Scale and the Origins of Structural Change*

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Abstract

This paper extends the three major facts of long run structural transformation – (i) sectoral reallocations, (ii) rich movements of productive activities between home and market, and (iii) an increase in the scale of productive units – and develops a model based on scale technologies to understand and explain them within a unified framework. The crucial distinction between industry, services, and home production is the scale of the productive unit. Scale technologies give rise to industrialization, and the marketization of previously home produced activities. The rise of mass consumption leads to an expansion of industry, but a reversal of the marketization process for service industries. Finally, the later growth in the scale of services leads to a decline in industry and a rise in services.

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1 Introduction

“The rate of structural transformation of the economy is high. Major aspects of structural change include the shift away from agriculture to non-agriculture pursuits, and, recently, away from industry to services; a change of the scale of productive units, and a related shift from personal enterprise to impersonal organization of economic firms, with a corresponding change in the occupational status of labor.” (Kuznets, 1971, 2)

The process of structural transformation is one of the most salient facts of economic development. As suggested by Kuznets, there are three key aspects of this transformation: (1) the movements of production across broadly defined economic sectors, (2) the shift of activity between home and market production, and (3) the increase in scale of productive units. This paper extends evidence on these three observations, and develops a model that unifies them.

Each of these facts can have important ramifications for growth, and overall secular trends of development. When production technologies differ across sectors, the distinction between sectors is economically meaningful, and reallocations across sectors can have important impacts on long run growth prospects, relative prices, and relative wages.1 The distinction between home and market production is also relevant, since the vast majority of home production is not included in national income accounts. Home production also affects labor supply decisions, and in turn marital and fertility decisions, all of which show interesting dynamics over development. The scale of productive units can have consequences for the effects of financial frictions on development. Specifically, fixed costs or a minimum scale can exacerbate the problems of financial frictions2

In our theory, scale technologies are the origin of structural change; a model designed to be consistent with cross-sectoral and secular evidence on the scale of productive units has strong predictions for the movement of production between sectors and between home and the market that are consistent with the data. Scale has this central role for two reasons.

First, we show that the scale of production is the primary technological difference distinguishing the goods and service sectors. Indeed, we argue that scale is the most consistent and meaningful basis of classification. Production that is most efficient when produced on a very large-scale tend to be categorized as goods, while smaller scale production is

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2Fixed costs or a minimum scale can exacerbate the problems of financial frictions. See, for example, Banerjee and Newman (1993), Lloyd-Ellis and Bernhardt (2001), and Buera and Shin (2006).
often categorized as services. So, for example, custom dress-making and tailoring have historically been services, while mass apparel production is industry.

Second, scale affects decisions on home vs. market production. The large scale of manufacturing makes modern home production of these goods irrelevant. In contrast, an important margin between home production and smaller-scale, market services exists. Market services lead to higher utilization of indivisible, specialized intermediates, but home production offers other utility benefits. So, for example, the public bus requires less capital per passenger than home production using a private automobile.

Thus, scale technologies are at the heart of both the reallocation of production between the home and market, and between sectors. Economic historians argue that the industrialization and the birth of the factory system was associated with the arrival of a series of economically viable, large-scale technologies (e.g., Chandler, 1990, Mokyr, 1994, 2001, Berg, 1994, Scranton, 1997). These modern scale technologies introduced new industries, but also moved existing traditional home production activities to the market (see Reid, 1935). We show that even early on market production involved both industry and services, however. As industrialization continued the range of production and industries in these sectors expands, traditional output, such as agriculture, falls as a share of production.

As development continues, the intermediate scale of services play another important role in structural transformation. As incomes rise, and the costs of intermediates in the production of fall, households begin purchasing these intermediates directly, and home producing using modern technologies, rather than indirectly through market services. The home to market product cycle is therefore reversed in later times with the diffusion of goods to households (see Buera and Kaboski, 2006, and Ramey and Francis, 2006). For example, an activity like laundry was originally performed using a traditional technology (hand-washing), later produced as market services using modern equipment, and has subsequently moved back into the home with the spread of the productive intermediate or durable to consumers.

This reversal of the home-market product cycles is therefore associated with the rise of mass consumption, when consumers purchase goods directly on a widespread scale.3 By increasing the demand for market goods and decreasing the purchase of services, this spread of intermediates to households not only affects the home vs. market production decision, but also the allocation of market production across sectors. Manufacturing experiences a boom, and the economy experiences a rise in manufacturing relative to services. This is

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3Katona (1964) described the mass consumption society as a society in which the “broad masses” consumed a wide range of goods, and generated most of the demand for them. Thus, the term has some presumption of heterogeneity in income or class. Matsuyama (2002) deals with the distribution and productivity conditions necessary for it spread. We instead look at its consequences for sectoral allocations, and home vs. market production.
consistent with U.S. experience, in which the peak in the output share of manufacturing corresponds with a peak in the share of consumption expenditures on non-food goods.

Finally, the model predicts that the relative size of the service sector is increasing in the scale of services. Home production of large-scale services is relatively more difficult/costly. We show evidence for the United States that the post-1950 growth in services has been driven by a growth in the scale of services.

This rest of this paper develops the evidence, and formally models this theory of structural transformation. In the next section, we review and extend the facts, including documenting the salient patterns of long run sectoral reallocations of output for a set of 29 developed and developing countries. There we also informally develop the argument of scale as a unifying factor behind these facts. Section 3 develops a model, based on Buera and Kaboski (2006) to crystallize the importance of scale economies, while Section 4 presents its implications for the early dynamics of structural transformation, the growth of industry, and its recent decline. Finally, Section 5 extends the model into a more neoclassical, dynamic setting, and Section 6 concludes.

2 Facts of Structural Change

This section documents key facts on three aspects of structural change: sectoral reallocations of production, rich dynamics between home and market production, and growth in the scale of productive establishments.

2.1 Sectoral Reallocations

Figure 1 shows an extended long run time series for the United States of the distribution of current price output across the three major sectors of the economy: agriculture, industry and services.

The U.S. exhibits several interesting stylized features of sectoral reallocations over development. First, the share of manufacturing in value-added is hump-shaped, with an extended rise followed by a late decline. Second, this peak also coincides well with the onset in the United States of what Katona (1964) described as the “mass consumption society”. This phenomenon was characterized by a rise in households’ discretionary spending and the expanded demand for a wide range of goods and services, durables in particular.

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4The bulk of models of sectoral reallocations have trouble producing a quantitatively meaningful rise and decline of industry (e.g., Kongsamut, Rebelo and Xie, 2001, Ngai and Pissarides, forthcoming) without resorting to arbitrary, mechanical assumptions (e.g. Foellmi and Zweimüller, 2005).
from the majority of households.\textsuperscript{5} Indeed, in the chart above the peak in manufacturing corresponds with a peak in the share of consumption expenditures on non-food goods.\textsuperscript{6} Third, the decline in manufacturing corresponds with a late rise/acceleration in the share of services. The fact that the rise in the output share of services occurs only late in development, while recognized by Kuznets, has been overlooked in the literature (e.g., Maddison, 1987).\textsuperscript{7}

The decline of agriculture, hump shape in manufacturing, and late acceleration in

\textsuperscript{5}Katona (1964)’s use of “mass consumption” has a double meaning in terms of both quantity of goods, and consumption of the “broad masses”. It therefore has some presumption of heterogeneity in income or class. Matsuyama (2002) and Murphy, Shleifer, and Vishny (1989) deal with the distribution and productivity conditions necessary for its spread. We instead focus on its consequences for sectoral allocations, and home vs. market production.

\textsuperscript{6}The consumption data from 1900-1929 is from Lebergott (1996). The latter data is from the National Income and Product Accounts. Consumption data prior to 1900 is not available at a sufficiently disaggregated level to make it comparable to the latter numbers.

