Moving House*

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Abstract

Using data on house sales and inventories, this paper shows that sales volume is driven mainly by listings and less so by transaction speed, thus the decision to move house is key to understanding sales volume. The paper builds a model where moving house is essentially an investment in match quality, implying that moving depends on variables such as interest rates and taxes. The endogeneity of moving means there is a cleansing effect — those at the bottom of the match quality distribution move first — which generates overshooting in aggregate variables. The model is applied to the 1995–2004 housing market boom.

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1 Introduction

The number of sales in the housing market is a widely watched statistic with implications for many sectors of the economy. The U.S. in particular has seen a dramatic rise and fall of sales volume in recent decades. For most housing-market transactions, at the origin is the decision of a homeowner to put a house up for sale. This is followed by a time-consuming and costly search for someone to sell to and for another house to buy. How frequently houses come on to the market for sale and how quickly they come off the market following a successful sale together determine the volume of sales.

The first contribution of this paper is to demonstrate that the main driver of changes in sales volume is how frequently houses come on to the market. Understanding the decision to move house is therefore crucial for explaining changes in housing-market activity. Following from this, the second contribution of the paper is to build a model of moving house. The model is based on the idea that moving house constitutes an investment in the quality of the match between a homeowner and a particular house.

The claim that the moving rate of existing homeowners is of much greater importance for sales volume than the selling rate of houses on the market can be understood with some minimal empirical discipline combined with a basic stock-flow accounting identity. Compared to the average time spent in a house (more than a decade), the average time taken to sell a house is very short (a few months). This means the average sales rate is around twenty to thirty times higher than the average moving rate, and the stock of houses for sale is about twenty to thirty times smaller than the stock of occupied houses. An increase in the sales rate with no change in the moving rate would rapidly deplete the stock of houses for sale leaving little overall impact on the volume of sales. On the other hand, an increase in the moving rate adds to the stock of houses for sale, which increases the volume of sales even with no change in the sales rate.

A simple counterfactual exercise confirms the claim that the moving rate is the dominant factor in explaining sales. Data on sales and inventories of existing single-family homes from the National Association of Realtors (NAR) are first used to construct a measure of listings (houses put up for sale) and the sales and moving/listing rates. The counterfactual exercise considers the sales volume that would have been obtained if the moving rate were constant at its sample average while the sales rate varied as in the data. This hypothetical sales series fails to capture any of the major movements in actual sales volume. On the other hand, if the sales rate were held constant at its sample average while the moving rate varied as in the data, the hypothetical sales volume series tracks closely the main changes in actual sales volume.

To understand what might drive changes in the moving rate, this paper presents a search-and-matching model of endogenous moving. Central to the model is the idea of match quality: the idiosyncratic values homeowners attach to the house they live in. This match quality is a persistent variable subject to occasional idiosyncratic shocks, representing life events such as changing jobs, marriage, divorce, and having children.[1] These shocks degrade existing match quality, following which homeowners decide whether to move. Eventually, after sufficiently many shocks, current

[1]These are the main reasons for moving according to the American Housing Survey.

match quality falls below a 'moving threshold' that triggers moving to a new house and a renewal of match quality.

Moving house is a process with large upfront costs that is expected to deliver long-lasting benefits, and is thus sensitive to macroeconomic and policy variables such as interest rates and taxes that influence other investment decisions. These variables affect the threshold for existing match quality that triggers moving. Thus, while idiosyncratic shocks are the dominant factor in moving decisions at the individual level, changes in the moving threshold lead to variation in the aggregate moving rate. In contrast, in a model of exogenous moving, the aggregate moving rate is solely determined by the arrival rate of an idiosyncratic shock that forces moving.

The endogeneity of moving generates new transitional dynamics that are absent from models imposing exogenous moving. Endogenous moving means that those who choose to move are not a random sample of the existing distribution of match quality: they are the homeowners who were only moderately happy with their match quality. Together with the persistence over time of existing match quality, endogenous moving thus gives rise to a 'cleansing effect'. An aggregate shock that changes the moving threshold leads to variation in the degree of cleansing of lower-quality matches. Since match quality is a persistent variable, more cleansing now leads to less cleansing in the future, which implies overshooting of the moving rate and other housing-market variables.

The modelling of the buying and selling process for houses on the market is close to the existing literature. There are search frictions in the sense that time is needed to view houses, and viewings are needed to know what idiosyncratic match quality would be in a particular house. Buyers and sellers face transaction costs and search costs. There is a 'transaction threshold' for match quality above which a buyer and a seller agree to a sale. The housing-market equilibrium is characterized by the moving threshold and the transaction threshold, and together with the distribution of existing match quality, these thresholds determine the sales and moving rates, and inflows to and outflows from the stock of houses for sale.

The model is applied to study the U.S. housing market in the decade from 1995 to 2004, which is noteworthy as a period of booming activity. Using NAR data, three stylized facts emerge during those years: sales volume rises, houses are selling faster, and houses are put up for sale more frequently. The increase in the listing rate is consistent with the independent empirical finding by Bachmann and Cooper (2014) of a substantial rise in the own-to-own moving rate using household-level PSID data.

The 1995–2004 period featured a number of developments in the U.S. that have implications for moving decisions according to the model. To name a few of these: the decline in mortgage rates, the post-1995 surge in productivity growth, and the rise of internet-based property search. These developments are represented in the model by changing parameters, and the model is solved analytically to conduct a series of comparative statics exercises. These illustrate the interpretation of moving house as an investment in match quality, and how incentives to invest are affected by macroeconomic conditions and changes in the housing market.

The implication of the model is that all three developments unambiguously increase the moving rate. This rise in the moving rate works through an increase in the moving threshold. Lower mortgage rates, interpreted as a fall in the rate at which future payoffs are discounted, create an incentive to invest in improving match quality because the capitalized cost of moving is reduced. An increase in productivity growth raises income and increases the demand for housing, which increases the marginal return to higher match quality. Finally, the adoption of internet technology reduces search frictions, making it cheaper for homeowners to move in order to invest in a better match.

The model is calibrated to study the quantitative effects of the three developments discussed above. Together they imply (depending on the time horizon) an 18% to 37% rise in the moving rate, which accounts for a substantial fraction of the rise in the moving rate derived from the NAR data. The model can also account for a large proportion of the rise in sales volume observed in the data and the rise in the stock of houses for sale.

There is a large literature (starting from Wheaton, 1990, and followed by many others) that studies frictions in the housing market using a search-and-matching model as done here. See, for example, Albrecht, Anderson, Smith and Vroman (2007), Anenberg and Bayer (2013), Caplin and Leahy (2011), Coles and Smith (1998), Díaz and Jerez (2013), Krainer (2001), Head, Lloyd-Ellis and Sun (2014), Moen, Nenov and Sniekers (2014), Ngai and Tenreyro (2014), Novy-Marx (2009), Piazzesi and Schneider (2009), Piazzesi, Schneider and Stroebel (2015), and the survey by Han and Strange (2014).[2] The key contribution of this paper to the literature is in studying moving house and showing its importance to understanding sales volumes. Moving is exogenous in the earlier papers with the exception of Guren (2014) and Hedlund (2013), but those papers focus on price fluctuations and foreclosures.[3]

An endogenous moving decision is analogous to the endogenous job-separation decision introduced in Mortensen and Pissarides (1994) for labour markets, but there is an important difference in the nature of the arrival process of new match quality. There, when an existing match is subject to an idiosyncratic shock, a new match quality is drawn independently of the match quality before the shock (the stochastic process for match quality is 'memory-less'). Here, idiosyncratic shocks degrade match quality, but an initially higher-quality match remains of a higher quality than a lower-quality match hit by the same shock (match quality is persistent).

This difference matters because it turns out that when match quality is persistent, the moving rate is affected by the transaction threshold as well as the moving threshold. More importantly, persistence of match quality makes the modelling of moving closer to an investment decision that is influenced by a predetermined stock of existing match quality. This means aggregate moving rates can display high volatility for similar reasons that capital investment rates do. Endogenous moving is also present in Guren (2014), though the focus there is on how the existence of a concave demand curve can amplify price insensitivity in housing market, helping to account for the positive autocorrelation of house-price changes (momentum). Endogenous moving is modelled there by assuming homeowners face idiosyncratic shocks to the cost of moving from their current houses. Like Mortensen and Pissarides (1994), the moving decision is effectively 'memory-less' (the focus is

[2]See Davis and Van Nieuwerburgh (2014) for a survey of housing and business cycles, including models without search frictions.

[3]Karahan and Rhee (2013) use a similar framework to Hedlund (2013) and focus on when a homeowner chooses to move to another region to look for a job.

mainly on how moving decisions react to short-run expectations of price changes).

The plan of the paper is as follows. First, section 2 uses NAR data to substantiate the claim that changes in the listing rate are the key determinant of housing-market activity. Section 3 presents the model of endogenous moving, and section 4 solves for the equilibrium analytically. Section 5 uses a calibration of the model to study developments in the U.S. economy that can rationalize the observed behaviour of the housing market during the 1995–2004 period. Section 6 concludes.

2 The importance of changes in the moving rate

The existing literature on housing markets has focused mainly on the decision processes of buyers and sellers that lead to sales of houses on the market. This section presents evidence showing that the decision of homeowners to put their houses up for sale is important not only for understanding the behaviour of listings, but also crucial for understanding sales and overall housing-market activity.

2.1 The basic idea

It is possible to grasp the relative importance of changes in sales and listing rates with some minimal empirical discipline combined with a basic stock-flow accounting identity.

A stock-flow accounting identity is a natural starting point when thinking about any market with search frictions:

$$\dot{u}_t = n(1 - u_t) - s u_t, \qquad [2.1]$$

where u_t is houses for sale as a fraction of the stock of all houses, $\dot{u}_t$ is the derivative of u_t with respect to time t, s is the rate at which houses for sale are sold, and n is the rate at which homeowners decide to move. Given s and n, steady-state houses for sale (where $\dot{u}_t = 0$) are

$$u = \frac{n}{s+n},$$

and the volume of sales S in the steady state (as a fraction of the stock of all houses) is:

$$S = \frac{sn}{s+n}. \qquad [2.2]$$

Convergence to the steady state occurs at rate $s+n$ (the coefficient of u_t in [2.1]), and given that houses sell relatively quickly (a few months on average), s is large enough that transitional dynamics are of limited importance. Therefore, understanding the evolution of the volume of sales over any period of time longer than a few months is mainly a question of understanding how changes in s and n affect equation [2.2]. The total derivative of sales volume S is:

$$\frac{\mathrm{d}S}{S} = \frac{s}{s+n}\frac{\mathrm{d}n}{n} + \frac{n}{s+n}\frac{\mathrm{d}s}{s}.$$

This implies the relative size of the effects on sales volume of a one percent change in n and a one percent change in s depends on the ratio of the sales rate s to the moving rate n.

The average time taken to sell a house is $1/s$, and the average time homeowners spend living in a house is $1/n$. The impact of a change in the moving rate relative to the same proportional change in the sales rate is therefore related to the ratio of the average time spent in a house to the average time to sell. The former is more than a decade and the latter is a few months, suggesting a ratio s/n of around 30. Since this means $n/(s+n)$ is very small, huge changes in sales rates would be required to have any significant lasting effect on sales volume.

Intuitively, with no change in the moving rate, the stock of houses for sale would be rapidly depleted by faster sales, leaving overall sales volume only very slightly higher. On the other hand, since $s/(s+n)$ is close to one, changes in the moving rate can have a large and lasting impact on the volume of sales. This is because even if the moving rate increased significantly, as the stock of potential movers is so large relative to the stock of houses for sale (the ratio $(1-u)/u$ is also equal to s/n), this can have a sustained impact on the number of homeowners who move. Therefore, any attempt to understand sustained changes in sales volume will founder without accounting for changes in the moving rate.

2.2 Empirical evidence

The quantitative importance of the claims above can be seen by using data on house sales and inventories of unsold houses to construct a measure of new listings (the number of houses put up for sale). Let N_t denote the inflow of houses that come on to the market during month t (new listings), and let S_t denote sales (the outflow from the market) during that month. If I_t denotes the beginning-of-month t inventory (or end-of-month $t-1$) then the stock-flow accounting identity is:

$$N_t = I_{t+1} - I_t + S_t. \qquad [2.3]$$

NAR provides monthly estimates of sales of houses during each month and inventories of houses for sale at the end of each month for existing homes including single-family homes and condominiums.[4] The focus here is on data for single-family homes, which represent 90% of total sales of existing homes. Monthly data on sales and inventories covering the period from January 1989 to June 2013 are first deseasonalized.[5] The data are then converted to quarterly series to smooth out excessive volatility owing to possible measurement error. Quarterly sales are the sum of the monthly sales numbers, and quarterly inventories are the level of inventories at the beginning of the first month of a quarter.

A quarterly listings series N_t is constructed that satisfies the accounting identity [2.3]. Assuming inflows N_t and outflows S_t both occur uniformly within a time period, the average number of houses

[4] The methodology and recent data are available at http://www.realtor.org/research-and-statistics/housing-statistics. The NAR data on inventories and sales are for existing homes, so newly constructed houses are excluded.

[5] Multiplicative monthly components are removed from the data.

U_t available for sale during quarter t is equal to:

$$U_t = I_t + \frac{N_t}{2} - \frac{S_t}{2} = \frac{I_t + I_{t+1}}{2}. \qquad [2.4]$$

Since the time series for inventories I_t is quite persistent, the measure U_t of the number of houses for sale turns out to be highly correlated with inventories (the correlation coefficient is equal to 0.99). Using the constructed series U_t for houses for sale, the sales rate is $s_t = S_t/U_t$.[6]

The listing rate is defined as the ratio of new listings N_t to the total stock of houses not already for sale. There is no comparable monthly or quarterly series for the total housing stock during the period covered by the sales and inventory data. The listing rate is calculated using a measure K of the average housing stock, that is, $n_t = N_t/(K - U_t)$.[7] The quantitative analysis later in the paper will make a correction for estimated changes in the housing stock over time. Figure 1 plots sales, listings, the sales rate, the listing rate, and houses for sale as differences in log points relative to the first quarter of 1989.

A simple counterfactual exercise is used to quantify the importance of the listing rate in understanding changes in housing-market activity. Given that convergence to the steady state in equation [2.2] occurs within a few months, the evolution of sales volume over time can be understood through the lens of the following equation:

$$S_t^* = \frac{s_t n_t}{s_t + n_t}, \qquad [2.5]$$

where s_t and n_t are the empirical sales and listing rates. The variable S_t^* is what the steady-state sales volume would be at each point in time given the (time-varying) sales and listing rates. The correlation between S_t^* and actual sales volume S_t is quite high (the correlation coefficient is 0.88), which is not surprising given that convergence to the steady state is fast.[8] To see the relative importance of the sales and listing rates, consider what [2.5] would be if each in turn of the two rates were held constant at its sample average:

$$S_{s,t}^* = \frac{s_t \bar{n}}{s_t + \bar{n}}, \quad \text{and} \quad S_{n,t}^* = \frac{\bar{s} n_t}{\bar{s} + n_t},$$

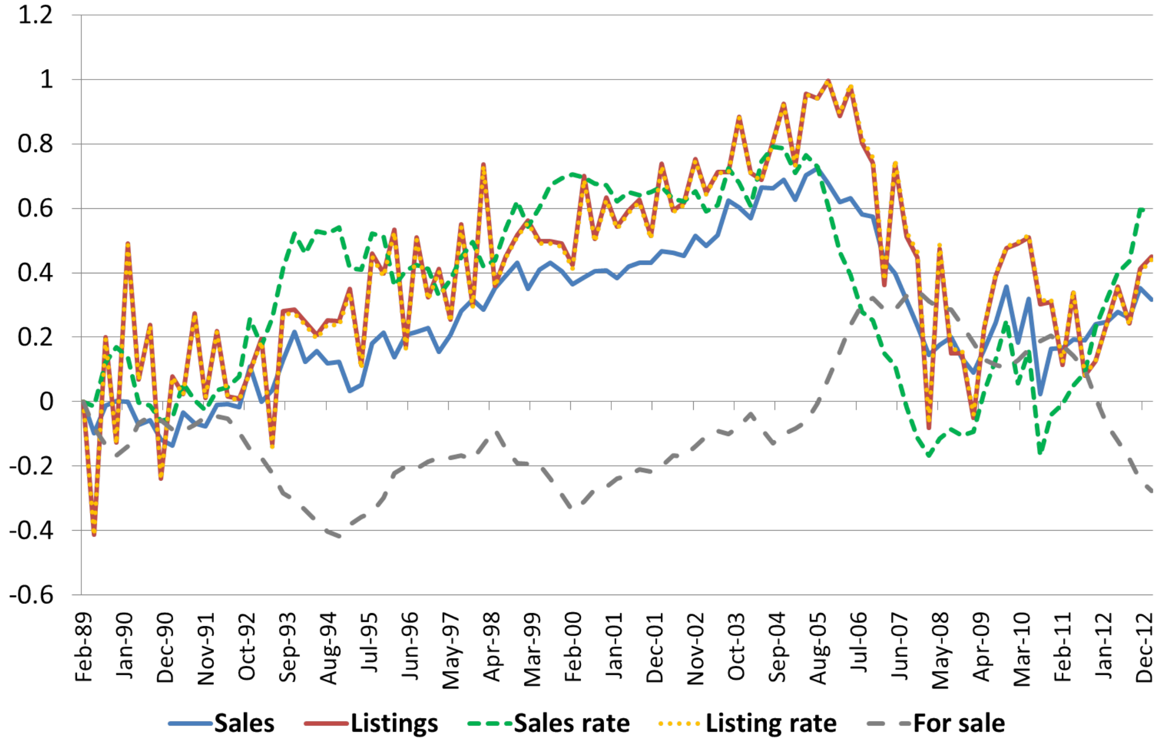
where $\bar{s}$ and $\bar{n}$ are the average sales and listing rates. The time series of S_t^*, $S_{s,t}^*$, and $S_{n,t}^*$ are plotted in Figure 2 below as log differences. It is striking, yet consistent with the basic idea set out above, that even large changes in the sales rate account for almost none of the large variation over time in

[6] The average time taken to a sell a house is the reciprocal of the sales rate. Time-to-sell is highly correlated with the 'months supply' variable reported by NAR, which is defined as inventories divided by sales. Mean time-to-sell is 6.7 months, compared to 6.5 for 'months supply'.

[7] The housing stock is the stock of single-family homes excluding those occupied by renters according to the American Housing Survey. More precisely, it is the sum of 'owner occupied' and 'vacant for sale' in Table 1A–1. The main effect of the value of K is on the average value of the listing rate. The expected time a homeowner will live in a newly purchased house is the reciprocal of the average listing rate, which is approximately 14.4 years. This is similar to direct estimates of the average time spent in a house from the American Housing Survey.

[8] As a robustness check, appendix A.1 performs the counterfactual exercise without any assumption on the speed of convergence to the steady state.

Figure 1: *Housing market activity*



Notes: Series are logarithmic differences from the initial data point. Monthly data (January 1989–June 2013), seasonally adjusted, converted to quarterly series. Definitions are given in section 2.2.
Source: National Association of Realtors

the volume of sales.

