UNSW Business School

Centre for Applied Economic Research

The Digital Economy, GDP and Consumer Welfare

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Background

- **There is much recent attention to the impact of the Digital Economy on welfare and GDP.**

- **Concerns about whether or not benefits from new goods and services are being appropriately measured.**

- **There are two features of the Digital Economy that we focus on here:**
  - **New goods and services**
  - **"Free" goods and services**

- **"Free" goods can be thought of as e.g. consumer entertainment and information from the Internet that is largely supported by advertising.**

Summary

- **Begin by defining a framework for measuring welfare change.**
- **Based on the work of Hicks (1941-42), Bennet (1920) and Diewert and Mizobuchi (2009).**
- **Derive an explicit term that is the value of a new good on welfare change.**
- **Provides the extent of welfare change mismeasurement if it is omitted from statistical agency collections.**
- **Work out a lower bound on the addition to real GDP growth from the introduction of a new good.**
- **Then re-work the theory allowing for there to be "free" goods (with an implicit or imputable price).**

Welfare Change and the New Goods Problem

Consumer's cost function:

$$C(u,p) \equiv \min_q \{p \cdot q \; ; \; f(q) \geq u\}$$

for each strictly positive price vector $p >> 0_N$ and each utility level u in the range of utility function, f(q), which is continuous, quasiconcave and increasing in the components of the nonnegative quantity vector $q \geq 0_N$.

Assume that the consumer minimizes the cost of achieving the utility level $u^t \equiv f(q^t)$:

$$p^t \cdot q^t = C(f(q^t),p^t) \text{ for } t = 0,1.$$

Welfare Change and the New Goods Problem

Valid measures of utility change over the two periods under consideration are the following Hicksian *equivalent and compensating variations*:

$$Q_E(q^0,q^1,p^0) \equiv C(f(q^1),p^0) - C(f(q^0),p^0)$$

$$Q_C(q^0,q^1,p^1) \equiv C(f(q^1),p^1) - C(f(q^0),p^1)$$

Note: Samuelson (1974):

$$Q_S(q^0,q^1,p) \equiv C(f(q^1),p) - C(f(q^0),p)$$

Hence there is an entire family of cardinal measures of utility change: one measure for each reference price vector p.

Welfare Change and the New Goods Problem

Hicks showed that the following provide a first-order approximation to equivalent and compensation variations, respectively:

$$V_L(p^0,p^1,q^0,q^1) \equiv p^0 \cdot (q^1 - q^0)$$

$$V_P(p^0,p^1,q^0,q^1) \equiv p^1 \cdot (q^1 - q^0)$$

These are difference counterparts to the usual Laspeyres and Paasche quantity indexes:

$$Q_L = \frac{p^0 \cdot q^1}{p^0 \cdot q^0} \qquad Q_P = \frac{p^1 \cdot q^1}{p^1 \cdot q^0}$$

Taking the geometric mean of these gives the superlative Fisher index:

$$Q_F = [Q_L Q_P]^{1/2}$$

Welfare Change and the New Goods Problem

The observable Bennet (1920) variation is the arithmetic average of the Laspeyres and Paasche variations:

$$V_B(p^0,p^1,q^0,q^1) \equiv \tfrac{1}{2}(p^0 + p^1)\cdot(q^1 - q^0) = p^0\cdot(q^1 - q^0) + \tfrac{1}{2}(p^1 - p^0)\cdot(q^1 - q^0)$$

$$= V_L + \tfrac{1}{2} \sum_{n=1}^{N} (pn^1 - pn^0)(qn^1 - qn^0)$$

Bennet variation is equal to the Laspeyres variation V_L plus a sum of N Harberger (1971) consumer surplus triangles of the form:

