The Race between Machine and Man: Implications of Technology for Growth, Factor Shares and Employment*

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Abstract

The advent of automation and the simultaneous decline in the labor share and employment among advanced economies, have raised concerns that labor will be marginalized and made redundant by new technologies. In this paper, we study these ideas in a task-based framework in which, in addition to the automation of tasks previously performed by labor, it is also possible to create more complex versions of existing tasks, and it is labor that tends to have a comparative advantage in these new tasks. We fully characterize the structure of equilibrium in this model, showing how the allocation of factors to tasks, and factor prices, are determined by the available technology and the endogenous choices of firms between capital and labor. We then show that while automation tends to reduce employment and the share of labor in national income, the creation of more complex tasks increases them, and both types of innovations contribute to economic growth.

Our full model endogenizes the direction of research and development towards automation and the creation of new complex tasks. Endogenous technology is consistent with a balanced growth path in which both types of innovations go hand-in-hand. More importantly, our analysis shows that the equilibrium self corrects: an increase in automation reduces the wage to rental rate ratio, discouraging further automation and encouraging greater creation of more labor-intensive tasks. This process restores the share of labor in national income and the employment to population ratio. Though the economy is self-correcting, we show that the equilibrium allocation of research effort is not optimal. Automation is attractive to firms because it reduces wage payments. To the extent that wages reflect quasi-rents for workers, this will create too much automation. Finally, we extend the model to include workers of different skills. We find that inequality may increase during transitions, but the self-correcting forces of the economy limit the increase in inequality over longer periods.

Still in Progress. Comments Welcome.

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1 Introduction

The accelerated automation of tasks performed by labor has intensified concerns that these new technologies will make labor redundant (e.g., Brynjolfsson and McAfee, 2012, Akst, 2014). The recent declines in the share of labor in national income and the employment to population ratio in the US economy, shown in Figure 1,[1] are often interpreted to support the claims that as digital technologies, robotics and artificial intelligence penetrate the economy more deeply, workers will find it increasingly difficult to compete and their compensation will experience a relative or even absolute decline. Nevertheless, a comprehensive framework where such effects, as well as countervailing forces, are present remains to be developed. The need for such a framework stems not only from the importance of understanding how and when automation will have these transformative effects on the labor market, but also from the fact that similar claims have been made, and yet have not always come true, about previous waves of new technologies. Keynes (1930), for example, famously foresaw the steady increase in per capita income in the 20th century from the introduction of new technologies, but incorrectly predicted that this would create widespread technological unemployment as machines replaced men. Economic historian Robert Heilbroner confidently stated in 1965 that "as machines continue to invade society, duplicating greater and greater numbers of social tasks, it is human labor itself — at least, as we now think of 'labor' — that is gradually rendered redundant" (quoted in Akst, 2014), while another observer of mid-century automation, economist Ben Seligman, similarly predicted a future of work without men (Seligman, 1966). Though more understated, Wassily Leontief was equally pessimistic about the implications of new machines, drawing an analogy with the technologies of the early 20th century making horses redundant, and speculating "Labor will become less and less important… More and more workers will be replaced by machines. I do not see that new industries can employ everybody who wants a job" (Leontief, 1952).

This paper is a first step in developing a conceptual framework which both shows how machines replaced human labor and why this may or may not translate into disappearance of work and stagnant wages. Our main conceptual innovation is to introduce not only automation that replaces tasks previously performed by labor, but also the creation of new complex tasks where labor has a comparative advantage.[2] This point is well illustrated by the technological and organizational changes during the Second Industrial Revolution, which not only involved the replacement of the

[1] Figure 1 presents the estimate trends in the employment to population ratio for potential workers aged 25-64, nonfarm business sector labor share and productivity. The trends are computed using the Hodrick-Prescott filter with parameter 6.25. See Karabarbounis and Neiman (2014), Piketty and Zucman (2014), and Oberfield and Raval (2014) for more detailed evidence on the decline of the share of labor in national income.

[2] And herein lies our answer to Leontief's analogy: the difference between human labor and horse labor is that humans have a comparative advantage in more complex, new tasks. Horses did not.

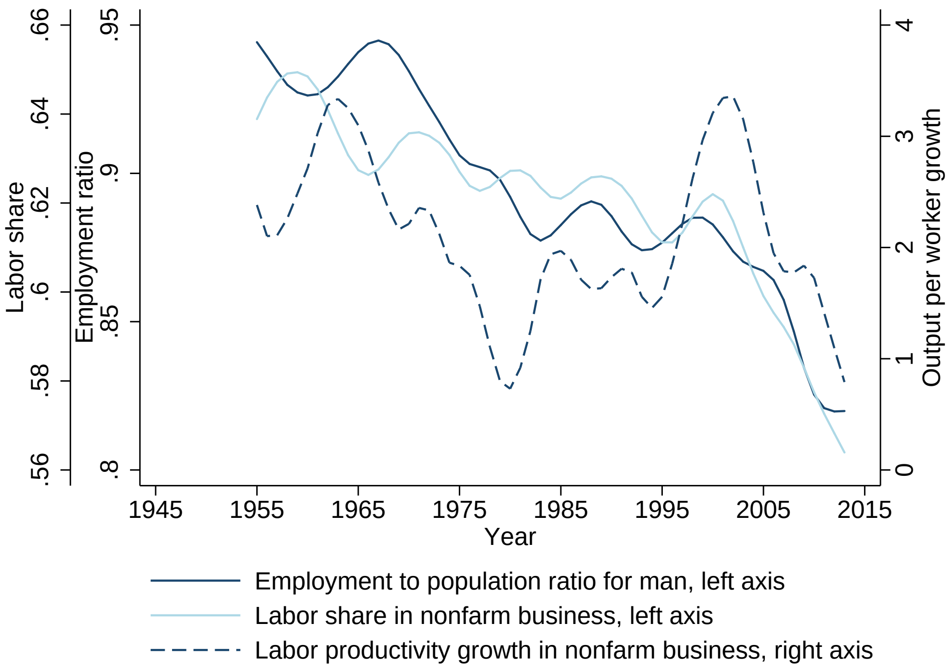

Figure 1: Trends in the employment to population ratio among men between 25-54 years, and the labor share in the nonfarm business sector in the United States.

stagecoach by the railroad, sailboats by steamboats, and of manual dock workers by cranes, but also the creation of new labor-intensive tasks — including a new class of engineers, machinists, repairmen, and conductors as well as modern managers and financiers involved with the introduction and operation of these new technologies (e.g., Landes, 1969, Chandler, 1977, and Mokyr, 1990). Similarly today, while digital technologies and computer-controlled machines replace labor, we are witnessing the emergence of new design tasks ranging from engineering functions based on new machines to programming and apps design.

After presenting a number of motivating facts, this paper develops a tractable but rich framework to study how different technologies impact factor prices, factor shares in national income and employment. In contrast to the more commonly-used models featuring factor-augmenting technological change, in this task-based framework new technologies that facilitate automation not only reduce the share of labor in national income, but may also reduce wages and employment. Conversely, the creation of new labor-intensive tasks increases wages, employment and the share of labor in national income, and may reduce the rate of return to capital. These comparative statics follow because factor prices are determined by the range of tasks performed by capital and labor (see also Acemoglu and Autor, 2011).

We then embed this framework in a dynamic economy in which capital accumulation is endogenous. We characterize restrictions under which the model delivers balanced growth — which we take to be a good approximation to economic growth in the United States and the United Kingdom over the last two centuries. The key restriction is that there is exponential productivity growth from the creation of new tasks and that the two types of technological changes — automation and creation of new labor-intensive tasks — ought to advance at equal paces.

Our main model endogenizes the rate of improvement of these two types of technologies by marrying our task-based framework with a canonical directed technological setup. We show that this full version of the model remains tractable and generates balanced growth — involving the equal advancement of the two types of technologies. More importantly, under natural restrictions, this balanced growth path is globally asymptotically stable, so that when one type of technological change runs ahead of the other, market forces induce advances in the other type of technology. The economics of these self-correcting forces are instructive and highlight a crucial new force: increased automation pushes wages down relative to the rental rate of capital, and when technology is endogenous, encourages the creation of new tasks.[3] Even though there may be an indirect market size effect due to an induced capital accumulation response, under our maintained assumptions this (factor) price effect dominates and makes it more profitable to use the now cheaper labor, thus triggering the creation of new labor-intensive tasks and a powerful force towards restoring employment and the labor share to their values before the increase in automation. Put differently, in our model where new technologies replace tasks, relative factor prices emerge as the key object regulating the future path of technological change.[4]

The most important implication of the stability of the balanced growth path is that, in this model economy, periods in which automation runs ahead of the creation of new more complex tasks will tend to self-correct. Thus, contrary to the increasingly widespread concerns discussed above, our model raises the (theoretical) possibility that the extended period of rapid automation may not signal the demise of labor, but may be a prelude to a new phase of new technologies favoring labor.[5]

Our framework also highlights that there are three types of elasticity of substitution that need

[3]Our analysis also reveals another (partially) self-correcting economic force, a *productivity effect*: automation substitutes the cheaper capital for labor, thus increasing productivity and the demand for all factors. This effect is present in our model throughout, and does not change the fact that automation reduces the share of labor in national income (and may even reduce the wage rate). However, it becomes more powerful in the long run when (and if) the interest rate is constant, e.g., due to capital accumulation, and we discuss its implications below.

[4]The role of technologies replacing tasks in this result can be seen by noting that with factor-augmenting technological changes, the impact on relative factor prices is ambiguous (depending on the elasticity of substitution between factors), and the incentives determining the direction of innovation may be dominated by a strong market size effect.

[5]Of course, in the model, there are other types of structural changes which may have different long-run consequences. For example, if the developments we observe are triggered by a change in the innovation possibilities frontier (the technology of creating technologies) making it easier than before to invent automation technologies, then the economy may undergo an extended period of automation and ultimately settling for new balance growth path with a greater share of tasks performed by capital and a lower share of labor in national income.

to be distinguished. The first simply reflects the elasticity of substitution between different tasks and applies when the allocation of tasks to capital and labor is held fixed (it also applies when all of the tasks that are technologically feasible to automate have been automated). The second, which is always larger than the first, incorporates the endogenous change in the range of tasks performed by capital and labor in response to changes in factor prices. The third, the long-run elasticity, also takes into account both the accumulation of capital and the endogenous change of technology in response to factor prices/supplies.

The final major implication of our framework concerns the efficiency of equilibrium. In addition to the standard and well-known inefficiencies due to monopoly markups and appropriability problems in endogenous technological change models, our analysis identifies a new source of inefficiency in the direction of technological advance, pushing towards too much automation and too little creation of new tasks. Intuitively, the market economy responds to factor prices, and thus when wages are high, automation becomes profitable as it enables firms to economize on wages. However, because some of the wage payments accruing to workers are rents (as highlighted by our quasi-labor supply), these do not represent cost savings, and firms would engage in too much automation. In contrast, the social planner's incentives to automate a task are determined by the opportunity cost of labor. Thus, the social planner automates less jobs, and conversely, her incentives for the creation of new tasks are always greater.

We consider two extensions of our model. In our baseline framework, all workers have the same skill level. In our first extension, we introduce heterogeneity in skills, and assume that skilled labor has a comparative advantage in newer tasks, which we deem as a natural assumption (in particular in view of the evidence presented in the next section).[6] Automation then tends to increase inequality by taking jobs from unskilled labor. The creation of new complex tasks also increases inequality at first, since skilled workers have comparative advantage in such tasks, but reduces it over longer periods as unskilled workers familiarize with the new technologies or tasks are standardized. This extension formalizes claims in the literature suggesting that both automation and new, more complex tasks, increase inequality, but also pointing out that short-run dynamics following such technological changes might be quite different — especially from their medium-term implications in the case of new labor-intensive tasks.

In our baseline framework we utilize a structure of patents and competition that eliminates the creative destruction of the profits of existing firms by new entrants (by assuming that new entrants have to buy the patents of the technologies they are replacing). This assumption, by removing the link between the net present discounted value of innovations and the future rate of replacement,

[6]This assumption builds on Schultz (1965) (see also Greenwood and Yorukoglu, 1997, Caselli, 1999, Galor and Moav, 2000, Acemoglu, Gancia and Zilibotti, 2010, and Beaudry, Green and Sand, 2013).

made our baseline model more tractable. Our second extension shows that, in the presence of this creative destruction of rents, the nature of the balance growth path remains largely unchanged, though dynamics become more complex (and there may also be room for multiple equilibria).

Our paper relates to several literatures. It can be viewed as a combination of task-based models of the labor market with directed technological change models.[7] Task-based models have been developed both in the economic growth and labor literatures, dating back at least to Roy's seminal work (1955). The first important recent contribution is Zeira (1998), which proposed a model of economic growth based on capital-labor substitution and constitutes a special case of our model when technology (both automation and the set of tasks) are held fixed. Acemoglu and Zilibotti (2000) developed a simple task-based model with endogenous technology and applied it to the study of productivity differences across countries resulting from mismatch between new technologies and the skills of developing economies (see also Zeira, 2006, Acemoglu, 2010). Autor, Levy and Murnane (2003) suggested that the increase in inequality in the US labor market reflects the replacement of routine, labor-intensive tasks by technology.[8] The static, exogenous-technology part of our model is most similar to Acemoglu and Autor's (2011) framework formalizing this notion. Our full model extends this framework not only because of the dynamic equilibrium incorporating directed technological change, but also because tasks are combined with a general elasticity of substitution (a feature that turns out to be important) and because the equilibrium allocation of tasks depends both on factor prices and the state of technology. Acemoglu and Autor's model, like ours, is one in which a discrete number of labor types are allocated to a continuum of tasks. Costinot and Vogel (2011) develop a complementary model in which skills and tasks form continuum sets. Also related is the recent paper by Hawkins, Ryan and Oh (2015), which shows how a task-based model is more successful than standard models in matching the co-movement of investment and employment at the firm level.

Three papers from the economic growth literature that are particularly related to our work are Acemoglu (2003a), Jones (2005), and Hemous and Olson (2014). The first two develop growth models in which the aggregate production function is endogenous and, in the long run, adapts to make balanced growth possible. In Jones (2005), this occurs because of endogenous choices about different combinations of activities/technologies being used. In Acemoglu (2003a), which is more closely related, this long-run behavior is a consequence of directed technological change. However, in contrast to the framework here, the two types of technologies that advance endogenously are both factor augmenting. The task-based framework developed here not only enables us to address

[7] On directed technological change and related models, see Acemoglu (1998, 2002, 2003a,b, 2007), Kiley (1999), Caselli and Coleman (2006), Gancia (2003), Thoenig and Verdier (2003) and Gancia and Zilibotti (2010).

[8] Acemoglu and Autor (2011), Autor and Dorn (2011), Jaimovich and Siu (2014), Foote and Ryan (2014), Burstein and Vogel (2012), and Burstein, Morales and Vogel (2014) provide various pieces of empirical evidence and quantitative evaluations on the importance of the endogenous allocation of tasks to factors in recent labor market dynamics.

questions related to automation and creation of new more complex tasks, which are our main focus here, but as already noted, also provides a more robust economic force ensuring the stability of a balanced growth path. As a result of these differences, in Acemoglu (2003a), a balanced growth path involving purely labor-augmenting technological change requires both somewhat more restrictive assumptions on the nature of the innovation possibilities frontier, and crucially also the elasticity of substitution between capital and labor to be less than 1. This is because, with factor-augmenting technologies, an elasticity of substitution greater than 1 implies that the factor that becomes more abundant commands a greater share of national income, potentially creating a force that might trigger further factor-augmenting improvements favoring the more abundant factor. In a task-based framework, in contrast, further automation increases the relative price of capital to labor, directly exerting a stabilizing force. Hemous and Olson (2014) develop a model of automation and horizontal innovation with endogenous technology and use it to study the income inequality consequences of different types of technologies. In their model too, high wages (but this time for low-skill workers) encourage automation, but they also show how this depresses growth in the short run and may be countered by horizontal innovation in the long run.

The rest of the paper is organized as follows. We start in Section 2 by presenting some recent facts motivating our approach. Section 3 presents our basic task-based framework in the context of a static economy. This enables us to explain the basic structure of our model, including the role of comparative advantage; highlight the roles of the two different types of technological advances (automation and creation of new tasks); and derive a complete set of comparative statics, showing how different types of technological changes impact factor shares, factor prices and employment. We also introduce two special cases of our model which lead to particularly transparent results. Section 4 introduces capital accumulation and clarifies the requisite structure of task productivity that is necessary for balanced growth in this economy. Section 5 introduces our full model. Here, we endogenize the decisions to automate certain tasks or create new more complex ones. We then show that equilibrium forces guarantee automation and the creation of new tasks advance together in a fashion that ensures balanced growth. Most importantly, our analysis in this section also establishes that this balanced growth path is globally stable, so that any imbalances in the composition of technologies will tend to self-correct. Section 6 compares the equilibrium composition of new technologies to the social planner's allocation, establishing that the equilibrium will tend to have too much automation and too little creation of new labor-intensive tasks. Section 7 considers the two extensions mentioned above. Section 8 concludes, while the Appendix contains the omitted proofs and empirical analysis described above.

2 Motivating Evidence

In this section, we provide a number of facts from the US labor market motivating our key assumption that the creation and expansion of new tasks play a central role in generating employment (and thus supporting the notion that labor has a comparative advantage in new tasks). We also provide some evidence suggesting that it is initially high-skill labor and then subsequently low-skill labor that mans these new occupations.

Our first exhibit uses data from Lin (2011) on "new tasks," in particular, measuring the extent to which there are novel jobs and tasks in each occupational group in 1980, 1990 and 2000 based on their appearance in the Dictionary of Occupational Titles in 1980 and 1990, and from US Census classification in 2000.[9] For example, in 2000, about 70% of the jobs and tasks performed by computer software developers (an occupational group employing 1 million people in 2000) were considered as new by the US Census (and did not appear in the 1990 Index of Occupations). In 1980, the same was true for jobs within the occupational group of management analysts, and in 1990 for workers involved with radiology technologies. Figure 2 focuses on 326 consistent occupational groups,[10] and shows that, in each decade between 1990 and 2008, employment growth has been faster in occupations with more novel jobs and tasks (1980-90 is shown in dark blue, 1990-2000 in blue, and 2000-2008 in light blue). The regression line shows the empirical relationship, which implies that occupational groups with 1 percentage point more novel jobs at the beginning of each decade grow .44% faster (standard error= .18%). To gauge the importance of new tasks in generating employment in the US labor market, we compute the counterfactual employment growth that would have resulted if occupational groups with more novel jobs did not grow any faster than a benchmark category with no novel jobs. This exercise implies that the contribution of the *additional* jobs created in occupational groups with more novel jobs is 29% of total employment growth between 1980 and 1990, 77% between 1990 and 2000, and 38% between 2000 and 2008.

As a second, complementary piece of evidence, we developed a measure of "complexity of occupations," which is particularly relevant for motivating the claim that labor has a comparative advantage in complex tasks. We first run a regression, separately by decade, of individual hourly log wages on various skills involved in occupations,[11] and then use the predicted wages as a complexity

[9]Lin uses new occupational titles added to new waves of the Dictionary of Occupational Titles to create measures of new jobs in each census occupational group for 1980 and 1990. He also compares the 1990 Census Index of Occupations with its 2000 counterpart, as well as technical documentation provided by the Census to determine the share of new job classes in each occupational category of the 2000 census. The data are available from his website https://sites.google.com/site/jeffrlin/newwork.

