Formal Contracts, Relational Contracts, and the Holdup Problem

Hideshi Itoh†  Hodaka Morita‡

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†Graduate School of Commerce and Management, Hitotsubashi University.
‡School of Economics, Australian School of Business, University of New South Wales.
Abstract

Can formal contracts help resolving the holdup problem? We address this important question by studying the holdup problem in repeated transactions between a seller and a buyer in which the seller can make relation-specific investments in each period. In contrast to previous findings, we demonstrate that a simple fixed-price contract based on product delivery can help mitigating the holdup problem even when relation-specific investment is purely cooperative. In particular, there is a range of parameter values in which a higher investment can be implemented only if a formal fixed-price contract is written and combined with an informal agreement on additional payments contingent upon investments. Furthermore, we show that under an additional natural assumption, focusing our attention on fixed-price contracts as a form of formal contracts is without loss of generality. The key driving force of our result is a possibility that relation-specific investment decreases the value of no-trade surplus. This possibility, although very plausible, has been largely ignored in previous theoretical analyses of the holdup problem.

Keywords: Holdup problem, formal contract, relational contract, cooperative investment, fixed-price contract, relation-specific investment, repeated transactions, long-term relationships.
1 Introduction

Relation-specific investments often cause holdup problems when contracting is incomplete. Suppose, as an example, that a seller has an opportunity to make an investment which creates more value inside its relationship to a particular buyer than outside. The relation-specific nature of the investment may result in the buyer’s opportunistic behavior. Contracts contingent upon investment-related information could protect the seller, but this is often difficult in reality. So, without adequate contractual protection, the seller’s anticipation of the buyer’s opportunistic behavior results in a less than socially optimal level of investment.

The holdup problem has played a central role in the economic analysis of organizations and institutions, and many authors have proposed various organizational interventions, such as vertical integration (Klein et al., 1978; Williamson, 1985), as remedies to the problem.

In the holdup literature, a fundamental driving force of the inefficiency has been the assumption that contracts contingent upon the nature of relation-specific investments are infeasible, which is a realistic assumption in a wide variety of real-world bilateral trade. At the same time, the courts can often verify delivery of the goods by the seller, and hence formal fixed-price contracts based on product delivery are often feasible. More general formal contracts that may be contingent upon the parties’ messages (i.e., reports on the state) should also be considered.

Can formal contracts help resolving or mitigating the holdup problem? The present paper offers new perspectives on this important question by incorporating two key ingredients in our model. The first ingredient concerns effects of relation-specific investments on the value of no-trade surplus. Most previous theoretical models in the holdup literature assume, implicitly or explicitly, that relation-specific investment weakly increases the value of the asset not only within the relationship but also in alternative uses (no-trade surplus). However, an equally plausible assumption is that the investment reduces the value of no-trade surplus, and our model incorporates this case. This is evident in site specificity. If a seller locates its plant adjacent to a buyer, the seller ends up increasing the distance of the plant to alternative buyers. That is, a site-specific investment decreases the value of no-trade surplus. See Section 2 for discussions on other types of asset specificity. Given this, relation-specific investment may increase or decrease the value of no-trade surplus in our model.

The second ingredient is repeated transactions between a seller and a buyer. In reality, relation-specific investments are often made under long-term and repeated interaction between parties. Coase (1988) pointed out that A.O. Smith, a large independent manufacturer of automobile frames, had invested in expensive equipment that was highly specific to its main customer, such as General Motors, for more than 50 years. Also, Coase (2000)
found that prior to the acquisition of Fisher Body by General Motors in 1926, Fisher Body had repeatedly made location-specific investments for General Motors. Regarding Japanese manufacturer-supplier relationships, Asanuma (1989) studied the Japanese automobile and the electric machinery industries and discovered that long-term relationships were more likely to be found in the transaction of intermediate products that require a high degree of relation-specific investments. According to Holmström and Roberts (1998, p.83), “Nucor [the most successful steel maker in the United States over the past 20 years] decided to make a single firm, the David J. Joseph Company (DJJ), its sole supplier of scrap. Total dependence on a single supplier would seem to carry significant hold-up risks, but for more than a decade, this relationship has been working smoothly and successfully.”

Despite the important connection between relation-specific investments and long-term relationships, there have been very few theoretical analyses, to the best of our knowledge, that have addressed the holdup problem under infinitely repeated interactions. This might be because, due to a reasoning based on the Folk Theorem, the holdup problem can obviously be resolved under infinitely repeated interactions if the discount factor is high enough. We show, however, that when the discount factor is not high enough, formal contracts can play a crucial role in resolving the holdup problem when relation-specific investment decreases the value of no-trade surplus.

The present paper is not the first one to explore the roles of formal contracts in resolving the holdup problem. Recently several articles have studied this research question under spot transaction. Edlin and Reichelstein (1996) considered a bilateral trade relationship in which the seller and the buyer can write a simple contract specifying a fixed trade price and quantity at a future date. The seller then decides how much to invest in a relation-specific asset that lowers the subsequent cost of producing the good. After the investment is made, the buyer and the seller are free to renegotiate the contract with exogenously specified bargaining power. Edlin and Reichelstein found that a well-designed fixed-price contract can give the seller efficient investment incentives.

Che and Hausch (1999) pointed out that these previous studies were limited by their restriction on the nature of the relation-specific investments; that is, these studies focused on “selfish” investments that benefited the investor (e.g., the seller’s investment reduces his/her production costs). Che and Hausch convincingly argued through a number of real-world examples that “cooperative” investments (e.g., the seller’s investment improves the buyer’s value of the good) were equally important, although cooperative investments had received

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1Note that while several recent papers introduce dynamic structures into the analysis of the holdup problem (Che and Sákovics, 2004; Gul, 2001; Pitchford and Snyder, 2004), they study repeated offers rather than repeated transactions.
little attention in the literature. For instance, the famous General Motors-Fisher Body example deals with Fisher Body’s decision of building a plant adjacent to General Motors. Such an arrangement involves a “selfish” as well as a “cooperative” aspect because it not only lowers the seller’s shipping costs but also improves its supply reliability.

Che and Hausch’s results for cooperative investments are very different from those of Edlin and Reichelstein for selfish investments although they considered a bilateral trade relationship similar to the one analyzed by Edlin and Reichelstein. The most important result of Che and Hausch concerns the case in which the parties cannot credibly commit not to renegotiate the contract. They showed that if investments are sufficiently cooperative, there exists an intermediate range of bargaining shares for which contracting has no value, i.e., contracting offers the parties no advantages over ex post negotiation. In particular, contracting has no value for any parameter range if both investments are purely cooperative (that is, the seller’s investment benefits the buyer only, and the buyer’s investment benefits the seller only).

Edlin and Reichelstein and Che and Hausch both assume that relation-specific investment has no effects on the value of no-trade surplus. We contribute to the literature by showing that, even when relation-specific investment is purely cooperative, formal contracts can still help resolving the holdup problem when relation-specific investment reduces the value of no-trade surplus.

We consider a standard setup of the holdup problem in which a buyer purchases 0 or 1 units of a product from a seller. The seller chooses a level of relation-specific investment, denoted $a$, by incurring private costs. The investment increases the product’s value for the buyer, but has no effects on the seller’s production cost. That is, the investment is purely cooperative. A formal contract can be signed prior to the seller’s investment decision. After the investment is made, some state uncertainty, denoted $\theta$, is revealed and observed. The buyer and the seller are then free to (re)negotiate the price with exogenously specified bargaining power, whether or not formal contracts have been signed. Let $\rho(a, \theta)$ denote the negotiation price in the absence of formal contracts. Unlike most previous models considered in the holdup literature, our model allows a possibility that relation-specific investment decreases the alternative-use value, and this in turn implies that $\rho(a, \theta)$ can be decreasing in the investment level $a$.

We first study the value of formal contracts under spot transaction. We find that formal contracts, even if we allow contracts to be contingent on messages, are of no value in resolving or mitigating the holdup problem under an assumption that $\rho(a, \theta)$ is either weakly increasing or weakly decreasing in $a$ for all realizations of uncertainty $\theta$. Although we believe that this

\[ \text{The contract can be a simple fixed-price contract based on product delivery or a more complex contract contingent upon the parties’ reports on the investment and the state.} \]
is a natural assumption, implying that the effects of uncertainty $\theta$ be not too large to alter the sign of the effects of investment $a$ on the negotiated price, our contribution actually does not rely on it: If this assumption does not hold, formal contracts can be of value even under spot transaction in our model. In other words, this is a conservative assumption in our attempt to show that formal contracts are of value in resolving the holdup problem even when relation-specific investment is purely cooperative.

Next we consider repeated transactions, and find that formal contracts can help resolving or mitigating the holdup problem even under the conservative assumption mentioned above. Furthermore, we find that it is without loss of generality to confine our attention to formal fixed-price contracts. That is, whenever a simple fixed-price contract can help resolving or mitigating the holdup problem, the parties cannot get strictly better off by signing a more complex contract that is contingent upon messages. We then show that the qualitative nature of our results remains unchanged under an extension of our model that allows multiple quantities (rather than 0 or 1) to be transacted between the seller and the buyer.

Below we illustrate the logic behind our main findings by using a simpler version of our model in which (i) the level of the seller’s investment $a$ is either $a = 0$ (“do not invest”) or 1 (“invest”) and the cost for the investment is $d > 0$, (ii) the buyer has the entire bargaining power (that is, the buyer can make a take-it-or-leave-it price offer to the seller), and (iii) there is no uncertainty. We focus on simple fixed-price contracts as the form of formal contracts. Let $v_a$ be the value for the buyer and $m_a$ the alternative-use value under investment level $a = 0, 1$, where $v_1 - v_0 > 0$ holds. As mentioned above, relation-specific investment may increase the alternative-use value (i.e. $m_1 - m_0 \geq 0$) or decrease it (i.e. $m_1 - m_0 < 0$) in our model. Assume (i) $v_1 > m_1$; (ii) $v_1 - v_0 > v_1 - m_0 > d$; and (iii) $m_1 - m_0 < d$. The first two assumptions imply that investment and trade are the efficient outcome while no trade is ex post efficient under no investment ($m_0 > v_0$). The third assumption implies that no investment is efficient if they do not trade, and we will see below that the same assumption will also imply “under-investment” under spot transaction.

Under spot transaction without formal contracts, the buyer’s take-it-or-leave-it price offer is $m_a$ if investment level is $a = 0, 1$. Since $m_0 - (m_1 - d) = -(m_1 - m_0) + d > 0$, the seller chooses not to invest (under-investment). By our assumption $m_0 > v_0$, no trade occurs and the seller obtains the alternative-use value $m_0$. The seller would choose to invest if the buyer could commit to cover the investment cost $d$ by paying $m_0 + d$ contingent upon investment. Such contingent price contracts, however, are often infeasible in reality, and hence we assume

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3Formal fixed-price contracts cannot be of value even without this assumption. However, more complex contracts contingent upon the parties’ reports on the state can be of value under spot transaction.

4Our analysis on more general versions of the model indicates that it is without loss of generality to confine our attention on fixed-price contracts in this simpler version of the model.
that they are not possible in our model. Simple fixed-price contracts are not helpful under spot transaction. This is because, under non-contingent fixed-price contracts, the buyer virtually commits herself to paying the same price whether or not the investment is made, and hence the seller chooses not to invest.

