Inflation Dynamics: Price Setting Models and Data

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Motivation

- Central questions in macroeconomics
  - what are the drivers of inflation?
  - what are the effects of shocks on output?
    - e.g. oil shocks, government spending shocks, monetary policy shocks
  - how should monetary policy respond?
- Answer depends on how firms adjust prices in response to shocks

Simple Example: Response to Monetary Shocks

- Suppose monetary policy targets nominal spending: $M_t = P_t Y_t$

- Consider a monetary policy shock: $\Delta m_t = \log M_t - \log M_{t-1}$

- The shock must be absorbed by either prices or output

$$\Delta m_t = \Delta p_t + \Delta y_t$$

- Effect on output depends on how strongly aggregate price level responds

  – if prices are flexible, P_t adjusts immediately, no effect on Y_t

  – if prices are sticky, P_t adjusts sluggishly, effect on Y_t

- *Key question*: how does aggregate price index respond shocks?

- *Answer*: depends on what we assume about price rigidity at micro level

Two Stark Assumptions

- Calvo (1983)
  - each period firms adjust prices with given probability
  - timing and which firms change prices is random
  - because some not selected to adjust, aggregate price response is modest
  - so monetary shocks have potentially large real effects

- Caplin-Spulber (1987)
  - assume (do not derive explicitly) firms follow (s, S) rules
  - timing and which firms change prices is chosen optimally
  - firms that want to change prices the most do
  - so aggregate price responds by a lot, monetary shocks are neutral

History of Thought

- Early literature postulated different models of price rigidity
  - time-dependent models
    - e.g., Calvo (1983), Taylor (1980), Rotemberg (1982)
  - state-dependent (menu cost) models
    - e.g., Sheshinski-Weiss (1977), Caplin-Spulber (1987), Caplin-Leahy (1991)
- The 1990s characterized by limited data and computational resources
  - cannot discriminate between models
  - favored time-dependent models, which are more tractable
- After 2000, new microdata on prices and better computers
  - revived interest in state-dependent models
  - microdata used to discriminate between models

This Lecture

1. Facts about price changes

2. Are models consistent with these facts? Does it matter?

3. Where does this leave us? Implications for policy and future work

Facts on Price Changes

1. How often do prices change?

2. By how much do prices change?

3. When do prices tend to change?

How Often Do Prices Change?

- Bils-Klenow (2004), Klenow-Kryvtsov (2008), Nakamura-Steinsson (2008)

- Answer using data from Bureau of Labor Statistics
  - product-level quotes for goods and services at monthly frequency
  - divided into consumption categories called entry-level items (ELI)
  - ELIs covered in BLS data account for 70% of expenditure

- Distinguish between regular price changes and sales
  - regular price change: exclude V-shaped sales that last less then 3 months
  - debate on whether sales should be excluded for macro purposes
  - theory says sales do not affect inflation dynamics (Kehoe-Midrigan, 2015)

How Often Do Prices Change?

statistic	all prices	regular prices
mean frequency (%)	36.2	29.9
median frequency	27.3	13.9
mean duration (months)	6.8	8.6
median duration	3.7	7.2

Source: Klenow-Kryvtsov (2008). Durations are computed within ELI categories and then averaged.

Prices change infrequently, every 7-8 months.

By How Much Do Prices Change?

- Let $\Delta p_{it}(j)$ be price change of quote i in category (ELI) j
- Absolute size of price changes $|\Delta p|$

standardize

statistic	all prices	regular prices
mean	0.140	0.113
median	0.115	0.097

Source: Klenow-Kryvtsov (2008).

When they change, prices change by a lot.

By How Much Do Prices Change?

- Share of price changes smaller than a size threshold

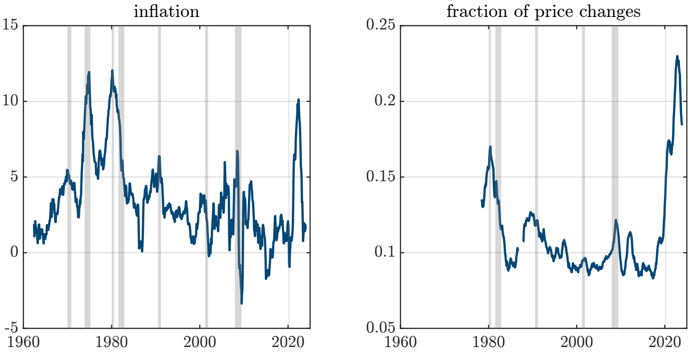
$\lvert\Delta p\rvert$	all prices	regular prices
$<5\%$	0.398	0.443
$<2.5\%$	0.234	0.254
$<1\%$	0.113	0.121

Source: Klenow-Kryvtsov (2008).

Many price changes are small.

When Do Prices Tend to Change?

- Evidence from the U.S. (consistent with Mexico, Argentina, U.K.)



Source: Blanco-Boar-Jones-Midrigan (2024). Data from Nakamura-Steinsson-Sun-Villar (2018) and Montag-Villar (2023). Inflation year-to-year changes, fraction monthly.

Fraction of price changes increases with inflation.

Taking Stock

- Prices change infrequently
- When they do, they change by a lot
- Yet, many price changes are small
- Prices change more often in times of high inflation

Model Framework

- Menu cost model is natural starting point
  - menu cost leads to infrequent adjustment
  - frequency endogenous, so may comove with inflation
- Add idiosyncratic, not just aggregate shocks (Golosov-Lucas, 2007)
  - inflation $\approx 2-3\%$, but prices change on average by 10%
- Menu cost technology: three commonly used specifications
- Follow analysis in Blanco-Boar-Jones-Midrigan (2024)
  - *Nonlinear Dynamics in Menu Cost Economies? Evidence from U.S. Data*

Model Overview

- Continuum of firms
  - produce with linear, labor-only technology
  - subject to idiosyncratic shocks
  - menu cost to change prices
- For simplicity, monetary policy targets nominal spending
  - only source of aggregate uncertainty
- Golosov-Lucas log-linear assumption on preferences

Consumers

- Life-time utility

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\log c_t - h_t\right)$$

- Budget constraint

$$P_t c_t + \frac{1}{1+i_t} B_{t+1} = W_t h_t + B_t + D_t$$

- First order conditions

$$\begin{aligned} c_t : \frac{1}{c_t} &= \lambda_t P_t \;\Rightarrow\; \lambda_t = \frac{1}{P_t c_t} \\ h_t : 1 &= \lambda_t W_t \end{aligned}$$

- Optimal labor supply implies

$$W_t = P_t c_t$$

Monetary Policy

- Monetary policy targets nominal spending (Nakamura-Steinsson, 2013)

$$M_t \equiv P_t c_t \quad \text{where} \quad \log \frac{M_{t+1}}{M_t} = \mu_{t+1},\ \mu_{t+1} \sim N\left(\mu, \sigma_m^2\right)$$

  - only source of aggregate uncertainty
  - isomorphic to cash-in-advance constraint or money in utility

- Combined with log-linear preferences implies

$$W_t = P_t c_t = M_t$$

Technology

- Firm $i \in [0,1]$ produces output with linear, labor-only technology

$$y_{it} = z_{it} l_{it} \quad \Rightarrow \quad \text{marginal cost} = \frac{W_t}{z_{it}}$$

- Final good produced with CES aggregator, used for consumption

$$c_t = y_t = \left(\int \left(\frac{y_{it}}{z_{it}} \right)^{\frac{\sigma-1}{\sigma}} \mathrm{d}i \right)^{\frac{\sigma}{\sigma-1}}$$