[10]We use David Dorn's occupational coding, made available at his website http://www.ddorn.net/data.htm. This provides consistent occupational groups throughout time and a balanced panel of 330 occupations from 1980 onward. The 326 occupations in our data are those appearing in the U.S. census. Finally, we use decennial census extracts for 1960, 1970, 1980, 1990, and 2000. For 2008 we use the American Community Survey.

[11]In particular, we use standardized measures from $O*NET$ for the use of: i. non-routine, cognitive and analytical

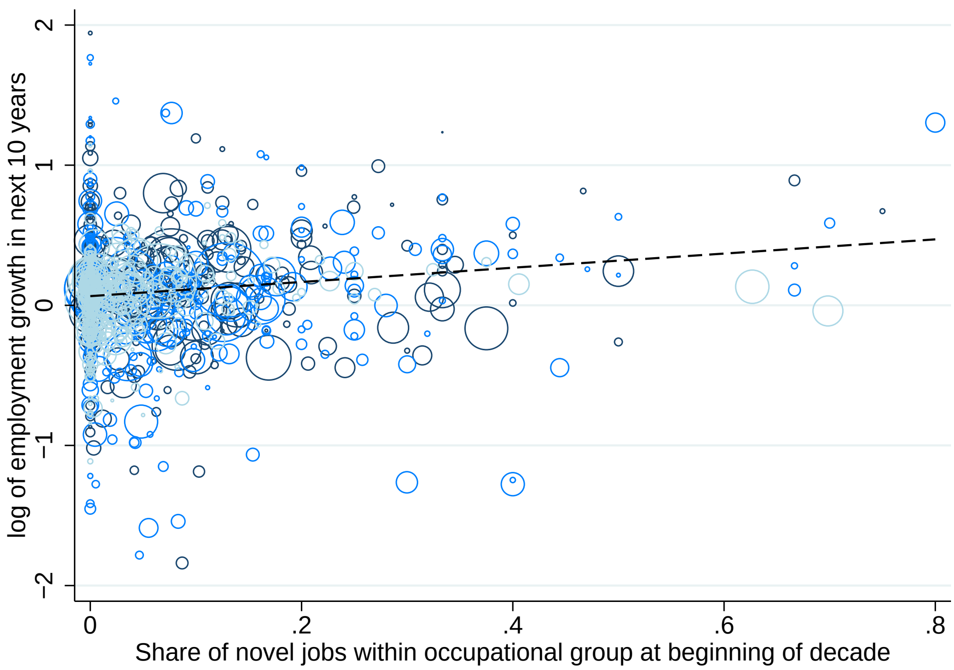

Figure 2: Scatter plot of employment growth within each occupational group and the share of novel jobs and tasks introduced in each, at the beginning of the decade. Data from 1980 to 1990 (in dark blue), 1990 to 2000 (in blue) and 2000 to 2008 (in light blue). See the text for details on sources and construction of variables.

index for our 326 occupational groups by decade. Figure 3 is similar to Figure 2, except for replacing the measure of novel jobs and tasks by this complexity index. The relationship is again similar, and with an analogous counterfactual exercise, we compute that the *additional* employment growth due to faster growth in complex occupations accounts for 12 percent of total employment growth between 1980 and 1990, 44 percent between 1990 and 2000, and 30 percent between 2000 and 2008.

To further understand the characteristics of novel, complex and fast growing occupations, Figure 4 presents the mean difference in several job and employee characteristics between these occupations and the rest. We present the differences separately for 2000 (light blue) 1990 (blue) and 1980 (navy blue), and standardize task contents so that the differences can be interpreted in terms of standard deviations. The figure shows a clear pattern: new jobs have been introduced in occupational groups intensive on interpersonal or non-routine analytical tasks. Such occupational groups also employ less high-school workers and more college-graduate and post-graduate workers, highlighting the skill requirements of these newer, more complex jobs.

Though Figure 4 suggests that the occupations favor high-skill workers, consistent with our

skills; ii. non-routine, cognitive and interpersonaL skills; iii. routine and cognitive skills; iv. routine and manual skills; v. non-routine, manual and interpersonal skills; vi. non-routine, manual and physical skills. The construction of these data is detailed in Acemoglu and Autor (2011).

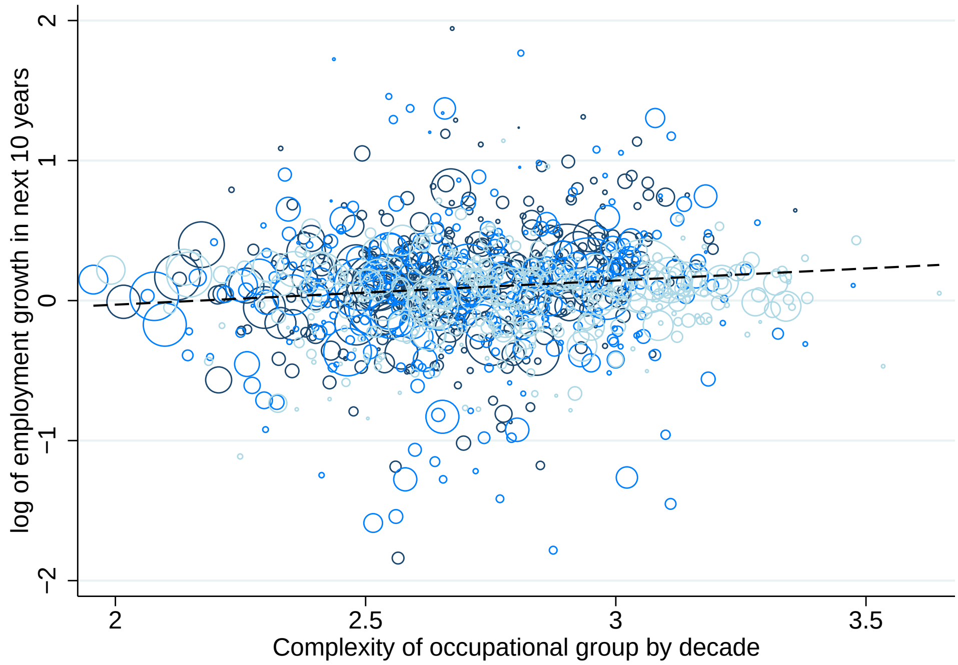

Figure 3: Scatter plot of employment growth within each occupational group and its complexity— determined by the tasks involved in such jobs— at the beginning of the decade. Data from 1980 to 1990 (in dark blue), 1990 to 2000 (in blue) and 2000 to 2008 (in light blue). See the text for details on sources and construction of variables.

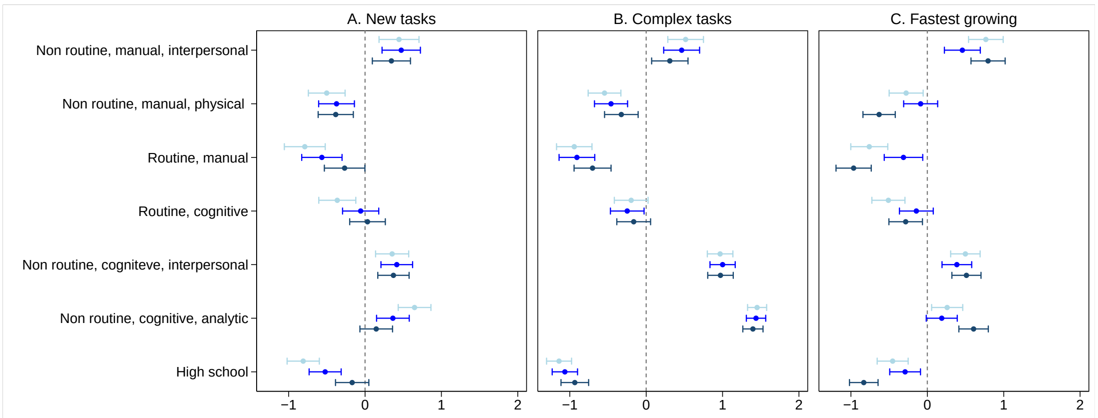

Figure 4: Comparison of job characteristics between new, complex and fast growing occupations and others. Panel A compares the occupational groups with more novel jobs than the mean in each decade to the remaining ones. Panel B compares the occupational groups with more complex jobs than the mean in each decade to the remaining ones. Panel C compares the occupational groups with faster employment growth than the mean in each decade to the remaining ones. We present the differences separately for 2000 (light blue) 1990 (blue) and 1980 (navy blue). See the text for details.

extension in Section 7.1, there is a pattern of "mean reversion" in skill requirements, enabling lower-skill workers to find employment in occupations they were previously underrepresented in. In particular, Figure 5 shows that occupational groups with lower fraction of high-school graduates in each decade tend to increase their share of high-school graduate workers over subsequent decades more than others.

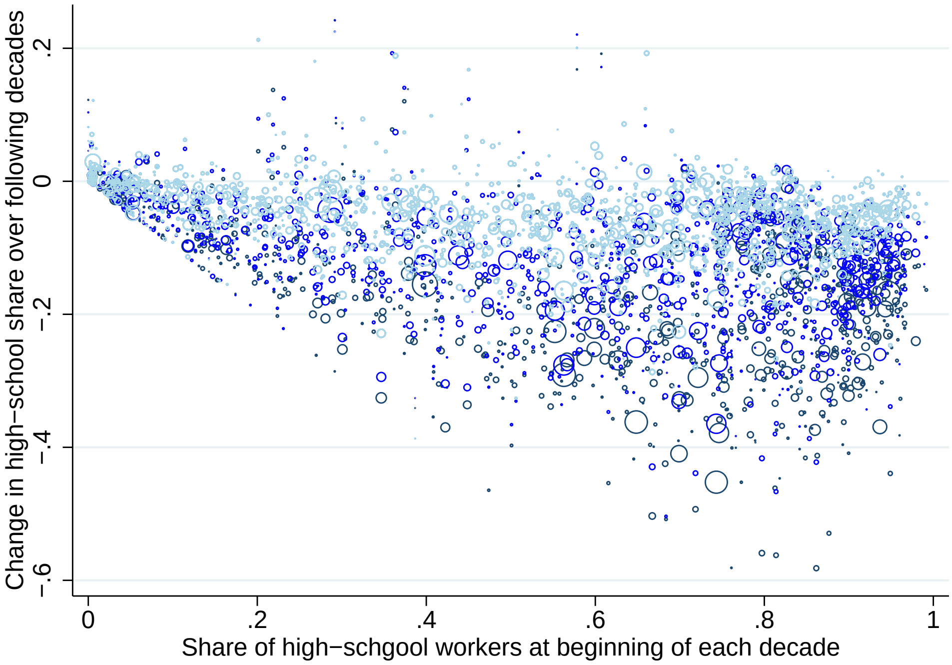

Figure 5: Scatter plot of the change in the share of high-school workers within each occupational group in subsequent decades (darker shades of blue indicate data for additional decades into the future) and its base level. Data from 1960 to 2008. See the text for details on sources and construction of variables.

3 Static Model

We start with a static environment with exogenous technology, which will enable us to introduce our main setup in the simplest fashion and characterize the impact of different types of technological change.

3.1 Environment

The economy contains a unique final good Y, produced by combining a continuum of tasks $y(i)$ with an elasticity of substitution $\sigma \in (0, \infty)$. Namely,

$$Y = \left(\int_{N-1}^{N} y(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

The final good and each task is produced competitively.

The new feature in the aggregate production function (1) is that the index of tasks runs from $N-1$ to N, guaranteeing that the total measure of tasks performed at any point in time is 1. As described in the Introduction, the economy will feature creation of new more complex tasks, represented here by an increase in N. By assuming that the range of tasks is between $N-1$ and N we are thus imposing that the creation of new tasks always corresponds to the destruction of the lowest-index task, capturing the replacement or upgrading of an existing task — a feature we model explicitly below.[12]

Each task is produced combining labor or capital with a task-specific intermediate $q(i)$, which embeds the technology used both for production and for the possible automation of tasks. In preparation for our full model in Section 5, we assume that property rights to each intermediate is held by a technology monopolist which can produce it at the marginal cost $\mu\psi$ in terms of the final good, where $\mu \in (0,1)$ and $\psi > 0$. The technology for each intermediate can be copied by a fringe of competitive firms, which can produce each at a higher marginal cost of ψ. We assume that μ is such that the unconstrained monopoly price of an intermediate would be greater than ψ, ensuring that the unique equilibrium price in the presence of the competitive fringe will be a limit price at ψ for all types of intermediates.

All tasks can be produced by labor. We model the technological constraints on automation by assuming that there exists $I \in [N-1, N]$ such that tasks $i \leq I$ are *technologically automated* in the sense that it is technologically feasible for them to be produced by capital as well. Conversely, tasks $i > I$ are not technologically automated, so cannot be produced by capital. Though tasks $i < I$ are technologically automated, the equilibrium may not involve all of those being produced by capital depending on factor prices as we will next describe.

Let us next describe the production function of tasks in greater detail. For tasks $i > I$, which are not technologically automated, the production function takes the form

$$y(i) = B\left[\eta q(i)^{\frac{\zeta-1}{\zeta}} + (1-\eta)\left(\gamma(i)l(i)\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}, \tag{2}$$

where $\gamma(i)$ denotes the productivity of labor in task i, $\zeta \in (0,\infty)$ is the elasticity of substitution between intermediates and labor, $\eta \in (0,1)$ is the distribution parameter of this constant elasticity of substitution production function, and finally, B is a normalizing constant, set equal to $B \equiv (1-\eta)^{\zeta/(1-\zeta)}$ to simplify the algebra.

In contrast, tasks $i \leq I$ can be produced using labor or capital, and their production function

[12] This formulation imposes that once a new task is created at N, it will automatically be utilized and as a consequence also replace the lowest available task, at $N-1$. In Section 4, we provide conditions under which firms will indeed prefer to do so.

takes the form

$$y(i) = B\left[\eta q(i)^{\frac{\zeta-1}{\zeta}} + (1-\eta)\left(k(i)+\gamma(i)l(i)\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}. \qquad (3)$$

All of the parameters are thus common between the production function of tasks above and below the threshold I, with the only difference that those below I can be produced by capital as well as labor. This feature is embedded in (3) via the assumption that capital and labor are perfect substitutes — so that capital can fully replace labor at the task level.[13] One simplifying feature of (3) is that capital has the same productivity in all tasks — while labor has different productivity. This is a very convenient simplifying assumption, and could be relaxed, though at the cost of additional complexity.

Though all of our main results apply with the task production functions (2) and (3), we sometimes illustrate our results with one of two special cases, which lead to easier-to-interpret and particularly insightful expressions (without sacrificing any of the qualitative effects in the model): either $\eta \to 0$ (so that the share of revenues going to intermediates is very low) or $\zeta \to 1$ (so that the production functions for tasks become Cobb-Douglas between factors and intermediates).

The key assumption we make throughout this paper, partly motivated by the facts presented in the previous subsection, is that $\gamma(i)$ is strictly increasing, so that labor has a *comparative advantage* in higher-indexed tasks. In the next section, we will strengthen this assumption by imposing a parametric form for $\gamma(i)$, which will ensure that productivity gains from the creation of new tasks is consistent with balanced growth (see in particular, equation (12)), but this functional form assumption plays no role in the analysis in this section. The important implication of strict comparative advantage is that, in equilibrium, there will exist some threshold task $I^* \leq I$ such that all tasks $i \leq I^*$ are produced using capital, while all tasks $i > I^*$ use labor (see Acemoglu and Zilibotti, 2001, and Acemoglu and Autor, 2011).[14] The argument for the existence of such a threshold in our model is provided in the next subsection.

Figure 1 diagrammatically represents the allocation of tasks to factors and also how the creation of new tasks replaces existing tasks from the bottom of the distribution, which was described above.

In the static model, we take the capital stock to be fixed at K (which will be endogenized via household decisions in Section 4). In addition, since we wish to study the impact of new technologies not just on factor prices but also on employment, we assume that the employment level is given by a quasi-labor supply taken to be an increasing function of the wage rate W relative to capital payments rK, i.e., $L^s\left(\frac{W}{rK}\right)$. This quasi-labor supply curve thus implies that as the wage rate increases relative

[13]The assumption implicit in writing this expression, that the same intermediate can be used regardless of whether this task is being produced by capital or labor, is for simplicity, and our results remain entirely unchanged if we have separate labor- and capital-specific intermediates.

[14]We impose without loss of any generality that when indifferent, firms use capital. This explains our convention of writing that all tasks $i \leq \widetilde{I}$ (rather than $i < \widetilde{I}$) are produced using capital.

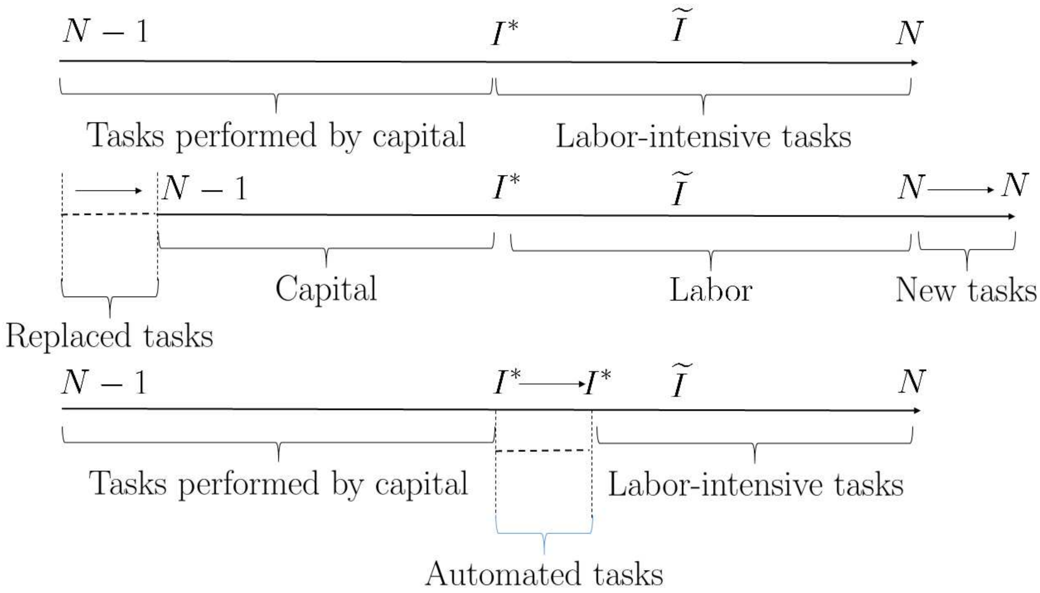

Figure 6: Task space, automation of existing tasks and introduction of new-complex tasks in which labor holds comparative advantage.

to payments to capital, the employment level increases as well. Though we impose this as a reduced-form in the text, it is straightforward to derive it from various microfoundations. In the Appendix, we show how an efficiency wage model generates this relationship, while in our companion paper, Acemoglu and Restrepo (2015), we derive this relationship from a search-matching model in a task-based framework. With this specification of the supply side, capital and labor market clearing can be written as

$$\begin{aligned} \int_{N-1}^{N} k(i)di &= K \\ \int_{N-1}^{N} l(i)di &= L^s\left(\frac{W}{rK}\right). \end{aligned}$$

We assume that $L^s(0) > 0$, so that labor never disappears from the economy entirely.

