Fiscal Policy

NBER Heterogeneous-Agent Macro Workshop

Ludwig Straub

Spring 2023

We just introduced the canonical HANK model.

Next: Focus on fiscal policy!

- Switch off all other shocks: TFP $X_t = 1$, no monetary shock $r_t = r = const$
- Focus on **first order** shocks to fiscal policy: $d\mathbf{G} = \{dG_t\}, d\mathbf{T} = \{dT_t\}$ such that

$$\sum_{t=0}^{\infty}(1+r)^{-t}(dG_t - dT_t) = 0$$

- Main reference for this class is Auclert et al. (2023b)

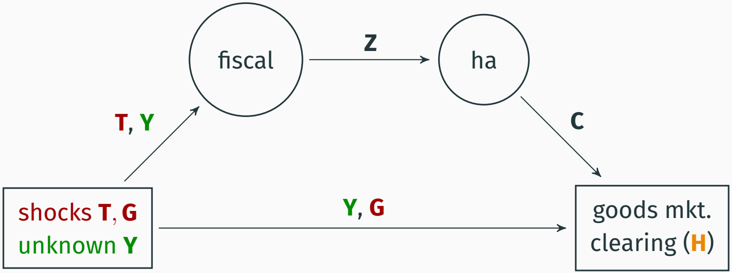
Roadmap

1. The intertemporal Keynesian cross
2. Three special cases
3. iMPCs in the HA model
4. Insights about Fiscal Multipliers
5. Takeaway

The intertemporal Keynesian cross

DAG for the economy with only fiscal shocks

Switching off monetary shocks, the DAG is simply:



In this case, $\mathbf{H} = 0$ simply corresponds to:

$$\mathbf{Y} = \mathbf{G} + \mathcal{C}(\mathbf{Z})$$

To emphasize that **C** is a function, write it as $\mathcal{C}$. **C** only a function of **Z** here!

Next: Analyze this equation “by hand”...

The aggregate consumption function

- We call $\mathcal{C}$ the **aggregate consumption function**

$$C_t = \mathcal{C}_t(Z_0, Z_1, Z_2, \ldots) = \mathcal{C}_t(\{Z_s\})$$

  It's a collection of ∞ many nonlinear functions of ∞ many Z's!

- It usually also depends on the path of real interest rates, but those are assumed to be constant
- Using the DAG, we can substitute out Z and write goods market clearing as

$$Y_t = G_t + \mathcal{C}_t(\{Y_s - T_s\})$$

Intertemporal MPCs

$$Y_t = G_t + \mathcal{C}_t\left(\{Y_s - T_s\}\right)$$

- Feed in small shock $\{dG_t, dT_t\}$

$$dY_t = dG_t + \sum_{s=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial Z_s} \cdot (dY_s - dT_s) \tag{1}$$

- Response dY_t **entirely** characterized by the **Jacobian** of $\mathcal{C}$ function, which we also call **intertemporal MPCs**

$$M_{t,s} \equiv \frac{\partial \mathcal{C}_t}{\partial Z_s} \qquad \left(= \mathcal{J}_{t,s}^{\mathbf{C},\mathbf{Z}}\right)$$

- $M_{t,s}$ = how much of an income change at date s is spent at date t
- Note: All income is spent at some point, hence $\sum_{t=0}^{\infty}(1+r)^{s-t} M_{t,s} = 1$

The intertemporal Keynesian cross

- Rewrite equation (1) in vector / matrix notation:

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y} \tag{2}$$

- This equation exactly corresponds to $d\mathbf{H} = 0$
- This is an **intertemporal Keynesian cross**
  - entire complexity of model is in $\mathbf{M}$
  - with $\mathbf{M}$ from data, could get $d\mathbf{Y}$ without model!
    (there is a "correct" $\mathbf{M}$ out there, but it's very hard to measure...)

Connecting to the standard Keynesian cross...

- Standard IS-LM theory postulates $C_t = \mathcal{C}(Y_t - T_t)$ plus market clearing, so

$$Y_t = G_t + \mathcal{C}(Y_t - T_t)$$

  Differentiate around steady state with constant Y, T, G:

$$dY_t = dG_t - mpc \cdot dT_t + mpc \cdot dY_t$$

  where $mpc = \mathcal{C}'(Y - T)$. This is the **static Keynesian cross**.

