Synthetic Controls: Methods and Practice

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SI 2021 Methods Lecture - Causal Inference Using Synthetic Controls and the Regression Discontinuity Design

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Introduction

- Synthetic control methods were originally proposed in Abadie and Gardeazabal (2003) and Abadie et al. (2010) with the aim to estimate the effects of aggregate interventions.
- Many events or interventions of interest naturally happen at an aggregate level affecting a small number of large units (such as cities, regions, or countries).
- Even in experimental settings micro-interventions may not be feasible (e.g., fairness) or effective (e.g., interference).

In this talk, I will use the terms "event", "intervention", and "treatment" interchangeably.

Applications

- Synthetic controls have been applied to study the effects of right-to-carry laws (Donohue et al., 2017), legalized prostitution (Cunningham and Shah, 2018), immigration policy (Bohn et al., 2014), corporate political connections (Acemoglu et al., 2016) and many other policy issues.
- They have also been adopted as the main tool for data analysis across different sides of the issues in recent prominent debates on the effects of immigration (Borjas, 2017; Peri and Yasenov, 2017) and minimum wages (Allegretto et al., 2017; Jardim et al., 2017; Neumark and Wascher, 2017; Reich et al., 2017).
- Synthetic controls are also applied outside economics in the social sciences, biomedical disciplines, engineering, etc. (see, e.g., Heersink et al., 2017; Pieters et al., 2017).

Applications

- Outside academia, synthetic controls have found considerable coverage in the popular press (see, e.g., Guo, 2015; Douglas, 2018) and have been widely adopted by multilateral organizations, think tanks, business analytics units, governmental agencies, and consulting firms.
- For example, the synthetic control method plays a prominent role in the official evaluation of the effects of the massive Bill & Melinda Gates Foundation's *Intensive Partnerships for Effective Teaching* program (Gutierrez et al., 2016).

The Washington Post

Wonkblog

Seriously, here's one amazing math trick to learn what can't be known

THE WALL STREET JOURNAL

REAL TIME ECONOMICS | ECONOMICS

How an Analysis of Basque Terrorism Helps Economists Understand Brexit

A method pioneered by an MIT professor has also been used to estimate the economic effect of a tobacco ban, German reunification, legalization of prostitution and gun rights

Plan for the talk

1. A primer on synthetic control estimation
2. Why use synthetic controls?
3. A penalized synthetic control estimator
4. Synthetic controls for experimental design
5. Closing remarks

Literature is large, and there is much I will not cover ...

- **Matrix/tensor completion:** Amjad, Shah, and Shen, (2018), Agarwal, Shah and Shen (2020), Athey, Bayati, Doudchenko, Imbens, and Khosravi (2018), Bai and Ng (2020)
- **Bias correction:** Abadie and L'Hour (2020), Arkhangelsky, Athey, Hirshberg, Imbens, and Wager, (2019), Ben-Michael, Feller, and Rothstein (2020)
- **Inference:** Cattaneo, Feng, Titiunik (2020), Chernozhukov, Wüthrich, and Zhu (2019a, 2019b), Firpo and Possebom (2018)
- **Functional and distributional outcomes:** Chernozhukov, Wüthrich, and Zhu (2019c), Gunsilius (2020)
- **Large-T:** Botosaru and Ferman (2019), Ferman (2019), Li (2020)
- **Other related methods:** Brodersen, Gallusser, Koehler, Remy, and Scott (2015)

... and many more (and many, many, empirical applications).

A primer on synthetic control estimation

- When the units of analysis are a few aggregate entities, a combination of comparison units (a "synthetic control") often does a better job reproducing the characteristics of a treated unit than any single comparison unit alone.
- The comparison unit in the synthetic control method is selected as the weighted average of all potential comparison units that best resembles the characteristics of the treated unit(s).

A primer on synthetic control estimation

- Suppose that we observe $J + 1$ units in periods $1, 2, \ldots, T$.
- Unit "one" is exposed to the intervention of interest (that is, "treated") during periods $T_0 + 1, \ldots, T$.
- The remaining J units are an untreated reservoir of potential controls (a "donor pool").
- Let Y_{it}^I be the outcome that would be observed for unit i at time t if unit i is exposed to the intervention in periods $T_0 + 1$ to T.
- Let Y_{it}^N be the outcome that would be observed for unit i at time t in the absence of the intervention.
- We aim to estimate the effect of the intervention on the treated unit,

$$\tau_{1t} = Y_{1t}^I - Y_{1t}^N = Y_{1t} - Y_{1t}^N$$

for $t > T_0$, and Y_{1t} is the outcome for unit one at time t.