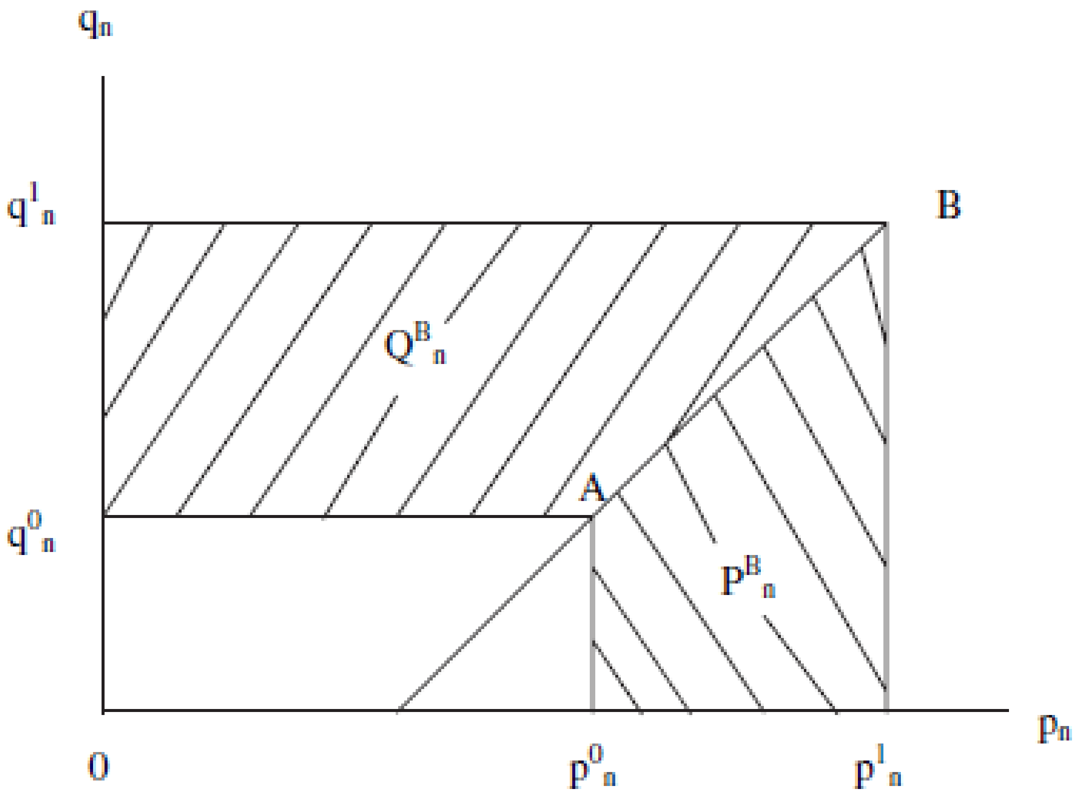
$$(1/2)(p_n^1 - p_n^0)(q_n^1 - q_n^0)$$

Also:

$$V_B(p^0,p^1,q^0,q^1) = V_P - \tfrac{1}{2} \sum_{n=1}^{N} (pn^1 - pn^0)(qn^1 - qn^0)$$

Welfare Change and the New Goods Problem

Bennet's Decomposition of Value Change



Diewert (2005, p. 325)

Welfare Change and the New Goods Problem

Recap:

Hicksian equivalent variation can be approximated by V_L

Hicksian compensating variation can be approximated by V_P

Hicks (1941-42) obtained the Bennet quantity variation V_B as an approximation to the arithmetic average of the equivalent and compensating variations.

Welfare Change and the New Goods Problem

So far, no economic justification for taking the average of V_L and V_P.

Diewert and Mizobuchi (2009) assumed that consumer preferences can be represented by a (flexible) normalized quadratic cost function:

$$C(u,p) \equiv b \cdot p + [c \cdot p + \tfrac{1}{2}(\alpha \cdot p)^{-1} p^T B p]u$$

where

$b \cdot p^* = 0$;

$c \cdot p^* = 1$;

$Bp^* = 0_N$ and $B = B^T$.