\textsuperscript{7}Labor allocations show somewhat different patterns. In particular, the share of agriculture is much larger in labor terms than in output terms in earlier periods. In addition, the fraction of labor in services grows even early on. The patterns for real output are difficult to compare across countries because of differences in base years and base year relative prices.
services are true not only for the U.S., and the small group of countries for which Kuznets had long time series, but are common to many countries. Utilizing recent independent work by economic historians, we have assembled reliable extended time series of current price value-added share data for 29 countries, covering six continents and different levels of current development. Of the 29, 22 countries – including all high income countries – have experienced an increase and then decline in manufacturing, while the remaining lower income countries have only (yet) experienced the increase in industry. For the 22 countries, the peak share averages 0.39 (std. dev: 0.04) and occurs at an average per capita income of $7500 (st. dev.: $1800). Pooling all countries, we divide the sample into country-year observations with real income per capita under $7500, and income per capita of at least $7500. Using country-specific fixed-effects, regressions of sector share on log real income per capita yield the following coefficients (with standard errors in parentheses):

<table>
<thead>
<tr>
<th>Sector</th>
<th>&lt;$7500</th>
<th>$\geq$7500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>-0.16</td>
<td>-0.07</td>
</tr>
<tr>
<td>Industry</td>
<td>0.12</td>
<td>-0.17</td>
</tr>
<tr>
<td>Services</td>
<td>0.04</td>
<td>0.23</td>
</tr>
</tbody>
</table>

The UN National Accounts Main Aggregates Database provides sector specific numbers over a shorter time period (1970-2000), but for a much larger cross-section of 161 countries that can be linked to Penn World Tables 6.1 GDP per capita data. The parallel fixed-effect regressions over this larger set of countries yield very similar results:

8These countries include Argentina, Australia, Austria, Brazil, Canada, Chile, China, Colombia, Denmark, Egypt, France, Germany, India, Indonesia, Israel, Italy, Japan, Korea, Mexico, Netherlands, Norway, Pakistan, Spain, Sri Lanka, Sweden, Switzerland, United Kingdom, United States, and Thailand. Based on Maddison (2005), our data covers: 66 percent of world population and 80 percent of world GDP in 2000; 69 percent and 74 percent, respectively, in 1950; and 40 percent and 60 percent, respectively in 1900. Although the numbers are lower for 1900, since the longer time series include Western Europe and its offshoots, we have cover a much larger share of the population and economic activity undergoing large structural change at the time.

9These figures and statistics may change slightly as we update the database, but we are confident of the overall picture presented. Furthermore, once the database is finalized we will provide full documentation of the data sources.

10The UN Nation Accounts are not strictly comparable to the historical series because of subtle differences in accounting techniques and sector definitions. Thus, to study long term trends it is preferable to pool historical national accounts constructed by economic historians with statistics from official individual country sources, rather than UN data, as we do in figure 2.
Graphically, Figure ?? shows these patterns of value-added shares vs. real income per capita for agriculture (top panel), industry (middle panel), and services (bottom panel).

### 2.2 Rich Dynamics Home vs. Market Movements

Historically, and even today in less developed economies, it has been difficult to construct truly meaningful national accounts, since they typically only encompass market activities.\textsuperscript{11} In these less developed economies, the advent and spread of industrialization involves the marketization of many formerly home-produced activities. In her seminal work on household production, Reid described this process\textsuperscript{12}:

“As factory production increased, tasks left the home. At first goods were made in both home and factory. The family gave up home production only as they were able to find a wider market for the products they had to sell. As time went on, one form of production after another, spinning, weaving, sewing, tailoring, baking, butchering, soap-making, candle-making, brewing, preserving, laundering, dyeing, gardening, care of poultry, and other tasks have wholly or in part been transferred to commercial production. In addition, child care, education, and the care of the sick are now to a large extent carried on by paid workers. At the present time the urban not the rural family is typical; and urban families are dependent on the market even for subsistence goods.” (Reid, 1935, p. 47)

Two important industries that Reid omits are transportation and trade, both of which became much less home produced over time. Canals, railroads, and, later, mass transportation gradually replaced walking and horse-driven transportation. Similarly, sale of

\begin{table}[h]
\begin{tabular}{|l|c|c|}
\hline
Sector & <\$7500 & \geq\$7500 \\
\hline
Agriculture & -0.11 & -0.06 \\
 & (0.00) & (0.00) \\
Industry & 0.07 & -0.12 \\
 & (0.00) & (0.01) \\
Services & 0.04 & 0.18 \\
 & (0.01) & (0.01) \\
\hline
\end{tabular}
\end{table}

\textsuperscript{11}Owner-occupied housing services and self-consumed agricultural output, particularly important in poorer, agrarian economies, are often imputed into national accounts, but home production of most other goods and services are not.

\textsuperscript{12}Reid’s observation was for the United States. Deane and Cole (1967) describe production in pre-industrial Britain, where market transactions were more prevalent, but small-scale production in the home still dominated. Even as industrialization increased market production of textiles, many productive activities were still contracted or "put out" to households.
Figure 2: Sectoral Shares vs. Log Income per Capita for Country Panels
home-produced output at markets became a smaller and smaller fraction of trade, as permanent retailers developed and distribution chains expanded.

Eventually, many of these marketized activities have moved back in the home. Buera and Kaboski (2006) show how many services declined in the twentieth century as important modern technologies and goods diffused to households. Important product cycles include the decline of transportation services, such as railroads, rail lines, and buses with the spread of the private automobile. The automobile was also related to the decline in neighborhood retail services (food, apparel, ice, fuel, dairy, five and dime stores), as was the spread of refrigerators and freezers.\footnote{Lagakos (2006) examines the relationship between automobiles and retailing consolidation and productivity in the context of developing countries.} Similarly, the spread of washers, dryers, vacuums, microwaves, and other home appliances (see Greenwood et al, 2005) was accompanied by declines in domestic servants, laundres, and dry cleaners. Francis and Ramey (2006) cite historical evidence that the spread of many household appliances were associated with increases in household production labor because activities (e.g., bread baking, laundry) moved from market to home production. Many newer activities that have started in the market have also moved toward home production. Examples include the relative decline of movie theaters (spread of televisions, VCRs, and DVD players), mail services (computers, fax machines), and recently internet cafes (computers, cable internet connections).

These examples are quantitatively important. Together, Buera and Kaboski (2006) show that 75 percent of all declining service industries between 1950 to 2000 are associated with identifiable movements toward home production.

2.3 Large Scale Technologies

In the model of the next section, scale could be captured by capital, output, or labor per establishment. In the data, we will focus on workers per establishment as our metric or definition of scale. We highlight several facts of scale technologies that are crucial in understanding structural transformation. First, industrialization is linked to the introduction of large scale production methods. Second, the scale of services are much smaller than the scale of goods production, and is the distinguishing feature between the two sectors.. Third, in recent decades, the growth in services has been driven by large scale services.

2.3.1 Scale technologies and the industrial revolution

Historians link the industrial revolution with an increase in scale and the rise of the factory system. Still, historians describe a slow process of increasing scale, characterized by the staggered arrival of a series of large-scale technologies (Mokyr, 2001, Scranton, 1997).
Even in the early 18th century, new technologies were increasing the scale in agriculture, and setting the stage for industrial growth. These scale technologies involved important tools and investments, seed drills, iron plows, and threshing machine, and, most importantly, enclosures on land. A bit later, the most influential technologies of the first industrial revolution, e.g. textile milling, iron production, mining, canals, and steam power became increasingly economically viable.\textsuperscript{14} All led to increases in the scale of production, and required large capital investments. Similarly, the technologies of the second industrial revolution in the late 19th and early 20th century, such as steel, concrete, paper and chemicals, internal combustion engines, electricity, and food processing, led to even larger scales of efficient production, as did increased mechanization in agriculture (tractors, harvesters, etc.).\textsuperscript{15}

Scale technologies were not particular only to manufacturing, however, nor was the industrial revolution solely a story of industry. New scale production methods required both manufacturing and services in their delivery (Chandler, 1990). The figures in the previous section indicate that services were a substantial share of output even early on in industrialization. Services, transportation, retail trade, and wholesale trade, in particular, were important elements even in early industrialization (Mokyr, 1990, Chandler, 1990). Broadberry (2006) argues that new office technologies later lead also to an increase in the scale of major services.