- The intertemporal Keynesian cross is a vector-valued version of this
- HANK models tend to revive & microfound IS-LM logic

Solving the intertemporal Keynesian cross

- How can we solve (2)? Rewrite as

$$(\mathbf{I}-\mathbf{M})\, d\mathbf{Y} = d\mathbf{G} - \mathbf{M} d\mathbf{T} \tag{3}$$

- Standard Keynesian cross solution:

$$dY_t = \frac{dG_t - mpc \cdot dT_t}{1 - mpc}$$

  Can we do the same, inverting $(\mathbf{I}-\mathbf{M})$? **Not so fast!**

- Why? Multiply both sides of (3) by: $\mathbf{q} \equiv (1, (1+r)^{-1}, (1+r)^{-2}, \ldots)'$

$$\mathbf{q}'(\mathbf{I}-\mathbf{M})\, d\mathbf{Y} = 0 \qquad \mathbf{q}' d\mathbf{G} - \mathbf{q}'\mathbf{M} d\mathbf{T} = \mathbf{q}' d\mathbf{G} - \mathbf{q}' d\mathbf{T} = 0$$

  both left and right hand side have "**zero NPV**" !

- Intuition: present value of *mpc* is 1, dY is 0/0... What to do?

Solving the intertemporal Keynesian cross

- So how can we solve the IKC? Just like with L'Hospital, we want to modify both numerator and denominator to avoid 0/0 issue ...
- Do this by pre-multiplying with a matrix $\mathbf{K}$

$$\mathbf{K}\left(\mathbf{I}-\mathbf{M}\right) d\mathbf{Y} = \mathbf{K}\left(d\mathbf{G}-\mathbf{M}d\mathbf{T}\right)$$

- Now for a clever choice of $\mathbf{K}$, $\mathbf{K}\left(\mathbf{I}-\mathbf{M}\right)$ may be invertible:

Theorem

There exists a unique solution to the IKC for any $d\mathbf{G}, d\mathbf{T}$ satisfying $\mathbf{q}'d\mathbf{G} = \mathbf{q}'d\mathbf{T}$, iff $\mathbf{K}\left(\mathbf{I}-\mathbf{M}\right)$ is invertible. Then, the solution is:

$$d\mathbf{Y} = \mathcal{M}\left(d\mathbf{G}-\mathbf{M}d\mathbf{T}\right)$$

where $\mathcal{M} \equiv \left(\mathbf{K}\left(\mathbf{I}-\mathbf{M}\right)\right)^{-1}\mathbf{K}$ is a bounded linear operator ("multiplier")

Which $\mathbf{K}$ are we using?

- Which $\mathbf{K}$ is needed?
- One natural choice:

$$\mathbf{K} = -\begin{pmatrix} 0 & (1+r)^{-1} & (1+r)^{-2} & (1+r)^{-3} & \cdots \\ 0 & 0 & (1+r)^{-1} & (1+r)^{-2} & \ddots \\ 0 & 0 & 0 & (1+r)^{-1} & \ddots \\ 0 & 0 & 0 & 0 & \ddots \\ \vdots & \ddots & \ddots & \ddots & \ddots \end{pmatrix} = -\sum_{t=1}^{\infty}(1+r)^{-t}\,\mathbf{F}^t$$

  where $\mathbf{F}$ is forward operator matrix.

- Then: $\mathbf{K}(\mathbf{I}-\mathbf{M})$ is the "asset jacobian" of the household block.
- When is $\mathbf{K}(\mathbf{I}-\mathbf{M})$ invertible? $\rightarrow$ see Auclert et al. (2023a) for a criterion.

The balanced budget multiplier

- Suppose $d\mathbf{G} = d\mathbf{T}$ (balanced budget)
- **Result**: We always have $d\mathbf{Y} = d\mathbf{G}$!
- Irrespective of **all** household heterogeneity, holds for any path of spending
- IS-LM antecedents: Gelting (1941), Haavelmo (1945)
- Proof is trivial: $d\mathbf{Y} = d\mathbf{G}$ is unique solution to

$$d\mathbf{Y} = (I - \mathbf{M}) \cdot d\mathbf{G} + \mathbf{M} \cdot d\mathbf{Y}$$

Deficit financed fiscal policy

- With deficit financing $d\mathbf{G} \neq d\mathbf{T}$ we have

$$d\mathbf{Y} = d\mathbf{G} + \underbrace{\mathcal{M} \cdot \mathbf{M} \cdot (d\mathbf{G} - d\mathbf{T})}_{d\mathbf{C}}$$

  Consumption $d\mathbf{C}$ depends on **primary deficits** $d\mathbf{G} - d\mathbf{T}$