3.2 Equilibrium in the Static Model

We now characterize the equilibrium in this static economy. As noted above, all intermediates will be priced at ψ, and strict comparative advantage ensures that there will exist some threshold task I^* below which all tasks will be produced using capital. Given these intermediate prices and the threshold structure, an equilibrium can be represented as a function of the wage rate, W, the rental rate, r, and the equilibrium threshold I^*.

It is most convenient to proceed by characterizing the unit cost of producing tasks as a function of factor prices and the automation technology represented by I. Since tasks are produced competitively, their prices will be equal to these units costs. Thus

$$p(i) = \begin{cases} c^u\left(\min\left\{r, \frac{W}{\gamma(i)}\right\}\right) \equiv \left[\left(\frac{\eta}{1-\eta}\right)^{\zeta} \psi^{1-\zeta} + \min\left\{r, \frac{W}{\gamma(i)}\right\}^{1-\zeta}\right]^{\frac{1}{1-\zeta}} & \text{if } i \leq I, \\ c^u\left(\psi, \frac{W}{\gamma(i)}\right) \equiv \left[\left(\frac{\eta}{1-\eta}\right)^{\zeta} \psi^{1-\zeta} + \left(\frac{W}{\gamma(i)}\right)^{1-\zeta}\right]^{\frac{1}{1-\zeta}} & \text{if } i > I, \end{cases} \tag{4}$$

Here c^u is the constant unit cost of production of task i derived from the task production functions, (2) and (3). This unit cost also depends on the price of intermediates, ψ, but we suppress this dependence to simplify notation. The reason why the unit cost for tasks $i \leq I$ is written as a function of $\min\left\{r, \frac{W}{\gamma(i)}\right\}$ is simply that, given perfect substitution between capital and labor, firms will choose whichever factor has a lower effective cost — where effective cost for labor is $W/\gamma(i)$ in view of the fact that the productivity of labor in task i is $\gamma(i)$. Notice also that this expression distinguishes between $i \leq I$ and $i > I$ (and not $i \leq I^*$ and $i > I^*$, since it refers to what is *technologically feasible*, not to the equilibrium allocation of tasks to capital and labor).

We choose the final good as the numeraire, which from (1) implies that the demand for task i is given by

$$y(i) = Yp(i)^{-\sigma}. \tag{5}$$

From equations (4) and (5), equilibrium levels of task production can be written as

$$y(i) = \begin{cases} Yc^u\left(\min\left\{r, \frac{W}{\gamma(i)}\right\}\right)^{-\sigma} & \text{if } i \leq I, \\ Yc^u\left(\frac{W}{\gamma(i)}\right)^{-\sigma} & \text{if } i > I. \end{cases}$$

The result that, because of the strict comparative advantage, there will exist a threshold $\widetilde{I}$ such that tasks below $I^* \equiv \min\{I, \widetilde{I}\}$ will be produced with capital and the remaining more complex tasks with labor, can now be derived as a consequence of this expression. In particular, whenever $\min\left\{r, \frac{W}{\gamma(i)}\right\}$ picks r, the relevant task is produced by capital, and whenever it picks $W/\gamma(i)$, it is produced by labor. Since $\gamma(i)$ is strictly increasing, this implies that there exists a threshold $\widetilde{I}$ at which, conditional on technological feasibility, firms are indifferent between using capital and labor. Namely, at task $\widetilde{I}$, we have that $r = W/\gamma(\widetilde{I})$, or that

$$\frac{W}{r} = \gamma(\widetilde{I}). \tag{6}$$

Put differently, this condition determines the cost-minimizing allocation of tasks between capital and labor. However, if $\widetilde{I} > I$, firms will not be able to use capital all the way up to task $\widetilde{I}$ and achieve this cost-minimizing allocation because of the constraints imposed by the available automation technology. For this reason, the equilibrium threshold below which tasks are produced using capital is given by

$$I^* = \min\{I, \widetilde{I}\},$$

meaning that $I^* = \widetilde{I}$ when this is technologically feasible, and $I^* = I$ otherwise.

To fully characterize the static equilibrium, we next need to derive the quantities of tasks produced, given that equilibrium threshold I^*. Factor demands from each intermediate task can be derived from (2) and (3) as

$$k(i) = \begin{cases} Yc^u(r)^{\zeta-\sigma} r^{-\zeta} & \text{if } i \le I^*, \\ 0 & \text{if } i > I^*. \end{cases}$$

and

$$l(i) = \begin{cases} 0 & \text{if } i \le I^*, \\ \gamma(i)^{\zeta-1} Y c^u \left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma} W^{-\zeta} & \text{if } i > I^*. \end{cases}$$

Capital and labor market clearing conditions then yield the following equilibrium conditions,

$$Y(\min\{I, \widetilde{I}\} - N + 1)c^u(r)^{\zeta-\sigma} r^{-\zeta} = K, \tag{7}$$

and

$$Y \int_{\min\{I,\widetilde{I}\}}^{N} \gamma(i)^{\zeta-1} c^u \left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma} W^{-\zeta} di = L^s\left(\frac{W}{rK}\right). \tag{8}$$

The following proposition summarizes our characterization of the equilibrium.

Proposition 1 (Equilibrium in the static model) *For any range of tasks $[N-1, N]$, automation technology $I \in (N-1, N]$, and capital stock K, there exists a unique equilibrium characterized by factor prices, W and r, and threshold tasks, $\widetilde{I}$ and I^*, such that: (i) $\widetilde{I}$ is determined by equation (6) and $I^* = \min\{I, \widetilde{I}\}$; (ii) all tasks $i \le I^*$ are produced using capital and all tasks $i > I^*$ are produced using labor; (iii) capital and labor market clearing conditions, equations (7) and (8), are satisfied; and (iii) factor prices satisfy:*

$$(I^* - N + 1)c^u(r)^{1-\sigma} + \int_{I^*}^{N} c^u \left(\frac{W}{\gamma(i)}\right)^{1-\sigma} di = 1. \tag{9}$$

Proof. All of the expressions in this proposition follow from the preceding derivations, and the full uniqueness proof is provided in the Appendix. ■

The equilibrium characterized in Proposition 1 is illustrated in Figure 2. The equilibrium is represented by the intersection of an upward and downward-sloping curve determining $\omega \equiv \frac{W}{rK}$. The downward-sloping curve, $\omega(I^*, N, K)$, corresponds to the relative demand for labor, which is obtained by combining the market clearing conditions for capital and labor, (7) and (8) together with the expression for the levels of factor prices, which is derived from the ideal price index, equation (9). The upward-sloping curve represents the cost-minimizing allocation of tasks to capital and labor, as represented by equation (6), with the constraint that the equilibrium level of automation can never exceed I (explaining the vertical portion).

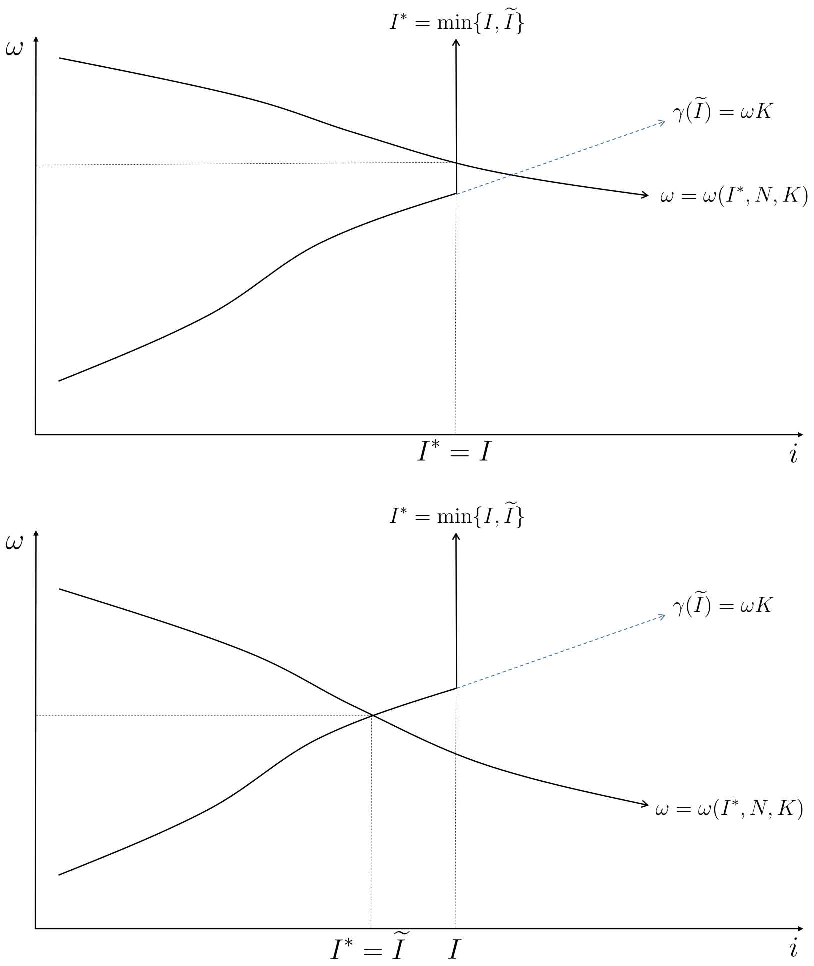

Figure 7: Static equilibrium of our model in the case in which $I^* = I$ and the allocation of factors is constrained by technology (top panel) and for the case in which $I^* = \widetilde{I}$ (bottom panel).

The figure distinguishes between the two cases already highlighted above. In the top panel, we have the case where $I^* = I < \widetilde{I}$ and the allocation of factors is constrained by technology, while the bottom panel plots the case where $I^* = \widetilde{I} < I$ and where the cost-minimizing allocation can be achieved. An immediate implication of our characterization and of Figure 7 is that an increase in N (the creation of new, more complex tasks) always expands the set of tasks performed by labor and contracts those performed by capital, and an increase in I (greater technological automation) expands the set of tasks performed by capital and contracts those performed by labor provided that $I < \widetilde{I}$. We will see the implications of these results in the comparative statics we present next.

The following proposition gives a complete characterization of comparative statics.

Proposition 2 (Comparative statics) *Let* $\omega \equiv \frac{W}{rK}$ *be the ratio of wages to capital payments, and* $\varepsilon \equiv \frac{d \ln \gamma(I)}{dI} > 0$ *be the semi-elasticity of the comparative advantage schedule. Then:*

- *If* $I^* = \widetilde{I} > I$ *— so that the allocation of tasks to factors is constrained by technology, we* *have:*

$$\frac{d\ln\omega}{dI}=\frac{d\ln(W/r)}{dI}=\frac{\partial\ln(W/r)}{\partial I^*}<0, \frac{d\ln\omega}{dN}=\frac{d\ln(W/r)}{dN}=\frac{\partial\ln(W/r)}{\partial N}>0$$

and

$$\frac{d\ln\omega}{d\ln K}+1=\frac{d\ln(W/r)}{d\ln K}=\frac{1}{\sigma_{SR}}>0.$$

Here, $\sigma_{SR}\in[0,\infty)$ *is the short-run elasticity of substitution between labor and capital holding the allocation of factors to tasks fixed, which in this model is a weighted average of* σ *and* ζ*. Moreover, if* σ_{SR} *is sufficiently large,* $\frac{d\ln W}{dI}<0$*, and* $\frac{d\ln W}{dI}>0$ *otherwise.*

- *If* $\widetilde{I}<I^*=I$ *— so that tasks are allocated to factors in the unconstrained cost minimizing fashion, we have*

$$dI^*=\frac{1}{\varepsilon}d\ln(W/r).$$

The resulting impact on factor prices and shares is given by

$$\frac{d\ln\omega}{dI}=\frac{d\ln(W/r)}{dI}=0, \frac{d\ln\omega}{dN}=\frac{d\ln(W/r)}{dN}=\frac{\frac{\partial\ln(W/r)}{\partial N}}{1-\frac{1}{\varepsilon}\frac{\partial\ln(W/r)}{\partial I^*}}>0 \text{ and}$$

$$\frac{d\ln\omega}{d\ln K}+1=\frac{d\ln(W/r)}{d\ln K}=\frac{\frac{1}{\sigma_{SR}}}{1-\frac{1}{\varepsilon}\frac{\partial\ln(W/r)}{\partial I^*}}>0.$$

Thus, when the allocation of tasks to factors is unconstrained, the aggregate elasticity of substitution between capital and labor becomes

$$\sigma_{MR}=\sigma_{SR}\left(1-\frac{1}{\varepsilon}\frac{\partial\ln(W/r)}{\partial I^*}\right)>\sigma_{SR}.$$

Moreover, if the medium-run elasticity of substitution between labor and capital, σ_{MR}*, is sufficiently large,* $\frac{d\ln r}{dN}<0$*, and* $\frac{d\ln W}{dN}>0$ *otherwise.*

- *Finally, in both parts of the proposition, the labor share and employment move in the same direction as* ω*.*

Proof. These results follow directly from differentiating the equilibrium conditions, and the details are given in the Appendix. ■

The most important implication of Proposition 2 is that the two types of technological changes — automation and creation of new, more complex tasks — have polar implications. Automation, corresponding to an increase in I, tends to reduce W/r, the labor share and employment (unless firms were deciding not to use capital in all of the tasks that were technologically automated), while the creation of new tasks, corresponding to an increase in N, increase W/r, labor share and employment.

It is also useful to note that these comparative static results can be derived using Figure 2: automation moves us along the relative labor demand curve in the technology-constrained case shown in the top panel (and has no impact in the bottom panel), while the creation of new tasks, shifts out the relative labor demand curve.

Another important implication of Proposition 2 is that, when $I^* = I$, automation — an increase in I — can reduce wages. For example, automation expands the range of tasks performed by capital and pushes labor into a fewer set of tasks, where the diminishing returns to the quantity of a task puts downward pressure on the wage, counteracted by a positive effect coming from the fact that tasks are (q-)complements in the aggregate production function (1). This positive effect is weaker when σ is greater, explaining why the overall impact of automation on the wage rate is negative when σ is large.[15] Similarly, again when σ is large, the creation of new tasks — that is, an increase in N — can reduce the rental rate on capital. Even more important is that automation is *always* capital-biased (that is, it reduces W/r), while the creation of new tasks is *always* labor-biased (that is, it increases W/r). Both of these are major consequences of the task-based framework developed here. With factor-augmenting technologies, technological improvements always increase the price of both factors, but this is no longer the case when technological change alters the range of tasks performed by the two factors (see Acemoglu and Autor, 2011).[16] Also, as is well known, with factor-augmenting technologies, whether different types of technological changes are biased towards one factor or the other depends on the elasticity of substitution, but this too is different in our task-based framework — again because different types of technological changes directly alter the range of tasks performed by the two factors. This last feature will play a critical role in our full model in Section 5.

A final implication of Proposition 2 is the difference between the short-run and the "medium-run" elasticities of substitution between capital and labor. The short-run elasticity, σ_{SR} is obtained when the range of tasks allocated to capital and labor is fixed (as in the case where $I^* = I$), and the medium-run elasticity, σ_{MR}, applies when the range of tasks responds to changes in factor prices (as in the case where $I^* = \widetilde{I}$).[17]

Though Proposition 2 provides a complete characterization of the responses of relative factor prices, factor shares and employment to automation and creation of new tasks, the results are

[15]This negative impact does not require σ to be unrealistically large. For example, if $\sigma = 1$, automation reduces the marginal product of labor if $K/Y < 2.7182$.

[16]For instance, an increase in capital-augmenting technology, from the viewpoint of other factors, is equivalent to an increase in the effective amount of capital and it increases the marginal product of labor because factors are q-complements in any production function with constant returns to scale and two factors. To see this, let $F(A_K K, A_L L)$ be such a production function. Then $W = F_L$, and $\frac{dW}{dA_K} = KF_{LK} = -LF_{LL} > 0$ because of constant returns to scale, establishing the claim.

[17]Another observation about the elasticity of substitution following from this proposition is that a long-run negative association between capital accumulation and the labor share is not sufficient to conclude that σ — the elasticity of substitution between labor and capital — is above 1 (as argued by Karabarbounis and Neiman, 2014). This reasoning would be valid in the special case when technology only takes a factor-augmenting form, but not in our task framework. For a stark counterexample, take $\sigma = 1$ in our model with $\eta \to 0$. Then, factor shares depend only on technology and are not informative about σ.

qualitative and the explicit expressions are complicated; this is because imperfect substitution between factors and intermediates (the $q(i)$'s) implies that as technology changes, the profits of intermediate producers change. As noted above, two special cases simplify this impact on profits and illustrate the workings of our model and the comparative statics more transparently. The first is when $\eta \to 0$, where these profits go to zero, and the second is when $\zeta \to 1$, where they become a constant fraction of revenue. We next provide the explicit expressions in these two special cases. We also simplify this illustration by taking $L(\omega) = L$, so that the quasi-labor supply coincides with the inelastic labor supply in the economy.

First, when $\widetilde{I} > I$, or equivalently when $I^* = \min\{I, \widetilde{I}\}$, in both cases we obtain a particularly revealing expression for aggregate output (or a "derived aggregate production function"):

$$Y = \left[(I^* - N + 1)^{\frac{1}{\hat{\sigma}}} K^{\frac{\hat{\sigma}-1}{\hat{\sigma}}} + \left(\int_{I^*}^{N} \gamma(i)^{\hat{\sigma}-1} di \right)^{\frac{1}{\hat{\sigma}}} L^{\frac{\hat{\sigma}-1}{\hat{\sigma}}} \right]^{\frac{\hat{\sigma}}{\hat{\sigma}-1}}, \tag{10}$$

where $\hat{\sigma} \equiv \eta + (1-\eta)\sigma$ (which also implies that when $\eta \to 0$, we have the particularly simple case with $\hat{\sigma} = \sigma$).

This expression emphasizes that aggregate output is a constant elasticity of substitution aggregate of capital and labor (with the short-run elasticity of substitution between capital and labor, σ_{SR}, simply being $\hat{\sigma}$), but the distribution parameters are endogenous and depend on the state of the two types of technologies in the economy. In particular, automation increases the importance of capital and reduces the importance of labor in the (derived) aggregate production function, while the creation of new, more complex tasks does the opposite.

Relative factor demands are also straightforward to derive since, simple differentiation of (10), implies

$$\ln \omega = \left(\frac{1}{\hat{\sigma}} - 1 \right) \ln K + \frac{1}{\hat{\sigma}} \ln \left(\frac{\int_{I^*}^{N} \gamma(i)^{\hat{\sigma}-1} di}{I^* - N + 1} \right). \tag{11}$$

In fact, equation (11) gives us an explicit expression for the relative labor demand plotted in Figure 2.

The next corollary provides a more explicit characterization of the comparative statics derived in Proposition 2 in the special cases.

