Fiscal Policy

NBER Heterogeneous-Agent Macro Workshop

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This session

We just introduced the canonical HANK model.

Next: Focus on fiscal policy!

- Switch off all other shocks: TFP $X_t = 1$, no monetary shock $r_t = r = const$
- Focus on **first order** shocks to fiscal policy: $d\mathbf{G} = \{dG_t\}, d\mathbf{T} = \{dT_t\}$ such that

$$\sum_{t=0}^{\infty} (1+r)^{-t} (dG_t - dT_t) = 0$$

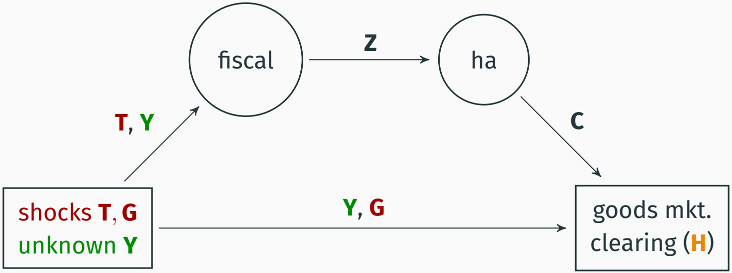
- Main reference for this class is Auclert et al. (2018)

Roadmap

1. The intertemporal Keynesian cross
2. Three special cases
3. Computing iMPCs in the HA model
4. Insights about Fiscal Multipliers
5. Takeaway

The intertemporal Keynesian cross

Switching off monetary shocks, the DAG is simply:



In this case, $\mathbf{H} = 0$ simply corresponds to:

$$\mathbf{Y} = \mathbf{G} + \mathcal{C}(\mathbf{Z})$$

To emphasize that $\mathbf{C}$ is a function, write it as $\mathcal{C}$. $\mathbf{C}$ only a function of $\mathbf{Z}$ here!

Next: Analyze this equation "by hand"...

The aggregate consumption function

- We call $\mathcal{C}$ the **aggregate consumption function**

$$C_t = \mathcal{C}_t(Z_0, Z_1, Z_2, \ldots) = \mathcal{C}_t(\{Z_s\})$$

  It's a collection of ∞ many nonlinear functions of ∞ many Z's!

- It usually also depends on the path of real interest rates, but those are assumed to be constant

- Using the DAG, we can substitute out Z and write goods market clearing as

$$Y_t = G_t + \mathcal{C}_t(\{Y_s - T_s\})$$

Intertemporal MPCs

$$Y_t = G_t + \mathcal{C}_t\left(\{Y_s - T_s\}\right)$$

- Feed in small shock $\{dG_t, dT_t\}$

$$dY_t = dG_t + \sum_{s=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial Z_s} \cdot (dY_s - dT_s) \tag{1}$$

- Response dY_t **entirely** characterized by the **Jacobian** of $\mathcal{C}$ function, which we also call **intertemporal MPCs**

$$M_{t,s} \equiv \frac{\partial \mathcal{C}_t}{\partial Z_s} \qquad \left(= \mathcal{J}_{t,s}^{\mathbf{C},\mathbf{Z}}\right)$$

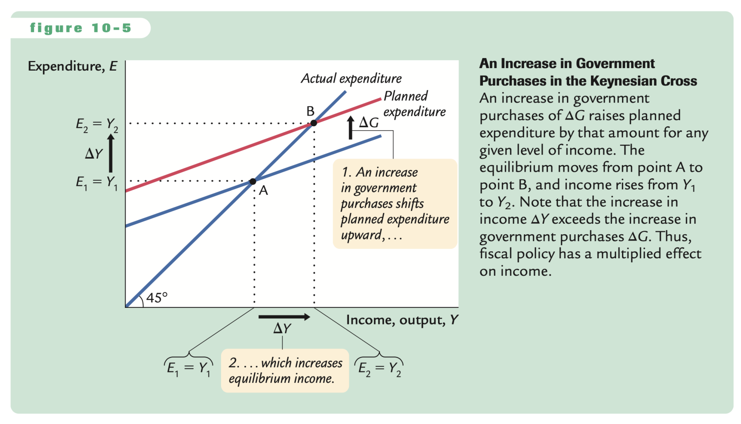
- $M_{t,s}$ = how much of an income change at date s is spent at date t
- Note: All income is spent at some point, hence $\sum_{t=0}^{\infty}(1+r)^{s-t}M_{t,s} = 1$

- Rewrite equation (1) in vector / matrix notation:

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y} \tag{2}$$

- This equation exactly corresponds to $\mathbf{H_Y}d\mathbf{Y} + \mathbf{H_G}d\mathbf{G} + \mathbf{H_T}d\mathbf{T} = 0$
- This is an **intertemporal Keynesian cross**
  - entire complexity of model is in $\mathbf{M}$
  - with $\mathbf{M}$ from data, could get $d\mathbf{Y}$ without model!
    (there is a "correct" $\mathbf{M}$ out there, but it's very hard to measure...)

