## Price-Setting with Menu Costs

Matthew Rognlie (based on Auclert, Rigato, Rognlie, Straub 2022)

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## Time dependent vs. state dependent pricing

### (TD) **Time dependent**: Pr(price change) depends on time since last adjustment

• tractable, e.g. for Calvo with constant probability get Phillips curve

$$\pi_t = \kappa \widehat{mc}_t + \beta \mathbb{E}_t \pi_{t+1}$$
 (NK-PC)

- $\kappa =$  slope of the Phillips curve, rises with probability
- $\widehat{\mathit{mc}}_{t} = \mathsf{arbitrary}$  real marginal cost  $\sim$  output gap  $\to$  easy to embed in DSGE
- (SD) **State dependent**: Pr(price change) depends on a state, eg price gap  $p_{it} p_{it}^*$ 
  - ullet better micro fit (e.g. menu cost), but hard to simulate o no NK-PC!
  - simpler experiments: e.g. permanent nominal MC shocks
  - key result: "selection effect", price level more flexible than Calvo
     [Golosov-Lucas, Klenow-Kryvtsov, Nakamura-Steinsson, Midrigan, Alvarez-Lippi...]

This paper characterizes the **analogue of the NK-PC** for **menu cost models** 

## The Phillips curve for menu cost models: 3 main results

• Introduce **generalized Phillips curve** (GPC): linear map from  $\{\widehat{\mathsf{mc}}_t\}$  to  $\{\pi_t\}$ , represented as matrix **K** in the space of  $MA(\infty)$  coefficients:

$$\pi = \mathbf{K} \cdot \widehat{\mathbf{mc}}$$
 (GPC)

- here,  $\pi$ ,  $\widehat{\mathbf{mc}}$  are coefficients of MA( $\infty$ ) representation, stacked in vector
- first order + certainty equivalence  $\Rightarrow$  can think of  $\widehat{\mathbf{mc}}$  as small MIT shock
- **K** exists for any pricing model, including menu cost models
- Calvo NK-PC is a special case of GPC for some K
- (1) **Menu cost GPC** = GPC of a mixture of **two TD models** 
  - gives exact sense in which SD and TD are "the same" for small shocks
  - ullet TD's depend on steady state moments o "exact sufficient statistics" for **K**

## **Implications**

- (1) **Generalized Phillips curve** (GPC) **K** shows how to embed menu cost models in GE, with three ways to obtain **K**
- (2) For quantitative macro literature, approximate equivalence result rationalizes the Calvo New Keynesian Phillips curve with better microfoundations
- (3) For literature trying to match both micro and macro, both optimism and caution
  - **Optimism**, because micro-based menu cost models can be taken to the macro data using the generalized Phillips curve
  - Caution, because these seem so close to the Calvo model that they suffer from the same macro deficiencies, like lack of internal persistence and extreme forward-lookingness

# Pricing models and GPC

#### Canonical menu cost model

- Discrete time, quadratic approximation to firm's objective function
- Firm *i* chooses **price gap**  $x_{it} = p_{it} p_{it}^*$ :
  - log price  $p_{it}$  net of idiosyncratic optimum  $p_{it}^* = p_{it-1}^* + \epsilon_{it}$ ,  $\epsilon_{it} \sim f(\epsilon)$  iid
  - ullet if  $p_{it}$  is unchanged,  $x_{it}$  inherits random walk,  $x_{it} = x_{it-1} \epsilon_{it}$
  - static optimum:  $x_{it} = \log MC_t$ , where  $\log MC_t$  is MIT shock to nominal marginal cost

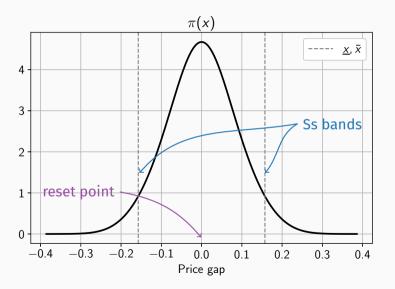
$$\min_{\{x_{it}\}} \mathbb{E}_{\mathsf{O}} \sum_{t=\mathsf{O}}^{\infty} \beta^{t} \left[ \frac{1}{2} \left( x_{it} - \log \mathsf{MC}_{\mathsf{t}} \right)^{2} + \xi_{it} \cdot \mathbf{1}_{\{x_{it} \neq x_{it-1} - \epsilon_{it}\}} \right]$$

- $\xi_{it} \in \{0, \xi\}$  iid random menu cost,  $\mathbb{P}(\xi_{it} = 0) = \lambda$ 
  - $\lambda = 0$  is Golosov-Lucas (GL),  $\lambda \in (0,1)$  is Nakamura-Steinsson (NS)
- Price index and inflation:  $\log P_t = \int x_{it} di$ ,  $\pi_t = \log P_t \log P_{t-1}$

