Electricity Spot Market Design

Martin Bichler*

Decision Sciences & Systems

Department of Computer Science

Technical University of Munich

*joint work with Mete Ahunbay and Johannes Knörr

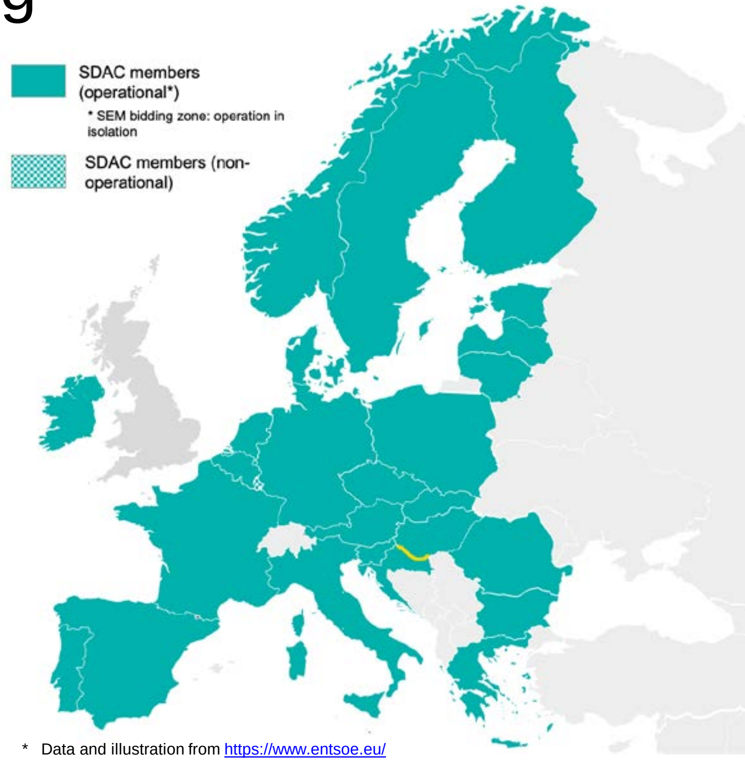
European Day-Ahead Market Coupling

Day-ahead market provides central price signals in the EU

- 98,6% of EU consumption is coupled
- 1.530 TWh / year coupled in one market solution
- 200 M€ average daily value of matched trades

All exchanges are cleared centrally once a day
using mixed-integer programming
(several problems for allocation and pricing)

- Size of the power grid for continental Europe and Ireland**
  ~16,000 generators and batteries
  ~25,000 nodes
  ~22,000 lines



* Data and illustration from https://www.entsoe.eu/
** Numbers from the ENTSO-E report on the LMP Study for the Bidding Zone Review Process

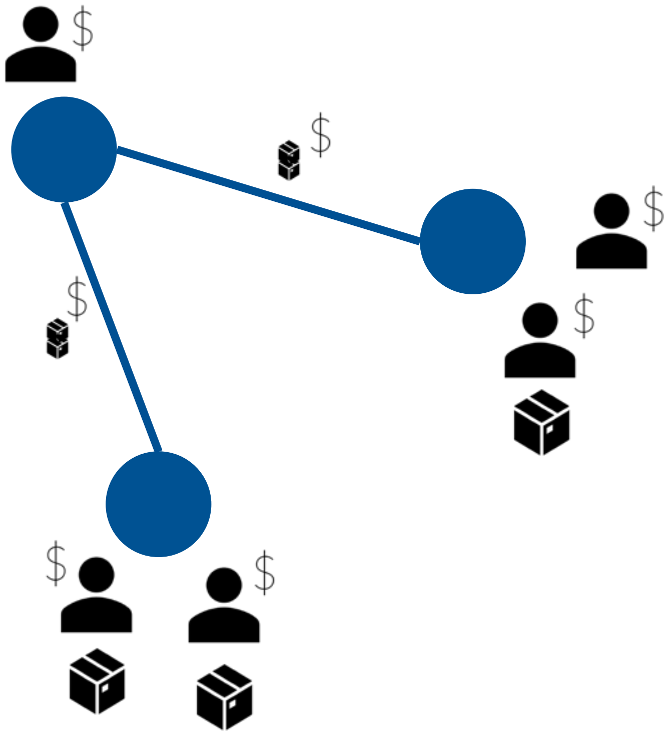
Electricity Spot Markets and Arrow-Debreu Markets

Not all countries or states are using markets!