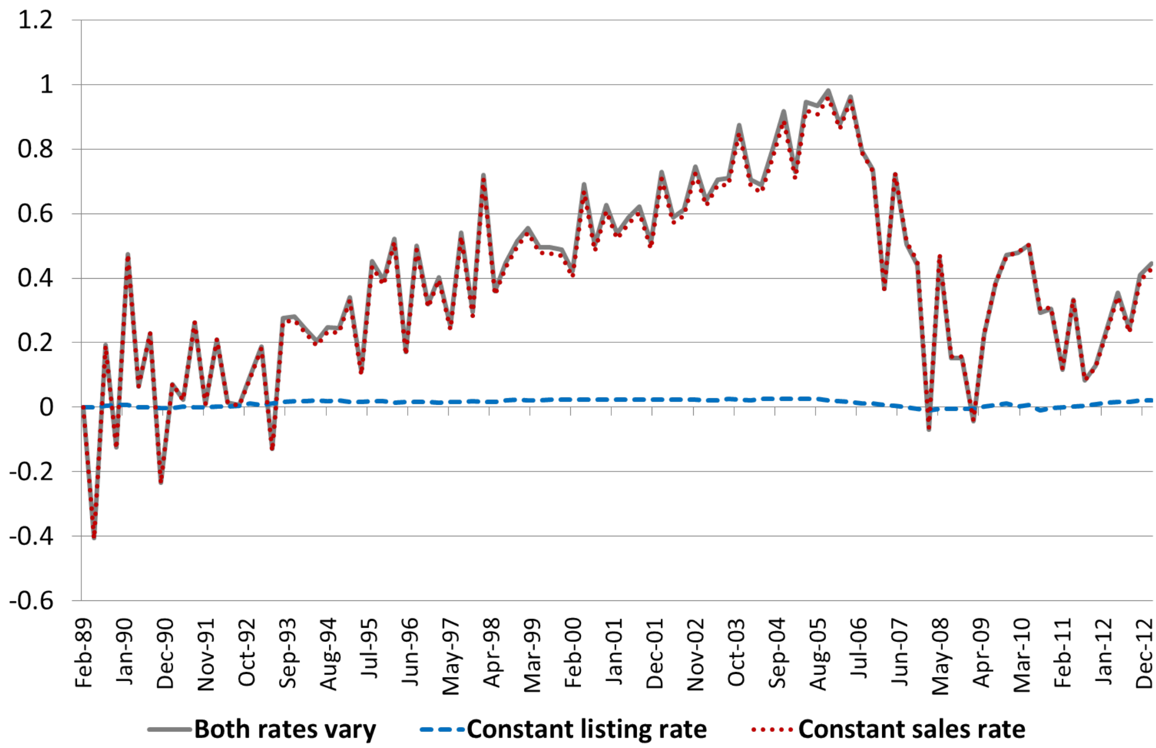
This section has shown that understanding changes in listings can be important not only for its own sake but also for understanding other important housing-market variables such as sales volume. The next section builds a tractable model with both endogenous listings and sales.

3 A model of investment in housing match quality

This section presents a search-and-matching model of the housing market that studies both the decision of when to move for an existing homeowner, and the buying and selling decisions of those in the market to buy a new house or sell their current house. The model focuses on the market for existing houses.[9]

[9]It abstracts from new entry of houses from either new construction or previously rented houses, and abstracts from the entry of first-time buyers to the market.

Figure 2: *Decomposition of steady-state sales volume*



Notes: The construction of these series is described in section 2.2. The series are reported as log differences relative to their initial values.

3.1 Match quality

The mere existence of an inventory of houses for sale together with a group of potential buyers indicates the presence of search frictions in the housing market. There are broadly two kinds of search frictions: the difficulty of buyers and sellers meeting each other, and the difficulty for buyers of knowing which properties would be a good match prior to viewing them. The first friction is usually modelled using a meeting function.[10] The second friction relates to the number of properties that buyers would need to view before a desirable property is found.

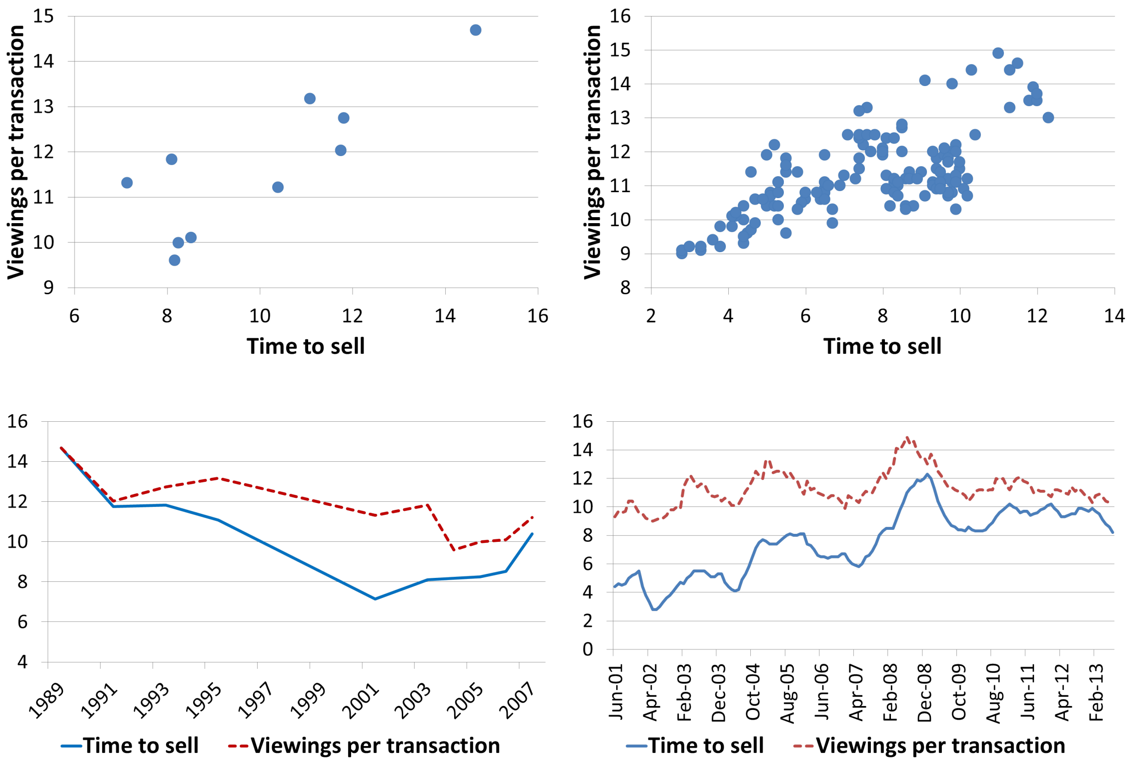
Whether a property is desirable is not easily determined simply by knowing objective features such as the number of bedrooms. What is desirable is a good match between the idiosyncratic preferences of the buyer and the idiosyncratic characteristics of the house for sale. This type of search friction can be modelled as a stochastic match-specific quality that only becomes known to a buyer when a house is actually viewed. The first friction can then be seen as an initial step in locating houses for sale that meet a given set of objective criteria such as size, and the second friction can be seen as the time needed to view the houses and judge the match quality between the buyer and the house.

A measure of the importance of the second friction is the average number of viewings needed

[10]The term 'matching function' is not used here because not all viewings will lead to matches.

before a house can be sold (or equivalently, before a buyer can make a purchase), referred to here as viewings-per-transaction. Genesove and Han (2012) report data on the number of homes visited using the 'Profile of Buyers and Sellers' surveys from the National Association of Realtors (NAR) in the U.S. for various years from 1989 to 2007. In the UK, monthly data on time-to-sell and viewings-per-sale are available from the Hometrack 'National Housing Survey' from June 2001 to July 2013.[11] The data are shown in Figure 3.[12] Viewings-per-transaction are far greater than one, indicating that there is substantial uncertainty about match quality prior to a viewing. The figure illustrates that variation in time-to-sell is associated with movements in viewings-per-transaction in the same direction, and is not simply due to variation in the time taken to meet buyers, in other words, a meeting function alone is not sufficient.

Figure 3: *Viewings per transaction and time to sell*



Notes: Left panels, U.S. data, annual frequency (years in sample: 1989, 1991, 1993, 1995, 2001, and 2003–2007); Right panels, U.K. data, monthly frequency (June 2001–July 2013). Time-to-sell is measured in weeks.
Source: U.S. data, Genesove and Han (2012); U.K. data, Hometrack (`www.hometrack.co.uk`).

The existence of multiple viewings per sale indicates that the quality of the match with a par-

[11] Hometrack data are based on a monthly survey starting in 2000. The survey is sent to estate agents and surveyors every month. It covers all postcodes of England and Wales, with a minimum of two returns per postcode. The results are aggregated over postcodes weighted by the housing stock.

[12] Correctly measured, both homes visited and viewings-per-sale are equal to viewings-per-transaction. However, if houses are listed with multiple realtors then viewings-per-sale might underestimate the number of viewings per transaction.

ticular house varies among potential buyers. Given an initial level of match quality when a buyer moves in, and the plausible assumption of some persistence over time in match quality, it is natural to think that moving is not simply exogenous: there is a comparison of what homeowners already have to what they might hope to gain.

3.2 Houses

There is an economy with a unit continuum of families and a unit continuum of houses. Each house is owned by one family (though families can in principle own multiple houses). Each house is either occupied by its owning family and yields a stream of utility flow values, or is for sale on the market while the family searches for a buyer.[13] A family can occupy at most one house at any time. If all a family's houses are on the market for sale, the family is in the market searching for a house to buy and occupy.

It is implicit in the model that families moving house might temporarily use the rental market in between selling and buying. However, there is no explicit modelling of the rental market: effectively, this is treated as a distinct segment of the housing market. This view is consistent with Glaeser and Gyourko (2007) who argue that there is little evidence in support of significant arbitrage between the rental and owner-occupied segments of the housing market because owner-occupied homes typically have different characteristics from rental units, as is also the case for homeowners themselves in comparison to renters. More recently, Bachmann and Cooper (2014) calculate gross flows across and within the owner and renter categories using PSID data. They conclude that rental and owner-occupied markets are distinct segments owing to the dominance of moves within the same tenure category. Moreover, between 1970 and 2009, they find that most house sales are associated with own-to-own moves, rather than own-to-rent moves (the former is 2.3 times the latter), suggesting the majority of owners selling their houses are buying another house.[14]

3.3 Behaviour of homeowners

To understand the decision to move, the key variable for homeowners is their match quality ϵ, which will be compared to owners' outside option of search. Match quality is the idiosyncratic utility flow value of an occupied house. This is match specific in that it is particular to both the house and the family occupying it. A homeowner with match quality ϵ receives a utility flow value of $\epsilon\xi$ over time while the house is occupied, where ξ is a variable representing the exogenous economy-wide level of housing demand. Homeowners also incur a flow maintenance cost M irrespective of whether houses

[13] The model abstracts from the possibility that those trying to sell will withdraw from the market without completing a sale.

[14] Using a different data source (AHS data from 2001), Wheaton and Lee (2009) find 42.6% of house purchases are by existing homeowners, as opposed to renters and newly formed households. To reconcile this with the conclusion drawn from Bachmann and Cooper's (2014) facts that the majority of owners selling their houses are buying another house, note the following observations. First, using Wheaton and Lee's (2009) data, 57.1% of listings of existing houses occur through own-to-own transitions, rather than through own-to-rent or owner exit. Second, some own-to-rent and rent-to-own transitions may be extremely short lived as part of what is effectively an own-to-own move, for example, someone who lives temporarily in a rented home while a newly purchased home is under refurbishment.

are occupied or on the market for sale. The variable ξ is common to all homeowners, whereas ϵ is match specific. Moving decisions will lead to an endogenous distribution of match quality across homeowners.

Match quality ϵ is a persistent variable. However, families are sometimes subject to idiosyncratic shocks that degrade match quality. These shocks can be thought of as life events that make a house less well suited to the family's current circumstances. The arrival of these shocks follows a Poisson process with arrival rate a (time is continuous). If a shock occurs, match quality ϵ is scaled down from ϵ to $\delta\epsilon$, where δ is a parameter that determines the size of the shocks ($\delta < 1$). If no shock occurs, match quality remains unchanged. Following the arrival of an idiosyncratic shock, a homeowner can decide whether or not to move. Those who move become both a buyer and a seller simultaneously.

Those who do not experience an idiosyncratic shock face a cost D if they decide to move.[15] For tractability, the model is set up so that a homeowner will always choose not to move in the absence of an idiosyncratic shock (formally, this is done by assuming the limiting case of $D \to \infty$).[16]

The decision of whether or not to move for those who receive shocks depends on all relevant variables including homeowners' own idiosyncratic match quality, and current and expected future conditions in the housing market. The value function for a homeowner occupying a house with match quality ϵ at time t (after a decision not to move has been made) is denoted by $H_t(\epsilon)$. The derivative of the value function with respect to time is denoted by $\dot{H}_t(\epsilon)$. The Bellman equation for $H_t(\epsilon)$ is

$$rH_t(\epsilon) = \epsilon\xi - M + a\left(\max\left\{H_t(\delta\epsilon), W_t\right\} - H_t(\epsilon)\right) + \dot{H}_t(\epsilon), \qquad [3.1]$$

where r is the discount rate, and W_t is the sum of the values of being a buyer and owning a house for sale. The value function $H_t(\epsilon)$ is increasing in ϵ. Thus, when a shock to match quality is received, a homeowner decides to move if match quality ϵ is now below a 'moving threshold' x_t defined by:

$$H_t(x_t) = W_t. \qquad [3.2]$$

This equates the value of a marginal homeowner to the outside option of selling an existing house and searching for a new one. The values of being a buyer and a seller are now characterized.

3.4 Search behaviour

The housing market is subject to two types of search frictions. First, it is time-consuming for buyers and sellers to arrange viewings of houses. Let u_t denote the measure of houses available for sale and b_t the measure of buyers. At any instant, each buyer and each house can have at most one viewing.

[15] This cost represents the 'inertia' of families to remain in the same house, which is in line with empirical evidence. According to the American Housing Survey and the Survey of English Housing, common reasons for moving include being closer to schools, closer to jobs, or because of marriage or divorce.

[16] The assumption of a positive D for those who do not receive idiosyncratic shocks has no consequences for the analysis of the steady state of the model. Furthermore, even in the analysis of the model's dynamics, if aggregate shocks are small in relation to the size of transaction costs then the assumption of a positive D has no consequences for those homeowners who have not yet received an idiosyncratic shock.

The arrival rate of viewings is determined by the meeting function $\mathcal{V}(u_t, b_t)$. For houses, viewings have Poisson arrival rate $\mathcal{V}(u_t, b_t)/u_t$. For buyers, the corresponding arrival rate is $\mathcal{V}(u_t, b_t)/b_t$. During this process of search, buyers incur flow search costs F (and homeowners continue to incur maintenance costs M). The meeting function $\mathcal{V}(u_t, b_t)$ is assumed to have constant returns to scale.

Given the unit measure of houses, there are $1 - u_t$ houses that are matched in the sense of being occupied by a family. As there is also a unit measure of families, there must be u_t families not matched with a house, and thus in the market to buy. This means the measures of buyers and sellers are the same ($b_t = u_t$). The arrival rates of viewings for buyers and sellers are then both equal to the constant $v = \mathcal{V}(u_t, u_t)/u_t$. This arrival rate summarizes all that needs to be known about the frictions in locating houses to view.[17]

The second aspect of the search frictions is heterogeneity in buyer tastes and the extent to which any given house will conform to these. As a result of this friction, not all viewings will actually lead to matches.[18] When a viewing takes place, match quality ϵ is realized from a distribution with cumulative distribution function $G(\epsilon)$. For analytical tractability, new match quality is assumed to be drawn from a Pareto distribution (with minimum value 1 and shape parameter λ satisfying $\lambda > 1$):

$$G(\epsilon) = 1 - \epsilon^{-\lambda}. \quad [3.3]$$

When a viewing occurs, the value of ϵ that is drawn becomes common knowledge among the buyer and the seller. The value to a family of occupying a house with match quality ϵ is $H_t(\epsilon)$. By purchasing and occupying this house, the buyer loses the option of continuing to search, with the value of being a buyer denoted by B_t. If the seller agrees to an offer to buy, the gain is the transaction price, and the loss is the option value of continuing to search, with the value of owning a house for sale denoted by U_t ('unsatisfied owner'). Finally, the buyer and seller face a combined transaction cost C. The total surplus resulting from a transaction with match quality ϵ at time t is given by

$$\Sigma_t(\epsilon) = H_t(\epsilon) - W_t - C, \quad [3.4]$$

where $W_t = B_t + U_t$ is the combined buyer and seller value function. Given that $H_t(\epsilon)$ is increasing in ϵ, purchases will occur if match quality ϵ is no lower than a threshold y_t, defined by $\Sigma_t(y_t) = 0$. This is the 'transaction threshold'. Intuitively, given that ϵ is observable to both buyer and seller, and the surplus is transferable between the two, the transactions that occur are those with positive

[17] There is no role for 'market tightness' (b/u) here. Limited data on time-to-buy from Genesove and Han (2012) show that there is a strong positive correlation at annual frequencies between time-to-buy and time-to-sell (years in sample: 1987, 1989, 1991, 1993, 1995, 2001, 2003–2007), suggesting market tightness cannot be the dominant factor. If it were, time-to-buy and time-to-sell would be negatively correlated.

[18] The two search frictions are also present in the labour-market model of Pissarides (1985), who combines the meeting function with match quality, where the latter is the focus of Jovanovic (1979). Both frictions also feature in Novy-Marx (2009).

surplus.[19] The transactions threshold y_t satisfies the following equation:

$$H_t(y_t) = W_t + C. \tag{3.5}$$

The combined value W_t satisfies the Bellman equation:

$$rW_t = -F - M + v \int_{y_t} (H_t(\epsilon) - W_t - C) \, \mathrm{d}G(\epsilon) + \dot{W}_t. \tag{3.6}$$

Intuitively, the first term on the right-hand side captures the flow costs of being a buyer and a seller, while the second term is the combined expected surplus from searching for a house and searching for a buyer.

3.5 Price determination

While the equations [3.5] and [3.6] for the transactions threshold y_t and the value function W_t do not depend on the specific price-setting mechanism, this section briefly discusses price determination under Nash bargaining.

Suppose the seller has bargaining power ω. The buyer and the seller directly bear transaction costs C_b and C_v (with $C_\mathrm{b} + C_\mathrm{v} = C$). The individual value functions of buyers and sellers are B_t and U_t respectively. If a house with match quality ϵ is sold at a price $p_t(\epsilon)$, the surpluses of the buyer and seller are:

$$\Sigma_{\mathrm{b},t}(\epsilon) = H_t(\epsilon) - p_t(\epsilon) - C_\mathrm{b} - B_t, \quad \text{and} \quad \Sigma_{\mathrm{u},t}(\epsilon) = p_t(\epsilon) - C_\mathrm{v} - U_t, \tag{3.7}$$

with $\Sigma_{\mathrm{b},t}(\epsilon) + \Sigma_{\mathrm{u},t}(\epsilon) = \Sigma_t(\epsilon)$ being the total surplus given in [3.4]. The Bellman equations for the buyer and seller value functions are:

$$rB_t = -F + v \int_{y_t} \Sigma_{\mathrm{b},t}(\epsilon) \mathrm{d}G(\epsilon) + \dot{B}_t, \quad \text{and} \quad rU_t = -M + v \int_{y_t} \Sigma_{\mathrm{u},t}(\epsilon) \mathrm{d}G(\epsilon) + \dot{U}_t. \tag{3.8}$$

The Nash bargaining solution implies the surplus-splitting equation $(1-\omega)\Sigma_{\mathrm{u},t}(\epsilon) = \omega \Sigma_{\mathrm{b},t}(\epsilon)$, which determines the transaction price $p_t(\epsilon)$. As shown in appendix A.3, the average transactions price P_t is

$$P_t = \frac{\omega}{1 - G(y_t)} \int_{y_t} H_t(\epsilon) \mathrm{d}G(\epsilon) + (1-\omega)C_\mathrm{v} - \omega C_\mathrm{b} + \frac{\omega F - (1-\omega)M}{r}. \tag{3.9}$$

The ratio of the seller's transaction cost C_v to the total transaction cost C is subsequently denoted by κ, and the model will be parameterized in terms of C and κ rather than C_b and C_v.