Now consider repeated transactions. Suppose that the buyer informally promises to pay \( m_0 + d \) contingent upon investment. The seller does invest, if it is anticipated that the buyer keeps the promise. But, once the seller invests, the buyer has a temptation to renege on the implicit promise and purchase the product at the price \( m_1 \). The buyer’s reneging temptation is \( (m_0 + d) - m_1 = d + (m_0 - m_1) > 0 \). Suppose relation-specific investment reduces the alternative-use value so that \( m_0 - m_1 > 0 \) holds. Then, a simple fixed-price contract eliminates \( m_0 - m_1 \) from the reneging temptation, because, under such a contract, the buyer credibly commits to pay a fixed price regardless of the level of investment. By the same reason, however, the seller would choose not to invest under the fixed-price contract. Therefore, in order to induce the seller to invest, the buyer needs to combine the fixed-price contract \( p = m_0 \) with an implicit promise of covering the investment cost \( d \) as a bonus. The buyer’s temptation to renege on this implicit promise is \( d \), which is less than \( d + (m_0 - m_1) \).

Hence, a formal fixed-price contract combined with an implicit bonus can help mitigating the holdup problem by reducing the buyer’s reneging temptation under repeated transactions. Similar basic logic carries over into more general versions of the model as we show in later sections. The rest of the paper is organized as follows: Section 2 offers detailed discussions on the effect of relation-specific investment on the value of no-trade surplus, and Section 3 relates the present paper to the existing literature. Section 4 presents and analyzes our base model in which a buyer purchases 0 or 1 units of a product from a seller. Section 6 explores an extension of the model that allows multiple quantities to be transacted, and Section 7 offers concluding remarks.

2 Relation-Specificity and the Value of No-Trade Surplus

In Introduction we argued, in the context of site-specific investment, the plausibility of the assumption that relation-specific investment decreases the value of no-trade surplus. This section offers detailed discussions on other types of asset specificity.

Concerning physical asset specificity, Rajan and Zingales (1998) pointed out, “The specialization of an asset implies almost by definition a reduction in the outside value of that asset” (p. 408). Iyer and Schoar (2008) used custom-printed lots of pens as an example of physical asset specificity in their recent field experiment regarding the holdup problem. A

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5Cobbaut and Lenoble (2003) pointed out that this is a natural assumption.
specific logo (e.g., company logo) printed on the pens increases the value of the pens for a specific buyer. However, it may decrease the value of no-trade surplus because, in order to sell the pens to alternative buyers, the seller may have to remove the logo by incurring adjustment costs. Similar idea is incorporated in Hart (1995)’s model, where he argues that, once a seller has made relation-specific investment, the seller will have to make some adjustments to turn its product into a general-purpose one before selling it to alternative buyers (Hart, 1995, p.36). See, for example, Andrabi et al. (2006) and Banerjee and Basu (2009) for recent theoretical analyses that incorporate the idea that relation-specific investment decreases the value of the no-trade surplus.

Also, even if relation-specific investment itself does not decrease the value of no-trade surplus, it may still end up decreasing the value of no-trade surplus if relation-specific investment and general-purpose investments are substitutes for investors. For example, consider a seller who can make two kinds of investments, a relation-specific investment (zero or one unit) that increases the value of its product only for a specific buyer, and a general-purpose investment (zero or one unit) that increases the value of its product for all potential buyers. If the seller can make only one unit of investment because of various resource constraints, then the seller’s relation-specific investment reduces the value of no-trade surplus by preventing itself from making the general-purpose investment. This idea can be applied, among other things, to human asset specificity: If a worker can spend only one hour per day for training, spending the limited time for acquiring skills specific to his employer could reduce the value of no-trade surplus by preventing him from acquiring general skills that are also applicable to some other potential employers.

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6One way to formalize this idea is as follows: Suppose that investment 0 represents a general-purpose investment, while 1 represents a relation-specific investment. If the seller makes the general-purpose investment, he/she can produce the general product that has value $m_0$ for alternative users. If the seller makes the relation-specific investment, he/she can produce the product that is customized to the buyer. And if an alternative user purchases the specific product, the user must incur an adjustment cost $k > 0$ in order to convert it to the general product, and hence the effective value of the specific product for the alternative user is $m_0 - k$, which is smaller than $m_0$.

7In Hart (1995)’s model, a seller’s relation-specific investment affects the seller’s cost. In particular, the seller’s cost to produce a widget for a specific buyer is $C(e)$ and the production cost is $c(e;B)$ for alternative buyers, where $e$ denotes the level of the seller’s relation-specific investment. Hart assumes that the investment $e$ is more effective for $C(e)$ than for $c(e;B)$ because, “If trade does not occur, M2 will sell her widget on the competitive spot market for $\bar{p}$, but will have to make some adjustments to turn it into a general-purpose widget.” In his model, relation-specific investment increases the value of no-trade surplus, because it assumes that $c(e;B)$ is decreasing in $e$. This, however, may not necessarily be the case. If adjustment costs are substantially high and increasing in $e$, $c(e;B)$ can also be increasing in $e$, implying that relation-specific investment decreases the value of no-trade surplus.

8See Cai (2003) who studies such a multi-dimensional investment model in which increasing relation-specific investment reduces general-purpose investment and hence reduces the outside value.
3 Relationship to the Literature

In Introduction we discussed how our paper is related to Edlin and Reichelstein and Che and Hausch. In this section we discuss relationships between our paper and some other previous theoretical papers on the holdup problem, and also our contribution to the empirical literature on the relationship between relational governance and formal contracts.

Our analysis of informal agreements builds on a general analysis of relational contracts by MacLeod and Malcomson (1989) and Levin (2003). Baker et al. (1994), Schmidt and Schnitzer (1995), and Pearce and Stacchetti (1998) study how formal contracting affects the self-enforceability of informal agreements. These three papers do not, however, analyze the holdup problem caused by the relation-specific nature of investments and incompleteness of contracting in the sense that they allow contingent formal contracts while they do not consider ex post price negotiation. In our model, on the other hand, formal contracts and ex post negotiation play crucial roles. Baker et al. (2001, 2002), Halonen (2002), and Morita (2001) analyze the holdup problem in infinitely repeated transactions, but their focus is quite different from ours. Baker et al. (2001, 2002) and Halonen (2002) study how asset ownership affects the self-enforceability of relational contracts, and Morita (2001) focuses on the role of partial ownership in resolving the holdup problem under repeated interaction. None of the studies capture the idea that formal contracts can play an important role in reducing reneging temptations under repeated transactions,\(^9\) nor do they identify whether the alternative-use value is increasing or decreasing in investment as an important factor in determining the value of formal contracting.

The present paper also sheds new light on recent empirical investigations on the relationship between relational governance and formal contracts. In the empirical literature of transaction cost economics, the majority of previous researchers have studied how several transactional properties (representing asset specificity, uncertainty, and transactional frequency) affect an organizational mode, conceptualized by market, hierarchy, or various hybrid and intermediate modes (see, for example, Shelanski and Klein (1995) and Boerner and Macher (2002) for surveys).

Several researchers have recently made an important contribution to this literature by investigating the relationship between relational governance and formal contracts (see, for example, Banerjee and Duflo (2000); Poppo and Zenger (2002); Kalnins and Mayer (2004)). It has often been argued that relational governance and formal contracts are substitutes rather than complements (see Dyer and Singh (1998) and Adler (2001), among others), and that the

\(^9\)Although Baker et al. (2001, 2002) employ a holdup model different from ours, integration in their model and fixed-price contracts in our model play a similar role of eliminating ex post opportunities for price negotiation. However, they do not consider more general formal contracts contingent on messages.
use of formal contracts may even have undesirable consequences under relational governance (see Macaulay (1963) for an empirical investigation and Bernheim and Whinston (1998) for a theoretical analysis). In contrast, Poppo and Zenger (2002) have recently presented evidence which suggests that relational governance and formal contracts can be complements. In their investigation of informational service outsourcing they found that, controlling for several transactional properties such as asset specificity, increases in the level of relational governance were associated with greater levels of complexity in formal contracts (see Ryall and Sampson (2008) for a related finding).

We contribute to this line of investigation by exploring the relationship between relational governance and formal contracts in the presence of the holdup problem. Our analysis identifies whether the alternative-use value is increasing or decreasing in investment as an important factor in determining the value of formal contracts. We find that relational governance and formal contracts can be complements or substitutes, and that the use of formal contracts may even have undesirable consequences. Our analysis indicates that they are complements when the relation-specific investment reduces the negotiated price, and a necessary condition for this is that the investment reduces the alternative-use value.

4 Model

We consider repeated transactions between an upstream party (seller) and a downstream party (buyer).\(^\text{10}\) In each period, the seller chooses an action (investment level) \(a \in A\) by incurring private cost \(d(a)\). Although the set of feasible actions \(A\) can be fairly general (e.g., multi-dimensional), to simplify exposition we assume \(A \subseteq \mathbb{R}\), and \(a \in A\) is measured in terms of the investment costs, and hence \(d(a) = a\). The feasible set of investment \(A\) can be finite with more than one elements, countably infinite, or continuous. We assume there exists the least costly action in \(A\), denoted by \(a \geq 0\).

The seller’s investment affects (i) the value of the seller’s product for the buyer and (ii) the alternative-use value of the product. Let \(v(a, \theta)\) be the value for the buyer when the seller’s investment is \(a\), where \(\theta\) is the state of nature drawn from support \(\Theta = [\underline{\theta}, \overline{\theta}]\) by a cumulative distribution function \(F(\cdot)\), and let \(m(a, \theta)\) be the alternative-use value, which we assume for simplicity to be obtained by the seller, and the payoff to an alternative user is zero. The buyer’s outside payoff is independent of the seller’s investment and normalized to zero, and hence the total no-trade surplus is equal to \(m(a, \theta)\). To highlight the value of formal contracting in repeated transaction, we assume that at most one unit of an indivisible

\(^{10}\)Throughout the paper we suppose the seller is a male and the buyer is a female for convenience.
product is traded, and the production cost is normalized to zero.\textsuperscript{11} Later in Section 6 we will extend our main results to the case in which different levels of quantity are traded. We assume \( v(a, \theta) \) is strictly increasing in \( a \) and \( v(a, \theta) > a \) for all \( a \) and \( \theta \).

The alternative-use value \( m(a, \theta) \) may be increasing or decreasing as discussed in the previous sections. We, however, follow the holdup literature by assuming that investment affects \( v(a, \theta) \) at least as much as \( m(a, \theta) \) at margins:

\[
v(a, \theta) - v(a', \theta) \geq m(a, \theta) - m(a', \theta) \quad \text{for all } \theta \text{ and } a > a'.
\] (1)

Denote the efficient investment by \( a^* \): \( a^* = \arg \max_a (E[v^+(a, \theta)] - a) \) where \( v^+(a, \theta) = \max \{v(a, \theta), m(a, \theta)\} \) and \( E[x(\theta)] = \int x(\theta) dF(\theta) \). We assume \( a^* \) is unique and \( a^* > a \). For simplicity, we assume \( v^+(a^*, \theta) = v(a^*, \theta) \) for all \( \theta \): under the first-best investment, trade is efficient in all states.

We assume that \( a, \theta, v, \) and \( m \) are observable to both parties but unverifiable, while delivery of the product and transfer payments are verifiable.

Suppose that at the beginning of each period the seller and the buyer can agree on a compensation plan, with the seller’s promising a particular investment level. The compensation plan consists of \( \{w, \mathbf{p}, (b(a, \theta))_{a \in A, \theta \in \Theta}\} \), where \( w \) is paid from the buyer to the seller at the beginning of each period and serves the distribution purpose only, \( \mathbf{p} \) is a formal fixed-price contract contingent on the delivery of the product, and \( b(a, \theta) \) is an additional informal payment made by the buyer when the seller’s investment is \( a \) and state \( \theta \) realizes (negative payments mean transfers from the seller to the buyer). Since delivery of the product is verifiable, we assume that \( \mathbf{p} \) is enforced with a specific performance damage clause. On the other hand, \( b(a, \theta) \) is not enforceable because neither \( a \) nor \( \theta \) is verifiable. In order to investigate the value of the formal fixed-price contract in resolution of the holdup problem, we compare it with the case of no formal contract, in which \( \mathbf{p} \) is not specified in a compensation plan. When \( \mathbf{p} \) is not specified, \( b(a, \theta) \) is the (informal) price paid by the buyer contingent on the level of the seller’s investment, the state of nature, and the delivery of the product.