- Demand for individual varieties

$$y_{it} = z_{it} \left(\frac{z_{it} P_{it}}{P_t} \right)^{-\sigma} y_t$$

- Aggregate price index

$$P_t \equiv \int P_{it} \frac{c_{it}}{c_t} \mathrm{d}i = \int P_{it} \frac{y_{it}}{y_t} \mathrm{d}i = \left(\int (z_{it} P_{it})^{1-\sigma} \, \mathrm{d}i \right)^{\frac{1}{1-\sigma}}$$

Idiosyncratic Shocks

- Idiosyncratic (quality) shocks z_{it} evolve according to

$$\log z_{it+1} = \log z_{it} + \varepsilon_{it+1}, \;\; \varepsilon_{it+1} \sim N\left(0, \sigma_z^2\right)$$

- An increase in z_{it} has two effects
  - increase firm's productivity
  - reduces demand for firm's product

- Changes firm's desired price, without need to keep track of z_{it} as state
  - trick introduced by Midrigan (2011), used by many others since
  - with flex prices, P_{it} would fall one-for-one, so that $z_{it}P_{it}$ unchanged

Price Adjustment Cost

- Aggregate and idiosyncratic costs change the firm's marginal cost W_t/z_{it}
  - and induce firms to want to change prices

- Menu cost ξ_{it} denominated in units of labor: price adjustment cost $W_t\xi_{it}$

- What we assume about the menu cost has implications for
  - how many firms adjust in response to shocks $\rightarrow$ fraction of price changes
  - by how much firms adjust in response to shocks $\rightarrow$ size of price changes

- And, in turn, for the aggregate effects of shocks

Menu Cost Technology

- Three widely used specifications

  1. Golosov-Lucas (2007) (GL)

  $$\xi = \bar{\xi}$$

  2. Nakamura-Steinsson (2010) (NS)

  $$\xi = \begin{cases} 0, & \text{with probability } 1-\lambda \\ \bar{\xi}, & \text{with probability } \lambda \end{cases}$$

  3. Blanco-Boar-Jones-Midrigan (2025a) (Uniform)

  $$\xi = \begin{cases} 0, & \text{with probability } 1-\lambda \\ \sim U[0,\bar{\xi}], & \text{with probability } \lambda \end{cases}$$

- Model increasingly able to match distribution of price changes in data

- But worse at matching comovement of frequency and inflation

Firm Objective

- Expected present value of profits

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{P_t c_t} \left((1+\tau) P_{it} y_{it} - W_t \frac{y_{it}}{z_{it}} - \xi_{it} W_t \mathbb{I}_{it} \right)$$

  - indicator $\mathbb{I}_{it} = 1$ if $P_{it} \neq P_{it-1}$
  - subsidy to eliminate flex-price markup distortion: $1 + \tau = \frac{\sigma}{\sigma - 1}$

- Define
  - firm price gap $x_{it} \equiv \frac{z_{it} P_{it}}{M_t}$
  - aggregate price gap $X_t \equiv \left(\int x_{it}^{1-\sigma} \mathrm{d}i \right)^{\frac{1}{1-\sigma}} = \frac{P_t}{M_t}$

- If price are flexible, then $x_{it} = X_t = 1$

Firm Objective

- Can express firm objective as function of x_{it} and X_t

$$\max_{x_{it}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(X_t^{\sigma-1} \left((1+\tau)\, x_{it}^{1-\sigma} - x_{it}^{-\sigma} \right) - \xi_{it} \mathbb{I}_{it} \right)$$

- Notice that z_{it} does not show up in firm objective. For example,

$$P_{it} y_{it} = P_{it} z_{it} \left(\frac{z_{it} P_{it}}{P_t} \right)^{-\sigma} y_t = \left(\frac{z_{it} P_{it}}{P_t} \right)^{1-\sigma} M_t = \underbrace{\left(\frac{z_{it} P_{it}}{M_t} \right)^{1-\sigma}}_{x_{it}^{1-\sigma}} \underbrace{\left(\frac{M_t}{P_t} \right)^{1-\sigma}}_{X_t^{\sigma-1}} M_t$$

- Choice of own price gap x_{it} only depends on aggregate price gap X_t
  - so all firms have the same reset price gap x_t^*
- Firm must forecast X_t, which depends on the distribution of price gaps

Recursive Formulation

- *Idiosyncratic state*: price gap absent a price change

$$s_{it} = \frac{z_{it} P_{it-1}}{M_t}$$

- Evolves according to

$$s_{it+1} = \frac{z_{it+1} P_{it}}{M_{t+1}} = \frac{z_{it} P_{it}}{M_t} \frac{z_{it+1}}{z_{it}} \frac{M_t}{M_{t+1}} = x_{it} \exp\left(\varepsilon_{it+1} - \mu_{t+1}\right)$$

- Adjustment decision

$$x_{it} = \begin{cases} s_{it}, & \text{if do not adjust} \\ x_t^*, & \text{if adjust} \end{cases} \quad \rightarrow \quad \text{adjust with probability } h_t(s)$$

- Aggregate price gap X_t depends on distribution $F_t\left(s\right) \rightarrow$ *aggregate state*

$$X_t = \left(\int \left[h_t\left(s\right)\left(x_t^*\right)^{1-\sigma} + \left(1 - h_t\left(s\right)\right) s^{1-\sigma} \right] \mathrm{d}F_t\left(s\right) \right)^{\frac{1}{1-\sigma}}$$

Recursive Formulation

- Value of not adjusting

$$v^n(s,F) = \pi(s,F) + \beta\mathbb{E}\max\{v^n(s',F'), v^a(F') - \xi\}$$

subject to

$$\begin{aligned} s' &= s\exp(\varepsilon' - \mu') \\ F' &= \Gamma(F,\mu') \end{aligned}$$

- Value of adjusting

$$v^a(F) = \max_{x^*} \pi(x^*,F) + \beta\mathbb{E}\max\{v^n(s',F'), v^a(F') - \xi\}$$

subject to

$$\begin{aligned} s' &= x^*\exp(\varepsilon' - \mu') \\ F' &= \Gamma(F,\mu') \end{aligned}$$

How to Solve Firm's Problem?

- Distribution $F_t(s)$ is state variable, infinitely dimensional
  - recall this is needed to compute aggregate price gap X_t

- Krusell-Smith (2008) approach
  - assume X_t depends on a single moment of $F_t(s)$: $S_t = \left(\int s^{1-\sigma} \mathrm{d}F_t(s)\right)^{\frac{1}{1-\sigma}}$
  - dependence give by: $X_t = \mathcal{X}(S_t)$
  - S_t evolves according to: $S_{t+1} = \frac{\mathcal{X}(S_t)}{\exp(\mu_{t+1})}$

- Find $\mathcal{X}(\cdot)$ by simulating shocks and decision rules
  - and iterating until convergence
  - use Chebyshev polynomials to account for non-linearity

Adjustment Probability

- Adjustment probability depends on assumptions about menu cost

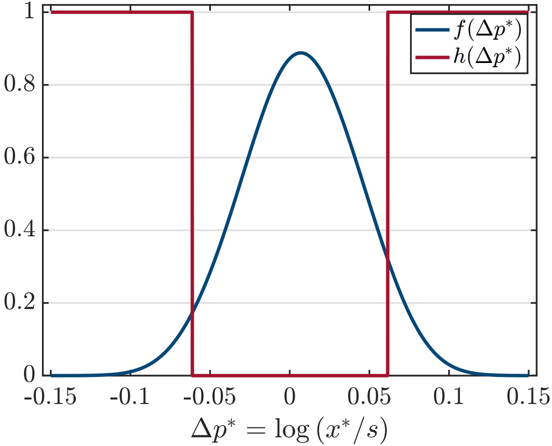
$$
h_t(s) = \begin{cases} \mathbb{I}\left(v_t^a - \bar{\xi} > v_t^n(s)\right), & \text{in GL} \\ 1 - \lambda + \lambda \mathbb{I}\left(v_t^a - \bar{\xi} > v_t^n(s)\right), & \text{in NS} \\ 1 - \lambda + \lambda \min\left\{\frac{v_t^a - v_t^n(s)}{\bar{\xi}}, 1\right\}, & \text{in Uniform.} \end{cases}
$$

- Has implications for the model implied distribution of price changes
  - which is observable, so can be used to discriminate between models

- And, importantly, it matters for the effect of aggregate shocks!