[19] Some extra assumptions are implicit in this claim, namely that there is no memory of past actions, so refusing an offer yields no benefit in terms of future reputation.

3.6 Stocks and flows

Let n_t denote the rate at which the stock $1-u_t$ of houses occupied by their owners are listed (put up for sale), and let s_t denote the rate at which the stock u_t of houses for sale are sold. The accounting identity that connects stocks and flows is

$$\dot{u}_t = n_t(1-u_t) - s_t u_t. \qquad [3.10]$$

The listing (inflow) and sales (outflow) rates n_t and s_t are endogenously determined by the moving decisions of individual homeowners and the transactions decisions of individual buyers and sellers.

Given that sales occur when the match quality ϵ from a viewing exceeds the transactions threshold y_t, and using the Pareto distribution of new match quality [3.3], the sales rate s_t is:

$$s_t = v\pi_t, \quad \text{with } \pi_t = y_t^{-\lambda}, \qquad [3.11]$$

where π_t is the proportion of viewings for which match quality is above the transactions threshold y_t. This term captures the second search friction due to buyers' idiosyncratic tastes. The first friction is captured by the viewing rate v.

The moving rate n_t is derived from the distribution of existing match quality among homeowners together with the moving threshold x_t. The evolution over time of the distribution of match quality depends on the idiosyncratic shocks and moving decisions. The derivation of the moving rate is much more complicated than the sales rate. Surviving matches differ along two dimensions: (i) the initial level of match quality, and (ii) the number of shocks received since the match formed. By using the Pareto distribution assumption [3.3] for new match quality, the following moving rate n_t is derived in appendix A.4:

$$n_t = a - \frac{a\delta^\lambda x_t^{-\lambda} v}{1-u_t} \int_{\tau \to -\infty}^{t} e^{-a(1-\delta^\lambda)(t-\tau)} u_\tau \mathrm{d}\tau. \qquad [3.12]$$

This equation demonstrates that given the moving threshold x_t, the moving rate n_t displays history dependence. The reason is the persistence in the distribution of match quality among existing homeowners.

The tractability that results from the Pareto distribution assumption comes from the property that a truncated Pareto distribution is also a Pareto distribution with the original shape parameter. Together with the nature of the idiosyncratic shock process, this is what allows the explicit expression [3.12] to be derived. This property of the truncated Pareto distribution is also useful in calculating the expected surplus from searching for a new house taking into account future moving decisions. This is because matches receiving idiosyncratic shocks will survive only if $\delta\epsilon > x$, so the calculation of the value function involves only an integral starting from x/δ. This integral can be easily obtained with the Pareto distribution $(\underline{\epsilon}, \lambda)$ because its probability density function is only a function of $\epsilon/\underline{\epsilon}$.[20]

[20] There is an analogy with Ss models of price adjustment where assumptions on the distribution of idiosyncratic shocks are often made so that the general shape of the distribution of prices is preserved after the truncations following

The occurrence of idiosyncratic shocks and the moving and buying decisions affect the distribution of match quality across all current homeowners. Summarizing this distribution by its first moment, let Q_t denote the average of match quality ϵ. The law of motion of Q_t is shown in appendix A.4 to be:

$$\dot{Q}_t = s_t \frac{u_t}{1-u_t}\left(\frac{\lambda}{\lambda-1}y_t - Q_t\right) - (a-n_t)\left(Q_t - \frac{\lambda}{\lambda-1}x_t\right). \qquad [3.13]$$

Average match quality Q_t is a state variable owing to the persistence of individual match qualities. The rate of change of match quality is increasing in both the moving and transaction thresholds x_t and y_t.

4 The equilibrium of the model

The equilibrium of the model is derived in two stages. First, moving and transaction thresholds x and y are obtained. The transactions threshold y determines the sales rate s, and both moving and transaction thresholds x and y determine the moving rate n. These then determine the volumes of sales and listings, and thus the stock of houses for sale. Throughout, the focus is on the case of perfect foresight with respect to the parameters of the model (no changes in these parameters are anticipated).

4.1 The moving and transactions thresholds

The analysis assumes a case where the idiosyncratic shock is large enough to induce a homeowner with match quality y (a marginal homebuyer) to move, that is, $\delta y < x$. This is true when the parameters of the model satisfy the condition in equation [4.8] below.

When $\delta y < x$, it follows from the homeowner's value function [3.1] that the value for a marginal homebuyer satisfies:

$$(r+a)H(y) = \xi y + aW. \qquad [4.1]$$

Using equation [3.1] again, the value for a marginal homeowner (in the sense of being indifferent between remaining a homeowner or moving) satisfies:

$$(r+a)H(x) = \xi x + aW. \qquad [4.2]$$

These two values are related as follows using the definitions of the moving and transaction thresholds in [3.2] and [3.5]:

$$H(y) = H(x) + C. \qquad [4.3]$$

price changes. See Gertler and Leahy (2008) for a recent example using the uniform distribution.

Equations [4.1]–[4.3] together imply that:

$$y - x = \frac{(r+a)C}{\xi}, \qquad [4.4]$$

which is the first equilibrium condition linking the moving and transaction thresholds x and y.

A second equilibrium condition connecting x and y is obtained by deriving the combined buyer and seller value W as a function of the moving and transactions thresholds. First, from the definition of the moving threshold x, equations [3.2] and [4.2] together imply that:

$$H(x) = W = \frac{x\xi}{r}. \qquad [4.5]$$

Second, the value W can be obtained directly from the flow value equation [3.6] by computing the surplus from a match. The expected surplus from a new match is shown in appendix A.2 to be given by:

$$\int_y (H(\epsilon) - W - C)\mathrm{d}G(\epsilon) = \frac{\xi\left(y^{1-\lambda} + \frac{a\delta^\lambda}{r+a(1-\delta^\lambda)}x^{1-\lambda}\right)}{(r+a)(\lambda-1)}. \qquad [4.6]$$

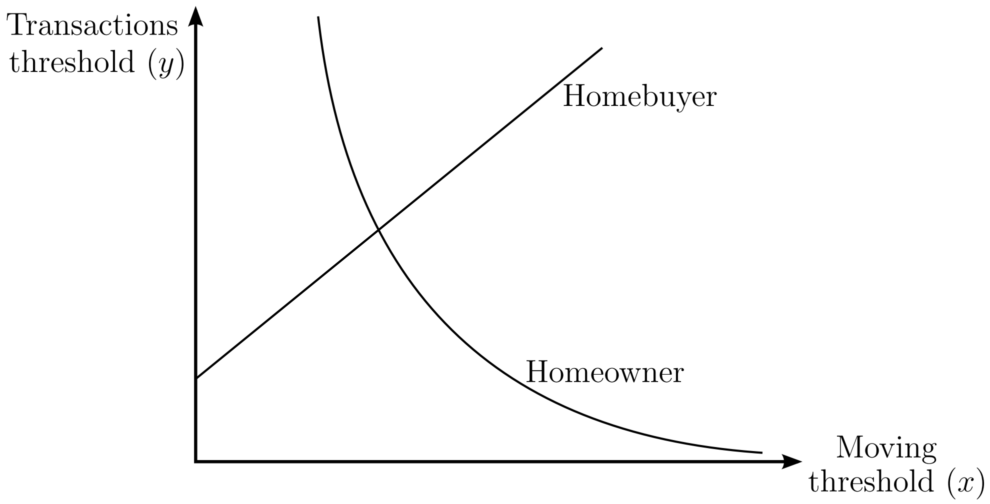
Allowing for moving decisions means that the expected surplus of a new match depends not only on the transactions threshold y but also on the moving threshold x. Combining this equation with [3.6] and [4.5] yields the second equilibrium condition linking x and y:

$$x = \frac{v\left(y^{1-\lambda} + \frac{a\delta^\lambda}{r+a(1-\delta^\lambda)}x^{1-\lambda}\right)}{(r+a)(\lambda-1)} - \frac{F}{\xi}. \qquad [4.7]$$

Together, equations [4.4] and [4.7] can be jointly solved for the thresholds x and y without reference to state variables such as the number of houses for sale or the distribution of existing match quality. Figure 4 depicts the determination of the moving and transaction thresholds as the intersection between an upward-sloping equation [4.4] and a downward-sloping equation [4.7]. Intuitively, the upward-sloping line ties the value of a marginal homebuyer to that of a marginal homeowner together with the transaction cost (which is sunk for someone who has decided to become a buyer, but not for an existing homeowner who can choose to stay). This line is referred to as the 'homebuyer' curve. The downward-sloping curve ties the value of the marginal homeowner to the expected value of becoming a buyer. This line is referred to as the 'homeowner' curve.

In (x, y) space, these two curves pin down the equilibrium values of x and y. If an equilibrium exists, it must be unique. It is shown in appendix A.2 that an equilibrium satisfying the conditions

Figure 4: *Determination of the moving (x) and transactions (y) thresholds*



Notes: The homebuyer and homeowner curves represent equations [4.4] and [4.7] respectively.

$y > 1$ and $\delta y < x$ exists if and only if:

$$\max\left\{1, \frac{(r+a)C}{(1-\delta)\xi}\right\}^{1-\lambda} + \frac{a\delta^\lambda}{r+a(1-\delta^\lambda)}\left(\max\left\{1, \frac{(r+a)C}{(1-\delta)\xi}\right\} - \frac{(r+a)C}{\xi}\right)^{1-\lambda} - \frac{(\lambda-1)(r+a)}{v}\left(\max\left\{1, \frac{(r+a)C}{(1-\delta)\xi}\right\} - \frac{(r+a)C}{\xi} + \frac{F}{\xi}\right) > 0. \quad [4.8]$$

4.2 Efficiency

The equilibrium of a search-and-matching model is often Pareto inefficient owing to the presence of externalities. For example, a 'congestion externality' occurs when individuals do not take account of the extra difficulty in finding a match faced by others when they enter the market. Furthermore, there is typically a 'hold-up' problem whereby one party is able to extract surplus from another after sunk costs of search have been incurred, which when anticipated reduces the incentive of the other to enter the market.

It turns out that equilibrium of this model is Pareto efficient. The equilibrium is the solution of the following social planner's problem with objective function

$$S_T = \int_{t=T}^{\infty} e^{-r(t-T)}\left(\xi(1-u_t)Q_t - Cs_t u_t - Fu_t - M\right) \mathrm{d}t, \quad [4.9]$$

where Q_t, s_t, and u_t are the average match quality, sales rate, and stock of unsold houses resulting from the planner's choice of the moving and transaction thresholds. It is shown in appendix A.5 that maximizing this objective function subject to the laws of motion derived earlier leads to exactly the same moving and transaction thresholds as the equilibrium of the model.

Intuitively, the usual congestion externality is not present here because homeowners who decide to move enter the market simultaneously on both sides as a buyer and as a seller. Owing to the constant-returns meeting function, entry has no effect on the likelihood of any other participant in the market meeting a buyer or a seller. Furthermore, the hold-up problem is also absent, again because homeowners enter on both sides of the market. If participants on one side of the market are able to extract surplus from participants on the other side then homeowners entering the market knows that they will face hold up, but will also be able to hold up others. Ex ante, when moving decisions are made, these two effects are expected to cancel out. Private moving and transaction decisions thus result in a socially efficient allocation.

4.3 Steady state

Taking as given the moving and transaction thresholds x and y, there exists a unique steady state for all stocks and flows. This steady state naturally has $\dot{u}_t = 0$, but also the distribution of existing match quality must have converged to its ergodic limit, which in practice requires that both u_t and n_t are constant over time.

First, the transaction threshold y directly pins down the sales rate s using equation [3.11]:

$$s = vy^{-\lambda}. \tag{4.10}$$

The steady-state moving rate can be derived from [3.10] and [3.12] using $\dot{u}_t = 0$ and $\dot{n}_t = 0$:

$$n = \frac{a}{1 + \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda}. \tag{4.11}$$

Note that the term in the denominator can be expressed as

$$1 + \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda = 1 + \left(\frac{\delta y}{x}\right)^\lambda + \left(\frac{\delta^2 y}{x}\right)^\lambda + \ldots, \tag{4.12}$$

which is equal to the sum of the conditional survival probabilities (starting from $\epsilon > y$) after receiving k shocks, summing over $k = 0, 1, 2, \ldots$. When no shocks have been received, the match is of quality $y > x$, so the survival probability is 1. After $k \geq 1$ shocks, the conditional survival probability is $(\delta^k y/x)^\lambda$.

Given the steady-state sales and moving rates s and n, the steady-state for houses for sale follows immediately from [3.10]:

$$u = \frac{n}{s+n}. \tag{4.13}$$

Average match quality in the steady state is obtained from equation [3.13]:

$$Q = \frac{\lambda}{\lambda - 1}\left(\frac{n}{a}y + \left(1 - \frac{n}{a}\right)x\right), \tag{4.14}$$

which is a weighted average of the transaction and moving thresholds y and x, with the weight on

x depending on the degree to which moving is endogenous (that is, how far the moving rate n is below the arrival rate a of idiosyncratic shocks). The average transaction price P is:

$$P = C_\mathrm{v} - \frac{M}{r} + \omega \left(\frac{1}{r} + \frac{y^\lambda}{v} \right) (\xi x + F). \qquad [4.15]$$

4.4 Transitional dynamics and overshooting

Following a change in the moving and/or transaction thresholds, the housing market will begin a process of convergence to the new steady state for the volume of transactions, the moving rate, and the stock of houses for sale. There are two facets of these transitional dynamics. First, there is convergence in the stock of houses for sale given the inflow and outflow rates, which is a common feature of most search models. Second, endogenous moving together with persistence in existing match quality gives rise to a novel source of transitional dynamics as the distribution of match quality converges to its ergodic limit.

First, abstracting from any transitions in the match-quality distribution, if the moving threshold is constant then the moving rate will also be constant. Given constant sales and moving rates s and n, the dynamics of houses for sale can be described as follows using the law of motion [3.10] and the steady-state equation [4.13]:

$$\dot{u}_t = -(s + n)(u_t - u). \qquad [4.16]$$

The gap between u_t and u is closed over time at rate $s + n$. Since time-to-sell (the reciprocal of s) is fairly short on average (a few months), the speed of convergence to the steady state is sufficiently rapid that these dynamics are of limited interest (as is also the case for unemployment in the labour-market analogue of this model).

The second source of transitional dynamics can be isolated using the following method. Denote by u_t^* the hypothetical level of houses for sale if the first source of transitional dynamics is ignored:

$$u_t^* = \frac{n_t^*}{s_t + n_t^*},$$

where n_t^* is the moving rate implied by [3.12] when $u_t = u_t^*$. Associated with u_t^* and n_t^* is the volume of sales and listings $S_t^* = N_t^* = s u_t^* = n_t^*(1 - u_t^*)$. It is shown in appendix A.6 that the transitional dynamics of these variables are given by:

$$\frac{\dot{S}_t^*}{S_t^* - S} = \frac{\dot{N}_t^*}{N_t^* - N} = \frac{\dot{u}_t^*}{u_t^* - u} = -a(1 - \delta^\lambda) \left(\frac{a}{n} \right) \left(\frac{s + n}{s + a} \right), \quad \text{and}$$
$$\frac{\dot{n}_t^*}{n_t^* - n} = -a(1 - \delta^\lambda) \left(\frac{a}{n} \right) \left(\frac{s + n_t^*}{s + a} \right).$$

Even ignoring the inflow-outflow dynamics affecting houses for sale, the housing market would still not reach its steady state immediately. The equation above shows that S_t^*, N_t^*, and u_t^* converge monotonically to the steady state at a common rate. For n_t^*, the differential equation is not linear, but qualitatively, the pattern of convergence is the same as the other variables. Quantitatively, the rate of convergence is largely determined by a, and also the ratio of a/n (the final term in parentheses is close to one because s is large relative to a and n). Since moving is infrequent (homeowners live in their houses for more than a decade on average), the arrival rate a of the idiosyncratic shock cannot be too large, and therefore convergence in the match quality distribution is relatively slow.

It is also shown in appendix A.6 that starting from a steady state, the moving rate n_t^* immediately following a change in x (with y held constant) is

$$n_{t_+}^* = \frac{a - \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda s u_{t_-}^*}{1 + \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda u_{t_-}^*},$$

where $n_{t_+}^*$ denotes the value of n_t^* on impact and $u_{t_-}^*$ denotes the value of u_t^* prior to the change. Consider a case where the moving threshold x rises, which increases the steady-state moving rate so that $n > n_{t_-}^*$. The appendix shows $n_{t_+}^* > n > n_{t_-}^*$, so there is overshooting of the moving rate on impact, and this overshooting gradually declines according to transitional dynamics characterized earlier. Intuitively, a higher moving threshold x cleanses the distribution of match quality. As this process takes place, there is more moving in the short term, but the subsequent improvement in the match quality distribution means the long-term effect of the moving threshold on the moving rate is smaller.

4.5 Exogenous moving model

The general model embeds an exogenous moving model as a special case when $\delta = 0$. In this case any homeowner will move house after an idiosyncratic shock because match quality will drop to zero. Thus, the moving rate is the same as the exogenous arrival rate a of the idiosyncratic shock. By substituting equation [4.4] into [4.7] and setting $\delta = 0$, the transaction threshold y is the solution of the equation:

$$\frac{vy^{1-\lambda}}{(r+a)(\lambda-1)} = \left(y + \frac{F-(r+a)C}{\xi}\right), \qquad [4.17]$$

where the first term is the expected surplus from a new match when future moving occurs at an exogenous rate a. This changes the effective discount rate to $a+r$, which is applied to the expected flow surplus $vy^{1-\lambda}/(\lambda-1)$ from a new match. The inflow rate n is now simply a, whereas the outflow rate s is the same as in [4.10].

4.6 The importance of transaction costs

In the special case of zero transactions costs, the model has the surprising feature that its steady-state equilibrium is isomorphic to the exogenous moving model with the parameter a redefined as $a(1-\delta^\lambda)$. The logic behind this is that equation [4.4] implies $y = x$ when $C = 0$. From [4.11], this means that $n = a(1-\delta^\lambda)$, so the moving rate is independent of the equilibrium moving and transactions thresholds. Hence, only those parameters directly related to the shocks received by homeowners affect the moving rate. The equilibrium value of y is then determined by replacing x by y in equation [4.7] and simplifying to:

$$\frac{vy^{1-\lambda}}{(r + a(1-\delta^\lambda))(\lambda - 1)} = y + \frac{F}{\xi}.$$

This equation is identical to [4.17] for the exogenous moving model with $a(1-\delta^\lambda)$ replacing a. Therefore, all steady-state predictions of the two models would be the same if $C = 0$.

5 Application: 1995–2004 boom in housing-market activity

This section presents an application of the model to the period of booming activity in the housing market between 1995 and 2004. The housing-market crash of 2007 has been the focus of much commentary, but Figure 1 reveals that the decade 1995–2004 was also a time of dramatic change. That period was characterized by a high level of housing-market activity: houses were selling faster, more houses were sold, and at the same time, more houses were put up for sale. The volume of sales rose by 51%, while the sales rate increased by 22%. Despite the rise in the sales rate, the stock of houses for sale did not fall, and in fact increased by 29%. The time series of listings plays a key role in reconciling the behaviour of sales, the sales rate, and houses for sale. During the 1995–2004 period, the volume of listings rose by 54% (the listing rate increased by 55%). Not only were houses selling faster (the rise in the sales rate), but at the same time homeowners decided to move more frequently. This increase in listings generated a rise in the stock of houses for sale, which also boosted sales volumes.