Corollary 1 *Suppose $\eta \to 0$ or $\zeta \to 1$. Then:*

- *If $I < \widetilde{I}$:*

$$\begin{aligned} \hat{\sigma} d \ln \omega &= (1-\hat{\sigma}) d \ln K - \left[\frac{\gamma(I)^{\hat{\sigma}-1}}{\int_I^N \gamma(i)^{\hat{\sigma}-1} di} + \frac{1}{I - N + 1} \right] dI \\ &\quad + \left[\frac{\gamma(N)^{\hat{\sigma}-1}}{\int_I^N \gamma(i)^{\hat{\sigma}-1} di} + \frac{1}{I - N + 1)} \right] dN. \end{aligned}$$

- *If* $\widetilde{I} < I$*:*

$$(\hat{\sigma} + \Lambda/\varepsilon)\, d\ln\omega \;=\; (1 - \hat{\sigma} - \Lambda/\varepsilon) d\ln K + \left[\frac{\gamma(N)^{\hat{\sigma}-1}}{\int_I^N \gamma(i)^{\hat{\sigma}-1} di} + \frac{1}{I - N + 1}\right] d\ln N,$$

where

$$\Lambda \equiv \frac{\gamma(\widetilde{I})^{\hat{\sigma}-1}}{\int_{\widetilde{I}}^N \gamma(i)^{\hat{\sigma}-1} di} + \frac{1}{\widetilde{I} - N + 1} > 0,$$

and $\hat{\sigma} \equiv \eta + (1-\eta)\sigma$*.*

The labor share and employment move in the same direction as ω*.*

In this corollary, the difference between the short-run and the medium-run elasticity of substitution can be seen quite clearly: $\sigma_{SR} = \hat{\sigma}$, and $\sigma_{MR} = \hat{\sigma} + \Lambda/\varepsilon$.

4 Dynamic Economy, Balanced Growth and the Productivity Effect

In this section, we extend our model to a dynamic economy in which the evolution of the capital stock is determined by households' saving decisions. We then investigate the conditions under which the economy admits a balanced growth path, where output, the capital stock and wages grow at a constant rate. We conclude this section by discussing the effect of automation on wages in the long run (when the interest rate is constant as in the balanced growth path), which highlights an important "productivity effect," creating a force from automation towards higher wages.

4.1 Balanced Growth

The most important assumption in this section will be to parametrize the comparative advantage schedule to ensure balanced growth. In particular since, as usual, balanced growth will be driven by technology, and in this model technological change comes in part from the creation of new tasks, exponential growth will require productivity improvements from new tasks to be exponential. In other words, we require

$$\gamma(i) = e^{Ai} \text{ with } A > 0, \tag{12}$$

which we impose in the remainder of the paper.[18]

Let also $\{K(t), N(t), I(t)\}_{t=0}^{\infty}$ denote the path of technology and capital. These are the state variables of our model. Also, let $\{r(t), W(t), Y(t)\}_{t=0}^{\infty}$ denote the path of factor prices and equilibrium output at each period. We start by assuming exogenous technological change, and define

$$n(t) \equiv N(t) - I(t)$$

[18] As usual we could impose this functional form only asymptotically, but simplify the analysis and exposition by imposing it throughout its range.

as a summary measure of the state of technology. A higher n corresponds to the state of technology favoring new tasks more than automation. Clearly, as automation increases, n declines, and conversely, as there are new tasks being created, n increases. We further simplify the discussion and notation by assuming that $I^*(t) = I(t)$. As noted in the next section (in particular footnote 22), with endogenous technology, this is the relevant region, since $I^*(t) < I(t)$ would imply that there are resources spent on automating tasks that will not be immediately produced with capital. We discuss conditions that ensure $I^*(t) = I(t)$ below.

The economy is assumed to admit a representative household. This representative household's preferences over consumption paths, $\{C(t)\}_{t=0}^{\infty}$, are given by

$$\int_0^{\infty} e^{-\rho t} \frac{C(t)^{1-\theta} - 1}{1-\theta} dt.$$

and the resource constraint faced by the household takes the form

$$\dot{K}(t) = Y(t) - C(t) - \delta K(t) - \psi\mu \int_{N-1}^{N} q(i,t) di,$$

where $Y(t)$ continues to be given by (1), and δ is the depreciation rate of capital. In addition, $\psi\mu$, with $\mu \in [0,1]$, parametrizes the marginal cost of producing intermediates. Thus, we allow for intermediaries to sell their products at a markup $1 - \mu \geq 0$. This markup does not play any role in this section, and these profits will only be important when we turn to the case with endogenous technology.

We characterize the equilibrium by defining the normalized variables $y(t) \equiv Y(t)/\gamma(I(t))$, $k(t) \equiv K(t)/\gamma(I(t))$, $c(t) \equiv C(t)/\gamma(I(t))$, and $w(t) \equiv W(t)/\gamma(I(t))$.

At each point in time, technology and capital, $n(t)$ and $k(t)$, fully determine output, $y(t)$, and factor prices $w(t)$ and $r(t)$ as in the static equilibrium (where, for consistency with our static analysis, $r(t)$, is taken to be the rental rate of capital, so that the interest rate is $r(t) - \delta$). Specifically, the market clearing conditions for capital and labor, (7) and (8), and the ideal price index condition, (9), give the following equilibrium conditions in this case:

$$\begin{aligned} k(t) &= y(t)(1 - n(t))c^u(r(t))^{\zeta-\sigma} r(t)^{-\zeta}, \\ L^s\left(\frac{w(t)}{r(t)k(t)}\right) &= y(t) \int_0^{n(t)} \gamma(i)^{\zeta-1} c^u \left(\frac{w(t)}{\gamma(i)}\right)^{\zeta-\sigma} w^{-\zeta} di, \\ 1 &= (1-n(t))c^u(r(t))^{1-\sigma} + \int_0^{n(t)} c^u \left(\frac{w(t)}{\gamma(i)}\right)^{1-\sigma} di \end{aligned}$$

The implied values for normalized output and factor prices can be written as $y(t) = y^E(n(t), k(t))$, $w(t) = w^E(n(t), k(t))$ and $r^E(t)(n(t), k(t))$, which are uniquely defined from Proposition 1. Importantly, we also have that $w^E(n, k) \geq r$, because the endogenous allocation of tasks to factors implies $W/\gamma(I^*) \geq r$ (or $\widetilde{I} \geq I^*$). We also denote by $f^E(n(t), k(t))$ the output net of intermediate costs.

Using this notation, we can describe the dynamic equilibrium of our model as a path for $c(t)$ and $k(t)$ satisfying the Euler equation

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}(r^E(n(t), k(t)) - \delta - \rho) - g \tag{13}$$

coupled with the household's transversality condition

$$\lim_{t\to\infty} k(t)e^{-\int_0^t (\rho-(1-\theta)g)ds} = 0, \tag{14}$$

and the resource constraint

$$\dot{k}(t) = f^E(n(t), k(t)) - c(t) - (\delta + g)k(t). \tag{15}$$

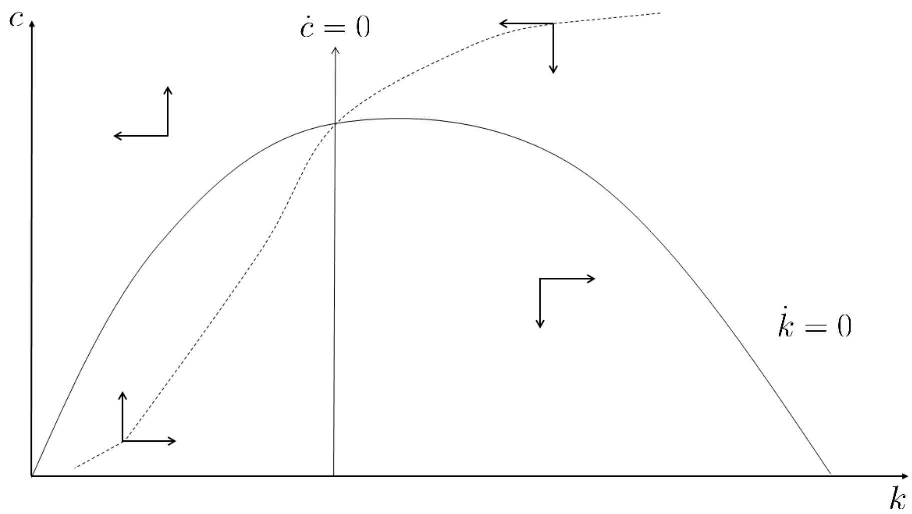

Figure 8: Steady state and dynamics for our model with exogenous technological change and $n(t) \to n$.

Figure 8 presents the phase diagram for this system for $n(t) \to n$. The structure of the above system is similar to the standard neoclassical growth model, with the slight exception that technology monopolists' markups create a wedge between r^E and f_k^E.

We define a balanced growth path as an allocation in which Y, C, K and w grow at a constant rate and r is constant. The next proposition characterizes the conditions under which the asymptotic behavior of this economy will converge to a balanced growth path and also establishes that this involves both types of technological change.

Proposition 3 (Dynamic equilibrium with exogenous technological change) *Suppose that technology evolves exogenously and that $I^*(t) = I(t)$.*

1. *Then a balanced growth path exists if and only if asymptotically* $\dot{N} = \dot{I} = \Delta$, $\lim_{t\to\infty} n(t) = n \in (0,1)$ *and* $A(1-\theta)\Delta < \rho$ *(so that net present discounted value of household income is finite). In this balanced growth path* Y, C, K *and* w *grow at a constant rate* $A\Delta$ *and* r *is constant.*

2. *Moreover, given such a path of technological change, the dynamic equilibrium is unique starting from any initial level of capital and converges to the balanced growth path.*

Proof. First suppose that $n(t) \to n \in (0,1)$. Then for any initial value of $k(0)$, the economy converges to its unique steady state, which depends only on n. This result can be proved straightforwardly by noting that equations (13), (14) and (15) are essentially identical to the two equations characterizing dynamics in the canonical neoclassical growth model. The condition $A(1-\theta)\Delta < \rho$, the transversality condition is satisfied when k capital converges to a constant (see, for example, Proposition 8.5 and 8.6 in Acemoglu (2009)). This proves part 2 of the proposition.

Since the normalized variables converge, the aggregate variables grow at the same rate as $\gamma(I)$, establishing the "if" direction of part 1.

We prove the "only if" part in the Appendix. ■

The most important implication of Proposition 3 is that balanced growth can emerge from the simultaneous process of automation and development of new tasks. But it also highlights that this process needs to be balanced itself: the race between machine and man cannot be dominated by either.

Note also that this proposition is stated under the assumption that $I^*(t) = I(t)$ (and also the implicit assumption we have maintained throughout as noted in footnote 12 that all new labor-intensive tasks are utilized immediately). Though this is the interesting configuration from an economic point of view, it does require some minimal conditions to be imposed. Intuitively, the condition necessary for this configuration takes the form $n > \bar{n}$ for some $\bar{n} \in (0,1)$, i.e., the level of automation relative to the set of new tasks should not be too high. However, as we will see in the next section, for this threshold $\bar{n}$ to be well-defined, we also need that the interest rate is not too low and also the growth rate is not too high. If the interest rate is very low, then it may be unprofitable to use these new labor-intensive tasks, and such a threshold $\bar{n}$ may not exist. If the growth rate of the economy is very high, then this creates an additional motive for automation (to save on the growth of future labor costs), and $\bar{n}$ may again fail to be well-defined. We will provide the expression for this threshold $\bar{n}$ in equation (17) in the next subsection.

Combining this proposition together with Proposition 2, we also see that when automation runs ahead of the creation of new tasks, i.e., $\dot{I} > \dot{N}$, so that $n(t)$ decreases, we will not only move away from balanced growth (presuming that we started at or near balanced growth), but also that this will reduce the share of labor in national income and employment. In light of this result, the patterns shown in Figure 1 in the Introduction can be interpreted as a consequence of automation outpacing the creation of new labor-intensive tasks over the last two decades.

Proposition 3 can also be further illustrated and strengthened in the two special cases considered in the previous section, where $\eta \to 0$ or $\zeta \to 1$. Supposing also that $\dot{N} = \dot{I} = \Delta$, the aggregate production function can be simplified to $Y(t) = f(K(t), A(t)L)$ as given in equation (10). We also have that

$$A(t) = \left(\int_{I(t)}^{N} (t)\gamma(i)^{\hat{\sigma}-1} di \right)^{\frac{1}{\hat{\sigma}-1}} = e^{AI(t)} \left(\frac{e^{A(\hat{\sigma}-1)n(t)} - 1}{A\hat{\sigma} - 1} \right)^{\frac{1}{\hat{\sigma}-1}},$$

so that $A(t)$ grows at a rate $A\Delta$. In this case, technology is purely labor augmenting on net because labor and capital perform a fixed share of tasks; while labor is used on tasks in which it is more productive over time. This provides a direct connection between our model and Uzawa's Theorem, which implies that balanced growth requires purely labor-augmenting technological change (e.g., Acemoglu, 2009). The condition $\dot{N} = \dot{I}$ ensures this in our economy.

4.2 The Productivity Effect

As just noted, the analysis in this section enables us to study the dynamic implications of automation running ahead of the creation of new tasks. Though many of the insights from our static model apply in this case, the dynamic economy also highlights another economic force, which we will call the *productivity effect*: automation, by enabling the substitution of the cheaper capital for labor, increases productivity and thus the demand for labor.[19] The productivity effect is implicitly present in our analysis so far. But it becomes more powerful in the balanced growth path because the interest rate is constant (Proposition 3) as we show next.

Consider the balanced growth path characterized in Proposition 3. We have $N(t) - I(t) \to n \in (\bar{n}, 1)$, and the long-run growth rates given by $g = A\Delta$. Moreover, in this balanced growth path, we have $r = \rho + \delta + \theta g$, and the long-run normalized wage as a function of the state of technology is

$$w^{LR}(n) = w^{E}(n, k(n)).$$

Given our normalization (where the wage, W, is divided by $\gamma(I) = \gamma(I^*)$), this is the wage per effective unit of labor paid in the least complex tasks performed by labor. The wage per effective unit of labor in the most complex task can then be written as $w^{LR}(n)/\gamma(n)$.

Recall the assumption in Proposition 3 that $I^*(t) = I(t)$ and that all new labor-intensive (new complex) tasks are utilized immediately. In terms of the notation we have just introduced, this is

[19] This is similar to the productivity or efficiency effect in models of offshoring such as Grossman and Rossi-Hansbereg (2008), Rodriguez-Clare (2010) and Acemoglu, Gancia and Zilibotti (2015), which results from the substitution of cheaper foreign labor for domestic labor in certain tasks.

equivalent to:

$$w^{LR}(n)/\gamma(n) \leq r \leq w^{LR}(n). \tag{16}$$

The first inequality implies that the wage per effective unit of labor in the most complex tasks is lower than the long-run rental rate of capital, ensuring that it is profitable to utilize these new tasks (with labor). The second inequality implies that the long-run rental rate of capital is lower than the wage per effective unit of labor in the least complex task, ensuring that it is profitable to automate this task when it is technologically feasible. Note also that the threshold $\bar{n}$ introduced in the previous subsection is defined implicitly as

$$w^{LR}(\bar{n}) = r = \rho + \delta + \theta g \tag{17}$$

(and the conditions mentioned above are used to guarantee that such a solution exists). Suppose in the remainder of the discussion in this subsection that (16) is satisfied.

The long-run productivity effect can now be seen from the ideal price condition, (9), in the balanced growth allocation where the interest rate is constant:

$$(I - N - 1)c^u(\rho + \delta + \theta g)^{1-\sigma} + \int_I^N c^u \left(\frac{W}{\gamma(i)}\right)^{1-\sigma} = 1, \tag{18}$$

where we have also used the fact that $I^*(t) = I(t)$. Or using the notation we have just introduced, this ideal price condition can also be written as

$$(1 - n)c^u(\rho + \delta + \theta g)) + \int_0^n c^u \left(\frac{w^{LR}(n)}{\gamma(i)}\right)^{1-\sigma} = 1. \tag{19}$$

There are three important implications from these equations. First, automation cannot reduce wages in the long run. This simply follows from (18) when we use (16). In particular, straightforward differentiation gives

$$\frac{dW}{dI^*} \propto \frac{1}{\sigma - 1}[c^u\,(r)^{1-\sigma} - c^u\left(w^{LR}(n)\right)^{1-\sigma}] \geq 0.$$

Intuitively, because the interest rate is constant in the long run, automation also increases the amount of capital used in production, and this effect always ensures that the equilibrium wage increases in response to additional automation (but the share of labor in national income continues to be decreasing in automation). Second, this time from (19), we also have that

$$\frac{dw^{LR}(n)}{dn} \propto \frac{dW}{dN} \propto -\frac{1}{\sigma - 1}\left[c^u\,(r)^{1-\sigma} - c^u\left(\frac{w^{LR}(n)}{\gamma(n)}\right)^{1-\sigma}\right] \geq 0,$$

so that automation (corresponding to a decrease in n) still reduces the wage per effective unit of labor in the least complex tasks, and the creation of new tasks still increases the equilibrium wage. Finally, once again using (18), we also have

$$\frac{dw^{LR}(n)/\gamma(n)}{dn} \propto \frac{1}{\sigma-1}[c^u\left(w^{LR}(n)\right)^{1-\sigma} - c^u(r)^{1-\sigma}] \leq 0,$$

so that the creation of new tasks reduces the wage per effective unit of labor in the most complex tasks; while increasing it in the least complex ones.

These observations thus establish that, provided that we are in the region where $n > \bar{n}$, several of the intuitive results from the static model continue to apply, but because of the productivity effect, the potential negative impact of automation on the equilibrium wage level does not. These conclusions do not necessarily hold, however, when there does not exist a well-defined $\bar{n}$, underscoring the importance of ensuring that this is the case in our analysis of endogenous technology in the next section.[20]

5 Full Model: Tasks and Endogenous Technologies

The analysis in the previous section established the existence of a balanced growth path under the assumption that $\dot{N} = \dot{I}$. But why should these two types of technologies advance at the same rate? This is the question at the center of our paper, and we now develop our full model, which endogenizes the pace at which automation and creation of new tasks proceeds.

5.1 Endogenous and Directed Technological Change

We endogenize technological change by allowing new intermediates to be introduced by technology monopolists. New firms can introduce either technologies automating previously non-automated tasks or create new tasks. We assume that successful innovations always achieve automation or the creation of new tasks in the order of the intermediate indices, $i \in [0, \infty)$, so that lower-indexed tasks will always be automated before higher-index tasks, and a new labor-intensive task will always correspond to the lowest-indexed task that has not been created yet (and the lower index of integration at $N-1$ in the aggregate production function, (1), already imposes that new tasks replace the lowest-indexed task currently in use). As a consequence, the two types of endogenous technological changes will correspond to an increase in I and to an increase in N, respectively. We continue to assume that all intermediates, including those that have just been invented, can be produced at the fixed marginal cost of $\mu\psi$, and that the fringe of competitive firms is still present, forcing the technology monopolists to price at ψ, which of course implies a per-unit profit of $(1-\mu)\psi$.

[20] The conditions listed above is sufficient for the existence of the threshold $\overline{n}$ are not very restrictive. For example, focusing on a standard annual parametrization of our model with $\theta = 1$, $\rho = 0.05$, $\delta = 0.06$, $g = 0.016$, $\sigma = 0.5$, $\zeta = 0.2$ (so that the elasticity of substitution between capital and labor lies between 0.5 and 0.2), $A = 2$, $\eta = 0.5$ and $\psi = 0.9$ satisfies all of the conditions and yields $\overline{n} = 0.56$.