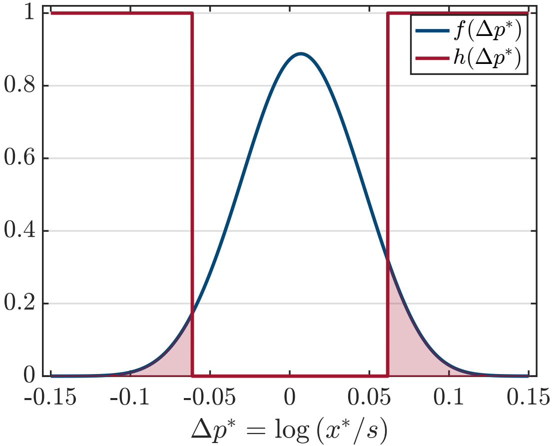
- Build intuition in Golosov-Lucas (2007)

Golosov-Lucas (2007)



Note: Distribution of desired price changes $f(\Delta p^*)$ and adjustment hazard $h(\Delta p^*)$.

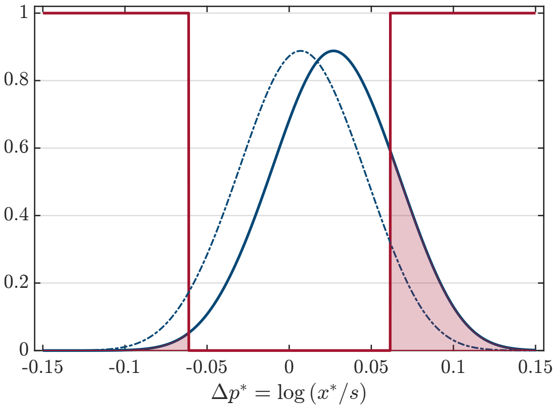
Golosov-Lucas (2007)



Note: Distribution of desired price changes $f(\Delta p^*)$ and adjustment hazard $h(\Delta p^*)$.

- Shaded areas are the distribution of price changes

Golosov-Lucas (2007)



$$\Delta P = \text{fraction adjust} \times \underbrace{\text{mean}\left(\Delta p | \text{adjust}\right)}_{\text{selection effect}}$$

Parameterization

- Assigned parameters: period 1 month
  - discount factor $\beta = 0.96$ (annualized) and elasticity of substitution $\sigma = 3$
  - probability of free price changes $1 - \lambda = 0.75$ (Nakamura-Steinsson, 2010)

- Calibrated parameters
  - mean and volatility of nominal spending growth μ and σ_m
  - volatility of idiosyncratic shocks σ_z and menu cost parameter $\bar{\xi}$

- Calibration targets
  - mean and standard deviation of inflation
  - mean fraction of price changes and median size of price changes

Calibration Results

A. Moments

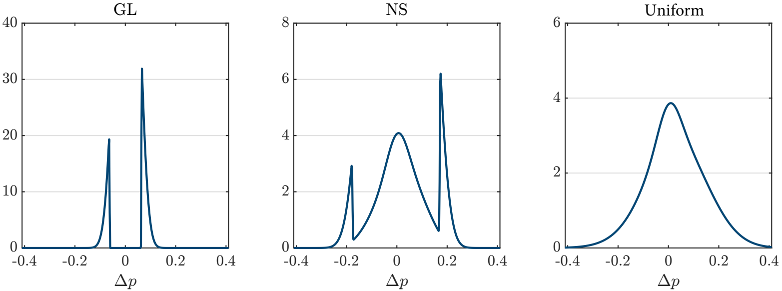
	Data	GL	NS	Uniform
fraction Δp	0.105	0.105	0.105	0.105
median $\lvert\Delta p\rvert$	0.075	0.075	0.075	0.075
mean inflation	0.034	0.034	0.034	0.034
std dev. inflation	0.026	0.026	0.026	0.026

B. Calibrated Parameter Values

		GL	NS	Uniform
μ	mean money growth rate	0.034	0.034	0.034
σ_m	s.d. monetary shocks	0.008	0.009	0.010
σ_z	s.d. idios. shocks	0.024	0.037	0.037
$\bar{\xi}$	menu cost	0.015	0.246	2.818

Note: Moments are calculated for the period 1979-2014. The money growth rate is annualized and the menu cost parameter $\bar{\xi}$ is expressed relative to total revenue.

Distribution of Price Changes



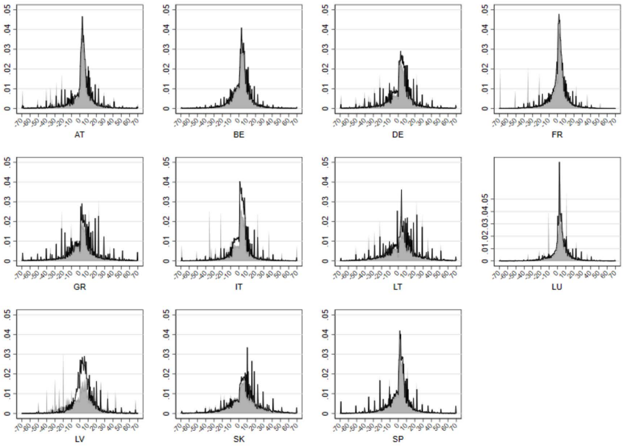
little dispersion
no small changes
no large changes
kurtosis = 1.5

more dispersion
small changes
jumps
kurtosis = 2.4

more dispersion
small changes
no jumps
kurtosis = 3.3

Empirical Distribution of Price Changes

- Evidence most consistent with the uniform specification



Source: Gautier, Conflitti, Faber, Fabo, Fadejeva, Jouvanceau, Menz, Messner, Petroulas, Roldan-Blanco, Rumler, Santoro, Wieland, Zimmer (2024).
Grey shaded histogram includes sales. Black line excludes sales.