Welfare Change and the New Goods Problem

Then, for normalized prices, we have the following exact equality:

$$V_B(p^0,p^1,q^0,q^1) = ½\, Q_E(q^0,q^1,p^0) + ½\, Q_C(q^0,q^1,p^1)$$

i.e., the observable Bennet variation is *exactly equal* to the arithmetic average of the unobservable equivalent and compensating variations.

Hence, a strong justification from an economic perspective for using the Bennet quantity variation. Also, it has strong justification from an axiomatic perspective (Diewert, 2005).

Welfare Change and the New Goods Problem

A decomposition of nominal GDP change into Bennet quantity and price variations:

$$p^1 \cdot q^1 - p^0 \cdot q^0 = V_B + I_B$$

where

$$V_B(p^0,p^1,q^0,q^1) \equiv \tfrac{1}{2}(p^0 + p^1) \cdot (q^1 - q^0)$$

$$I_B(p^0,p^1,q^0,q^1) \equiv \tfrac{1}{2}(q^0 + q^1) \cdot (p^1 - p^0)$$

Welfare Change and the New Goods Problem

Introduction of a new good (or service) to a consumer who cannot purchase the good in period 0 but can purchase it in period 1.

Assume (as per Hicks 1940) that there is a shadow price for the new good in period 0 that will cause the consumer to consume 0 units of the new good in period 0.

Let the new good be indexed by the subscript 0 and let the N dimensional vectors of period t prices and quantities for the continuing commodities be denoted by p^t and q^t for $t = 0,1$.

Period 0 shadow price for commodity 0 is not observed but we make some sort of *estimate* for it, denoted as $p_0^{0*} > 0$.

The period 0 quantity is observed and is equal to 0; i.e., $q_0^0 = 0$.

Welfare Change and the New Goods Problem

Bennet variation measure of welfare change becomes:

$$V_B = ½(p^0 + p^1)\cdot(q^1 – q^0) + ½(p_0^{0*} + p_0^1)(q_0^1 – 0)$$

$$= p^1\cdot(q^1 – q^0) – ½(p^1 – p^0)\cdot(q^1 – q^0) + p_0^1q_0^1 – ½(p_0^1 – p_0^{0*})q_0^1$$

Terms:

1. **$p^1\cdot(q^1 – q^0)$: change in consumption valued at the prices of period 1**
2. **$– ½(p^1 – p^0)\cdot(q^1 – q^0)$: sum of the consumer surplus terms associated with the continuing commodities**
3. **$p_0^1q_0^1$: the usual price times quantity contribution term to the value of real consumption of the new commodity in period 1 which would be recorded as a contribution to period 1 GDP**

Welfare Change and the New Goods Problem

$V_B = p^1 \cdot (q^1 - q^0) - \frac{1}{2}(p^1 - p^0) \cdot (q^1 - q^0) + p_0^1 q_0^1 - \frac{1}{2}(p_0^1 - p_0^{0*}) q_0^1$

The last term, $-\frac{1}{2}(p_0^1 - p_0^{0*}) q_0^1 = \frac{1}{2}(p_0^{0*} - p_0^1) q_0^1$, is the additional consumer surplus contribution of commodity 0 to overall welfare change (which would not be recorded as a contribution to GDP).

If we assume that $p_0^{0*} = p_0^1$, then the downward bias in the resulting Bennet measure of welfare change will be equal to a Harberger-type triangle, $\frac{1}{2}(p_0^{0*} - p_0^1) q_0^1$.

Welfare Change and the New Goods Problem

So, can we just add something to GDP growth to fully capture the introduction of the new good?

A decomposition of nominal GDP can be written as follows (Diewert 2005):

$$p^1 \cdot q^1 - p^0 \cdot q^0 = p^0 \cdot q^0[½(1+Q)(P-1) + ½(1+P)(Q-1)]$$

where P and Q are price and quantity indexes, respectively, that satisfy
$P \times Q = p^1 \cdot q^1 / p^0 \cdot q^0$

Economic Price and Quantity Change Indicators:

$$I_E = ½\, p^0 \cdot q^0(1+Q)(P-1) \quad \text{and} \quad V_E = ½\, p^0 \cdot q^0(1+P)(Q-1)$$

Welfare Change and the New Goods Problem

Adapting Proposition 9 of Diewert (2005):

If a superlative index number is chosen for P and Q, V_B approximates V_E to the second order for $q^0=q^1$ and $p^0=p^1$.