A primer on synthetic control estimation

- Let $\boldsymbol{W} = (w_2, \ldots, w_{J+1})'$ with $w_j \geq 0$ for $j = 2, \ldots, J+1$ and $w_2 + \cdots + w_{J+1} = 1$. Each value of $\boldsymbol{W}$ represents a potential synthetic control.
- Let $\boldsymbol{X}_1$ be a $(k \times 1)$ vector of pre-intervention characteristics for the treated unit. Similarly, let $\boldsymbol{X}_0$ be a $(k \times J)$ matrix which contains the same variables for the unaffected units.
- The vector $\boldsymbol{W}^* = (w_2^*, \ldots, w_{J+1}^*)'$ is chosen to minimize $\|\boldsymbol{X}_1 - \boldsymbol{X}_0 \boldsymbol{W}\|$, subject to our weight constraints.
- Let Y_{jt} be the value of the outcome for unit j at time t. For a post-intervention period t (with $t \geq T_0$) the synthetic control estimator is:

$$\widehat{\tau}_{1t} = Y_{1t} - \sum_{j=2}^{J+1} w_j^* Y_{jt}.$$

A primer on synthetic control estimation

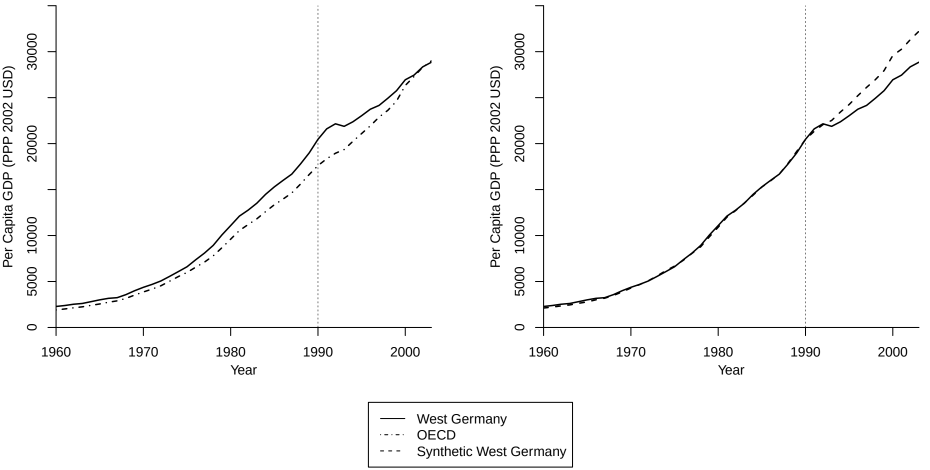
- Typically,

$$\|\boldsymbol{X}_1 - \boldsymbol{X}_0 \boldsymbol{W}\| = \left(\sum_{h=1}^{k} v_h \left(X_{h1} - w_2 X_{h2} - \cdots - w_{J+1} X_{hJ+1} \right)^2 \right)^{1/2}$$

- The positive constants $v_1, \ldots, v_k$ reflect the predictive power of each of the k predictors on Y_{1t}^N.
- $v_1, \ldots, v_k$ can be chosen by the analyst or by data-driven methods.

A primer on synthetic control estimation

Application: German reunification



A primer on synthetic control estimation

Application: German reunification

	West Germany (1)	Synthetic West Germany (2)	OECD Sample (3)
GDP per-capita	15808.9	15802.24	13669.4
Trade openness	56.8	56.9	59.8
Inflation rate	2.6	3.5	7.6
Industry share	34.5	34.5	34.0
Schooling	55.5	55.2	38.7
Investment rate	27.0	27.0	25.9

Note: First column reports $\boldsymbol{X}_1$, second column reports $\boldsymbol{X}_0\boldsymbol{W}^*$, and last column reports a simple average for the 16 OECD countries in the donor pool. GDP per capita, inflation rate, and trade openness are averages for 1981–1990. Industry share (of value added) is the average for 1981–1989. Schooling is the average for 1980 and 1985. Investment rate is averaged over 1980–1984.