Our focus on fixed-price contracts as a form of formal contracts can be justified by our objective to show that even writing a simple fixed-price formal contract can help mitigate the holdup problem under repeated transactions but it is not the case under spot transaction (Proposition 4 (a)). Furthermore, in subsection 5.5 we identify a natural condition under which more general formal contracts that may be contingent on messages are of no value, and hence confining our attention to fixed-price contracts as a form of formal contracts is without loss of generality.

\textsuperscript{11}Hence the investment is purely “cooperative” (Che and Hausch, 1999).
In each period, the timing is as follows. First, the seller and the buyer can agree on a compensation plan. If \( \mathcal{F} \) is specified in the plan, the agreement includes signatures by the seller and the buyer on the formal fixed-price contract. Second, the seller chooses investment. Third, the state of nature realizes. Fourth, after observing the seller’s investment and the state of nature, the buyer and the seller negotiate to an ex post efficient outcome if there is inefficiency. This applies either when trade is inefficient under a fixed-price contract, or when trade is efficient under no formal contract. In these cases, we assume that the transfer is determined by the generalized Nash bargaining solution. Let \( \alpha \in [0, 1) \) be the seller’s share of the surplus, and hence the buyer’s share is \( 1 - \alpha \). Finally, the seller produces and sells the product to the buyer according to the compensation plan or at the negotiated price, or in the outside market.

5 Analysis

This section explores the value of formal contracts in resolving or mitigating the holdup problem under both spot and repeated transactions. In subsections 5.1–5.4, we focus on fixed-price as a form of formal contracts. In subsection 5.5, we show that this focus is without loss of generality when \( \rho(a, \theta) \) is either weakly increasing or weakly decreasing in \( a \) for all realizations of uncertainty \( \theta \), where \( \rho(a, \theta) \) denotes the negotiation price in the absence of formal contracts (see subsection 5.1 for the definition of \( \rho(a, \theta) \)).

5.1 Spot Transaction

When the seller and the buyer meet only once, or they do not use history dependent strategies, a standard holdup problem can arise. Since \( b(a, \theta) \) does not play any role in spot transaction, we simply set \( b(a, \theta) \equiv 0 \) in this subsection.

Suppose that no formal fixed-price contract is written at the beginning. If state \( \theta \) satisfying \( v(a, \theta) \geq m(a, \theta) \) realizes, the seller and the buyer negotiate trade and a price. The negotiated price, denoted by \( \rho(a, \theta) \), is given by

\[
\rho(a, \theta) = m(a, \theta) + \alpha(v(a, \theta) - m(a, \theta)) = \alpha v(a, \theta) + (1 - \alpha) m(a, \theta).
\]

The seller’s payoff is then \( \rho(a, \theta) - a \). On the other hand, if \( v(a, \theta) < m(a, \theta) \) holds in the realized state, there is no negotiation and trade does not occur. The seller’s payoff is \( m(a, \theta) - a \).
Define $\rho^+(a, \theta)$ by

$$
\rho^+(a, \theta) = \max\{\rho(a, \theta), m(a, \theta)\} = m(a, \theta) + \alpha \max\{v(a, \theta) - m(a, \theta), 0\}.
$$

Then the seller chooses the investment that maximizes $E[\rho+(a, \theta)] - a$. Denote the optimal investment by $a^o$:

$$
a^o \in \arg \max_a \left( E[\rho^+(a, \theta)] - a \right) \quad (2)
$$

In this setup it is easy to show that the seller does not overinvest.

**Proposition 1** If no formal fixed-price contract is written at the beginning, the seller does not overinvest under spot transaction: $a^* \geq a^o$.

**Proof** Suppose instead $a^* < a^o$. Since $a^*$ is uniquely efficient, $E[v(a^*, \theta)] - a^* > E[v^+(a^o, \theta)] - a^o$, or

$$
a^o - a^* > E[v^+(a^o, \theta)] - E[v(a^*, \theta)]
$$

holds. On the other hand, since $a^o$ is optimal under spot transaction,

$$
a^o - a^* \leq E[\rho^+(a^o, \theta)] - E[\rho^+(a^*, \theta)]
$$

By $\alpha < 1$, $a^o > a^*$, and (1),

$$
\rho^+(a^o, \theta) - \rho^+(a^*, \theta) \leq v(a^o, \theta) - v(a^*, \theta) \leq v^+(a^o, \theta) - v(a^*, \theta)
$$

holds for all $\theta$. Therefore

$$
a^o - a^* \leq E[v^+(a^o, \theta)] - E[v(a^*, \theta)]
$$

must hold, which is a contradiction. Q.E.D.

Since the seller cannot reap all the returns from the investment, his optimal investment choice is at most $a^*$. To make the analysis interesting, we hereafter assume $a^* > a^o$: There exists $a < a^*$ such that the following inequality holds:

$$
a - a^* < E[\rho^+(a, \theta)] - E[\rho^+(a^*, \theta)].
$$

We denote the joint surplus from trade at action $a$ by $\pi(a) \equiv E[v^+(a, \theta)] - a$, and the joint surplus at no trade by $\pi \equiv \max_a(E[m(a, \theta)] - a)$. To simplify analysis (particularly under repeated transaction), we assume $v(a^o, \theta) \leq m(a^o, \theta)$ for all $\theta$: for all states no trade
is efficient under action $a^o$. This implies $v^+(a^o, \theta) = m(a^o, \theta)$ for all $\theta$, and hence $\pi = \pi(a^o)$: the equilibrium outcome under spot transaction without formal contracting is that, in neither state the seller and the buyer trade.

Next, suppose that the buyer and the seller sign a formal fixed-price contract $\overline{p}$ at the beginning. If $v(a, \theta) \geq m(a, \theta)$, there is no room for negotiation and the parties trade with price $\overline{p}$. The seller’s payoff is $\overline{p} - a$. If $v(a, \theta) < m(a, \theta)$, however, they negotiate to cancel the contract and break off trade. The seller’s payoff is then $\overline{p} + \alpha(m(a, \theta) - v(a, \theta)) - a$. Define $\rho^-(a, \theta)$ by

$$\rho^-(a, \theta) = \rho(a, \theta) - \rho^+(a, \theta) = -\alpha \max\{m(a, \theta) - v(a, \theta), 0\}.$$ 

Note it is negative and increasing in $a$. The seller chooses $a$ to maximize $E[\rho^-(a, \theta)] - a$, the solution of which is obviously $a = \underline{a}$ by (1). The outcome is no better than the case with no formal fixed-price contract, where although the seller underinvests, he may choose an investment higher than $\underline{a}$. Fixed-price contracts are of no value under spot transaction.

### 5.2 Relational Contract without Formal Fixed-price Contract

We now consider the case in which the seller and the buyer engage in infinitely repeated transactions, with the common discount factor $\delta \in (0, 1)$. Since they have symmetric information in our model, we follow MacLeod and Malcomson (1989) and Levin (2003) that provide the definite treatment of models of ongoing contractual relationships. A relational contract is a complete plan for the relationship describing the compensation plan and the seller’s investment for every period and history. Since a relational contract is in general contingent on the seller’s investment which is observable but unverifiable, it must satisfy conditions under which it is neither party’s interest to renege on the contract: it must be self-enforcing, i.e., a subgame perfect equilibrium of the repeated game.

The optimal contract is a self-enforcing relational contract that maximizes the joint surplus. Without loss of generality we can focus on stationary contracts under which in every period the parties agree on the same compensation plan and the seller chooses the same investment on the equilibrium path, and furthermore, if either party reneges on the payment or investment, they negotiate to determine the trade in the current period, and, from the next period on, they revert to no trade, which is the worst equilibrium outcome. Furthermore, for any optimal stationary contract, there is an optimal stationary contract with the same equilibrium behavior and the property that even off the equilibrium path the seller and the buyer cannot jointly benefit from renegotiating to a new self-enforcing contract.\textsuperscript{12} This implies

\textsuperscript{12}Although Levin (2003) does not analyze a case where the parties engage in ex post price negotiation in each period, it is straightforward to generalize his results to such a situation.
that, for any optimal stationary contract, there is an optimal renegotiation-proof stationary contract with the same equilibrium behavior.\textsuperscript{13} That is, although our focus on stationary contracts means that we are assuming that the parties can commit not to renegotiate the relational contract, this assumption is for simplicity and not critical for our results.

In this subsection, we assume no formal fixed-price contract is written. The effects of writing a formal fixed-price contract are analyzed in the next subsection. We obtain conditions under which there exists a self-enforcing (stationary) relational contract that implements a given investment $\hat{a} > a_o$ satisfying $\pi(\hat{a}) > \Pi$.

The relational contract in this subsection includes the following efficient trade decision $e(a, \theta) \in \{0, 1\}$ and compensation plan $b(a, \theta)$:

- If $v(a, \theta) \geq m(a, \theta)$, then trade ($e(a, \theta) = 1$) and the buyer pays $b(a, \theta)$.

- If $v(a, \theta) < m(a, \theta)$, then no trade ($e(a, \theta) = 0$) and the buyer pays $b(a, \theta)$.

If the parties follow the trade decision and compensation plan as above, the seller’s incentive compatibility constraints are given as follows.

$$E[b(\hat{a}, \theta) + (1 - e(\hat{a}, \theta))m(\hat{a}, \theta)] - \hat{a} \geq E[b(a, \theta) + (1 - e(a, \theta))m(a, \theta)] - a$$ for all $a$ \quad (3)

Note that future payoffs do not appear in the constraints.\textsuperscript{14}

We next derive the buyer’s self-enforcing condition. If $v(a, \theta) \geq m(a, \theta)$ holds, the buyer’s short-term gain from not paying $b(a, \theta)$ and negotiating to trade by price $\rho(a, \theta)$ instead is $b(a, \theta) - \rho(a, \theta)$. If $v(a, \theta) < m(a, \theta)$ holds, then the buyer’s gain is $b(a, \theta)$ since trade is inefficient. The buyer’s reneging temptation is hence written as $\max_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\}$. The buyer will then lose her future per period gain $E[e(\hat{a}, \theta)v(\hat{a}, \theta)] - w - E[b(\hat{a}, \theta)]$. The buyer therefore honors the agreement if and only if

$$\max_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} \leq \frac{\delta}{1 - \delta} (E[e(\hat{a}, \theta)v(\hat{a}, \theta)] - w - E[b(\hat{a}, \theta)])$$ \quad (4)

The seller’s self-enforcing condition is obtained in a similar fashion. The seller honors the

\textsuperscript{13}A result in Levin (2003) implies that, for any optimal stationary contract, there is an optimal stationary contract in which from the next period on after a deviation, the term of transaction is altered to hold the deviating party to the same payoff as the one in the spot transaction outcome but still keep them on the ex post efficiency frontier. Such an optimal stationary contract is renegotiation proof.

\textsuperscript{14}A logic similar to the existence of optimal stationary contracts can be applied to show that we can further restrict our attention to contracts that provide the seller’s investment incentives with discretionary payments alone.
agreement if and only if

$$- \min_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} \leq \frac{\delta}{1 - \delta} (w + E[b(\hat{a}, \theta) + (1 - e(\hat{a}, \theta))m(\hat{a}, \theta)] - \hat{a} - \pi). \quad (5)$$

Combining (4) and (5) yields a single necessary condition:

$$\max_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} - \min_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} \leq \frac{\delta}{1 - \delta} (\pi(\hat{a}) - \pi). \quad (6)$$

And (3) and (6) are also sufficient for investment $\hat{a}$ to be implemented: one can find an appropriate $w$ such that (4), (5), and the parties’ participation constraints are satisfied.