The U.S. uses the superlative Fisher Quantity Index for GDP, so:

$$V_E^F \equiv ½\, p^0 \cdot q^0(1+P^F)(Q^F-1) \approx ½\,(p^0 + p^1)\cdot(q^1 - q^0) = V_B$$

Re-arranging:

$$Q^F \approx [(p^0 + p^1)\cdot(q^1 - q^0)]/[p^0 \cdot q^0(1+P^F)] + 1$$

Note that the numerator is 2 x $V_{B.}$

Welfare Change and the New Goods Problem

$$2V_B = (p^0 + p^1)\cdot(q^1 - q^0)$$
$$= 2p^1\cdot(q^1 - q^0) - (p^1 - p^0)\cdot(q^1 - q^0) + 2p_0{}^1q_0{}^1 - (p_0{}^1 - p_0{}^{0*})q_0{}^1$$

Assuming that the approximation holds exactly, then:

$$Q^F = [(p^0 + p^1)\cdot(q^1 - q^0)]/[p^0\cdot q^0(1+P^F)] + 1$$

If Q^F omits the new good in period 0, and we assume that P^F (the aggregate GDP deflator between adjacent periods) is unaffected by the introduction of the new good, then the (approximate) amount missing from Q^F is:

$$(p_0{}^{0*} - p_0{}^1)q_0{}^1 / [p^0\cdot q^0(1+P^F)]$$

which can simply be added to Q^F if $p_0{}^{0*}$ is known or can be estimated. P^F will typically fall with the inclusion of the new good, so this is a lower bound on the amount to add.

Welfare Change and the Free Goods Problem

Consider a consumer whose preferences over N market goods and services and M commodities that are available to the household with no visible charge.

Utility function f(x,z): where $x \geq 0_N$ and $z \geq 0_M$ are vectors which represent the consumption of market commodities and of free commodities respectively.

We assume that f(x,z) is defined over the nonnegative orthant in R^{N+M} and has the following properties:

(i) continuity,

(ii) quasiconcave in x and y, and

(iii) f(x,z) is increasing if all components of x increase and increasing if all components of z increase.

Welfare Change and the Free Goods Problem

Define two cost functions that are dual to f. The consumer's *regular cost function* assumes (hypothetically) that the household faces positive prices for market and free goods and services so that $p >> 0_N$ and $w >> 0_M$:

$$C(u,p,w) \equiv \min_{x,z} \{p \cdot x + w \cdot z: f(x,z) \geq u, x \geq 0_N, z \geq 0_M\}.$$

The *conditional cost function* minimizes the cost of market goods and services needed to achieve utility level u, conditional on having the vector $z \geq 0_M$ of free goods and services at its disposal:

$$c(u,p,z) \equiv \min_{x} \{p \cdot x: f(x,z) \geq u, x \geq 0_N\}.$$

Welfare Change and the Free Goods Problem

Decompose the first cost function into a two-stage minimization problem using the second cost function:

$$C(u,p,w) \equiv \min_{x,z} \{p\cdot x + w\cdot z\colon f(x,z) \geq u;\ x \geq 0_N,\ z \geq 0_M\}$$
$$= \min_{z} \{c(u,p,z) + w\cdot z\colon z \geq 0_M\}.$$

Suppose $z^* \geq 0_M$ solves this cost minimization problem and suppose further that $c(u,p,z^*)$ is differentiable with respect to the components of z at $z = z^*$.