- Interaction term: Deficits matter precisely when $\mathbf{M}$ is "large" (which will mean very different from RA model)
- **Next:** Go over our three examples and then compare multipliers to full HA model
- Define:
  - initial multiplier: dY_0/dG_0
  - cumulative multiplier: $\frac{\sum(1+r)^{-t}dY_t}{\sum(1+r)^{-t}dG_t}$

Three special cases

Let's get an intuition for all this in the RA model. Last lecture we derived consumption function for RA model when $\beta(1+r)=1$

$$C_t = (1-\beta)\sum_{s\geq 0}\beta^s Z_s + ra_{-1}$$

In particular

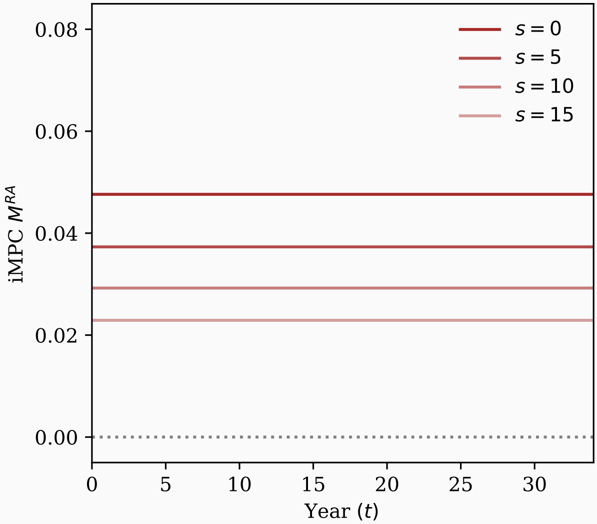
$$M_{t,s} = \frac{\partial C_t}{\partial Z_s} = (1-\beta)\beta^s$$

Thus iMPC matrix is given by

$$\mathbf{M}^{RA} = \begin{pmatrix} 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} = \frac{\mathbf{1}\mathbf{q}'}{\mathbf{1}'\mathbf{q}}$$

Easy to verify that $\mathbf{q}'\mathbf{M} = \mathbf{q}'$, and also that $\mathbf{M}\mathbf{w} = 0$ for any zero NPV $\mathbf{w}$

Representative-agent model



Fiscal policy in RA model

- Let's solve the IKC for the RA model
- Calculate:

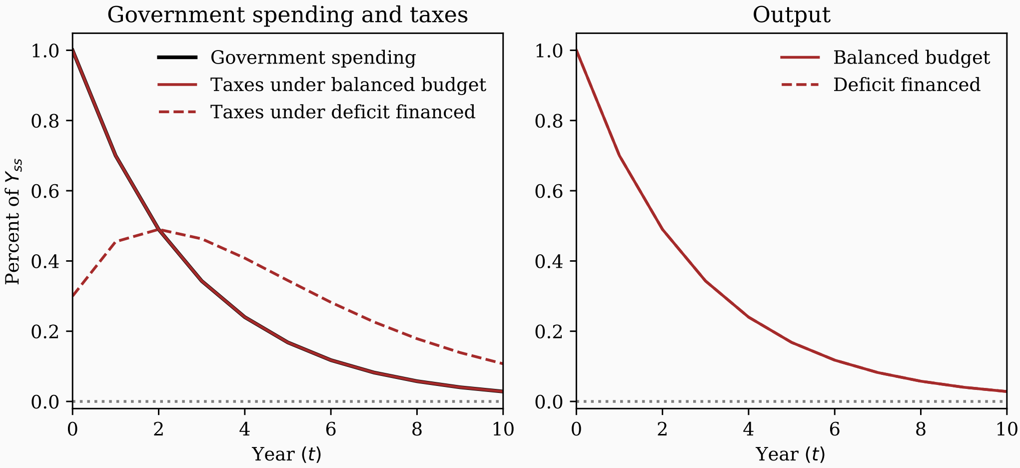
$$\begin{aligned} d\mathbf{C} &= \mathcal{M} \cdot \mathbf{M} \cdot (d\mathbf{G} - d\mathbf{T}) \\ &= \mathcal{M} \cdot (1-\beta)\,\mathbf{1q}'\,(d\mathbf{G} - d\mathbf{T}) \end{aligned}$$

  But government budget balance implies $\mathbf{q}'\,(d\mathbf{G} - d\mathbf{T}) = 0$! So:

$$d\mathbf{Y} = d\mathbf{G}$$

- Can prove this directly, too (eg Woodford 2011).
- **Deficits are irrelevant in RA!**

