Monetary policy

NBER Heterogeneous-Agent Macro Workshop

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Yesterday: The canonical HANK model & fiscal policy

Today: Monetary policy

We start by studying consumption in the closed economy

We keep our focus on real interest rate rules (also see what Taylor rules do)
1. Review of monetary policy in the standard NK model
2. Monetary policy in the canonical HANK model
3. Direct and indirect effects of monetary policy
4. Cyclical income risk
Review of monetary policy in the standard NK model
The NK model

• Recall the standard 3-equation NK model
  • separable preferences, sticky prices or wages, perfect foresight

\[ c_t = c_{t+1} - \sigma^{-1} (i_t - \pi_{t+1}) \]  
\[ \pi_t = \kappa c_t + \beta \pi_{t+1} \]  
\[ i_t = \epsilon_t + \pi_{t+1} \]  

(EE) \hspace{1cm} (NKPC) \hspace{1cm} (r-rule)

• Taylor rule instead of (r-rule): \( i_t = \epsilon_t + \phi \pi_t \) (usually \( \phi > 1 \))

• What does a monetary policy shock do, e.g. \( \epsilon_t \downarrow \)?

  1. expansion in \( c_t \) so output \( y_t \), inflation \( \pi_t \uparrow \), nominal rate \( i_t \) ambiguous

  2. far out shocks to \( \epsilon_t \) with large \( t \) have large effects (forward guidance “puzzle”)
(1) One-time expansion

- Consider shock $\epsilon_t = (-\Delta, 0, 0, \ldots)$. Forward looking: $\pi_t = c_t = 0$ for $t > 0$
- From (EE), (NKPC), (r-rule)
  \[ c_0 = \sigma^{-1} \Delta > 0 \quad \pi_o = \kappa \sigma^{-1} \Delta > 0 \quad i_o = -\Delta < 0 \]
- With Taylor rule, instead
  \[ c_0 = -\sigma^{-1} i_o \quad \pi_o = \kappa c_0 \quad i_o = -\Delta + \phi \pi_o \]
  \[ c_0 = \frac{\sigma^{-1} \Delta}{1 + \kappa \phi \sigma^{-1}} > 0 \quad \pi_o = \frac{\sigma^{-1} \kappa \Delta}{1 + \kappa \phi \sigma^{-1}} > 0 \quad i_o = -\frac{\Delta}{1 + \kappa \phi \sigma^{-1}} < 0 \]
- A plausible outcome. But is the **transmission mechanism** also plausible?
  - happens *entirely* through the Euler equation (intertemporal substitution)
  - no debt, no redistribution, no feedback from $y_t$ to $c_t$ ← **want to fix this**
  - also: no investment, no exchange rate ← **fixes exist, but want to revisit them**
Where the Euler equation really matters: **forward guidance**. E.g:

\[ r_t \equiv i_t - \pi_{t+1} = \begin{cases} -\Delta & t = T \\ 0 & t \neq T \end{cases} \]

- At time of shock \( \Rightarrow c_T = \sigma^{-1}\Delta, \pi_T = \kappa\sigma^{-1}\Delta \)

Then, solving (EE) and (NKPC) backwards from \( T \)

\[ c_t = \sigma^{-1}\Delta \quad \pi_t = \kappa \sum_{k=0}^{T-t} \beta^k \sigma^{-1}\Delta \quad \text{for all } t \leq T \]

- Transmission to \( c_t \) independent of \( T \). Transmission to \( \pi_t \) growing in \( T \! \! \! \! \! \! \! \)!
  - e.g. \( \beta = 0.99 \) and \( T = 20 \) quarters then \( \frac{\pi_0}{\pi_T} = \frac{1-\beta^T}{1-\beta} = 18! \)
  - “Forward guidance puzzle” from Del Negro et al. (2013)
  - gets even worse at ZLB!
Summary: two issues

- To summarize, two key issues with the standard NK model:
  - **transmission channel:** 100% through Euler equation, seems implausible
  - **output response:** Euler equation “too forward looking”