Aggregate Dynamics

1. How does the fraction of price changes fluctuate in the time-series?

– recall in the data fraction of price changes is high when inflation is high

2. What are the real effects of aggregate shocks in these economies?

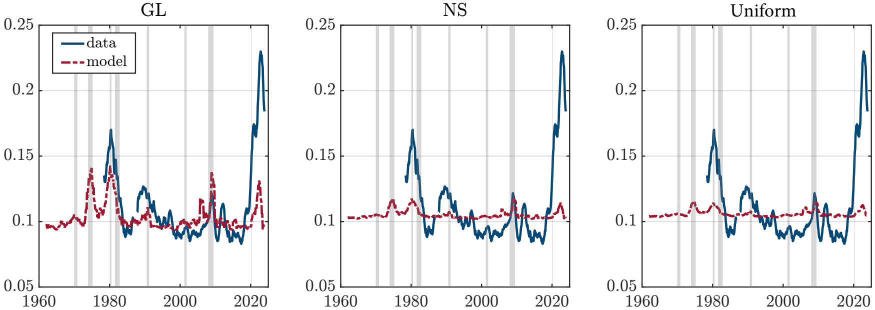
– depends on how price index responds to shocks

$$\Delta P = \underbrace{\text{fraction adjust}}_{\text{extensive margin}} \times \underbrace{\text{mean}\,(\Delta p|\text{adjust})}_{\text{intensive margin}}$$

Inflation and The Fraction of Price Changes

- Use model solution to back out shocks μ_t to match inflation series
- In response to μ_t, model implies time-series for fraction of price changes
- Compare model-implied fraction of price changes with data

Inflation and The Fraction of Price Changes



Intuition

- NS and Uniform can match the distribution of price changes
- Idiosyncratic rather than aggregate shocks drive repricing decisions
  - recall larger σ_z in calibration, as well as randomness in menu cost
- So fraction of price change $\approx$ invariant to shocks that drive inflation
- Need larger aggregate shocks to generate more fluctuations in frequency
  - but would imply inflation more volatile than observed

Taking Stock

- None of the models can simultaneously reproduce
  - micro-price distribution
  - comovement between inflation and the fraction of price changes

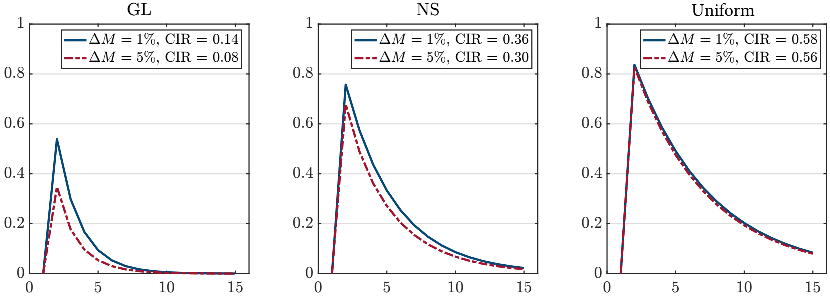
- But, as shown next, both facts are important!

- Dispersion in size of price changes $\rightarrow$ size of effects of monetary shocks
  - see also analytical results in Alvarez-Le Bihan-Lippi (2016)

- Fluctuations in fraction of price changes $\rightarrow$ non-linearity of the effects
  - i.e., do small and large shocks affect the economy differently?

Output Response to Monetary Shocks

- Initialize economy at stochastic steady state
- Consider 1% and 5% monetary shock



Note: IRFs scaled by shock size. CIR relative to Calvo model with frequency equal to average frequency in menu cost model.

money ≈ neutral non-linear effects	larger real effects less non-linearity	even larger real effects ≈ no non-linearity

Multi-Product Model

- Standard multi-product model has similar issues

- Key ingredients (Midrigan, 2011; Alvarez-Lippi, 2014)
  - firms sell continuum of products and face product idiosyncratic shocks z_{ikt}
  - economies of scope: menu cost $\xi_{it} \sim U\left[0, \bar{\xi}\right]$ to change all prices
  - so no need for additional free price changes

- Idiosyncratic shocks generate costly price dispersion inside the firm
  - akin to lower productivity
  - need large menu cost to justify firms not changing prices
  - so fraction of price changes fluctuates little with inflation

A Solution

- Blanco-Boar-Jones-Midrigan (2025a) propose a solution
  - *Nonlinear Inflation Dynamics in Menu Cost Economies*

- Add two ingredients to multi-product menu cost model
  - low elasticity of substitution between products of a firm
  - specific factor (e.g. managerial input) mobile across products within firm

- Within-firm price dispersion from idiosyncratic shocks is now less costly
  - price adjustment decisions relatively more responsive to aggregate shocks

- *Product*: set of highly substitutable goods facing correlated shocks
  - e.g. coffee vs pastries sold by Starbucks

Technology

- Only focus on new elements in the model

- Composite output of firm i is CES aggregator of individual varieties k

$$y_{it} = \left(\int \left(\frac{y_{ikt}}{z_{ikt}} \right)^{\frac{\gamma-1}{\gamma}} \mathrm{d}k \right)^{\frac{\gamma}{\gamma-1}}$$

- Individual varieties produced using labor and specific factor m_{ikt}

$$y_{ikt} = z_{ikt} \left(m_{ikt}\right)^{1-\eta} \left(l_{ikt}\right)^{\eta}, \quad \text{where}$$

- Specific factor mobile across products, fixed at firm level

$$\int m_{ikt} \mathrm{d}k = 1 \qquad \text{vs.} \qquad m_{ikt} = 1$$

Firm Objective

- Define product, firm and aggregate price gap

$$x_{ikt} = \frac{z_{ikt}P_{ikt}}{M_t}, \quad x_{it} = \left(\int x_{ikt}^{1-\gamma}\mathrm{d}k\right)^{\frac{1}{1-\gamma}} \quad \text{and} \quad X_t = \left(\int x_{it}^{1-\sigma}\mathrm{d}i\right)^{\frac{1}{1-\sigma}}$$

- Can derive firm-level production function

$$y_{it} = \phi_{it} l_{it}^{\eta}$$

- Productivity losses from misallocation from price dispersion

$$\phi_{it} = \left(\int \left(\frac{x_{ikt}}{x_{it}}\right)^{1-\gamma}\mathrm{d}k\right)^{-1}$$

- As before, can express firm objective in terms of price gaps

$$\max_{x_{it}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(X_t^{\sigma-1}\left((1+\tau)x_{it}^{1-\sigma} - X_t^{\left(\frac{1}{\eta}-1\right)(\sigma-1)} x_{it}^{-\frac{\sigma}{\eta}} \phi_{it}^{-\frac{1}{\eta}}\right) - \xi_{it}\mathbb{I}_{it}\right)$$

Misallocation from Price Dispersion

- Recursive formulation now has two idiosyncratic states
  - price gap absent a price change s_{it}, as before
  - duration since last price change d, determines losses from misallocation

- Misallocation increasing in γ and σ_z, given duration d

$$\phi_{it} = \exp\left(-d\gamma\frac{\sigma_z^2}{2}\right)$$

- So price dispersion less costly, given d and σ_z
  - so idiosyncratic shocks are relatively less important in repricing decision

Calibration

- Assigned parameters: $\gamma = 1$, $\sigma = 6$ and $\eta = 2/3$

A. Moments

	data	our model	standard
fraction Δp	0.105	0.105	0.105
median $\lvert\Delta p\rvert$	0.075	0.075	0.075
mean inflation	0.034	0.034	0.034
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B. Calibrated Parameter Values