A primer on synthetic control estimation

Application: German reunification

country j	W_j^*	country j	W_j^*
Australia	0	Netherlands	0.10
Austria	0.42	New Zealand	0
Belgium	0	Norway	0
Denmark	0	Portugal	0
France	0	Spain	0
Greece	0	Switzerland	0.11
Italy	0	United Kingdom	0
Japan	0.16	United States	0.22

A primer on synthetic control estimation

- Abadie et al. (2010) establish a bias bound under the factor model

$$Y_{it}^{N} = \boldsymbol{\theta}_t \boldsymbol{Z}_i + \boldsymbol{\lambda}_t \boldsymbol{\mu}_i + \varepsilon_{it},$$

  where $\boldsymbol{Z}_i$ are observed features, $\boldsymbol{\mu}_i$ are unobserved features, and ε_{it} is a unit-level transitory shock, modeled as random noise.

- Suppose that we can choose $\boldsymbol{W}^*$ such that:

$$\sum_{j=2}^{J+1} w_j^* \boldsymbol{Z}_j = \boldsymbol{Z}_1, \ \sum_{j=2}^{J+1} w_j^* Y_{j1} = Y_{11}, \ \cdots \ , \ \sum_{j=2}^{J+1} w_j^* Y_{jT_0} = Y_{1T_0}.$$

  In practice, these may hold only approximately.

A primer on synthetic control estimation

Suppose that $E|\varepsilon_{jt}|^p < \infty$ for some $p > 2$. Then,

$$\boxed{|E[\widehat{\tau}_{1t} - \tau_{1t}]| < C(p)^{1/p} \left(\frac{\bar{\lambda}^2 F}{\underline{\xi}}\right) J^{1/p} \max\left\{ \frac{\bar{m}_p^{1/p}}{T_0^{1-1/p}}, \frac{\bar{\sigma}}{T_0^{1/2}} \right\}}$$

where F is the number of unobserved factors,

$$\sigma_{jt}^2 = E|\varepsilon_{jt}|^2, \quad \sigma_j^2 = \frac{1}{T_0}\sum_{t=1}^{T_0} \sigma_{jt}^2, \quad \bar{\sigma}^2 = \max_{j=2,\dots,J+1} \sigma_j^2,$$

$$m_{pjt} = E|\varepsilon_{jt}|^p, \quad m_{pj} = \frac{1}{T_0}\sum_{t=1}^{T_0} m_{pjt}, \quad \bar{m}_p = \max_{j=2,\dots,J+1} m_{pj},$$

for p even, $|\lambda_{tf}| \leq \bar{\lambda}$ for all $t = 1, \dots, T$ and $f = 1, \dots, F$, and

$$\underline{\xi} \leq \xi(M) = \text{smallest eigenvalue of } \frac{1}{M} \sum_{t=T_0-M+1}^{T_0} \lambda_t' \lambda_t.$$

A primer on synthetic control estimation

- The bias bound is predicated on close fit, and controlled by the ratio between the scale of ε_{it} and T_0.
- In particular, the credibility of a synthetic control depends on the extent to which it is able to fit the trajectory of Y_{1t} for an extended pre-intervention period.

A primer on synthetic control estimation

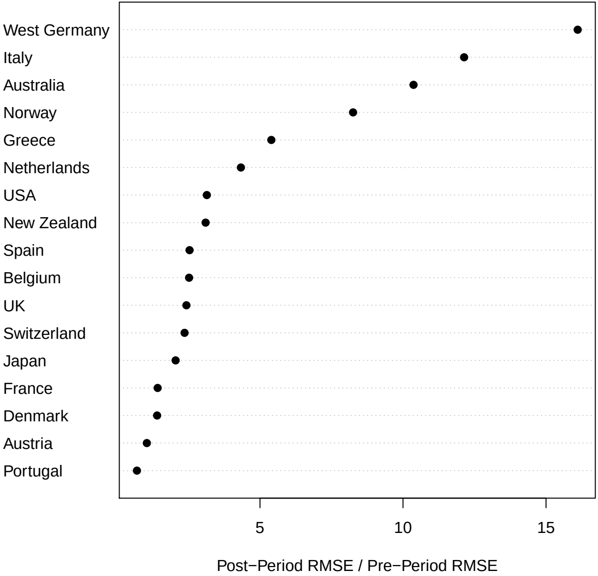
- There are no ex-ante guarantees on the fit. If the fit is poor, Abadie et al. (2010) recommend against the use of synthetic controls.
- Settings with small T_0, large J, and large noise create substantial risk of overfitting.
- To reduce interpolation biases and risk of overfitting, restrict the donor pool to units that are similar to the treated unit.