- A major goal of the early HANK literature was to solve these two issues
  - **Auclert (2019), Kaplan et al. (2018):** wealth distribution + high MPCs $\Rightarrow$ redistribution channels of m.p., substitution effects less important for $C$
  - **McKay et al. (2016):** borrowing constraints imply that $C$ is less forward looking. Could help deliver “discounting” in the Euler equation? Something like:
    \[
    c_t = \delta c_{t+1} - \sigma^{-1} (i_t - \pi_{t+1}) \quad \text{with } \delta < 1
    \] (DEE)

**Next:** What HANK actually does!
Monetary policy in the canonical HANK model
Setting up the model

- Take canonical HANK model, but abstract from fiscal policy
  - \( T = \tau = G = B = 0 \)
  - but allow agents to borrow from each other: \( a < 0 \) (as in Huggett model)
  - Later bring back govt to study monetary-fiscal interactions

- Real rate rule: monetary policy sets \( r_t^{ante} \) directly

- Related to the above, want to ask two questions:
  1. What’s the output response relative to RA? (Magnitude? Any “discounting”?)
  2. What are the transmission channels relative to RA?

We’ll start with 1.
Back to our equilibrium conditions

Under these assumptions, the canonical HANK model can just be written as:

\[
\max_{c_{it}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (u(c_{it}) - v(N_t))
\]

\[
c_{it} + a_{it} \leq (1 + r_{t-1}^{ante}) a_{i_{t-1}} + s_{it} Y_t
\]

\[
a_{it} \geq a
\]

with

\[
C_t \equiv \int c_{it} \, di = Y_t = N_t
\]

\[
A_t \equiv \int a_{it} \, di = 0
\]

That’s it!
Let’s visualize this as a DAG:

Here again, simple fixed point:

\[ C_t \left( \{ r_{s}^{ante}, Y_s \} \right) = Y_t \]
Ex-ante vs ex-post $r$

• In practice, we usually write HetBlocks with “ex-post $r$” convention, i.e. here:

$$\max_{c_{it}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (u(c_{it}) - v(N_t))$$

$$c_{it} + a_{it} \leq (1 + r_t)a_{it-1} + s_{it}Y_t$$

$$a_{it} \geq a$$

• This is more general: allows us to handle valuation effects (see next lecture)

• Here there are no valuation effects, so we just have

$$r_t = r_{ante}^{t-1} \quad t \geq 1$$

$$r_0 = r_{ss}$$

• This adds one “no valuation” block to the DAG
Our new DAG is:

Using a CombineBlock, we can just let SSJ do the convolution for us, ie define

$$C_t (\{r_s^{ante}, Y_s\}) \equiv C_t^{post} (\{r_j (r_s^{ante}), Y_s\})$$

So that we are back to our simple fixed point:

$$C_t (\{r_s^{ante}, Y_s\}) = Y_t$$
Jacobians again

• As in fiscal lecture, let’s linearize this sequence space equation

• Define \( dr \equiv (dr_0^{ante}, dr_1^{ante}, \ldots) \), and let \( dY = (dY_0, dY_1, \ldots) \) as before. Define Jacobian \( M' \equiv (\partial C_t/\partial r_s^{ante})_{t,s} \) capturing direct effect of \( r \) on \( C \). Then:

\[
dY = M' dr + MdY
\]

• Almost like the IKC, except that partial eqbm demand shock is no longer coming from fiscal policy, \( dG - MdT \), but instead from monetary policy!

• Just as with fiscal, the PE demand shock has zero NPV (Why?)

• General solution uses same linear mapping \( M \) (recall \( "(I - M)^{-1}" \))

\[
dY = MM' dr
\]

Next: Let’s visualize \( M' \); then the solution \( dY \) for an AR(1) shock to \( dr \)
Columns of Jacobian $M^r$

- $M^r$ is our old friend from Lecture 2
Monetary policy shock in HA (AR(1) with $\rho = 0.7$)

Output response

- HA > RA! Interesting! But why?
• One way to make progress is to simplify the model ⇒ ZL model: $a \to 0$