Now suppose $\hat{a} > a^o$ can be implemented: There exists a compensation plan $(b(a, \theta))_{a \in A, \theta \in \Theta}$ satisfying (3) and (6). The left-hand side of (6) is rewritten as follows:

$$\max_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} - \min_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} \geq \max_{\theta} \{b(\hat{a}, \theta) - e(\hat{a}, \theta)\rho(\hat{a}, \theta)\} - \min_{\theta} \{b(a^o, \theta) - e(a^o, \theta)\rho(a^o, \theta)\}$$

$$\geq E[b(\hat{a}, \theta) - e(\hat{a}, \theta)\rho(\hat{a}, \theta)] - E[b(a^o, \theta) - e(a^o, \theta)\rho(a^o, \theta)]$$

$$\geq \hat{a} - a^o - E[(1 - e(\hat{a}, \theta))m(\hat{a}, \theta)] + E[(1 - e(a^o, \theta))m(a^o, \theta)]$$

$$- E[e(\hat{a}, \theta)\rho(\hat{a}, \theta)] + E[e(a^o, \theta)\rho(a^o, \theta)]$$

$$= \hat{a} - a^o - \hat{a}^+ + \rho^+(\hat{a}, \theta) - \rho^+(a^o, \theta)]$$

Thus the following condition must follow.

$$\hat{a} - a^o - \Delta^+(\hat{a}, a^o) \leq \frac{\delta}{1 - \delta} (\pi(\hat{a}) - \pi) \quad \text{(DE-NC)}$$

where we define $\Delta^+(a, a') \equiv E[\rho^+(a, \theta) - \rho^+(a', \theta)]$. Note that the optimality of $a^o$ under spot transaction without formal contracts (2) implies

$$\Delta^+(a^o, a') \geq a^o - a \quad \text{for all } a. \quad (7)$$

The next proposition shows that condition (DE-NC) is necessary and sufficient for the implementation of $\hat{a}$.

**Proposition 2** Suppose no formal fixed-price contract is written. Investment $\hat{a}$ satisfying $\pi(\hat{a}) > \pi$ can be implemented by a relational contract if and only if (DE-NC) holds.

**Proof** We only need to prove the sufficiency part. Supposing (DE-NC), we construct a
compensation plan that satisfies (3) and (6). Define \( b(a, \theta) \) as follows:\(^{15}\)

\[
\begin{align*}
  b(\hat{a}, \theta) & = e(\hat{a}, \theta)\rho(\hat{a}, \theta) - \Delta^+(\hat{a}, a^o) + \hat{a} - a^o \\
  b(a, \theta) & = e(a, \theta)\rho(a, \theta), \quad \text{for all } a \neq \hat{a}
\end{align*}
\]

(3) is satisfied for \( a = a^o \):

\[
\begin{align*}
  E[b(\hat{a}, \theta)] + E[(1 - e(\hat{a}, \theta))m(\hat{a}, \theta)] & = E[b(a^o, \theta)] - E[(1 - e(a^o, \theta))m(a^o, \theta)] \\
  & = \hat{a} - a^o + \Delta^+(\hat{a}, a^o) + \hat{a} - a^o - E[\rho^+(a^o, \theta)] \\
  & = \hat{a} - a^o
\end{align*}
\]

For \( a \neq a^o \), (3) holds because

\[
\begin{align*}
  E[b(\hat{a}, \theta)] + E[(1 - e(\hat{a}, \theta))m(\hat{a}, \theta)] - E[b(a, \theta)] & = E[(1 - e(a, \theta))m(a, \theta)] \\
  & = \hat{a} - a^o + \Delta^+(a^o, a) \\
  & \geq \hat{a} - a^o + a^o - a = \hat{a} - a
\end{align*}
\]

where the last inequality follows from (7).

We next show

\[
\max_{a, \theta \in \Theta} \{ b(a, \theta) - e(a, \theta)\rho(a, \theta) \} = \max_{\theta} \{ b(\hat{a}, \theta) - e(\hat{a}, \theta)\rho(\hat{a}, \theta) \}. \tag{9}
\]

First, \( b(\hat{a}, \theta) - e(\hat{a}, \theta)\rho(\hat{a}, \theta) = \Delta^+(a^o, \hat{a}) + \hat{a} - a^o \geq 0 \) holds by (7). And for \( a \neq \hat{a} \), \( b(a, \theta) - e(a, \theta)\rho(a, \theta) = 0 \), and hence we obtain (9). Similarly, we can show

\[
\min_{a, \theta \in \Theta} \{ b(a, \theta) - e(a, \theta)\rho(a, \theta) \} = \min_{\theta} \{ b(a^o, \theta) - e(a^o, \theta)\rho(a^o, \theta) \}.
\]

Therefore

\[
\max_{a, \theta} \{ b(a, \theta) - e(a, \theta)\rho(a, \theta) \} - \min_{a, \theta \in \Theta} \{ b(a, \theta) - e(a, \theta)\rho(a, \theta) \} \\
= \hat{a} - a^o - \Delta^+(\hat{a}, a^o)
\]

which completes the proof.

\[\text{Q.E.D.}\]

---

\(^{15}\)The fixed payment \( w \) is only used to guarantee that (4), (5), and the participation constraints are satisfied.
The necessary and sufficient condition (DE-NC) depends only on the parameters under the investment which is to be implemented \( \hat{a} \) and the investment which is most preferred by the seller under spot transaction \( a^o \). Intuitively, the seller’s incentive compatibility constraints are binding at \( a = a^o \), and the buyer must pay the seller sufficiently higher \( \hat{a} - a^o \) for investment \( \hat{a} \) than for \( a^o \). However, the higher pay for \( \hat{a} \) results in reneging temptations for both parties. The buyer faces the temptation not to pay informal price \( b(\hat{a}, \theta) \) but to pay the negotiated price \( \rho^+(\hat{a}, \theta) \). The seller faces the temptation to choose \( a^o \), and not to pay penalty \(-b(a^o, \theta)\) but to receive \( \rho^+(a^o, \theta) \). The total reneging temptation is thus equal to the left-hand side of (DE-NC), which must be at most as large as the total future loss.

Note that the right-hand side of (6) or (DE-NC) does not depend on the compensation plan. There is hence no compensation plan that makes the total reneging temptation given in the left-hand side of (6) smaller than the left-hand side of (DE-NC). Therefore, the compensation plan that satisfies (8) in the proof of the proposition minimizes the left-hand side of (6), and in this sense, it is an optimal contract implementing a given investment \( \hat{a} \).

### 5.3 Relational Contract with Formal Fixed-Price Contract

As discussed in Subsection 5.1, formal fixed-price contracts play no roles in resolving the holdup problem under spot transaction. The story is however different for repeated transactions.

In this subsection, we continue considering the case in which the seller and the buyer engage in infinitely repeated transactions, focusing on stationary contracts as in the previous subsection. Unlike in the previous subsection, however, we consider the case in which the buyer and the seller sign a formal fixed-price contract \( \overline{p} \) at the beginning of each period in the equilibrium path. Note that if either party reneges on additional payments \( b(a, \theta) \) when trade is actually efficient, no price negotiation arises because formal fixed-price contract \( \overline{p} \) is enforced. From the next period on, the parties revert to no trade. We derive conditions for a self-enforcing (stationary) relational contract implementing a given investment \( \hat{a} \) to exist.

The relational contract along with a formal fixed-price contract \( \overline{p} \) includes the following efficient trade decision and compensation plan:

- If \( v(a, \theta) \geq m(a, \theta) \), then trade following the formal fixed-price contract \( e(a, \theta) = 1 \) and the buyer pays \( b(a, \theta) \) in addition to \( \overline{p} \).

- If \( v(a, \theta) < m(a, \theta) \), then cancel the formal contract to no trade \( e(a, \theta) = 0 \) and the buyer pays \( b(a, \theta) \).
The seller’s incentive compatibility constraints are given as follows.

\[
E[b(\hat{a}, \theta)] + E[e(\hat{a}, \theta)\overline{p}] + E[(1 - e(\hat{a}, \theta))m(\hat{a}, \theta)] - \hat{a} \\
\geq E[b(a, \theta)] + E[e(a, \theta)\overline{p}] + E[(1 - e(a, \theta))m(a, \theta)] - a 
\text{ for all } a
\] (10)

The buyer’s reneging temptation is derived as follows. First, when \(v(a, \theta) \geq m(a, \theta)\), the buyer can refuse to pay \(b(a, \theta)\) though she has to follow the formal contract and pay \(\overline{p}\). Her short-term gain is \(b(a, \theta)\). Next when \(v(a, \theta) < m(a, \theta)\), the buyer can refuse to cancel the formal contract and to pay \(b(a, \theta)\), and instead negotiate to obtain

\[
v(a, \theta) - \overline{p} + (1 - \alpha)(m(a, \theta) - v(a, \theta)) = \rho(a, \theta) - \overline{p}.
\]

The buyer’s reneging temptation is thus written as \(\max_{a, \theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \overline{p})\}\).

The buyer honors the agreement if and only if

\[
\max_{a, \theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \overline{p})\} \\leq \frac{\delta}{1 - \delta} (E[e(\hat{a}, \theta)v(\hat{a}, \theta)] - w - \overline{p} - E[b(\hat{a}, \theta)]) .
\] (11)

Similarly, the seller honors the agreement if and only if

\[
-\min_{a, \theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \overline{p})\} \leq \frac{\delta}{1 - \delta} (w + \overline{p} + E[b(\hat{a}, \theta) + (1 - e(\hat{a}, \theta)m(\hat{a}, \theta)] - \hat{a} - \overline{p}) .
\] (12)

Combining these conditions yields

\[
\max_{a, \theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \overline{p})\} - \min_{a, \theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \overline{p})\} \\leq \frac{\delta}{1 - \delta} (\pi(\hat{a}) - \overline{p})
\] (13)
Then the incentive compatibility constraints (10) are satisfied:

\[
\max_{a,\theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \overline{p})\} - \min_{a,\theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \overline{p})\} \\
\geq \max_{\theta} \{b(\hat{a}, \theta) + (1 - e(\hat{a}, \theta))(\rho(\hat{a}, \theta) - \overline{p})\} \\
- \min_{\theta} \{b(\underline{a}, \theta) + (1 - e(\underline{a}, \theta))(\rho(\underline{a}, \theta) - \overline{p})\} \\
\geq E[b(\hat{a}, \theta) + (1 - e(\hat{a}, \theta))(\rho(\hat{a}, \theta) - \overline{p})] - E[b(\underline{a}, \theta) + (1 - e(\underline{a}, \theta))(\rho(\underline{a}, \theta) - \overline{p})] \\
\geq \hat{a} - \underline{a} - E[e(\hat{a}, \theta)\overline{p} + (1 - e(\hat{a}, \theta))(m(\hat{a}, \theta) + \overline{p} - \rho(\hat{a}, \theta))] \\
+ E[e(\underline{a}, \theta)\overline{p} + (1 - e(\underline{a}, \theta))(m(\underline{a}, \theta) + \overline{p} - \rho(\underline{a}, \theta))] \\
= \hat{a} - \underline{a} + E[\rho^-(\hat{a}, \theta) - \rho^-(\underline{a}, \theta)].
\]

We thus obtain the following necessary condition.

\[
\hat{a} - \underline{a} + \Delta^- (\hat{a}, \underline{a}) \leq \frac{\delta}{1 - \delta} (\pi(\underline{a}) - \overline{p}) \quad \text{(DE-FP)}
\]

where we define \(\Delta^- (a, a') = E[\rho^-(a, \theta) - \rho^-(a', \theta)]\).

