Fiscal Policy

NBER Heterogeneous-Agent Macro Workshop

Ludwig Straub

Spring 2023
This session

We just introduced the canonical HANK model.

**Next:** Focus on fiscal policy!

- Switch off all other shocks: TFP $X_t = 1$, no monetary shock $r_t = r = \text{const}$
- Focus on **first order** shocks to fiscal policy: $dG = \{dG_t\}$, $dT = \{dT_t\}$ such that
  \[
  \sum_{t=0}^{\infty} (1 + r)^{-t}(dG_t - dT_t) = 0
  \]
- Main reference for this class is Auclert et al. (2023b)
Roadmap

1. The intertemporal Keynesian cross
2. Three special cases
3. iMPCs in the HA model
4. Insights about Fiscal Multipliers
5. Takeaway
The intertemporal Keynesian cross
DAG for the economy with only fiscal shocks

Switching off monetary shocks, the DAG is simply:

In this case, \( H = 0 \) simply corresponds to:

\[
Y = G + C(Z)
\]

To emphasize that \( C \) is a function, write it as \( C \). \( C \) only a function of \( Z \) here!

Next: Analyze this equation “by hand”...
The aggregate consumption function

- We call $C$ the **aggregate consumption function**

$$C_t = C_t (Z_0, Z_1, Z_2, \ldots) = C_t (\{Z_s\})$$

It's a collection of $\infty$ many nonlinear functions of $\infty$ many $Z$'s!

- It usually also depends on the path of real interest rates, but those are assumed to be constant

- Using the DAG, we can substitute out $Z$ and write goods market clearing as

$$Y_t = G_t + C_t (\{Y_s - T_s\})$$
Intertemporal MPCs

\[ Y_t = G_t + C_t \left( \{ Y_s - T_s \} \right) \]

- Feed in small shock \( \{dG_t, dT_t\} \)

\[ dY_t = dG_t + \sum_{s=0}^{\infty} \frac{\partial C_t}{\partial Z_s} \cdot (dY_s - dT_s) \] (1)

- Response \( dY_t \) **entirely** characterized by the **Jacobian** of \( C \) function, which we also call **intertemporal MPCs**

\[ M_{t,s} \equiv \frac{\partial C_t}{\partial Z_s} \quad \left( = J_{t,s}^{C,Z} \right) \]

- \( M_{t,s} = \) how much of an income change at date \( s \) is spent at date \( t \)

- Note: All income is spent at some point, hence \( \sum_{t=0}^{\infty} (1 + r)^{s-t}M_{t,s} = 1 \)
The intertemporal Keynesian cross

• Rewrite equation (1) in vector / matrix notation:

\[ dY = dG - Mt + My \] (2)

• This equation exactly corresponds to \( dH = 0 \)

• This is an intertemporal Keynesian cross
  • entire complexity of model is in \( M \)
  • with \( M \) from data, could get \( dY \) without model!
    (there is a “correct” \( M \) out there, but it’s very hard to measure...)
Connecting to the standard Keynesian cross...

- Standard IS-LM theory postulates $C_t = C (Y_t - T_t)$ plus market clearing, so

$$Y_t = G_t + C (Y_t - T_t)$$

Differentiate around steady state with constant $Y, T, G$:

$$dY_t = dG_t - mpc \cdot dT_t + mpc \cdot dY_t$$

where $mpc = C' (Y - T)$. This is the static Keynesian cross.

- The intertemporal Keynesian cross is a vector-valued version of this

- HANK models tend to revive & microfound IS-LM logic
Solving the intertemporal Keynesian cross

- How can we solve (2)? Rewrite as

\[(I - M) dY = dG - MdT\]  \hspace{1cm} (3)

- Standard Keynesian cross solution:

\[dY_t = \frac{dG_t - mpc \cdot dT_t}{1 - mpc}\]

Can we do the same, inverting \((I - M)\)? **Not so fast!**

- Why? Multiply both sides of (3) by: \(q \equiv (1, (1 + r)^{-1}, (1 + r)^{-2}, \ldots)'

\[q' (I - M) dY = 0 \qquad q' dG - q' MdT = q' dG - q' dT = 0\]

both left and right hand side have “zero NPV”!

- Intuition: present value of mpc is 1, \(dY\) is o/o... What to do?
Solving the intertemporal Keynesian cross

- So how can we solve the IKC? Just like with L’Hospital, we want to modify both numerator and denominator to avoid $0/0$ issue...
- Do this by pre-multiplying with a matrix $K$

$$K(I - M)\, dY = K(dG - MdT)$$

- Now for a clever choice of $K$, $K(I - M)$ may be invertible:

**Theorem**

*There exists a unique solution to the IKC for any $dG, dT$ satisfying $q'dG = q'dT$, iff $K(I - M)$ is invertible. Then, the solution is:*

$$dY = M(dG - MdT)$$

*where $M \equiv (K(I - M))^{-1}K$ is a bounded linear operator (“multiplier”)
Which \( \mathbf{K} \) are we using?

