# Balanced Growth Despite Uzawa

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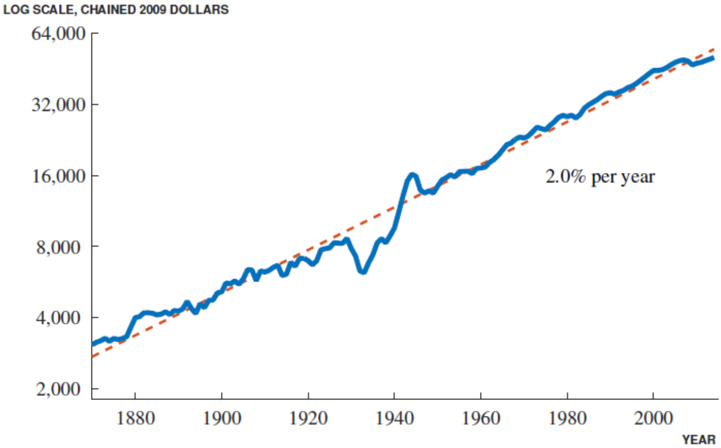
# Stylized Facts of Economic Growth

- Kaldor (1961) facts:
  - Constant growth rates of output/worker, capital/worker
  - Constant capital/output ratio, real return to capital
  - Roughly constant factor shares (maybe until 2000?)
- Updated by Jones (2015)

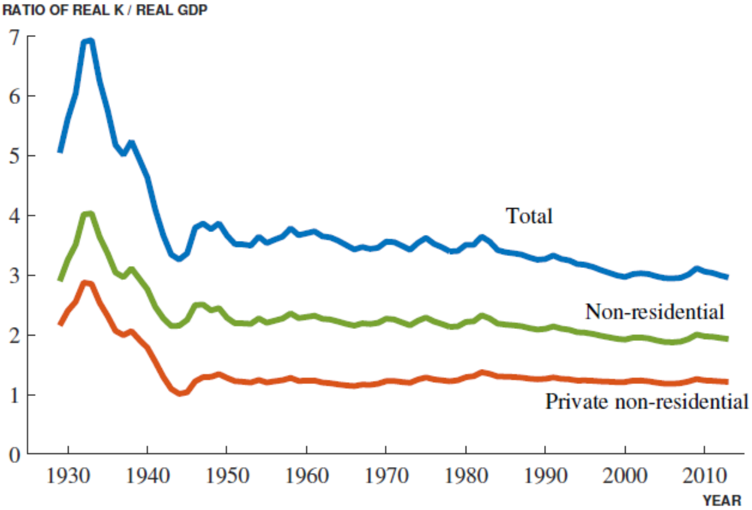
# Steady Growth of US Per Capita Income for 150 Years