Electric power systems are often operated by vertically integrated utilities, which own the generation, transmission, and distribution assets.

Arguments for markets are often based on the Welfare theorems, but power markets are different from the original Arrow-Debreu model:

1. markets are coupled
2. preferences are not convex



Pricing in Convex Markets

The welfare maximization problem for the market is

$$\max_x \sum_{i\in P} v_i(x_i) \ \ s.t. \sum_{i\in P} x_i = 0.$$

where $\sum_{i\in P} v_i(x_i) = \sum_b v_b(x_b) - \sum_s c_s(x_s)$.

When valuations and costs of buyers and sellers are all concave and a maximum exists, prices may be found by the Lagrangian dual

$$\min_p \max_x \sum_{i\in P} v_i(x_i) - p^T x_i$$

This insight extends to distributed and coupled markets with transmission service operators.

The Welfare Theorems for Coupled and Convex Markets

Theorem 1 (Welfare Theorems for Coupled Markets with Quasilinear and Convex Preferences).
Let price vector $p^* \in \mathbb{R}^{M \cup F}$ and the allocation ${(z_l)^*}_{l \in L}$ be a Walrasian equilibrium, then this allocation maximizes social welfare. Conversely, if ${(z_l)^*}_{l \in L}$ is a welfare-maximizing allocation, then it can be supported by a Walrasian price vector p that forms a Walrasian equilibrium.

Notes (Ahunbay, Bichler, Knoerr, 2023):

- Theorem 1 does not require differentiability of the functions.
- Proof via Fenchel-Young inequalities.
- The version with quasilinear utility allows for fast computation via (discrete) convex optimization.

Properties of Convex (Coupled) Markets

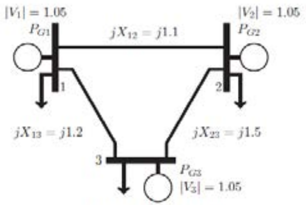
For a convex market, a primal solution x^* and a dual solution p^* assemble to form a *Walrasian equilibrium.*

Such a pair (x^*, p^*) satisfy certain desirable properties:

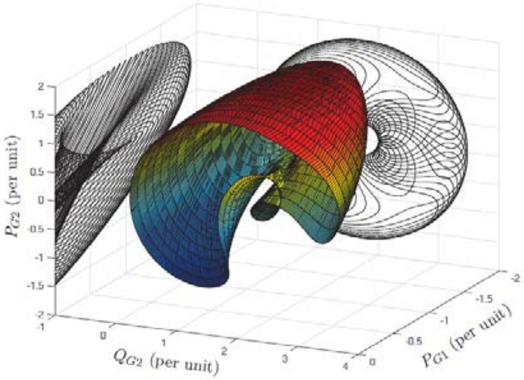
Efficiency	Supply-Demand Balance	Envy-Freeness	Budget Balance
→ Primal Optimality	→ Primal Feasibility	→ Strong duality	→ C. Slackness

Non-Convexities due to Power Flows

- Non-linear AC (Alternating Current) power flow equations
  - ACOPF (AC Optimal Power Flow) is intractable for realistic problem sizes.



(a) One-line diagram



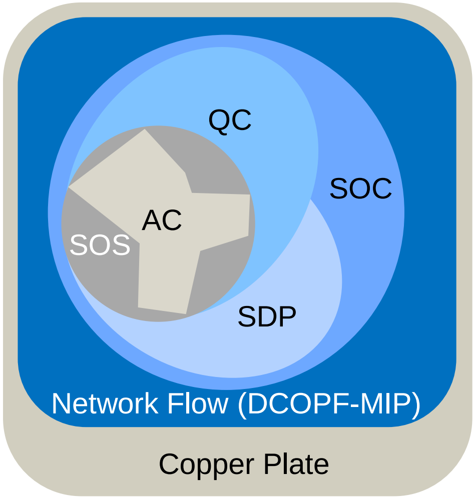
(b) Feasible space

I. A. Hiskens and R. J. Davy, "Exploring the Power Flow Solution Space Boundary," *IEEE Transactions on Power Systems*, vol. 16, no. 3, pp. 389–395, August 2001.