Impulse response to dG shock in RA model

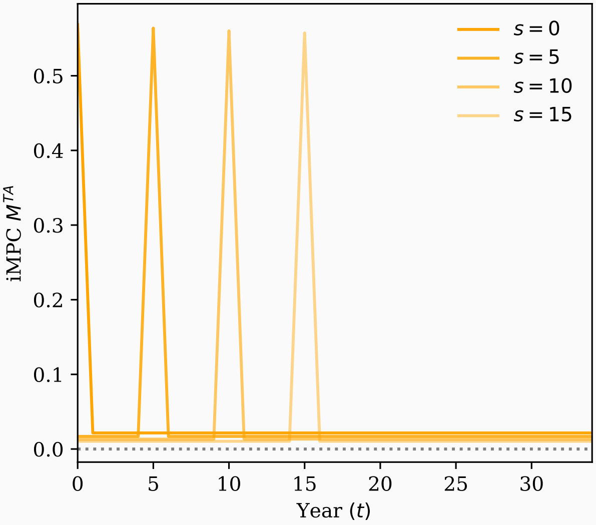


- $1 - \mu$ share of agents behave like RA agent, μ are hand to mouth $\Rightarrow$ $\mathbf{M}$ matrix is simple linear combination

$$\mathbf{M}^{TA} = (1 - \mu)\mathbf{M}^{RA} + \mu\mathbf{I}$$

- Issue: Only strong **contemporaneous** spending effect

iMPCs in TA model



Fiscal policy in TA model

- In Keynesian cross:

$$\left(\mathbf{I} - \mathbf{M}^{TA}\right) d\mathbf{Y} = d\mathbf{G} - \mathbf{M}^{TA} d\mathbf{T} \quad \Leftrightarrow \quad \left(\mathbf{I} - \mathbf{M}^{RA}\right) d\mathbf{Y} = \frac{1}{1-\mu}\left[d\mathbf{G} - \mu d\mathbf{T}\right] - \mathbf{M}^{RA} d\mathbf{T}$$

  This equation has same shape as for RA, hence:

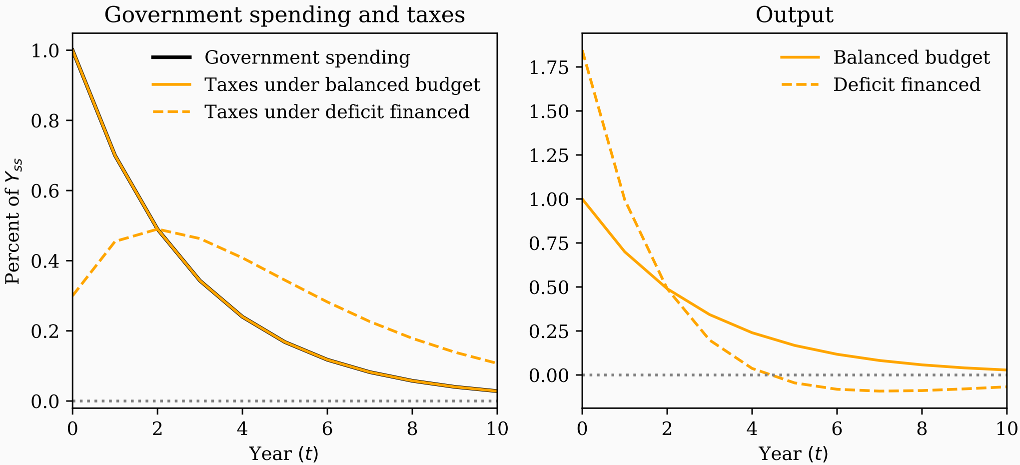
$$d\mathbf{Y} = \frac{1}{1-\mu}\left[d\mathbf{G} - \mu d\mathbf{T}\right]$$

- Results from undergrad: Spending multiplier $1/(1-\mu)$ and transfer multiplier $\mu/(1-\mu)$. So: μ is "effective" MPC, ignoring RA
- Can also write:

$$d\mathbf{Y} = d\mathbf{G} + \frac{\mu}{1-\mu} \underbrace{\left[d\mathbf{G} - d\mathbf{T}\right]}_{\text{primary deficit}}$$

- Only **current** deficit matters. Initial multiplier can be large $\in [1, \frac{1}{1-\mu}]$, but cumulative multiplier is always equal to 1!