A primer on synthetic control estimation

- Abadie et al. (2010) propose a mode of inference for the synthetic control framework that is based on permutation methods.
- A permutation distribution can be obtained by iteratively reassigning the treatment to the units in the donor pool and estimating "placebo effects" in each iteration.
- The effect of the treatment on the unit affected by the intervention is deemed to be significant when its magnitude is extreme relative to the permutation distribution.

A primer on synthetic control estimation

Application: German reunification

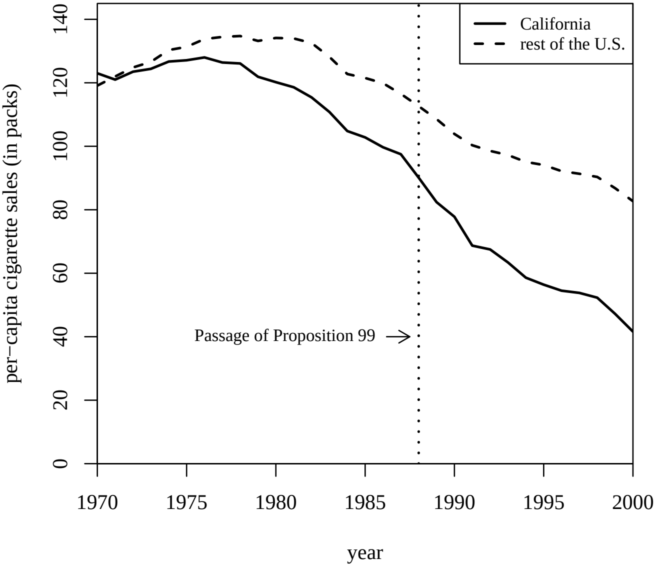


A primer on synthetic control estimation

- The permutation distribution is more informative than mechanically looking at p-values alone.
- Depending on the number of units in the donor pool, conventional significance levels may be unrealistic or impossible.
- Often, one sided inference is most relevant.

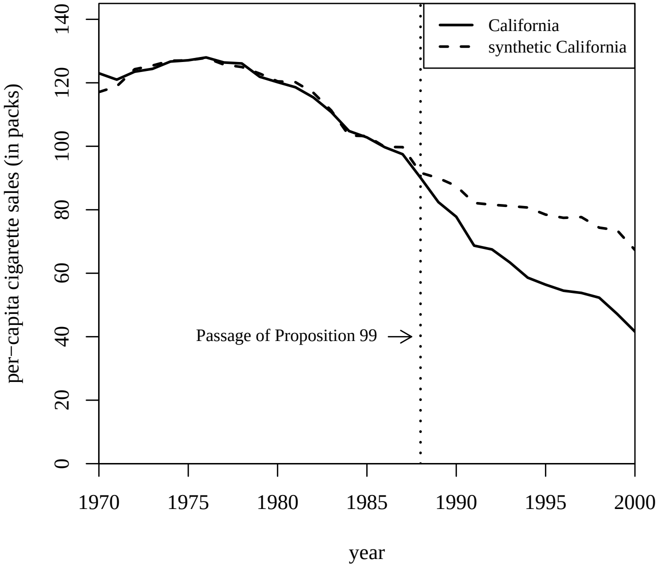
A primer on synthetic control estimation

Application: California tobacco control program



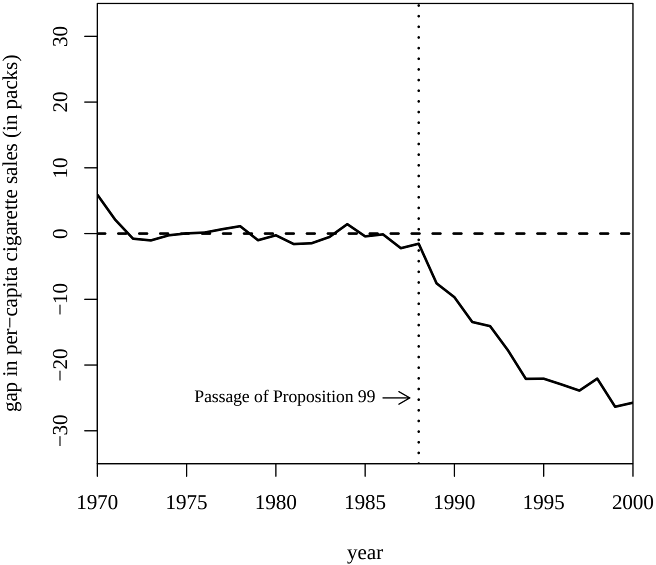
A primer on synthetic control estimation