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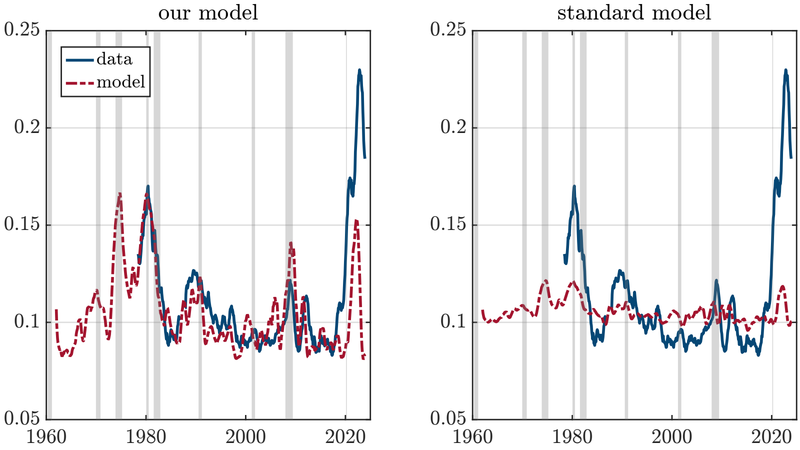
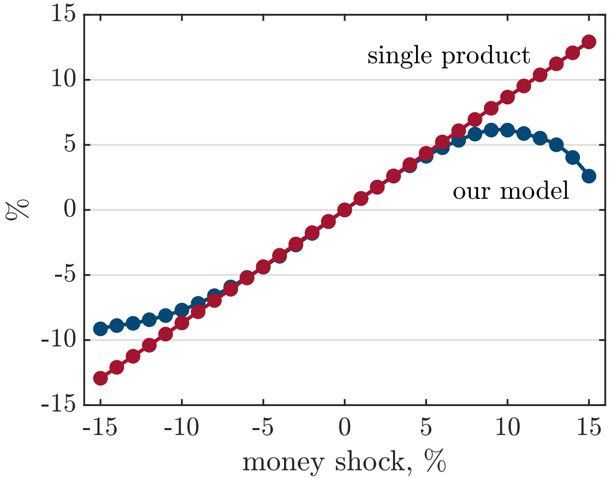
μ	mean money growth rate	0.034	0.034
σ_m	s.d. monetary shocks	0.010	0.010
σ_z	s.d. idios. shocks	0.039	0.038
$\bar{\xi}$	upper bound menu cost	0.632	10.59

Inflation and the Fraction of Price Changes

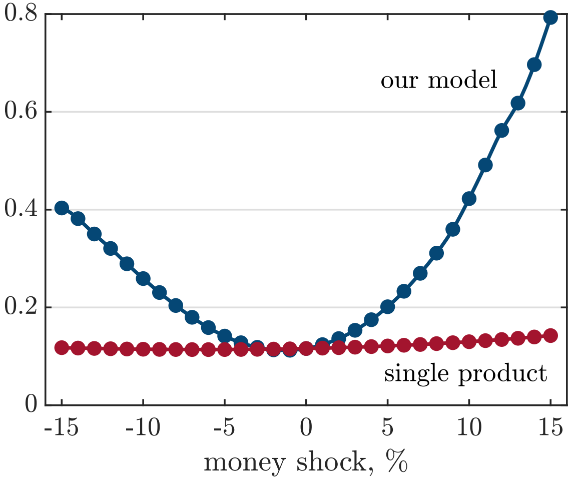


Output Response to Monetary Shocks

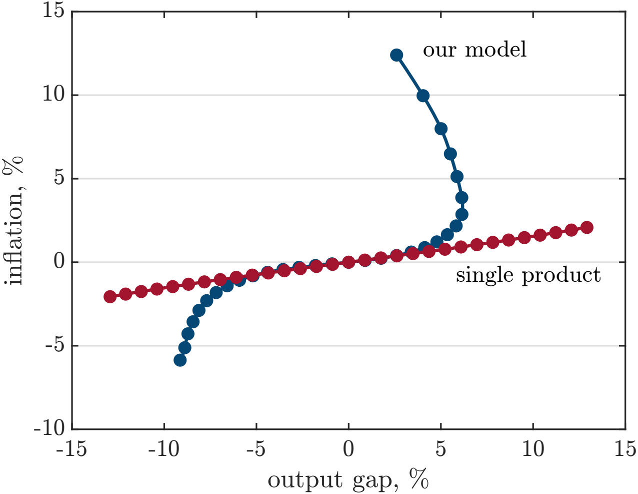


Note: The figure plots the output response on impact to monetary shocks of different sizes.

Fraction of Price Changes on Impact



Non-Linear Phillips Curve



Taking Stock

- We can write a menu cost model that simultaneously reproduces
  - distribution of price changes
  - comovement between inflation and the fraction of price changes
- Model predicts Phillips curve is non-linear
  - suggesting that monetary non-neutrality is time varying
  - and so is tradeoff between inflation and output stabilization
- But solving such a model is hard
  - because distribution of firms is a state variable
  - makes it difficult to use it for more applied or policy relevant questions

How to Make Progress?

- Will next show a model with the same predictions as menu cost model
  - but is much easier to solve

- Will start backwards
  - ignore first the distribution of micro price changes
  - focus on comovement between inflation and fraction of price changes

- Follow Boar-Blanco-Jones-Midrigan (2025b): *The Inflation Accelerator*
  - multi-product firms choose *how many*, but not *which*, prices to change
  - exact aggregation: reduces to one-equation extension of Calvo

Model

- Log-linear preferences, monetary policy targets nominal spending
  - so $W_t = P_t c_t = M_t$
  - $\log M_{t+1}/M_t = \mu + \varepsilon_{t+1}$ only aggregate shock (robust to Taylor rule, etc.)

- Multi-product firms i sell continuum of goods k
  - each produced with DRS technology $y_{ikt} = l_{ikt}^{\eta}$
  - aggregated into final output by a competitive final good producer

$$c_t = y_t = \left(\int_0^1 \int_0^1 (y_{ikt})^{\frac{\theta-1}{\theta}} \, \mathrm{d}k \mathrm{d}i \right)^{\frac{\theta}{\theta-1}}$$

  - demand for individual variety

$$y_{ikt} = \left(\frac{P_{ikt}}{P_t} \right)^{-\theta} y_t, \qquad P_t = \left(\int_0^1 \int_0^1 (P_{ikt})^{1-\theta} \, \mathrm{d}k \mathrm{d}i \right)^{\frac{1}{1-\theta}}$$

Price Adjustment Costs

- Firm chooses fraction of prices to change n_{it} ▸ profits
  - but not which prices to change (similar to Greenwald 2018)
- Price adjustment cost, denominated in units of labor

$$\frac{\xi}{2}\left(n_{it}-\bar{n}\right)^2, \quad \text{if } n_{it}>\bar{n}$$

  - when $\xi \to \infty$, $n_{it}=\bar{n}$ and model collapses to Calvo
- If adjust $P_{ikt}=P_{it}^*$, otherwise $P_{ikt}=P_{ikt-1}$

Firm-Level Aggregation

- Firm-level output y_{it} and labor l_{it}

$$y_{it} = \left(\int_0^1 (y_{ikt})^{\frac{\theta-1}{\theta}} \, \mathrm{d}k \right)^{\frac{\theta}{\theta-1}} \qquad \text{and} \qquad l_{it} = \int_0^1 l_{ikt} \mathrm{d}k$$

- Firm-level production function

$$y_{it} = \left(\frac{X_{it}}{P_{it}} \right)^{\theta} l_{it}^{\eta}$$

  - firm price index P_{it} and losses from misallocation X_{it}

$$P_{it} = \left(\int_0^1 (P_{ikt})^{1-\theta} \, \mathrm{d}k \right)^{\frac{1}{1-\theta}} \qquad \text{and} \qquad X_{it} = \left(\int_0^1 (P_{ikt})^{-\frac{\theta}{\eta}} \, \mathrm{d}k \right)^{-\frac{\eta}{\theta}}$$

