Comments on "The constraint on public debt when $r < g$ but $g < m$" by Ricardo Reis

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February 26 2021

Important!

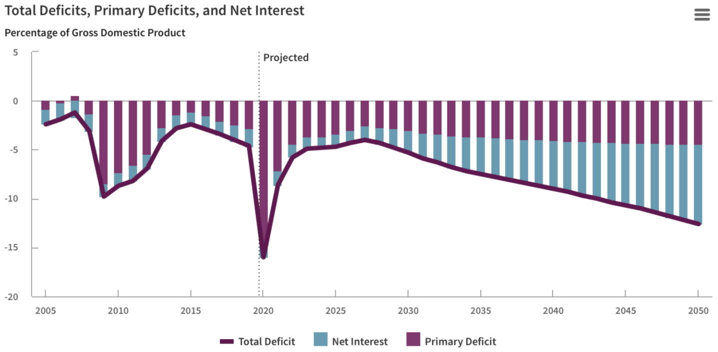
- The single most important macro question of our time.

$$\frac{d}{dt}\left(\frac{b_t}{y_t}\right) = (r_t - g_t)\frac{b_t}{y_t} - \frac{s_t}{y_t}.$$

- $r < g$: Raise debt (with $s < 0$) then roll over ($s = 0$), b/y gently declines? "Fiscal expansion" with "no fiscal cost?"
- Washington understands logic better than economists!
- Obviously not.
- Theory wall between $r < g$ manna and $r > g$ austerity? 0.01%?
- Obviously not.
- Conventional response: 1) r will rise. An upper bound on b/y. Crowding out? Liquidity premium?
- 2) 50-100 years of large b/y threatens doom loop. What if there is another crisis, a WWII budget need *starting* at $b/y = 200\%$?
- Today: $r < g$ *is irrelevant to US fiscal policy issues.*

Radical objection – perpetual deficits with steady b/y?

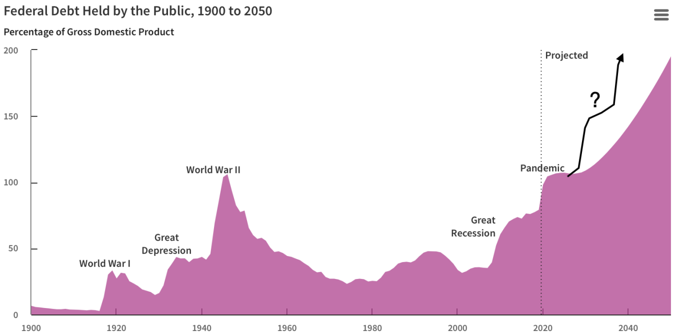
The whole $r - g$ debate is irrelevant to current US fiscal policy issues.



- $r < g$ by 1% and $b/y = 100\%$ allow $s/y = -1\%$.
- not $s/y = -5\%$ in good times, $s/y = -25\%$ in 1/10 year crisis, and then entitlements, *and then* "one-time" expansion.

Radical objection – one time expansion and grow out?

The whole $r - g$ debate is irrelevant to current US fiscal policy issues.



- $r < g$ by 1% allows "one-time b/y expansion" *and then* $s = 0$.
- $s = 0$ would be big austerity / conservative dream!
- $r < g$ by 1% does not allow exponentially growing b/y!

Discontinuity at $r = g$?

$r = g$ divides bond vigilantes from garden of eden? Look at flows.

- $r - g = +0.01\%$ with $b/y = 100\%$ means $s/y = 0.01\% =$ \$2 billion.
- $r - g = -0.01\%$ means $s/y = -0.01\% =$ -\$2 billion.
- *This* transition is clearly continuous.

Look at growing out of "one time" expansion

- $r - g = -0.01\%$, means b/y=150% resolves with $s = 0$ back to $b/y = 50\%$ in $11,000$ years.
- $r - g = +0.01\%$ means b/y=150% grows to 450% in $11,000$ years, on the way to ∞.
- "Wealth effect" in transversality condition, is likely the same.

Lessons

- Economic meaning of solving integrals forward vs. backward should be continuous.
- Economically sensible reading: Small $r < g$ is not discontinuously different from small $r > g$.

R, g and present values

Summary: $r < g$ in perfect foresight modeling

- $r < g \approx 1\%$ shifts the *average* surplus to a slight perpetual deficit $s/y \approx -1\%$, while it lasts.
- Any substantial *variation* in deficits about that average must be met by a substantial period of above average surpluses, to bring back debt to GDP in a reasonable time.
- Like seigniorage.
- A quantitative question. $r < g$ of 10% would be different.
- *The $r < g$ debate is irrelevant to current US fiscal policy issues.*

Liquidity, uncertainty, paper?