Impulse response to dG shock in TA model



Zero-liquidity model

- What about iMPCs in the ZL model?
- We can calculate (see IKC paper)

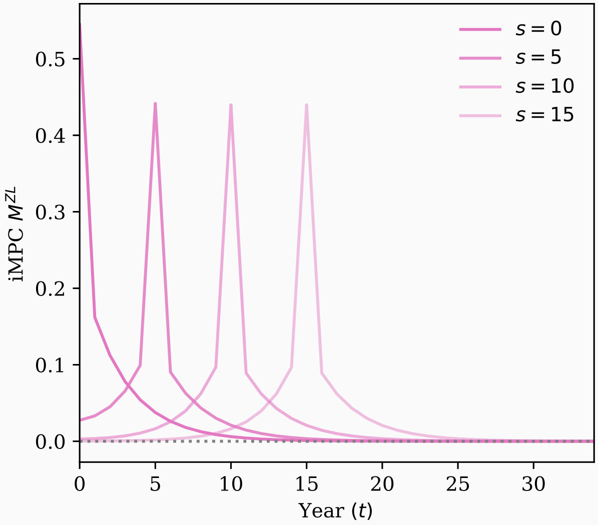
$$M_{t0}^{ZL} = \mu \mathbf{1}_{t=0} + (1-\mu)\left(1 - \frac{\lambda}{1+r}\right) \cdot \lambda^t$$

$$M_{0s}^{ZL} = (1-\mu)\frac{1-\beta\lambda}{\beta(1+r)} \cdot (\beta\lambda)^s$$

- Intuitively, it's a mix of a "constrained agent" with mass μ and agents that spend down assets at constant rate λ
  - Latter are also the iMPCs of a bond-in-utility model (and an OLG model!)
- Note, given known M_{00} and M_{10}, can solve for μ and λ

iMPCs in ZL model

see also Bilbiie (2021)



Fiscal policy in ZL model

- Can solve above model explicitly

$$dY_t = \underbrace{\frac{1}{1-\mu}\left[dG_t - \mu dT_t\right]}_{\text{as in TA model}} + \underbrace{\frac{\beta(1+r)-1}{1-\mu} dB_t + (1+r)\frac{1-\beta\lambda}{1-\mu}\left(\frac{1}{\lambda}-1\right)\sum_{s=0}^{\infty} dB_{t+s}}_{\text{new terms}}$$

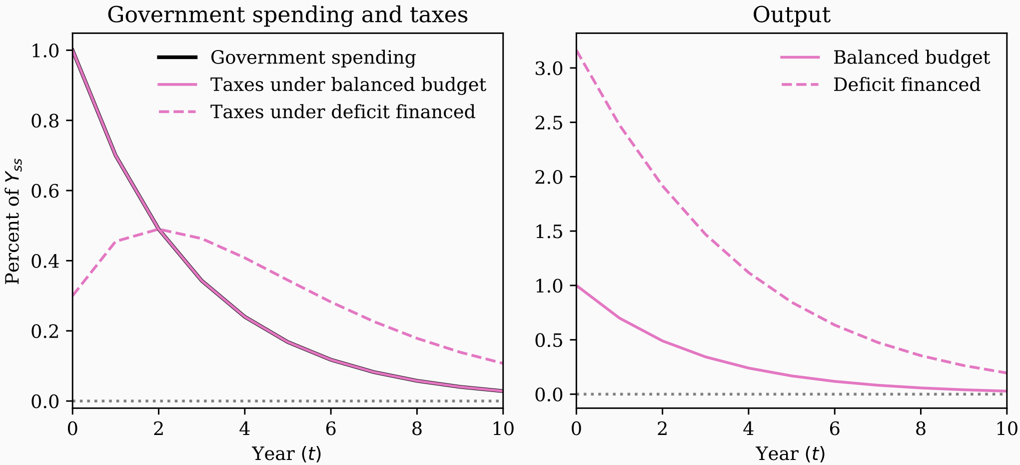
  Future fiscal policy extremely powerful here.

- Why? Dynamic income-consumption feedback from “spending down” effect
- In particular, can show:

Theorem

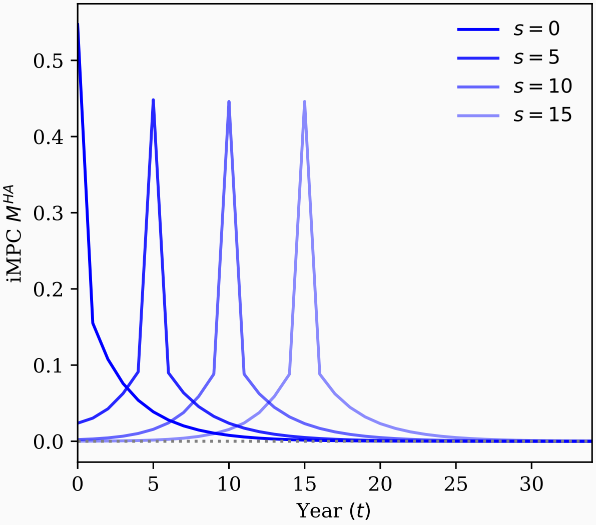
Holding β, r, and M_{00} fixed in the ZL model, a higher M_{10} increases the cumulative multiplier whenever $d\mathbf{B} \geq 0$ and $dB_t > 0$ for some t.