  - absent price dispersion $X_{it}/P_{it} = 1$, otherwise $X_{it}/P_{it} < 1$

Firm Problem

- Choose reset price P_{it}^* and fraction of price changes n_{it} to maximize

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\underbrace{\left(\frac{P_{it+s}}{P_{t+s}} \right)^{1-\theta}}_{\text{sales}} - \underbrace{\tau \left(\frac{X_{it+s}}{P_{t+s}} \right)^{-\frac{\theta}{\eta}} y_{t+s}^{\frac{1}{\eta}}}_{\text{labor costs}} - \underbrace{\frac{\xi}{2} \left(n_{it+s} - \bar{n} \right)^2}_{\text{repricing costs}} \right]$$

- P_{it}^* and n_{it} affect price index and misallocation at all future dates

$$\begin{aligned} \left(P_{it+s}\right)^{1-\theta} &= n_{it+s} \left(P_{it+s}^*\right)^{1-\theta} + \left(1 - n_{it+s}\right) n_{it+s-1} \left(P_{it+s-1}^*\right)^{1-\theta} + \cdots \\ &+ \prod_{j=1}^{s} \left(1 - n_{it+j}\right) n_{it} \left(P_{it}^*\right)^{1-\theta} + \prod_{j=1}^{s} \left(1 - n_{it+j}\right) \left(1 - n_{it}\right) \left(P_{it-1}\right)^{1-\theta} \end{aligned}$$

- History encoded in two state variables: P_{it-1} and X_{it-1}
  - exact aggregation because adjustment hazard does not depend on P_{ikt-1}

Optimal Choices

- Reset price depends on present value of revenue and marginal costs
  - weighted by the probability that the price is still in effect
  - similar to Calvo, except that n_{it} time-varying ▸ P_{it}^*

- Fraction of price changes equates marginal cost to marginal benefit
  - higher n_{it} changes firm price index and reduces misallocation ▸ n_{it}

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- Symmetric equilibrium has $P_{it}^* = P_t^*$, $n_{it} = n_t$, ...
  - model collapes to one-equation extension of Calvo (is Calvo when $\xi \to \infty$)
  - two state variables: previous price and misallocation
  - solve the model globally, but third order perturbation reasonably accurate

Calibration Strategy

- Assigned parameters
  - period 1 quarter so $\beta = 0.99$
  - demand elasticity $\theta = 6$ and span of control $\eta = 2/3$
- Calibrated parameters
  - mean and standard deviation of nominal spending growth μ and σ
  - fraction of free price changes $\bar{n}$ and price adjustment cost ξ
- Calibration targets
  - mean and standard deviation of inflation
  - mean fraction of price changes
  - slope of fraction of price changes on absolute value of inflation

Calibrated Parameters

Targeted Moments

	Data	BBJM model	Calvo
mean inflation	3.517	3.517	3.517
s.d. inflation	2.739	2.739	2.739
mean fraction	0.297	0.297	0.297
slope of n_t on $\|\pi_t\|$	0.016	0.016	–

Calibrated Parameters

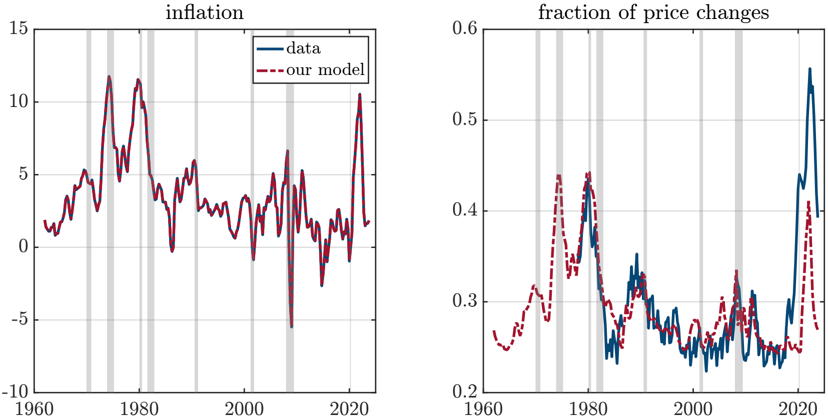
		BBJM model	Calvo
μ	mean spending growth rate	0.035	0.035
σ	s.d. monetary shocks	0.022	0.022
$\bar{n}$	fraction free price changes	0.241	0.297
ξ	adjustment cost	1.767	–

- Price adjustment costs account for 0.65% of all labor costs

Approach

- Use non-linear solution to back out shocks that match U.S. inflation series
  - initialize 1962 in stochastic steady state
- Compare fraction of price changes predicted by the model with data
- Derive Phillips curve by perturbing equilibrium conditions at each date

Fraction of Price Changes



Reproduces fraction well, except post-Covid

Towards the Slope of the Phillips Curve

- First order perturbation around equilibrium point at each date t
  - hats denote deviations from equilibrium at that date

- Aggregate price index

$$\hat{\pi}_t = \underbrace{\frac{1}{(1-n_t)\,\pi_t^{\theta-1}} \frac{\pi_t^{\theta-1}-1}{\theta-1}}_{\mathcal{M}_t}\,\hat{n}_t + \underbrace{\frac{1-(1-n_t)\,\pi_t^{\theta-1}}{(1-n_t)\,\pi_t^{\theta-1}}}_{\mathcal{N}_t}\,\hat{p}_t^*$$

- Elasticity $\mathcal{N}_t$ to reset price: identical to Calvo
  - increases with n_t, decreases with π_t (lower weight on new prices)

- Elasticity $\mathcal{M}_t$ to frequency: zero if $\pi_t = 1$, increases with inflation

Intuition

- Why is inflation more sensitive to changes in n_t when inflation is high?

$$\mathcal{M}_t = \frac{1}{(1-n_t)\,\pi_t^{\theta-1}} \frac{\pi_t^{\theta-1}-1}{\theta-1}$$

- Inflation $\approx$ average price change $\times$ fraction of price changes
  - when $\pi_t = 1$ the average price change $= 0$
    - so fraction inconsequential
  - when π_t is high the average price change is large
    - so Δn_t increases inflation considerably

The Inflation Accelerator

- Recall aggregate price index

$$\hat{\pi}_t = \mathcal{M}_t \hat{n}_t + \mathcal{N}_t \hat{p}_t^*$$

  - elasticity $\mathcal{M}_t$ increases with inflation, zero if $\pi_t = 1$

- Optimal fraction of price changes

$$\hat{n}_t = \mathcal{A}_t \hat{\pi}_t + \mathcal{B}_t \hat{p}_t^* - \mathcal{C}_t \hat{x}_{t-1} + \frac{n_t - \bar{n}}{n_t} \hat{b}_{1t}$$

  - elasticities $\mathcal{A}_t$ and $\mathcal{B}_t$ also increase with π_t

- Feedback loop amplifies inflation response to changes in reset price

$$\hat{\pi}_t = \frac{\mathcal{M}_t \mathcal{B}_t + \mathcal{N}_t}{1 - \mathcal{M}_t \mathcal{A}_t} \hat{p}_t^* - \frac{\mathcal{M}_t \mathcal{C}_t}{1 - \mathcal{M}_t \mathcal{A}_t} \hat{x}_{t-1} + \frac{\mathcal{M}_t}{1 - \mathcal{M}_t \mathcal{A}_t} \frac{n_t - \bar{n}}{n_t} \hat{b}_{1t}$$