#### Solution to menu cost model

- Optimal pricing policy consists of three objects:  $(\underline{x}_t, \overline{x}_t, x_t^*)$ 
  - $[\underline{x}_t, \overline{x}_t] = \text{Ss band, } x_t^* = \text{reset point}$
- Law of motion based on these policies:
  - $x_{it}$  follows random walk (no adjustment)
  - ... until it leaves  $[\underline{x}_t, \overline{x}_t]$  or free adjustment is drawn
  - ... then price gap jumps to  $x_t^*$
- Steady state:  $\underline{\mathbf{x}} = -\overline{\mathbf{x}}$ ,  $\mathbf{x}^* = \mathbf{MC}_{ss} = \mathbf{o}$ . Distribution:  $\pi(\mathbf{x})$  before adjustment.

## Ss bands and steady state price gap distribution



## General time dependent model

• Exogenous probability of adjusting after s periods without adjustment

[Whelan, Sheedy, Carvalho-Schwartzman, Alvarez-Borovičková-Shimer]

- Parametrize with survival function  $\Phi_s$ : Prob. that price survives for s periods
- When resetting at *t*, firm *i* solves

$$\min_{\left\{x_{it}\right\}} \mathbb{E}_{t} \sum_{s=0}^{\infty} \beta^{s} \left[ \frac{1}{2} \Phi_{s} \left( x_{it+s} - \log MC_{t+s} \right)^{2} \right]$$

- Calvo:  $\Phi_s = (1 \lambda)^s$  (constant adjustment hazard  $\lambda$ )
- Hazard rate can have any shape: increasing (e.g. Taylor model), decreasing...

## Aggregate dynamics: pass-through matrix

- Start in steady state, consider MIT shock to nominal cost  $\{MC_s\}_{s\geq 0}$
- ullet Both models boil down to functions  $\mathcal{P}_t$  such that

$$P_t = \mathcal{P}_t (\{MC_s\})$$
  $\Rightarrow$  for small shocks:  $\hat{P}_t = \sum_{s=0}^{\infty} \frac{\partial \log \mathcal{P}_t}{\partial \log MC_s} \widehat{MC}_s$ 

• Define the pass-through matrix  $\Psi$  as sequence-space Jacobian with elements  $\Psi_{t,s} \equiv \frac{\partial \log P_t}{\partial \log MC_s}$ . Then:

$$\hat{\boldsymbol{P}} = \boldsymbol{\Psi} \cdot \widehat{\boldsymbol{MC}}$$
 where  $\hat{\boldsymbol{P}} \equiv \left(\hat{P}_{o}, \hat{P}_{1}, \hat{P}_{2}, \ldots\right)'$ ,  $\widehat{\boldsymbol{MC}} \equiv \left(\widehat{MC}_{o}, \widehat{MC}_{1}, \ldots\right)'$ 

- column s = IRF of price level to small aggregate nominal cost shock at date s
- IRF to permanent shock:  $\hat{\mathbf{P}} = \mathbf{\Psi} \cdot \mathbf{1}$  [Golosov-Lucas, Alvarez-Le Bihan-Lippi, ...]
- flexible prices  $\Leftrightarrow \Psi = I$

• For TD model with survival curve  $\{\Phi_s\}$ , optimal reset point at t:

$$x_t^* = \frac{\sum_{s \ge 0} \beta^s \Phi_s \widehat{MC}_{t+s}}{\sum_{s \ge 0} \beta^s \Phi_s}$$
 (Policy equation)

• Price level: (notice the same  $\Phi_s$  appears!)

$$\hat{P}_t = \frac{\sum_{s=0}^t \Phi_s X_{t-s}^*}{\sum_{s>0} \Phi_s}$$
 (Law of motion)

Implies rank-one fake news matrix:

$$\mathbf{F}^{\Phi} \equiv \frac{1}{\left(\sum_{s \geq 0} \Phi_{s}\right) \left(\sum_{s \geq 0} \beta^{s} \Phi_{s}\right)} \begin{pmatrix} \Phi_{0} \\ \Phi_{1} \\ \Phi_{2} \\ \vdots \end{pmatrix} \begin{pmatrix} \Phi_{0} & \beta \Phi_{1} & \beta^{2} \Phi_{2} & \cdots \end{pmatrix}$$

• For TD model with survival curve  $\{\Phi_s\}$ , optimal reset point at t:

$$x_{t}^{*} = \frac{\sum_{s \geq o} \beta^{s} \Phi_{s} \widehat{MC}_{t+s}}{\sum_{s \geq o} \beta^{s} \Phi_{s}}$$
 (Policy equation)

• Price level: (notice the same  $\Phi_s$  appears!)