As is well known, there was a boom in construction during the period in question, with the stock of single-family homes increasing by an estimated 20%.[21] The NAR data on sales and inventories of existing single-family homes do not directly include newly constructed houses. However, by increasing the total stock of houses, new construction will subsequently show up in future data on sales and inventories when the first owners decide to sell. Given a sales rate and a listing rate, increases in the housing stock would be expected to increase the volumes of sales and listings and the stock of houses for sale by the same proportion. Thus, net of new construction over the 1995–2004 period, sales volume rose by 31%, the volume of listings increased by 34%, and the stock of houses for sale by 9%. For consistency, new construction is also subtracted from the measured listing rate (calculated for a constant housing stock) to leave an increase of 35%, but no adjustment of the sales rate is needed. There remains a substantial rise in sales volume even after accounting for the effects of new construction. Interestingly, the rise in the listing rate mirrors the 33% rise in the own-to-own moving rate found by Bachmann and Cooper (2014) using household-level data from the PSID.

There are at least three features of the economic environment during the period 1995–2004 that have implications for moving decisions according to the model and which are consistent with

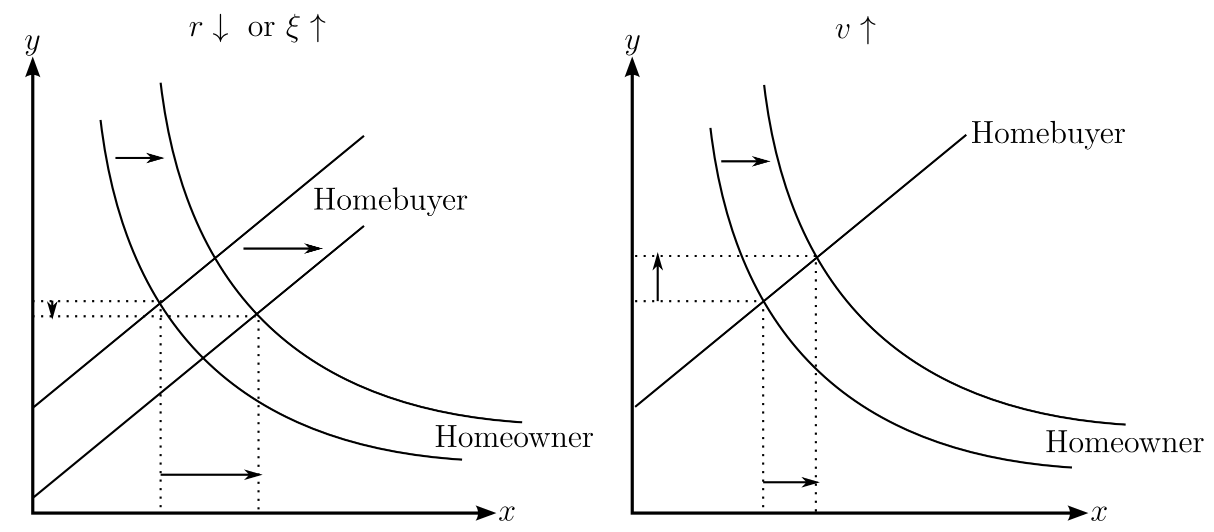
[21] The measurement of the housing stock is described in footnote 7.

the rise in listings. These are the fall in mortgage rates, the increase in productivity growth, and the adoption of internet technology. Lower mortgage rates reduce the opportunity cost of capital and thus lower the discount rate r in the model. An increase in productivity raises incomes and increases the demand for housing ξ. It is explained in appendix A.7 why changes in productivity and market interest rates can be interpreted as changes in the preference parameters ξ and r. Finally, the adoption of internet technology reduces search frictions in arranging viewings by distributing information about available houses and their general characteristics more widely among potential buyers. This raises the meeting rate v implied by the meeting function.

Using Figure 4, which depicts equations [4.4] and [4.7], both a fall in r and a rise in ξ imply the two curves shift to the right (this is shown in the left panel of Figure 5 below), while a rise in v implies the curve representing [4.7] shifts to the right (as shown in the right panel of Figure 5). In all cases, the moving threshold x increases, and increases proportionately more than y. Then using [4.11], a rise in x/y leads to an increase in the moving rate n.

The intuition for the effects on the moving threshold is the following. Take the case of a reduction in r. This increases the present discounted value of flows of housing services, so it increases the incentive to invest in match quality, reducing the tolerance for low current match quality (higher moving threshold x), resulting in more frequent moves. However, there are two offsetting effects on transactions decisions. On the one hand, buyers are more keen to make a purchase to receive the higher discounted sum of flow values, so they become less picky (lower transaction threshold y). On the other hand, owing to the reduced tolerance for low quality matches as a homeowner, the expected duration of a match is shortened, which goes against the interest-rate effect and makes buyers more picky (higher y). The intuition is essentially the same for the effects of higher housing demand ξ. In the case of the higher viewing rate v, the effect is to increase the expected surplus from searching. This increases the incentive to search both for existing homeowners and homebuyers, making both more picky (higher moving and transaction thresholds x and y).

Figure 5: *Comparative statics of moving and transaction thresholds*



Quantifying the predictions of the model requires a calibration of the parameters. This is the goal of the next section.

5.1 Calibration

The model contains a total of 11 parameters $\{a, \delta, \lambda, v, C, F, M, \omega, \kappa, r, \xi\}$. Some parameters are directly matched to the data, while others are determined indirectly by choosing values that make the predictions of the model consistent with some empirical targets. Finally, for some parameters, reasonable values are directly imposed.

Starting with the parameters that have values directly imposed, the (annual) discount rate r is set to 7% ($r = 0.07$).[22] Buyers and sellers are assumed to have equal bargaining power ($\omega = 0.5$).

The parameters C, F, M, and κ are calibrated to match the costs of owning a house and the costs involved in buying and selling houses, and how those costs are distributed across buyers and sellers. Let $c = C/P$, $f = F/P$, and $m = M/P$ denote the costs C, F, and M relative to the average house price P in the steady state of the model. The data provide information on costs relative to price, so calibration targets for c, f, and m are adopted that will determine C, F, and M indirectly. As shown in appendix A.8, the cost-to-price ratios predicted by the model are:

$$c = \frac{\frac{C}{\xi}}{\kappa \frac{C}{\xi} - \frac{M}{r\xi} + \omega\left(\frac{1}{r} + \frac{y^\lambda}{v}\right)\left(x + \frac{F}{\xi}\right)}; \qquad [5.1a]$$

$$f = \frac{\frac{F}{\xi}}{\kappa \frac{C}{\xi} - \frac{M}{r\xi} + \omega\left(\frac{1}{r} + \frac{y^\lambda}{v}\right)\left(x + \frac{F}{\xi}\right)}; \qquad [5.1b]$$

$$m = \frac{\frac{M}{\xi}}{\kappa \frac{C}{\xi} - \frac{M}{r\xi} + \omega\left(\frac{1}{r} + \frac{y^\lambda}{v}\right)\left(x + \frac{F}{\xi}\right)}. \qquad [5.1c]$$

Observe that C, F, and M appear only as ratios to ξ, so the value of ξ can be normalized to 1.

Following Poterba (1991), the flow cost M of owning a house is set so that in equilibrium it is 4.5% of the average house price ($m = 0.045$). This cost is made up of a 2.5% maintenance cost and a 2% property tax. The maintenance cost is interpreted as the cost required perpetually to maintain a house in the same physical condition as when it was first purchased.

The costs incurred in buying and selling houses comprise the one-off transactions cost C and the flow costs of search F. For transaction costs, Quigley (2002) estimates the total costs as being in the range 6–12% of price in the U.S., with about 3–6% being the realtor's fee paid by the seller. Ghent (2012) summarizes recent research and uses a total transaction cost of 13.1%, where 5.1% is the realtor's fee borne by the seller. In light of these findings, the total transaction cost C is set so that it is 10% of the price ($c = 0.1$), and the share κ of these costs borne by the seller is set to be 1/3.

For the flow cost parameter F, unfortunately there are almost no estimates of the flow costs of

[22] This is close to the 6.9% 30-year real mortgage rate in 1995 (9.2% nominal, 2.3% PCE inflation).

searching. The approach taken here is to base an estimate of F on the opportunity cost of the time spent searching. Assuming one house viewing entails the loss of a day's income, the value of F can be calibrated by adding up the costs of making the expected number of viewings. In the model, time-to-buy is equal to time-to-sell, so buyers will incur search costs $T_{\mathrm{s}}F$ per housing transaction on average, where T_{s} denotes time-to-sell. With viewings-per-sale equal to the average number of viewings made by a buyer, the total search cost should be equated to $V_{\mathrm{s}}Y/365$, where V_{s} denotes average viewings-per-sale and Y denotes average annual income. Thus, the calibration assumes $T_{\mathrm{s}}F = V_{\mathrm{s}}I/365$, and by dividing both sides by PT_{s}, this implies an equation for $f \equiv F/P$:

$$f = \frac{1}{365}\frac{Y}{P}\frac{V_{\mathrm{s}}}{T_{\mathrm{s}}}.$$

Using a house-price to income ratio of 2 as a reasonable average value (Case and Shiller, 2003) together with the values of $T_{\mathrm{s}} = 6.5/12$ and $V_{\mathrm{s}} = 10$ discussed below, the ratio of the flow cost of search to the average price is calibrated to be 2.5% ($f = 0.025$). Note that 2.5% should be interpreted as the hypothetical cost of spending a whole year searching.

While the data described above provide information about the ratios of the parameters C, F, and M to the average house price P, since determination of the price P depends in general on all the other parameters, the calibration must be done jointly with that for the remaining parameters $\{a, \delta, \lambda, v\}$. These four parameters are calibrated using four additional empirical targets: the average time to sell a house, the average number of viewings per sale, the expected tenure (expected duration of new matches), and the average years of ownership (the average time existing home-owners have spent in their homes).

The average time to sell is the reciprocal of the average sales rate obtained using data from the National Association of Realtors (NAR) on sales and inventories (for existing single-family homes), as described in section 2.2. Average time-to-sell over the period 1991–2013 is 6.5 months.

Previous research on housing markets has used a variety of sources for data on time-to-sell, and there is a considerable dispersion in these estimates. Using the 'Profile of Buyers and Sellers' survey collected by NAR, Genesove and Han (2012) report that for the time period 1987–2008, the average time-to-sell is 7.6 weeks, the average time-to-buy is 8.1 weeks, and the average number of homes visited by buyers is 9.9. They also discuss other surveys that have reported similar findings.

However, the estimates of time-to-sell and time-to-buy derived from survey data are likely to be an underestimate of the actual time a new buyer or seller would expect to spend in the housing market. The reason is that the survey data include only those buyers and sellers who have successfully completed a house purchase or sale, while the proportion of buyers or sellers who withdraw from the market (at least for some time) without a completed transaction is substantial. Genesove and Mayer (1997) estimate the fraction of withdrawals at 50%, and Levitt and Syverson (2008) report a withdrawal rate of 22%. In comparing the efficiency of different platforms for selling properties, Hendel, Nevo and Ortalo-Magné (2009) explicitly control for withdrawals and report a time-to-sell of 15 weeks (using the Multiple Listing Service for the city of Madison).[23]

[23]For the U.K., Merlo and Ortalo-Magné (2004) obtain data from four real estate agencies that contain 780

An alternative approach to estimating time-to-sell that does not face the problem of withdrawals is to look at the average duration for which a home is vacant using data from the American Housing Survey. In the years 2001–2005, the mean duration of a vacancy was 7–8 months. However, that number is likely to be an overestimate of the expected time-to-sell because it is based on houses that are 'vacant for sale'. Houses that are for sale but currently occupied would not be counted in this calculation of average duration. A further approach that avoids the problem of withdrawals is to look at the average time taken to sell newly built houses. Díaz and Jerez (2013) use the Census Bureau 'New Residential Sales' report to find that the median number of months taken to sell a newly built house is 5.9 (for the period 1991–2012). This is only slightly shorter than the average of the time-to-sell number constructed using NAR data on existing single-family homes, but there is reason to believe that newly built homes should sell faster than existing homes owing to greater advertising expenditure and differences in the target groups of buyers.

In summary, most studies find that average time-to-sell is less than three months in cases where there is a potential withdrawal bias that is not controlled for. Most studies that are not subject to this bias, or attempt to control for it, find times-to-sell of more than four months. Since the predictions of the model will be compared to variables constructed from the NAR sales and inventories data, a measure of time-to-sell consistent with this data is used. The calibration target is a time-to-sell of 6.5 months (the average of the NAR 'months supply' number and the time-to-sell number derived from the NAR data), hence $T_{\text{s}} = 6.5/12$. The calibration target for viewings per sale is set to 10 ($V_{\text{s}} = 10$) on the basis of the studies discussed above. In the model, average time-to-sell is the reciprocal of the sales rate in [4.10], and average viewings per sale is the reciprocal of the probability π that a viewing leads to a sale:

$$T_{\text{s}} = \frac{1}{vy^{-\lambda}}, \quad \text{and} \quad V_{\text{s}} = \frac{1}{y^{-\lambda}}. \qquad [5.2]$$

The remaining calibration targets are for the number of years a buyer expects to remain in the same house (expected tenure, denoted T_{n}), and the average number of years existing homeowners have lived in their current houses (average years of ownership, denoted T_{h}). Note that these two numbers are not necessarily the same when the hazard rate of moving is not independent of the time already spent in a house. An estimate of both expected tenure and average years of ownership can be derived from the data in Table 2.9 (Year Householder Moved into Unit) of the American Housing Survey, which gives a frequency distribution for the time since owners moved into their homes. The data are supplied in 5-year bins for durations of less than 40 years, and in 10-year bins for longer durations.[24] In calculating the expected tenure and the average years of ownership, the frequency

completed transaction histories between 1995–1998 for Greater London and for South Yorkshire. They report an average time-to-sell of 11 weeks, but this number does not control for withdrawals, which they find occur at a rate of 25% in their data. They also report an average of 9.5 viewings per transaction for a sub-sample of 199 properties in their data.

[24] The first bin requires special treatment because it covers a five-year interval that does not generally coincide with the survey year, and because the survey itself is conducted in the middle of the year (between mid-April and mid-September during a survey year). For example, in 2005, the first bin starts in the survey year, so this bin effectively covers only one tenth of the time spanned by the other bins. The frequency in the first bin is scaled up accordingly.

in each bin is assumed to be equally distributed within the bin. Elderly owners (over 65 years) are removed from the data because such individuals are less likely to consider moving. Using the 2005 survey, the average years of ownership is found to be 11 years ($T_\mathrm{h} = 11$).

The expected tenure is found from the same data by calculating the hazard function for moving house consistent with the frequency distribution of the years of ownership (this assumes that the empirical distribution is the stationary distribution implied by the hazard function). The method leads to an estimate of expected tenure of 12.2 years ($T_\mathrm{n} = 12.2$). That the expected tenure is longer than the average years of ownership is consistent with the model's prediction of a hazard rate for moving that is increasing in time spent in a house. It is shown in appendix A.4 that the model implies expected tenure T_n and average years of ownership T_h are given by:

$$T_\mathrm{n} = \frac{1}{a}\left(1 + \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda\right), \quad \text{and} \quad T_\mathrm{h} = \frac{\frac{1}{a}\left(1 + \frac{\delta^\lambda}{1-\delta^\lambda}\left(1 + \frac{1}{1-\delta^\lambda}\right)\left(\frac{y}{x}\right)^\lambda\right)}{1 + \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda}. \qquad [5.3]$$

The seven calibration targets used to determine the parameters $\{a, \delta, \lambda, v, C, F, M\}$ are listed in Table 1. Intuitively, expected tenure and average years of ownership provide information about the arrival rate a of idiosyncratic shocks and the size of those shocks (the parameter δ). Both a lower arrival rate and smaller idiosyncratic shocks would lead to longer expected tenure and greater average years of ownership. The two parameters can be separately identified because having data on both T_n and T_h provides information not only about the average hazard rate of moving, but also its dependence on duration. Furthermore, the parameters a and δ have very different effects on the hazard function. A decrease in the arrival rate a of shocks uniformly decreases the hazard rate for all durations, while a decrease in the size of the shocks also tilts the hazard function so that its slope increases. The reason is that with very large idiosyncratic shocks, the moving decision would essentially depend only on receiving one shock. With smaller idiosyncratic shocks, homeowners who start with a high match quality would require more than one shock to persuade them to move, making moving more likely for longer-duration homeowners who have had time to receive multiple shocks than for those who have moved more recently.

Table 1: *Targets used to calibrate parameters*

Target description	Notation	Value
Time to sell (time to buy)	T_s	6.5/12
Viewings per sale (viewings per purchase)	V_s	10
Expected tenure of homeowners	T_n	12.2
Average years of homeownership	T_h	11
Ratio of transaction cost to average price	c	0.10
Ratio of flow search costs to average price	f	0.025
Ratio of flow maintenance costs to average price	m	0.045

Notes: The data sources for these empirical targets are discussed in section 5.1.

There is also an intuitive connection between the parameter λ and the calibration target time-to-sell. The value of λ determines the amount of dispersion in the distribution of potential match quality, and thus the incentive to continue searching. A low value of λ indicates a high degree of dispersion, in which case families will be willing to spend longer searching for an ideal house. For the final parameter υ, the average time between viewings can be found by dividing time-to-sell by viewings-per-sale, which directly provides information about the arrival rate υ of viewings, as can be seen from equation [5.2].

A simple method for exactly matching the seven parameters $\{a, \delta, \lambda, \upsilon, C, F, M\}$ to the seven empirical targets in Table 1 is described in appendix A.8. The parameters matching the targets and those directly calibrated are all shown in Table 2.

5.2 Quantitative results

This section presents a quantitative analysis of the three shocks discussed earlier for the period 1995–2004: the decline in mortgage rates, the productivity boom, and the rise of internet-based property search.

Mortgage rates (30-year conventional) declined from 9.2% in 1995 to 5.8% in 2004, while inflation (PCE) increased from 2.3% to 2.8%.[25] Real mortgage rates therefore fell from 6.9% to 3%. A simple measure of productivity growth is the increase in real GDP per person, which grew by a total of 25% over the decade. A rise in income naturally leads to an increase in housing demand (the parameter ξ in the model), the extent of the increase also depending on the income-elasticity of housing, which is assumed to be unity here.[26] Finally, the rise of internet-based property search would be expected to improve the efficiency of search as captured by the meeting function (the parameter υ in the model). Since $\upsilon = T_s/V_s$, data on time-to-sell and viewings-per-sale can be used to infer the rise in υ. Using data from Genesove and Han (2012), the maximum rise in T_s/V_s over the period in question is 33%, and this is taken as an upper bound on the efficiency gains.

The quantitative effects of each of these changes individually and taken together are shown in Table 3. The table reports both the steady-state effects and the effects in the short term (in brackets) before any transitional dynamics have occurred. The effects are large: the calibrated model shows that macroeconomic variables can have a large impact on moving and buying decisions. As discussed earlier, it is changes in moving rates that are the main driver of sales volume in the longer term. The model is consistent with large changes in sales volume precisely because it is able to explain large changes in moving rates. The comparison of short-run and long-run effects in the table also reveals interesting dynamics working through the endogenous distribution of existing match quality. The moving rate displays significant overshooting because of the cleansing effect on the distribution of match quality following changes in the moving and transaction thresholds.[27]

[25] The 10-year Treasury rate declined by slightly more over this period from 7.8% to 4.2%. Inflation expectations as measured by the Michigan survey began and ended the period at 3.0%.

[26] Harmon (1988) finds an income elasticity of housing demand in the range 0.7–1.

[27] There is no overshooting in the sales rate because the distribution of existing match quality is irrelevant for buying decisions.