Figure 1: GDP per person in the United States



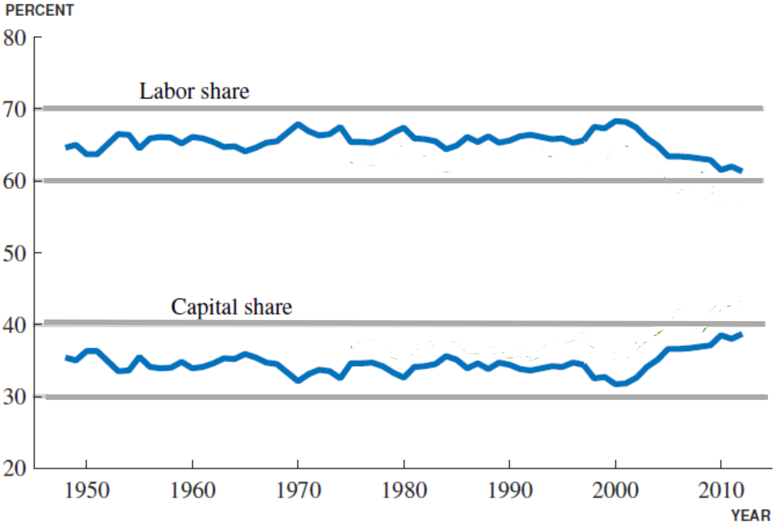
# Constant Capital/Output Ratio

Figure 3: The Ratio of Physical Capital to GDP



# Constant Factor Shares (until 2000)

Figure 6: Capital and Labor Shares of Factor Payments, United States



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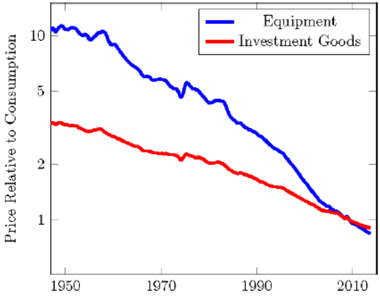
- But ... "all is not well"
- Uzawa Growth Theorem:
  - Balanced growth requires either Cobb Douglas aggregate production function or an absence of capital augmenting technical progress (embodied or disembodied)
  - How do these requirements square with the evidence?

# Uzawa Growth Theorem and the Uncomfortable Evidence

- Preponderance of evidence suggests $\sigma_{KL} < 1$
  - See Chirinko (2008) for survey. Oberfield & Raval (2014), Lawrence (2015), Herrendorf et al. (2015), Chirinko & Mallick (2014) etc.
  - Exception: Karabarbounis & Neiman (2014)
- Falling investment-good prices indicative of investment-specific technical change



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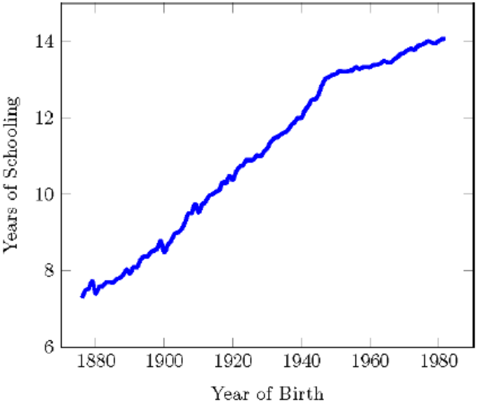
Education by birth cohort (Goldin and Katz):



# Plan of Paper

- Show why endogenous schooling might allow BGP with $\sigma_{KL} < 1$ and $\gamma_K > 0$ under certain circumstances
- Models with optimizing behavior and "short lifespans"
  - Planner's problem with reduced-form tradeoff between labor force and measure of economy's education level
  - Show restrictions on $F(\cdot)$ that are sufficient and (essentially) necessary for existence of BGP when $\gamma_K > 0$
  - Two market economies that yield such a reduced-form:
    - (i) Time-in-school model;
    - (ii) Manager-worker model
- OLG model with time in school

# Extended Uzawa Growth Theorem

- Let $Y = F(AK, BL, s)$ prod fct with CRS in $K$ and $L$ and increasing in $s$, where $s$ is scalar representation of educational attainment (e.g., average years of schooling or fraction with college degree)
- Can convert one unit of output into $q_t$ units of investment good
- Let $\gamma_K = g_A + g_q$: disembodied plus embodied capital-augmenting technological progress
  - $g_q$ is "investment specific technical change"
  - Price of capital falls at constant rate $g_q$
- Suppose $\gamma_K$, $g_B$, $g_L$ are constant
- BGP: Define as $Y$, $K$, and $C$ growing at constant rates and factor shares constant and strictly positive.

# Extended Uzawa Growth Theorem

Proposition 1: Suppose $q$ grows at constant rate. If there exists a BGP, then

$$(1-\sigma_{KL})\,\gamma_K = \sigma_{KL}\frac{F_L}{F_K}\frac{\partial\left(F_s/F_L\right)}{\partial K}\dot{s}$$

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2. Human Capital: $\exists$ measure of human capital $H\left(BL,s\right)$ such that $F\left(AK,BL,s\right)\equiv\tilde{F}\left[AK,H\left(BL,s\right)\right]\Rightarrow$ BGP requires $\sigma_{KL}=1$ or $\gamma_K=0$.