$$\hat{P}_t = \frac{\sum_{s=0}^t \Phi_s X_{t-s}^*}{\sum_{s>0} \Phi_s}$$
 (Law of motion)

$$\Psi^{\Phi} \equiv \frac{1}{\left(\sum_{s \geq 0} \Phi_{s}\right) \left(\sum_{s \geq 0} \beta^{s} \Phi_{s}\right)} \begin{pmatrix} \Phi_{0} & 0 & 0 & \cdots \\ \Phi_{1} & \Phi_{0} & 0 & \cdots \\ \Phi_{2} & \Phi_{1} & \Phi_{0} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \Phi_{0} & \beta \Phi_{1} & \beta^{2} \Phi_{2} & \cdots \\ 0 & \Phi_{0} & \beta \Phi_{1} & \cdots \\ 0 & 0 & \Phi_{0} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Can read off  $\{\Phi_s\}$  from IRF to permanent shock:  $(\Psi^{\Phi} \cdot \mathbf{1})_t = \sum_{s=0}^t \Phi_s / \sum_{s=0}^\infty \Phi_s$ 

- In simple GE models,  $\hat{\mathbf{P}} = \mathbf{\Psi} \cdot \mathbf{1}$  gives IRF of price level to money shock
- In std NK models, want response of  $\pi_t$  to real marginal cost  $\widehat{mc}_t = \widehat{MC}_t \hat{P}_t$
- Get  $\hat{P}_t$  via fixed point equation

$$\hat{\mathbf{P}} = \mathbf{\Psi} \cdot \left(\widehat{\mathbf{mc}} + \hat{\mathbf{P}}\right)$$

solution

$$\hat{\mathbf{P}} = \left(\sum_{k=1}^{\infty} \mathbf{\Psi}^{k}\right) \cdot \widehat{\mathbf{mc}} = (\mathbf{I} - \mathbf{\Psi})^{-1} \mathbf{\Psi} \cdot \widehat{\mathbf{mc}}$$

• Get inflation  $\pi_t$  using lag matrix **L**. Find **Generalized Phillips Curve (GPC) K** 

$$\pi = (\mathbf{I} - \mathbf{L})(\mathbf{I} - \mathbf{\Psi})^{-1}\mathbf{\Psi} \cdot \widehat{\mathbf{mc}} \equiv \mathbf{K} \cdot \widehat{\mathbf{mc}}$$

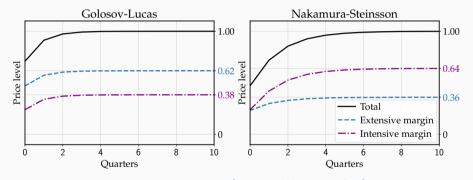
• Models with the same  $\Psi$  also have the same K.

# Exact equivalence: Menu cost $model = 2 \times TD$

## Are menu cost and TD models exactly the same?



• Permanent nominal shock:  $(\underline{x}, \overline{x}, x^*)$  all shift up by 1 (infinitesimal unit)



- Split up into only shift in **Ss bands ("extensive margin")**  $\rightarrow \{\Phi_t^e\}$
- ullet ... and only shift in reset point ("intensive margin")  $o \left\{ \Phi_t^i 
  ight\}$
- ullet Let lpha be the long-run price level in the extensive margin experiment

## Equivalence result

• Our first result shows that  $\Phi^e$  and  $\Phi^i$  are "structural": we can use them to obtain the impulse response to **any other shock** 

#### Proposition

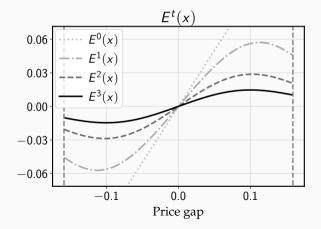
The pass-through matrix  $\Psi$  of the canonical menu cost model with any  $\lambda, \xi$  and any symmetric f is the weighted average of the two TD pass-through matrices

$$\Psi = \alpha \Psi^{\Phi^e} + (1 - \alpha) \Psi^{\Phi^i}$$

- Menu cost model = 2  $\times$  TD model. Also: Menu cost GPC = GPC of 2  $\times$  TD
- Next: Proof idea + what  $\Phi^e$  and  $\Phi^i$  look like



- Key objects in the proof: expected price gaps
- $E^{t}(x) \equiv \mathbb{E}[x_{t}|x_{0} = x]$  is the expected price gap in t periods starting from any x



## Why does the extensive margin behave like a TD model?

- Start from  $\log P_t = \mathbb{E}\left[x_{it}\right]$
- Consider a shock that only affects  $\underline{x}_0, \overline{x}_0$ . What is its effect on price at t?

$$\log P_{t} = \int_{\underline{X}_{0}}^{\overline{X}_{0}} E^{t}(x) \pi(x) dx + \underbrace{\left(1 - \int_{\underline{X}_{0}}^{\overline{X}_{0}} \pi(x)\right)}_{\text{freq}} \underbrace{E^{t}(0)}_{\text{o}}$$