Table 2: *Calibrated parameters*

Parameter description	Notation	Value
Parameters matching calibration targets		
Arrival rate of shocks	a	0.131
Size of shocks	δ	0.862
Steady-state distribution of match quality	λ	13.0
Arrival rate of viewings	v	22.8
Total transaction cost	C	0.565
Flow search costs	F	0.141
Flow maintenance costs	M	0.254
Directly chosen parameters		
Share of total transaction cost directly borne by seller	κ	1/3
Bargaining power of seller	ω	1/2
Discount rate	r	0.07
Common component of homeowner flow value (normalization)	ξ	1

Notes: The parameters are chosen to match exactly the calibration targets in Table 1.

Take the effect of lower mortgage rates. Homeowners find it cheaper to invest in improving their match quality, so they have a higher threshold for remaining in the same house, and as a result the moving rate increases and there is a cleansing effect on the distribution of match quality. As the match quality distribution improves, the extent of cleansing subsequently declines, which explains why overshooting occurs.

Comparing the effects across different shocks also reveals another consequence of persistence in match quality, namely that the magnitude of overshooting depends on what happens to the transaction threshold. If there is a decrease in the transaction threshold then the improvement in match quality that results from moving is smaller (even if cleansing of the existing match quality distribution is occurring), which means that cleansing remains important for longer, so there is less overshooting. This can be seen from a comparison of the productivity boom to the case of declining mortgage rates. It is worth noting that in the case of internet search, the transaction threshold is actually higher, but the effect on the sales rate is offset by the direct effect of the higher meeting rate, which is consistent with there being more overshooting if the transaction threshold is higher.

Overall, the model does a fairly good job in explaining a significant fraction of the rise in the moving rate, and the volumes of sales and listings in the data. For average match quality, the (long-run) responses to the three shocks are 3%, 1%, and 2% respectively, and 6% in total. The (long-run) responses of the average transactions price are 134%, 38%, and 3% respectively, and 233% in total. The model is consistent with substantial movements in prices because the productivity and interest-rate shocks have large direct impacts. It fails to match the increase in the sales rate observed in the data, but note that this failure would also be present in a model with exogenous moving, where the opposing force that leads to the rise in the transactions threshold is also present

Table 3: *Effects of macroeconomic developments in the long run and short run*

Shock	Sales rate	Moving rate	Sales volume	Listing volume	Houses for sale
Mortgage rates ($r \downarrow 57\%$)	−18% [−18%]	10% [23%]	9% [−18%]	9% [23%]	33% [0%]
Productivity boom ($\xi \uparrow 25\%$)	5% [5%]	10% [12%]	9% [5%]	9% [12%]	4% [0%]
Internet search ($v \uparrow 33\%$)	1% [1%]	1% [16%]	1% [1%]	1% [16%]	0% [0%]
Combined	−15% [−15%]	18% [37%]	16% [−15%]	16% [37%]	36% [0%]
Data (net change 1995–2004)	22%	35%	31%	34%	9%

Notes: Figures in brackets are the short-run effects. The combined effects are not the sum of the individual effects because the model is not linear.

(as seen from equation [4.17]). The advantage of the endogenous-moving model is that it can still generate a large rise in sales volume through the increase in the moving rate.

There are two other potential factors that might have contributed to the rise in the sales rate during the period in question: a decrease in buyers' transaction costs C_b, possibly due to a fall in the fixed cost of obtaining a mortgage, and a rise in housing demand owing to demographics (in particular, the baby-boomer generation). The effects of the former can be understood using Figure 4, where there would be a downward shift of the homebuyer curve, resulting in a higher moving threshold x and a lower transaction threshold y (and a lower value of y/x). This would imply that both the sales and moving rates would increase (which would also boost sales volume). The effects of the baby boomers (a 25% rise in the age group 25–64) can be seen as an increase in the demand for housing, with consequences similar to those of the productivity boom discussed earlier. This may help to explain the rise in the sales rate observed in the data.

6 Conclusions

This paper presents evidence that changes in sales volume are largely explained by changes in the frequency at which houses are put up for sale rather than changes in the length of time taken to sell them. Except for a relatively short transitional period (less than one year), even large changes in the sales rate have very little impact on the volume of sales.

The paper builds a tractable model to analyse moving house where a homeowner's decision to move is an investment in housing match quality. Since moving house is an investment with upfront costs and potentially long-lasting benefits, the model predicts that the aggregate moving rate depends on macroeconomic variables such as interest rates. The endogeneity of moving means that those who move come from the bottom of the existing match quality distribution, and the non-random selection of movers gives rise to a cleansing effect that leads to overshooting of housing-market variables. The model is applied to understand the booming housing market during the decade 1995–2004, and it successfully accounts for some of the key features in the data such as rising moving rates and sales volumes.

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A Appendices

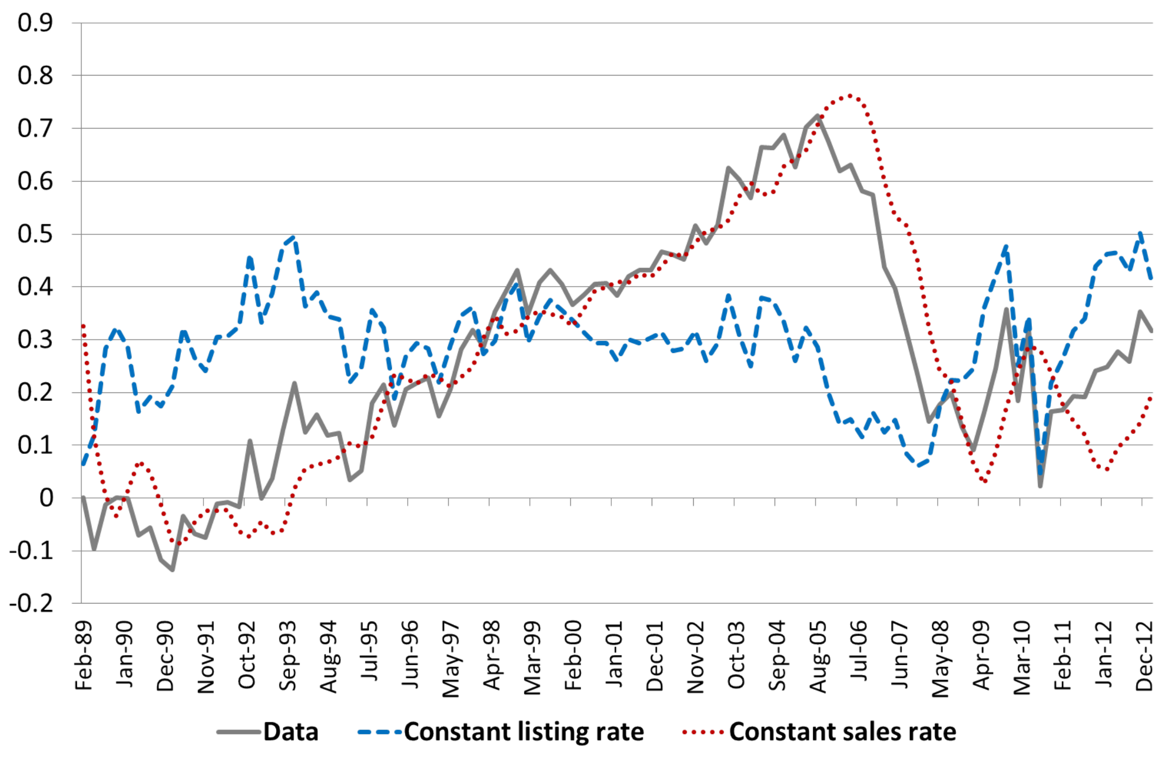
A.1 Counterfactuals out of steady state

Note that equations [2.3] and [2.4] and the definitions of the sales and listing rates imply the following identity:

$$I_{t+1} = \frac{\left(1 - \frac{s_t + n_t}{2}\right) I_t + n_t K}{1 + \frac{s_t + n_t}{2}}. \qquad \text{[A.1.1]}$$

Two counterfactuals are considered using this identity. The first holds the listing rate constant at its sample average ($n_t = \bar{n}$) while s_t varies as in the data, and the second holds the sales rate constant at its sample average ($s_t = \bar{s}$) while n_t varies as in the data. Equation [A.1.1] is used to construct a hypothetical inventories series I_t given the sales and listing rates s_t and n_t (starting from the initial inventories given in the data). With this series, houses for sale U_t can be calculated using [2.4], which yields a counterfactual series for sales volume using $S_t = s_t U_t$. The log differences of the actual sales volume series and the two counterfactuals are shown in Figure 6. The constant listing-rate counterfactual is largely unrelated to the actual sales volume series (the correlation coefficient is 0.11). On the other hand, the counterfactual that completely ignores all variation in the sales rate is still able to reproduce most of the variation seen in the actual sales volume series (the correlation coefficient is 0.89).

Figure 6: *Actual sales volume and two counterfactual sales volumes*



Notes: The calculation of the two counterfactual sales volume series is described in section 2.2. The series are plotted as log differences relative to the initial value of actual sales volume.

A.2 Value functions and thresholds

Moving and transaction thresholds

The value functions $H_t(\epsilon)$ and W_t and the thresholds x_t and y_t satisfy the equations [3.1], [3.2], [3.5], and [3.6]. No other variables appear in these equations. Given constant parameters, there is a time-invariant solution $H_t(\epsilon) = H(\epsilon)$, $W_t = W$, $x_t = x$, and $y_t = y$. The time-invariant equations are:

$$rH(\epsilon) = \epsilon\xi - M + a\left(\max\{H(\delta\epsilon), W\} - H(\epsilon)\right); \qquad \text{[A.2.1]}$$

$$H(x) = W; \qquad \text{[A.2.2]}$$

$$rW = -F - M + v\int_y (H(\epsilon) - W - C)\mathrm{d}G(\epsilon); \qquad \text{[A.2.3]}$$

$$H(y) = W + C. \qquad \text{[A.2.4]}$$

Attention is restricted to parameters where the solution will satisfy $\delta y < x$.

Evaluating [A.2.1] at $\epsilon = x$, noting that $\delta < 1$ and $H(\epsilon)$ is increasing in ϵ:

$$rH(x) = \xi x - M + a(W - H(x)).$$

Since $H(x) = W$ (equation [A.2.2]), it follows that:

$$W = H(x) = \frac{\xi x - M}{r}. \qquad \text{[A.2.5]}$$

Next, evaluate [A.2.1] at $\epsilon = y$. With the restriction $\delta y < x$, it follows that $H(\delta y) < H(x) = W$, and hence:

$$rH(y) = \xi y - M + a(W - H(y)).$$

Collecting terms in $H(y)$ on one side and substituting the expression for W from [A.2.5]:

$$(r + a)H(y) = \xi y - M + \frac{a}{r}(\xi x - M) = \xi(y - x) + \left(1 + \frac{a}{r}\right)(\xi x - M),$$

and thus $H(y)$ is given by:

$$H(y) = \frac{\xi x - M}{r} + \frac{\xi(y - x)}{r + a}. \qquad \text{[A.2.6]}$$

Combining the equation above with [A.2.4] and [A.2.5], it can be seen that the thresholds y and x must be related as follows:

$$y - x = \frac{(r + a)C}{\xi}. \qquad \text{[A.2.7]}$$

Using the expression for the Pareto distribution function [3.3] and using [A.2.4] to note $H(\epsilon) - W - C = H(\epsilon) - H(y)$, the Bellman equation [A.2.3] can be written as:

$$rW = -F - M + vy^{-\lambda}\int_{\epsilon=y}^{\infty} \frac{\lambda}{y}\left(\frac{\epsilon}{y}\right)^{-(\lambda+1)}(H(\epsilon) - H(y))\mathrm{d}\epsilon, \qquad \text{[A.2.8]}$$

which assumes $y > 1$. In solving this equation it is helpful to define the following function $\Psi(z)$ for all $z \le y$:

$$\Psi(z) \equiv \int_{\epsilon=z}^{\infty} \frac{\lambda}{z}\left(\frac{\epsilon}{z}\right)^{-(\lambda+1)}(H(\epsilon) - H(z))\mathrm{d}\epsilon. \qquad \text{[A.2.9]}$$

Since $\delta y < x$ is assumed and $z \le y$, it follows that $\delta z < x$, and thus $H(\delta z) < H(x) = W$. Equation [A.2.1] evaluated at $\epsilon = z$ therefore implies:

$$rH(z) = \xi z - M + a(W - H(z)).$$

Subtracting this equation from [A.2.1] evaluated at a general value of ϵ leads to:

$$\begin{aligned} r(H(\epsilon) - H(z)) = \xi(\epsilon - z) + a\left(\max\{H(\delta\epsilon), W\} - H(\epsilon)\right) - a(W - H(z)) \\ = \xi(\epsilon - z) - a(H(\epsilon) - H(z)) + a\max\{H(\delta\epsilon) - W, 0\}. \end{aligned}$$

Noting that $W = H(x)$ and solving for $H(\epsilon) - H(z)$:

$$H(\epsilon) - H(z) = \frac{\xi}{r+a}(\epsilon - z) + \frac{a}{r+a}\max\{H(\delta\epsilon) - H(x), 0\}. \qquad \text{[A.2.10]}$$

The equation above can be substituted into [A.2.9] to deduce:

$$\varPsi(z) = \frac{\xi}{r+a}\int_{\epsilon=z}^{\infty}\frac{\lambda}{z}\left(\frac{\epsilon}{z}\right)^{-(\lambda+1)}(\epsilon - z)\mathrm{d}\epsilon + \frac{a}{r+a}\int_{\epsilon=z}^{\infty}\frac{\lambda}{z}\left(\frac{\epsilon}{z}\right)^{-(\lambda+1)}\max\{H(\delta\epsilon) - H(x), 0\}\mathrm{d}\epsilon. \quad \text{[A.2.11]}$$

First, observe that:

$$\int_{\epsilon=z}^{\infty}\frac{\lambda}{z}\left(\frac{\epsilon}{z}\right)^{-(\lambda+1)}(\epsilon - z)\mathrm{d}\epsilon = \frac{z}{\lambda - 1}. \qquad \text{[A.2.12]}$$

Next, make the change of variable $\epsilon' = \delta\epsilon$ in the second integral in [A.2.11] to deduce:

$$\begin{aligned} \int_{\epsilon=z}^{\infty}\frac{\lambda}{z}\left(\frac{\epsilon}{z}\right)^{-(\lambda+1)}\max\{H(\delta\epsilon) - H(x), 0\}\mathrm{d}\epsilon = \int_{\epsilon'=\delta z}^{\infty}\frac{\lambda}{\delta z}\left(\frac{\epsilon'}{\delta z}\right)^{-(\lambda+1)}\max\{H(\epsilon') - H(x), 0\}\mathrm{d}\epsilon' \\ = \int_{\epsilon'=\delta z}^{x}\frac{\lambda}{\delta z}\left(\frac{\epsilon'}{\delta z}\right)^{-(\lambda+1)}0\mathrm{d}\epsilon' + \int_{\epsilon'=x}^{\infty}\frac{\lambda}{\delta z}\left(\frac{\epsilon'}{\delta z}\right)^{-(\lambda+1)}(H(\epsilon') - H(x))\mathrm{d}\epsilon' \\ = \left(\frac{\delta z}{x}\right)^{\lambda}\int_{\epsilon=x}^{\infty}\frac{\lambda}{x}\left(\frac{\epsilon}{x}\right)^{-(\lambda+1)}(H(\epsilon) - H(z))\mathrm{d}\epsilon = \left(\frac{\delta z}{x}\right)^{\lambda}\varPsi(x), \end{aligned}$$

where the second line uses $\delta z < x$ (as $z \leq y$ and $\delta y < x$) and $H(\epsilon') < H(x)$ for $\epsilon' < x$, and the final line uses the definition [A.2.9]. Putting the equation above together with [A.2.11] and [A.2.12] yields the following for all $z \leq y$:

$$\varPsi(z) = \frac{\xi z}{(r+a)(\lambda - 1)} + \frac{a}{r+a}\left(\frac{\delta z}{x}\right)^{\lambda}\varPsi(x). \qquad \text{[A.2.13]}$$

Evaluating this expression at $z = x$ (with $x < y$):

$$\varPsi(x) = \frac{\xi x}{(r+a)(\lambda - 1)} + \frac{a}{r+a}\delta^{\lambda}\varPsi(x),$$

and hence $\varPsi(x)$ is given by:

$$\varPsi(x) = \frac{\xi x}{(r + a(1 - \delta^{\lambda}))(\lambda - 1)}. \qquad \text{[A.2.14]}$$

Next, evaluating [A.2.13] at $z = y$ and using [A.2.14] to substitute for $\varPsi(x)$:

$$\varPsi(y) = \frac{\xi y}{(r+a)(\lambda - 1)} + \frac{a}{r+a}\left(\frac{\delta z}{x}\right)^{\lambda}\left(\frac{\xi x}{(r + a(1 - \delta^{\lambda}))(\lambda - 1)}\right),$$

and simplifying this equation yields the following expression for $\Psi(y)$:

$$\Psi(y) = \frac{\xi}{(r+a)(\lambda-1)}\left(y + \frac{a\delta^{\lambda}y^{\lambda}x^{1-\lambda}}{r+a(1-\delta^{\lambda})}\right). \quad \text{[A.2.15]}$$

Using the definition [A.2.9], equation [A.2.8] can be written in terms of $\Psi(y)$:

$$rW = -F - M + vy^{-\lambda}\Psi(y),$$

and substituting from [A.2.5] and [A.2.15] yields:

$$\xi x - M = -F - M + vy^{-\lambda}\left(\frac{\xi}{(r+a)(\lambda-1)}\left(y + \frac{a\delta^{\lambda}y^{\lambda}x^{1-\lambda}}{r+a(1-\delta^{\lambda})}\right)\right).$$

This equation can be simplified as follows:

$$x + \frac{F}{\xi} = \frac{v}{(\lambda-1)(r+a)}\left(y^{1-\lambda} + \frac{a\delta^{\lambda}}{r+a(1-\delta^{\lambda})}x^{1-\lambda}\right). \quad \text{[A.2.16]}$$

The two equations [A.2.7] and [A.2.16] can be solved for the thresholds x and y.

Existence and uniqueness

By using equation [A.2.7] to replace x with a linear function of y, the equilibrium threshold y is the solution of the equation:

$$\mathcal{I}(y) \equiv \frac{v}{(\lambda-1)(r+a)}\left(y^{1-\lambda} + \frac{a\delta^{\lambda}}{r+a(1-\delta^{\lambda})}\left(y - \frac{(r+a)C}{\xi}\right)^{1-\lambda}\right) - y + \frac{(r+a)C}{\xi} - \frac{F}{\xi} = 0. \quad \text{[A.2.17]}$$

It can be seen immediately (since $\lambda > 1$) that $\mathcal{I}'(y) < 0$, so any solution that exists is unique. A valid solution must satisfy $x > 0$, $y > 1$, and $\delta y < x$. Using equation [A.2.7], the inequality $\delta y < x$ is equivalent to:

$$\delta y < y - \frac{(r+a)C}{\xi},$$

which is in turn equivalent to:

$$y > \frac{(r+a)C}{(1-\delta)\xi}.$$

Thus, to satisfy $y > 1$ and $\delta y < x$, the equilibrium must feature:

$$y > \max\left\{1, \frac{(r+a)C}{(1-\delta)\xi}\right\}. \quad \text{[A.2.18]}$$

Observe that $\lim_{y\to\infty}\mathcal{I}(y) = -\infty$ (using [A.2.17] and $\lambda > 1$), so an equilibrium satisfying [A.2.18] exists if and only if:

$$\mathcal{I}\left(\max\left\{1, \frac{(r+a)C}{(1-\delta)\xi}\right\}\right) > 0. \quad \text{[A.2.19]}$$

If the condition [A.2.18] is satisfied then by using [A.2.7]:

$$x > \max\left\{1, \frac{(r+a)C}{(1-\delta)\xi}\right\} - \frac{(r+a)C}{\xi} > \frac{(r+a)C}{(1-\delta)\xi} - \frac{(r+a)C}{\xi} = \frac{\delta(r+a)C}{(1-\delta)\xi} > 0,$$

confirming that $x > 0$ must hold. Therefore, [A.2.19] is necessary and sufficient for the existence of a unique equilibrium satisfying all the required conditions. Using equation [A.2.17], [A.2.19] is equivalent to the condition in [4.8].