Application: California tobacco control program



A primer on synthetic control estimation

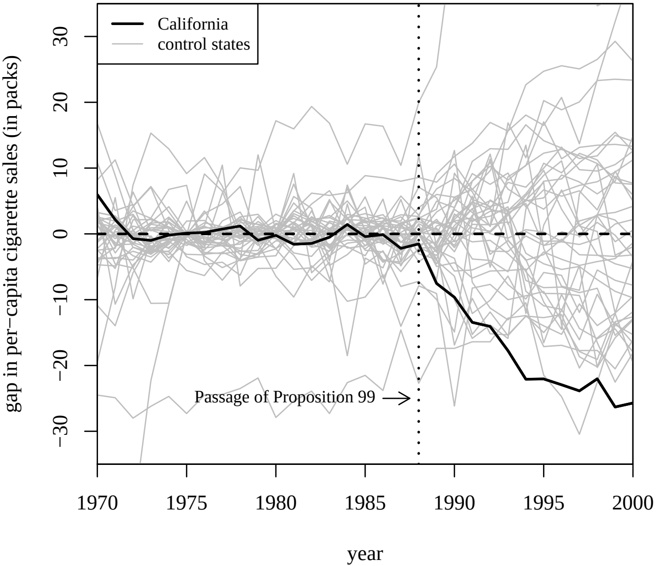
Application: California tobacco control program



A primer on synthetic control estimation

Application: California tobacco control program

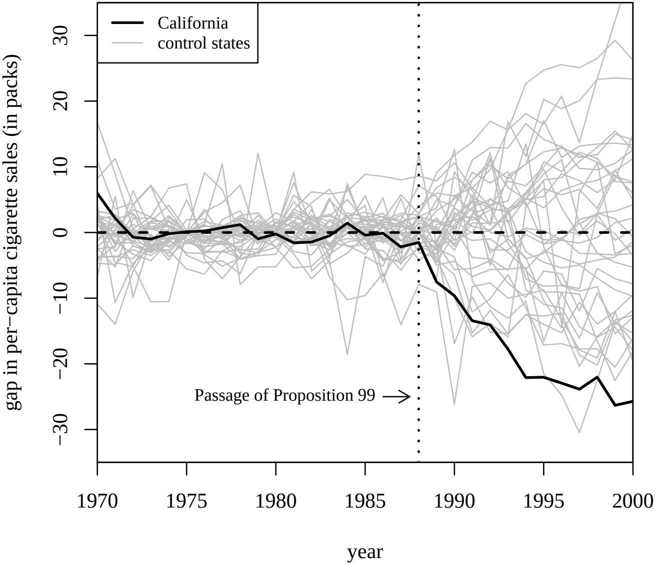
(All States in Donor Pool)



A primer on synthetic control estimation

Application: California tobacco control program

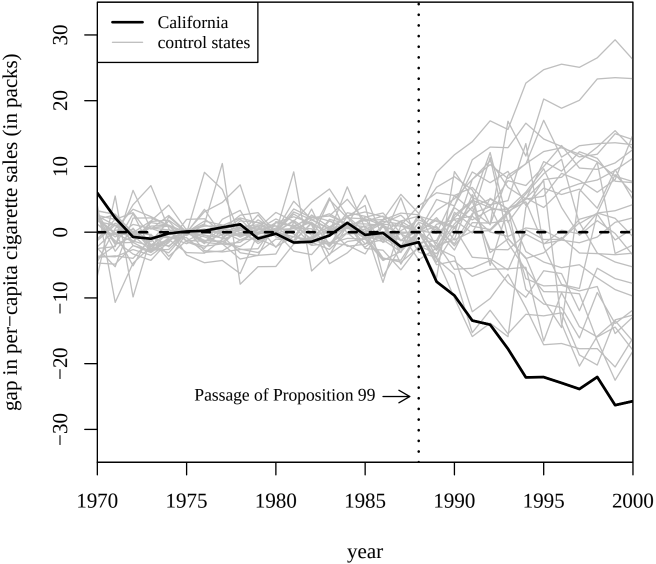
(Pre-Prop. 99 MSPE $\leq$ 20 Times Pre-Prop. 99 MSPE for CA)