Impulse response to dG shock in ZL model

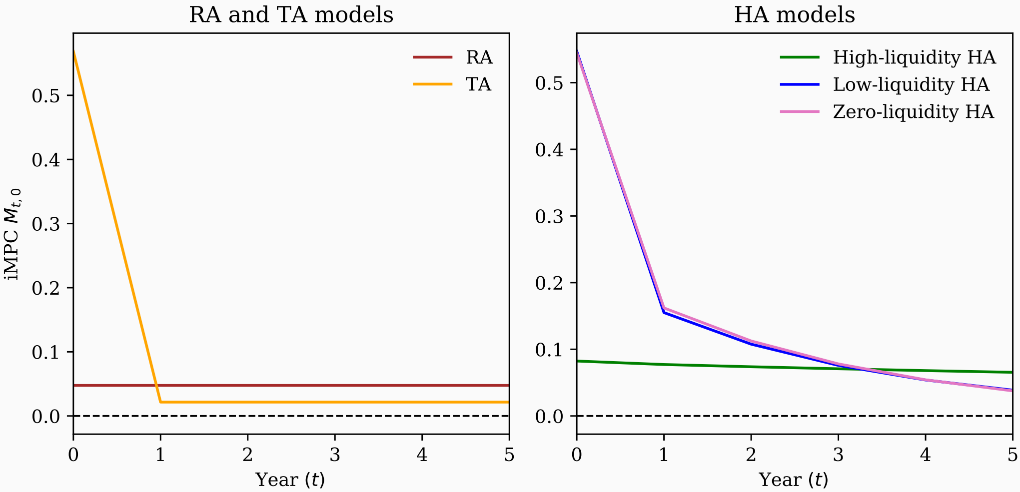


iMPCs in the HA model

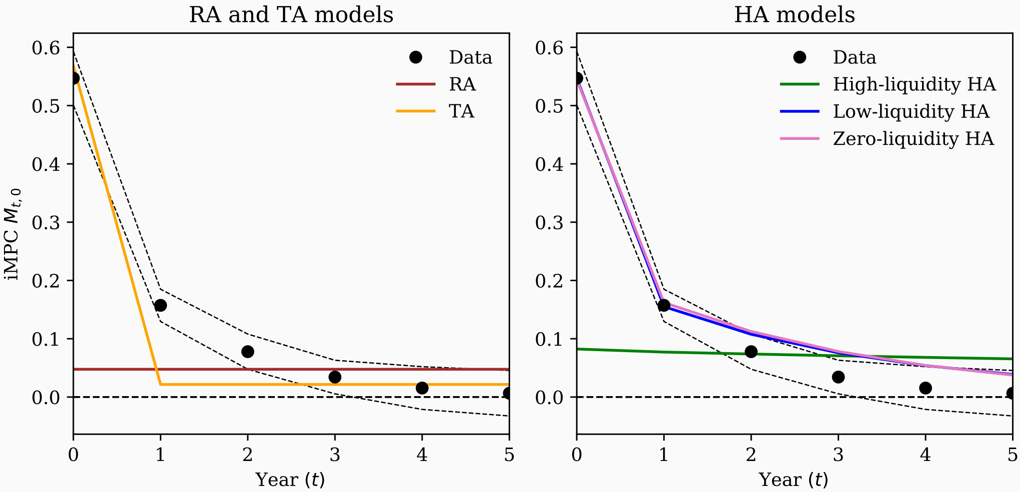
iMPCs in the HA model (computed using fake news algorithm)