Bringing back memories from undergrad ...



- The intertemporal Keynesian cross is the same ... just in vectors
- **Bigger** theme in this workshop: HANK models are able to revive IS-LM logic

Solving the intertemporal Keynesian cross

- How can we solve (2)? Rewrite as

$$(\mathbf{I}-\mathbf{M})\, d\mathbf{Y} = d\mathbf{G} - \mathbf{M} d\mathbf{T}$$

Can't we just invert $(\mathbf{I}-\mathbf{M})$?

- **Not so easy:** multiply both sides by $\mathbf{q} \equiv (1, (1+r)^{-1}, (1+r)^{-2}, \ldots)'$

$$\mathbf{q}'(\mathbf{I}-\mathbf{M})\, d\mathbf{Y} = 0 \qquad \& \qquad \mathbf{q}' d\mathbf{G} - \mathbf{q}'\mathbf{M} d\mathbf{T} = \mathbf{q}' d\mathbf{G} - \mathbf{q}' d\mathbf{T} = 0$$

both left and right hand side are **"zero NPV"** (why RHS?)

- General solution is then of the form

$$d\mathbf{Y} = \sum_{k=0}^{\infty} \mathbf{M}^k (d\mathbf{G} - \mathbf{M} d\mathbf{T}) + d\lambda \cdot \mathbf{v}$$

where $d\lambda \in \mathbb{R}$ and $\mathbf{v}$ is right eigenvector of $\mathbf{M}$ with EV 1. Pick $d\lambda$ such that $\lim_{t\to\infty} dY_t = 0$

Solving the intertemporal Keynesian cross

- We can summarize solution as

$$d\mathbf{Y} = \mathcal{M}\left(d\mathbf{G} - \mathbf{M}d\mathbf{T}\right)$$

  for some linear map $\mathcal{M}$ that ensures $dY_t \to 0$ as $t \to \infty$

- **Note:** When solving this on the computer, inverting a truncated version of $\mathbf{I} - \mathbf{M}$ will automatically give you (essentially) a truncated version of $\mathcal{M}$. So this does not cause trouble in SSJ...

- Can we say more about the solution? **Yes!**

The balanced budget multiplier

- Suppose $d\mathbf{G} = d\mathbf{T}$ (balanced budget)
- **Result**: We always have $d\mathbf{Y} = d\mathbf{G}$!
- Irrespective of **all** household heterogeneity, holds for any path of spending
- IS-LM antecedents: Gelting (1941), Haavelmo (1945)
- Proof is trivial: $d\mathbf{Y} = d\mathbf{G}$ is unique solution to

$$d\mathbf{Y} = (I - \mathbf{M}) \cdot d\mathbf{G} + \mathbf{M} \cdot d\mathbf{Y}$$

Deficit financed fiscal policy

- With deficit financing $d\mathbf{G} \neq d\mathbf{T}$ we have

$$d\mathbf{Y} = d\mathbf{G} + \underbrace{\mathcal{M} \cdot \mathbf{M} \cdot (d\mathbf{G} - d\mathbf{T})}_{d\mathbf{C}}$$

  Consumption $d\mathbf{C}$ depends on **primary deficits** $d\mathbf{G} - d\mathbf{T}$

- Interaction term: Deficits matter precisely when $\mathbf{M}$ is "large" (which will mean very different from RA model)
- **Next:** Go over our three examples and then compare multipliers to full HA model
- Define:
  - initial multiplier: dY_0/dG_0
  - cumulative multiplier: $\frac{\sum(1+r)^{-t}dY_t}{\sum(1+r)^{-t}dG_t}$

Three special cases

Let's get an intuition for all this in the RA model. Last lecture we derived consumption function for RA model when $\beta(1+r)=1$

$$C_t = (1-\beta)\sum_{s\geq 0}\beta^s Z_s + ra_{-1}$$

In particular

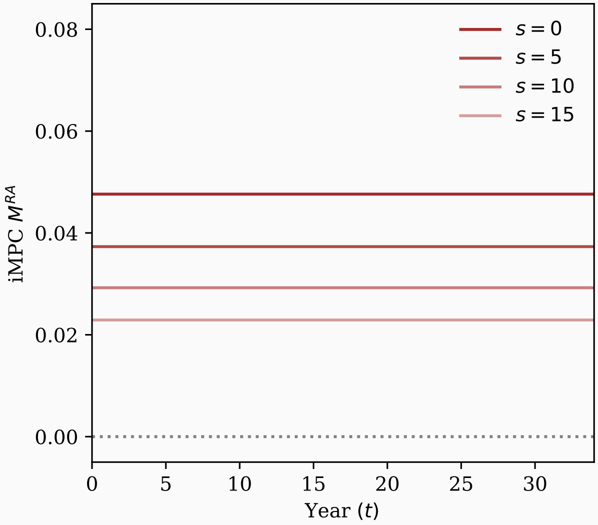
$$M_{t,s} = \frac{\partial C_t}{\partial Z_s} = (1-\beta)\beta^s$$