The historical accounts of the scale of early production is supported by 19th century U.S. censuses of manufacturing made available by Atack and Bateman (1999). Most manufacturers were still small-scale, with the median establishment employing just three workers in 1850. Still, there were larger scale producers – means are substantially greater than the means, and scale grew in most industries between 1850 and 1870. Also, the scales of industries associated with the new technologies (steel, textiles, paper, engines, farming machinery) were an order of magnitude larger. These industries also tend to have experienced the largest increases in scale from 1850 to 1870. Appendix A presents these data from major industries that can be compared over time data from the 1850 and 1870 census of manufacturers.\textsuperscript{16}

\textsuperscript{14}Textiles poses as an example of the staggered arrival of technologies, which took over a century to fully move to large scale production. As Mokyr (2001) describes, cotton spinning, carding, bleaching, and printing were mechanized relatively early and moved to factory production, while weaving production remained in the home until the power looms arrival in the 1820s. Combed wool spinning was mechanized early, but the combing process was not mechanized until the mid-19th century. Hand production of worsted wool and linen lasted even longer.

\textsuperscript{15}Berg (1994) provides an excellent description of the early development of the factory system. Mokyr (1990) and Chandler (1990) give detailed accounts of technological innovations in the second industrial revolution and how they lead to large scale production .

\textsuperscript{16}We include only “major” industries for the sake of brevity, and define these as industries with at
In contrast, although the census is only of manufacturers, the smallest scale industries are those most commonly associated with services (dairy, bakeries, crop services, repair shops). For example, there was a large increase in the scale of meat products from 1850 to 1870 that may reflect a transformation of this industry from butchers to meat packers.

2.3.2 Large scale goods, smaller-scale services

The fact that manufacturing involves large-scale production, while services utilizes much smaller scale technologies is a salient characteristic in the data.\textsuperscript{17} Production performed on a very large-scale yields goods (e.g. commercial software), while smaller-scale production yields services (e.g., custom software). The histograms of establishment size in Figure 3 show services are overwhelmingly small scale relative to industry. Despite the wide variance of scale in industry, the distributions overlap very little. This distinction is true across each broad industry in the goods sector (including agriculture, mining, utilities, and manufacturing) and services sectors (transportation, services, public administration) with the exception of construction, which is typically in the industrial sector, but has many service-like characteristics.\textsuperscript{18}

Indeed, we view scale as the most consistent, economically-meaningful, distinction between the goods and service sectors. Over time, classifications of producers have changed as the scale of production has changed. Dean and Cole (1967, pp. 138-139) describe the problems of classification that arose from the “radical transformation” of the structure of the British economy. Many occupations were classified in “retail trade and handicraft” in the 1831 census (e.g., wood and furniture, shipbuilding, printing, fur and leather, dressmaking, watches, toys and musical instruments, food/drink and also iron founders, weavers, dyers, and paper) were classified as manufacturing in later censuses. At times, scale has been used as an explicit basis for classification.\textsuperscript{19}

The scale distinction is economically meaningful because it highlights a technological difference between the sectors.\textsuperscript{20} Also, scale affects the home production margin. The

\textsuperscript{17}The distinction between the size of firms, determined by contractual arrangements, and the size of establishments/productive units, determined by the efficient scale, is important here.

\textsuperscript{18}For example, construction is non-tradable, and much of construction consists of small-scale contractors for which their is a home production margin.

\textsuperscript{19}For example, in the 1927 census, producers of confectionaries, ice cream and sheet iron were deemed to be manufacturers (as opposed to services) if annual production was at least $20,000.

\textsuperscript{20}We argue that the scale distinction is more fundamental than other distinctions. The examples given
economic advantages of large-scale manufacturing pushes production out of small-scale traditional household production. Goods output, which is large scale, has no quantitatively important home production alternative, while small-scale services often involve decisions between home and market production. As the price of goods that lead to scale economies in services fall, many services move back into the home using modern production techniques. These decisions to home produce services affect not only labor supply, and the demand for services, but also the demand for complimentary inputs used in home production. Thus, the introduction of scale technologies, and their falling costs over time, drive both the increase in productive scale, but also changes in output across sectors, and movements between home and market production.
2.3.3 Growth in scale of services

During the post-1950 growth in services, the average scale of services has grown, while that of manufacturing has actually declined. Figure 4 displays this by plotting log workers per firm/estabishment for the goods and service sub-sectors over the period.\textsuperscript{21} Moreover, at a disaggregate level the growth in the service sector has been dominated by services whose scale has grown, and who are now among the largest scale services. Using scale and payroll information by 3-digit level from the 1959 and 1997 County Business Patterns, OLS show that \textit{tangibility} of output is not exclusive to the goods sector. Moreover, the \textit{tradability} and \textit{storability} of output, two characteristics often cited as distinguishing manufacturing from services, are related to scale, since both are required for centralized large-scale production. Related, Reid (1935) argues that manufacturing is production of form (an object), whereas services are production of circumstance (location, condition, etc.). Clearly, production of circumstance is related to customization, which requires smaller-scale production. Along this line, Locay (1990) partially endogenizes the differences in scale across sectors by modeling a multi-stage production process for final consumption, where downstream processes tend to be more customized and therefore smaller scale.

\textsuperscript{21}Data on Figure 4 is from the County Business Patterns. In 1974 there is a change from a "reporting unit" (firm) concept to establishment. A vertical line signals this break in the series.
regressions yield the following estimates (with standard errors in parentheses):

\[
\Delta \text{share}_i = 0.20 + 0.69 \Delta \log \text{scale}_i
\]

where \(i\) represents 3-digit SIC industry (based on IPUMS 1950 coding, which allows us to link it to IPUMS data on schooling levels of workers in each industry), \(\Delta \text{share}_i\) is the absolute change in the percentage share of industry in total payroll payments between 1959 and 1997. The positive coefficient on \(\Delta \log \text{scale}_i\), the change in log employees per establishment, is significant at a one percent level. That is, industries that have grown in share have been the industries whose scale has increased.

This result is robust in two important ways. First, excluding the five largest and five smallest changes in shares still yields an estimate that is positive and still significant at a five percent level. Second, the relationship is not a mere correlate with the relationship between growth and skill intensity observed in Buera and Kaboski (2006). Controlling for \(\text{skill}_i\), the fraction of labor in an industry that was college-educated in 1940\(^{22}\), yields the following estimates:

\[
\Delta \text{share}_i = -0.31 + 0.71 \Delta \log \text{scale}_i + 5.01 \text{skill}_i
\]

The coefficient on \(\Delta \log \text{scale}_i\) is nearly identical and still significant at a one percent level. Thus, growth in scale appears to be independently related to the growth of disaggregate services.

2.4 Summary

We have established seven important facts:

**Fact 1** The hump shape in the value-added share of manufacturing.

**Fact 2** A peak in the consumption share of non-food goods coinciding with the peak in manufacturing.

**Fact 3** A late rise in the value-added share of services.

**Fact 4** Rich product cycles between home and market production of activities, including the marketization and later demarketization of many services.

\(^{22}\)Using the fraction that was college-educated in 2000 yields similar results for the role of scale, though the coefficient on skill is somewhat smaller given the higher education levels.
Fact 5 The introduction of large scale technologies identified with the onset of the industrial revolution.

Fact 6 The difference in the scale of productive establishments distinguishing manufacturing (large-scale) from services (small-scale).

Fact 7 A growth in the scale of services during the period of service sector growth.

Fact 8 A strong relationship between the growth in services and their growth in scale.

In the next section we present a model consistent with Facts 5-8, which yields Facts 1-4.

3 A Theory of Structural Change

We model the consumption decision over a continuum of discrete wants. Individuals also choose whether to home produce or to procure these wants from the market. Production can be done using a traditional or a modern technology. Production using the modern technology requires the use of fixed amount of intermediate manufactured goods in combination with labor to produce up to a maximum scale. To satiate each want requires the use of both manufactured goods and services. In the model economy, as in the data, manufacturing differ from services by requiring a larger fixed cost and operating technologies with a larger scale.