Though per-unit profits of technology monopolists are constant, their net present discounted value is a complex object for two reasons. First, the fact that I and N will grow at some fixed rate, for example in the balanced growth path as characterized in Proposition 3, implies that there will be a deterministic component to the length of time during which a monopolist will be able to enjoy profits from its technology. Despite this first complication, we will see that the dynamics of endogenous technology can be characterized, though this will involve somewhat different arguments than in the standard endogenous technological change models. Second, as in other models of quality improvements (e.g., Aghion and Howitt, 1992; Grossman and Helpman, 1991), new intermediates replace some existing ones. This generates the "creative destruction" of profits of existing producers by new firms, at least under the assumptions used in the literature, which is that new firms do not have to respect the intellectual property rights of the technology on which they are building. This assumption, however, creates more complex dynamics, especially coupled with the deterministic replacement of products in our model. For this reason, we adopt an alternative (and arguably equally plausible) structure of protection of intellectual property rights whereby building and replacing an existing technology is viewed as infringement of the patent of that technology. This implies that the inventor of a new technology will have to buy this existing patent (or license the technology). We assume that this takes place with the inventor making a take-it-or-leave-it offer to the holder of the patent on the technology on which it is building. Consequently, a firm automating a task previously performed by labor will have to license or buy the relevant patent from an existing firm supplying the intermediate to this task, and a firm creating a new task, which is effectively creating a more complex, labor-intensive version of an existing task, will have to obtain the patent from an existing firm for the intermediate used in this task (which, in any equilibrium with automation, will be an automated task, since it is the lowest-indexed task currently in use. This game form ensures that each technology monopolist will receive the same flow of revenues regardless of whether its own product is replaced or not (either as profits when he is continuing to operate or as payments for its patent when it is replaced). We return to the analysis of how the results change when we allow for the creative destruction of profits in Section 7.

We are now in a position to describe the innovation possibilities frontier (the technology of creating new technologies). We assume that innovation requires scientists, and there is a fixed (inelastic) supply of S scientists in this economy.[21] At each point in time, $S_I(t) \geq 0$ of these scientists are hired by monopolists at a competitive wage W^S for automation, and $S_N(t) \geq 0$ of

[21]Focusing on an innovation possibilities frontier using just scientists, rather than variable factors such as in the lab-equipment specifications, is convenient because it enables us to focus on the direction of technological change — and not on the overall amount of technological change — especially when we turn to the welfare analysis in the next section.

them are hired at the same wage for creating new tasks. The market clearing condition for scientists is

$$S_I(t) + S_N(t) \leq S,$$

with the wage W^S being equal to zero if this inequality if strict.

We assume that advances in automation and creation of new tasks follow the next two differential equations

$$\dot{I}(t) = \kappa_I \phi(n(t)) S_I(t), \tag{20}$$

and

$$\dot{N}(t) = \kappa_N S_N(t), \tag{21}$$

where κ_I and κ_N are positive constants, representing the difficulty/ease of the corresponding type of technological change. The function $\phi(n(t))$ in (20) is included to capture the fact that automaking tasks closer to the frontier (the highest available task) may be more difficult. In particular, if $n(t)$ is close to 0, then it will be the recently invented tasks that are being automated, which may be more difficult than the case in which $n(t)$ is close to 1. For this reason, we assume that ϕ is nondecreasing. For most of our results, we do not need ϕ to be increasing, but we will invoke this feature to guarantee existence of a balanced growth path in the next subsection.

5.2 Equilibrium with Endogenous Technological Change

The key objects we need to compute to characterize the equilibrium with endogenous technological change are value functions determining the net present discounted value of new automation and labor-intensive innovations. We denote these by $V_I(t)$ and $V_N(t)$. More specifically, $V_I(t)$ is the value of a new technology automating the task at $i = I(t)^+$ (i.e., the highest-indexed task that has not yet been automated, or more formally $i = I(t) + \varepsilon$ for ε arbitrarily small and positive). Likewise, $V_N(t)$ is the value of a new technology creating a more complex task at $i = N(t)^+$.

Given these value functions, an equilibrium with endogenous technology is given by paths $\{K(t), N(t), I(t)\}_{t=0}^{\infty}$ for capital and technology (starting from an initial values $K(0), N(0), I(0)$), paths $\{r(t), W(t), W^S(t)\}_{t=0}^{\infty}$ for factor prices, paths $\{V_N(t), V_I(t)\}_{t=0}^{\infty}$ for the value functions of technology monopolists, and paths $\{S_N(t), S_I(t)\}_{t=0}^{\infty}$ for the allocation of scientists such that all markets clear, all firms, including prospective technology monopolists, maximize profits, the representative household maximizes its utility, and $N(t)$ and $I(t)$ evolve endogenously according to equations (20) and (21).

We start by characterizing the value functions for technology monopolists. Suppose also that in this equilibrium, $I^*(t) = I(t)$, so that a new automated task starts being used immediately.[22]

[22] As already noted, it can be proved that, when $n > \bar{n}$ (where $\bar{n}$ is defined in (17) in the previous section),

Let us next compute the flow profits from automation, which naturally replaces a task previously performed by labor (i.e., $i > I(t)$) and can be written as

$$\pi_I(t,i) = (1-\mu)\left(\frac{\eta}{1-\eta}\right)^{\zeta}\psi^{1-\zeta}Y(t)c^u(r(t))^{\zeta-\sigma}.$$

Intuitively, these profits come from the ability of firms to produce task i using capital (which is necessarily profitable given our assumption that $I^*(t) = I(t)$). Similarly, the flow profits of producing such task using labor are

$$\pi_N(t,i) = (1-\mu)\left(\frac{\eta}{1-\eta}\right)^{\zeta}\psi^{1-\zeta}Y(t)c^u\left(\frac{W(t)}{\gamma(i)}\right)^{\zeta-\sigma}.$$

It is then straightforward to compute the offer that a monopolist with a new technology automating task I at time t needs to make to the firm currently holding the patent for the (labor-intensive) technology of that intermediate. This offer will be given by the net present discounted value of the profit streams, discounted using the path of future interest rates, that the existing patent holder would obtain

$$(1-\mu)\left(\frac{\eta}{1-\eta}\right)^{\zeta}\psi^{1-\zeta}\int_t^{\infty} e^{-\int_0^{\tau}(r(s)-\delta)ds}Y(\tau)c^u\left(\frac{W(\tau)}{\gamma(I)}\right)^{\zeta-\sigma}d\tau.$$

Since this is a take-it-or-leave-it offer, the best response of the patent holder is to accept it.[23]

Similarly, the offer of the technology monopolist with a new technology for creating a new labor-intensive task while replacing task $N-1$ (which is necessarily automated in any equilibrium with automation) is given by

$$(1-\mu)\left(\frac{\eta}{1-\eta}\right)^{\zeta}\psi^{1-\zeta}\int_t^{\infty} e^{-\int_0^{\tau}(r(s)-\delta)ds}Y(\tau)c^u\left(r(\tau)\right)^{\zeta-\sigma}d\tau.$$

Both of these offers will be accepted by the patent-holders with the current technologies. Incorporating this, we can then compute the values of firms that innovate (respectively with automation and creation of new tasks):

$$V_I(t) = (1-\mu)\left(\frac{\eta}{1-\eta}\right)^{\zeta}\psi^{1-\zeta}\int_t^{\infty} e^{-\int_t^{\tau}(r(s)-\delta)ds}Y(\tau)\left(c^u\left(r(\tau)\right)^{\zeta-\sigma} - c^u\left(\frac{W(\tau)}{\gamma(I(t))}\right)^{\zeta-\sigma}\right)d\tau, \quad (22)$$

and

$$V_N(t) = (1-\mu)\left(\frac{\eta}{1-\eta}\right)^{\zeta}\psi^{1-\zeta}\int_t^{\infty} e^{-\int_t^{\tau}(r(s)-\delta)ds}Y(\tau)\left(c^u\left(\frac{W(\tau)}{\gamma(N(t))}\right)^{\zeta-\sigma} - c^u\left(r(\tau)\right)^{\zeta-\sigma}\right)d\tau. \quad (23)$$

$I^*(t) < I(t)$ is not possible in an equilibrium with R&D directed at automation, since this would induce technology monopolists wishing to automate tasks previously performed by labor to delay until $I^*(t') = I(t')$.

[23] This expression is written by assuming that the patent-holder will also turn down subsequence less generous offers in the future. Writing it in the value function form, using the one-step ahead deviation principle, leads to the same conclusion.

Both of these expressions have a common form: they subtract the lower cost of producing a task with the factor for which the new technology is designed from the higher cost of producing the same task with the other factor (working with the older technology). Note also that both of these expressions factor in the fact that, because of the same structure of offers that will be forthcoming in the future, the innovator will continue to earn a flow of revenues in the future, regardless of when its technology is replaced — and this is the reason why the time of replacement does not feature in these expressions. Observe also, for future reference, that these values are positive only when $\sigma > \zeta$. This can be seen from (22), by virtue of the fact that a task performed by labor is being automated, $c^u(r(\tau)) < c^u\left(\frac{w(\tau)}{\gamma(I(t))}\right)$. Thus if we had $\zeta > \sigma$, the profit stream would be negative. The same applies to (23). The intuitive reason for this is that, in the case where $\zeta > \sigma$, profits are lower when the firm is more productive, and thus when the holder of the new technology buys out the patent-holder of the less productive technology, it ends up with negative net profits.

Once these value functions are derived, the allocation of scientists to the two different types of technological change follows immediately by noting that the market wage of scientists will be equal to their value in the activity where their productivity is greater. Therefore,

$$
\begin{array}{lll}
S_N(t) = S, & S_I(t) = 0 & \text{if } \kappa_N V_N(t) > \kappa_I \phi(n(t)) V_I(t) \\
S_N(t) = 0, & S_I(t) = S & \text{if } \kappa_N V_N(t) < \kappa_I \phi(n(t)) V_I(t) \\
S_N(t) \in [0, S] \quad , & S_I(t) = S - S_N(t) & \text{if } \kappa_N V_N(t) = \kappa_I \phi(n(t)) V_I(t).
\end{array}
$$

Intuitively, whenever one of the two types of technologies (automation versus creation of new tasks) is more profitable, all scientists will be allocated to this activity, and their wage, W^S, will be equal to their value in this activity. But this also implies that this wage will exceed their value in the other technological activity, unless we are in the case where $\kappa_N V_N(t) = \kappa_I \phi(n(t)) V_I(t)$.

These observations enable us to represent, using the same normalizations is in the previous section, the equilibrium path with endogenous technology by the time path of the tuple $\{n(t), k(t), c(t), S_I(t)\}_{t=0}^{\infty}$ such that:

- The evolution of the state variables is given by

$$
\begin{aligned}
\dot{k}(t) &= f^E(k(t), n(t)) - c(t) - (\delta + A\kappa_I \phi(n(t)) S_I(t)) k(t) \\
\dot{n}(t) &= \kappa_N (S - S_I(t)) - \kappa_I \phi(n(t)) S_I(t).
\end{aligned}
$$

- Consumption satisfies the Euler equation (13) coupled with the transversality condition in equation (14).

- The allocation of scientists satisfies:

$$S_I(t) = \begin{cases} 0 & \text{if } \kappa_I \phi(n(t)) V_I(t) < \kappa_N V_N(t) \\ \in [0, S] & \text{if } \kappa_I \phi(n(t)) V_I(t) = \kappa_N V_N(t) \\ S & \text{if } \kappa_I \phi(n(t)) V_I(t) > \kappa_N V_N(t) \end{cases},$$

  with $V_N(t)$ and $V_I(t)$ given by equations (22) and (23).

We next characterize the dynamic equilibrium with endogenous technology. A balanced growth path is defined as in Proposition 3, as an allocation in which normalized capital $k(t)$ and the interest rate $r(t)$ are constant, except that now n will be determined endogenously. The next proposition gives another one of the main results of the paper. It establishes conditions for the existence of a unique balance growth path in which there are both types of technological changes and also shows that, under the same set of conditions, it is (saddle-path) stable.

Proposition 4 (Equilibrium with endogenous technological change) *Suppose that $\sigma > \zeta$, $\rho > \underline{\rho}$ and $S < \overline{S}$ (where $\underline{\rho}$ and $\overline{S}$ are suitably defined thresholds). Then:*

1. *There is at most one balanced growth path. Along this path, we have $N(t) - I(t) = n^D$, with n^D determined endogenously from the condition $\kappa_N V_N = \kappa_I \phi(n^D) V_I$, and satisfies $n^D \in (\bar{n}, 1)$, where $\bar{n}$ is as defined in (17). In this balanced growth path, $\dot{N} = \dot{I} = \frac{\kappa_I \kappa_N \phi(n^D)}{\kappa_I \phi(n^D) + \kappa_N} S$, and Y, C, K and W grow at the constant rate $g = A \frac{\kappa_I \kappa_N \phi(n^D)}{\kappa_I \phi(n^D) + \kappa_N} S$, and r, the labor share and employment are constant.*

2. *There exist $\underline{\phi} < \overline{\phi}$ such that if $\phi(x)$ satisfies $\lim_{x \to 0} \phi(x) < \underline{\phi}$ and $\lim_{x \to 1} \phi(x) > \overline{\phi}$, then the balanced growth path described above always exists.*

3. *The dynamic equilibrium is unique in the neighborhood of the balanced growth path and is locally (saddle-path) stable. Moreover, when $\theta \to 0$, the dynamic equilibrium is globally stable.*

Proof. The proofs are presented in the appendix. ■

The first important result is the existence and uniqueness of the balanced growth path. The second critical result, established in the third part, is that this balanced growth path is locally stable and also globally stable when θ is small (so that preferences are approximately linear or equivalently have an infinite elasticity of intertemporal substitution). This result implies that there are powerful market forces pushing the economy towards the balance growth path.

These results are established under several conditions. First, we have imposed that $\sigma > \zeta$. This condition is critical in ensuring that innovations are directed towards technologies using the cheaper factors.[24] Recall from Section 3 that more tasks are allocated to the factor that is cheaper. This creates a natural force that tends to push innovations to be directed towards the same cheaper factor. One way of understanding this effect is that as a factor becomes cheaper, the range of

[24] By the term "innovation directed towards the cheaper factor", we mean a comparative static statement: as the relative price of a factor declines, is innovation directed more or less towards this factor?

activities in which it is used expands. Holding the proportions at which this factor is combined with intermediates in the task production functions, (2) and (3), this implies that the quantity of the corresponding intermediate, $q(i)$, also increases. This makes technologies working with this factor more profitable, encouraging innovation beneficial for this factor. The extent of this positive force is regulated by the elasticity of substitution σ: the greater is σ, the more powerful is this effect directing innovation towards the cheaper factor. There is a countervailing effect as well, however: as a factor becomes cheaper, it is substituted for the intermediate it is combined with, so that the quantity of the corresponding intermediate declines holding the level of task production fixed. This creates a negative force, discouraging innovations directed towards the cheaper factor. Task production functions, (2) and (3), clarify that the extent of the substitution effect will depend on the elasticity of substitution between the factor in question and the intermediates, ζ. The condition $\sigma > \zeta$ guarantees the positive effect dominates so that innovations are directed towards the cheaper factor.[25]

We should note that this condition is quite plausible: in our model, when $\sigma < \zeta$, there will be no research at all since, as observed above, the net present discounted values from innovation will be negative (recall equations (22) and (23)). This is because, as explained above, somewhat pathologically profits are higher when the producer is less productive. Put differently, in this case, there is such a strong substitution effect allowing the substitution of the cheaper factor for intermediates that there is no incentive to innovate on intermediates working with cheaper factors. Because there will be no technological change with a negative net present discounted value from innovation, this condition is imposed even for the existence result in part 1 of the proposition. Moreover, the condition $\sigma > \zeta$ is also empirically plausible. We expect the elasticity of substitution between factors and intermediates, ζ, to be very low — in the limit,zero as in the Leontief case since new technologies, for example enabling automation, are embedded in these intermediates.[26]

Second, we have also imposed that $\rho > \underline{\rho}$ and $S < \overline{S}$. The role of these assumptions was already discussed in the previous section, where we pointed out that for an equilibrium in which technologically automated tasks will be immediately produced with capital and all available new complex tasks will be immediately produced with labor, we need $n > \bar{n}$, which in turn requires the interest rate not to be too low and the growth rate not to be too high. Here, $\rho > \underline{\rho}$ ensures that the interest rate is indeed not too low, and $S < \overline{S}$ is to guarantee that the growth rate is not too high (and also ensures that the net present discounted value of the representative household is

[25]Our assumption ruling out the creative destruction of profits of existing producers is also playing a role in the stability result as we discussed further Section 7, which shows that though the balanced growth path equilibrium is very similar, stability is no longer guaranteed.

[26]Observe also that this condition does not impose any restrictions on the short-run elasticity of substitution between capital and labor, which can be less than one as in many of the studies reviewed in Acemoglu and Robinson (2015).

finite). Figure 9 draws the relative net present discounted values of creating new tasks relative to automation as a function of n. In the region where $n > \bar{n}$, this schedule is diminishing, reflecting the key economic forces of our model: greater automation reduces wages relative to the interest rate (the rental rate of capital), encouraging further creation of new tasks. Without the assumptions that $\rho > \underline{\rho}$ and $S < \overline{S}$, the productivity effects discussed in the previous section may dominate. The figure makes it clear that there can at most be one intersection, defining a unique balanced growth path with innovation directed towards both types of technologies. However, such an intersection may fail to exist depending on the location of the ϕ schedule. This motivates the final assumption used in part 2 of the proposition, ensuring that ϕ satisfies a weaker form of the Inada conditions, ensuring that such an intersection exists. In the absence of these conditions, the long-run equilibrium may be one in which only one type of technology is developed.

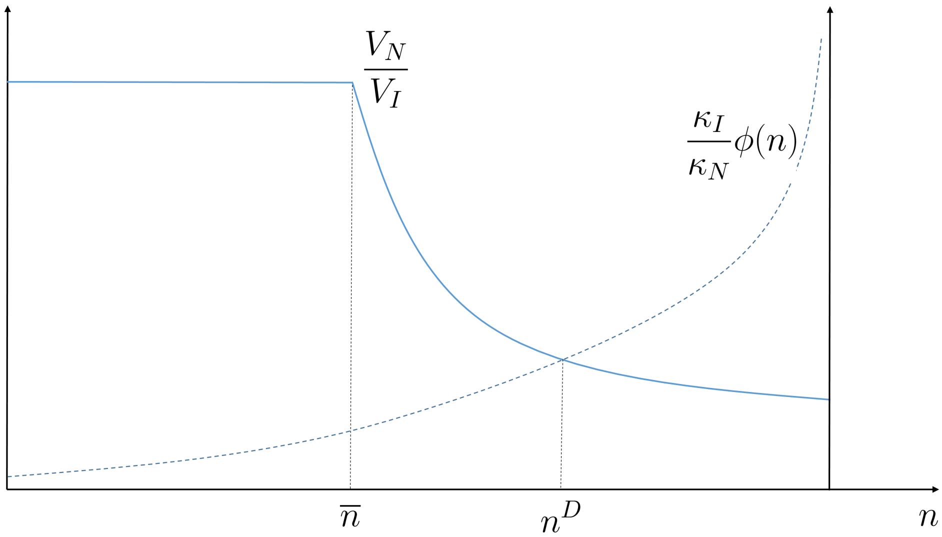

Figure 9: Determination of n^D in steady state.