Thus iMPC matrix is given by

$$\mathbf{M}^{RA} = \begin{pmatrix} 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} = \frac{\mathbf{1}\mathbf{q}'}{\mathbf{1}'\mathbf{q}}$$

Easy to verify that $\mathbf{q}'\mathbf{M} = \mathbf{q}'$, and also that $\mathbf{M}\mathbf{w} = 0$ for any zero NPV $\mathbf{w}$

Representative-agent model



- Let's solve the Keynesian cross for the RA model
- Right eigenvector of $\mathbf{M}$ with EV 1 is $\mathbf{1}$, and so

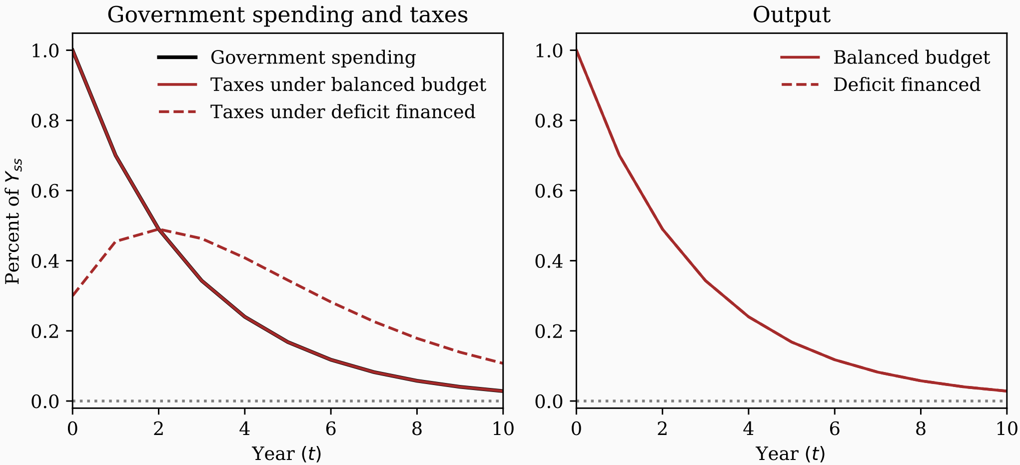
$$d\mathbf{Y} = \underbrace{\sum_{k=0}^{\infty} \mathbf{M}^k \left(d\mathbf{G} - \mathbf{M}d\mathbf{T}\right)}_{=d\mathbf{G}-\mathbf{M}d\mathbf{T}} + d\lambda \cdot \mathbf{1}$$

- Here, $\mathbf{M}d\mathbf{T}$ is vector with all elements equal to $(1-\beta)\mathbf{q}'d\mathbf{T}$
- Choose $d\lambda$ to ensure $dY_t \to 0$: $d\lambda = (1-\beta)\mathbf{q}'d\mathbf{T}$. Hence

$$d\mathbf{Y} = d\mathbf{G}$$

- One can prove this directly, too (eg Woodford 2011).
  Deficits are irrelevant in RA!

Impulse response to dG shock in RA model



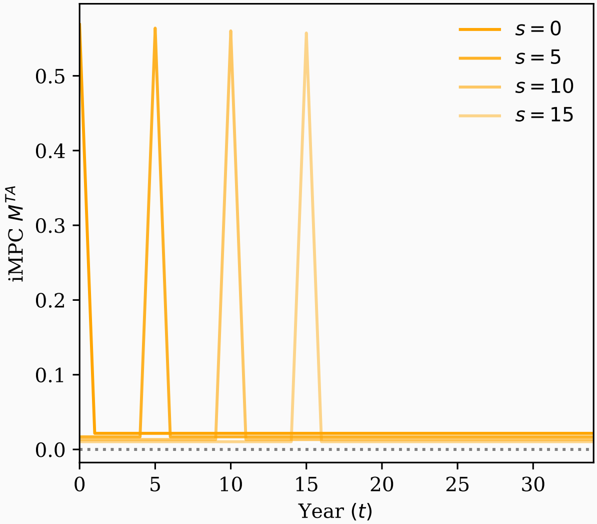
Two agent model

- $1-\mu$ share of agents behave like RA agent, μ are hand to mouth $\Rightarrow$ **M** matrix is simple linear combination