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3. GHOS: $\dot{s} > 0$, $\gamma_K > 0$ and $\sigma_{KL} < 1 \Rightarrow$ BGP requires $\partial\left(F_s/F_L\right)/\partial K > 0$ (capital-schooling complementarity)

# Short Lifespans

- Unit measure of identical family dynasties. $N_t = N_0 e^{nt}$
- Infinitessimal lives $\Rightarrow$ $s$ is a jump variable
- Reduced form trade-off between education and labor supply:

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- Planner problem

$$\max_{\{c_t, s_t, L_t\}} \int_{t_0}^{\infty} N_t e^{-\rho(t-t_0)} \frac{c_t^{1-\eta} - 1}{1-\eta} dt \qquad \text{subject to ...}$$

$$\begin{aligned} Y_t &\leq F(A_t K_t, B_t L_t, s_t) \\ L_t &\leq D(s_t) N_t \\ \dot{K}_t &= q_t (Y_t - N_t c_t) - \delta K_t \end{aligned}$$

# Assumptions

- Assumption 1 Production function can be written as $F(AK, BL, s) = \tilde{F}\left[D(s)^a AK, D(s)^{-b} BL\right]$, with $a > 0$, $b > 0$, and

  (i) $\tilde{F}$ strictly increasing, smooth, concave in first argument

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Example: $F(AK, BL, s) = (BL)^{1-\beta}\left\{(AK)^{\alpha} + \left[D(s)^{-(a+b)} BL\right]^{\alpha}\right\}^{\beta/\alpha}$,

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- More Assumptions: Parameter restrictions to ensure
  - $F_s > 0$
  - $s > 0$
  - utility is finite

# Planner's Problem

- Choice of $s_t, L_t$ is a static problem

$$Y_t = \max_{s,L} \tilde{F}\left[D\left(s\right)^a A_t K_t, D\left(s\right)^{-b} B_t L\right] \qquad \text{s.t.}$$

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- FOCs imply

$$\frac{A_t K_t}{B_t L_t} D\left(s_t\right)^{a+b} = z^* = \mathcal{E}_h^{-1}\left(\theta\right)$$

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- Substitute into $Y_t$:

$$Y_t = (A_t K_t)^{\theta} (B_t N_t)^{1-\theta} z^{*-\theta} h(z^*)$$

$\Rightarrow$ Optimal education and Assumption 1 imply output Cobb-Douglas in capital and population!

# Balanced Growth

Proposition 2 Suppose $L = D(s)N$ and Assumptions 1-3 hold. Then along the optimal trajectory from any initial capital stock $K_{t_0}$ the economy converges to a unique BGP. On the BGP

1. aggregate output and aggregate consumption grow at the common rate
$$g_Y = n + \gamma_L + \frac{b-1}{a}\gamma_K$$
2. schooling evolves to satisfy
$$g_D = -\frac{\gamma_K}{a};$$
3. the capital share is constant and equal to
$$\theta_K = \frac{b-1}{a+b-1}$$

# Role of Functional Form

- Assumption 1:

$$F\left[AK, BL, s\right] = \tilde{F}\left[AKD\left(s\right)^{a}, BLD\left(s\right)^{-b}\right]$$

  - Schooling as if augments $L$, while curtailing $K$
  - Combined effect is positive: $\partial F / \partial s > 0$
  - Decline in productivity of $K$ (given $LD\left(s\right)^{-b}$) just what is needed to keep schooling-plus-technology augmented $K$ stock growing in line with output.
    - $D\left(s_t\right)^{a} q_t A_t$ is constant along BGP

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- Race between education and effective capital:
  - More abundant effective $K \Rightarrow \theta_K \downarrow$
  - $F$ log-supermodular in $K, s$ and $\dot{K} > 0 \Rightarrow$ return to schooling $\uparrow$
  - Capital-schooling complementarity and $\dot{s} > 0 \Rightarrow \theta_K \uparrow$

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  - $F$ log-supermodular in $K, s$ and $\dot{K} > 0 \Rightarrow$ return to schooling $\uparrow$
  - Capital-schooling complementarity and $\dot{s} > 0 \Rightarrow \theta_K \uparrow$
- Can we dispense with Assumption 1? Essentially NO.
  - If $\exists$ BGP with $\gamma_K > 0$, technology must have representation as $\tilde{F}$.