Given **steady state policies**, transition dynamics are governed by  $E^{t}(x)$ 

[Alvarez-Le Bihan-Lippi, Alvarez-Lippi]

• For a small shock, using symmetry

$$d \log P_t = \pi \left( \overline{x} \right) \left( d\underline{x}_{\mathsf{O}} + d\overline{x}_{\mathsf{O}} \right) E^t \left( \overline{x} \right)$$

• With many changes at dates t - s, get law of motion:

$$d \log P_t = \pi(\bar{x}) \sum_{s>0} E^s(\bar{x}) \cdot (d\underline{x}_{t-s} + d\bar{x}_{t-s})$$

## Extensive margin policies and summary



- How are  $d\bar{x}_t$ ,  $d\underline{x}_t$  optimally determined? (Policy equation?)
- Using envelope theorem, can show:

$$d\underline{x}_{t} = d\overline{x}_{t} = \frac{\sum_{s \geq 0} \beta^{s} E^{s}(\overline{x}) \cdot \widehat{MC}_{t+s}}{\sum_{u \geq 0} \beta^{u} E^{u}(\overline{x})}$$

The same "virtual survival rate" matters as for l.o.m., just with extra  $\beta$ 

• Use to rewrite law of motion as

$$d\log P_{t} = 2\pi(\bar{x}) \sum_{s \geq 0} E^{s}(\bar{x}) \frac{\sum_{s \geq 0} E^{s}(\bar{x}) \cdot d\bar{x}_{t}}{\sum_{s \geq 0} E^{s}(\bar{x})}$$

• Extensive margin acts like a TD model, scaled by  $\alpha$ , with  $\Phi_t^e \equiv E^t(\overline{x})/\overline{x}$ .

## Why does the intensive margin behave like a TD model?

- Intensive margin is similar. Consider first shock that only affects  $x_0^*$ .
- Mass equal to fraction freq of prices adjusts to  $dx_0^*$  rather than o at t = 0
- Raises price level by  $E^t(o + dx_o^*) E^t(o) = (E^t)'(o) dx_o^*$  and so

$$d \log P_t = \operatorname{freq} \cdot \left( E^t \right)'(o) \, dx_o^*$$

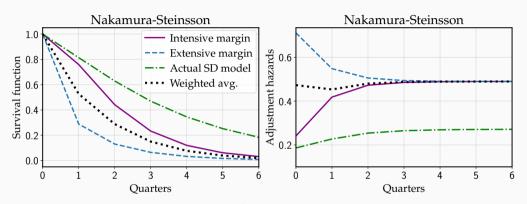
• With many changes at dates  $s \le t$ , get TD law of motion

$$d \log P_{t} = \text{freq} \cdot \sum_{s \geq 0} (E^{s})'(0) dx_{t-s}^{*} = (1 - \alpha) \frac{\sum_{s \geq 0} (E^{s})'(0) \cdot dx_{t-s}^{*}}{\sum_{s \geq 0} (E^{s})'(0)}$$

Meanwhile, envelope theorem shows policy is

$$dx_{t}^{*} = \frac{\sum_{s \geq 0} \beta^{s}(E^{s})'(0) \cdot \widehat{MC}_{t+s}}{\sum_{u \geq 0} \beta^{u}(E^{u})'(0)}$$

• Intensive margin acts like a TD model, scaled down by  $(1 - \alpha)$ ,  $\Phi_t^i \equiv (E^t)'(0)$ .



- "virtual" survival functions  $\Phi_t^e$ ,  $\Phi_t^i$  + implied hazards  $\neq$  actual ones! The difference is the "selection effect"
- Average survival function  $\alpha \Phi_t^e + (1 \alpha) \Phi_t^i$  is close to exponential in practice

Numerical equivalence: Menu cost model pprox Calvo

- Ultimately interested in the menu cost GPC  $\mathbf{K} = (\mathbf{I} \mathbf{L}) (\mathbf{I} \mathbf{\Psi})^{-1} \mathbf{\Psi}$
- To compare, consider Calvo NK-PC:

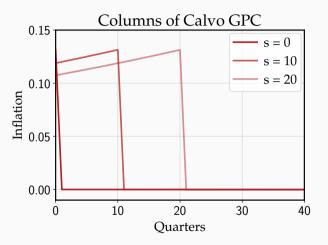
$$\pi_{\mathsf{t}} = \kappa \widehat{m\mathsf{c}}_{\mathsf{t}} + \beta \mathbb{E}_{\mathsf{t}} \pi_{\mathsf{t+1}} = \sum_{\mathsf{s}=\mathsf{o}}^{\infty} \kappa \beta^{\mathsf{s}} \mathbb{E}_{\mathsf{t}} \widehat{m\mathsf{c}}_{\mathsf{t+s}}$$

which gives the GPC

$$\mathbf{K}^{Calvo}(\kappa) = \left(\frac{\partial \pi_t}{\partial \widehat{mc}_{t+s}}\right)_{t,s} = \begin{pmatrix} \kappa & \kappa\beta & \kappa\beta^2 & \cdots \\ 0 & \kappa & \kappa\beta & \cdots \\ 0 & 0 & \kappa & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