Surplus and selling rate

Given x and y, the value functions W, $H(x)$, and $H(y)$ can be obtained from [A.2.5] and [A.2.6]. The average surplus can be obtained by combining [A.2.5] and [A.2.8] to deduce:

$$\int_{\epsilon=y}^{\infty} \frac{\lambda}{y}\left(\frac{\epsilon}{y}\right)^{-(\lambda+1)}(H(\epsilon)-H(y))\mathrm{d}\epsilon = \frac{y^{\lambda}}{v}(\xi x + F). \tag{A.2.20}$$

Given x and y, the probability π that a viewing leads to a sale, and the expected number of viewings before a sale $V_{\mathrm{s}} = 1/\pi$ are:

$$\pi = y^{-\lambda}, \quad \text{and} \quad V_{\mathrm{s}} = y^{\lambda}. \tag{A.2.21}$$

The selling rate s and the expected time-to-sell $T_{\mathrm{s}} = 1/s$ are given by:

$$s = vy^{-\lambda}, \quad \text{and} \quad T_{\mathrm{s}} = \frac{y^{\lambda}}{v}. \tag{A.2.22}$$

A.3 Prices

Nash bargaining

The price $p_t(\epsilon)$ is determined by combining the Nash bargaining solution $\omega\Sigma_{\mathrm{b},t}(\epsilon) = (1-\omega)\Sigma_{\mathrm{u},t}(\epsilon)$ with the expressions for the buyer and seller surpluses in [3.7]:

$$\omega(H_t(\epsilon) - p_t(\epsilon) - (1-\kappa)C - B_t) = (1-\omega)(p_t(\epsilon) - \kappa C - U_t),$$

from which it follows that:

$$p_t(\epsilon) = \omega H_t(\epsilon) + (\kappa - \omega)C + ((1-\omega)U_t - \omega B_t). \tag{A.3.1}$$

The surplus-splitting condition implies $\Sigma_{\mathrm{b},t}(\epsilon) = (1-\omega)\Sigma_t(\epsilon)$ and $\Sigma_{\mathrm{u},t}(\epsilon) = \omega\Sigma_t(\epsilon)$, with $\Sigma_t(\epsilon) = \Sigma_{\mathrm{b},t}(\epsilon) + \Sigma_{\mathrm{u},t}(\epsilon)$ being the total surplus from [4.6]. The Bellman equations in [3.8] can thus be written as:

$$rB_t = -F + (1-\omega)v\int_{y_t}\Sigma_t(\epsilon)\mathrm{d}G(\epsilon) + \dot{B}_t, \quad \text{and} \quad rU_t = -M + \omega v\int_{y_t}\Sigma_t(\epsilon)\mathrm{d}G(\epsilon) + \dot{U}_t,$$

and a multiple ω of the first equation can be subtracted from a multiple $1-\omega$ of the second equation to deduce:

$$r((1-\omega)U_t - \omega B_t) = \omega F - (1-\omega)M + ((1-\omega)\dot{U}_t - \omega\dot{B}_t).$$

The stationary solution of this equation is:

$$(1-\omega)U_t - \omega B_t = \frac{\omega F - (1-\omega)M}{r},$$

and by substituting this into [A.3.1]:

$$p_t(\epsilon) = \omega H_t(\epsilon) + (\kappa - \omega)C + \frac{\omega F - (1-\omega)M}{r}. \tag{A.3.2}$$

Integrating this equation over the distribution of new match quality yields equation [3.9] for the average transaction price.

Average transactions price

In an equilibrium where the moving and transactions thresholds x_t and y_t are constant over time, the value function $H_t(\epsilon)$ is equal to the time-invariant function $H(\epsilon)$. This means that prices $p_t(\epsilon) = p(\epsilon)$ are also time invariant. Using the Pareto distribution function [3.3] and equation [3.9], the average price is:

$$P = \omega \int_y \frac{\lambda}{y}\left(\frac{\epsilon}{y}\right)^{\lambda} H(\epsilon)\mathrm{d}\epsilon + (\kappa-\omega)C + \frac{\omega F - (1-\omega)M}{r}.$$

By using the expression for $H(y)$ in [A.2.6], the above can be written as:

$$P = \omega \int_y \frac{\lambda}{y}\left(\frac{\epsilon}{y}\right)^{\lambda} (H(\epsilon)-H(y))\mathrm{d}\epsilon + \omega\left(\frac{\xi x - M}{r} + \frac{\xi(y-x)}{r+a}\right) + (\kappa-\omega)C + \frac{\omega F - (1-\omega)M}{r},$$

and substituting from [A.2.7] and [A.2.20] yields:

$$P = \omega\frac{y^{\lambda}}{v}(\xi x + F) + \omega C + \frac{\omega\xi x}{r} + (\kappa-\omega)C + \frac{\omega F - M}{r}.$$

Therefore, the following expression for the average price is obtained:

$$P = \kappa C - \frac{M}{r} + \omega\left(\frac{1}{r} + \frac{y^{\lambda}}{v}\right)(\xi x + F). \qquad \text{[A.3.3]}$$

This leads to expressions for the ratios of costs (search, transactions, and maintenance) to the average price:

$$f = \frac{\frac{F}{\xi}}{\kappa\frac{C}{\xi} - \frac{M}{r\xi} + \omega\left(\frac{1}{r} + \frac{y^{\lambda}}{v}\right)\left(x + \frac{F}{\xi}\right)}; \qquad \text{[A.3.4a]}$$

$$c = \frac{\frac{C}{\xi}}{\kappa\frac{C}{\xi} - \frac{M}{r\xi} + \omega\left(\frac{1}{r} + \frac{y^{\lambda}}{v}\right)\left(x + \frac{F}{\xi}\right)}; \qquad \text{[A.3.4b]}$$

$$m = \frac{\frac{M}{\xi}}{\kappa\frac{C}{\xi} - \frac{M}{r\xi} + \omega\left(\frac{1}{r} + \frac{y^{\lambda}}{v}\right)\left(x + \frac{F}{\xi}\right)}. \qquad \text{[A.3.4c]}$$

A.4 Stocks and flows

The moving rate

The formula [3.12] for the moving rate can also be given in terms of inflows $N_t = n_t(1-u_t)$, where u_t is the stock of unsold houses:

$$N_t = a(1-u_t) - a\delta^{\lambda}x_t^{-\lambda}v\int_{\tau\to-\infty}^{\infty} e^{-a(1-\delta^{\lambda})(t-\tau)}u_{\tau}\mathrm{d}\tau. \qquad \text{[A.4.1]}$$

The first term $a(1-u_t)$ is the quantity of existing matches that receive a shock (arrival rate a). The second term is the quantity of existing matches that receive a shock now, but decide not to move. The difference between these two numbers (under the assumption that only those who receive a shock make a moving decision) gives inflows N_t.

Now consider the derivation of the second term in [A.4.1]. The distribution of existing matches (measure $1-u_t$) can be partitioned into vintages τ (when matches formed) and the number k of previous shocks that have been received. At time τ, a quantity u_τ of houses were for sale, and viewings arrived at rate v. Viewings were draws of match quality ϵ from a Pareto$(1,\lambda)$ distribution, and those draws with $\epsilon \geq y_\tau$ formed new matches, truncating the distribution at y_τ. In the interval between τ and t, those matches that have received k shocks now have match quality $\delta^k \epsilon$. Some of these matches will have been destroyed as a result of these shocks, truncating the distribution of surviving match quality. Because the distribution of initial match quality is a Pareto distribution, these truncations also result in Pareto distributions with the same shape parameter λ.

Consider the matches of vintage τ. All of these were originally from a Pareto distribution truncated at $\epsilon \geq y_\tau$. Subsequently, depending on the arrival of idiosyncratic shocks (both timing and number), this distribution may have been truncated further. Let z denote the last truncation point in terms of the original match quality ϵ (at the time of the viewing). This is $z = y_\tau$ if no shocks have been received, or $z = \delta^{-k} x_T$ if k shocks have been received and the last one occurred at time T when the moving threshold was x_T. Conditional on this last truncation point z, it is shown below that the measure of surviving matches is $z^{-\lambda} v u_\tau$. Furthermore, the original match quality of these surviving matches must be a Pareto(z, λ) distribution.

Now consider the distribution of the number of previous shocks k between τ and t. Given the Poisson arrival rate a, k has a Poisson distribution, so the probability of k is $e^{-a(t-\tau)}(a(t-\tau))^k/k!$. If a shock arrives at time t, matches of current quality greater than x_t survive. If these have received k shocks earlier, this means the truncation threshold in terms of original match quality ϵ is $\epsilon \geq \delta^{-(k+1)} x_t$. Of these matches that have accumulated k earlier shocks, suppose last relevant truncation threshold (in terms of original match quality) was z (this will vary over those matches even with the same number of shocks because the timing might be different), so the distribution of surviving matches in terms of their original match quality is Pareto(z, λ). The probability that these matches then survive the shock at time t is given by $(\delta^{-(k+1)} x_t / z)^{-\lambda}$, and multiplying this by $z^{-\lambda} v u_\tau$ gives the number that survive:

$$(\delta^{-(k+1)} x_t / z)^{-\lambda} z^{-\lambda} v u_\tau = (\delta^\lambda)^{k+1} x_t^{-\lambda} v u_\tau,$$

noting that the terms in z cancel out. This is conditional on z, k, and τ, but since z does not appear above, the distribution of the past truncation thresholds is not needed. Averaging over the distribution of k yields:

$$\sum_{k=0}^{\infty} e^{-a(t-\tau)} \frac{(a(t-\tau))^k}{k!} (\delta^\lambda)^{k+1} x_t^{-\lambda} v u_\tau = \delta^\lambda x_t^{-\lambda} v u_\tau e^{-a(t-\tau)} \sum_{k=0}^{\infty} \frac{(a\delta^\lambda (t-\tau))^k}{k!}$$
$$= \delta^\lambda x_t^{-\lambda} v u_\tau e^{-a(t-\tau)} e^{a\delta^\lambda (t-\tau)} = \delta^\lambda x_t^{-\lambda} v u_\tau e^{-a(1-\delta^\lambda)(t-\tau)},$$

where the penultimate expression uses the Taylor series expansion of the exponential function $e^z = \sum_{k=0}^{\infty} z^k/k!$ (valid for all z). Next, integrating over all vintages τ before the current time t leads to:

$$\int_{\tau \to -\infty}^{t} \delta^\lambda x_t^{-\lambda} e^{-a(1-\delta^\lambda)(t-\tau)} \mathrm{d}\tau = \delta^\lambda x_t^{-\lambda} \int_{\tau \to -\infty}^{t} e^{-a(1-\delta^\lambda)(t-\tau)} v u_\tau \mathrm{d}\tau.$$

Multiplying this by the arrival rate a of the idiosyncratic shocks confirms the second term of the expression for N_t in [A.4.1].

This leaves only the claim that the measure of vintage-τ surviving matches with truncation point z (in terms of the original match quality distribution ϵ) has measure $z^{-\lambda} v u_\tau$. When these matches first form, they have distribution Pareto(y_t, λ) and measure $y_\tau^{-\lambda} v u_\tau$, so the formula is correct if no shocks have occurred and $z = y_\tau$. Now suppose the formula is valid for some z and truncation now occurs at a new point $w > z$ (in terms of original match quality). Since matches surviving truncation at z have distribution Pareto(z, λ), the proportion of these that survive the new truncation is $(w/z)^{-\lambda}$, and so the measure becomes $(w/z)^{-\lambda} z^{-\lambda} v u_\tau = w^{-\lambda} v u_\tau$ (with the term in z cancelling out), which confirms the claim.

The distribution of match quality

Now consider the derivation of the law of motion for average match quality Q_t in [3.13]. Let total match quality across all families be denoted by $\mathcal{E}_t$ (those not matched have match quality equal to zero), with $\mathcal{E}_t = (1 - u_t) Q_t$ by definition. Total match quality $\mathcal{E}_t$ changes over time as new matches form, when matches are hit by shocks, and when moving decisions are made. With transaction threshold y_t and new match quality drawn from a Pareto$(1;\lambda)$ distribution, new matches have average quality $(\lambda/(\lambda-1))y_t$. The contribution to the rate of change of total match quality is that average multiplied by $s_t u_t$. Shocks to existing matches arrive randomly at rate a. If no shock is received then there is no change to match quality and no moving decision. For those who receive a shock, let $\underline{\mathcal{E}}_t$ denote the total match quality of those matches that survive (with matches that dissolve counted as having zero match quality). The contribution of the shocks and moving decisions to the rate of change of total match quality is to subtract $a(\mathcal{E}_t - \underline{\mathcal{E}}_t)$. The differential equation for $\mathcal{E}_t$ is therefore:

$$\dot{\mathcal{E}}_t = \frac{\lambda}{\lambda-1} y_t s_t u_t - a(\mathcal{E}_t - \underline{\mathcal{E}}_t). \quad \text{[A.4.2]}$$

Using this formula requires an expression for $\underline{\mathcal{E}}_t$.

Consider the distribution of all matches that formed before time t, survived until time t, and now receive an idiosyncratic shock at time t, but one that is not sufficient to trigger moving. The distribution of surviving matches can be partitioned into vintages τ (when the match formed) and the number of shocks k that have been received previously (not counting the shock at time t). At time τ, a quantity u_τ of houses were for sale, and viewings arrived at rate v. Viewings were draws of match quality ϵ from a Pareto$(1;\lambda)$ distribution, and those draws with $\epsilon \geq y_\tau$ formed new matches, truncating the distribution at y_τ. Subsequently, a number k of idiosyncratic shocks have occurred, with k having a Poisson$(a(t-\tau))$ distribution, and these shocks resulting in the distribution of surviving match quality being truncated. With a shock now occurring at time t after k earlier shocks, match quality is now $\delta^{k+1}\epsilon$, and the distribution is truncated at x_t. In terms of the original match quality ϵ, survival requires $\epsilon \geq \delta^{-(k+1)}x_t$.

Consider matches of vintage τ that have previously accumulated k shocks for which the last truncation threshold was z in terms of original match quality (this threshold will depend on when the previous shocks occurred). Since the Pareto distribution is preserved after truncation with the same shape parameter, these matches have a Pareto$(z;\lambda)$ distribution in terms of their original match quality. It was shown above that the measure of surviving vintage-τ matches with truncation point z is $z^{-\lambda}vu_\tau$ (conditional on z, the number of shocks k is irrelevant, though the number of shocks may be related to the value of z). The measure that remain ($\epsilon \geq \delta^{-(k+1)}x_t$) after moving decisions are made at time t is:

$$\left(\delta^{-(k+1)}x_t/z\right)^{-\lambda} z^{-\lambda}vu_\tau = (\delta^\lambda)^{k+1}x_t^{-\lambda}vu_\tau,$$

noting that the terms in z cancel out. The probability of drawing k shocks in the interval between τ and t is $e^{-a(t-\tau)}(a(t-\tau))^k/k!$, and hence averaging over the distribution of k for vintage-τ matches implies that the surviving measure is:

$$\sum_{k=0}^{\infty} e^{-a(t-\tau)}\frac{(a(t-\tau))^k}{k!}(\delta^{k+1})^\lambda x_t^{-\lambda}vu_t = \delta^\lambda x_t^{-\lambda}vu_\tau e^{-a(1-\delta^\lambda)(t-\tau)},$$

which is confirmed by following the same steps as in the derivation of the moving rate above. Integrating these surviving measures over all cohorts:

$$\int_{\tau\to-\infty}^{t} \delta^\lambda x_t^{-\lambda}vu_\tau e^{-a(1-\delta^\lambda)}\mathrm{d}\tau = v\delta^\lambda x_t^{-\lambda}\int_{\tau\to-\infty}^{t} e^{-a(1-\delta^\lambda)(t-\tau)}u_\tau \mathrm{d}\tau,$$

and since the average match quality among the survivors after the shock at time t is $(\lambda/(\lambda-1))x_t$ for all cohorts, it follows that:

$$\underline{\mathcal{E}}_t = \frac{v\delta^\lambda\lambda}{\lambda-1}x_t^{1-\lambda}\int_{\tau\to-\infty}^{t} e^{-a(1-\delta^\lambda)(t-\tau)}u_\tau \mathrm{d}\tau.$$

Substituting this and equation [3.11] into [A.4.2] implies:

$$\dot{\mathcal{E}}_t = \frac{v\lambda}{\lambda-1}y_t^{1-\lambda}u_t - a\left(\mathcal{E}_t - \frac{v\delta^\lambda\lambda}{\lambda-1}x_t^{1-\lambda}\int_{\tau\to-\infty}^{t} e^{-a(1-\delta^\lambda)(t-\tau)}u_\tau \mathrm{d}\tau\right). \quad \text{[A.4.3]}$$

The integral can be eliminated by defining an additional variable I_t:

$$I_t = \int_{\tau\to-\infty}^{t} e^{-a(1-\delta^\lambda)(t-\tau)}u_\tau \mathrm{d}\tau, \quad \text{[A.4.4]}$$

and hence [A.4.3] can be written as follows:

$$\dot{\mathcal{E}}_t = \frac{v\lambda}{\lambda-1}y_t^{1-\lambda}u_t - a\mathcal{E}_t + \frac{av\delta^\lambda\lambda}{\lambda-1}x_t^{1-\lambda}I_t. \quad \text{[A.4.5]}$$

The evolution of the state variables u_t is determined by combining equations [3.10], [3.11], and [3.12]:

$$\dot{u}_t = a(1-u_t) - av\delta^\lambda x_t^{-\lambda}\int_{\tau\to-\infty}^{t} e^{-a(1-\delta^\lambda)(t-\tau)}u_\tau \mathrm{d}\tau - vy_t^{-\lambda}u_t,$$

where the integral can also be eliminated by writing the equation in terms of the new variable I_t from [A.4.4]:

$$\dot{u}_t = a(1-u_t) - av\delta^\lambda x_t^{-\lambda}I_t - vy_t^{-\lambda}u_t. \quad \text{[A.4.6]}$$

Differentiating the integral in [A.4.4] shows that I_t must satisfy the differential equation:

$$\dot{I}_t = u_t - a(1-\delta^\lambda)I_t. \quad \text{[A.4.7]}$$

These results can be used to obtain the differential equation for average match quality Q_t in [3.13]. Since the definition implies $Q_t = \mathcal{E}_t/(1-u_t)$, it follows that:

$$\dot{Q}_t = \frac{\dot{\mathcal{E}}_t}{1-u_t} + \frac{\mathcal{E}_t\dot{u}_t}{(1-u_t)^2} = \frac{\dot{\mathcal{E}}_t}{1-u_t} + Q_t\frac{\dot{u}_t}{1-u_t}.$$

Substituting from the differential equations [A.4.5] and [A.4.6] leads to:

$$\dot{Q}_t = \left(\frac{v\lambda}{\lambda-1}y_t^{1-\lambda}\frac{u_t}{1-u_t} - aQ_t + \frac{av\delta^\lambda\lambda}{\lambda-1}x_t^{1-\lambda}\frac{I_t}{1-u_t}\right) + Q_t\left(a - av\delta^\lambda x_t^{-\lambda}\frac{I_t}{1-u_t} - vy_t^{-\lambda}\frac{u_t}{1-u_t}\right),$$

noting that the terms in Q_t on the right-hand side cancel out, so $\dot{Q}_t$ can be written as:

$$\dot{Q}_t = vy_t^{-\lambda}\left(\frac{\lambda}{\lambda-1}y_t - Q_t\right)\frac{u_t}{1-u_t} - \frac{av\delta^\lambda x_t^{-\lambda}I_t}{1-u_t}\left(Q_t - \frac{\lambda}{\lambda-1}x_t\right).$$

Comparison with equations [3.11], [3.12], and the definition of I_t in [A.4.4] confirms the differential equation for Q_t in [3.13].