**Proposition 3** Investment \(\hat{a}\) satisfying \(\pi(\hat{a}) > \overline{p}\) can be implemented by combining a formal fixed-price contract and a relational contract if and only if (DE-FP) holds.

**Proof** To show the sufficiency part, define \(b(a, \theta)\) as follows:

\[
b(\hat{a}, \theta) = \hat{a} - a - (1 - e(\hat{a}, \theta))(\rho(\hat{a}, \theta) - \overline{p}) + E[\rho^-(\hat{a}, \theta)] \\
b(a, \theta) = -(1 - e(a, \theta))(\rho(a, \theta) - \overline{p}) + E[\rho^-(a, \theta)] \quad \text{for all } a \neq \hat{a}
\]

Then the incentive compatibility constraints (10) are satisfied:

\[
E[b(\hat{a}, \theta)] + E[e(\hat{a}, \theta)\overline{p}] + E[(1 - e(\hat{a}, \theta))m(\hat{a}, \theta)] \\
- E[b(a, \theta)] - E[e(a, \theta)\overline{p}] - E[(1 - e(a, \theta))m(a, \theta)] \\
= \hat{a} - a \geq \hat{a} - a \quad \text{for all } a \neq \hat{a}
\]

We next show the following:

\[
\max_{a,\theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \overline{p})\} = \hat{a} - a + E[\rho^-(\hat{a}, \theta)] \\
\min_{a,\theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \overline{p})\} = E[\rho^-(a, \theta)]
\]
First, for \( a = \hat{a} \),

\[
b(\hat{a}, \theta) + (1 - e(\hat{a}, \theta))(\rho(\hat{a}, \theta) - \overline{p}) = \hat{a} - \underline{a} + E[\rho^{-}(\hat{a}, \theta)]
\]

holds for all \( \theta \). Second, for \( a \neq \hat{a} \),

\[
b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \overline{p}) = E[\rho^{-}(a, \theta)] \geq E[\rho^{-}(\underline{a}, \theta)]
\]

holds for all \( \theta \) since \( \underline{a} \) is the minimum level of investment and \( \rho^{-}(a, \theta) \) is increasing in \( a \). Finally, \( \hat{a} - \underline{a} + E[\rho^{-} (\hat{a}, \theta)] - E[\rho^{-}(\underline{a}, \theta)] > 0 \) is satisfied. Therefore,

\[
\max_{a, \theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \overline{p})\} - \min_{a, \theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \overline{p})\}
\]

\[
= \hat{a} - \underline{a} + E[\rho^{-}(\hat{a}, \theta) - \rho^{-}(\underline{a}, \theta)]
\]

holds, which completes the proof. Q.E.D.

The necessary and sufficient condition (DE-FP) depends only on two investment levels \( \hat{a} \) and \( \underline{a} \), the latter of which is the one most preferred by the seller under spot transaction with a fixed-price contract. Intuitively, the buyer faces the temptation not to pay informal price \( b(\hat{a}, \theta) \) but to pay \( \alpha(m(\hat{a}, \theta) - v(\hat{a}, \theta))^+ \) in addition to \( \overline{p} \). The seller faces the temptation to choose \( \underline{a} \) and not to pay penalty \(-b(a^o, \theta)\) but to receive \( \alpha(m(\underline{a}, \theta) - v(\underline{a}, \theta))^+ \) in addition to \( \overline{p} \). The total reneging temptation is thus equal to the left-hand side of (DE-FP).

### 5.4 Comparison

We can analyze the value of writing a formal fixed-price contract in repeated transactions by comparing two conditions, (DE-NC) for the case of no formal fixed-price contract, and (DE-FP) for the case of writing a formal fixed-price contract.

The conditions differ only in terms of the reneging temptations given on the left-hand sides, and the reneging temptations are different in two respects. One difference is captured by the term \( \Delta^+(\hat{a}, a^o) \) in (DE-NC) and \( \Delta^-(\hat{a}, \underline{a}) \) in (DE-FP). \( \Delta^+(\hat{a}, a^o) \) is nonzero when trade is efficient under either \( \hat{a} \) or \( a^o \) so that after reneging, the seller and the buyer negotiate the price to trade the product under no formal contract. On the other hand, \( \Delta^-(\hat{a}, \underline{a}) \) is nonzero when trade is inefficient under either \( \hat{a} \) or \( \underline{a} \) so that after reneging, they negotiate the transfer to cancel trade under a formal fixed-price contract. Through these terms the no-trade surplus affects the parties’ reneging temptations.

The other difference, captured by the term \( \hat{a} - a^o \) in (DE-NC) and \( \hat{a} - \underline{a} \) in (DE-FP), arises because the seller’s optimal investment under spot transaction may be different. It
is \( a \) under a formal fixed-price contract, while \( a^o \), the optimal investment under no formal contract, may be higher than \( a \). Without formal contracting, the seller may choose an investment higher than the least costly level because the investment has positive effects on the price determined by negotiation, which depends on the seller’s share \((\alpha)\), the value for the buyer \((v(a, \theta))\), and the alternative-use value \((m(a, \theta))\). Since the value for the buyer is increasing in investment, it provides the seller with an incentive to choose higher investment if the seller’s share is positive. Furthermore, if the alternative-use value increases with investment, it provides an additional incentive to increase investment, although the effect is not as large as that of the value for the buyer because of (1). And even if the alternative-use value is decreasing, the marginal benefit of investment for the buyer captured by the seller may be so large that the seller is induced to choose \( a^o > a \). On the other hand, the investment incentive via the price determined by negotiation is never positive under a fixed-price contract. If trade is always efficient, there will be no negotiation and hence the seller is paid a constant amount equal to the fixed price. If trade is inefficient, more investment reduces the total surplus and hence the negotiated price. The seller hence chooses the least costly action under formal fixed-price contracts.

The following comparative result is now immediate.

**Proposition 4** Consider the implementation of \( \hat{a} \) satisfying \( \pi(\hat{a}) > \pi \).

(a) Suppose \((a^o - a) + \Delta^+(\hat{a}, a^o) + \Delta^-(\hat{a}, a) < 0 \) holds. If \( \hat{a} \) can be implemented under repeated transaction without any formal contract, the same investment can be implemented under repeated transaction with an appropriate fixed-price contract. And there is a range of parameter values in which \( \hat{a} \) can be implemented only if a formal fixed-price contract is written.

(b) Suppose \((a^o - a) + \Delta^+(\hat{a}, a^o) + \Delta^-(\hat{a}, a) > 0 \) holds. If \( \hat{a} \) can be implemented under repeated transaction with a formal fixed-price contract, the same investment can be implemented under repeated transaction without any formal contract. And there is a range of parameter values in which \( \hat{a} \) can be implemented only if no formal fixed-price contract is written.

Proposition 4 (a) shows that in contrast to a well-known result in the case of spot transaction that “formal contracting has no value,” a simple formal fixed-price contract, combined with an informal compensation plan, can help mitigate the holdup problem under repeated transactions. The condition reflects two sources of differences in the reneging temptation explained above. To better understand the condition, we first suppose \( a^o = a \); under spot transaction, the seller faces no incentive to invest higher than the least costly investment.
This is the case in our two-investment example in Introduction. Then the condition in (a) is equivalent to

$$\Delta^+(\hat{a}, a) + \Delta^-(\hat{a}, a) = E[\rho(\hat{a}, \theta) - \rho(a, \theta)] < 0,$$

(14)

that is, the total negotiated price is lower for the larger action $\hat{a}$ than for $a$.

Suppose for the moment that trade is always efficient, that is, $v(\hat{a}, \theta) \geq m(\hat{a}, \theta)$ and $v(a, \theta) \geq m(a, \theta)$ for all $\theta$.\(^{16}\) Then $\Delta^- (\hat{a}, a) = 0$, that is, there is no price negotiation under fixed-price contracts while $\Delta^+(\hat{a}, a) < 0$ implies price negotiation brings negative investment incentive under no formal contract. By eliminating the effect of the price negotiation on the reneging temptation, a well-designed formal fixed-price contract reduces the reneging temptation from $(\hat{a} - a) - E[\rho(\hat{a}, \theta) - \rho(a, \theta)]$ to $(\hat{a} - a)$. Therefore, there is a range of parameter values in which (DE-FP) holds while (DE-NC) does not.

More generally, price negotiation can happen under a fixed-price contract when trade is inefficient, and under no formal contract when trade is efficient. The total difference of reneging temptation is thus always summarized by (14), and this being negative implies the reneging temptation is smaller under a fixed-price contract.

Since the post-reneging price is determined by the generalized Nash bargaining solution, $E[\rho(\hat{a}, \theta) - \rho(a, \theta)]$ is rewritten as

$$E[\rho(\hat{a}, \theta) - \rho(a, \theta)] = E[\alpha (v(\hat{a}, \theta) - v(a, \theta)) + (1 - \alpha) (m(\hat{a}, \theta) - m(a, \theta))].$$

A necessary condition for (14) is thus $E[m(\hat{a}, \theta)] < E[m(a, \theta)]$: the expected alternative-use value must be lower under the higher investment $\hat{a}$ than under $a$. We have already argued in Section 2 that this is plausible under some settings. Under repeated transactions, this marginal change of the expected alternative-use value brings a new negative effect of raising the total reneging temptation under no formal fixed-price contract. The formal fixed-price contract can reduce this negative “market incentive” and hence can be valuable.

On the other hand, Proposition 4 (b) shows that if the marginal effect of investment on the expected alternative-use value is positive, the formal fixed-price contract has no value even under repeated interactions. Furthermore, reducing such a positive “market incentive” by writing a formal fixed-price contract may decrease the total surplus under repeated transactions. Note that the result follows even though the marginal benefit on the expected alternative-use value is not large enough to increase the seller’s investment from the least costly level under spot transaction. The formal fixed-price contract has a negative value.

\(^{16}\)Although this last condition violates an assumption we adopted in Section 5.1, this violation does not affect our analysis.
because of the increasing reneging temptation under repeated transactions.\textsuperscript{17}

The two-investment example in Introduction corresponds to $\alpha = 0$ (the buyer’s take-it-or-leave-it offer), and hence the sign of $\Delta_m = m_1 - m_0$ is the same as that of $E[\rho(\hat{a}, \theta)] - E[\rho(a, \theta)]$: whether the alternative-use value is increasing or decreasing fully determines the value of writing a formal fixed-price contract. In more general settings analyzed in this section, not only the marginal effect of investment on the alternative-use value but also the marginal effect on the value for the buyer matters.

We have so far developed intuition under assumption $a^o = a$, in order to clarify how crucial is the marginal effect of investment on the post-reneging price, and in particular the alternative-use value, for the value of writing a formal fixed-price contract. Now consider a more general case of $a^o \geq a$. Suppose the investment incentive through price negotiation is so strong that the seller is induced to choose an investment higher than the least costly level even under spot transaction ($a^o > a$). This advantage of not writing a formal fixed-price contract under spot transaction plays an additional beneficial role of reducing the reneging temptation under repeated transactions, because the incentive necessary to induce the seller to choose $\hat{a}$ decreases from $\hat{a} - a$ to $\hat{a} - a^o$. The condition for writing a formal fixed-price contract to be valuable is now $(a^o - a) + \Delta^+(\hat{a}, a^o) + \Delta^-(\hat{a}, a) < 0$: the value of writing a formal fixed-price contract thus may not be positive even if the expected post-reneging price is decreasing.

However, writing a formal fixed-price contract can still be beneficial if $\Delta^+(\hat{a}, a^o) + \Delta^-(\hat{a}, a)$ is sufficiently negative and dominates the effect of increasing the reneging temptation by $a^o - a$. It is easy to construct an example in which despite $a^o > a$, writing a formal fixed-price contract is of value under repeated transaction.