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- Individuals choose $s$ to maximize $(1-s)\,W_t(s)$
- Capital deepening raises $W_t'(s)\,/\,W_t(s)$ (return to education) due to $K-s$ complementarity
  - Wage schedule gives incentives for schooling to grow over time

# Manager-Worker

- Individuals face discrete choice: Devote fixed fraction $m$ of time to train as manager, or work full-time as production worker.
- Workers and equipment generate output. Productivity depends on $s$ ("monitoring by managers").
  - $s = M/L$, ratio of manager hours to worker hours
- Schooling/hours tradeoff: $N = L + \frac{M}{1-m} = \left(1 + \frac{s}{1-m}\right) L$

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- In equilibrium $(1-m) W_{Mt} = W_{Lt}$
- This gives incentives for $s = M/L$ to grow over time
  - Capital deepening raises $F_M / F_L$, due to $K - s$ complementarity
  - Incentive for greater fraction of population to be trained as managers as effective capital grows

# Overlapping Generations

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  - composition of labor force changes over time
  - labor force participation rate changes over time
- Challenge: How to maintain balance in face of evolving composition of labor force and different amounts of capital allocated to different workers?
- Answer: Technology with Mincer (1974) wage equation

# Elements of the Model

- Size of representative dynasty is $N_t = e^{(\lambda - \upsilon)t} N_0$, where $\lambda$ is instantaneous probability of offspring for any individual and $\upsilon$ is instantaneous hazard of death
- Production function $F\left(A_t K, B_t L, s, u\right)$, where $F\left(A_t K, B_t L, s, u\right) = 0$ for $u \geq \bar{u}$
  - Wage of individual with schooling $s$ and experience $u$ at $t$ is $W_t(s, u)$
  - Firm hires workers with $\{s, u\}$, allocates capital to each
- Maximize dynastic welfare s.t. intertemporal budget constraint
  - Dynasties choose schooling for individual born at $b$ to maximize expected pdv of lifetime wages:

$$\int_{b+s}^{\infty} e^{-(\iota+\upsilon)(t-b)} W_t\left(s, t-b-s\right) dt$$

# Assumptions

- Assumption 4 The production function can be expressed as $F(AK, BL, s, u) = \tilde{F}\left(e^{-as}AK, e^{bs}BL, u\right)$, with $a > 0$ and $b > 0$, s.t.

  (i) $\tilde{F}$ is strictly increasing, smooth, concave in first argument
  (ii) $F(AK, BL, s, u) = 0$ for all $u \geq \bar{u}$; and
  (iii) $\sigma_{KL}(K, L, s, u) < 1$

- More Assumptions Parameter restrictions that ensure (i) $F_s > 0$, (ii) $s$ is interior, (iii) $\dot{s} > 0$, (iv) finite lifetime budget

# Main Results

Proposition 3 Suppose that Assumptions 4 and 5 hold. Then the OLG economy has a unique balanced growth path. On the BGP

(i) aggregate output, consumption, and wages grow at rate

$$g_Y = n + \gamma_L + \frac{b - \lambda}{a} \gamma_K \ ;$$

(ii) the educational attainment of new cohorts rises linearly over time

$$\dot{s}_b = \frac{\gamma_K}{a - \gamma_K} \ ;$$

(iii) aggregate $K$ share constant ($\theta_K$ varies with $s, u$ in cross section)

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- BGP? Linear increase in $s$ generates constant decline in $e^{-as}$, offsets growth in $A$ and $q$

# Additional Results for BGP

- The labor force participation rate $L/N$ declines exponentially (longer time spent in school)
- Distribution of experience in labor force is stable
- Density of $s$ in labor force shifts right at constant rate per year
- Mincerian wage equation for log wages as function of $s$ and $u$

# Determinants of Capital Share

- At given $R$, those with higher $s$ produce with higher $K$ share
  - Aggregate capital share is an average
  - No closed form for $\theta_K$:

$$\theta_K = \frac{\int_0^{\bar{u}} e^{-\left[\lambda + \frac{b-\lambda}{a}\gamma_K\right]u} e^{-\gamma_K u} x^* \Phi\left[e^{-\gamma_K u} x^*, u\right] du}{\int_0^{\bar{u}} e^{-\left[\lambda + \frac{b-\lambda}{a}\gamma_K\right]u} h\left\{\Phi\left[e^{-\gamma_K u} x^*, u\right], u\right\} du}$$