3.1 Preferences

There is a continuum of consumption wants indexed by z. For each z, households make discrete decisions of whether to consume c(z), and, if so, home produce h(z), each want. Preferences over these decisions are represented by the following utility function:

\[ \tilde{u}(c, h) = \int_{z_A}^{+\infty} [h(z) + \gamma(1 - h(z))] c(z) \, dz + z_A \]  

(3)

where wants \( z \leq z_A \) correspond to subsistence needs (A is for agriculture) that must always be satisfied, and \( h(z) \leq c(z) \in \{0, 1\} \). As will be clear with the discussion of technologies, \( z \) indexes the complexity associated with the production of a want.\(^{23}\)

\(^{23}\)These preferences over a continuum of satiable wants are related to Matsuyama (2000, 2002) and Murphy, Shleifer and Vishny (1989). On the preference side, the innovation is to incorporate the home-production decision.
Since $\gamma \in (0, 1)$, home production yields more utility, perhaps because it avoids the disutility of public consumption (e.g., sitting next to others on the bus instead of driving your own car), or because it allows to customize final consumption to the particular needs of an individual (e.g., driving your own car allows to use the preferred scheduled and route).\footnote{An alternative way to motivate home-production is to introduce transaction cost. See Buera and Kaboski (2006) from a discussion of the implication of this alternative model.}

### 3.2 Technologies

Individual wants can be produced using a traditional or a modern (scale) technology. The traditional technology requires only labor as an input and experiences no productivity growth. The modern technology uses both labor and a fixed input of intermediate manufactured inputs to produce up to a maximum scale. Overtime, the productivity associated with the modern technology increases at a constant rate $g$.

#### 3.2.1 Traditional Technology

Individual wants can be produced using a traditional technology that requires only labor as an input and experiences no productivity growth:

$$y_0(z) = e^{-z}l$$

Labor productivity declines with the index of wants $z$, so that high $z$ goods and services are more complex, and therefore more difficult to produce. The traditional technology does not require manufactured inputs, and therefore exhibits no scale economies. Therefore, all production using the traditional technology is done at home.

#### 3.2.2 Modern (Scale) Technology

We also consider a modern production technology that requires a fixed input and is characterized by an efficient scale of production. In particular, production of goods and services associated with a want $z$ requires a specialized intermediate manufactured input of size $q$. Given the intermediate input, the technology is linear in labor $l$ up to a capacity of $n$:

$$y(z, t) = \begin{cases} 0 & \text{if } k < q \\ e^{gt} \min \{n, e^{-\lambda z}l\} & \text{if } k = q \end{cases}$$

Furthermore, $\lambda < 1$, i.e., the modern technology is relatively more productive than the traditional technology for more complex goods. The modern technology becomes relatively
more attractive over time because of technological change at a constant rate $g$, and as consumption moves towards more complex wants.

Here $n$ represents both the capacity and the efficient scale. For example, if a particular $z$ were laundry, a service, then $q$ might represent the cost of the laundry machine, which enables one to wash $n$ loads of laundry when used at capacity.

At home, individuals will produce only one unit of output, and therefore underutilize purchased intermediates, i.e., produce at an "inefficient" scale. For this implication, it is important that the intermediates are indivisible (one cannot be half as productive with half a laundry machine) and specialized (a car cannot substitute for a laundry machine in doing laundry).

### 3.2.3 Distinguishing Sectors

The first distinction between goods and services is made primarily for modeling simplicity. We assume that only goods are used as intermediate inputs, but services are solely final consumption. Goods consumption by households solely represents the purchase of intermediate inputs to home production.

The second, and more substantive, distinction we make between sectors is to assume that goods production is much larger scale than services production. This is consistent with the evidence presented in Section 2.

As we show in the following section, production requiring large intermediate inputs $q$ and/or done on a large scale $n$ will tend to be performed on the market. For simplicity we model the extreme limiting case as $q \to \infty$, so that manufactures are exclusively market produced. A further assumption of $n \to \infty$, and $q/n \to 0$ bounds the cost of goods. Thus, manufacturing production in the market simplifies to a constant return to scale technology:

$$y_M(z, t) = e^{gt - \lambda z} l_M$$

We also make the further simplification that goods are only intermediates and not valued directly in the utility function. Goods will nevertheless be purchased as final consumption to be used in household production of services. Including goods as direct final consumption

$$y_m(z, t) = e^{gt - \lambda z} \min \left\{ (1 - \alpha) l_m, \alpha k_m \right\}.$$  

\textsuperscript{25}Alternatively, we can assume that $\frac{q}{n} \to \alpha$, a constant that equals the intermediate goods’ share in manufacturing. In this case, manufacturing production in the market simplifies to a constant return to scale technology with fixed factor proportions:

$$y_m(z, t) = e^{\gamma t - \lambda z} \min \left\{ (1 - \alpha) l_m, \alpha k_m \right\}.$$  

17
is feasible, but complicates the analysis without yielding much insight. Thus, for every $z$ there is an intermediate good and a final service.

Finally, within the goods sector we distinguish agriculture as being the least complex goods, those below $z_A$.

The assumption that goods production is large scale makes it market rather than home produced.

### 3.3 Equilibrium

We can now state the household’s problem and the competitive equilibrium. For each want $z$, the household makes three linked binary decisions: whether to consume or not $c(z)$, if so whether to home produce or not $h(z)$, and again if so, whether to use the modern technology in home production $m(z)$.

Normalizing labor as the numeraire, the household takes the wage and the prices of each good $p_M(z)$ and service $p_S(z)$ as given, and solves the following static problem at each point in time:

$$
\max_{m(z) \leq h(z) \leq c(z)} \int_{z_a}^{+\infty} \left[ h(z) + \gamma \left( 1 - h(z) \right) \right] c(z) \, dz + z_A
$$

s.t.

$$
\int_{-\infty}^{\infty} c(z) \left[ \frac{h(z) m(z) p_M(z) + (1 - h(z)) p_S(z,t)}{\text{manuf. cons.}} + \frac{1 - m(z) p_M(z) + (1 - h(z)) p_S(z,t)}{\text{service, cons.}} \right] \, dz =
$$

$$
1 - \int_{-\infty}^{\infty} h(z) \left[ \frac{m(z) e^{\delta t + \lambda z}}{\text{modern home production}} + \frac{1 - m(z) e^{\lambda z}}{\text{trad. home production}} \right] \, dz
$$

The left-hand side of the budget constraint is total market expenditures, while the right-hand side is income/labor supply.

The first-order condition of whether to home produce or market purchase output of a particular $z$ yields the central intuition for the model:

$$
\mu \left[ p_M q \left( 1 - \frac{1}{n} \right) \right] > 1 - \gamma
$$

The assumption that goods production is large scale makes it market rather than home produced. This can be seen clearly from the the household’s first-order condition, i.e.,
the first order conditions associated with the maximization problem in (5): where, \( \mu \) is the marginal utility of income. The bracketed term represents the cost-savings of market production. Both market and home production use labor (valued at the opportunity cost of time \( w = 1 \)), but the market service requires paying only a fraction \( (1/n) \) of the intermediate goods cost, as opposed to the full goods cost from purchasing the input. Households will use the market if the utility value of this cost-savings (left-hand side) exceeds the lost utility from consuming market- rather than home-produced output (right-hand side). Output that requires large or expensive intermediates (high \( q \) or \( p_M \)), or has a large efficient scale \( n \) will be home produced. Hence, our assumption that manufacturing requires large intermediates inputs \( q \) and is done on a large scale \( n \) justify the statement that manufacturing is market produced.\(^{26}\)

The first-order condition with respect to the decision of whether to use the modern technology simply yields that the modern technology is used if the time cost of traditional production sum of the goods and time cost for modern production.

A competitive equilibrium is given by price functions \( p_M(z,t) \), \( p_S(z,t) \), consumption, home production, and technology decisions \( c(z) \), \( h(z) \) and \( m(z) \) (associated with purchases of goods and services by households) such that: i) given prices \( p_M(z,t) \) and \( p_S(z,t) \), \( c(z) \), \( h(z) \) and \( m(z) \) solve (5) ; ii) prices solve zero profits conditions, i.e.,

\[
p_M(z,t) = e^{-gt+\lambda z}
\]

and

\[
p_S(z,t) = \left(1 + \frac{q}{n}\right) p_M(z,t);
\]

iii) markets (i.e., for labor, each \( z \) good, and each \( z \) service) clear.