Slope of the Phillips Curve

- Let $\widehat{mc}_t = \frac{1}{\eta}\hat{y}_t$ aggregate real marginal cost

- Can derive Phillips curve

$$\hat{\pi}_t = \mathcal{K}_t \widehat{mc}_t + \ldots$$

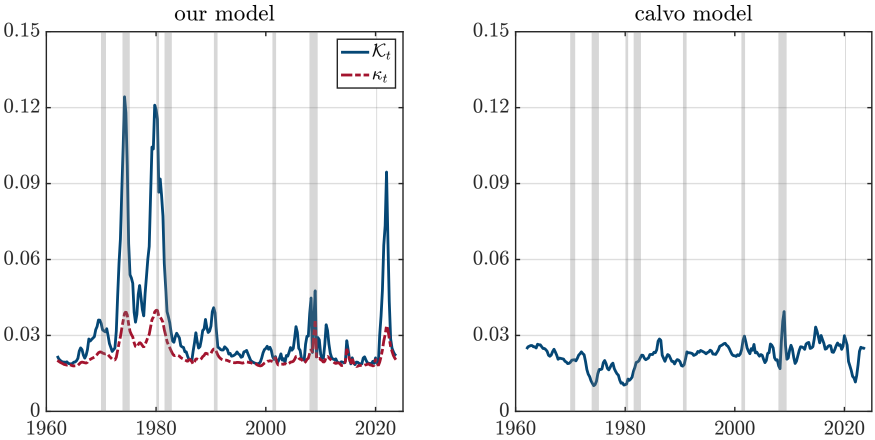
- Slope of the Phillips curve

$$\mathcal{K}_t = \underbrace{\frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}}_{\text{complementarities}} \times \underbrace{\frac{y_t^{\frac{1}{\eta}}}{b_{2t}}}_{\text{horizon}} \times \underbrace{\frac{\mathcal{M}_t\mathcal{B}_t + \mathcal{N}_t}{1-\mathcal{M}_t\mathcal{A}_t}}_{\text{reset price}}$$

- Absent endogenous frequency response ($\mathcal{A}_t = \mathcal{B}_t = \mathcal{M}_t = 0$)

$$\kappa_t = \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)} \times \frac{y_t^{\frac{1}{\eta}}}{b_{2t}} \times \underbrace{\frac{1-(1-n_t)\,\pi_t^{\theta-1}}{(1-n_t)\,\pi_t^{\theta-1}}}_{\mathcal{N}_t}$$

Time-Varying Slope of the Phillips Curve

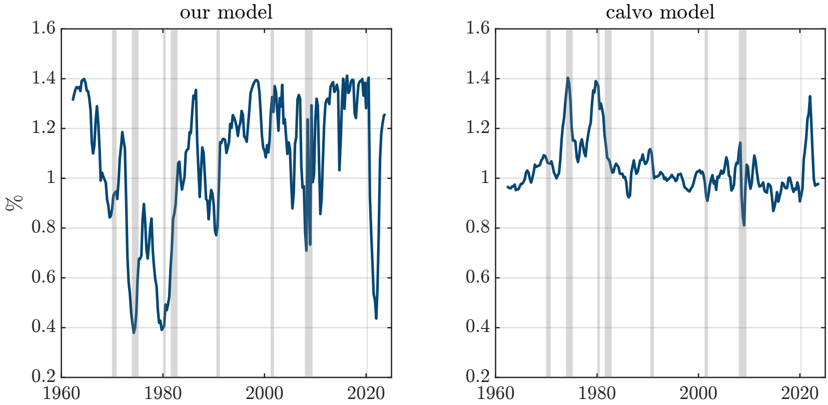


Ranges from 0.02 to 0.12, mostly due to inflation accelerator

In Calvo model slope falls in periods of high inflation

Sacrifice Ratio

- Calculate average drop in output needed to reduce π by 1% over a year



Ranges from 0.4% (high inflation) to 1.4% (low inflation), opposite of Calvo

Three Practical Extensions

1. Idiosyncratic shocks ▸ idiosyncratic shocks
   - to match distribution of micro price changes
2. Taylor rule for monetary policy ▸ taylor rule
   - standard in NK models
3. Multiple aggregate shocks ▸ aggregate shocks
   - to study drivers of inflation

Taking Stock (one last time)

- Data shows
  - large dispersion in price changes
    - many small price changes, as well as very large price changes
  - many more prices changes in times of high inflation
- Both frequency and size of price changes important
  - to understand real effects of shocks
  - and design the appropriate policy response
- Menu cost model natural framework to think about these issues
  - but hard to reconcile with facts above and not so easy to solve
- Alternative and simpler framework that captures most of the intuition
  - and is more amenable to applied and policy relevant questions

Standardized Price Changes

- Calculate standardized price change

$$\hat{\Delta} p_{it}(j) = \frac{\Delta p_{it}(j) - \mu_{\Delta(j)}}{\sigma_{\Delta(j)}} \sigma_\Delta + \mu_\Delta$$

 where

 - $\mu_{\Delta(j)}$ and $\sigma_{\Delta(j)}$ are ELI-level mean and standard deviation of price changes
 - μ_Δ and σ_Δ are overall mean and standard deviation of price changes

back

Flow Profits

- Demand for individual product

$$y_{ikt} = \left(\frac{P_{ikt}}{P_t}\right)^{-\theta} y_t$$

- Real flow profits of firm i

$$\int_0^1 \left(\left(\frac{P_{ikt}}{P_t}\right)^{1-\theta} y_t - \tau \frac{W_t}{P_t} \left(\frac{P_{ikt}}{P_t}\right)^{-\frac{\theta}{\eta}} y_t^{\frac{1}{\eta}} \right) \mathrm{d}k$$

  - subsidy to eliminate markup distortion $\tau = 1 - 1/\theta$

back

Optimal Reset Price

- Optimal reset price

$$\frac{P^*_{it}}{P_t} = \left(\frac{1}{\eta}\frac{b_{2it}}{b_{1it}}\right)^{\frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}}$$

- Depends on present value of output and production costs in future dates

$$b_{1it} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \prod_{j=1}^{s} (1 - n_{it+j}) \left(\frac{P_{t+s}}{P_t}\right)^{\theta-1}$$

$$b_{2it} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \prod_{j=1}^{s} (1 - n_{it+j}) \left(\frac{P_{t+s}}{P_t}\right)^{\frac{\theta}{\eta}} (y_{t+s})^{\frac{1}{\eta}}$$

- Can be equivalently written as present value of future marginal costs
  - similar to Calvo, except n_{it} time-varying

back

Optimal Fraction of Price Changes

- Equate marginal cost to marginal benefit

$$\xi\left(n_{it}-\bar{n}\right)=b_{1it}\left(\left(\frac{P_{it}^{*}}{P_{t}}\right)^{1-\theta}-\left(\frac{P_{it-1}}{P_{t}}\right)^{1-\theta}\right)-\tau b_{2it}\left(\left(\frac{P_{it}^{*}}{P_{t}}\right)^{-\frac{\theta}{\eta}}-\left(\frac{X_{it-1}}{P_{t}}\right)^{-\frac{\theta}{\eta}}\right)$$

- Marginal benefit: higher n_{it}
  - changes firm price index
  - and reduces misallocation
  - weighted by the same terms b_{1it} and b_{2it} that determine P_{it}^{*}

back

Idiosyncratic Shocks

- Individual goods produced with technology

$$y_{ikt} = z_{ikt} l_{ikt}^{\eta}, \quad \text{where} \quad \log z_{ikt} = \log z_{ikt-1} + \sigma_z \epsilon_{ikt},\ \epsilon_{ikt} \sim N(0,1)$$