A primer on synthetic control estimation

Application: California tobacco control program

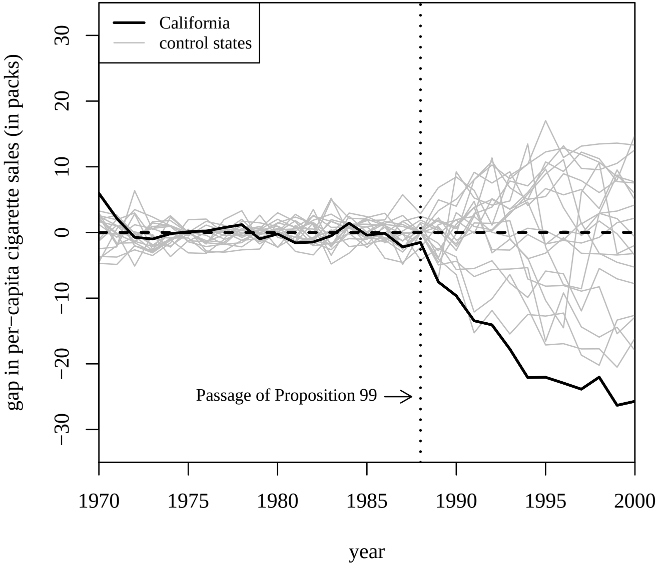
(Pre-Prop. 99 MSPE $\leq$ 5 Times Pre-Prop. 99 MSPE for CA)



A primer on synthetic control estimation

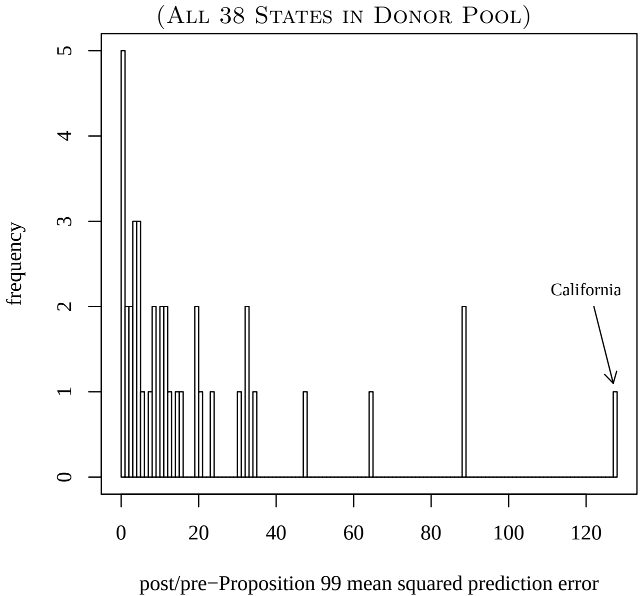
Application: California tobacco control program

(PRE-PROP. 99 MSPE $\leq$ 2 TIMES PRE-PROP. 99 MSPE FOR CA)



A primer on synthetic control estimation

Application: California tobacco control program



A primer on synthetic control estimation

- The availability of a well-defined procedure to select the comparison unit makes the estimation of the effects of placebo interventions feasible.
- The permutation method we just described does not attempt to approximate the sampling distributions of test statistics.
- Sampling-based inference is often complicated in a synthetic control setting, sometimes because of the absence of a well-defined sampling mechanism and sometimes because the sample is the same as the population.

A primer on synthetic control estimation

- This mode of inference reduces to classical randomization inference (Fisher, 1935) when the intervention is randomly assigned, a rather improbable setting.
- More generally, this mode of inference evaluates significance relative to a benchmark distribution for the assignment process, one that is implemented directly in the data.

The uniform benchmark is often employed in practice, but departures from uniformity are possible (see, Firpo and Possebom, 2018).

Why use synthetic controls?

- Compare to linear regression. Let:
  - $\boldsymbol{Y}_0$ be the $(T - T_0) \times J$ matrix of post-intervention outcomes for the units in the donor pool.
  - $\overline{\boldsymbol{X}}_1$ and $\overline{\boldsymbol{X}}_0$ be the result of augmenting $\boldsymbol{X}_1$ and $\boldsymbol{X}_0$ with a row of ones.
  - $\widehat{\boldsymbol{B}} = (\overline{\boldsymbol{X}}_0 \overline{\boldsymbol{X}}_0')^{-1} \overline{\boldsymbol{X}}_0 \boldsymbol{Y}_0'$ collects the coefficients of the regression of $\boldsymbol{Y}_0$ on $\overline{\boldsymbol{X}}_0$.
- $\widehat{\boldsymbol{B}}' \overline{\boldsymbol{X}}_1$ is a regression-based estimator of the counterfactual outcome for the treated unit without the treatment.
- Notice that $\widehat{\boldsymbol{B}}' \overline{\boldsymbol{X}}_1 = \boldsymbol{Y}_0 \boldsymbol{W}^{reg}$, with

$$\boldsymbol{W}^{reg} = \overline{\boldsymbol{X}}_0' (\overline{\boldsymbol{X}}_0 \overline{\boldsymbol{X}}_0')^{-1} \overline{\boldsymbol{X}}_1.$$