Next, we characterize the evolution of the structure of production of the economy. This process includes a shift from traditional technologies to modern (scale) technologies, changes in the wants that are home vs. market produced, and a transformation of the sectoral composition of output and employment.

4 Evolution of Structural Change

This section presents the results of the paper, which tie in closely with the facts presented in Section 2, given our assumption of large scale modern technologies (Fact 5), and the

\(^{26}\)Strictly speaking, if manufactured goods are only intermediate goods there will not be a utility advantage associated with home-production of manufactures. The following heuristic argument should be understood within generalized model in which there is a utility gain associated with the home-production of manufactures, e.g., because of the possibility of customizing its design.
larger relative scale of manufacturing (Fact 6). Proposition 1 describes the early transition from the pre-industrial to industrial scale economies and the marketization of previously home production activities, while Proposition 2 describes the later phase of industrialization in which activities return to the home as households begin mass consumption of modern technology intermediates. Thus, together the two propositions lead to rich product cycles (Fact 4), and a growth in manufacturing that is tied to the growth in the consumption of non-food goods (Fact 2). Finally, Proposition 3 shows how the share of the service sector is increasing in its efficient scale of production, thus the model matches the relationship between scale and share (Fact 8). Given the recent growth in the scale of services (Fact 7), Proposition 3 predicts a recent growth in the share of services (Fact 3). The corresponding decline in manufacturing, coupled with the earlier increase yield the hump shape in manufacturing (Fact 1).

4.1 Early Structural Transformation

In early times, i.e., for a low enough $t$, only the traditional technology is utilized. Since production using the traditional technology requires no specialized inputs, all production is done at home. Households consume the low $z$ goods first, since all $z$ are valued symmetrically, but the least complex output is cheapest to produce. An upper bound $z_0(t)$ defines the range of goods that are produced using the traditional technology. Early on, $z_0(t)$ also equals the most complex want that is satiated $z(t)$. This upper bound remains fixed until industrialization.\footnote{27This meshes with the historical evidence of the pre-industrial economy: relatively stagnant, with a very high fraction of production at home, and at a small scale (Reid, 1935, Deane and Cole, 1967, Mokyr, 1990, 2001).}

As productivity improves, the modern technology eventually becomes economically viable. The frontier $z = z_0$ is the first to be replaced by the modern technology, but over time the modern technology becomes more productive for even the less complex output. During this period, the upper range of consumption $z(t)$ increases, and the upper range of consumption produced using the old technology $z_0(t)$ declines. In particular, there exists a point in time at which the modern technology overtakes the traditional technology for the most complex want that is satiated, $z = z_0$:

$$t_0 = \frac{1}{g} \log \left( \frac{1 + \frac{q}{n}}{\gamma} \right)$$

The timing of the onset of industrialization in the model depends positively on the share of intermediate specialized inputs in the modern technologies, $q/n$, and negatively on the
rate of productivity growth in the modern technology and the disutility associated with market consumption.\textsuperscript{28}

The rise of scale technologies is associated with an increase in $\bar{z}(t)$ i.e., an expansion of the wants that are satiated, and a decrease in $z_0(t)$ (a decline of the range of wants satisfied through the traditional technology). Figure 5 illustrates this process. It describe the average cost per util as a function of the complexity of wants for the traditional (dotted) and modern (solid) technologies. Over time, the average cost per util for the modern technology declines.

Whether the new modern production that was previously traditional occurs as market or home production depends on the efficient scale of services relative to the utility advantage of home-production. If the scale of services is sufficiently small relative to the utility advantage of home-production, $1 + q/n > \gamma (1 + q)$, the advent of the modern technology is associated with a rise in the consumption of intermediate manufactured goods by households to be used as input in the home production of services. For these wants, services remain home produced, and there is just a transition from a traditional to a modern technology that

\textsuperscript{28}In modelling the onset of the industrial revolution as the moment in which a modern technology overcomes a traditional technology we follow Hansen and Prescott (2002). See also Stokey (2001).
utilizes intermediate inputs produced with a large scale technology. In the case of wants for which the scale of service production is large relative to the utility advantage of home-production, \(1 + q/n < \gamma (1 + q)\), service production using the modern technology occurs on the market.

The model could be generalized so that different individual wants \(z\) had different parameter \(q, n, \) and \(\gamma\), and one can think of examples for the two cases. The provision of clothing services would be an example of the former wants that move straight to being satisfied through home production.\(^{29}\) Examples of the latter might be laundry, which was washed initially by hand at home, but later washed at a larger scale market laundry utilizing modern laundry equipment, or transportation, which was initially self-produced but increasingly market provided with the introduction of mass transit and rail. For these latter examples, the modern technology requires both market services and manufacturing, and so with growth, both of these sectors increase relative to agriculture.\(^{30}\) Thus, the model can explain a sizable share for services even early in industrialization, consistent with the evidence shown previously in Figure 1 for the United States.

We summarize the previous discussion in the following proposition.

**Proposition (Industrialization):** There exist two critical periods \(t_0\) and \(t_1\), \(t_0 < t_1\), such that:

i) for \(t < t_0\), only the traditional technology is utilized, the set of wants that are satiated remains fixed, and all production is done at home, i.e., \(z_0(t) = z(t) = \bar{z}(t) = 0\); 

ii) for \(t_0 \leq t < t_1\),

(a) the most complex wants are produced using the modern technology, \(z_0(t) \leq z \leq \bar{z}(t)\), the set of wants satiated expands, \(\partial \bar{z}(t)/\partial t > 0\), the set of wants produced using the traditional technology contracts, \(\partial z_0(t)/\partial t < 0\); and

(b) if \(\frac{1+q/n}{1+q} < (>)\gamma\), the most complex wants are satisfied in the market (at home) using the modern technology, and the service and industrial sectors (only the industrial sector) grow relative to agriculture.

### 4.2 The Rise of Mass-Consumption

Eventually the goods cost of producing any particular service \(z\) fall enough to induce direct household purchase of the market good and the home production of this service. Services

\(^{29}\)Nevertheless, in the case of very specialized clothing services, e.g., tuxedo rentals, we do observe the market provision of these services.

\(^{30}\)Agricultural production, together with housing, are the only home production that are included in National Income accounts.
begin returning to home production, but this time using the modern technology. This leads to the mass consumption of manufactured goods that are used in the production of services, and therefore is associated with sectoral reallocations in output: the return of market production to the home increases the demand for the given market good (by a factor of n), and decreases the purchase of the related service. Thus, the manufacturing sector experiences a boom relative to the service sector, and this contributes to the rising section of the hump-shaped manufacturing trend found in the data.

**Proposition (Mass Consumption):** Assume \((1 + q/n)/(1 + q) < \gamma\). Then, for \(t \geq t_1\), the most complex home-produced wants are produced using the modern technology, \(z_0(t) < z \leq \bar{z}(t) < \bar{z}(t)\), the set of wants satiated expands, \(\partial \bar{z}(t)/\partial t > 0\), the set of home-produced wants using the modern technology expands, \(\partial \bar{z}(t)/\partial t > 0\) and \(\partial z_0(t)/\partial t < 0\); and the industrial sector grows relative to the service sector.

The model’s prediction on this front is consistent with Fact 2, the rise in the consumption of non-food goods. Also, this consumption could be interpreted as driven by a particular understanding of “discretionary spending”, which Katona (1964) claimed characterized the mass consumption society. That is, \(t_1\) is the point in time at which households first satisfy consumption in ways that are more expensive than alternatives (i.e., the cost of modern household production exceeds the cost of market production). The threshold \(t_1\) is also the point in time in which households consumption of market goods expands.

### 4.3 Large Scale Services and the Decline of Manufacturing

The previous sections have developed the model’s ability to deliver a long extended rise of industry. This section focuses on the model’s implications for the later decline in manufacturing, and corresponding rise in services.\(^{31}\)

The model predicts that the larger the scale of services, the larger the relative size of services sector. There are two intuitive reasons. First, the larger the scale, the smaller the goods cost per unit. That is, keeping \(q\) constant, the share of intermediate goods is decreasing in scale. Second, the larger the scale, the larger the cost savings of market production of services (which produces at this efficient scale) relative to home production. The following proposition formalizes this.