• Recall that for low $\beta$ only Euler equation of agents in income state $\bar{s}$ holds

$$ (Y_t \bar{s})^{-\sigma} = \beta (1 + r_t) \mathbb{E}_t \left[ (Y_{t+1} \bar{s}')^{-\sigma} | \bar{s} \right] $$

where $\bar{s}$ is income state attaining $\bar{\rho} \equiv \max_s \mathbb{E} \left[ (s' / s)^{-\sigma} | s \right]$

• Hence, we always have

$$ Y_t^{-\sigma} = \beta (1 + r_t) \bar{\rho} (Y_{t+1})^{-\sigma} \Rightarrow y_t = y_{t+1} - \sigma^{-1} (r_t - \log (\beta \bar{\rho})) $$

• **This is like our representative agent Euler equation!**

  • just with effective discount factor $\beta \bar{\rho}$

  $\Rightarrow$ Werning (2015)'s **neutrality result** for zero liquidity and acyclical income risk

• In particular: No discounting in log-linearized Euler equation!
Neutrality for monetary policy in the ZL limit

Output response

Real rate shock

% deviation from ss

pp deviation from ss

Year (t)
Neutrality also implies the forward guidance puzzle is not solved by HA
Summary: Output response of monetary policy in HA

- No robust result that HA ≠ RA!
  - In fact here, with zero liquidity, we show that HA = RA
  - Forward guidance can be equally powerful

- But how can that be, given that HA breaks the Euler equation?

- Next: study transmission channels
Direct and indirect effects of monetary policy
Direct and indirect effects

• To see what’s going on, let’s go back to our IKC-like equation:

\[ dY = dC = M' \cdot dr + M \cdot dY \]

Direct effect Indirect effect

• **Two competing effects** of market incompleteness! direct ↓, indirect ↑

[Kaplan et al. (2018) showed this in their two-asset HA model]

• Why? High MPCs make \( C \) more sensitive to \( Y \) but also **less sensitive to \( r \)**!
  • cf Auclert (2019): substitution effect of \( dr \) scales with \(-\sigma^{-1}(1 - MPC)\)
  • In ZL model above, can actually prove that \( M' = -\sigma^{-1}(I - M) \) so [new result!]

\[ dC = -\sigma^{-1}(I - M) \cdot dr + M \cdot dY \]
Decomposition into direct and indirect effects

- Let’s implement $dC = M'r + M \cdot dY$ in our canonical HA model:

![Graph showing decomposition into direct and indirect effects for HA and RA models over time.](two.osf/one.osf)
Takeaway so far

1. HA model does not imply robustly different output response
2. But it does change transmission: indirect effects are more important!
   • This is the main result in KMV. Why do we care about that per se?
     • KMV: labor & financial market institutions matter more than we thought
     • We’ll see other reasons for why we should care in the next lecture
   • Before going there, let’s see how we can make HA and RA really different
   • Recall that so far, we have assumed **acyclical income risk**, i.e.
     \[ \text{sd}(\log y_{it}) = \text{sd}(\log s_i), \text{ independent of } Y_t \]
   • This turns out to be a critical assumption. Let’s relax it now.
Cyclical income risk
Introducing cyclical income risk

- We can introduce cyclical income risk by adopting different allocation rule. For instance take Auclert and Rognlie (2018)

\[ n_{it} = Y_t \frac{\left( s_{it} \right)^\zeta \log Y_t}{\mathbb{E} \left[ s_{it}^{1+\zeta} \log Y_t \right]} \equiv Y_t \Gamma (s_{it}, Y_t) \]

- Distribution of income \( y_{it} \equiv s_{it} n_{it} \) now reacts to monetary policy

\[ \text{sd} (\log y_{it}) = (1 + \zeta \log Y_t) \text{sd} (\log s_i) \]

  - \( \zeta > 0 \): procyclical inequality and income risk
  - \( \zeta < 0 \): countercyclical inequality and income risk
  - \( \zeta = 0 \) is benchmark from above (acyclical inequality & risk)

- Matters because:
  - current shocks redistribute between different MPCs (“cyclical inequality”)
  - future shocks change income risk (“cyclical risk”)
How does cyclical income risk change forward guidance?

- Consider a $r_T$ shock with three calibrations for $\zeta$ in HA model.