Overall, the critical economic force highlighted by this result is that, differently from models with factor-augmenting technologies, it is factor prices that guide the direction of technological change, and there are stronger incentives to undertake the type of innovation that will work with the factor that is relatively cheaper.

We can also observe that the long-run elasticity of substitution between capital and labor, σ_{LR}, which allows both for the endogeneity of technology and for capital accumulation, is equal to 1 because following a shock to technology or capital stock, the economy returns back to its balanced growth path, where the share of labor in national income is constant. This implies that, interestingly, the long run elasticity of substitution need not be larger than the medium-run and short-run elasticities σ_{MR} and σ_{SR} defined above. This is because it is not only technology but also the capital stock of the economy that adjusts in the long run (and thus bringing in the productivity effects discussed in the previous section).

Finally, the emphasis of our main result in this section, Proposition 4, has been to show that shocks to technology, for example in the form of a series of new automation technologies, will set in motion self-correcting forces, so that in the long run the economy returns back to its pre-shock balanced growth path with the same employment level and labor share in national income. This does not, however, imply that all changes will leave the long-run prospects of labor unchanged. The next corollary shows that if there is a change in the innovation possibilities frontier, making automation easier than before, then there will be a new balanced growth path with lower employment and lower share of labor in national income.

Corollary 2 *Suppose that there is a one-time permanent increase in κ_I relative to κ_N. Then the economy converges to a new balanced growth path with lower n^D, lower employment and lower share of labor in national income.*

This corollary follows immediately because an increase in κ_I/κ_N shifts the upward sloping curve in Figure 9 further up, leaving to a lower value of n^D in the balanced growth path.

One implication of this corollary, in conjunction with Proposition 4, is that it clearly delineates the types of changes in technology that will set in motion self-correcting dynamics: those driven by faster than usual arrival of automation technologies. In contrast, those which changed the ability of the society to create new automation technologies will not create such self-correcting dynamics and will result in lower prospects for labor in the future.

6 Welfare

In this section we turn to an analysis of the efficiency of the equilibrium described in Proposition 4. Our main finding is that the presence of rents for workers, as captured by our quasi-labor supply, distorts the composition of equilibrium technology towards too much automation and too little creation of new, more complex (labor-intensive) technologies — and this is in addition to other distortions that exist in this class of models. We present two complementary results shedding light on this inefficiency. First, we characterize the constrained efficient allocation of a social planner who is subject to the same quasi-labor supply schedule, as well as to the constraint that wages have to be given by (possibly subsidized) marginal product of labor of firms and technologies evolve according to the same innovation possibilities frontier. We then show how this constrained efficient allocation can be decentralized by a set of taxes and subsidies. This exercise shows that, in addition to the usual wedges (taxes/subsidies) between the social planner's allocation and the decentralized equilibrium, workers' rents create an additional reason to subsidize the creation of new tasks relative to automation. Second, for a particular set of parameters that help us isolate this novel inefficiency, we show the decentralized equilibrium could be improved by altering the composition of R&D in the direction of the creation of new tasks.

We start by characterizing the constrained efficient allocation, which we will use in deriving both results. In this constrained efficient allocation. Let us denote by $F^P(N, I, K, L)$ the *net* aggregate output (net of the costs of producing intermediates) when the level of employment is L, the capital stock is K, the state of technologies is represented by $\{N, I\}$, and intermediates are priced at their marginal cost (which is the relevant net aggregate output expression for the social planner, since she would always price all intermediates at marginal cost). Also, let $W^P(N, I, K)$ and $r^P(N, I, K)$ denote the resulting marginal products of labor and capital (corresponding to the wage and interest rates in the decentralized allocation) with the level of employment given by the quasi-labor supply schedule, $L = L^s(\omega)$. Finally, let $\omega^P(N, I, K)$ denote the equilibrium value for W/rK in this case. It is straightforward to prove that these variables satisfy the same comparative statics described in Proposition 2.

The constrained efficient allocation solves the problem

$$\max_{\{C(t),L(t),S_N(t),S_I(t)\}} \int_0^\infty e^{-\rho t} \frac{C(t)^{1-\theta}-1}{1-\theta} dt,$$

subject to the endogenous evolution of the state variables:

$$\begin{aligned}
\dot{K}(t) =& F^P(N(t), I(t), K(t), L(t)) - C(t) - \delta K(t), \\
\dot{N}(t) =& \kappa_N S_N(t), \\
\dot{I}(t) =& \kappa_I S_I(t) \phi(N(t) - I(t)).
\end{aligned}$$

In using the net aggregate production function, F^P, we have already incorporated that the planner will price all intermediates at marginal cost, $\mu\psi$. Furthermore, we have written the objective function of the social planner as just maximizing the net present discounted value of consumption streams, thus imposing that there is no disutility or opportunity cost of labor supply and all wages received by workers are "quasi rents" — which, as noted above, will be an additional source of deviation between the social planner's allocation and the equilibrium. This formulation is justified by the microfoundation provided for the quasi-labor supply schedule in the Appendix, and we also note that the results are entirely analogous if there is a positive opportunity cost of labor lower than the market wage (and assuming that this opportunity cost is equal to zero is merely for notational simplicity). The most important implication of this structure is that, all else equal, the social planner would like to maximize employment as this increases net output and wage payments without any disutility cost.[27]

[27] This program also imposes that the threshold that the social planner will set for automation, $I^*(t)$ in the terminology used in the previous section, is given by $I(t)$, that is, by the available automation technology. As noted in footnote 22 for the equilibrium, in any allocation with positive research effort directed to automation, including the limiting allocation, this has to be the case, and we impose it throughout to simplify the exposition.

Because the planner faces the same quasi-labor supply schedule and labor demand relations, we also have:[28]

$$L(t) \leq L^s(\omega^P(N, I, K)).$$

The relationship imposes that the planner will take into account the impact of technology and capital accumulation on employment.

Let μ_N and μ_I denote the shadow values of the two types of technology, respectively, and μ_L and μ_K the shadow values of labor and capital. The maximum principle (see Acemoglu, 2009, Theorem 7.9) implies these satisfy the necessary conditions:

$$\rho\mu_N - \dot{\mu}_N = \mu_K F_N^P + \mu_L L_\omega^s \omega_N^P + \mu_I \phi'(N - I), \quad \rho\mu_I - \dot{\mu}_I = \mu_K F_I^P + \mu_L L_\omega^s \omega_I^P - \mu_I \phi'(N - I),$$
$$\rho\mu_K - \dot{\mu}_K = \mu_K(r^P - \delta) + \mu_L L_\omega^s \omega_K^P, \qquad \mu_L = \mu_K W^P.$$

All the functions in the above equations are evaluated at their corresponding arguments at time t, and subscripts denote partial derivatives. Moreover, we show in the Appendix that the current value Hamiltonean associated with the planner's problem is concave, so these conditions (plus the Euler equation for consumption and the transversality condition) are sufficient for characterizing the constrained efficient allocation.

Let $\Psi_N(t) \equiv \mu_N(t)/\mu_K(t)$ and $\Psi_I \equiv \mu_I(t)/\mu_K(t)$ be the shadow discounted net present values of new technologies (in terms of additional net output they create). The optimal allocation of scientists to two different types of research then satisfies

$$\begin{array}{lll} S_N(t) = S, & S_I(t) = 0 & \text{if } \kappa_N \Psi_N(t) > \kappa_I \Psi_I(t) \\ S_N(t) = 0, & S_I(t) = S & \text{if } \kappa_N \Psi_N(t) < \kappa_I \Psi_I(t) \\ S_N(t) \in [0, S] \quad , & S_I(t) = S - S_N(t) & \text{if } \kappa_N \Psi_N(t) = \kappa_I \Psi_I(t). \end{array}$$

Thus, intuitively, Ψ_N and Ψ_I play an analogous role to V_N and V_I in the decentralized allocation, and can be also written as integrals of future net benefits:

$$\Psi_N = \int_t^\infty e^{-\int_0^\tau (r^P - \delta + W^P L_\omega^s \omega_K^P) ds} \left(F_N^P + W^P L_\omega^P \omega_N^P + \Psi_I \phi'(N - I)\right) d\tau,$$
$$\Psi_I = \int_t^\infty e^{-\int_0^\tau (r^P - \delta + W^P L_\omega^s \omega_K^P) ds} \left(F_I^P + W^P L_\omega^P \omega_I^P - \Psi_I \phi'(N - I)\right) d\tau.$$

These equations are clearly analogous to the expressions for V_N and V_I in the decentralized equilibrium given by equations (22) and (23).

To complete our characterization, let $f^P(n, k) = F^P(N, I, K, L(\omega^P(N, I, K)))/\gamma(I)$ denote the normalized net output; $w^P(n, k) = W^P(N, I, K)/\gamma(I)$ the normalized wages; and $r^P(n, k)$ the normalized interest rate obtained when intermediates are priced at their marginal cost. These are

[28] Recall that $W^P(N, I, K), r^P(N, I, K)$ and $\omega^P(N, I, K)$ are defined in terms of the allocation $\{N, I, K\}$.

defined in the same way as in the decentralized equilibrium as detailed in Section 5. Summarizing, the constrained efficient allocation can be represented as $\{n(t), k(t), c(t), S_I(t)\}_{t=0}^{\infty}$ — where we normalize $n(t) = N(t) - I(t)$, $k(t) = K(t)/\gamma(I(t))$, $c(t) = C(t)/\gamma(I(t))$, such that:

- The evolution of the state variables is given by

$$\begin{aligned} \dot{k}(t) &= f^P(n(t), k(t)) - c(t) - (\delta + A\kappa_I S_I(t))k(t) \\ \dot{n}(t) &= \kappa_N(S - S_I(t)) - \kappa_I \phi(n(t)) S_I(t). \end{aligned} \tag{26}$$

- Normalized consumption satisfies the Euler equation

$$\dot{c}(t) = c(t)\left(\frac{1}{\theta}(r^P(n(t), k(t))\left(1 - \omega^P L^s \frac{\partial \ln L^s}{\partial \ln \omega} \frac{\partial \ln \omega^P}{\partial \ln K}\right) - \delta - \rho) - \kappa_I S_I(t)\right). \tag{27}$$

- The allocation of scientists satisfies:

$$S_I(t) = \begin{cases} 0 \text{ if } \kappa_I \phi(n) \Psi_I(t) < \kappa_N \Psi_N(t) \\ \in [0, S] \text{ if } \kappa_I \Psi_I(t) \phi(n) = \kappa_N \Psi_N(t) \\ S \text{ if } \kappa_I \Psi_I(t) \phi(n) > \kappa_N \Psi_N(t) \end{cases}, \tag{28}$$

  with $S_N(t) = S - S_I(t)$.

- The transversality condition holds, i.e.,

$$\lim \mu_k e^{-\rho t} = 0 \tag{29}$$

  holds.

This characterization implies that the constrained efficient allocation has a similar structure to the equilibrium described in Proposition 4. The next proposition summarizes this result and shows that it also has the same asymptotic and stability properties. Most importantly, it also characterizes the set of taxes and subsidies that can be used in the equilibrium to decentralize this constrained efficient allocation.

Proposition 5 (Constrained efficient allocation and decentralization) *Suppose that $\sigma > \zeta$ and $\rho > \underline{\rho}$. Then:*

- *The constrained efficient allocation is uniquely defined by the solution to (26)-(29). Moreover, under the same conditions derived in Proposition 4 for $\phi(n)$, this allocation locally converges to the unique constrained efficient balanced growth path, and if $\theta \to 0$, it globally converges to this efficient balance growth path.*
- *The constrained efficient allocation can be decentralized by using the following sets of taxes and subsidies:*

1. *a proportional subsidy at the rate $1 - \mu$ on intermediate prices to remove the monopoly markups;*

2. *a proportional tax/subsidy of* $\tau_k = -\omega^P L^s \frac{\partial \ln L^s}{\partial \ln \omega} \frac{\partial \ln \omega^P}{\partial \ln K}$ *on savings to correct for the impact of capital on employment (this expression is positive, i.e., tax, when* $\sigma_{SR} > 1$*, zero when* $\sigma_{SR} = 1$*, and a subsidy, i.e., negative, when* $\sigma_{SR} < 1$*);*

3. *additive taxes/subsidies for successful innovators who entered the market at time* t_0*, which correct for the technological externality and spillovers generated by the two different types of innovation;*

4. *an additive subsidy* $W^P L^P \frac{\partial \ln L^s}{\partial \ln \omega} \frac{\partial \ln \omega^P}{\partial N} \geq 0$ *for successful innovators of new more complex tasks, and an additive tax* $W^P L^P \frac{\partial \ln L^s}{\partial \ln \omega} \frac{\partial \ln \omega^P}{\partial I} \leq 0$ *on successful innovators of new automation technologies; this tax and subsidy correct for the fact that technology monopolists do not take into account the effect of technologies on the level of equilibrium employment.*

Proof. We present explicit formulas for all of the taxes and subsidies in the appendix. ■

This proposition contains several important results. First, it characterizes the constrained efficient allocation, establishing that it has a similar structure to the equilibrium. Second, in contrast to neoclassical models of capital taxation (e.g., Chamley, 1986 and Judd, 1985, but also see Straub and Werning, 2014), the decentralization of the constrained efficient allocation requires taxing or subsidizing capital accumulation. This is because the capital stock affects wages and thus the level of employment through the quasi-labor supply schedule. For instance, if $\sigma_{SR} < 1$, capital increases employment in the short run (see Proposition 2), which is, as noted above, beneficial. Thus in this case, the social planner would set $\tau_K < 0$, further encouraging capital accumulation, while when $\sigma_{SR} > 1$, the opposite applies.

Third, the quality ladder structure in the creation of new labor-intensive complex tasks introduces a technological externality. By undertaking this type of innovation and thus increasing N, a technology monopolist also allows new entrants to create more productive new tasks (because $\gamma(N)$ is increasing). The externality created by automation is somewhat more subtle. Because capital has the same productivity in all automated tasks, this direct technological externality is absent. But automation today forces future innovators to automate higher-indexed tasks, which are the ones where labor has a comparative advantage (because $\gamma(I)$ is increasing), and this reduces the profits of future innovators. Though in different environments, some of these externalities could be internalized through a more sophisticated patent system, here we have focused on taxes and subsidies, which explains the wedges introduced in part 3 of the proposition. Likewise, the function ϕ introduces spillovers directly into the technology possibilities frontier, which also contributes to the wedges in part 3.

Finally, the quasi-labor supply schedule creates an additional, and novel, distortion in the equilibrium relative to the constrained efficient allocation. Because firms do not internalize the quasi-rents received by workers, they automate tasks taking into account the wage rate. In contrast, the social planner internalizes these quasi-rents, and thus at the margin prefers to create more employment as we have already noted (or equivalently, at the margin she uses the opportunity cost of labor rather than the market wage in the automation decision). The resulting greater incentives of firms to automate tasks with given technology then translate into a stronger impetus for R&D directed towards automation and too little towards the creation of new, more complex tasks. For this reason, the social planner would like to encourage more R&D towards creation of labor-intensive new tasks and less automation, and she achieves this by using taxes on automation innovations and subsidies to innovations creating new labor-intensive tasks as outlined in part 4 of the proposition.

Proposition 5 outlined how the constrained efficient allocation can be decentralized. A key result, as we have just emphasized, is that conditional on the other taxes and subsidies necessary for dealing with markups and technological externalities, there needs to be an additional set of taxes and subsidies to encourage less automation and more effort towards the creation of new, more complex tasks. The complementary question is whether starting from a decentralized allocation, and without this full set of subsidies, the social planner would still like to discourage automation. The next proposition answers this question (in the affirmative), focusing on the configuration where $\zeta \to 1$ which, as we have already emphasized, is a particularly tractable special case of our model, and assuming that the proportional subsidy at the rate $1-\mu$ removing the main effect of monopoly markups is present.

Proposition 6 (Excessive automation) *Suppose that $\rho > \underline{\rho}$ and $S < \overline{S}$ as in Proposition 4, and that $\sigma > \zeta \to 1$. Moreover, suppose intermediate goods are subsidized and can be purchased at their marginal cost (or equivalently $\mu \to 1$). Consider the decentralized equilibrium path starting from some initial level of capital, $K(0)$, and technologies, $N(0)$ and $I(0)$, converging to the balanced growth path described in Proposition 4 (i.e., $n^D(t) = N(t) - I(t)$ converging to n^D). Then there exists a feasible allocation satisfying $n^P(t) \geq n^D(t)$ with $\lim_{t\to\infty} n^P(t) > n^D$ that achieves strictly greater welfare than the decentralized equilibrium.*

Proof. The proof is constructive, and proceeds by showing that slightly reducing $S_I(t)$ whenever $S_I(t) \in (0, 1)$ produces a welfare improvement. All the details and derivations are presented in the appendix. ■

This proposition therefore establishes that even without the full set of other taxes and subsidies, departing from the equilibrium in the direction of discouraging automation and further encouraging the creation of new, more complex tasks will be welfare improving. The assumption that $\zeta \to 1$ plays an important in this result. Because in this special case, the production function for intermediates becomes Cobb-Douglas, and monopoly profits are proportional to revenues. This, coupled with the assumption that monopoly markups are removed, ensures that incentives to undertake different types of innovations, as summarized by the value functions V_N and V_I, are proportional to social values except for the distortion working through the quasi-labor supply schedule, and thus enables us to focus on this novel source of distortion in the composition of R&D and direction of technological change.[29]

7 Extensions

In this section, we discuss two extensions. First we introduce heterogeneous skills, enabling us to analyze the impact of the two types of technological changes we have studied in this paper on inequality between different skill types. Second, we reintroduce the creative destruction of profits, and show how similar balanced growth path results continue to apply in this case, though there may also exist other balance growth path or steady states.

7.1 Automation, New Tasks and Inequality

In this subsection, we introduce heterogeneous skills — namely, two types of labor, high skill and low skill — and study how automation and creation of new tasks impact inequality. Let us suppose that there is a quasi-labor supply of low-skill labor given by $L^s(\frac{w_L}{rK})$, and a quasi-labor supply of high-skill labor given by $H_s(\frac{w_H}{rK})$. The respective wages of these two types of labor are denoted by w_L and w_H. For simplicity, we focus on the dynamic economy with exogenous technology.

In this economy with three types of factors (capital, low-skill labor and high-skill labor), the pattern of comparative advantage is slightly more complicated. We first assume that high-skilled labor has productivity analogous to what we have assumed so far for labor overall:

$$\gamma_H(i) = e^{A_H i}.$$

For low-skilled workers, we assume

$$\gamma_L(i,t) = e^{A_L i + (A_H - A_L)\Delta(t - t_0(i))},$$

where $A_L < A_H$ and t is calendar time and $t_0(i)$ the date at which task i was introduced. This structure thus implies that the productivity of low-skill labor increases as time passes from the initial date at which a task was first invented/introduced. This assumption captures the feature that new technologies and tasks are standardized over time (e.g., Acemoglu, Gancia and Zilibotti, 2010) or that low-skill workers may not be good at adapting to a changing environment or new technologies (e.g., Schultz, 1965, Nelson and Phelps, 1966, Greenwood and Yorukoglu, 1997, Caselli, 1999, Galor and Moav, 2000, and Beaudry, Green and Sand, 2013). The implication of this assumption for our setup is that while capital has a comparative advantage in low-indexed tasks that have

[29]The same result can be established without these assumptions if the quasi-labor supply curve is sufficiently elastic, so that the benefits from small increases in wages (in terms of expanding employment) outweigh costs that may come from other nonlinear effects that are not removed by taxes and subsidies in this case.

been automated, high-skill labor will have a comparative advantage in high-indexed tasks that have recently been introduced; while low-skill labor will perform intermediate-indexed tasks. In particular, it follows straightforwardly that there exists a threshold task M such that high-skill labor performs tasks in $(M, N]$, low-skill labor performs tasks in $(I, M]$, and tasks in $[N-1, M]$ are performed by capital.