**Proposition:** Both the share of market services (relative to market goods) and the ratio

\(^{31}\)Buera and Kaboski (2006) focus on a related, and complementary explanation for the growth in services: their increasing skill intensity.
of market labor to home labor are increasing in the scale of services, $n$.

The proposition is relevant to the recent growth of the service sector and changes in the scale of services. First, given the increasing scale of services (Fact 7), the model would predict a corresponding decline of the manufacturing sector, and rise in services. Second, a model with heterogeneity would predict that aggregations of services with growing scale $n$ would also have growing service shares (Fact 8).

### 4.4 Summary

We have presented three phases of growth consistent with (1) an early introduction of scale technologies leading to industrialization and a relative decline in the importance of agricultural output; (2) a somewhat later expansion of industry associated with mass consumption, and (3) still later expansion of services with the growth in their scale. Figure 6 illustrates the three phases of structural change in the model economy.
5 Explicit Durability/Capital

In this section, we extend the basic model to allow for the durability of intermediate manufactured (capital) inputs. This is more in line with much of the earlier motivation which involves home durables and market capital goods. It also shows how the model maps into a more standard dynamic model that has similarities to the standard neoclassical growth model, but also yields insight into sectoral allocations.

In particular, we assume that each intermediate input faces one-hoss-shay depreciation at a constant hazard rate $\delta$. As before there are continuum of wants indexed by $z$ that are provided using labor and capital as inputs. For simplicity, we only consider the limit case where the modern technology is used in the production of all wants. The preferences over the various wants within a period are still represented by the utility function (1), while the intertemporal preferences are represented the following time-separable utility function:

$$\int_0^\infty e^{-\rho t} U(C(t)) \, dt \quad (7)$$

where $C(t) = \int_{z_A}^{h(t)} \left[ h(z, t) + \gamma \left(1 - h(z, t)\right)\right] c(z, t) \, dz$ and $U(\cdot)$ is a strictly increasing and concave function.

To simplify the exposition, we consider a decentralization in which households own the durable goods used in home-production while the capital used by the market sector is own by a competitive holding company. Under this assumption, the household’s problem simplifies to maximize (7) by choosing the stock of durable goods used in home-production $z(t)$, the purchases of durable goods $d(t)$, the most complex want that is purchase in the market $\tilde{z}(t)$, and the stock of bonds $B(t)$ subject to the time-$t$ budget constraint

$$\frac{\partial B(t)}{\partial t} + \int_{\tilde{z}(t)}^{z(t)} p_s(z, t) \, dz + d(t) = rB(t) + 1 - \int_{-\infty}^{\tilde{z}(t)} \frac{dz}{A(z, t)}$$

where the left-hand-size gives the purchases of new bonds, market services and durable goods and the right-hand-size the capital and labor income; and the law of motion for the stock of durable goods

$$\frac{\partial K^d(t)}{\partial t} = \delta K^d(t) + d(t)$$

where $K^d(t) = q_s \int_{-\infty}^{\tilde{z}(t)} p_m(z, t) \, dz$, as all wants with complexity $z < \tilde{z}(t)$ are home-produced and therefore $q_s$ united of capital is required for production.

Standard optimal control methods can be used to derive the dynamic system imply by the consumer’s problem. In what follows we describe a balance growth path of this system.
Provided $U(C) = -\frac{\varepsilon \sigma C}{\sigma}$, a balanced growth path exists and is characterized by the following two equations:

$$r = \rho + \sigma \frac{g}{\lambda}$$

and

$$p_m(\bar{z}, t) q_s(r + g + \delta) + \frac{1}{A(\bar{z}, t)} - p_s(\bar{z}, t) = (1 - \gamma) \frac{p_s(\bar{z}, t)}{\gamma}$$

The first condition, the Euler equation, equates the interest rate to the rate of time preference plus a multiple of the growth rate, $\frac{g}{\lambda}$. The second condition equates the marginal cost of expanding the set of home-produced goods to the marginal return. The marginal cost (left-hand side) is given by the sum of the rental cost, $p_m(\bar{z}, t) q_s(r + g + \delta)$ and the labor costs, $\frac{1}{A(\bar{z}, t)}$, netted of the savings associated with not having to satisfied this want in the market, $p_s(\bar{z}, t)$. The marginal return (right-hand side) is proportional to the utility gain of home-production relative to market consumption of a given want, $1 - \gamma$. This last condition determines the (constant) width of the set of services that are provided by the market, $\bar{z}(t) - \bar{z}(t)$.

The model with durability allows us to study the effect of an increase in the cost of capital on the structural composition of consumption of this economy.

**Proposition:** The share of services in consumption $c_s$ is a decreasing function of the cost of capital $r$.

A larger cost of capital, due to a larger discount rate $\rho$ or capital distortions, leads to a bigger cost advantage of market services that use more “efficiently” the capital input. Interestingly, this result is independent of whether services are more or less capital intensive than manufactures.

## 6 Conclusions

This paper has tried to incorporate the efficient scale of productive units into theory, particularly the distinction between the scale of production in manufacturing, market services, and home produced services, and secular patterns on the efficient scale of production. These factors help provide a unified explanation for broad trends of structural transformation, including not only scale, but also sectoral movements, and rich product cycles between home and market production.
We have also presented a potentially important explanatory factor in understanding the recent growth of the service economy: the increasing scale of services, and the increasing importance of large scale services. To the extent, that these large scale may be improperly classified as services, these trends have implications for revisiting sectoral definitions in the national income accounts.

Our emphasis on the importance of scale is relevant to the definition of service sector in National Accounting classification schemes. In particular, the NAICS system, which was instituted in the 1990s, moved in principle to a production method concept of industry. Still, it moved many large-scale information industries such as software publishing, printing, and motion pictures were classified into the service sector, while smaller scale activities such as bakeries and custom “manufactures” were moved into manufacturing. Such classifications based on the content of what is produced rather than the production method lead to a less meaningful distinction between the sectors. Perhaps such classifications need to be revisited.
A Proof of the Results in the Paper

The various results in the paper follows from the characterization of the household’s problem. In this appendix we provide a characterization of this problem and we relate this characterization to the propositions in the paper.

Household choose the set of wants to home-produced using the traditional technology, \( z \in (-\infty, z_0] \), the set of wants to home-produced using the modern technology, \( z \in (z_0, \bar{z}] \), and the set of want to be market produced, \( z \in (\underline{z}, \bar{z}] \), where \( z_0 \leq \underline{z} \leq \bar{z} \). Thus, households choose thresholds \( z_0, \underline{z} \) and \( \bar{z} \) to maximize

\[
\max_{z_0 \leq \underline{z} \leq \bar{z}} (1 - \gamma) \max_{\underline{z}} \gamma \bar{z}
\]

subject to the budget constraint

\[
\int_{z_0}^{\bar{z}} q_{PM}(z, t) \, dz + \int_{\underline{z}}^{\bar{z}} p_{S}(z, t) \, dz = 1 - \int_{-\infty}^{z_0} e^{\gamma z} \, dz - \int_{z_0}^{\bar{z}} e^{-gt+\lambda z} \, dz
\]

where \( p_{M}(z, t) = e^{-gt+\lambda z} \) and \( p_{S}(z, t) = (1 + \frac{q}{n}) p_{M}(z, t) \). The first order conditions are

\[
\gamma + \theta_2 = \mu p_{S}(\bar{z}, t)
\]

\[
(1 - \gamma) + \theta_1 - \theta_2 = \mu \left[ e^{-gt+\lambda \bar{z}} + q_{PM}(\bar{z}, t) - p_{S}(\bar{z}, t) \right]
\]

and

\[
-\theta_1 = \mu \left[ e^{z_0} - e^{-gt+\lambda z_0} - q_{PM}(z_0, t) \right]
\]

where \( \mu \) is the Lagrange multiplier of the budget constraint, while \( \theta_1 \) and \( \theta_2 \) are the Lagrange multipliers of the inequality constraints, \( z_0 \leq \underline{z} \leq \bar{z} \).