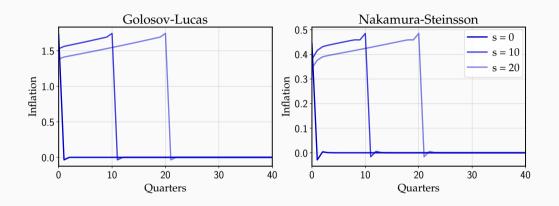
ightarrow inflation is purely & strongly forward looking, no "intrinsic" persistence

## Visualizing GPC for Calvo model



• **Q**: how "far" are our menu cost models from a simple Calvo in practice?

#### GPC in our two calibrated menu cost models



• Menu cost GPCs "look" very similar to Calvo with different slope parameters!

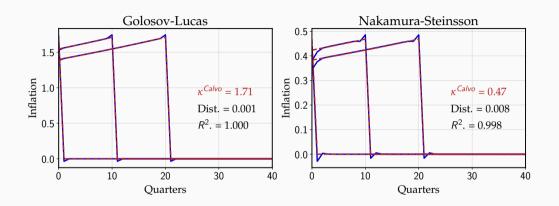
## Finding closest-distance Calvo model

• Let's look for  $\kappa$  that minimizes

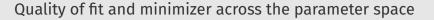
$$\mathsf{dist} = \min_{\kappa} \| \| \mathbf{K} - \mathbf{K}^{\mathsf{Calvo}} \left( \kappa \right) \|_{2} / \| \| \mathbf{K} \|_{2}$$

- if  $\mathbf{K} = \mathbf{K}^{Calvo}(\tilde{\kappa})$ , then dist  $= (\tilde{\kappa} \kappa)/\tilde{\kappa}$
- Recall that two models that share the exact same K also share the same:
  - pass-through matrix Ψ
  - IRF to any shock to MC or mc
  - IRF to any fundamental shock once integrated in a broader macro model

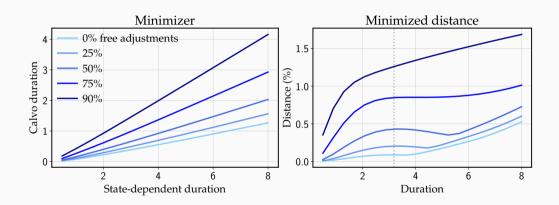
(so, they are also indistinguishable in estimation based on macro data)



[Reported  $R^2$  from predicting  $\pi_t$  with  $\kappa \widehat{mc}_t + \beta \mathbb{E}_t \pi_{t+1}$  on **K** simulated data]

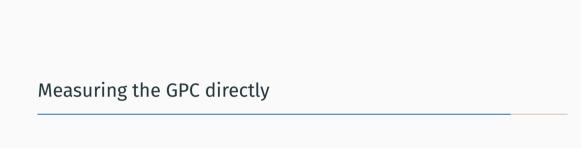






#### Extensions

- Strategic complementarities  $\rightarrow$  •
- Steady state inflation  $\rightarrow$   $\square$
- Infrequent shocks  $\rightarrow$  •
- Multi-product models  $\rightarrow$  •
- Multi-sector models →
- Large shocks  $\rightarrow$   $\square$



## Measuring the GPC exactly using $E^{t}(x)$

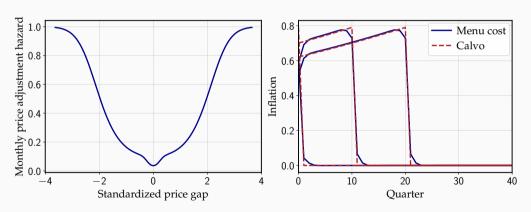
- For **K**, we can measure  $E^t(x)$  in the data.
- One option: use data on price changes alone + model law of motion
- To do this, first enrich model to allow for general cdf  $\xi_{it} \sim G(\cdot)$ 
  - $\rightarrow$  leads to a generalized state-dependent adjustment hazard  $\Lambda(x)$ [Caballero-Engel, Alvarez-Lippi-Oskolkov, Karadi-Schoenle-Wursten]
- $\Lambda(x)$ ,  $\pi(x)$ ,  $\sigma_{\epsilon}$  can all be backed out from data on price changes
  - $\rightarrow$  recover expected price gaps  $E^{t}(x)$  from this
- Plug into generalized decomposition

$$\Psi = \alpha \int \frac{\Lambda'(x)\pi(x)G(x)}{\int \Lambda'(\tilde{x})\pi(\tilde{x})G(\tilde{x})d\tilde{x}} \cdot \Psi^{\Phi^{e}(x)}dx + (1-\alpha) \cdot \Psi^{\Phi^{i}}$$

where  $\Phi_t^e(x) = E^t(x)/x$  and  $\Phi_t^i = (E^t)'(0)$  similar to before  $G(x) \equiv \sum_t E^t(x)$  26