- Final output

$$y_t = \left(\int_0^1 \int_0^1 \left(\frac{y_{ikt}}{z_{ikt}} \right)^{\frac{\theta-1}{\theta}} \mathrm{d}k \mathrm{d}i \right)^{\frac{\theta}{\theta-1}}$$

- Firm price index P_{it} and misallocation X_{it} depend on $z_{ikt} P_{ikt}$

- Expressions similar to benchmark, with scaling terms that depend on σ_z
  - e.g., terms involving $\pi_t^{\theta-1}$ scaled by $\exp\left(\frac{\sigma_z^2}{2}(1-\theta)^2\right)$

Calibration

- Because idiosyncratic shocks motive to change prices, assume $\bar{n} = 0$

A. Targeted Moments

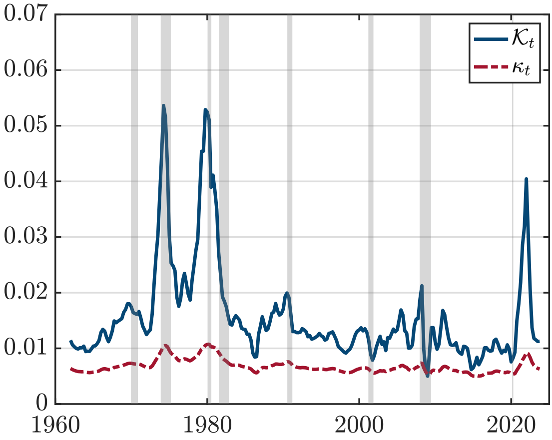
	Data	Model
mean inflation	3.517	3.517
s.d. inflation	2.739	2.739
mean frequency	0.297	0.297
slope of n_t on $\|\pi_t\|$	0.016	0.015
s.d. price changes	0.129	0.129

B. Calibrated Parameter Values

		Model
μ	mean spending growth rate	0.035
σ	s.d. monetary shocks	0.023
ξ	adjustment cost	17.00
σ_z	s.d. idiosyncratic shocks	0.068

Note: The mean nominal spending growth rate is annualized. S.d. of price changes is from Morales-Jimenez-Stevens (2024).

Slope of the Phillips Curve



Smaller with idiosyncratic shocks, but fluctuates as much

▸ back

Taylor Rule

- Replace nominal spending target with Taylor rule

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i}\right)^{\phi_i} \left(\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{y_t}{y_{t-1}}\right)^{\phi_y}\right)^{1-\phi_i} \exp\left(u_t\right)$$

- Two versions
  - u_t shocks iid
  - serially correlated with persistence ρ to match autocorrelation inflation
- Use Justiniano-Primiceri (2008) estimates
  - $\phi_i = 0.65$, $\phi_\pi = 2.35$, $\phi_y = 0.51$

Calibration of Economy with a Taylor Rule

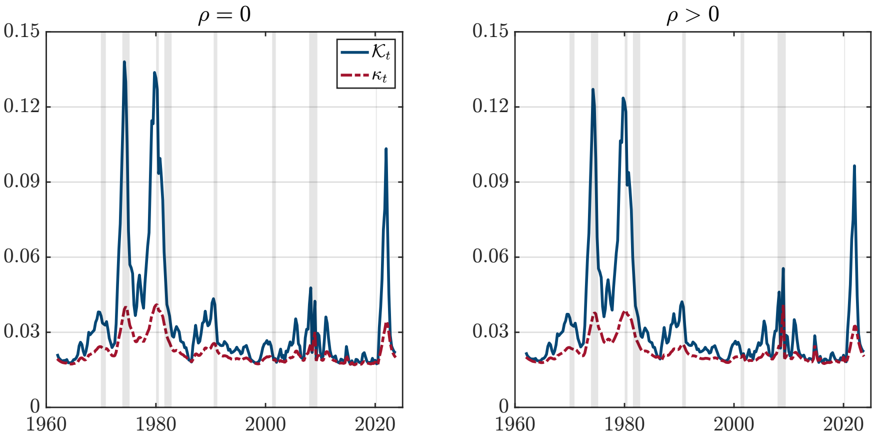
Targeted Moments

	Data	$\rho = 0$	$\rho > 0$
mean inflation	3.517	3.517	3.517
s.d. inflation	2.739	2.739	2.739
mean frequency	0.297	0.297	0.297
slope of n_t on $\|\pi_t\|$	0.016	0.016	0.016
autocorr. inflation	0.942	*0.913*	0.942

Calibrated Parameters

		$\rho = 0$	$\rho > 0$
$\log \pi$	inflation target	0.040	0.037
σ	s.d. monetary shocks $\times 100$	2.626	0.551
ρ	persistence monetary shocks	–	0.685
$\bar{n}$	fraction free price changes	0.241	0.241
ξ	adjustment cost	1.671	1.688

Slope of the Phillips Curve



Results are robust to assuming a Taylor rule

back

Additional Aggregate Shocks

- Three sources of aggregate uncertainty, all follow AR(1)
  - aggregate productivity shocks

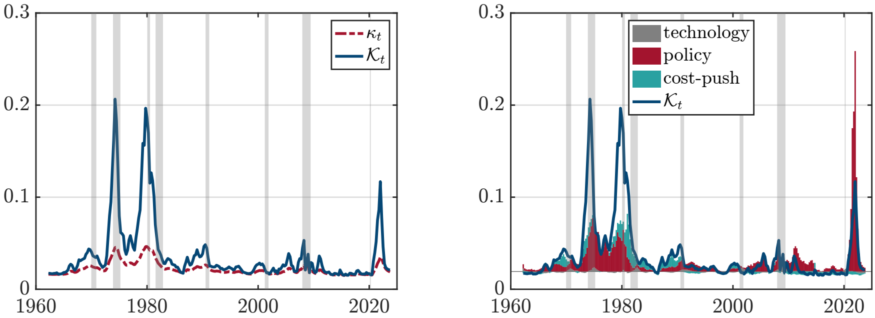
    $$y_{ikt} = z_t l_{ikt}^{\eta}$$

  - time-varying tax on labor (cost-push shock)

    $$P_{ikt} y_{ikt} - \tau_t W_t l_{ikt}$$

  - interest rate shocks in Taylor rule
- Bayesian estimation, as typicall in NK literature
- Back out productivity, cost-push and monetary shocks
  - so that model matches path of inflation, output growth and interest rate
- Compute slope of Phillips curve as in bechmark

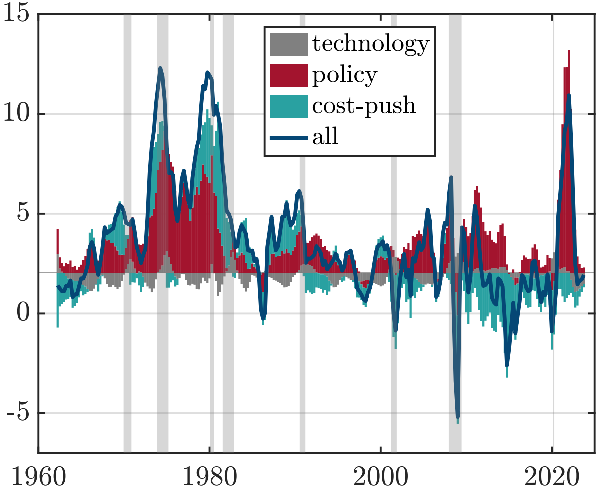
Slope of the Phillips Curve



Results are robust to adding multiple aggregate shocks

back

Causes of Inflation



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