Comparing iMPCs across models

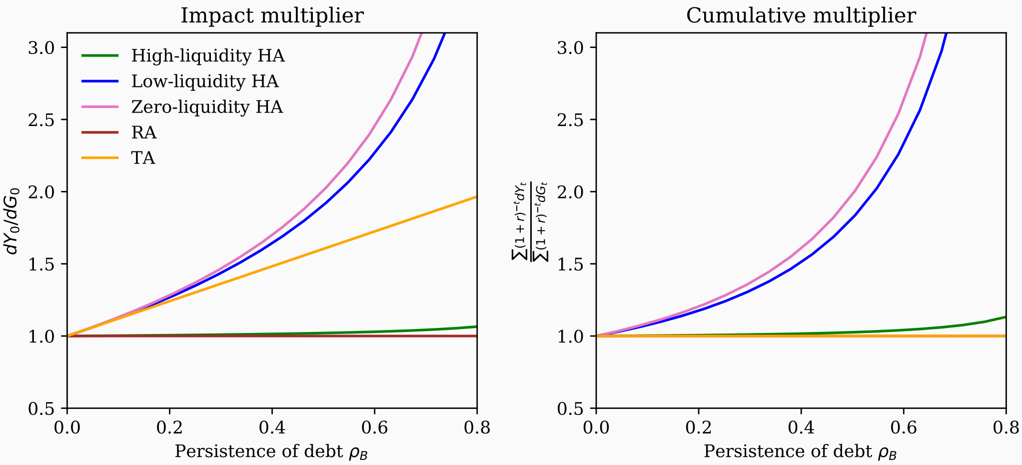


Comparison with the data

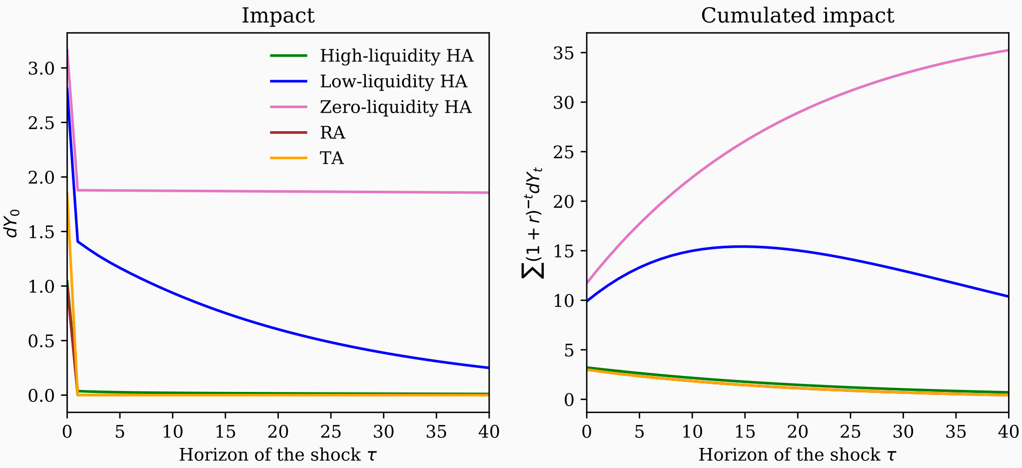


Insights about Fiscal Multipliers

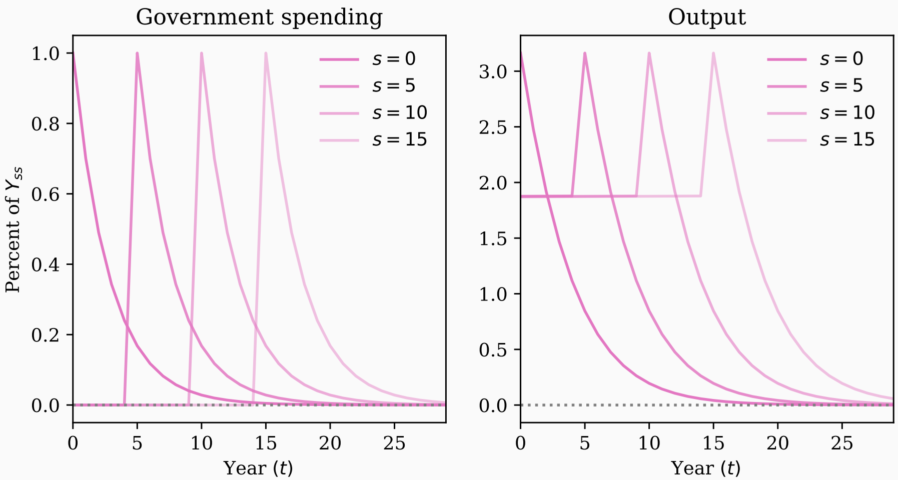
Fiscal stimulus more powerful when deficit financed