$$\mathbf{M}^{TA} = (1-\mu)\mathbf{M}^{RA} + \mu\mathbf{I}$$

- Issue: Only strong **contemporaneous** spending effect

iMPCs in TA model



Fiscal policy in TA model

- In Keynesian cross:

$$\left(\mathbf{I}-\mathbf{M}^{TA}\right) d\mathbf{Y} = d\mathbf{G} - \mathbf{M}^{TA} d\mathbf{T} \quad \Leftrightarrow \quad \left(\mathbf{I}-\mathbf{M}^{RA}\right) d\mathbf{Y} = \frac{1}{1-\mu}\left[d\mathbf{G} - \mu d\mathbf{T}\right] - \mathbf{M}^{RA} d\mathbf{T}$$

  This equation has same shape as for RA, hence:

$$d\mathbf{Y} = \frac{1}{1-\mu}\left[d\mathbf{G} - \mu d\mathbf{T}\right]$$

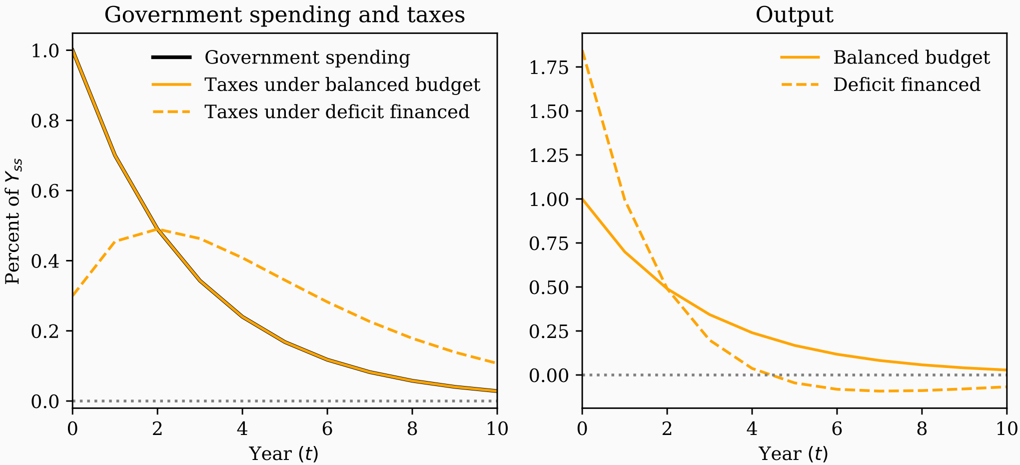
- Results from undergrad: Spending multiplier $1/(1-\mu)$ and transfer multiplier $\mu/(1-\mu)$. So: μ is "effective" MPC, ignoring RA

- Can also write:

$$d\mathbf{Y} = d\mathbf{G} + \frac{\mu}{1-\mu} \underbrace{\left[d\mathbf{G} - d\mathbf{T}\right]}_{\text{primary deficit}}$$

- Only **current** deficit matters. Initial multiplier can be large $\in [1, \frac{1}{1-\mu}]$, but cumulative multiplier is always equal to 1!

Impulse response to dG shock in TA model



- What are iMPCs in the ZL model?
- Feed in small shocks to after-tax income $\{dZ_t\}$ and figure out consumption + assets
- Consider an average agent in state $\overline{e}$. It saved da_{t-1} at date $t-1$, but only $\Pi_{\overline{e}\leftarrow\overline{e}}$ of that still in hands of $\overline{e}$ agents at date t.
- What do they plan on saving then? Linearized date-t Euler equation:

$$(1+r)\Pi_{\overline{e}\leftarrow\overline{e}}da_{t-1} - da_t + \overline{e}dZ_t = \beta(1+r)\cdot$$

$$\left[\Pi_{\underline{e}\leftarrow\overline{e}}\frac{(\underline{e})^{-\sigma-1}}{\overline{e}^{-\sigma-1}}\left[(1+r)da_t + e'dZ_{t+1}\right] + \Pi_{\overline{e}\leftarrow\overline{e}}\left[(1+r)da_t - da_{t+1} + \overline{e}dZ_{t+1}\right]\right]$$

- Define: $\tilde{\rho} \equiv \mathbb{E}\left[\left(e'/\overline{e}\right)^{-\sigma-1} | e = \overline{e} \right]$ and $\mu \equiv 1 - \frac{\pi_{\overline{e}} \overline{e}}{\Pi_{\overline{e} \leftarrow \overline{e}}}$
- Aggregate assets are $dA_t = \pi_{\overline{e}} da_t$. Simplifying the Euler $\Rightarrow$

$$dA_{t+1} - \frac{\overline{\rho} + (1+r)\tilde{\rho}}{\Pi_{\overline{e} \leftarrow \overline{e}}} dA_t + \frac{1}{\beta} dA_{t-1} = \overline{\rho}\left(1-\mu\right)\left[dZ_{t+1} - dZ_t\right]$$