Then the first order necessary conditions for z^* to solve the cost minimization problem imply that:

$$\nabla_z c(u,p,z^*) = -w\ .$$

Welfare Change and the Free Goods Problem

With $z = z^*$, we can find an x solution which we denote by x^*; i.e., x^* is a solution to:

$$c(u,p,z^*) \equiv \min_x \{p \cdot x : f(x,z^*) \geq u, x \geq 0_N\}.$$

It can be seen that (x^*,z^*) is a solution to the regular cost minimization problem so that:

$$\begin{aligned} C(u,p,w) &\equiv \min_{x,z} \{p \cdot x + w \cdot z : f(x,z) \geq u, x \geq 0_N, z \geq 0_M\} \\ &= p \cdot x^* + w \cdot z^*. \end{aligned}$$

Thus the imputed marginal valuation prices $w \equiv -\nabla_z c(u,p,z^*) \geq 0_M$ are appropriate prices to use when valuing the services of free goods in order to construct cost of living indexes or measures of money metric utility change.

Welfare Change and the Free Goods Problem

CASE 1

Consumer holding no free goods has utility $u^* = f(x^*,0_M)$.

"Global" willingness to pay function **for the acquisition of z^* as follows:**

$$W_P(u^*,p,z^*) \equiv c(u^*,p,0_M) - c(u^*,p,z^*)$$

CASE 2

Consumer holding $Z^{**} > 0$ free goods has utility $u^{**} \equiv f(x^{**},z^{**})$.

"Global" willingness to sell function **for the disposal of z^{**} as follows:**

$$W_S(u^{**},p,z^{**}) \equiv c(u^{**},p,0_M) - c(u^{**},p,z^{**})$$

Welfare Change and the Free Goods Problem

Marginal willingness to sell function **for free good m:**

$W_m(u,p,z) \equiv c(u,p,z-e_m) - c(u,p,z)$; $m = 1,\ldots,M$.

where e_m is a unit vector of dimension M with a 1 in component m and zeros elsewhere for m = 1,...,M.

Survey methods could be used in order to obtain approximate measures for these marginal willingness to sell functions.

Let W(u,p,z) denote the vector $[W_1(u,p,z),\ldots, W_M(u,p,z)]$.

It can be seen that W(u,p,z) is a discrete approximation to the marginal valuation price vector $w \equiv -\nabla_z c(u,p,z)$

Welfare Change and the Free Goods Problem

Welfare change including the free good:

$$V_B = p^1\cdot(q^1 - q^0) - \tfrac{1}{2}(p^1 - p^0)\cdot(q^1 - q^0) + p_0^1 q_0^1 - \tfrac{1}{2}(p_0^1 - p_0^{0*})q_0^1$$

$$+ w^1\cdot(z^1 - z^0) - \tfrac{1}{2}(w^1 - w^0)\cdot(z^1 - z^0) + w_0^1 z_0^1 - \tfrac{1}{2}(w_0^1 - w_0^{0*})z_0^1$$

Can the make an appropriate adjustment to real GDP growth, as before.

Summary

- **Begin by defining a framework for measuring welfare change.**
- **Based on the work of Hicks (1941-42), Bennet (1920) and Diewert and Mizobuchi (2009).**
- **Derive an explicit term that is the value of a new good on welfare change.**
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EXTRA SLIDES

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Welfare Change and the Free Goods Problem

Attempts to find prices for “free” goods include:

Brynjolfsson and Oh (2012)

“We develop a new framework to measure the value of free services using the insight that even when people do not pay cash, they must still pay “attention,” or time.”