- Liquidity, uncertainty: Many r to choose from.
- Do *not* measure r from a world with liquidity and uncertainty, and use it in perfect foresight modeling!
- Two examples: $r =$ rate of return on government debt $< g$, but present values converge and no fiscal opportunity if done right.

Liquidity value of government debt

- Example: All debt is money. $r = -\pi < g$.
- Steady state can finance small deficit.

$$(\pi + g)\frac{M}{Py} = -\frac{s}{y}$$

  but big deficits need to be repaid by later surpluses.
- PV? Discount with r^f, i.e. with $e^{-\delta t}u'(c_t)$:

$$\frac{M_t}{P_t y_t} = \int_{\tau=t}^{T} e^{-\left(r^f - g\right)(\tau - t)} \left(\frac{s_\tau}{y_\tau} + i_\tau \frac{M_\tau}{P_\tau y_\tau}\right) d\tau + e^{-(r^f - g)(T-t)} \frac{M_T}{P_T y_T}$$

  Debt = PV of surpluses, including seignorage. Terminal value converges. Can fund $s < 0$. Big $s < 0$ need to be repaid by $s > 0$.
- Discount with gov't debt return $r = -\pi$:

$$\frac{M_t}{P_t y_t} = \int_{\tau=t}^{T} e^{(\pi+g)(\tau-t)} \left(\frac{s_\tau}{y_\tau}\right) d\tau + e^{(\pi+g)(T-t)} \frac{M_T}{P_T y_T}.$$

  Explosive "bubble," negative PV.
- Same $M/(Py) < \infty$. Which is *useful*? "Mine bubble"? See s limit?

The technical problem

- You can discount one-period payoffs with ex-post returns.

$$1 = E_t \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} R_{t+1} \right) = E_t \left(R_{t+1}^{-1} R_{t+1} \right).$$

- You cannot always discount infinite payoffs with ex-post returns.

$$p_t = E_t \sum_{j=1}^{T} \frac{\beta^j u'(c_{t+j})}{u'(c_t)} d_{t+j} + E_t \frac{\beta^j u'(c_{t+j})}{u'(c_t)} p_{t+T}$$

each term converges, yet

$$p_t = E_t \sum_{j=1}^{T} \prod_{k=1}^{j} \frac{1}{R_{t+k}} d_{t+j} + E_t \prod_{k=1}^{T} \frac{1}{R_{t+k}} p_{t+T}$$

terms *can* explode in opposite directions.

Bohn's (1995) example – uncertainty

- $\frac{c_{t+1}}{c_t} \sim$ iid. $\frac{1}{1+r^f} = E\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\right]$. $r^f < g$ is possible.
- Government keeps a constant debt/GDP. Borrows c_t, repays $(1+r^f)c_t$ at time $t+1$. $b_t = c_t$. See as present value?
- $s_t = (1+r^f)c_{t-1} - c_t$. Discounting with marginal utility,

$$b_t = E_t \sum_{j=1}^{T} \beta^j \left(\frac{c_{t+j}}{c_t}\right)^{-\gamma} s_{t+j} + E_t \beta^T \left(\frac{c_{t+T}}{c_t}\right)^{-\gamma} c_{t+T}$$

$$= E_t \sum_{j=1}^{T} \beta^j \left(\frac{c_{t+j}}{c_t}\right)^{-\gamma} \left[(1+r^f)c_{t+j-1} - c_{t+j}\right] + E_t \beta^T \left(\frac{c_{t+T}}{c_t}\right)^{-\gamma} c_{t+T}$$

$$b_t = \left[c_t - E_t \beta^T \left(\frac{c_{t+T}}{c_t}\right)^{-\gamma} c_{t+T}\right] + E_t \beta^T \left(\frac{c_{t+T}}{c_t}\right)^{-\gamma} c_{t+T}$$

Bohn's (1995) example

- Discounting with gov't bond return $= r^f$,

$$b_t = \sum_{j=1}^{T} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j} + \left(\prod_{k=1}^{T} \frac{1}{R_{t+k}} \right) c_{t+T} =$$

$$= \sum_{j=1}^{T} \frac{(1+r^f)c_{t+j-1} - c_{t+j}}{(1+r^f)^j} + \frac{1}{(1+r^f)^T} c_{t+T}$$

$$b_t = \left(c_t - \frac{c_{t+T}}{(1+r^f)^T} \right) + \frac{c_{t+T}}{(1+r^f)^T}.$$