- No clear relationship between rates/form of technological progress and capital share!
- Resort to numerical simulation of parameterized version of model
- Use production function

$$F(A_t K, B_t L, s, u) = \tilde{h}(u)\left(B_t L\right)^{1-\beta}\left[A_t K^{\alpha} + \left(e^{\mu s} B_t L\right)^{\alpha}\right]^{\beta/\alpha}$$

- Use quadratic experience profile for $u \leq \bar{u}$:

$$\tilde{h}(u) = 1 + 0.2\left[1 - \left(2u/\bar{u} - 1\right)^2\right]$$

# Simulation Parameters

- Working life: $\bar{u} = 40$
- Birth and death rates: $\lambda = \upsilon = 0.01$
- Production function parameters: $\alpha, \beta, \mu$ so that capital share is 0.35, average local elasticitity of substitution between $K$ and $L$ is 0.6, and educational attainment grows one year per decade in baseline scenario with $\gamma_K = 0.02$ and $\gamma_L = 0.01$
- Discount rate and elasticity of substitution?
  - Sensitivity of $\theta_K$ to $\gamma_K$ and $\gamma_L$ governed by real interest rate
  - Low riskless rate of return suggests targeting low interest rate
  - High rate of return on schooling suggests targeting high interest rate
  - Cannot match both low riskless rate and high internal rate of return on schooling in our model
  - Do not take strong stand: Present low-interest rate and high-interest rate scenarios.

# Simulation Results

| Low Interest Rate: $\rho = .01,\ \eta = 1$ | | | | | |
|---|---|---|---|---|---|
| $\gamma_K$ | $\gamma_L$ | Growth in per capita Income | Annual Increase in Schooling | Capital Share | Interest Rate |
| 0.03 | 0.01 | 0.028 | 0.158 | 0.348 | 0.038 |
| 0.02 | 0.01 | 0.022 | 0.1 | 0.35 | 0.032 |
| 0.01 | 0.01 | 0.016 | 0.048 | 0.352 | 0.026 |
| | | | | | |
| 0.02 | 0.02 | 0.032 | 0.1 | 0.35 | 0.042 |
| 0.02 | 0.01 | 0.022 | 0.1 | 0.35 | 0.032 |
| 0.02 | 0 | 0.012 | 0.1 | 0.35 | 0.022 |

| High Interest Rate: $\rho = .01,\ \eta = 3$ | | | | | |
|---|---|---|---|---|---|
| $\gamma_K$ | $\gamma_L$ | Growth in per capita Income | Annual Increase in Schooling | Capital Share | Interest Rate |
| 0.03 | 0.01 | 0.038 | 0.158 | 0.288 | 0.123 |
| 0.02 | 0.01 | 0.028 | 0.1 | 0.35 | 0.095 |
| 0.01 | 0.01 | 0.019 | 0.048 | 0.402 | 0.068 |
| | | | | | |
| 0.02 | 0.02 | 0.038 | 0.1 | 0.303 | 0.125 |
| 0.02 | 0.01 | 0.028 | 0.1 | 0.35 | 0.095 |
| 0.02 | 0 | 0.018 | 0.1 | 0.394 | 0.065 |

# Conclusions

- Can generate balanced growth in neoclassical growth model with endogenous education, provided capital is more complementary with schooling than it is with raw labor
- Mechanism is straightforward:
  - Over time, growth of effective capital stock due to $\dot{K} > 0$ and $\gamma_K > 0$ raise returns to schooling
  - Individuals induced to spend more time in school.
  - Capital accumulation tends to lower capital share with $\sigma_{KL} < 1$.
  - Schooling offsets. With Assumption 1, it neutralizes.
- OLG model captures salient trends in US growth experiences, including linear growth in educational attainment
- For reasonable parameter values, capital share grows when technological progress slows.
- BGP requires delicate functional-form restrictions, as in any balanced growth model.