Nakamura et al. (2016)

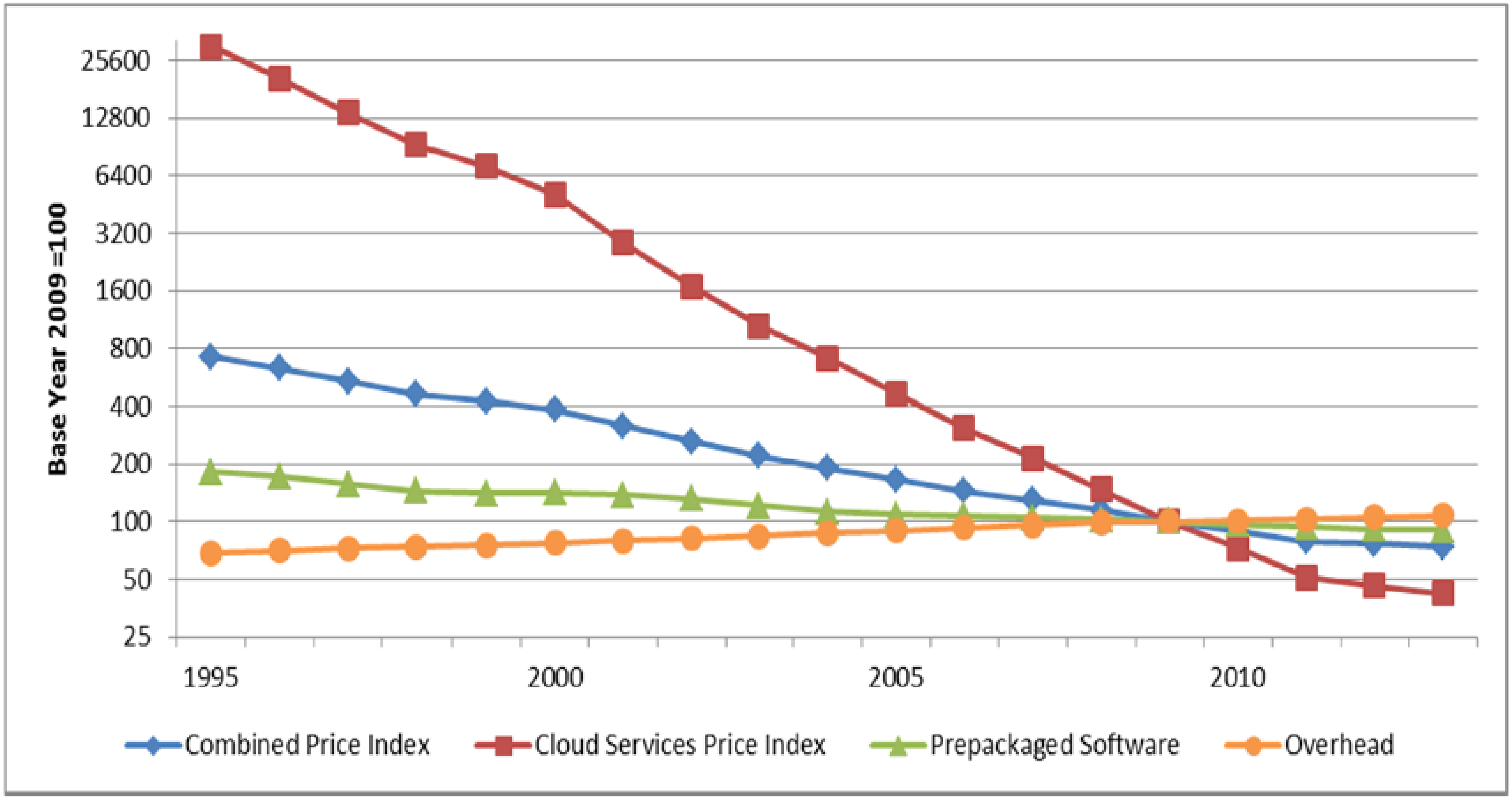
“Our method for accounting for ‘free media’ is production oriented in the sense that it is a measure of the resource input into the entertainment (or other content) of the medium…”

“we use the BEA’s price indexes for prepackaged software (Table 5.6.4, line 3) as a proxy for software costs; and a price index for cloud computing services reported in ‘ICT Prices and ICT Services: What Do They Tell Us About Productivity and Technology’ (Byrne and Corrado 2016) as a proxy for computer costs; and BEA’s price index for personal consumption services (Table 1.1.4, line 6) as a proxy for overhead costs.”