![Impulse response on output to forward guidance](image-url)
Zero liquidity limit with cyclical income risk

• What’s going on? In ZL limit, we get an exact discounted Euler equation

\[ y_t = \delta \mathbb{E}_t [y_{t+1}] - \sigma^{-1} \cdot \text{Cst} \cdot (r_{t+1} - \log (\beta \rho)) \]

where \( \delta \) depends on \( \gamma(s) \), elasticity of \( \Gamma \) in state \( s \) wrt \( Y \), relative to that for \( \bar{s} \)

1. Dynamic discounting \( (\delta < 1) \Leftrightarrow \gamma(s) < \gamma(\bar{s}) \) on average
   - “procyclical income risk”, not commonly used, exception below

2. Dynamic amplification \( (\delta > 1) \Leftrightarrow \gamma(s) > \gamma(\bar{s}) \) on average
   - “countercyclical risk”, more common
   - Microfound w/ \( u \): Ravn and Sterk (2017), den Haan et al. (2018), Challe (2020)
   - Lots of evidence: Storesletten et al. (2004), Guvenen et al. (2014)

3. Dynamic neutrality \( (\delta = 1) \Leftrightarrow \gamma(s) = \gamma(\bar{s}) \)
   - “acyclical risk”, ie Werning benchmark

• Why? Precautionary savings. Think about logic of discounted Euler equation.
In the empirically plausible case, the fwd guidance puzzle is **aggravated!**

Bilbiie (2019), Acharya and Dogra (2020)
Indirect ways to make income risk cyclical

- In richer models income of agents typically involves multiple components,
  \[ y_{it} = \frac{W_t}{P_t} n_{it} s_{it} - \tau_{it} + T_{it} \]

  \( \begin{array}{c}
  \text{taxes} \\
  \text{transfers}
  \end{array} \)

- These also matter for cyclicality of income risk

- For example, suppose taxes are set to keep balanced budget,
  \( \tau_t \equiv \int \tau_{it} di = r_t B \) and transfers \( T_t \) are dividends from firms with sticky prices
  \[ \Rightarrow \text{both } \tau_t \text{ and } T_t \text{ fall after expansionary } r_t \text{ (why?)} \]

- If \( \tau_t \) allocated to highest income state and \( T_t \) to all \( \Rightarrow \) procyclical risk!

- These are the assumptions in McKay et al. (2016).
  - Reason why that paper “solves” the forward guidance puzzle!
Summary

• Cyclical income risk matters

• Procyclical income risk ⇒
  • smaller effects of monetary policy in HA relative to RA
  • solves forward guidance puzzle
  • not empirically supported

• Countercyclical income risk is empirically more plausible, amplifies HA relative to RA, but aggravates forward guidance puzzle!


Zero liquidity limit with cyclical income risk

- Take ZL model with cyclical income risk. Euler for $\bar{s}$:
  \[ (Y_t \Gamma (\bar{s}, Y_t))^{-\sigma} = \beta (1 + r_t) \mathbb{E}_t \left[ (Y_{t+1} \Gamma (s', Y_{t+1}))^{-\sigma} | \bar{s} \right] \]

- Log-linearize around steady state $\Rightarrow$
  \[ y_t = \delta \mathbb{E}_t [y_{t+1}] - \sigma^{-1} \gamma(\bar{s})^{-1} (r_t - \log (\beta \rho)) \]
  where, if $\gamma (s) \equiv 1 + \frac{r_Y Y}{t}$ is the elasticity of income wrt $Y$ for agent in $s$:
  \[ \delta \equiv \rho^{-1} \mathbb{E} \left[ (s/\bar{s})^{-\sigma} \frac{\gamma (s)}{\gamma (\bar{s})} | \bar{s} \right] = \sum_s \omega (s) \frac{\gamma (s)}{\gamma (\bar{s})} \text{ where } \sum_s \omega(s) = 1 \]

- What matters is cyclicality of $y(\bar{s})$ relative to other income states

- Example with two states: $\delta = 1 - \omega + \omega \frac{\gamma_L}{\gamma_H}$ with $\omega \in (0, 1)$