The main implications of this model with heterogeneous labor are summarized in the next proposition.

Proposition 7 (Automation, new tasks and inequality) *Suppose technology evolves exogenously:*

1. *Then a balanced growth path exists if and only if asymptotically $\dot{N} = \dot{I} = \Delta$ (and $A_H(1-\theta)\Delta < \rho$ so that net present discounted value of household income is finite). In this balanced growth path Y, C, K, w_H and w_L grow at a constant rate $A_H\Delta$ and r is constant. Moreover, the wage ratio between high-skilled and low-skilled workers (w_H/w_L) is constant but depends on $n = N - I$*

2. *Given such a path of technological change, the dynamic equilibrium is unique starting from any initial condition and converges to the balanced growth path.*

3. *The immediate effect of increases in both I and N is to increase w_H/w_L. But the medium-run impact of an increase in N is to reduce inequality.*

Proof. As explained above, high-skill workers will perform tasks in $(M, N]$; while low-skilled workers perform tasks in $(I, M]$.

This threshold is given by

$$\frac{e^{A_H M(t)}}{w_H(t)} = \frac{e^{A_L M(t) + (A_H - A_L)\Delta(t - t_0(M))}}{w_L(t)},$$

which grows at a rate Δ over time as well, along the balanced growth path.

Likewise, the threshold for automation is defined as

$$\frac{1}{r(t)} = \frac{e^{A_L \widetilde{I}(t) + (A_H - A_L)\Delta(t - t_0(I))}}{w_L(t)},$$

which also grows at the rate Δ over time maintaining the economy balanced.

Notice that if learning took place at a speed below $(A_H - A_L)\Delta$, low-skilled workers would get squeezed over time and perform a decreasing fraction of tasks at lower wages.

Finally, for the comparative statics notice that a temporal shock to I reduces the amount of tasks performed by low-skill workers. Since they haven't had enough time to learn how to perform more complex tasks, M grows by less than I and their wages fall. Importantly, their wages also fall relative to high-skill workers, and inequality increases. Thus, in this case, we would observe a declining labor share coinciding with more inequality, but this would eventually revert in steady state.

On the other hand, a temporal shock to N increases the range of tasks performed by high-skill workers, raising their wages. However, N only increases mildly and w_H/w_L falls because low-skill workers haven't had enough time to gain comparative advantage in complex tasks. Thus, in this case, we would observe an increasing labor share coinciding also with more inequality, but this would revert in steady state. ■

A number of features are worth noting. First, this extended model generates not only an endogenous distribution of income between capital and labor, but also inequality between high-skill and low-skill workers. Moreover, this latter inequality also reflects comparative advantage — now the comparative advantage of high-skill workers relative to their low-skill brethren. This comparative advantage structure also implies that automation, by squeezing out tasks previously performed by low-skill labor increases inequality between the two types of skills. Interestingly, however, the creation of new tasks also tends to increase inequality at first because it is high-skilled labor that has a comparative advantage in the higher-index tasks (i.e., new tasks). However, given our standardization assumption, that as tasks become standardized (as more time passes from their introduction), the productivity of low-skill workers increases, the medium-term implications of automation and creation of new tasks are very different. The first, just like in the short-run, tends to increase inequality in the medium-run also. In contrast, the creation of new tasks increase inequality in the short run, but not in the medium run. In fact, low-skill workers gain relative to capital in the medium run from the creation of new tasks.

Interestingly, inequality may be particularly high following a period of adjustment in which the labor share first declines — due to increases in automation— and then recovers — due to the introduction of new complex tasks. Inequality may remain large for a while, until learning by low-skilled workers pushes their wages up.

7.2 Creative Destruction of Profits

In this subsection, we modify the assumption we have made on the structural intellectual property rights, reverting to the assumption that new technologies destroy the rents/profits of existing technologies. We will show that this has little effect on the balanced growth path in our model, but makes dynamics and stability more complicated.

Formally, we follow the standard models of quality improvements such as Aghion and Howitt (1992) and Grossman and Helpman (1991), and assume that a new innovation building on the previous technology directly replaces the previous technology without making any licensing nor patent payments.

Let us now compute the behavior of $V_N(t)$ and $V_I(t)$ under this assumption. To do so, let us first define $V_N(t,i)$ and $V_I(t,i)$ as the values at time t of having introduced different technologies for the production of task i (respectively, new labor-intensive tasks and automation). As before, flow profits from introducing new technologies are given by $\pi_I(t,i)$ and $\pi_N(t,i)$, respectively for automation and creation of new tasks. Since firms need not purchase production rights as before, their value functions while producing satisfy the Bellman equations:

$$\begin{aligned} r(t)V_N(t,i) - \dot{V}_N(t,i) &= \pi_N(t,i) \\ r(t)V_I(t,i) - \dot{V}_I(t,i) &= \pi_I(t,i). \end{aligned}$$

For a firm creating a labor-intensive technology for task i, let $T^N(i)$ denote the time at which it will be replaced by a technology allowing the automation of this task. Likewise, for a firm automating task i at time t, let $T^I(i)$ denote the time at which it will be replaced by a more complex technology using labor. Given that firms anticipate these deterministic replacement dates, their value functions also satisfy the boundary conditions $V_N(T^N(i),i) = 0$ and $V_I(T^I(i),i) = 0$.

Using the Bellman equations together with the boundary conditions derived above, we find the following formula for these value functions:

$$\begin{aligned} V_N(t) = V_N(N(t),t) &= \int_t^{T^N(N(t))} e^{-\int_t^\tau r(s)ds}\psi(1-\mu)Y(\tau)Bc^u\left(\frac{W(\tau)}{\gamma(N(t))}\right)^{\zeta-\sigma}, \\ V_I(t) = V_I(I(t),t) &= \int_t^{T^I(I(t))} e^{-\int_t^\tau r(s)ds}\psi(1-\mu)\kappa Y(\tau)Bc^u\left(\min\left\{r(\tau), \frac{w(\tau)}{\gamma(I(t))}\right\}\right)^{\zeta-\sigma} d\tau. \end{aligned}$$

Once these expressions are derived, the rest of the analysis follows the first part of Proposition 4 to establish the existence of the balanced growth path characterize there, but there can also be other balanced growth paths or steady states as shown in the next proposition.

Proposition 8 (Equilibrium with creative destruction) *Suppose that $\sigma > \zeta$, $\rho > \underline{\rho}$ and $S < \overline{S}$ (where $\underline{\rho}$ and $\overline{S}$ are suitably defined thresholds). If there is creative destruction of profits of existing technologies. Then:*

1. *There exist $\underline{\phi} < \overline{\phi}$ such that if $\phi(0) < \underline{\phi}$ and $\phi(1) > \overline{\phi}$, there exists at least one balanced growth path with both automation and creation of new tasks as in Proposition 4. In this balanced growth path, we have $N(t) - I(t) = n^{DR}$, $\kappa_N V_N(t) = \kappa_I \phi(n^{DR}) V_I(t)$ and $\dot{N} = \dot{I} = \frac{\kappa_I\kappa_N\phi(n^{DR})}{\kappa_I\phi(n^{DR})+\kappa_N}S$. Also, Y, C, K and w grow at the constant rate $g = A\frac{\kappa_I\kappa_N\phi(n^{DR})}{\kappa_I\phi(n^{DR})+\kappa_N}S$, r is constant, and the labor share and employment are constant.*

2. *There may also exist other steady states or balanced growth paths..*

The first part of the proposition follows using analogous lines of argument to the proof of Proposition 4. In particular, in a balanced growth path, we have $T^N(N(t)) - t = \frac{n^{DR}}{\Delta}$, and

$T^I(I(t)) - t = \frac{1-n^{DR}}{\Delta}$. Thus, both types of innovations are replaced at a fixed length of time, ensuring that the creative destruction of rents does not change the balance of the incentives for innovation.

However, in the presence of the creative destruction of rents V_N/V_I is an increasing function of n because a higher n reduces the wage per effective unit of labor paid in new complex tasks (while the interest rate remains roughly constant), and also because an increase in n implies firms creating new jobs will be automated With greater delay, creating another force towards higher V_N/V_I. Then from Figure 9, multiple intersections of the loci for V_N/V_I and $\frac{\kappa_I}{\kappa_N}\phi(n)$ are possible.

This proposition shows that most of the qualitative results from our main analysis continue to apply in this case. However, the creative destruction of profits and the productivity effect keeping interest rates constant and raising wages, introduce the possibility of multiple steady states. Though this source of multiplicity is interesting in and of itself, we chose to emphasize the major forces towards the stability of the balanced growth path in which automation and creation of new tasks proceed hand-in-hand.

8 Conclusion

As the pace of new technological advances automating tasks previously performed by labor has accelerated, concerns that these new technologies will make labor increasingly redundant have also intensified. This paper has attempted to develop a comprehensive framework in which these forces can be analyzed and contrasted with countervailing effects. At the center of our model is a task-based framework in which an endogenous set of tasks are allocated between capital and labor. Automation is modeled as (endogenous) expansion of the set of tasks that can be performed by capital, thus replacing labor in tasks that it previously controlled. The main new feature of our framework is that in addition to automation, there is another type of technological change enabling the creation of new, more complex versions of existing tasks, and it is labor that tends to have a comparative advantage in these new tasks. We fully characterize the structure of equilibrium in such a model, showing how the allocation of tasks between capital and labor is determined both by available technology and the endogenous choices of firms between capital and labor given factor prices. One attractive feature of task-based models is the link they highlight between factor prices and the range of tasks allocated to the two factors. More generally, as the equilibrium range of tasks allocated to capital increases (for example as a result of automation), the wage relative to the rental rate of capital and the share of labor in national income decline, and the equilibrium wage rate may also decline. Conversely, as the equilibrium range of tasks allocated to labor increases, the opposite result obtains. In our model, we also make the supply of labor potentially elastic by introducing a quasi-labor supply curve (which also implies that equilibrium wage may be greater than the opportunity cost of labor). Given this relationship, automation also tends to reduce employment, while the creation of new tasks increase employment. These results highlight that, while both types of technological changes underpin economic growth, they have very different implications for the factor distribution of income and also for employment.

Our full model endogenizes the direction of research and development towards automation and the creation of new complex tasks, showing how this framework naturally leads to a (unique) balanced growth path in which both types of innovations go hand-in-hand. Moreover, the dynamic equilibrium is also unique and, starting from any initial conditions, converges to the (unique) balanced growth path. Underpinning this global stability result is the impact of relative factor prices on the direction of technological change. The task-based framework (differently from the standard models of directed technological change which are based on factor-augmenting technologies) implies that as a factor becomes cheaper, this not only expands the range of tasks allocated to this factor in equilibrium, but also generates stronger incentives for the type of technological change working this factor. These economic incentives then imply that automation, by reducing wages relative to the rental rate of capital, and encourages the creation of new labor-intensive tasks, generating a powerful self-correcting force towards stability.

Though market forces ensure the stability of the balanced growth path, they do not necessarily generate the efficient composition of technology. In particular, the presence of the quasi-labor supply, by creating a wedge between the market wage and the opportunity cost of labor, creates an equilibrium distortion in the type of new technologies that are created. Firms tend to have an excessive bias for automation, because they derive profits by replacing labor by cheaper capital. The social planner, on the other hand, recognizes that part of the wage is rent captured by workers, and has weaker incentives to replace labor by capital. Put differently, the social planner prefers to choose a different composition of technologies (specifically, biased towards the creation of new tasks and away from automation) because she would like to expand employment, which generates greater rents for workers along the equilibrium path.

In addition to claims about automation leading to the demise of labor, several commentators are concerned about the inequality implications of automation the new technologies. In one of our extensions, we have studied this question by introducing a distinction between low-skilled and high-skilled labor, where the latter has a comparative advantage in producing with newer technologies. This structure implies that both automation, which squeezes out tasks previously performed by low-skill labor, and the creation of new tasks, which directly benefits high-skill labor, will increase inequality between the two labor types. Nevertheless, we show that the medium-term implications of creation of new tasks could be very different, because these tasks are later standardized and used by low-skill labor. As a result of this effect, we show that there exists a unique balance growth path in which not only the factor distribution of income (between capital and labor) but also inequality between the two skill types is constant.

Our second extension reintroduces the creative destruction of profits of existing technologies by new innovations, which was eliminated by assuming that new technologies have to buy the patents from the technologies on which they are building. This extension shows that the presence of this creative destruction effect has little impact on the balance growth path, but complicates and enriches dynamics.

We consider our paper to be a first step towards a systematic investigation of different types of technological changes that impact capital and labor differentially. Several areas of research appear fruitful based on this first step. First, rather than the reduced-form quasi-labor supply curve, a richer model of the labor market based on search and matching can be introduced and combined with this task-based framework. Such a model is developed in our companion paper, Acemoglu and Restrepo (2015). Second, there may be major differences in the ability of technology to automate and also to create new tasks across industries (e.g., Polanyi, 1966, Autor, Levy and Murnane, 2003). An interesting step is to construct realistic models in which the sectoral composition of the allocation of capital and labor and technological change evolve endogenously and subject to industry-level ecological and automation constraints. Third and perhaps most importantly, our model highlights the need for additional empirical evidence on how automation takes place and incentives for automation and creation of new tasks respond to incentives and policies. One interesting direction would be to construct measures of automation and creation of new tasks, potentially at the industry level, and then exploit industry-level variation in wages and institutional restrictions on capital-labor substitution on technology choices and innovation.

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Appendix: Omitted Proofs and Additional Results

Proof of Proposition 1

We proceed in three steps: first, we show that a given I^*, N and Kuniquely determine r, W, Y. This allows us to define the function $\omega(I^*, N, K)$ introduced in the text as the relative demand for labor. Second, we show that $\omega(I^*, N, K)$ is decreasing in I^* near any equilibrium. Third, we show that $\min\{I, \widetilde{I}\}$ is weakly increasing in ω and conclude that there is a unique pair $\{\omega^*, I^*\}$ such that $I^* = \min\{I, \widetilde{I}\}$ and $\omega^* = \omega(I^*, N, K)$. This pair uniquely determines the equilibrium.

Before proceding with the proof, we establish a useful lemma, which guarantees factor demands are downward slopping for a given output.

Lemma 1 *For all $x > 0$, we have that $c^u(x)^{\zeta-\sigma}x^{-\zeta}$ is decreasing in x and converges to 0 when $x \to \infty$ and to ∞ when $x \to 0$.*

Proof. Differentiating the expression, we find that the elasticity of $c^u(x)^{\zeta-\sigma}x^{-\zeta}$ with respect to x is:

$$\frac{x^{1-\zeta}}{x^{1-\zeta} + \left(\frac{\eta}{1-\eta}\right)^{\zeta} \psi^{1-\zeta}}(\zeta - \sigma) - \zeta < 0,$$

thus establishing the first part of the result.

The fact that this elasticity is negative implies that $c^u(x)^{\zeta-\sigma}x^{-\zeta}$ is decreasing in x and converges to 0 when $x \to \infty$ and to ∞ when $x \to 0$. ■

We now present the three steps mentioned above in detail.

- **Step 1:** Take I^*, N, K as given (with $I^* \in (N-1, N)$). Then, r, W, Y satisfy the system of equations given by the capital and labor demand (equations (7) and (8) in the main text) and the ideal price index (equation (9) in the main text).

  Dividing the labor and capital demand equations yields:

  $$\frac{\int_{I^*}^{N} \gamma(i)^{\zeta-1} c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma} W^{-\zeta} di}{L^s\left(\frac{W}{rK}\right)(I^* - N + 1)c^u(r)^{\zeta-\sigma}r^{-\zeta}} = \frac{1}{K} \tag{A.1}$$

  This equation gives an upward slopping locus between r and W, since lemma 1 implies the left-hand side is decreasing in W and increasing in r.

  On the other hand, the ideal price index condition (equation (9)) gives a downward slopping locus between r and W.

  These observations imply there is at most one interception between the locus determined by equation (A.1) and the ideal price index condition in the (W, r) space. Thus, there is at most one equilibrium presented diagramatically in Figure A.1.

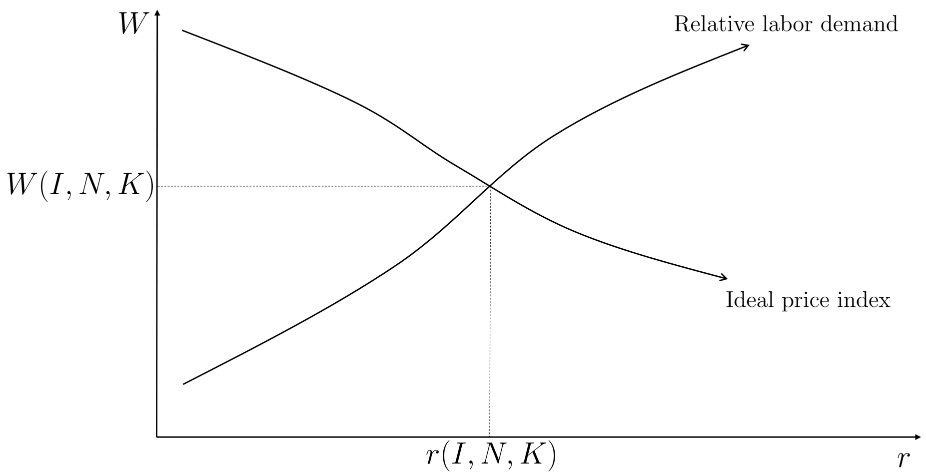

Figure A.1: Construction of function $\omega(I^*, N, K)$.

To prove the existence, note that as $W \to 0$, the ideal price index condition requires a bounded interest rate $r_0 > 0$ to be valid. However, equation (A.1) requires $r \to 0$ to hold, since the demand for labor grows without bound (as stated in Lemma 1). Moreover, as $W \to \infty$, equation (A.1) requires $r \to \infty$ as well, since the demand for labor becomes arbitrarily small (see again Lemma 1). This implies that the relative demand curve and the ideal price index condition intercept at a unique point (W, r) by the intermediate value theorem.

This also implies the equilibrium object $\omega(I^*, N, K) = \frac{W}{rK}$ is well defined.

- **Step 2:** Differentiating the ideal price index condition and equation (A.1), we obtain:

$$d\ln\left(\frac{W}{r}\right) \propto -dI\left(\frac{\gamma(I)^{\zeta-1}c^u\left(\frac{W}{\gamma(I)}\right)^{\zeta-\sigma}}{\int_{I^*}^{N}\gamma(i)^{\zeta-1}c^u\left(\frac{W}{\gamma(i)}\right)^{\zeta-\sigma}di} + \frac{1}{(I^*-N+1)} + \frac{\sigma-\zeta}{1-\sigma}\left(c^u\left(\frac{W}{\gamma(I)}\right)^{1-\sigma} - c^u(r)^{1-\sigma}\right)\right).$$

Since $\frac{W}{\gamma(I)} \geq r$ near any equilibrium, automation reduces W/r, and hence ω as wanted.