- Denote by $\lambda_1 < 1 < \lambda_2$ the two roots of $X^2 - \frac{\overline{\rho}+(1+r)\tilde{\rho}}{\Pi_{\overline{e} \leftarrow \overline{e}}} X + \frac{1}{\beta} = 0$. Define $m \equiv 1 - \frac{\lambda_1}{1+r}$.
- We can then solve for assets and consumption

$$dA_t = \left(1-m\right)\left(1+r\right)dA_{t-1} + \left(1-m\right)\left(1-\mu\right)dZ_t - \left(1-\mu\right)\left[\overline{\rho} - 1 + m\right] \sum_{u=1}^{\infty} \lambda_2^{-u} dZ_{t+u}$$

$$dC_t = m(1+r)dA_{t-1} + \left(\mu + m\left(1-\mu\right)\right)dZ_t + \left(1-\mu\right)\left[\overline{\rho} - 1 + m\right] \sum_{u=1}^{\infty} \lambda_2^{-u} dZ_{t+u}$$

Zero-liquidity model (3)

- Here, special cases for intuition: **first column** and **first row**
- **First column** is purely “backward looking”: only $dZ_0 = 1$, rest 0. Then:

$$M_{0,0} = \mu + (1-\mu)\, m$$

$$M_{t,0} = (1-\mu)\, m\, ((1-m)\,(1+r))^t$$

This is a linear combination between hand to mouth with share μ and an exponentially decaying spending profile. Sanity check: $\sum(1+r)^{-t} M_{t,0} = 1$

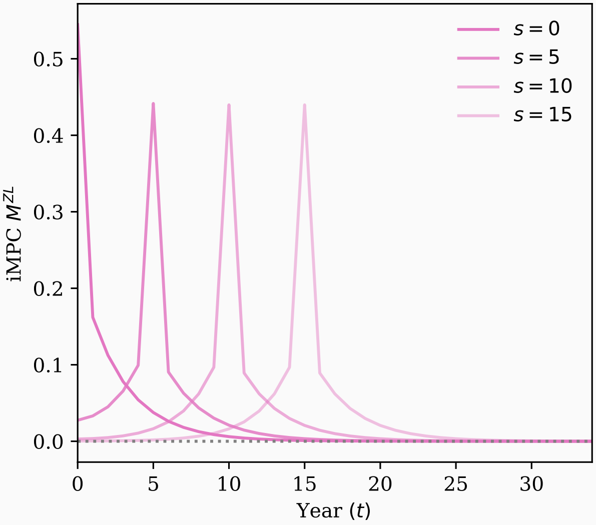
- **First row** is purely “anticipatory”:

$$M_{0,s} = (1-\mu)\,[\overline{\rho} - 1 + m]\,(\beta\,(1-m)\,(1+r))^s$$

Again exponential. Faster decay rate than first column by β.

iMPCs in ZL model

see also Bilbiie (2019)



Fiscal policy in ZL model

- Can solve above model explicitly

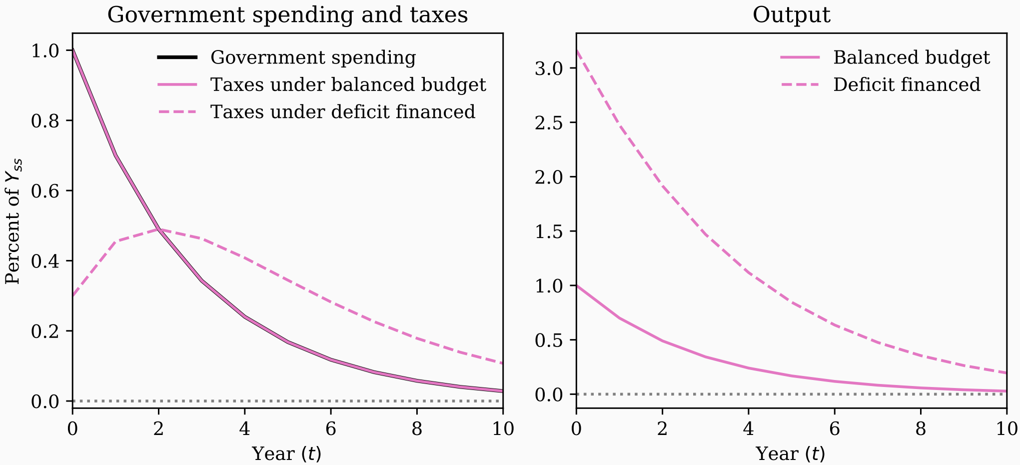
$$dY_t = \underbrace{\frac{1}{1-\mu}\left[dG_t - \mu dT_t\right]}_{\text{as in TA model}} + \underbrace{\frac{1}{1-\mu}\alpha_0 dB_t + \frac{1}{1-\mu}\alpha\sum_{k=1}^{\infty} dB_{t+k}}_{\text{new terms}}$$