## Fitted hazard function $\Lambda(x)$ and (GPC)

• Apply this to Israeli price change distribution [Bonomo-Carvalho-Kryvtsov-Ribon-Rigato]





#### Conclusion

• Calvo:

$$\pi_{\mathsf{t}} = \kappa^{\mathsf{Calvo}} \widehat{\mathsf{mc}}_{\mathsf{t}} + \beta \mathbb{E}_{\mathsf{t}} \pi_{\mathsf{t+1}}$$

• Menu cost:

$$\pi_t = \sum_{s>o} \mathbf{K}_{t,s} \cdot \widehat{mc}_s \approx \kappa \widehat{mc}_t + \beta \mathbb{E}_t \pi_{t+1}, \qquad \kappa > \kappa^{\text{Calvo}}$$

- Sequence-space Jacobians Ψ and K give new insights!
- ightarrow Menu cost models suffer from similar shortcomings as Calvo....
  - ... more work needed to get model that matches micro prices and macro inflation

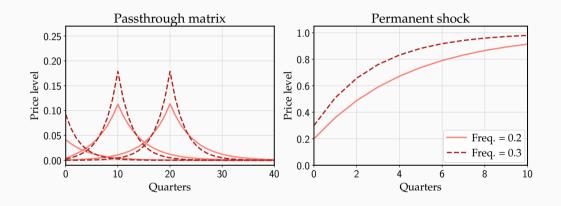


#### Calibration of random menu cost model

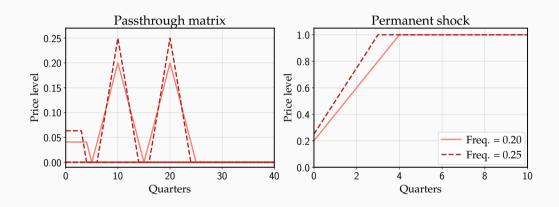


- For calibration, assume idiosyncratic shock distribution is  $\phi \sim \mathcal{N}\left(\mathsf{O}, \sigma\right)$
- Given  $\lambda$ ; calibrate  $\xi$ ,  $\sigma$  to match:
- Average frequency of price change of 23.9% quarterly ("freq")
  - Median price adjustment of 8.5%
     [regular price changes for median sector in US CPI, see Nakamura-Steinsson]
- Two benchmarks:  $\lambda = 0$  (GL) and  $\lambda = 0.75 \cdot \text{freq (NS)}$
- Notes:
  - only two effective parameters are  $\lambda/\text{freq}$  and  $\xi/\sigma^2$ ,  $\xi$  then determines scale
  - for convenience, we reparameterize by  $\lambda/\text{freq}$  and freq (or duration=1/freq)

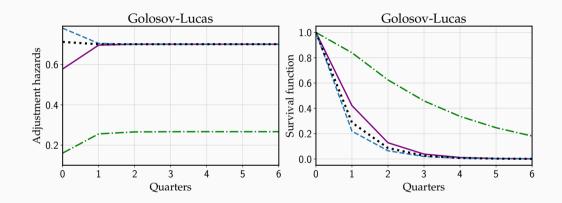












# Alvarez-Lippi-Souganidis



- Another use of  $\Psi$ : permanent cost shock but strategic complementarities
- ullet As in Alvarez-Lippi-Souganidis (2022): parameterize by heta
  - from either Kimball demand or I-O with common input
- Get  $\hat{P}_t$  via fixed point equation

$$\hat{\mathbf{P}} = \mathbf{\Psi} \cdot \left( \mathbf{1} + \theta \hat{\mathbf{P}} \right)$$

solution

$$\hat{\mathbf{P}} = \left(\sum_{k=0}^{\infty} (\theta \mathbf{\Psi})^k\right) \cdot \mathbf{\Psi} \mathbf{1} = (\mathbf{I} - \theta \mathbf{\Psi})^{-1} \cdot \hat{\mathbf{P}}_0$$

where  $\hat{\textbf{P}}_{\text{O}}$  is response without strategic complementarities

- ullet ALS use self-adjointness of  $\Psi$  to write with eigenvalues-eigenfunctions
- When  $\theta =$  1, we get the GPC K