There are 4 cases to be considered.

Case 1: \( z_0 = \underline{z} = \bar{z} \) In this case, all production is done at home using the traditional technology. The most complex want that is satisfied using the traditional technology solves:

\[
\int_{-\infty}^{z_0} e^{\gamma z} \, dz = 1
\]

or

\[
z_0 = 0.
\]
This corresponds to the pre-industrial economy in which the set of wants that are satisfied remain constant over time. This will be the optimal solution as long as the following inequality is satisfied

\[
e^{z_0} < \min \left\{ (1 + q) e^{-gt + \lambda z_0}, \left(1 + \frac{q}{n}\right) \frac{e^{-gt + \lambda z_0}}{\gamma} \right\}
\]

\[= e^{-gt + \lambda z_0} \min \left\{ (1 + q), \frac{1}{\gamma} \left(1 + \frac{q}{n}\right) \right\}
\]

This inequality holds for a sufficiently early date, i.e.,

\[t < t_0 = \frac{1}{g} \log \left( \min \left\{ (1 + q), \frac{1}{\gamma} \left(1 + \frac{q}{n}\right) \right\} \right)
\]

**Case 2: \(z_0 = z < \bar{z}\)** The first order conditions simplify to

\[\gamma = \mu \left(1 + \frac{q}{n}\right) e^{-gt + \lambda \bar{z}}, \quad (8)
\]

\[(1 - \gamma) = \mu \left[e^{z_0} - \left(1 + \frac{q}{n}\right) e^{-gt + \lambda z_0}\right] \quad (9)
\]

and

\[
\left(1 + \frac{q}{n}\right) \int_{z_0}^{\bar{z}} e^{-gt + \lambda z} dz = 1 - \int_{-\infty}^{z_0} e^{\bar{z}} dz \quad (10)
\]

Conditions (8), (9) and (10) simplify to two equations in \(\bar{z}\) and \(z_0\)

\[
\left(1 + \frac{q}{n}\right) e^{-gt + \lambda \bar{z}} = \frac{\gamma}{1 - \gamma} \left[e^{z_0} - \left(1 + \frac{q}{n}\right) e^{-gt + \lambda z_0}\right] \quad (11)
\]

and

\[
\frac{1}{\lambda} \left(1 + \frac{q}{n}\right) e^{-gt + \lambda \bar{z}} + e^{z_0} - \frac{1}{\lambda} \left(1 + \frac{q}{n}\right) e^{-gt + \lambda z_0} = 1 \quad (12)
\]

Equations (11) and (12) define an upward and a downward sloping curve in the \((z_0, \bar{z})\) space respectively. It is straightforward to see that \(\partial \bar{z} / \partial t > 0\) as both curves move upwards with productivity. The effect of technological progress on the upper bound of the set of wants that are home produced using the traditional technology \(z_0\) is given by

\[
\frac{\partial z_0}{\partial t} = -\frac{g (1 - \gamma) \left(1 + \frac{q}{n}\right) e^{-gt + \lambda z_0}}{\frac{\lambda (1-\gamma) + \gamma}{\lambda} e^{z_0} - \left(1 + \frac{q}{n}\right) e} < 0,
\]

29
This corresponds to the optimal solution if the following set of inequalities are satisfied:

\[
\left(1 + \frac{q}{n}\right) \frac{e^{-gt+\lambda z_0}}{\gamma} < e^{\bar{z}_0} < (1 + q) e^{-gt+\lambda z_0}
\]  

(13)

Alternatively, this is the solution if \(1 + \frac{q}{n}\frac{1}{\gamma} < (1 + q)\) and \(t_0 < t < t_1\) where

\[
t_1 = \frac{1 - \lambda}{g} \log \left\{ \frac{\gamma + (1 - \gamma) \lambda}{(1 - \gamma) \lambda} - \left(1 + \frac{q}{n}\right) \frac{1}{(1 - \gamma) \lambda} \right\} + \frac{\lambda}{g} \log (1 + q).
\]

**Case 3: \(z_0 < \bar{z} < \bar{z}\)** This correspond to the situation after the rise of mass consumption. In this case, the first order conditions simplify to

\[
\gamma = \mu \left(1 + \frac{q}{n}\right) e^{-gt+\lambda \bar{z}},
\]

(14)

\[
(1 - \gamma) = \mu \left[e^{-gt+\lambda \bar{z}} + q e^{-gt+\lambda \bar{z}} - \left(1 + \frac{q}{n}\right) e^{-gt+\lambda \bar{z}}\right],
\]

(15)

\[
e^{\bar{z}_0} - (1 + q) e^{-gt+\lambda z_0} = 0,
\]

(16)

and

\[
e^{\bar{z}_0} + (1 + q) \left[e^{-gt+\lambda \bar{z}} - e^{-gt+\lambda \bar{z}}\right] + \left(1 + \frac{q}{n}\right) \left[e^{-gt+\lambda \bar{z}} - e^{-gt+\lambda \bar{z}}\right] = 1.
\]

(17)

This correspond to the optimal solution if the following set of inequalities are satisfied:

\[
\left(1 + \frac{q}{n}\right) \frac{e^{-gt+\lambda z_0}}{\gamma} < (1 + q) e^{-gt+\lambda z_0} < e^{\bar{z}_0}
\]

Equation (16) can be solved for \(z_0\)

\[
z_0 = \frac{1}{1 - \lambda} \log (1 + q) - \frac{g}{1 - \lambda} t
\]

Using (14) and (15) we obtain a linear relationship between \(\bar{z}\) and \(\bar{z}\)

\[
\bar{z} = \frac{1}{\lambda} \log \left(\frac{(1 - \gamma) \left(1 + \frac{q}{n}\right)}{\gamma q \left(1 - \frac{1}{n}\right)}\right) + \bar{z}
\]

Finally, using (17) it is straightforward to see that \(\bar{z}\) and \(\bar{z}\) increase over time.
Case 4: \( z_0 < \bar{z} = \ddot{z} \) For this case, the first order conditions simplify to

\[
1 = \mu \left[ e^{-gt + \lambda \ddot{z}} + q e^{-gt + \lambda \bar{z}} \right],
\]

\[
e^{z_0} - e^{-gt + \lambda z_0} - q e^{-gt + \lambda \bar{z}_0} = 0
\]
and

\[
e^{z_0} + (1 + q) \frac{1}{\lambda} \left[ e^{-gt + \lambda z} - e^{-gt + \lambda \bar{z}_0} \right] = 1
\]
As in case 3, \( z_0 = \frac{1}{1-\lambda} \log (1 + q) - \frac{g}{1-\lambda} t \) and \( \ddot{z} \) increases overtime.

A.1 Explicit Durability

Appendix

In the model with durable intermediate (capital) input the household’s problem simplifies to

\[
\max_{B(t), \bar{z}(t), d(t), \bar{z}(t)} \int_{0}^{\infty} e^{-\rho t} U \left((1 - \gamma) \bar{z}(t) + \gamma \ddot{z}(t)\right) dt
\]

s.t.

\[
\frac{\partial B(t)}{\partial t} + \int_{0}^{\bar{z}(t)} p_s(z,t) dz + d(t) = rB(t) + 1 - \int_{-\infty}^{\bar{z}(t)} \frac{dz}{A(z,t)}
\]
and

\[
\frac{\partial K^d(t)}{\partial t} = \delta K^d(t) + d(t)
\]
where \( K^d(t) = q_s \int_{-\infty}^{\bar{z}(t)} p_m(z,t) dz \).