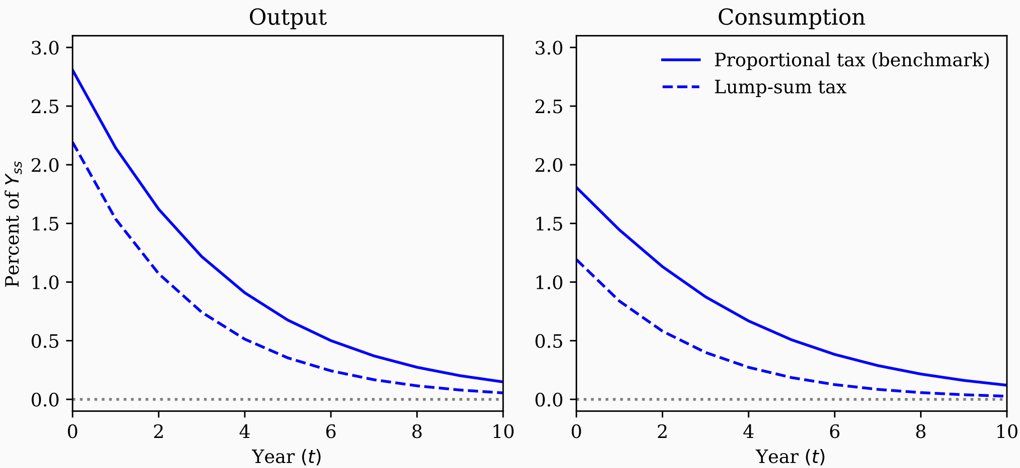
Fiscal policy is more powerful if front loaded...



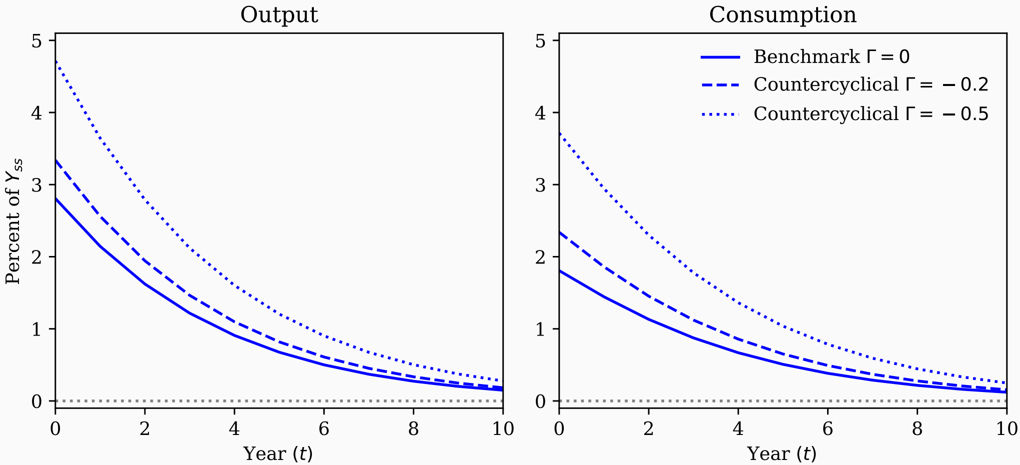
… but not in the zero-liquidity model (a fiscal policy forward guidance puzzle?)



Fiscal policy is less powerful if financed by lump-sum taxes (Why?)



Fiscal policy is more powerful if income risk is countercyclical (Why?)



Auclert-Rognlie "incidence function". More negative Γ means incomes more dispersed in recessions, Π is fixed.

Takeaway

Fiscal policy in HANK

- First exploration of shocks & policies in HANK
- One key difference already emerged: in HANK, households have very different **iMPCs**
- This matters for fiscal policy:
    - deficit financing & front loading amplifies initial and cumulative multipliers
    - not the case in RA, and not even in TA

References

Auclert, A., Rognlie, M., and Straub, L. (2023a). Determinacy and Existence in the Sequence Space. *Manuscript*. 00000.

Auclert, A., Rognlie, M., and Straub, L. (2023b). The Intertemporal Keynesian Cross. *Manuscript*.

Bilbiie, F. O. (2021). Monetary Policy and Heterogeneity: An Analytical Framework. *Manuscript*.

Fagereng, A., Holm, M. B., and Natvik, G. J. (2021). MPC Heterogeneity and Household Balance Sheets. *American Economic Journal: Macroeconomics*, 13(4):1–54.

Gelting, J. (1941). Nogle Bemærkninger Om Finansieringen Af Offentlig Virksomhed. *Nationaløkonomisk Tidsskrift*, 3.

Haavelmo, T. (1945). Multiplier Effects of a Balanced Budget. *Econometrica*, 13(4):311–318.

Woodford, M. (2011). Simple Analytics of the Government Expenditure Multiplier. *American Economic Journal: Macroeconomics*, 3(1):1–35.