### 5.5 General Formal Contracts

We have so far restricted analysis to two polar cases of formal contracts, either no formal contract or a fixed-price contract. In this subsection, we consider general formal contracts that may be contingent on the messages sent by the buyer and the seller. We first extend the well-known result of Che and Hausch (1999) concerning the foundation of incomplete contracting to our setting where the seller’s action affects no-trade surplus. We then consider

\textsuperscript{17}This result has a flavor of an endogenous incomplete contract. Bernheim and Whinston (1998) show that parties may optimally leave some verifiable aspects of performance unspecified (“strategic ambiguity”) in order to alter the set of feasible self-enforcing informal agreements. Not writing a formal fixed-price contract in our model may be classified as one form of strategic ambiguity, although the underlying models and logics are different. While we model the dynamic contracting problem in the context of infinitely repeated interaction and emphasize the effect on the alternative-use values, they consider two-period dynamic models with or without intertemporal payoff linkages.
general formal contracts under repeated transaction and show that under some conditions it is without loss of generality to confine our attention to two polar cases: either not writing a formal contract or writing a fixed-price contract attains the highest investment incentives.

**Spot Transaction**

A general formal contract is written as \{p(\eta_b, \eta_s), q(\eta_b, \eta_s)\}, where \(\eta_b = (a_b, \theta_b)\) is the buyer’s report, \(\eta_s = (a_s, \theta_s)\) is the seller’s report. For all reports \((\eta_b, \eta_s)\), the contract specifies trade decision \(q(\eta_b, \eta_s) \in \{0, 1\}\) and payment from the buyer to the seller \(p(\eta_b, \eta_s)\). If trade decision is inefficient for action and state \(\eta = (a, \theta)\), the parties renegotiate the contract to the efficient trade decision \(e(\eta)\), where \(e(\eta) = 1\) if \(v(\eta) - m(\eta) \geq 0\) and \(e(\eta) = 0\) if \(v(\eta) - m(\eta) < 0\). The ex post payoffs in \(\eta\), resulting from the contract and renegotiation, are as follows:

\[
\begin{align*}
    u_B(\eta_b, \eta_s | \eta) &= v(\eta)q(\eta_b, \eta_s) - p(\eta_b, \eta_s) - (1 - \alpha)e(\eta)(v(\eta) - m(\eta))(1 - q(\eta_b, \eta_s)) \\
    &+ (1 - \alpha)(1 - e(\eta))(m(\eta) - v(\eta))q(\eta_b, \eta_s) \\
    u_S(\eta_b, \eta_s | \eta) &= p(\eta_b, \eta_s) + m(\eta)(1 - q(\eta_b, \eta_s)) + \alpha e(\eta)(v(\eta) - m(\eta))(1 - q(\eta_b, \eta_s)) \\
    &+ \alpha(1 - e(\eta))(m(\eta) - v(\eta))q(\eta_b, \eta_s)
\end{align*}
\]

Note \(u_B(\eta_b, \eta_s | \eta) + u_S(\eta_b, \eta_s | \eta) = \max\{v(\eta), m(\eta)\}\) holds for all \(\eta\).

For each \(\eta = (a, \theta)\), truth telling must form a Nash equilibrium:

\[
\begin{align*}
    u_S(\eta) &\equiv u_S(\eta, \eta | \eta) \geq u_S(\eta, \hat{\eta} | \eta), \quad \forall \hat{\eta} \\
    u_B(\eta) &\equiv u_B(\eta, \eta | \eta) \geq u_B(\hat{\eta}, \eta | \eta), \quad \forall \hat{\eta}
\end{align*}
\]

Using the zero-sum feature of the payoffs yields \(u_B(\hat{\eta}) \geq u_B(\eta, \hat{\eta} | \hat{\eta})\) if and only if \(u_S(\hat{\eta}) \leq u_S(\eta, \hat{\eta} | \hat{\eta})\). Thus we must have

\[
\begin{align*}
    u_S(\hat{\eta}) - u_S(\eta) &\leq u_S(\eta, \hat{\eta} | \hat{\eta}) - u_S(\eta, \hat{\eta} | \eta) \\
    &= (1 - q(\eta, \hat{\eta})) (\rho^+(\hat{\eta}) - \rho^+(\eta)) - q(\eta, \hat{\eta}) (\rho^- (\hat{\eta}) - \rho^- (\eta))
\end{align*}
\]

**Proposition 5** Suppose \(\rho(a, \theta)\) is either weakly increasing in \(a\) for all \(\theta\), or weakly decreasing in \(a\) for all \(\theta\). Then formal contracts are of no value under spot transaction.

**Proof** Suppose instead there is a contract under which the seller chooses \(\hat{a} > a^o\), where \(\hat{a}\) is not optimal under no contract. Then by the seller’s incentive compatibility constraints the following inequality must hold.

\[
E[u_S(\hat{a}, \theta)] - E[u_S(a^o, \theta)] \geq \hat{a} - a^o
\]
By specificity (1) and \( \hat{a} > a^o \), \( \rho^-(\hat{\eta}) - \rho^-(\eta) \geq 0 \) holds for all \( \theta \) where \( \hat{\eta} = (\hat{a}, \theta) \) and \( \eta = (a^o, \theta) \). Hence by (15) \( u_S(\hat{\eta}) - u_S(\eta) \leq (1 - q(\eta, \hat{\eta})) (\rho^+(\hat{\eta}) - \rho^+(\eta)) \) for all \( \theta \). We thus obtain

\[
E[u_S(\hat{a}, \theta) - u_S(a^o, \theta)] \leq E \left[ (1 - q((a^o, \theta), (\hat{a}, \theta))) (\rho^+(\hat{a}, \theta) - \rho^+(a^o, \theta)) \right]
\]

Now suppose first \( \rho(a, \theta) \) is weakly increasing in \( a \) for all \( \theta \). Then

\[
\hat{a} - a^o \leq E \left[ (1 - q((a^o, \theta), (\hat{a}, \theta))) (\rho^+(\hat{a}, \theta) - \rho^+(a^o, \theta)) \right] \leq \Delta^+(\hat{a}, a^o)
\]

which contradicts \( \hat{a} \neq \arg \max_a E [\rho^+(a, \theta)] - a \).

Next suppose \( \rho(a, \theta) \) is weakly decreasing in \( a \) for all \( \theta \). Then

\[
\hat{a} - a^o \leq E \left[ (1 - q((a^o, \theta), (\hat{a}, \theta))) (\rho^+(\hat{a}, \theta) - \rho^+(a^o, \theta)) \right] \leq 0
\]

which contradicts \( \hat{a} > a^o \). \( \Box \)

Formal contracts cannot improve investment incentives from the no contract case if the effects of uncertainty \( \theta \) are not so drastic that the sign of the effects of investment \( a \) on the negotiated price is unaffected. For example, this condition holds if \( \theta \) does not affect the negotiated price. The condition also holds if alternative-use value \( m(a, \theta) \) does not depend on \( a \), which is the case studied by Che and Hausch (1999). Proposition 5 thus extends their well-known result to the case where alternative-use value depends on \( a \).

If the condition is violated, formal contracts contingent on messages may be of value, although fixed-price contracts are not, as the following example shows.

**Example 1**

Let \( \theta \in \{\theta_L, \theta_H\} \) and \( \beta = \Pr\{\theta = \theta_H\} \). The agent’s action is either \( a = 0 \) (no investment) or \( a = 1 \) (investment), the latter of which costs him \( d > 0 \). Denote \( v_{at} = v(a, \theta_t) \), \( m_{at} = m(a, \theta_t) \), \( \rho_{at} = \rho(a, \theta) \), and so on, for \( a = 0, 1 \) and \( t = L, H \). The marginal effects of the seller’s action on these values are denoted by \( \Delta_{vt} = v_{1t} - v_{0t} \) and \( \Delta_{mt} = m_{1t} - m_{0t} \).

The key feature of the example is that the negotiated price is increasing in investment in state \( \theta_H \) while it is decreasing in state \( \theta_L \). We assume \( \alpha = 0 \) so that the negotiated price is \( \rho_{at} = m_{at} \). And we assume no-trade surplus satisfies the following conditions: \( m_{0L} = m_{0H} = m_0 \) and \( m_{1H} > m_0 > m_{1L} \). The first assumption is for simplicity: state does not affect no-trade surplus under no investment. The second assumption implies \( \Delta_{mH} > 0 \) and \( \Delta_{mL} < 0 \): Investment increases no-trade surplus in state \( \theta_H \), but reduces the surplus in state
As for $v_{at}$, we assume $v_{1H} > m_{1H}$, $m_0 \geq v_{0H}$, and $v_{1L} > m_0 \geq v_{0L}$. These assumptions imply $\Delta_{vt} > 0$ and $\Delta_{vt} > \Delta_{mt}$. And for all states trade is efficient when the seller invests while it is inefficient when he does not.

We assume $\beta \Delta_{mH} + (1 - \beta) \Delta_{mL} < d < \beta \Delta_{mH}$. The first inequality implies underinvestment occurs if no contract is written. And the seller obviously chooses $a = 0$ under fixed-price contract.

Now consider the following form of formal contracts: $\{p_i^j, q_i^j\}$ where $i \in \{0, 1\}$ is the buyer's report concerning the seller's action, $j \in \{L, H\}$ is the seller's report concerning state, $p_i^j$ is the payment by the buyer to the seller, and $q_i^j \in \{0, 1\}$ is the decision of trade (1) or no trade (0). Note fixed-price contract corresponds to $p_i^j \equiv \overline{p}$ and $q_i^j \equiv 1$. The trade decision and the payment are specified as follows. $q_0^L = q_1^L = 1$, $p_0^L = p_1^L = m_0$, $q_0^H = q_1^H = 0$, and $p_0^H = p_1^H = 0$. The idea is to utilize positive “market incentive” by not specifying trade when state is $\theta_H$, while the formal contract specifying trade in state $\theta_L$ prevents the negotiated price from affecting the seller’s incentive negatively.

We first show that this contract induces investment by the seller, provided that both the buyer and the seller report truthfully. If the seller invests, his expected payoff is $\beta m_{1H} + (1 - \beta) m_0 - a$. If the seller does not invest, his expected payoff is $\beta m_0 + (1 - \beta) m_0 = m_0$. It is optimal for the seller to invest because of assumption $\beta \Delta_{mH} > d$.

We next show that the buyer reports truthfully. Suppose the seller’s investment is $a$, the true state is $\theta_t$, and the seller reports the state truthfully. The buyer’s payoff is $v_{at} q_i^t - p_i^t + (1 - q_i^t)(v_{at} - m_{at})$, which does not depend on the buyer’s report $i$ by the construction of the formal contract. Hence she has no incentive to misreport.

The remaining task is to show that the seller reports truthfully. Suppose the seller’s investment is $a$, the true state is $\theta_t$, and the buyer reports $a$ truthfully. If the seller reports $\theta_H$, his payoff is $m_a - da$. If the seller reports $\theta_L$, his payoff is $m_0 - da$. When the true state is $\theta_H$, $m_{aH} - da \geq m_0 - da$ must hold, which conditions are satisfied for $a = 0, 1$ because $m_{1H} > m_{0H} = m_0$. Finally, when the true state is $\theta_L$, $m_{aL} - da \leq m_0 - da$ must hold, which inequalities are satisfied because $m_{1L} < m_{0L} = m_0$.

