$ is

$$\frac{\partial a^*}{\partial \gamma} = \frac{m\{k_A k_F(k_F - \bar{m}) + m^2(\gamma + \delta)[k_F(\gamma - \delta) - \bar{m}(\gamma + \delta)]\}}{[k_A k_F - m^2(\gamma + \delta)^2]^2}.$$ 

The assumption that $k_F > \frac{\gamma + \delta}{\gamma} \bar{m}$ implies that

$$\frac{\partial a^*}{\partial \gamma} > \frac{m\{k_A k_F\bar{m}(\gamma + \delta)\delta k_F\}}{[k_A k_F - m^2(\gamma + \delta)^2]^2} = \frac{k_F m^2 \delta\{k_A - \bar{m}(\gamma + \delta)\gamma\}}{\gamma[k_A k_F - m^2(\gamma + \delta)^2]^2},$$

which is positive given the assumption that $k_A > (\gamma + \delta)\bar{m}$. The derivative of $a^*$ with respect to $\delta$ is

$$\frac{\partial a^*}{\partial \delta} = -\frac{m^2\{k_A k_F + \bar{m}(\gamma + \delta)\{-2k_F\gamma + \bar{m}(\gamma + \delta)\}\}}{[k_A k_F - m^2(\gamma + \delta)^2]^2}.$$ 

Since $k_A k_F > 0$ and $m^2(\gamma + \delta)^2$, the expression in curly brackets is positive (and so $\frac{\partial a^*}{\partial \delta}$ is negative) when $\gamma$ is close to zero, as then $-2k_F\gamma$ is also close to zero. The comparative statics for $q^*$ can be established similarly. ■

**Proof of Proposition 4**

Given the agent’s problem in (19), the first-order conditions for the agent’s and the firm’s maximization problems are

$$a = \frac{(1 - \phi)(1 - q)(\gamma + \delta)\rho_A - \delta b}{k_A}$$

and

$$q = \frac{(1 - \phi)[1 - a(\gamma + \delta)]\rho_F}{k_F}.$$ 

Direct substitution yields (20) and (21). Given the relationships in (22) and (23), and the fact that the law can induce first-best, it must be the case that $\rho_A^A > \rho_A^*$ and $\rho_F^F < \rho_F^*$. ■
Proof of Proposition 5

The government seeks to solve

$$\max_{\rho_A, \rho_F} W = n_hm - n_lm - \frac{k_Fq^2}{2} - \frac{k_Aa^2}{2},$$

subject to (9), (10) and $\rho_A + \rho_F \leq \bar{m} + \bar{w}$, where $n_h$ and $n_L$ are given by (3) and (4). Since we know that first-best cannot be achieved, it is never optimal for the government to set $\rho_A$ and $\rho_F$ such that $\rho_A + \rho_F < \bar{m} + \bar{w}$. As such, the third constraint must be satisfied with equality, that is, $\rho_A + \rho_F = \bar{m} + \bar{w}$ or, using (6) and (8),

$$\frac{ak_A}{(1-q)(1-\phi)(\gamma+\delta)} + \frac{qk_F}{(1-\phi)[1-a(\gamma+\delta)]} = \bar{m} + \bar{w}. \tag{A2}$$

Recall from Proposition 1 that $a^*$ in (9) is increasing in $\rho_A$ and decreasing in $\rho_F$ while $q^*$ in (10) is increasing in $\rho_F$ and decreasing in $\rho_A$. This implies that both $\rho_A$ and $\rho_F$ must be increased in order to increase $a^*$ without affecting $q^*$ or in order to increase $q^*$ without affecting $a^*$. Thus the government’s maximization problem is equivalent to

$$\max_{a, q} W = n_h\bar{m} - n_L\bar{m} - \frac{k_Fq^2}{2} - \frac{k_Aa^2}{2},$$

subject to $a + q = t$, where $t$ is chosen so that this constraint is equivalent to (A2).

For any given $t > 0$, it is easy to show that the solution to this problem is given by $q = A + Bt$ and $a = -A + (1 - B)t$ with

$$A = \frac{(1-\gamma)\bar{m}}{k_A + k_F - 2\gamma\bar{m}}, \quad \text{and}$$

$$B = \frac{\bar{m} - \gamma\bar{m}}{k_A + k_F - 2\gamma\bar{m}}.$$ 

Using these expressions in (A2) and manipulating yields a quadratic expression in $t$ with a unique positive root. We can use this root to get $a = -A + (1 - B)t$, insert the resulting expression for $a$ in (6), and solve for $\rho_A$. This yields (27). The solution for $\rho_F$ in (28) comes from the constraint that $\rho_A + \rho_F = \bar{m} + \bar{w}$. \[\blacksquare\]
References