Non-Convexities due to Power Flows

- Non-linear AC (Alternating Current) power flow equations
  - ACOPF (AC Optimal Power Flow) is intractable
    for realistic problem sizes.

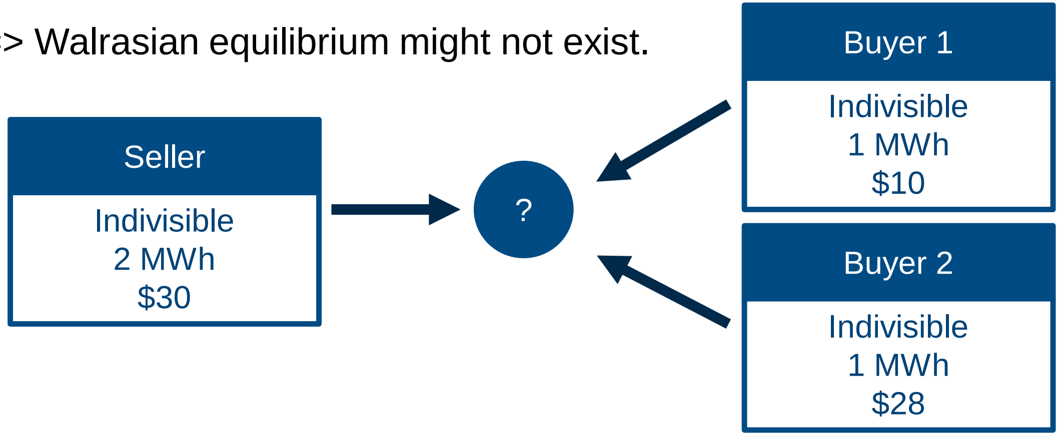
- Linear approximations used today provide poor solutions
  - US ISO markets are based on approximations (DCOPF) via MIPs.
  - SOC relaxations are often feasible in the ACOPF.
  - DCOPF can lead to large but unnecessary price peaks
    (Bichler & Knörr, 2023).



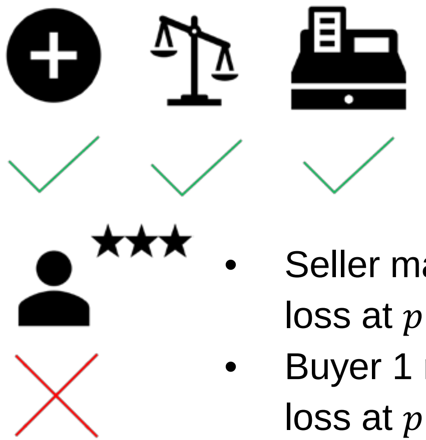
Non-Convexities due to Preferences

Non-convexities due to start-up and shutdown costs of gas turbines, curtailment, transmission and distribution costs, etc.

=> Walrasian equilibrium might not exist.

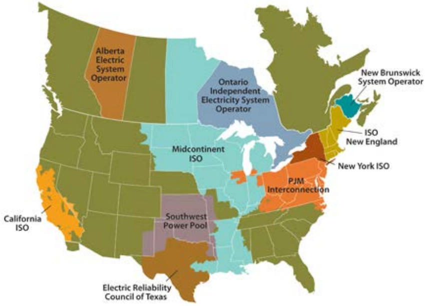


Clearing the market



- Seller makes a loss at $p \leq \$10$
- Buyer 1 makes a loss at $p > \$10$

How to Deal with Non-Convexities?



In the USA real-time market:
Welfare-maximizing outcome, make-whole payments and penalties.

In the EU day-ahead market:
Suboptimal outcome, no make-whole payments.

Maintain the Efficient Outcome

Properties of Walrasian equilibria:

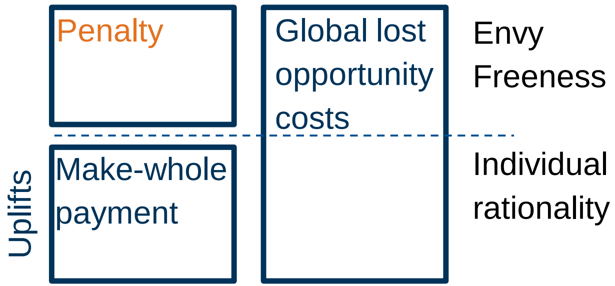
Look magnitude of deviations.