**Repeated Transaction**

We next extend the analysis of general formal contracts to repeated transaction. Consider formal (short-term) contracts $\{p(\eta_b, \eta_s), q(\eta_b, \eta_s)\}$ along with the relational contract that consists of the following promises:

- The seller chooses $\hat{a}$ and both the buyer and the seller report truthfully.
• If $v(\eta) \geq m(\eta)$ and $q(\eta, \eta) = 1$, then trade following the formal contract ($e(\eta) = 1$) and the buyer pays $b(\eta)$ in addition to $p(\eta, \eta)$

• If $v(\eta) \geq m(\eta)$ and $q(\eta, \eta) = 0$, then cancel the formal contract to trade ($e(\eta) = 1$) and the buyer pays $b(\eta)$

• If $v(\eta) < m(\eta)$ and $q(\eta, \eta) = 1$, then cancel the formal contract to no trade ($e(\eta) = 0$) and the buyer pays $b(\eta)$

• If $v(\eta) < m(\eta)$ and $q(\eta, \eta) = 0$, then no trade following the formal contract ($e(\eta) = 0$) and the buyer pays $b(\eta)$ in addition to $p(\eta, \eta)$

The ex post payoffs in state $\eta$ when both report truthfully are given as follows:

$$u_B(\eta) = e(\eta)v(\eta) - b(\eta) - p(\eta, \eta)R(\eta)$$
$$u_S(\eta) = (1 - e(\eta))m(\eta) + b(\eta) + p(\eta, \eta)R(\eta)$$

where $R(\eta) = e(\eta)q(\eta, \eta) + (1 - e(\eta))(1 - q(\eta, \eta))$ Note that $u_B(\eta) + u_S(\eta) = e(\eta)v(\eta) + (1 - e(\eta))m(\eta) = \max\{v(\eta), m(\eta)\}$ holds for all $\eta$.

If the parties follow the informal promises, the seller’s incentive compatibility constraints are given as follows.

$$E[(1 - e(\hat{a}, \theta))m(\hat{a}, \theta) + b(\hat{a}, \theta) + p((\hat{a}, \theta), (\hat{a}, \theta))R(\hat{a}, \theta)] - \hat{a}$$
$$\geq E[(1 - e(a, \theta))m(a, \theta) + b(a, \theta) + p((a, \theta), (a, \theta))R(a, \theta)] - a \quad \text{for all } a \quad (16)$$

The buyer’s payoff from deviating in state $\hat{\eta}$ by reporting $\eta$ is written as

$$u_B(\eta, \hat{\eta} \mid \eta) = e(\hat{\eta})v(\hat{\eta}) + (1 - e(\hat{\eta}))q(\eta, \hat{\eta})m(\hat{\eta}) - p(\eta, \hat{\eta})$$
$$- (1 - q(\eta, \hat{\eta}))\rho^+(\hat{\eta}) + q(\eta, \hat{\eta})\rho^-(\hat{\eta})$$

For example, suppose $e(\hat{\eta}) = 1$ and $q(\eta, \hat{\eta}) = 1$. The buyer deviates by reporting $\eta$ and not paying $b(\hat{\eta})$, and hence her payoff is $v(\hat{\eta}) - p(\eta, \hat{\eta})$. Note there is no negotiation after reneging in this case. As another case, suppose $e(\hat{\eta}) = 0$ and $q(\eta, \hat{\eta}) = 1$. In this case the buyer reports $\eta$, does not pay $b(\hat{\eta})$ but negotiate to obtain $v(\hat{\eta}) - p(\eta, \hat{\eta}) + (1 - \alpha)(m(\hat{\eta}) - v(\hat{\eta}))$. Her payoff is thus $m(\hat{\eta}) - p(\eta, \hat{\eta}) - \alpha(m(\hat{\eta}) - v(\hat{\eta})) = m(\hat{\eta}) - p(\eta, \hat{\eta}) + \rho^-(\hat{\eta})$. One can check the other two cases similarly to obtain the buyer’s payoff as above. The buyer’s reneging temptation is thus $\max_{\eta', \eta''}[u_B(\eta', \eta \mid \eta) - u_B(\eta)]$.
Similarly, the seller’s payoff from deviating in state $\eta$ by reporting $\hat{\eta}$ is written as

$$u_S(\eta, \hat{\eta} | \eta) = (1 - q(\eta))(1 - q(\eta, \hat{\eta}))m(\eta) + p(\eta, \hat{\eta})$$

$$+ (1 - q(\eta, \hat{\eta}))\rho^+(\eta) - q(\eta, \hat{\eta})\rho^-(\eta)$$

The seller’s reneging temptation is hence $-\min_{\eta, \eta'}[u_S(\eta', \eta | \eta') - u_S(\eta')]$.

The sum of these reneging temptations are rewritten as follows:

$$\max_{\eta, \eta'}[u_B(\eta', \eta | \eta) - u_B(\eta)] - \min_{\eta, \eta'}[u_S(\eta', \eta | \eta') - u_S(\eta')]$$

$$\geq \max_{\eta} u_B((a, \theta), (\hat{a}, \theta) | (\hat{\eta}, \theta)) - u_B(\hat{a}, \theta)] - \min_{\eta} u_S((a, \theta), (\hat{a}, \theta) | (a, \theta)) - u_S(a, \theta)]$$

$$\geq E[u_B((a, \theta), (\hat{a}, \theta) | (\hat{\eta}, \theta)) - u_B(\hat{a}, \theta)] - E[u_S((a, \theta), (\hat{a}, \theta) | (a, \theta)) - u_S(a, \theta)]$$

$$\geq \hat{\alpha} - a - E[(1 - q((a, \theta), (\hat{a}, \theta)))(\rho^+(\hat{a}, \theta) - \rho^+(a, \theta)) - q((a, \theta), (\hat{a}, \theta)))(\rho^-(\hat{a}, \theta) - \rho^-(a, \theta))]$$

$$= \hat{\alpha} - a - E[(\rho^+(\hat{a}, \theta) - \rho^+(a, \theta)) - q((a, \theta), (\hat{a}, \theta)))(\rho(\hat{a}, \theta) - \rho(a, \theta))]$$

Now suppose $\rho(a, \theta)$ is increasing in $a$ for all $\theta$. By setting $a = a^o$, we obtain

$$\max_{\eta, \eta'}[u_B(\eta', \eta | \eta) - u_B(\eta)] - \min_{\eta, \eta'}[u_S(\eta', \eta | \eta') - u_S(\eta')]$$

$$\geq \hat{\alpha} - a^o - \Delta^+(\hat{a}, a^o)$$

The right-hand side is attained by no contract. Hence the reneging temptation is minimized by not writing a formal contract.

Next suppose $\rho(a, \theta)$ is decreasing in $a$ for all $\theta$. By setting $a = a^o$, we obtain

$$\max_{\eta, \eta'}[u_B(\eta', \eta | \eta) - u_B(\eta)] - \min_{\eta, \eta'}[u_S(\eta', \eta | \eta') - u_S(\eta')]$$

$$\geq \hat{\alpha} - a + \Delta^-(\hat{a}, a)$$

The right-hand side is attained by a fixed-price contract. Hence the reneging temptation is minimized by writing a fixed-price contract. We have hence shown the following result.

**Proposition 6** Suppose $\rho(a, \theta)$ is either weakly increasing in $a$ for all $\theta$, or weakly decreasing in $a$ for all $\theta$. Then it is without loss of generality to confine attention to no contract or fixed-price contracts under repeated transaction.

### 6 Multiple Quantities

In the main model analyzed in the previous section, we have assumed that at most one unit of the product is traded, in order to highlight the value of formal contracting in repeated
transactions. In this section, we introduce the possibility of trading more than one unit into our model, and derive a sufficient condition for writing a fixed-price contract to be of no value under spot transaction. We then illustrate through an example that if the condition does not hold, formal fixed-price contracts can be valuable even under spot transaction. More importantly, however, we show, using the same example, that if the transaction is repeated infinitely, formal fixed-price contracts can be valuable under a broader range of parameter values because of the role that formal contracts can play under repeated transactions in mitigating the parties’ reneging temptation. As in the previous section, it can be shown that confining our attention to fixed-price contracts as a form of formal contracts is without loss of generality under a natural condition that is analogous to the one identified in the previous section. Given this, we focus on formal fixed-price contracts in what follows.

Let $Q$ be the set of feasible quantity levels. We assume $Q$ is countable (either finite or infinite): $Q = \{0, q_1, q_2, \ldots \}$ with $0 < q_1 < q_2 < \cdots$, where 0 corresponds to no trade. The value to the buyer is written as $v(q, a, \theta)$. We assume $v(0, a, \theta) = 0$, $v(q, a, \theta)$ is strictly increasing in $q$ for all $(a, \theta)$, and $v(q, a, \theta)$ is strictly increasing in $a$ for all $q > 0$ and $\theta$. Since there are multiple quantities to be traded, we introduce the seller’s production cost $c(q, \theta)$.

We assume $c(0, \theta) = 0$, and $c(q, \theta)$ is strictly increasing in $q$ for all $\theta$.

Define $\phi(q, a, \theta) = v(q, a, \theta) - c(q, \theta)$. We assume for each $(a, \theta)$ there exists a unique efficient quantity $q^*(a, \theta) = \arg \max_{q \in Q} \phi(q, a, \theta)$, and $0 < q^*(a, \theta) < +\infty$ for all $(a, \theta)$. We assume quantity and action are complementary:

$$\phi(q, a, \theta) - \phi(q', a, \theta) \geq \phi(q', a', \theta) - \phi(q', a', \theta) \quad \text{for all } q > q', a > a', \text{and } \theta. \quad (17)$$

This assumption is sufficient for $q^*(a, \theta)$ to be increasing in $a$.

We write $\phi(a, \theta) = \phi(q^*(a, \theta), a, \theta)$, $v(a, \theta) = v(q^*(a, \theta), a, \theta)$, and $c(a, \theta) = c(q^*(a, \theta), \theta)$.

The assumption of “specificity,” corresponding to (1), is written as

$$\phi(a, \theta) - \phi(a', \theta) \geq m(a, \theta) - m(a', \theta) \quad \text{for all } \theta \text{ and } a > a'. \quad (18)$$

The first-best investment $a^*$ is assumed to be unique, and is defined similarly by $a^* = \arg \max_a E[\phi^+(a, \theta)] - a$ where $\phi^+(a, \theta) = \max\{\phi(a, \theta), m(a, \theta)\}$. As before, we assume $\phi^+(a^*, \theta) = \phi(a^*, \theta)$ for all $\theta$.

We obtain a sufficient condition for fixed-price contracts to be of no value under spot transaction. First we analyze the case in which no formal contract is written. As before, let
\( e(a, \theta) \in \{0, 1\} \) indicates the efficiency of trade:

\[
e(a, \theta) = \begin{cases} 
1 & \text{if } \phi(a, \theta) \geq m(a, \theta) \\
0 & \text{if } \phi(a, \theta) < m(a, \theta) 
\end{cases}
\]

Then if \( e(a, \theta) = 1 \), the buyer and the seller negotiate transfer to trade \( q^*(a, \theta) \). The buyer’s payoff is then \( (1 - \alpha)(\phi(a, \theta) - m(a, \theta)) \), and the seller’s payoff is \( \rho(a, \theta) - a = m(a, \theta) + \alpha(\phi(a, \theta) - m(a, \theta)) - a \). On the other hand, if \( e(a, \theta) = 0 \), there is no negotiation and no trade occurs. The buyer’s payoff is 0 and the seller’s payoff is \( m(a, \theta) - a \).

The seller’s decision is written as follows:

\[
\max_a E[\rho^+(a, \theta)] - a = \max_a E[m(a, \theta) + \alpha \max\{\phi(a, \theta) - m(a, \theta), 0\}] - a.
\]

Denote the optimal investment by \( a^o \). Proposition 1 applies here, with minor modification, to show \( a^o \leq a^* \). We hereafter assume \( a^o < a^* \).