The distribution of time spent in a house

Now consider an equilibrium where parameters are expected to remain constant. In this case, the moving and transaction thresholds x and y are constant over time. Let $\psi(\tau)$ denote the survival function for new matches, in the sense of the fraction of matches forming at time t that survive until at least $t+\tau$. Each cohort starts with a match quality distribution $\epsilon \sim \text{Pareto}(y;\lambda)$ at $\tau = 0$. Now consider some $\tau > 0$. Moving occurs only if the value of ϵ after shocks have occurred (ϵ') is such that $\epsilon' < x$. Shocks arrive at a Poisson rate a, so the number k of shocks that would occur to a match over an interval of time τ has a Poisson($a\tau$) distribution, which means the probability that k shocks occur is $e^{-a\tau}(a\tau)^k/k!$. If no shocks occur, $\epsilon' = \epsilon$, so no moving occurs. If $k \geq 1$ shocks have occurred then $\epsilon' = \delta^k\epsilon$, where ϵ is the initial draw of match quality. These matches survive only if $\epsilon' \geq x$, that is, $\epsilon \geq x/\delta^k$. Since the original values of ϵ are drawn from a Pareto distribution truncated at $\epsilon = y$ with shape parameter λ, this probability is $((x/\delta^k)/y)^{-\lambda}$ (this expression is valid for all $k \geq 1$ since $\delta y < x$). Therefore, the survival function is given by:

$$\psi(\tau) = e^{-a\tau} + \sum_{k=1}^{\infty} e^{-a\tau} \frac{(a\tau)^k}{k!} \left(\frac{x/\delta^k}{y}\right)^{-\lambda} = e^{-a\tau} + \left(\frac{y}{x}\right)^{\lambda} e^{-a\tau} \sum_{k=1}^{\infty} \frac{(a\delta^\lambda \tau)^k}{k!} = e^{-a\tau} + \left(\frac{y}{x}\right)^{\lambda} e^{-a\tau} e^{a\delta^\lambda \tau},$$

where the final equality uses the (globally convergent) series expansion of the exponential function. The survival function is thus:

$$\psi(\tau) = \left(1 - \left(\frac{y}{x}\right)^{\lambda}\right) e^{-a\tau} + \left(\frac{y}{x}\right)^{\lambda} e^{-a(1-\delta^\lambda)\tau}, \qquad \text{[A.4.8]}$$

observing that $\psi(0) = 1$.

Given the survival function $\psi(\tau)$, the hazard function $\phi(\tau)$ for moving is defined by $\phi(\tau) = -\psi'(\tau)/\psi(\tau)$. Using equation [A.4.8]:

$$\phi(\tau) = -\frac{\mathrm{d}}{\mathrm{d}\tau} \log e^{-a\tau} \left(1 + \left(\frac{y}{x}\right)^{\lambda} \left(e^{a\delta^\lambda \tau} - 1\right)\right) = a - \frac{a\delta^\lambda \left(\frac{y}{x}\right)^{\lambda} e^{a\delta^\lambda \tau}}{1 + \left(\frac{y}{x}\right)^{\lambda} \left(e^{a\delta^\lambda \tau} - 1\right)}.$$

Therefore, the hazard function is given by:

$$\phi(\tau) = a \left(1 - \frac{\delta^\lambda \left(\frac{y}{x}\right)^{\lambda}}{\left(\frac{y}{x}\right)^{\lambda} + \left(1 - \left(\frac{y}{x}\right)^{\lambda}\right) e^{-a\delta^\lambda \tau}}\right), \qquad \text{[A.4.9]}$$

which is increasing in τ.

For new matches, the distribution $\mu(\tau)$ of the time τ until the next move is obtained from the survival function $\psi(\tau)$ using $\mu(\tau) = -\psi'(\tau)$. Hence, by using [A.4.8]:

$$\mu(\tau) = a \left(1 - \left(\frac{y}{x}\right)^{\lambda}\right) e^{-a\tau} + a(1 - \delta^\lambda) \left(\frac{y}{x}\right)^{\lambda} e^{-a(1-\delta^\lambda)\tau}. \qquad \text{[A.4.10]}$$

The expected time T_{n} until the next move for a new match (expected tenure) can be derived from this distribution:

$$T_{\mathrm{n}} = \int_{\tau=0}^{\infty} \tau \mu(\tau) \mathrm{d}\tau = \left(1 - \left(\frac{y}{x}\right)^{\lambda}\right) \int_{\tau=0}^{\infty} a\tau e^{-a\tau} \mathrm{d}\tau + \left(\frac{y}{x}\right)^{\lambda} \int_{\tau=0}^{\infty} a(1-\delta^\lambda)\tau e^{-a(1-\delta^\lambda)\tau} \mathrm{d}\tau$$
$$= \left(1 - \left(\frac{y}{x}\right)^{\lambda}\right) \frac{1}{a} + \left(\frac{y}{x}\right)^{\lambda} \frac{1}{a(1-\delta^\lambda)} = \frac{1}{a} \left(1 - \left(\frac{y}{x}\right)^{\lambda} + \frac{1}{1-\delta^\lambda} \left(\frac{y}{x}\right)^{\lambda}\right).$$

Thus, an expression for expected tenure is:

$$T_{\mathrm{n}} = \frac{1}{a} \left(1 + \frac{\delta^\lambda}{1 - \delta^\lambda} \left(\frac{y}{x}\right)^{\lambda}\right). \qquad \text{[A.4.11]}$$

The stationary age distribution $\theta(\tau)$ in the cross-section of surviving matches is proportional to the survival function $\psi(\tau)$:

$$\theta(\tau) = \frac{\psi(\tau)}{\int_{\tau=0}^{\infty} \psi(\tau) \mathrm{d}\tau}. \qquad \text{[A.4.12]}$$

The expression for $\psi(\tau)$ in [A.4.8] can be used to deduce that:

$$\int_{\tau=0}^{\infty} \psi(\tau)\mathrm{d}\tau = \left(1 - \left(\frac{y}{x}\right)^{\lambda}\right)\frac{1}{a} + \left(\frac{y}{x}\right)^{\lambda}\frac{1}{a(1-\delta^{\lambda})} = \frac{1}{a}\left(1 + \frac{\delta^{\lambda}}{1-\delta^{\lambda}}\left(\frac{y}{x}\right)^{\lambda}\right). \qquad \text{[A.4.13]}$$

Combining equations [A.4.12] and [A.4.13], the stationary age distribution is:

$$\theta(\tau) = \frac{a\left(1 - \left(\frac{y}{x}\right)^{\lambda}\right)e^{-a\tau} + a\left(\frac{y}{x}\right)^{\lambda}e^{-a(1-\delta^{\lambda})\tau}}{1 + \frac{\delta^{\lambda}}{1-\delta^{\lambda}}\left(\frac{y}{x}\right)^{\lambda}}. \qquad \text{[A.4.14]}$$

This distribution can be used to calculate the average years of ownership T_{h} of existing homeowners:

$$T_{\mathrm{h}} = \int_{\tau=0}^{\infty} \tau\theta(\tau)\mathrm{d}\tau = \frac{\left(1 - \left(\frac{y}{x}\right)^{\lambda}\right)\int_{\tau=0}^{\infty} a\tau e^{-a\tau}\mathrm{d}\tau + \frac{1}{1-\delta^{\lambda}}\left(\frac{y}{x}\right)^{\lambda}\int_{\tau=0}^{\infty} a(1-\delta^{\lambda})\tau e^{-a(1-\delta^{\lambda})\tau}\mathrm{d}\tau}{1 + \frac{\delta^{\lambda}}{1-\delta^{\lambda}}\left(\frac{y}{x}\right)^{\lambda}}$$

$$= \frac{\left(1 - \left(\frac{y}{x}\right)^{\lambda}\right)\frac{1}{a} + \frac{1}{1-\delta^{\lambda}}\left(\frac{y}{x}\right)^{\lambda}\frac{1}{a(1-\delta^{\lambda})}}{1 + \frac{\delta^{\lambda}}{1-\delta^{\lambda}}\left(\frac{y}{x}\right)^{\lambda}} = \frac{\frac{1}{a}\left(1 - \left(\frac{y}{x}\right)^{\lambda} + \frac{1}{(1-\delta^{\lambda})^2}\left(\frac{y}{x}\right)^{\lambda}\right)}{1 + \frac{\delta^{\lambda}}{1-\delta^{\lambda}}\left(\frac{y}{x}\right)^{\lambda}} = \frac{\frac{1}{a}\left(1 + \left(\frac{(1-\delta^{\lambda})^2-1}{(1-\delta^{\lambda})^2}\right)\left(\frac{y}{x}\right)^{\lambda}\right)}{1 + \frac{\delta^{\lambda}}{1-\delta^{\lambda}}\left(\frac{y}{x}\right)^{\lambda}}.$$

Simplifying leads to the following formula for T_{h}:

$$T_{\mathrm{h}} = \frac{\frac{1}{a}\left(1 + \frac{\delta^{\lambda}}{1-\delta^{\lambda}}\left(1 + \frac{1}{1-\delta^{\lambda}}\right)\left(\frac{y}{x}\right)^{\lambda}\right)}{1 + \frac{\delta^{\lambda}}{1-\delta^{\lambda}}\left(\frac{y}{x}\right)^{\lambda}}. \qquad \text{[A.4.15]}$$

The average moving rate n can be calculated from the hazard function $\phi(\tau)$ and the age distribution $\theta(\tau)$:

$$n = \int_{\tau=0}^{\infty} \phi(\tau)\theta(\tau)\mathrm{d}\tau = \int_{\tau=0}^{\infty}\left(-\frac{\psi'(\tau)}{\psi(\tau)}\right)(\theta(0)\psi(\tau))\mathrm{d}\tau = \theta(0)\int_{\tau=0}^{\infty}(-\psi'(\tau))\mathrm{d}\tau = \theta(0),$$

where the second equality uses the definition $\phi(\tau) = -\psi'(\tau)/\psi(\tau)$, and $\theta(\tau) = \theta(0)\psi(\tau)$, which follows from [A.4.12] noting $\psi(0) = 1$. Evaluating [A.4.14] at $\tau = 0$ implies the following expression for n:

$$n = \frac{a}{1 + \frac{\delta^{\lambda}}{1-\delta^{\lambda}}\left(\frac{y}{x}\right)^{\lambda}}, \qquad \text{[A.4.16]}$$

which is the reciprocal of expected tenure ($T_{\mathrm{n}} = 1/n$), and thus consistent with equation [A.4.11].

Given the moving rate n and the sales rate s, the steady-state stock of houses for sale is:

$$u = \frac{n}{s+n}. \qquad \text{[A.4.17]}$$

The expression for n given in [A.4.16] is consistent with the formula [3.12] in the steady state:

$$n = a - a\delta^{\lambda}x^{-\lambda}v\frac{u}{1-u}\int_{\tau=0}^{\infty} e^{-a(1-\delta^{\lambda})\tau}\mathrm{d}\tau = a - a\delta^{\lambda}\left(\frac{y}{x}\right)^{\lambda}vy^{-\lambda}\frac{n}{s}\frac{1}{a(1-\delta^{\lambda})},$$

where the final equality uses $u/(1-u) = n/s$, as implied by [A.4.17]. Since $s = vy^{-\lambda}$ according to [A.2.22], the equation above becomes:

$$n = a - \frac{\delta^{\lambda}}{1-\delta^{\lambda}}\left(\frac{y}{x}\right)^{\lambda}n,$$

which yields the same solution for n as [A.4.16].

A.5 Efficiency

The social planner's objective function from [4.9] can be written in terms of total match quality $\mathcal{E}_t$, the transaction threshold y_t, and houses for sale u_t by substituting $\mathcal{E}_t = (1-u_t)Q_t$ and using equation [3.11]:

$$S_T = \int_{t=T}^{\infty} e^{-r(t-T)} \left(\xi\mathcal{E}_t - Cvy_t^{-\lambda}u_t - Fu_t - M\right) \mathrm{d}t, \tag{A.5.1}$$

This is maximized by choosing x_t, y_t, $\mathcal{E}_t$, u_t, and I_t subject to the differential equations for $\mathcal{E}_t$, u_t, and I_t in [A.4.5], [A.4.6], and [A.4.7]. The problem is solved by introducing the (current-value) Hamiltonian:

$$\mathcal{J}_t = \xi\mathcal{E}_t - Cvy_t^{-\lambda}u_t - Fu_t - M + \varphi_t\left(\frac{v\lambda}{\lambda-1}y_t^{1-\lambda}u_t - a\mathcal{E}_t + \frac{av\delta^\lambda\lambda}{\lambda-1}x_t^{1-\lambda}I_t\right) \\ + \vartheta_t\left(a(1-u_t) - av\delta^\lambda x_t^{-\lambda}I_t - vy_t^{-\lambda}u_t\right) + \gamma_t\left(u_t - a(1-\delta^\lambda)I_t\right), \tag{A.5.2}$$

where φ_t, ϑ_t, and γ_t are the co-state variables associated with $\mathcal{E}_t$, u_t, and I_t. The first-order conditions with respect to x_t and y_t are:

$$\frac{\partial\mathcal{J}_t}{\partial x_t} = av\delta^\lambda\lambda x_t^{-\lambda-1}I_t\vartheta_t - av\delta^\lambda\lambda x_t^{-\lambda}I_t\varphi_t = 0; \tag{A.5.3a}$$

$$\frac{\partial\mathcal{J}_t}{\partial y_t} = v\lambda Cy_t^{-\lambda-1}u_t - v\lambda y_t^{-\lambda}u_t\varphi_t + v\lambda y_t^{-\lambda-1}u_t\vartheta_t = 0, \tag{A.5.3b}$$

and the first-order conditions with respect to the state variables $\mathcal{E}_t$, u_t, and I_t are:

$$\frac{\partial\mathcal{J}_t}{\partial\mathcal{E}_t} = \xi - a\varphi_t = r\varphi_t - \dot{\varphi}_t; \tag{A.5.3c}$$

$$\frac{\partial\mathcal{J}_t}{\partial u_t} = -Cvy_t^{-\lambda} - F + \frac{v\lambda}{\lambda-1}y_t^{1-\lambda}\varphi_t - (a + vy_t^{-\lambda})\vartheta_t + \gamma_t = r\vartheta_t - \dot{\vartheta}_t; \tag{A.5.3d}$$

$$\frac{\partial\mathcal{J}_t}{\partial I_t} = \frac{av\delta^\lambda\lambda}{\lambda-1}x_t^{1-\lambda}\varphi_t - av\delta^\lambda x_t^{-\lambda}\vartheta_t - a(1-\delta^\lambda)\gamma_t = r\gamma_t - \dot{\gamma}_t. \tag{A.5.3e}$$

By cancelling common terms from [A.5.3a], the following link between the moving threshold x_t and the co-states φ_t and ϑ_t can be deduced:

$$\varphi_t = \frac{\vartheta_t}{x_t}. \tag{A.5.4}$$

Similarly, cancelling common terms from [A.5.3b] implies a link between the transaction threshold y_t and φ_t and ϑ_t:

$$\frac{C}{y_t} + \frac{\vartheta_t}{y_t} = \varphi_t. \tag{A.5.5}$$

The differential equation for φ_t in [A.5.3c] is:

$$\dot{\varphi}_t = (r+a)\varphi_t - \xi,$$

and since $r + a > 0$, the only solution satisfying the transversality condition is the following constant solution:

$$\varphi_t = \frac{\xi}{r+a}. \tag{A.5.6}$$

With this solution for φ_t, equation [A.5.5] implies that ϑ_t is proportional to the moving threshold x_t:

$$\vartheta_t = \frac{\xi}{r+a}x_t. \tag{A.5.7}$$

Eliminating both φ_t and ϑ_t from [A.5.5] by substituting from [A.5.6] and [A.5.7] implies that y_t and x_t must satisfy:

$$y_t - x_t = \frac{(r+a)C}{\xi}. \tag{A.5.8}$$

Using [A.5.3d] to write a differential equation for ϑ_t and substituting the solution for φ_t from [A.5.6]:

$$\dot{\vartheta}_t = (r+a+vy_t^{-\lambda})\vartheta_t - \gamma_t + F + Cvy_t^{-\lambda} - \frac{\xi}{r+a}\frac{v\lambda}{\lambda-1}y_t^{1-\lambda}. \tag{A.5.9}$$

Similarly, [A.5.3e] implies a differential equation for γ_t, from which φ_t can be eliminated using [A.5.6]:

$$\dot{\gamma}_t = (r+a(1-\delta^\lambda))\gamma_t + av\delta^\lambda x_t^{-\lambda}\left(\frac{\xi}{r+a}x_t\right) - \frac{av\delta^\lambda\lambda}{\lambda-1}x_t^{1-\lambda}\left(\frac{\xi}{r+a}\right),$$

which can be simplified as follows:

$$\dot{\gamma}_t = (r+a(1-\delta^\lambda))\gamma_t - \frac{\xi}{r+a}\frac{av\delta^\lambda}{\lambda-1}x_t^{1-\lambda}. \tag{A.5.10}$$

It is now shown that there is a solution of the constrained maximization problem where the co-states ϑ_t and γ_t are constant over time. In this case, equations [A.5.7] and [A.5.8] require that x_t and y_t are constant over time and related as follows:

$$y - x = \frac{(r+a)C}{\xi}. \tag{A.5.11}$$

With $\dot{\vartheta}_t = 0$ and $\dot{\gamma}_t = 0$, [A.5.9] and [A.5.10] imply the following pair of equations:

$$(r+a+vy^{-\lambda})\vartheta - \gamma + F + Cvy^{-\lambda} - \frac{\xi}{r+a}\frac{v\lambda}{\lambda-1}y^{1-\lambda} = 0; \tag{A.5.12}$$

$$(r+a(1-\delta^\lambda))\gamma - \frac{\xi}{r+a}\frac{av\delta^\lambda}{\lambda-1}x^{1-\lambda} = 0. \tag{A.5.13}$$

Equation [A.5.13] yields the following expression for γ in terms of x:

$$\gamma = \frac{\xi av\delta^\lambda}{(\lambda-1)(r+a)(r+a(1-\delta^\lambda))}x^{1-\lambda},$$

and substituting this and [A.5.7] into [A.5.12] leads to:

$$\frac{\xi(r+a+vy^{-\lambda})}{r+a}x - \frac{\xi av\delta^\lambda}{(\lambda-1)(r+a)(r+a(1-\delta^\lambda))}x^{1-\lambda} + F + Cvy^{-\lambda} - \frac{\xi v\lambda}{(\lambda-1)(r+a)}y^{1-\lambda} = 0.$$

Since $(r+a)C = \xi(y-x)$ according to [A.5.11], multiplying the equation above by $(r+a)$ and substituting for $(r+a)C$ implies:

$$\xi(r+a+vy^{-\lambda})x - \frac{\xi av\delta^\lambda}{(\lambda-1)(r+a(1-\delta^\lambda))}x^{1-\lambda} + (r+a)F + \xi v(y-x)y^{-\lambda} - \frac{\xi v\lambda}{(\lambda-1)}y^{1-\lambda} = 0.$$

Dividing both sides by ξ and grouping terms in $(r+a)$ on the left-hand side:

$$(r+a)\left(x+\frac{F}{\xi}\right)=\frac{v\lambda}{\lambda-1}y^{1-\lambda}-vy^{1-\lambda}+\frac{av\delta^{\lambda}}{(\lambda-1)(r+a(1-\delta^{\lambda}))}x^{1-\lambda},$$

and dividing both sides by $r+a$ and simplifying the terms involving $y^{1-\lambda}$ leads to:

$$x+\frac{F}{\xi}=\frac{v}{(\lambda-1)(r+a)}\left(y^{1-\lambda}+\frac{a\delta^{\lambda}}{r+a(1-\delta^{\lambda})}x^{1-\lambda}\right). \qquad \text{[A.5.14]}$$

The pair of equations [A.5.11] and [A.5.14] for x and y are identical to the equations [4.4] and [4.7] characterizing the equilibrium values of x and y. The equilibrium is therefore the same as the solution to the social planner's problem, establishing that it is efficient.