$$\alpha_0 \equiv \overline{\rho}^{-1}\left[(\lambda_1+\lambda_2) - \overline{\rho} - \frac{1}{\beta}\right] > 0$$

$$\alpha \equiv \overline{\rho}^{-1}\left[(\lambda_1+\lambda_2) - 1 - \frac{1}{\beta}\right] > \alpha_0 > 0$$

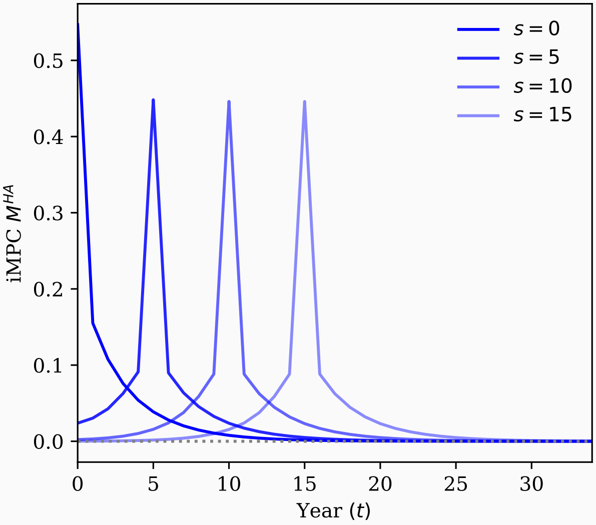
Future fiscal policy extremely powerful here, cumulative multiplier from deficit financed policy easily above 1.

Impulse response to dG shock in ZL model

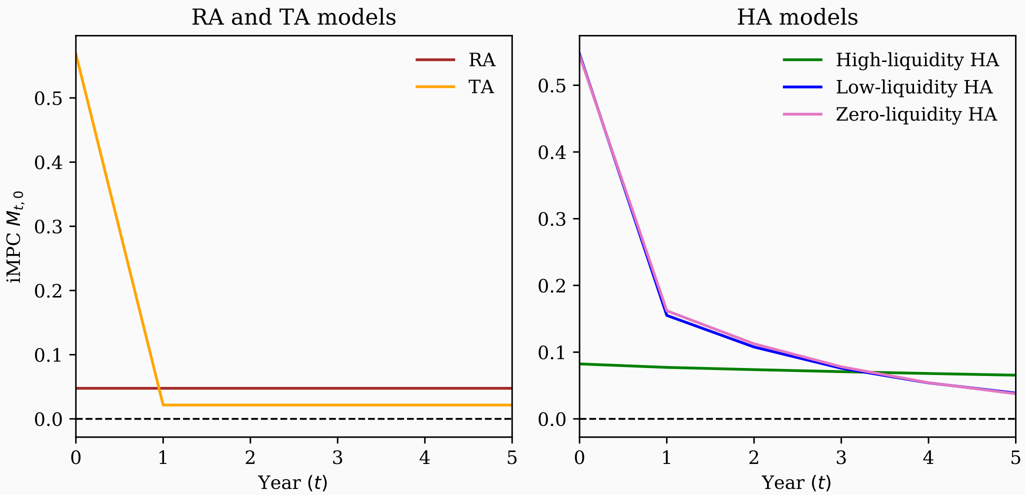


Computing iMPCs in the HA model

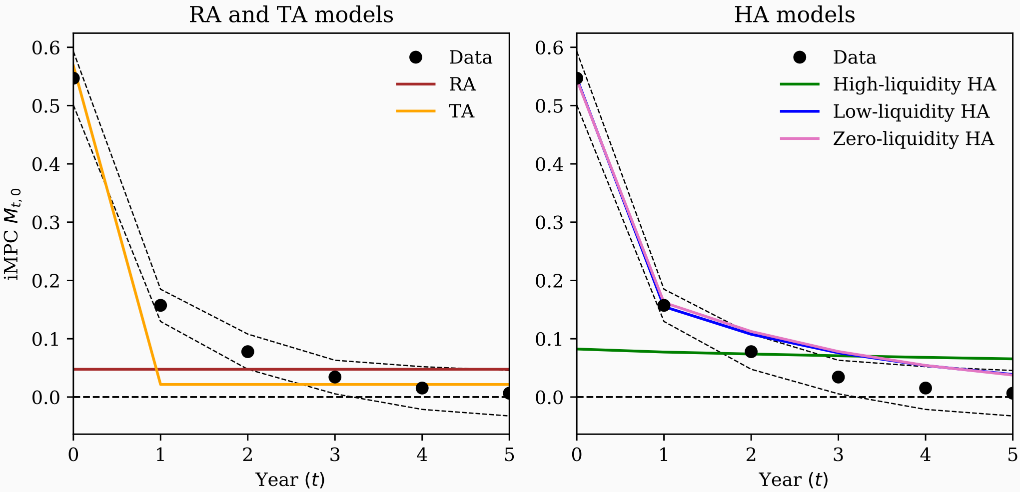
iMPCs in the HA model (computed using fake news algorithm)