Taking expected value,

$$b_t = c_t \left(1 - \frac{(1+g)^T}{(1+r^f)^T} \right) + c_t \frac{(1+g)^T}{(1+r^f)^T}$$

Bohn's (1995) example

- Discounting with marginal utility $c_t^{-\gamma}$,

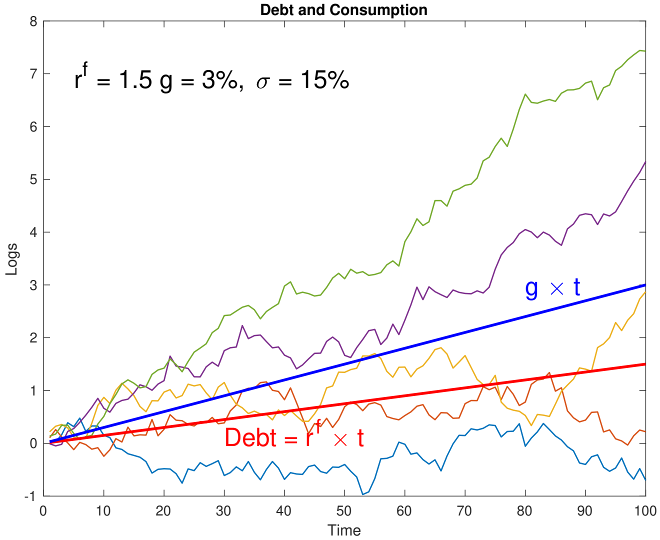
$$b_t = \left[c_t - E_t \beta^T \left(\frac{c_{t+T}}{c_t} \right)^{-\gamma} c_{t+T} \right] + E_t \beta^T \left(\frac{c_{t+T}}{c_t} \right)^{-\gamma} c_{t+T}.$$

- Discounting with government bond return $= r^f$,

$$b_t = c_t \left(1 - \frac{(1+g)^T}{(1+r^f)^T} \right) + c_t \frac{(1+g)^T}{(1+r^f)^T}.$$

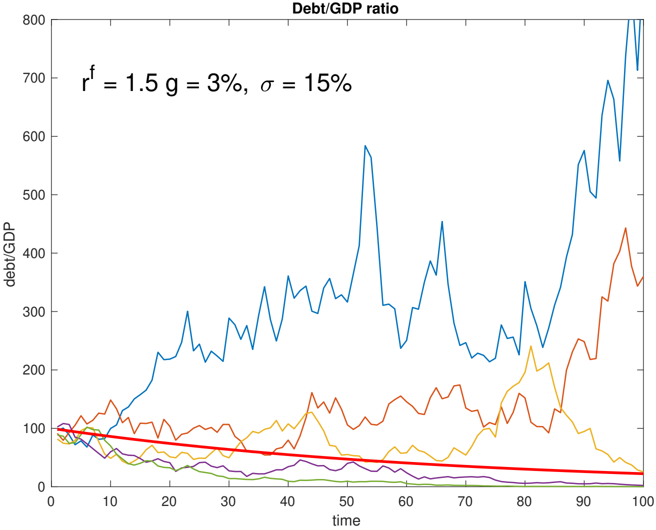
- Both right. Which is more useful?
- At least be careful about offsetting infinite limits! Can miss $b/y = 1$, not ∞, that deficits *are* repaid in PV terms. No "mineable bubbles"!

Ex post consumption and debt paths



- Borrow 1, roll over at $r^f < g$ forever. Certainty: g beats r^f. Uncertainty: there are sample paths of low consumption (high u').

Ex post debt to GDP ratio



- Certainty: b/y declines. Uncertainty: Some low growth sample paths lead to huge b/y, fiscal adjustment in low c, high u' state.
- If you weight by u', transversality is violated. *Like writing puts.*

Bottom line

Paper

- Paper has liquidity from frictions, $mpk \neq r^f \neq r$ from uncertainty. Important for realistic values. Same basic point (?) in many, simpler, models.

Lessons

- $r < g \approx 1\%$ is fun but irrelevant for US fiscal problems.
- $r < g \approx 1\%$ allows steady small deficits like seignorage. Larger deficits need to be repaid with subsequent surpluses.
- Grow out of debt opportunity is like writing out of the money put options and calling it arbitrage.
- With liquidity or uncertainty, discounting with ex post return *can* lead to terminal condition and PV that explode in opposite directions, while discounting with marginal utility is well behaved.
- If you do it, be careful. Discounting with marginal utility is safer.
- Do not pluck r measures from the world and use risk free models for quantitative questions.