Efficiency	Supply-Demand Balance	Envy-Freeness	Budget Balance
→ Primal Optimality	→ Primal Feasibility	→ Strong Duality	→ C. Slackness

State-of-the-Practice

Pricing on US ISO markets:

- Convex-Hull Pricing (Hogan and Ring, 2003) and ELMP pricing (MISO, 2019)
- IP-Pricing (O'Neill et al, 2005)



Central concerns:

High make-whole payments (MWPs):
"The use of side-payments can undermine the market's ability to send actionable price signals." (U.S. FERC, 2018)

Wrong congestion signals (LLOCs):
"Convex Hull Pricing may produce positive congestion prices for transmission lines that are not congested as dispatched." (Schiro et al. 2015)
=> IP-Pricing achieves zero LLOCs

Pricing as Multi-Objective Optimization Problem*

Minimize a convex combination of
MWPs and LLOCs

$$\min_p \sum_l max\{\lambda_l^{MWP}(p|z_l^*), \lambda_l^{LLOC}(p|z_l^*)\}$$

Experiments based on data from the ENTSO-E bid zone review (4538 generators, 1687 nodes)

1) guaranteed to have lower MWPs than IP-Pricing
2) lower LLOCs compared to min-MWP
3) Prices are participant-wise Pareto optimal

	MWPs ($)	LLOCs ($)
Min GLOC (ELMP)	24,577	44,726
Min LLOC (IP-Pricing)	22,487	0
Min MWP	0	8,933,860
Min LLOC v MWP	326	1292

*M. Ahunbay, M. Bichler, and J. Knoerr. Pricing optimal outcomes in coupled and non-convex markets: Theory and applications to electricity markets. In: Proceedings of the ACM Conference on Economics and Computation. ACM, 2023.

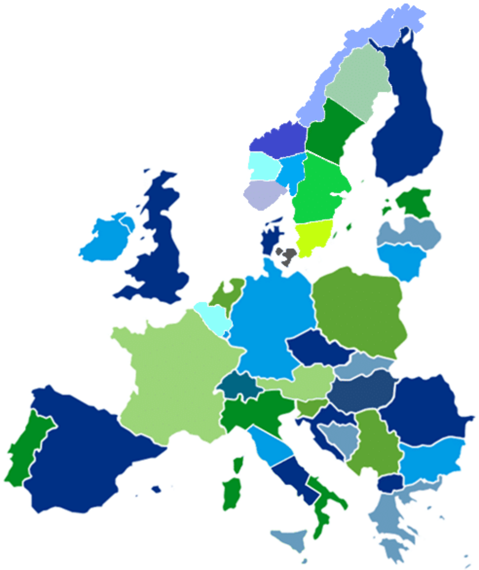
Relax Welfare Maximization

- The EU day-ahead markets use an iterative algorithm (PCR EUPHEMIA) to find an allocation that allows for linear and anonymous prices.

- Bichler, Fux, and Goeree (ISR, 2018) compute constrained welfare-maximizing outcomes and provide welfare bounds decreasing with the number of agents.

- Milgrom and Watt (2022) introduce a mechanism with two price vectors, which is nearly efficient, nearly IR and IC that only relies on convex optimization.
  => The *Bound-Form First Welfare Theorem* bounds the welfare loss
  of non-convex markets by a constant indep. of the # of agents.

- Implementations require domain-specific rounding procedures.
  Experimental results show <2% welfare loss with realistic data from the ARPA-E Grid Optimization Competition.



Summary

Electricity spot markets need to solve very large non-convex problems and determine prices.

- US ISO markets relax budget balance
  - High make-whole payments
  - Wrong congestion signals (high LLOCs)

→ Both costs can be reduced significantly with adequate optimization models.

- EU day-ahead markets relax efficiency
  - Expensive price computations (PCR EUPHEMIA) (focus of current SDAC revisions)
  - No bounds on the welfare loss

→ Even very large problems can be computed via convex relaxations. Welfare losses are small on realistic data sets.