- Which \( \mathbf{K} \) is needed?
- One natural choice:

\[
\mathbf{K} = - \begin{pmatrix}
  0 & (1 + r)^{-1} & (1 + r)^{-2} & (1 + r)^{-3} & \cdots \\
  0 & 0 & (1 + r)^{-1} & (1 + r)^{-2} & \ddots \\
  0 & 0 & 0 & (1 + r)^{-1} & \ddots \\
  \vdots & \vdots & \vdots & \ddots & \ddots \\
\end{pmatrix}
\]

\[
= - \sum_{t=1}^{\infty} (1 + r)^{-t} \mathbf{F}^t
\]

where \( \mathbf{F} \) is forward operator matrix.

- Then: \( \mathbf{K} (\mathbf{I} - \mathbf{M}) \) is the “asset jacobian” of the household block.

- When is \( \mathbf{K} (\mathbf{I} - \mathbf{M}) \) invertible? \( \rightarrow \) see Auclert et al. (2023a) for a criterion.
The balanced budget multiplier

- Suppose $dG = dT$ (balanced budget)
- Result: We always have $dY = dG$
- Irrespective of all household heterogeneity, holds for any path of spending
- IS-LM antecedents: Gelting (1941), Haavelmo (1945)
- Proof is trivial: $dY = dG$ is unique solution to

$$dY = (I - M) \cdot dG + M \cdot dY$$
Deficit financed fiscal policy

• With deficit financing \( dG \neq dT \) we have

\[
dY = dG + M \cdot M \cdot (dG - dT)
\]

Consumption \( dC \) depends on primary deficits \( dG - dT \)

• Interaction term: Deficits matter precisely when \( M \) is “large” (which will mean very different from RA model)

• **Next:** Go over our three examples and then compare multipliers to full HA model

• Define:
  • initial multiplier: \( \frac{dY_0}{dG_0} \)
  • cumulative multiplier: \( \frac{\sum (1+r)^{-t}dY_t}{\sum (1+r)^{-t}dG_t} \)
Three special cases
Let’s get an intuition for all this in the RA model. Last lecture we derived consumption function for RA model when $\beta(1 + r) = 1$

$$C_t = (1 - \beta) \sum_{s \geq 0} \beta^s Z_s + ra_{-1}$$

In particular

$$M_{t,s} = \frac{\partial C_t}{\partial Z_s} = (1 - \beta)\beta^s$$

Thus iMPC matrix is given by

$$M^{RA} = \begin{pmatrix}
1 - \beta & (1 - \beta)\beta & (1 - \beta)\beta^2 & \cdots \\
1 - \beta & (1 - \beta)\beta & (1 - \beta)\beta^2 & \cdots \\
1 - \beta & (1 - \beta)\beta & (1 - \beta)\beta^2 & \cdots \\
\vdots & \vdots & \vdots & \ddots \\
\end{pmatrix} = \frac{1q'}{1'q'}$$

Easy to verify that $q'M = q'$, and also that $Mw = 0$ for any zero NPV $w$. 
Representative-agent model
Fiscal policy in RA model

- Let’s solve the IKC for the RA model
- Calculate:

\[ dC = \mathcal{M} \cdot M \cdot (dG - dT) \]

\[ = \mathcal{M} \cdot (1 - \beta) q' (dG - dT) \]

But government budget balance implies \( q' (dG - dT) = 0 \)! So:

\[ dY = dG \]

- Can prove this directly, too (eg Woodford 2011).
- **Deficits are irrelevant in RA!**
Impulse response to dG shock in RA model

Government spending and taxes

- Government spending
- Taxes under balanced budget
- Taxes under deficit financed

Output

- Balanced budget
- Deficit financed
Two agent model

• $1 - \mu$ share of agents behave like RA agent, $\mu$ are hand to mouth $\Rightarrow$ $M$ matrix is simple linear combination

\[ M^{TA} = (1 - \mu)M^{RA} + \mu I \]

• Issue: Only strong \textit{contemporaneous} spending effect
iMPCs in TA model

![Graph showing iMPCs in TA model with different values of s (0, 5, 10, and 15).]
Fiscal policy in TA model

• In Keynesian cross:

\[
\left( I - M^{TA} \right) dY = dG - M^{TA} dT \quad \Leftrightarrow \quad \left( I - M^{RA} \right) dY = \frac{1}{1-\mu} [dG - \mu dT] - M^{RA} dT
\]