#### Gertler-Leahy



• Gertler and Leahy (2008 JPE) assume the mixture distribution

$$\phi = (\mathbf{1} - \eta) \cdot \mathbf{0} + \eta \cdot \mathcal{U}[-\mathbf{M}, \mathbf{M}]$$

where M is large

• This implies

$$E^{t}\left(x\right)=\left(1-\eta\right)^{t}x$$

SO

$$\Phi_t^e = \frac{E^t(\overline{X})}{\overline{X}} = (1 - \eta)^t$$
  $\Phi_t^i = (E^t)'(0) = (1 - \eta)^t$ 

so pass-through matrix  $\Psi$  is a Calvo with reset frequency 1  $-\eta$ 

# Response of Ss bands



- Reason for shock at s affecting date o, then sum across s and shift
- Start with upper Ss band. Value matching implies

$$V_{o}\left(\overline{X_{o}}\right) = V_{o}\left(X_{o}^{*}\right) + \xi$$

Differentiate and use  $V'(o) = dV_o(o) = o$ 

$$dV_{0}\left(\overline{x}\right)+V'\left(\overline{x}\right)d\overline{x_{t}}=0$$

Next, envelope theorem implies

$$V'(x) = \sum_{t} \beta^{t} E^{t}(x)$$

$$dV_{o}(x) = -\beta^{s} E^{s}(x) dM \hat{C}_{s}$$

Conclude that

$$d\overline{x_0} = \frac{\beta^s E^s(\overline{x})}{\sum_{u} \beta^u E^u(\overline{x})} dM \hat{C}_s$$

## Response of reset points



• For reset point, FOC is

$$V_{o}^{\prime}\left( x_{o}^{st}\right) =\mathsf{o}$$

Differentiate

$$dV_{o}^{\prime}\left(o\right)+V^{\prime\prime}\left(o\right)dx_{o}^{*}=o$$

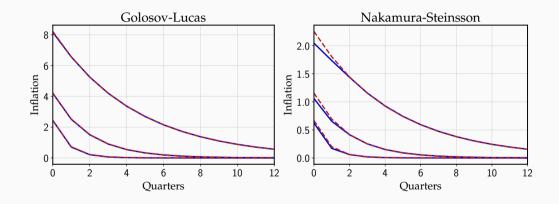
• Envelope theorem again

$$V''(x) = \sum_{t} \beta^{t} (E^{t})'(x)$$
$$dV'_{0}(x) = -\beta^{s} (E^{s})'(x) dM \hat{C}_{s}$$

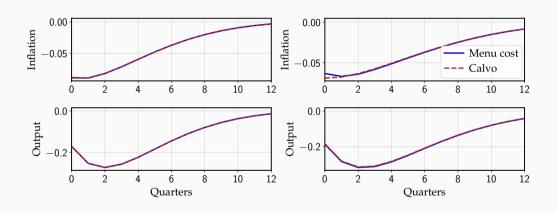
Conclude that

$$dx_{o}^{*} = \frac{\beta^{s} (E^{s})'(o)}{\sum_{u} \beta^{u} (E^{u})'(o)} d\hat{MC}_{s}$$

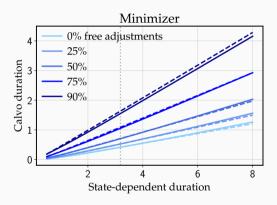




#### Menu costs in a Smets-Wouters model (Back)







### What determines $\kappa$ ? A sufficient statistic approach



• Implementing with  $\beta = 0.99$ , find  $\kappa$  to be: • performance vs model

$$Freq (\Delta p) \begin{vmatrix} Kur (\Delta p) \\ 2 & 3 & 4 \\ 0.2 & 0.40 & 0.17 & 0.09 \\ 0.3 & 1.02 & 0.40 & 0.22 \\ 0.4 & 2.26 & 0.77 & 0.40 \end{vmatrix}$$

- For reference:
  - In data, quarterly Freq ( $\Delta p$ )  $\simeq$  0.2 to 0.3 (model = 0.24)
  - In data:  $Kur(\Delta p)$  between 3 and 4

[Alvarez-Le Bihan-Lippi, Bonomo-Carvalho-Kryvtsov-Ribon-Rigato]

- In models:  $Kur(\Delta p)$  is 1.3 for GL, 2.3 for NS, 2 for Midrigan
- Contrast to recent macro full-sample IV estimate of  $\kappa = 0.0031!$

### Strategic complementarities



- Standard resolution to adjust size: **strategic complementarities**.
- These work very well with GPCs. Suppose now:

$$p_{it}^{*\text{compl}} = \zeta p_{it}^* + (1 - \zeta) \log P_t$$

- $\zeta \in (0,1)$  implies firms like to set price close to aggregate price level
- ullet can microfound in GE with intermediate input share 1  $-\zeta$