Next consider formal contract \( (p, q) \) meaning the parties trade quantity \( q \) and the buyer pays \( p \) to the seller. Under this contract, if \( e(a, \theta) = 1 \), they negotiate to trade \( q^*(a, \theta) \). The buyer’s payoff and the seller’s payoff are, respectively, obtained as follows:

The buyer’s payoff: \( v(q, a, \theta) - p + (1 - \alpha)(\phi(a, \theta) - \phi(q, a, \theta)) \);

The seller’s payoff: \( p - c(q, \theta) + \alpha(m(a, \theta) - \phi(q, a, \theta)) - a \).

Note that the special case of \( q = q^*(a, \theta) \) is covered here as well.

On the other hand, if \( e(a, \theta) = 0 \), the parties cancel the contract and negotiate to no trade. The payoffs are then as follows:

The buyer’s payoff: \( \phi(a, \theta) - p + (1 - \alpha)(m(a, \theta) - \phi(q, a, \theta)) \);

The seller’s payoff: \( p - c(q, \theta) + \alpha(m(a, \theta) - \phi(q, a, \theta)) - a \).

Using \( \rho^-(a, \theta) = -\alpha(1 - e(a, \theta))(m(a, \theta) - \phi(a, \theta)) \), we can write the seller’s investment decision as follows:

\[
\max_a \alpha E[\phi(a, \theta) - \phi(q, a, \theta)] - E[\rho^-(a, \theta)] - a
\]

Denote the optimal action under fixed-price contract \( (\bar{p}, \bar{q}) \) by \( a^1(\bar{q}) \). The optimal fixed-price contract is the one that maximizes \( a^1(\bar{q}) \). (20) implies the optimal contract minimize the marginal effect of \( E[\phi(q, a, \theta)] \), which is attained at \( q = q_1 \) under assumption (17): Specifying the minimum positive quantity in the contract provides the strongest incentive.
Define $a^1 = a^1(q_1)$.

**Proposition 7** Define $\rho(q,a,\theta) = m(a,\theta) + \alpha(\phi(q,a,\theta) - m(a,\theta))$

(a) $a^o \geq a^1$ if $E[\rho(q_1,a,\theta)]$ is increasing in $a$.

(b) $a^o \leq a^1$ if $E[\rho(q_1,a,\theta)]$ is decreasing in $a$

**Proof** Rewrite the objective function in (20) as follows:

$$\alpha E[\phi(a,\theta) - \phi(q_1,a,\theta)] - E[\rho^-(a,\theta)] - a$$
$$= E[\alpha(\phi(a,\theta) - \phi(q_1,a,\theta)) + \alpha(1 - e(a,\theta))(m(a,\theta) - \phi(a,\theta))] - a$$
$$= E[m(a,\theta) + \alpha(\phi(a,\theta) - e(a,\theta)m(a,\theta)) - m(a,\theta) - \alpha(\phi(q_1,a,\theta) - m(a,\theta))] - a$$
$$= E[\rho^+(a,\theta) - \rho(q_1,a,\theta)] - a$$

(21)

Now the conclusion follows from (19) and (21).

Q.E.D.

Applying Proposition 7 to our model in Section 4 where $Q = \{0,1\}$ and $q_1 = 1 = q^*(a,\theta)$ yields the following. There the condition that $E[\rho(q_1,a,\theta)]$ is increasing in $a$ is equivalent to $E[\rho(a,\theta)]$ being increasing in $a$, and hence $a^o \geq a^1 = a^1$. On the other hand, the condition that $E[\rho(q_1,a,\theta)]$ is decreasing in $a$ is equivalent to $E[\rho(a,\theta)]$ being decreasing in $a$, and hence $a^o = a^1 = a^1$: the least costly action is chosen either under no formal contracting or under fixed-price contract. Fixed-price contract hence cannot improve investment incentive from the case of no formal contract under spot transaction.

However, if $|Q| \geq 3$, $a^1 > a^o$ is possible. In the following example we first illustrate this possibility. More importantly, we then use the same example to illustrate that under repeated transactions, formal fixed-price contracts can be valuable under a broader range of parameter values when $a^1 > a^o$, and there appears a range of parameter values in which fixed-price contracts are of value even though writing a fixed-price contract is not valuable under spot transaction ($a^1 \leq a^o$).

**Example 2**

Let $Q = \{0,q_1,q_2\}$ and $A = \{0,1\}$. There is no uncertainty and hence we drop $\theta$. We also use notations $\phi_a(q) = \phi(q,a)$, $\phi_a = \phi(q^*(a),a)$, $m_a = m(a)$, and so on, for action $a = 0,1$. Assume $\phi_1 = \phi_1(q_2) > \phi_1(q_1) > m_1$, and $m_0 = \phi_0(q_2) > \phi_0(q_1)$. Hence $q^*(1) = q^*(0) = q_2$ holds, and trading $q_2$ is efficient under $a = 1$ while no trade is efficient under action 0. We can choose these values so as to satisfy the other assumptions (17) and
(18). The efficient action is assumed to be \( a^* = 1 \):

\[
\phi_1 - m_0 > d
\]

where \( d > 0 \) is the cost of investment \( (a = 1) \). Define \( \rho_a(q) = m_a + \alpha(\phi_a(q) - m_a) \) and \( \rho_a^+ = m_a + \alpha \max\{\phi_a - m_a, 0\} \). Suppose

\[
\Delta_m + \alpha(\phi_1 - m_1) < d
\]

(22)

where \( \Delta_m = m_1 - m_0 \). This assumption implies \( \rho_1^+ - \rho_0^+ < d \), that is, the seller chooses not to invest \( (a = 0) \) under no contract.

There are two kinds of fixed-price contracts. First, consider a fixed-price contract specifying quantity \( q_2 \): \( (\overline{p}, \overline{q} = q_2) \). Since \( \overline{q} = q_2 \) is the efficient quantity, there is no negotiation under investment, and hence the seller chooses no investment.

Next consider a fixed-price contract specifying quantity \( q_1 \): \( (\overline{p}, \overline{q} = q_1) \). Under this contract, if the seller invests, the parties negotiate to \( q_2 \), and hence the seller’s payoff is \( \overline{p} - c(q_1) + \alpha(\phi_1(q_2) - \phi_1(q_1)) - d \). If the seller does not invest, they cancel the contract and the seller’s payoff is \( \overline{p} - c(q_1) + \alpha(m_0 - \phi_0(q_1)) \). The seller chooses to invest if

\[
\alpha(\phi_1 - m_0) - \alpha(\phi_1(q_1) - \phi_0(q_1)) \geq d
\]

(23)

which is rewritten as

\[
(\rho_1^+ - \rho_0^+) - (\rho_1(q_1) - \rho_0(q_1)) \geq d.
\]

(24)

By (22) and (24), fixed-price contracts cannot improve investment incentives if \( \rho_1(q_1) \geq \rho_0(q_1) \) or \( \alpha(\phi_1(q_1) - \phi_0(q_1)) \geq -(1 - \alpha)\Delta_m \). However, if

\[
\alpha(\phi_1(q_1) - \phi_0(q_1)) < -(1 - \alpha)\Delta_m
\]

(25)

holds, fixed-price contracts may be of value. For example, suppose \( \Delta_m < 0 \) holds and \( \alpha(\phi_1 - m_0) \) satisfies \( d - \Delta_m > \alpha(\phi_1 - m_0) > d \). Then if \( \phi_1(q_1) - \phi_0(q_1) > 0 \) is sufficiently close to zero, (23) holds, and there is a fixed-price contract implementing investment.

Now consider repeated transactions. The reneging temptation under no formal contract is

\[
d - (\rho_1^+ - \rho_0^+) = d - \Delta_m - \alpha(\phi_1 - m_1),
\]

(26)

similar to the one in the previous model. The reneging temptation under fixed-price contract
is also similar and equal to
\[ d + \alpha(m_0 - \phi_0). \] (27)

The reneging temptation under fixed-price contract \((p, q) = q_2\) is derived as follows. The buyer and the seller agree to cancel the contract and trade \(q_2\) with the buyer’s payment \(b_1\) if the seller invests, and not to trade with the buyer’s payment \(b_0\) if the seller does not invest. The payoffs are \(v_1(q_2) - b_1\) and \(b_1 - c(q_2)\) for the buyer and the seller, respectively, if \(a = 1\), and \(-b_0\) and \(b_0 + m_0\) if \(a = 0\). The seller’s incentive compatibility constraint is hence \(b_1 - c(q_2) - b_0 - m_0 \ge d\).

When the seller chooses \(a = 1\), the buyer can deviate by not paying \(b_1\) but negotiating transfer. The gain in her payoff is then
\[ v_1(q_2) + (1 - \alpha)(\phi_1(q_1) - \phi_1(q_1)) - (v_1(q_2) - b_1) = b_1 - p + (1 - \alpha)(\phi_1(q_1)). \]

When the seller chooses \(a = 0\), he can refuse to cancel the contract and instead negotiate transfer. The gain in his payoff is then
\[ \bar{p} - c(q_1) + \alpha(m_0 - \phi_0(q_1)) - (b_0 + m_0) = -b_0 + \bar{p} - c(q_1) + \alpha(m_0 - \phi_0(q_1)). \]

Summing up these gains and using the seller’s incentive compatibility constraint yield the reneging temptation as follows.
\[ d - \alpha(\phi_1 - m_0) + \alpha(\phi_1(q_1) - \phi_0(q_1)). \] (28)

By comparing (27) and (28) we find that the reneging temptation is at least as high under fixed-price contract \((\bar{p}, \bar{q} = q_2)\) than under \((\bar{p}, \bar{q} = q_1)\), using (17). Fixed-price contract is thus of value if (28) is smaller than (26), which condition is written as follows:
\[ \alpha(\phi_1 - m_0) - \alpha(\phi_1(q_1) - \phi_0(q_1)) \ge \rho_1 - m_0 = \rho_1^+ - \rho_0^+. \] (29)

Now first suppose \(\rho_1(q_1) \ge \rho_0(q_1)\) or equivalently \(\alpha(\phi_1(q_1) - \phi_0(q_1)) \ge -(1-\alpha)\Delta_m\). Fixed-price contracts are then of no value under spot transaction. However, since \(d > \rho_1^+ - \rho_0^+\) holds and the left-hand side of (29) is smaller than \(d\), there is a range of parameter values in which (29) is satisfied, and hence fixed-price contract is valuable under repeated transaction. Next suppose \(\rho_1(q_1) < \rho_0(q_1)\). Fixed-price contracts are valuable even under spot transaction if
(23) holds. However, since the right-hand side of (29) is smaller than that of (23), fixed-price contract is valuable in a broader range of parameter values under repeated transaction than spot transaction.

In summary, our main assertion that repeated transaction, along with decreasing no-trade surplus, makes crucial contribution to the value of formal contracting turns out to be valid for the case of multiple quantities.

7 Concluding Remarks

This paper has offered a new perspective on the role of formal contracts in resolving the holdup problem. In situations where formal fixed-price contracts have no value under spot transaction due to the cooperative nature of the relation-specific investment, we have shown that writing a simple fixed-price contract can be valuable under repeated transactions. In our model, there is a range of parameter values in which a formal fixed-price contract combined with an informal agreement can help mitigate the holdup problem, while under another parameter range not writing a formal fixed-price contract but entirely relying on an informal agreement increases the total surplus of the buyer and the seller. Furthermore, we have shown that under an additional natural assumption, more complex formal contracts contingent upon messages cannot provide higher investment incentives than an appropriate simple fixed-price contract. The key driving force of our result is a possibility that relation-specific investment decreases the value of no-trade surplus. This possibility, although very plausible, has been largely ignored in previous theoretical analyses of the holdup problem.

Our analysis contributes to empirical literature that investigates the relationship between relational governance and formal contracts by offering new theoretical hypotheses. We identify whether the alternative-use value is increasing or decreasing in investment as an important factor in determining whether relational governance and formal contracts are complements or substitutes. Testing our theoretical hypotheses by data either from fields or laboratories will be a next important agenda for future research.

References