Comparing iMPCs across models

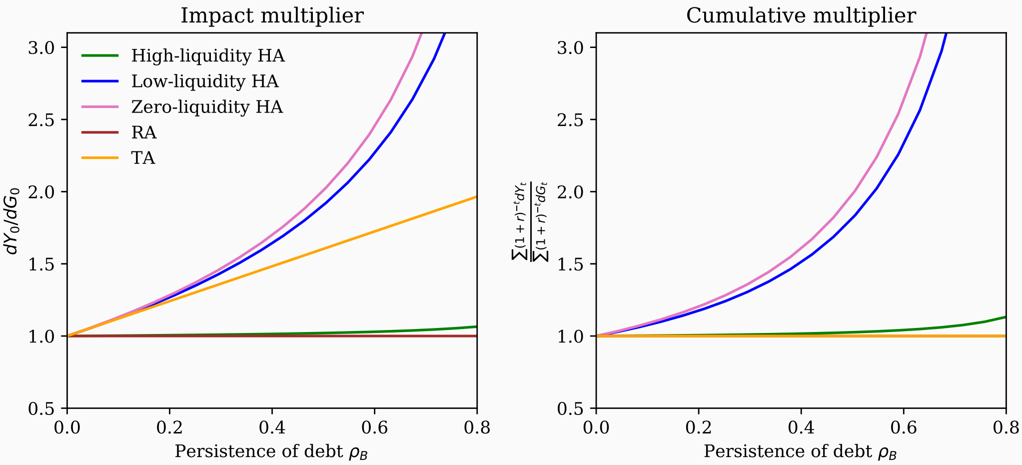


Comparison with the data

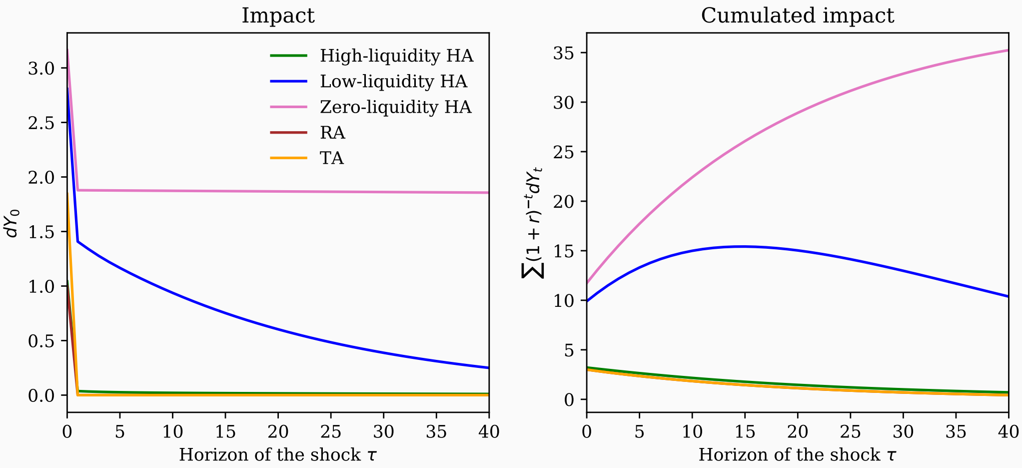


Insights about Fiscal Multipliers

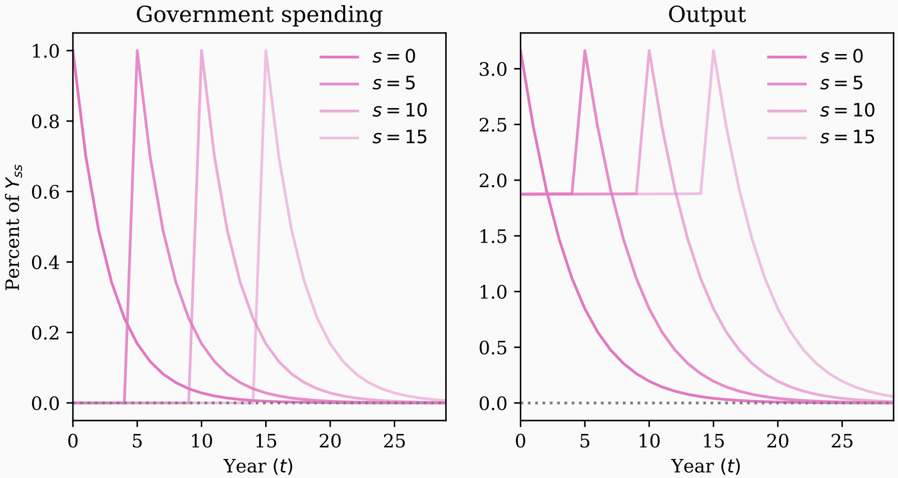
Fiscal stimulus more powerful when deficit financed



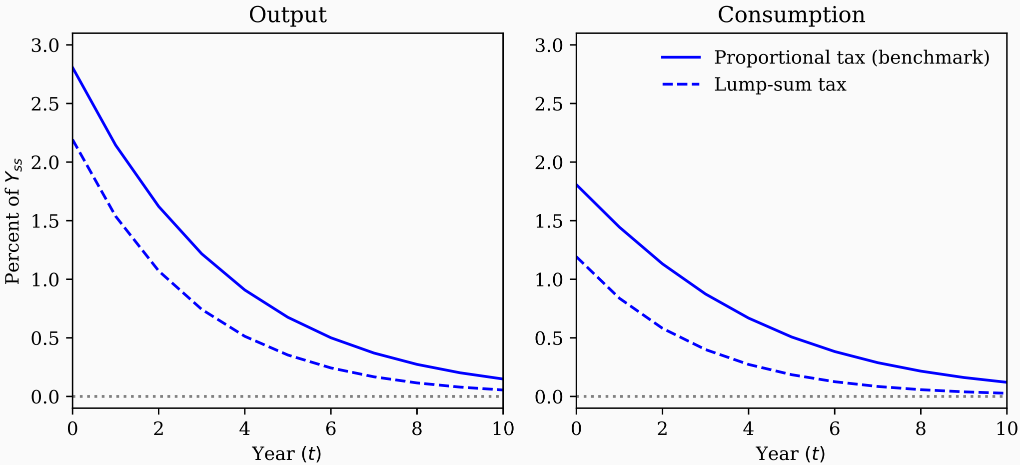
Fiscal policy is more powerful if front loaded...



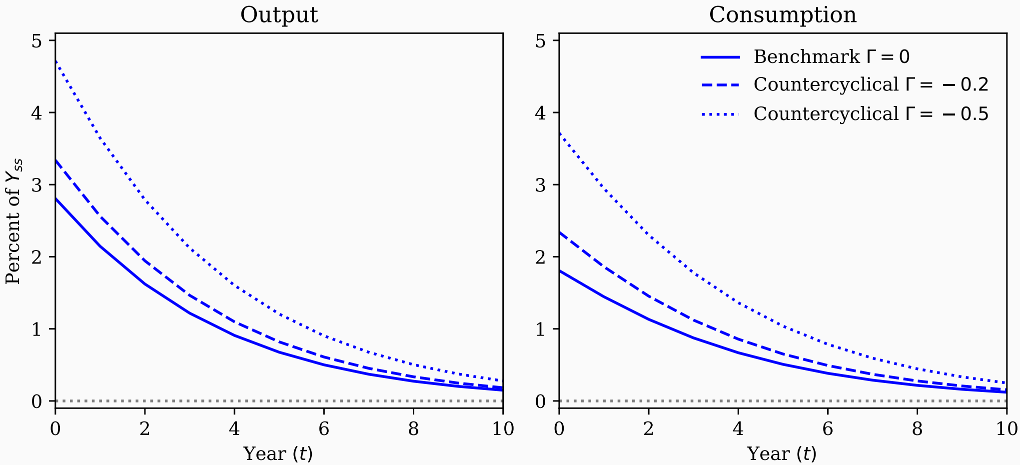
… but not in the zero-liquidity model (a fiscal policy forward guidance puzzle?)



Fiscal policy is less powerful if financed by lump-sum taxes (Why?)



Fiscal policy is more powerful if income risk is countercyclical (Why?)



Auclert-Rognlie "incidence function". More negative Γ means incomes more dispersed in recessions, Π is fixed.

Takeaway

Fiscal policy in HANK

- First exploration of shocks & policies in HANK
- One key difference already emerged: in HANK, households have very different **iMPCs**
- This matters for fiscal policy:
  - deficit financing & front loading amplifies initial and cumulative multipliers
  - not the case in RA, and not even in TA

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