This equation has same shape as for RA, hence:

\[
dY = \frac{1}{1-\mu} [dG - \mu dT]
\]

• Results from undergrad: Spending multiplier \( 1/(1-\mu) \) and transfer multiplier \( \mu/(1-\mu) \). So: \( \mu \) is “effective” MPC, ignoring RA

• Can also write:

\[
dY = dG + \frac{\mu}{1-\mu} \left[ dG - dT \right]
\]

• Only current deficit matters. Initial multiplier can be large \( \in [1, \frac{1}{1-\mu}] \), but cumulative multiplier is always equal to 1!
Impulse response to dG shock in TA model

Government spending and taxes

- Government spending
- Taxes under balanced budget
- Taxes under deficit financed

Output

- Balanced budget
- Deficit financed
Zero-liquidity model

• What about iMPCs in the ZL model?
  • We can calculate (see IKC paper)

  \[
  M_{t=0}^{ZL} = \mu 1_{t=0} + (1 - \mu) \left(1 - \frac{\lambda}{1+r}\right) \cdot \lambda^t
  \]

  \[
  M_{os}^{ZL} = (1 - \mu) \frac{1 - \beta \lambda}{\beta(1+r)} \cdot (\beta \lambda)^s
  \]

• Intuitively, it’s a mix of a “constrained agent” with mass $\mu$ and agents that spend down assets at constant rate $\lambda$
  • Latter are also the iMPCs of a bond-in-utility model (and an OLG model!)
• Note, given known $M_{oo}$ and $M_{1o}$, can solve for $\mu$ and $\lambda$
iMPCs in ZL model

see also Bilbiie (2021)
Fiscal policy in ZL model

• Can solve above model explicitly

\[
dY_t = \frac{1}{1 - \mu} [dG_t - \mu dT_t] + \frac{\beta (1 + r) - 1}{1 - \mu} dB_t + (1 + r) \frac{1 - \beta \lambda}{1 - \mu} \left( \frac{1}{\lambda} - 1 \right) \sum_{s=0}^{\infty} dB_{t+s}
\]

as in TA model

Future fiscal policy extremely powerful here.

• Why? Dynamic income-consumption feedback from “spending down” effect

• In particular, can show:

Theorem

*Holding \( \beta, r, \) and \( M_{00} \) fixed in the ZL model, a higher \( M_{10} \) increases the cumulative multiplier whenever \( dB \geq 0 \) and \( dB_t > 0 \) for some \( t \).*
Impulse response to dG shock in ZL model

Government spending and taxes

- Government spending
- Taxes under balanced budget
- Taxes under deficit financed

Output

- Balanced budget
- Deficit financed
iMPCs in the HA model
iMPCs in the HA model (computed using fake news algorithm)
Comparing iMPCs across models

**RA and TA models**

- Red line: RA
- Orange line: TA

**HA models**

- Green line: High-liquidity HA
- Blue line: Low-liquidity HA
- Pink line: Zero-liquidity HA
Comparison with the data

RA and TA models

Data
RA
TA

HA models

Data
High-liquidity HA
Low-liquidity HA
Zero-liquidity HA

Data from Fagereng et al. (/two.osf/zero.osf/two.osf/one.osf), estimating consumption response to lottery winnings
Insights about Fiscal Multipliers
Fiscal stimulus more powerful when deficit financed

Impact multiplier

Cumulative multiplier

\[
\frac{dY_t}{dG_t} = (1 + r)^t \frac{dY_0}{dG_0}
\]

\[
\sum_{k=0}^{t} (1 + r)^k dY_t = (1 + r)^t \frac{dY_0}{dG_0}
\]
Fiscal policy is more powerful if front loaded...

Impact

- High-liquidity HA
- Low-liquidity HA
- Zero-liquidity HA
- RA
- TA

Cumulated impact

\[
\sum (1 + r)^{-t} dY_t
\]
... but not in the zero-liquidity model (a fiscal policy forward guidance puzzle?)

![Graph showing Government spending and Output over time with different fiscal policy options.](three.osf/one.osf)
Fiscal policy is less powerful if financed by lump-sum taxes (Why?)

![Output Graph](image1)

![Consumption Graph](image2)
Fiscal policy is more powerful if income risk is countercyclical (Why?)

Auclert-Rognlie “incidence function”. More negative $\Gamma$ means incomes more dispersed in recessions, $\Pi$ is fixed.
Takeaway
Fiscal policy in HANK

- First exploration of shocks & policies in HANK

- One key difference already emerged: in HANK, households have very different iMPCs

- This matters for fiscal policy:
  - deficit financing & front loading amplifies initial and cumulative multipliers
  - not the case in RA, and not even in TA
References