#### **Proposition**

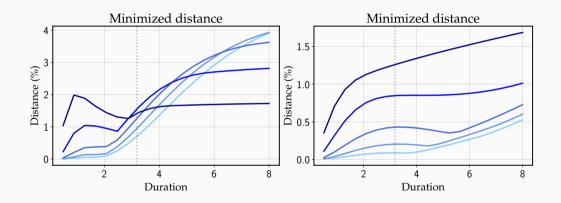
Generalized Phillips Curve scales with  $\zeta$ :

$$\mathbf{K}^{compl} = \zeta \mathbf{K}$$

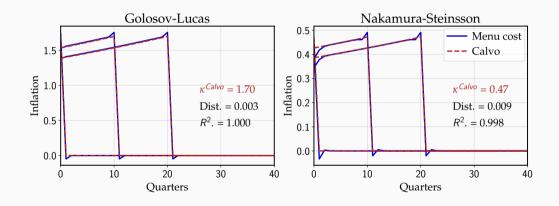
• Note **shape** of Phillips curve is unchanged by  $\zeta$ , e.g. no more persistence

## Arbitrary parameters

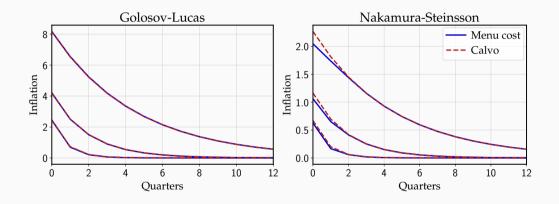




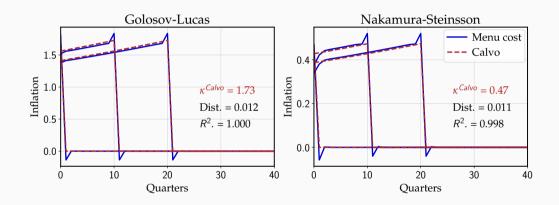




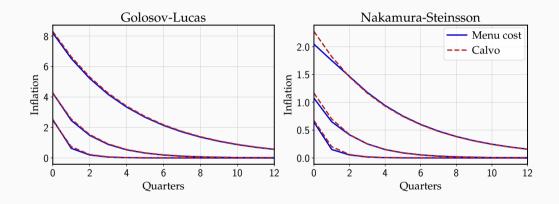
# Steady state inflation of 2% - Impulse responses





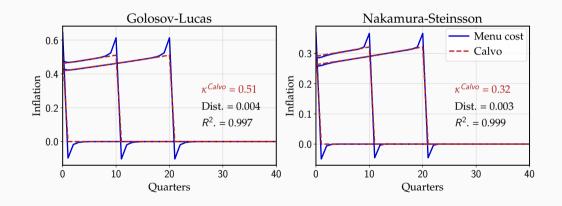


## Steady state inflation of 5% - Impulse responses

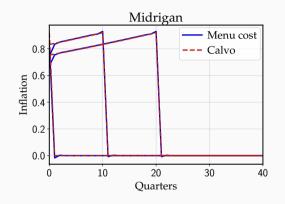


### Infrequent shocks



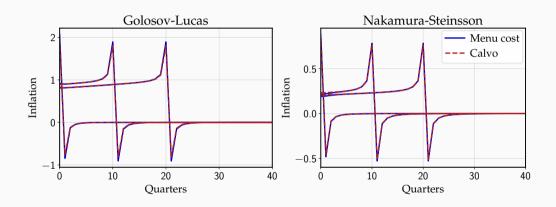






• Midrigan model: 2 products.





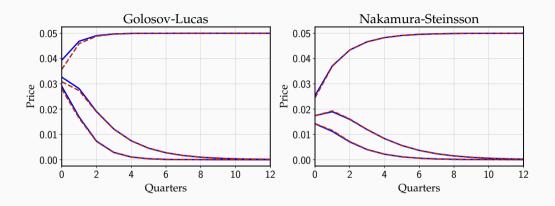
### Multi-sector models



Sectors	Golosov-Lucas		Nakamura-Steinsson		Sectors	Golosov-Lucas		Nakamura-Steinsson	
	Real Norm	$\kappa^{\sf Calvo}$	Real Norm	$\kappa^{\it Calvo}$		Real Norm	$\kappa$ Calvo	Real Norm	$\kappa^{\it Calvo}$
Vehicle fuel, used cars	-	-	-	-	Services (2)	0.001	1.60	0.010	0.44
Utilities	0.212	618.8	0.003	98.82	Hh. furnishings	0.002	0.97	0.010	0.26
Travel	0.071	294.6	0.001	44.13	Services (3)	0.002	0.89	0.010	0.23
Unprocessed food	0.002	23.24	0.003	5.19	Recreation goods	0.002	0.86	0.010	0.23
Transp. goods	0.001	13.31	0.004	3.27	Services (4)	0.003	0.56	0.010	0.15
Services (1)	0.001	14.07	0.004	3.42	Apparel	0.007	0.31	0.012	0.08
Processed food, other	0.001	3.23	0.009	0.90	Services (5)	0.011	0.20	0.015	0.05

# Large nominal cost shock and the price level





• 5% shock size with persistence  $\in$  {0.3, 0.6, 1}.