- The components of $\boldsymbol{W}^{reg}$ sum to one, but may be outside $[0, 1]$, allowing extrapolation, and will not be sparse.

Why use synthetic controls?

Application: German reunification

country j	W_j^{reg}	country j	W_j^{reg}
Australia	0.12	Netherlands	0.14
Austria	0.26	New Zealand	0.12
Belgium	0.00	Norway	0.04
Denmark	0.08	Portugal	-0.08
France	0.04	Spain	-0.01
Greece	-0.09	Switzerland	0.05
Italy	-0.05	United Kingdom	0.06
Japan	0.19	United States	0.13

Why use synthetic controls?

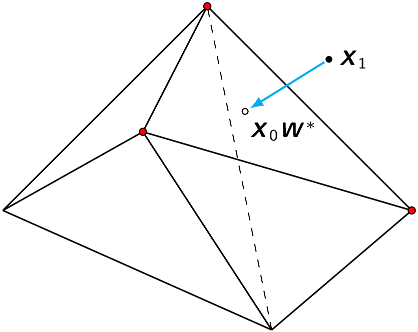
- **No extrapolation.** Synthetic control estimators preclude extrapolation outside the support of the data.
- **Transparency of the fit.** Linear regression uses extrapolation to obtain $\boldsymbol{X}_0 \boldsymbol{W}^{reg} = \boldsymbol{X}_1$, even when the untreated units are completely dissimilar in their characteristics to the treated unit. In contrast, synthetic controls make transparent the actual discrepancy between the treated unit and the convex hull of the units in the donor pool, $\boldsymbol{X}_1 - \boldsymbol{X}_0 \boldsymbol{W}^*$.
- **Safeguard against specification searches.** Synthetic controls do not require access to post-treatment outcomes in the design phase of the study, when synthetic control weights are calculated. Therefore, all design decisions can be made without knowing how they affect the conclusions of the study.

Why use synthetic controls?

- **Safeguard against specification searches (cont.)** Synthetic control weights can be calculated and pre-registered before the post-treatment outcomes are realized, or before the actual intervention takes place, providing a safeguard against specification searches and p-hacking.
- **Transparency of the counterfactual.** Synthetic controls make explicit the contribution of each comparison unit to the counterfactual of interest.
- **Sparsity.** Because the synthetic control coefficients are proper weights and are sparse, they allow a precise interpretation of the nature of the estimate of the counterfactual of interest (and of potential biases).

Why use synthetic controls?

Sparsity: Geometric interpretation



- If $\boldsymbol{X}_1$ does not belong to the convex hull of the columns of $\boldsymbol{X}_0$, the synthetic control $\boldsymbol{X}_0 \boldsymbol{W}^*$ is unique and sparse.
- If $\boldsymbol{X}_1$ belongs to the convex hull of the columns of $\boldsymbol{X}_0$, the synthetic control $\boldsymbol{X}_0 \boldsymbol{W}^*$ may not be unique and candidate $\boldsymbol{W}^*$'s may not be sparse, although sparse solutions always exist (by Carathéodory's theorem).

A penalized synthetic control estimator

Penalized synthetic control (Abadie and L'Hour, 2020): $\boldsymbol{W}^*(\lambda)$ solves

$$\min_{\boldsymbol{W}} \left\| \boldsymbol{X}_1 - \sum_{j=2}^{J+1} W_j \boldsymbol{X}_j \right\|^2 + \lambda \sum_{j=2}^{J+1} W_j \left\| \boldsymbol{X}_1 - \boldsymbol{X}_j \right\|^2$$

$$\text{s.t.} \quad W_j \geq 0, \quad \sum_{j=2}^{J+1} W_j = 1.$$

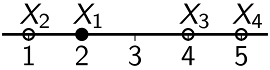
- $\lambda > 0$ controls the trade-off between fitting well the treated and minimizing the sum of pairwise distances to selected control units.
- $\lambda \to 0$: **pure synthetic control** *i.e.* synthetic control that minimizes the pairwise matching discrepancies among all solutions for the unpenalized estimator.
- $\lambda \to \infty$: **nearest neighbor** matching.