The Hamiltonian of this problem is given by

\[
H(t) = u \left((1 - \gamma) \bar{z} + \gamma \ddot{z}\right) + \theta \left[ 1 + rB - \int_{-\infty}^{\bar{z}} \frac{dz}{A(z,t)} - \delta q_s \int_{-\infty}^{\bar{z}} p_m(z,t) dz - \int_{\ddot{z}}^{\bar{z}} p_s(z,t) dz - d \right]
+ \mu \frac{d}{q_s p_m(\bar{z},t)}
\]

The Principle of the Maximum implies

\[
u'(C) \gamma = \theta p_s(\bar{z}, t)
\]
\[-\dot{\theta} = (r - \rho) \theta\]  

(19)

\[-\dot{\mu} = -\rho \mu + u'(C) (1 - \gamma) - \theta \left[ \frac{1}{A(z,t)} + \delta q_s p_m (z,t) - p_s (z,t) \right] - \mu \frac{d}{\delta p_m (z,t)} \]  

(20)

and

\[\theta(t) = \frac{1}{q_s p_m (z,t)} \mu(t)\]  

(21)

Performing standard manipulations we obtain the Euler equation

\[\sigma \left[ (1 - \gamma) \frac{\partial z(t)}{\partial t} + \gamma \frac{\partial z(t)}{\partial t} \right] = r - \rho - \frac{1}{p_s (z,t)} \frac{\partial p_s (z,t)}{\partial z} \frac{\partial z}{\partial t} + g\]  

(22)

and an equation for the equality of marginal cost of durables to the marginal return of durables

\[q_s p_m (z,t) (r + g + \delta) + \frac{1}{A(z,t)} - p_s (z,t) = \frac{1 - \gamma}{\gamma} p_s (z,t)\]  

(23)

**Holding Company’s Problem**

We assume that there is a competitive holding company that owns the capital stock used by the market sector. In particular, the holding company purchase manufacturing goods and rent these for a rental price \(R(z,t)\) to maximize the present value of profits

\[\int_0^\infty \int_{-\infty}^\infty e^{-rt} [R(z,t) k(z,t) - I(z,t) p_m (z,t)] dz dt\]

subject to the low of motion for each type of capital

\[\dot{k} (z,t) = I(z,t) - \delta k(z,t)\]

The firm’s problem solve the following Hamiltonian problem,

\[H (t) = \int_{-\infty}^\infty \{ [R(z,t) k(z,t) - I(z,t) p_m (z,t)] + \kappa(z,t) [I(z,t) - \delta k(z,t)] \} dz\]
Necessary conditions are:

\[ \kappa(z, t) = p_m(z, t) \]

\[ -\dot{\kappa}(z, t) = R(z, t) - r\kappa(z, t) - \kappa(z, t) \delta \]

implying

\[ R(z, t) = p_m(z, t) \left( r + \delta - \frac{1}{p_m(z, t)} \frac{\partial p_m(z, t)}{\partial t} \right) \tag{24} \]

**Producer’s Problem**

Competitive firms produce market services and manufacturing goods. Zero profits imply

\[ p_m(z, t) = \frac{1}{A(z, t)} + \frac{q_m}{n_m} R(z, t) \]

and

\[ p_s(z, t) = \frac{1}{A(z, t)} + \frac{q_s}{n_s} R(z, t) \]

using (24)

\[ p_m(z, t) = \frac{1}{A(z, t)} + \frac{q_m}{n_m} p_m(z, t) \left( r + \delta - \frac{1}{p_m(z, t)} \frac{\partial p_m(z, t)}{\partial t} \right) \]

guessing \( \frac{1}{p_m(z, t)} \frac{\partial p_m(z, t)}{\partial t} = -g \) and using \( A(z, t) = e^{gt} \lambda z \),

\[ p_m(z, t) = \frac{e^{\lambda z - gt}}{1 - \frac{q_m}{n_m} (r + \delta + g)} \tag{25} \]

and

\[ p_s(z, t) = \frac{1 + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + \delta + g)}{1 - \frac{q_m}{n_m} (r + \delta + g)} e^{\lambda z - gt} \tag{26} \]

where a bounded price of manufactured goods requires \( 1 - \frac{q_m}{n_m} (r + \delta + g) > 0 \). In this economy capital shares equal

\[ \alpha_m = \frac{q_m}{n_m} (r + \delta + g) \]
\[ \alpha_s = \frac{q_s n_s (r + \delta + g)}{1 + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + \delta + g)} \]

Thus, as long as \( r(t) = r \) we get constant factor shares. Furthermore, if \( \frac{q_s}{n_s} = \frac{q_m}{n_m} \)
\( \alpha_s = \alpha_m = \alpha. \)

**Balance Growth Path**

For a balance growth path we need to have \( \bar{z}(t) = \bar{z}(0) + \frac{q}{\lambda} t \) and \( \bar{z}(t) = \bar{z}(0) + \frac{q}{\lambda} t. \)
Substituing these conditions into (22)

\[ \sigma \left[ (1 - \gamma) \frac{g}{\lambda} + \gamma \frac{g}{\lambda} \right] = r - \rho - g + g \]

or

\[ r = \rho + \sigma \frac{g}{\lambda} \]

From (23)

\[ q_s p_m(\bar{z}, t) (r + g + \delta) + \frac{1}{A(\bar{z}, t)} - p_s(\bar{z}, t) = \frac{1 - \gamma}{\gamma} p_s(\bar{z}, t) \]

or

\[ q_s (r + g + \delta) + \frac{1}{p_m(\bar{z}, t) A(\bar{z}, t)} - \frac{p_s(\bar{z}, t)}{p_m(\bar{z}, t)} = \frac{1 - \gamma}{\gamma} p_s(\bar{z}, t) \]

Using (25) and (26),

\[ q_s \left( 1 - \frac{1}{n_s} \right) (r + g + \delta) \]

\[ = \frac{1 - \gamma}{\gamma} \left[ 1 + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + \delta + g) \right] e^{\lambda(\bar{z}(0) - \bar{z}(0))} \]

or

\[ q_s \left( 1 - \frac{1}{n_s} \right) \]

\[ = \frac{1 - \gamma}{\gamma} \left[ \left( \frac{1}{r + g + \delta} + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) \right) e^{\lambda(\bar{z}(0) - \bar{z}(0))} \]
The share of services in consumption equals

\[ c_s = \frac{\int_{-\infty}^{\infty} p_s(z, t) \, dz}{\delta q_s \int_{-\infty}^{\infty} p_m(z, t) \, dz + q_s \frac{\partial q_s}{\partial t} p_m(z, t) + \int_{-\infty}^{\infty} p_s(z, t) \, dz} \]

or

\[ c_s = \frac{\left[ 1 + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + \delta + g) \right] \left( e^{\lambda(\bar{z}(0) - \bar{z}(0))} - 1 \right)}{q_s (\delta + \frac{g}{\lambda}) + \left[ 1 + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + \delta + g) \right] \left( e^{\lambda(\bar{z}(0) - \bar{z}(0))} - 1 \right)} \]

(27)

Proof of Proposition: Differentiating (27) with respect to the cost of capital we obtain

\[ \frac{\partial c_s}{\partial r} = \frac{q_s (\delta + \frac{g}{\lambda})}{[q_s (\delta + \frac{g}{\lambda}) + A]^2} \frac{\partial A}{\partial r} \]

where

\[ A = \left[ 1 + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + \delta + g) \right] \left( e^{\lambda(\bar{z}(0) - \bar{z}(0))} - 1 \right) \]

and

\[ \frac{\partial A}{\partial r} = \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + g + \delta) \frac{q_s \left( 1 - \frac{1}{n_s} \right)}{1 - \frac{1-\gamma}{\gamma} \left[ 1 + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + g + \delta) \right]} - \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) \]

\[ + \frac{q_s \left( 1 - \frac{1}{n_s} \right)}{1 - \frac{1-\gamma}{\gamma} \left[ 1 + \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + g + \delta) \right]} \]

If \( \frac{q_s}{n_s} - \frac{q_m}{n_m} > 0 \) then \( \frac{\partial A}{\partial r} > 0 \) as the first two terms on the right-hand-side are equal to \( \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) \left[ e^{\lambda(\bar{z}(0) - \bar{z}(0))} - 1 \right] \) and are therefore positive. In the case \( \frac{q_s}{n_s} - \frac{q_m}{n_m} < 0 \), we know that the sum of the first and third terms are positive as \( \left( \frac{q_s}{n_s} - \frac{q_m}{n_m} \right) (r + g + \delta) + 1 > 0 \).
References