A.6 Transitional dynamics and overshooting

Transitional dynamics

If the moving and selling rates n_t and s_t were equal to constants n and s, the stock of houses for sale evolves according to the differential equation:

$$\dot{u}_t=n(1-u_t)-su_t=-(s+n)(u_t-u), \quad \text{where } u=\frac{n}{s+n}.$$

Using $s=1/T_{\text{s}}$ and $n=1/T_{\text{n}}$, the rate of convergence to the steady state is determined by:

$$\dot{u}_t=-\left(\frac{1}{T_{\text{s}}}+\frac{1}{T_{\text{n}}}\right)(u_t-u).$$

Now suppose the moving and transaction thresholds are constant over time at x and y. This means the sales rate is constant at $s=vy^{-\lambda}$. Let u_t^*, n_t^*, S_t^*, and N_t^* denote houses for sale, the moving rate, and the volumes of sales and listings, ignoring transitional dynamics in houses for sale. This means that the number of houses for sale satisfies:

$$u_t^*=\frac{n_t^*}{s+n_t^*}. \qquad \text{[A.6.1]}$$

Given u_t^*, the listing rate n_t^* can be obtained using the formula in [3.12]:

$$n_t^*=a-\frac{a\delta^{\lambda}vx^{-\lambda}}{1-u_t^*}\int_{\tau\to-\infty}^{t}e^{-a(1-\delta^{\lambda})(t-\tau)}u_\tau^*\mathrm{d}\tau. \qquad \text{[A.6.2]}$$

The volumes of sales and listings are:

$$S_t^*=su_t^*, \quad \text{and } N_t^*=n_t^*(1-u_t^*), \quad \text{with } S_t^*=N_t^*, \qquad \text{[A.6.3]}$$

where the latter claim follows from the definition of u_t^* in [A.6.1]. Multiplying both sides of [A.6.2] by $1-u_t^*$ leads to an equation for N_t^*:

$$N_t^*=a(1-u_t^*)-a\delta^{\lambda}vx^{-\lambda}\int_{\tau\to-\infty}^{t}e^{-a(1-\delta^{\lambda})(t-\tau)}u_\tau^*\mathrm{d}\tau, \qquad \text{[A.6.4]}$$

and differentiating with respect to time:

$$\dot{N}_t^*=-a\dot{u}_t^*-a\delta^{\lambda}vx^{-\lambda}u_t^*+a(1-\delta^{\lambda})\left(a\delta^{\lambda}vx^{-\lambda}\int_{\tau\to-\infty}^{t}e^{-a(1-\delta^{\lambda})(t-\tau)}u_\tau^*\mathrm{d}\tau\right).$$

Using [A.6.4] to substitute for the integral above:

$$\dot{N}_t^* = -a\dot{u}_t^* - a\delta^\lambda v x^{-\lambda} u_t^* + a(1-\delta^\lambda)(a(1-u_t^*) - N_t^*),$$

and since [A.6.3] implies $u_t^* = N_t^*/s$ for all t for which the sales rate s is constant (and thus $\dot{u}_t^* = \dot{N}_t^*/s$), the differential equation above can be written solely in terms of N_t^*:

$$\dot{N}_t^* = -\frac{a}{s}\dot{N}_t^* - \frac{a}{s}\delta^\lambda v x^{-\lambda} N_t^* + a(1-\delta^\lambda)\left(a\left(1-\frac{N_t^*}{s}\right) - N_t^*\right).$$

Collecting terms in $\dot{N}_t^*$ on the left-hand side and grouping terms in N_t^* on the right-hand side:

$$\left(\frac{a+s}{s}\right)\dot{N}_t^* = a(1-\delta^\lambda)a - a(1-\delta^\lambda)\left(\frac{a+s}{s} + \frac{\delta^\lambda}{1-\delta^\lambda}\frac{vx^{-\lambda}}{s}\right)N_t^*,$$

and using $s = vy^{-\lambda}$ it follows that:

$$\dot{N}_t^* = a(1-\delta^\lambda)\left(\frac{as}{a+s} - \left(1 + \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda \frac{s}{a+s}\right)N_t^*\right). \qquad \text{[A.6.5]}$$

Now observe that:

$$\frac{\frac{as}{a+s}}{1 + \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda \frac{s}{a+s}} = \frac{a}{\frac{a+s}{s} + \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda} = \frac{a}{\frac{a}{s} + \left(1 + \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda\right)} = \frac{a}{\frac{a}{s} + \frac{a}{n}} = \frac{sn}{s+n} = N,$$

which uses the expression for n from [A.4.16], hence the differential equation [A.6.5] can be written as:

$$\dot{N}_t^* = -a(1-\delta^\lambda)\left(1 + \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda \frac{s}{a+s}\right)(N_t^* - N). \qquad \text{[A.6.6]}$$

The coefficient of $N_t^* - N$ gives the rate of convergence to the long-run steady state. Since $S_t^* = N_t^*$ and $u_t^* = N_t^*/s$, it follows that the rate of convergence is the same for S_t^* and u_t^*:

$$\dot{S}_t^* = -a(1-\delta^\lambda)\left(1 + \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda \frac{s}{a+s}\right)(S_t^* - S);$$
$$\dot{u}_t^* = -a(1-\delta^\lambda)\left(1 + \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda \frac{s}{a+s}\right)(u_t^* - u).$$

The result is similar for the moving rate n_t^*, though this is not a linear function of the other variables:

$$\dot{n}_t^* = -a(1-\delta^\lambda)\left(1 + \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda \frac{s}{a+s}\right)\left(\frac{s+n_t^*}{s+n}\right)(n_t^* - n).$$

Overshooting

Now consider the effect of a change in the moving threshold x at time t while the transaction threshold y is held constant, starting from a steady state. Beginning in a steady state implies

$$\int_{\tau\to-\infty}^{t} e^{-a(1-\delta^\lambda)(\tau-t)} u_\tau^* \mathrm{d}\tau = \frac{u_{t_-}^*}{a(1-\delta^\lambda)},$$

and substituting this expression into equation [A.6.2] shows that $n_{t_+}^*$ must satisfy:

$$n_{t_+}^* = a - \frac{a\delta^\lambda v x^{-\lambda}}{a(1-\delta^\lambda)}\frac{u_{t_-}^*}{1-u_{t_+}^*} = a - \frac{\delta^\lambda}{1-\delta^\lambda}\frac{vx^{-\lambda}}{s}(s+n_{t_+}^*)u_{t_-}^*,$$

where the second equality uses $1 - u_t^* = s/(s + n_t^*)$ implied by [A.6.1]. Noting that $s = vy^{-\lambda}$, the equation becomes:

$$n_{t_+}^* = a - \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda (s + n_{t_+}^*)u_{t-}^*,$$

which can be solved for $n_{t_+}^*$:

$$n_{t_+}^* = \frac{a - \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda s u_{t_-}^*}{1 + \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda u_{t_-}^*}.$$

This confirms the equation given in the text. Equation [4.11] implies that the new steady state moving rate n satisfies:

$$\frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda = \frac{a}{n} - 1,$$

which can be substituted into the formula for $n_{t_+}^*$ above to obtain:

$$n_{t_+}^* = \frac{a - \left(\frac{a}{n} - 1\right) s u_{t_-}^*}{1 + \left(\frac{a}{n} - 1\right) u_{t_-}^*}.$$

Note that $n_{t_+}^* > n$ is equivalent to:

$$a - \left(\frac{a}{n} - 1\right) s u_{t_-}^* > \left(1 + \left(\frac{a}{n} - 1\right) u_{t_-}^*\right) n,$$

and dividing both sides by n and factorizing leads to:

$$\left(\frac{a}{n} - 1\right)\left(1 - \frac{u_{t_-}^*}{u}\right) > 0.$$

Since $a > n$, this can hold only if $u > u_{t_-}^*$. With $u = n/(s + n)$ and $u_{t_-}^* = n_{t_-}^*/(s + n_{t_-}^*)$, this occurs when $n > n_{t_-}^*$, which confirms the claim that there is overshooting.

A.7 Productivity and interest rates

Suppose that a family's flow utility is $\mathcal{C}_t^{1-\alpha}\mathcal{H}_t^\alpha$, where $\mathcal{C}_t$ denotes consumption and $\mathcal{H}_t$ denotes housing, and where α indicates the importance of housing in the utility function ($0 < \alpha < 1$). This adds non-housing goods to the model and replaces the flow utility $\xi\epsilon$ assumed earlier. The housing variable $\mathcal{H}_t$ that enters the utility function is equal to the match quality ϵ of a family with its house, and the evolution of this variable in response to idiosyncratic shocks and moving and transaction decisions is the same as before. The discount rate for future utility flows is the rate of pure time preference ϱ. The life-time utility function from time T onwards is therefore:

$$\mathcal{U}_T = \int_{t=T}^{\infty} e^{-\varrho(t-T)}\mathcal{C}_t^{1-\alpha}\mathcal{H}_t^\alpha \mathrm{d}t. \qquad \text{[A.7.1]}$$

Suppose there are complete financial markets for securities with consumption payoffs contingent on any state of the world, and suppose all families receive the same real income and initially all have equal financial wealth. It follows that the marginal utility of consumption must be equalized across all families. The marginal utility of consumption is $z_t^{-\alpha}$, where $z_t = \mathcal{C}_t/\mathcal{H}_t$ is the ratio of consumption to housing match quality. If r is the real interest rate (in terms of consumption goods) then maximization of utility [A.7.1] subject to the life-time budget constraint requires that the following consumption Euler equation holds:

$$\alpha\frac{\dot{z}_t}{z_t} = r - \varrho. \qquad \text{[A.7.2]}$$

In equilibrium, the sum of consumption $\mathcal{C}_t$ across all families must be equal to aggregate real income Y_t, which is assumed to be an exogenous endowment growing at rate g over time. Given equalization of $z_t = \mathcal{C}_t/\mathcal{H}_t$ across all families at a point in time and given a stationary distribution of match quality $\mathcal{H}_t = \epsilon$ across all families, it follows that all families have a value of z_t proportional to aggregate real income Y_t at all times:

$$z_t = \varkappa Y_t, \tag{A.7.3}$$

for some constant $\varkappa$. Substituting this into the consumption Euler equation [A.7.2] implies that the equilibrium real interest rate is:

$$r = \varrho + \alpha g. \tag{A.7.4}$$

Since [A.7.3] implies $\mathcal{C}_t = \varkappa Y_t \mathcal{H}_t$, it follows that lifetime utility [A.7.1] can be expressed as follows:

$$\mathcal{U}_T = \varkappa^{1-\alpha} \int_{t=T}^{\infty} e^{-\varrho(t-T)} Y_t^{1-\alpha} \mathcal{H}_t \mathrm{d}t.$$

With Y_t growing at rate g, income at time t can be written as $Y_t = e^{g(t-T)Y_T}$. By substituting this into the lifetime utility function and using the expression for the real interest rate r in [A.7.4]:

$$\mathcal{U}_T = \varkappa^{1-\alpha} Y_T^{1-\alpha} \int_{t=T}^{\infty} e^{-(r-g)(t-T)} \mathcal{H}_t \mathrm{d}t. \tag{A.7.5}$$

Lifetime utility is a discounted sum of match quality $\mathcal{H}_t = \epsilon$. The coefficient of match quality (this is the parameter ξ in the main text) is increasing in the current level of real income, and the discount rate (denoted r in the main text) is the difference between the market interest rate and the real growth rate. This provides a justification for interpreting a rise in real incomes as an increase in ξ and a fall in the market interest rate as a lower discount rate.

A.8 Calibration method

This section shows how the 10 parameters a, δ, λ, v, C, F, M, κ, ω, and r can be obtained from observables. Three of the parameters (κ, ω, and r) are set directly. The other seven are obtained indirectly from seven calibration targets. These are time-to-sell T_{s}, viewings per sale V_{s}, expected tenure T_{n}, average years of ownership T_{h}, the transaction cost to price ratio c, the flow search cost to price ratio f, and the flow maintenance cost to price ratio m. Note that the model contains one other parameter ξ, but in all equations determining observables, ξ enters only as a ratio to other parameters (this can be seen from equations [5.1a]–[5.1c]). This parameter is therefore normalized to $\xi = 1$.

The calibration method begins by setting κ, ω, and r directly. Next, consider a guess for T_δ, the expected time until an idiosyncratic shock occurs. This conjecture determines the parameter a using:

$$a = \frac{1}{T_\delta}. \tag{A.8.1}$$

The admissible range for T_δ is $0 < T_\delta < T_{\mathrm{n}}$.

Define the following variable ζ:

$$\zeta \equiv \frac{\delta^\lambda}{1-\delta^\lambda}\left(\frac{y}{x}\right)^\lambda. \tag{A.8.2}$$

Using equations [A.8.1] and [A.8.2], the expressions for T_{n} and T_{h} in [A.4.11] and [A.4.15] can be written as:

$$T_{\mathrm{n}} = (1+\zeta)T_\delta, \quad \text{and} \quad T_{\mathrm{h}} = \frac{\left(1 + \left(1 + \frac{1}{1-\delta^\lambda}\right)\zeta\right)T_\delta}{1+\zeta}. \tag{A.8.3}$$

This equation confirms it is necessary that $0 < T_\delta < T_\mathrm{n}$ otherwise ζ would not be positive, as required in [A.8.2]. Solving for ζ using the expression for T_n in [A.8.3]:

$$\zeta = \frac{T_\mathrm{n} - T_\delta}{T_\delta}. \qquad \text{[A.8.4]}$$

Equations [A.8.3] and [A.8.4] can be used to deduce:

$$\frac{T_\mathrm{n}}{T_\delta} + \left(\frac{T_\mathrm{n} - T_\delta}{T_\delta}\right)\frac{1}{1-\delta^\lambda} = 1 + \left(1 + \frac{1}{1-\delta^\lambda}\right)\left(\frac{T_\mathrm{n} - T_\delta}{T_\delta}\right) = (1+\zeta)\frac{T_\mathrm{h}}{T_\delta} = \frac{T_\mathrm{n}}{T_\delta}\frac{T_\mathrm{h}}{T_\delta},$$

and multiplying both sides by T_δ/T_n and rearranging yields:

$$\left(\frac{T_\mathrm{n} - T_\delta}{T_\mathrm{n}}\right)\frac{1}{1-\delta^\lambda} = \frac{T_\mathrm{h} - T_\delta}{T_\delta}.$$

This leads to an expression for $\delta^\lambda/(1-\delta^\lambda)$:

$$\frac{\delta^\lambda}{1-\delta^\lambda} = \frac{1}{1-\delta^\lambda} - 1 = \frac{(T_\mathrm{h} - T_\delta)T_\mathrm{n}}{(T_\mathrm{n} - T_\delta)T_\delta} - 1, \qquad \text{[A.8.5]}$$

and using this in conjunction with [A.8.2] and [A.8.4]:

$$\left(\frac{y}{x}\right)^\lambda = \frac{\frac{T_\mathrm{n} - T_\delta}{T_\delta}}{\frac{(T_\mathrm{h} - T_\delta)T_\mathrm{n}}{(T_\mathrm{n} - T_\delta)T_\delta} - 1}. \qquad \text{[A.8.6]}$$

Now take equation [A.3.3] for the average price and divide both sides by P (recalling the normalization $\xi = 1$):

$$\kappa c - \frac{m}{r} + \omega\left(\frac{1}{r} + \frac{y^\lambda}{v}\right)\left(\frac{x}{P} + f\right) = 1.$$

Noting the expression for T_s in [A.2.22], the equation above can be solved for x/P as follows:

$$\frac{x}{P} = \frac{1 - \kappa c + \frac{m}{r}}{\omega\left(\frac{1}{r} + T_\mathrm{s}\right)} - f. \qquad \text{[A.8.7]}$$

Now take the linear equation [A.2.7] involving the thresholds x and y and divide both sides by P (again, recalling $\xi = 1$):

$$\frac{y}{P} = \frac{x}{P} + (r + a)c,$$

and then dividing both sides by x/P and using [A.8.7]:

$$\frac{y}{x} = \frac{y/P}{x/P} = 1 + \frac{(r+a)c}{\frac{1-\kappa c + \frac{m}{r}}{\omega\left(\frac{1}{r} + T_\mathrm{s}\right)} - f}. \qquad \text{[A.8.8]}$$

With formulas for both $(y/x)^\lambda$ from [A.8.6] and y/x from [A.8.8], the value of λ can be deduced from the identity $\log(y/x)^{\lambda} = \lambda \log(y/x)$:

$$\lambda = \frac{\log\left(\frac{\frac{T_{\mathrm{n}}-T_{\delta}}{T_{\delta}}}{\frac{(T_{\mathrm{h}}-T_{\delta})T_{\mathrm{n}}}{(T_{\mathrm{n}}-T_{\delta})T_{\delta}}-1}\right)}{\log\left(1+\frac{(r+a)c}{\frac{1-\kappa c+\frac{m}{r}}{\omega\left(\frac{1}{r}+T_{\mathrm{s}}\right)}-f}\right)}. \qquad [\text{A.8.9}]$$

Given λ, the transactions and moving thresholds y and x can be obtained from viewings per sale V_{s} using [A.2.21] and [A.8.8]:

$$y = V_{\mathrm{s}}^{\frac{1}{\lambda}}, \quad \text{and} \quad x = \frac{V_{\mathrm{s}}^{\frac{1}{\lambda}}}{1+\frac{(r+a)c}{\frac{1-\kappa c+\frac{m}{r}}{\omega\left(\frac{1}{r}+T_{\mathrm{s}}\right)}-f}}. \qquad [\text{A.8.10}]$$

Using [A.2.21] and [A.2.22], the ratio of viewings per sale V_{s} and time to sell T_{s} determines the meeting rate v:

$$v = \frac{V_{\mathrm{s}}}{T_{\mathrm{s}}}. \qquad [\text{A.8.11}]$$

Combining equations [A.8.7] and [A.8.10] leads to the following expression for P:

$$P = \frac{V_{\mathrm{s}}^{\frac{1}{\lambda}}}{\frac{1-\kappa c+\frac{m}{r}}{\omega\left(\frac{1}{r}+T_{\mathrm{s}}\right)} + (r+a)c - f},$$

and this can be used to obtain the parameters C, F, and M using $C = cP$, $F = fP$, and $M = mP$. The parameter δ is derived from [A.8.5] and the value of λ in [A.8.9]:

$$\delta = \left(1 - \frac{(T_{\mathrm{n}}-T_{\delta})T_{\delta}}{(T_{\mathrm{h}}-T_{\delta})T_{\mathrm{n}}}\right)^{\frac{1}{\lambda}}.$$

Finally, equation [A.2.16] must also hold, which requires:

$$x + \frac{F}{\xi} = \frac{v}{(\lambda-1)(r+a)}\left(y^{1-\lambda} + \frac{a\delta^{\lambda}}{r+a(1-\delta^{\lambda})}x^{1-\lambda}\right),$$

and this is used to verify the initial conjecture for T_{δ}.