A penalized synthetic control estimator

Advantages of the penalized estimator:

1. For any $\lambda > 0$, **solution is unique and sparse** provided that untreated observations are in general position.
2. The presence of the penalization term reduces the **interpolation bias** that occurs when averaging units that are far away from each other.
3. Same **computational complexity** as the unpenalized estimator.

A penalized synthetic control estimator



- $X_1 = 2$ and $X_0 = [1\ 4\ 5]$.
- The (unpenalized) synthetic control has two sparse solutions: $W_1^* = (2/3,\ 1/3,\ 0)$ and $W_1^{**} = (3/4,\ 0,\ 1/4)$.
- W_1^* dominates W_1^{**} in terms of matching discrepancy. Infinite number of non-sparse solutions from convex combinations of these two.
- However, when $\lambda > 0$, the penalized synthetic control has a unique solution:

$$W_1^*(\lambda) = \begin{cases} (2 + \lambda/2,\ 1 - \lambda/2,\ 0)/3 & \text{if } 0 < \lambda \leq 2, \\ (1,\ 0,\ 0), & \text{if } \lambda > 2. \end{cases}$$

- As $\lambda \to 0$, $W_1^*(\lambda) \to W_1^*$, the pure synthetic control. The penalized synthetic control never uses the "bad" match X_4.

Synthetic controls for experimental design

- Suppose a ridesharing company wants to assess the impact of a new incentive pay program for drivers.
- To do so, the new incentive pay treatment will be applied as a pilot program in one market/city or in a few markets/cities.
- Which market or markets should they treat?
- Which market or markets should they use as a comparison/control?
- This is a setting where randomization of treatment may create defective designs where:
  - The treated market/markets are non-representative of the entire set of markets of interest.
  - Treated and control markets are very different in their characteristics.

Synthetic controls for experimental design

- In these contexts, Abadie and Zhao (2021) propose:
  - Find a first set of weights that make a synthetic treated unit reproduce the features of the population of units of interest.
  - Create a synthetic control for the synthetic treated unit.
- In contrast to the observational case, in the experimental settings we have two synthetic control units: one treated and one untreated.
- Related ongoing work by Doudchenko and co-authors. Synthetic controls are widely used as experimental designs by business analytics units (e.g., Uber).

Closing remarks

- Synthetic controls provide many practical advantages for the estimation of the effects of policy interventions and other events of interest.
- Like for any other statistical procedure (and especially for those aiming to estimate causal effects), the credibility of the results depends crucially on the level of diligence exerted in the application of the method and on whether contextual and data requirements are met in the empirical application at hand (see Abadie, *JEL* 2021).

Closing remarks

- Some open areas of research: sampling-based inference, external validity, sensitivity to model restrictions, estimation with multiple interventions, data driven selectors of v_h, mediation analysis ...
- An area of recent heightened interest regarding the use of synthetic controls is the design of experimental interventions.
- Results on robust and efficient computation of synthetic controls are scarce, and more research is needed on the computational aspects of this methodology.
- On the empirical side, many of the events and the policy interventions economists care about take place at an aggregate level, affecting entire aggregate units.

Resources

The material in this presentation comes from:

- Abadie, A. and J. Gardeazabal. 2003. "The Economic Costs of Conflict: A Case Study of the Basque Country." American Economic Review, 93(1): 112-132.
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Code: `synth` (Matlab, Stata and R)

`http://web.stanford.edu/~jhain/synthpage.html`

`pensynth` (R)

`https://github.com/jeremylhour/pensynth`

Resources

While this talk has mostly focused on my work, many have contributed to the literature on synthetic control estimators and related methods. Some references:

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Resources

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Resources

- Doudchenko, N. and G. W. Imbens. 2016. "Balancing, Regression, Difference-in-Differences and Synthetic Control Methods: A Synthesis." arXiv:1610.07748.
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- Samartsidis, P., S. R. Seaman, A. M. Presanis, M. Hickman, and D. De Angelis. 2019. "Assessing the Causal Effect of Binary Interventions from Observational Panel Data with Few Treated Units. Statistical Science, forthcoming.

Thank you!

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