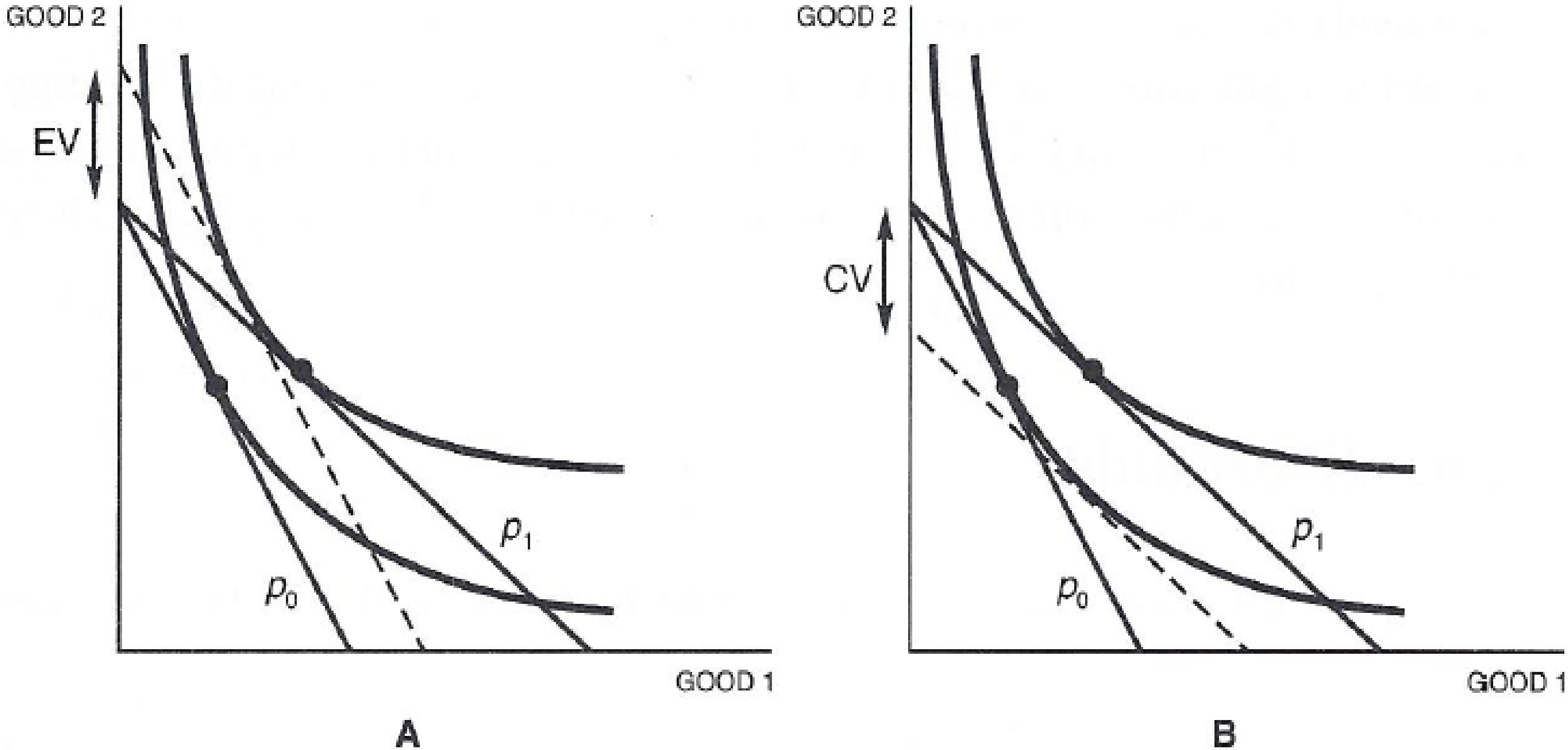
Welfare Change and the Free Goods Problem

Nakamura et al. (2016)

Figure 6: Prices for Online Media



Welfare Change and the New Goods Problem



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