- **Step 3:** We have that $\gamma(\widetilde{I}) = \omega K$. Thus, $\widetilde{I}$ is increasing in ω, and so is $\min\{I, \widetilde{I}\}$.

Therefore, there is at most a pair (ω, I^*) solving $\omega = \omega(I^*, N, K)$ and $I^* = \min\{I, \widetilde{I}\}$, as plotted in Figure 7, because $\omega(I^*, N, K)$ is decreasing at any interception.

To prove such pair exists, take $I^* \to N-1$. Then, the locus for $I^* = \min\{I, \widetilde{I}\}$, gives $\omega \to \gamma(N-1)/K$, while the locus for $\omega = \omega(I^*, N, K)$ gives $\omega = \infty$ since $r \to 0$. Likewise, take $I^* \to N$. Then, the locus for $I^* = \min\{I, \widetilde{I}\}$, gives $\omega \to \gamma(N)/K$, while the locus for $\omega = \omega(I^*, N, K)$ gives $\omega = 0$, since wages go to zero. Thus, both curves most cross at some unique point by the intermediate value theorem, establishing Proposition 1.

Proof of Proposition 2

We proceed in four steps: First we prove the comparative statics results for I, then for N and finally for K. In the last step we prove that all the above changes move total employment and the labor share in the same direction.

- **Comparative statics for I:** Clearly I is only binding when $I^* = I$. In this case, an increase in I shifts the curve $I^* = \min\{I, \widetilde{I}\}$ to the right in Figure 7, increasing I^*in the same amount and reducing ω as stated in the proposition.

- **Comparative statics for N:** The same argument used in Step 2 in the proof of Proposition 1 establishes that $\omega(I^*, N, K)$ increases with N near the equilibrium. Thus, N shifts the locus for $\omega = (I^*, N, K)$ upwards in Figure 7, weakly increasing I^* and always increasing ω.

  It also follows that N increases W/r as stated in the proposition.

  When $I^* = I$, a change in N only has a direct effect on ω since it does not change the allocation of factors. Thus

$$\frac{d\ln\omega}{dN} = \frac{d\ln(W/r)}{dN} = \frac{\partial\ln(W/r)}{\partial N} > 0,$$

  and its total effect equal its partial effect.

  When $I^* = \widetilde{I}$, a change in N also increases I^* by $\frac{1}{\varepsilon}d\ln\omega$ (from equation (6)). Therefore, the total change in ω is given by:

$$d\ln\omega = \frac{\partial\ln\omega}{\partial N}dN + \frac{\partial\ln\omega}{\partial I^*}\frac{1}{\varepsilon}d\ln\omega.$$

  Solving for $d\ln\omega$ yields:

$$\frac{d\ln\omega}{dN} = \frac{\frac{\partial\ln\omega}{\partial N}}{1 - \frac{1}{\varepsilon}\frac{\partial\ln\omega}{\partial I^*}}.$$

- **Comparative statics for K:** The definition of ω gives the identity:

$$\frac{d\ln\omega}{d\ln K} = \frac{d\ln(W/r)}{d\ln K} - 1.$$

  Consider an increase in K holding ω fixed — so that we are computing the effect of K on W/r. The increase shifts the locus of points (r, W) satisfying equation (A.1) upwards, so that a given r requires a higher wage to be consistent with market clearing. Therefore, K reduces r and increases W, and

$$\frac{\partial\ln(W/r)}{\partial\ln K} = \frac{1}{\sigma_{SR}} > 0.$$

When $I^* = I$, the partial effect of K equals the total effect since it does not affect I^*. However, when $I^* = \widetilde{I}$, we have

$$d\ln(W/r) = \frac{\partial \ln(W/r)}{\partial \ln K} d\ln K + \frac{\partial \ln(W/r)}{\partial \ln K} \frac{1}{\varepsilon} d\ln(W/r).$$

Solving for $d\ln W/r$ yields:

$$\frac{d\ln(W/r)}{d\ln K} = \frac{\frac{\partial \ln(W/r)}{\partial \ln K}}{1 - \frac{1}{\varepsilon}\frac{\partial \ln(W/r)}{\partial I^*}},$$

as stated in the text.

Proof of Proposition 3

First suppose that $n(t) \to n \in (0,1)$. Then for any initial value of $k(0)$, the economy converges to its unique steady state, which depends only on n. This result can be proved straightforwardly by noting that equations (13), (14) and (15) are essentially identical to the two equations characterizing dynamics in the canonical neoclassical growth model. The condition $A(1-\theta)\Delta < \rho$, the transversality condition is satisfied when k capital converges to a constant (see, for example, Proposition 8.5 and 8.6 in Acemoglu (2009)). This proves part 2 of the proposition.

Since the normalized variables converge, the aggregate variables grow at the same rate as $\gamma(I)$, establishing the "if" direction of part 1.

For the "only if" part, recall that in a balanced growth path, Y, C, K and w grow at a constant rate g. Therefore, y, c, k and w also grow at some constant rate $\widetilde{g}$ and r is constant. We will show that $\widetilde{g} = 0$.

First, suppose by way of contradiction that $\widetilde{g} < 0$. Then w is eventually below r, contradicting the fact that $w \geq r$ (since otherwise, task I^* would be cheaper to produce with labor, contradicting the definition of I^*).

Second, suppose by way of contradiction that $\widetilde{g} > 0$. This implies that, eventually, $\frac{w(t)}{\gamma(n(t))} \geq \frac{w(t)}{\gamma(1)} > r$ (recall r is fixed). When $\frac{w(t)}{\gamma(n(t))} > r$ and r is fixed, the ideal price index condition requires $n(t)$ to decrease over time in order to keep $w(t)$ increasing. However, since $n(t) \geq 0$ this cannot go on indefinitely and $n(t)$ must reach zero. At this point, all tasks are automated and use capital, so the economy converges to an AK economy. Thus, labor is not used along the equilibrium and $w = 0$. However, our assumption that $L(0) > 0$ rules out this equilibrium, contradicting our initial assumption that $\widetilde{g} > 0$.

These contradictions imply $\widetilde{g} = 0$, as wanted. In this case, w is constant, and the ideal price index condition implies $n(t) = n \in (0,1)$ is fixed. Here, $n(t) = 0$ is ruled out as above by noting that $L(0) > 0$. Also, $n(t) = 1$ requires $r(t) = 0$, but $r(t) = \rho + \delta + \theta g > 0$. Finally, since $n(t) = n \in (0,1)$, $\dot{N} = \dot{I}$ as wanted, thus establishing the "only if" direction.

Proof of Proposition 4

We start with two lemmas which we will use throughout the rest of the Appendix.

Lemma 2 (Asymptotic behavior of wages w^{LR}) *There exists a positive threshold $\underline{\rho}$ and $\overline{n}$, such that:*

1. *For $\rho > \underline{\rho}$, we have $I^* = I$ for $n > \overline{n}$, and $I^* < I$ for $n \leq \overline{n}$.*
2. *For $n > \overline{n}$, $w^{LR}(n)$ is increasing and $w^{LR}(n)/\gamma(n)$ decreasing in n.*

Lemma 3 (Asymptotic behavior of value functions V_N, V_I) *Suppose $\sigma > \zeta$. Then, there exist positive thresholds $\underline{\rho}$, and $\overline{S}$, such that:*

1. *For $\rho > \underline{\rho}$, $\overline{S} > S$ and $n \leq \overline{n}$, we have $V_N > 0$. Moreover, as $S \to 0$, $V_I \to 0$ for all $n \leq \overline{n}$.*
2. *For $n > \overline{n}$, both V_N/Y and V_I/Y are increasing in n, and V_N/V_I is (strictly) decreasing in n.*

Using these lemmas, we are in a position to prove Proposition 4. We start by characterizing the existence of a BGP.

Conditions for existence and uniqueness of a balanced growth path

The existence of a balanced growth path follows by noting that, as stated in Proposition 3, it emerges if and only if $\dot{I} = \dot{N}$, and $n(t) = n^D$. Since all scientists are allocated to developing one of the two available technologies, we must have:

$$S_I = \frac{\kappa_N}{\kappa_I \phi(n^D) + \kappa_N} S \qquad\qquad S_N = \frac{\kappa_I \phi(n^D)}{\kappa_I \phi(n^D) + \kappa_N} S,$$

and the growth rate of the economy is therefore $g = \frac{\kappa_N \kappa_I \phi(n^D)}{\kappa_I \phi(n^D) + \kappa_N} S$.

In the balanced growth path, the Euler equation, (13), implies the interest rate equals $r = \rho + \delta + \theta g$,and wages are then given by $w^{LR}(n)$. Moreover, when $\rho > A \frac{\kappa_I \phi(1) \kappa_N}{\kappa_I \phi(1) + \kappa_N} S(1-\theta)$, the transversality condition will be automatically satisfied.

The key equilibrium condition is for research and both types of technologies to be profitable (so that $S_I > 0$ and $S_N > 0$):

$$\kappa_I \phi(n^D) V_I = \kappa_N V_N,$$

Then from Lemma 3, this condition can be rearranged as

$$\frac{\kappa_I}{\kappa_N} \phi(n^D) = \frac{\int_0^\infty e^{-(\rho-(1-\theta)g)\tau} \left(c^u \left(\frac{w^{LR}(n^D) e^{g\tau}}{\gamma(n^D)} \right)^{\zeta-\sigma} - c^u \left(\rho + \delta + \theta g \right)^{\zeta-\sigma} \right) d\tau}{\int_0^\infty e^{-(\rho-(1-\theta)g)\tau} \left(c^u \left(\rho + \delta + \theta g \right)^{\zeta-\sigma} - c^u \left(w^{LR}(n^D) e^{g\tau} \right)^{\zeta-\sigma} \right) d\tau}. \tag{A.2}$$

In the balanced growth path, the right-hand side of equation (A.2) is solely a function of n. Moreover, Lemma 3 implies there is a threshold $\overline{n}$ such that, for $n^D < \overline{n}$, $w^{LR}(n^D) = r$, and for

$n > n^D \geq \overline{n}$, $w^{LR}(n^D)$ is increasing in n^D and $w^{LR}(n^D)/\gamma(n^D)$ is decreasing. Now note that a balanced growth path cannot have $n^D < \overline{n}$, because.... Beyond $\overline{n}$, V_N/V_I (the right-hand side of equation (A.2)) is decreasing in n. This implies that the balanced growth path must be given by the intersection of the locus for V_N/V_I, the solid curve in Figure 9, and the locus for $\frac{\kappa_I}{\kappa_N}\phi(n^D)$. Moreover, since the latter is nondecreasing, that can at most be one intersection, establishing that if a balanced growth path exists, it is also unique. Finally, the conditions in the proposition ensure that this latter locus starts below the curve for V_N/V_I at $n = \overline{n}$ and and above it at $n = 1$, as shown in Figure 9 in the text.

Heuristic argument for stability

We start with the heuristic argument for stability, and then provide the details. Consider an exogenous and instantaneous increase in I (the argument applies similarly for exogenous changes to N), taking the economy out of its balanced growth path.

The instantaneous impact of this change is to reduce w/r (since capital is fixed; see Proposition 2). Since $\sigma > \zeta$, such a change in factor prices *increases* $\pi_N(t, i)$ relative to $\pi_I(t, i)$, providing incentives for the creation of new labor-intensive tasks relative to automation. However, in the long run (or immediately if $\theta \to 0$) capital adjusts to keep the interest rate constant, and creates a force towards greater wage per effective unit of labor in the most complex tasks, reducing incentives for introducing new tasks. Yet it also increases the wage per effective unit of labor paid at the least complex tasks, reducing incentives for automation. The conditions $\rho > \underline{\rho}$ and $\overline{S} > S$ in Lemma 3 guarantees the second effect dominates, and V_N/V_I is higher than its balanced growth path value when n is below n^D, creating an economic force towards returning back to the balanced growth path.

A further intuition can be gained by noting that when $\rho > \underline{\rho}$, the interest rate is close to $w^{LR}(n)$ (or I^* is close to $\widetilde{I}$), so automation does not create a large productivity effect, and does not have a large impact on long-run wages. Consequently, the impact of automation on V_N is small, while its impact on V_I is large because it does significantly reduce the wage per effective unit of labor in the least complex tasks (that is, $w^{LR}(n)$). Hence, automation above its balanced growth path level reduces V_I relative to V_N, ensuring stability.

Global stability with risk neutrality

We next prove the global stability of the balanced growth path when $\theta \to 0$. We continue to assume $\rho > \underline{\rho}$, and also focus on the case in which $S \to 0$. The results then generalize to the case in which $S < \overline{S}$.

In this case,, normalized capital adjusts immediately and only depends on n, which becomes the unique state variable of the model. Moreover, $r = \rho + \delta$ at each point in time, and wages are given by $w^{LR}(n)$, as defined in the text in the case $\theta = 0$. Moreover, define $v = \kappa_I \phi(n(t)) V_I/Y - \kappa_N V_N/Y$.

In this case, starting from any $n(0)$ the equilibrium path with endogenous technology is given by a tuple $\{n(t), S_I(t)\}_{t=0}^{\infty}$ such that:

- The evolution of the state variable is given by

$$\dot{n}(t) \quad = \quad \kappa_N S - (\kappa_N + \kappa_I) S_I(t).$$

- The allocation of scientists satisfies:

$$S_I(t) = \begin{cases} 0 \text{ if } v < 0 \\ \in [0, S] \text{ if } v = 0 \\ S \text{ if } v > 0 \end{cases},$$

  with v satisfying the forward looking differential equation:

$$rv - \dot{v} = \kappa_I \phi(n(t)) \left(c^u (\rho + \delta)^{\zeta - \sigma} - c^u \left(w^{LR}(n) \right)^{\zeta - \sigma} \right) - \kappa_N \left(c^u \left(\frac{w^{LR}(n)}{\gamma(n)} \right)^{\zeta - \sigma} - c^u (\rho + \delta)^{\zeta - \sigma} \right).$$

  This expression is derived for the limit in which $g \to 0$, which is implied by our assumption that $S \to 0$.

Suppose n^D is the unique equilibrium plotted in Figure 9. We now prove it is globally stable. Figure A.2 presents the phase diagram of the system in (v, n). Importantly, the locus for $\dot{v} = 0$ crosses $v = 0$ at n^D from below only once. This follows from the fact that V_N/V_I is decreasing in n as established in the Lemma 3. Thus, n^D is saddle path stable, and for each $n(0)$ there is a unique $v(0)$ in the stable arm of the system.

In order to show all equilibria must be along the stable arm, we need to rule out other paths. From the figure it is clear that either the equilibrium settles at n^D, or it reaches the region with $\dot{v} > 0$ and $\dot{n} < 0$, or the region with $\dot{v} < 0$ and $\dot{n} > 0$. In the first case, v is strictly increasing and n is strictly decreasing, and hence there are no interior limit points. Moreover, n cannot cross $\overline{n}$ because in this region we have $\dot{n} > 0$ (there are no incentives for automation). This implies $v \to \infty$ along any such path, which violates the transversality condition for households entitled to profits from automation. In the second case, $v \to -\infty$ and $n \to 1$; which again violates the transversality condition for households entitled to profits from the creation of new tasks.

A.0.0.1 Local stability in the general case

Finally, we establish local saddle path stability in the general case with $\theta > 0$. As above, we work with the limit $S \to 0$, and then generalize the results to the case in which $S < \overline{S}$.

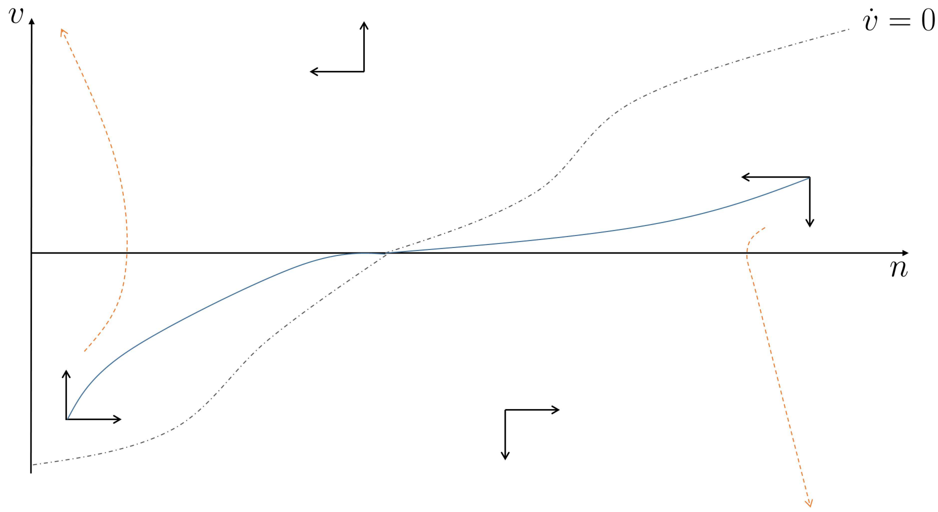

Figure A.2: Phase diagram and global saddle path stability when $\theta = 0$.

Let $\pi_N(n,k) = c^u \left(\frac{w^E(n,k)}{\gamma(n)}\right)^{\zeta-\sigma} - c^u \left(r^E(n,k)\right)^{\zeta-\sigma}$ and $\pi_I(n,k) = c^u \left(r^E(n,k)\right)^{\zeta-\sigma} - c^u \left(w^E(n,k)\right)^{\zeta-\sigma}$ be the flow profits for innovators. Also, let $-Q_k = \kappa_N \frac{\partial \pi_N}{\partial k} - \kappa_I \phi(n^D) \frac{\partial \pi_I}{\partial k}$ and $-Q_n = \kappa_N \frac{\partial \pi_N}{\partial n} - \kappa_I \phi(n^D) \frac{\partial \pi_I}{\partial n} - \kappa_I \phi'^D) \pi_I$ evaluated at their balanced growth path values. Using this notation and applying a first-order Taylor expansion, the equilibrium conditions for v, c, k, n can be expressed around their balance growth path values as

$$
\begin{aligned}
\dot{n} =& -Q_v v \\
\dot{v} =& rv - Q_k[k(t) - k^D] - Q_n[n(t) - n^D], \\
\dot{c} =& \frac{\theta}{c^D} r_n^E [n(t) - n^D] + \frac{\theta}{c^D} r_k^E [k(t) - k^D] \\
\dot{k} =& f_n^E [n(t) - n^D] + (f_k^E - \delta)[k(t) - k^D] - c.
\end{aligned}
$$

Here, $Q_v > 0$ is a constant capturing the fact that $\dot{n}$ changes discontinuously at $v = 0$, with $\dot{n} > 0$ if $v < 0$ and $\dot{n} < 0$ if $v > 0$. As $Q_v \to \infty$, the above system approximates the local behavior of the dynamic economy near the steady state.

The characteristic polynomial of this system of differential equations (with all derivatives still evaluated at their balance growth path values) can be written as

$$
P(\lambda) = \left| \begin{pmatrix} -\lambda & -Q_v & 0 & 0 \\ -Q_n & r - \lambda & 0 & -Q_k \\ \frac{\theta}{c^D} r_n^E & 0 & -\lambda & \frac{\theta}{c^D} r_k^E \\ f_n^E & 0 & -1 & f_k^E - \delta - \lambda \end{pmatrix} \right|,
$$

or expanding it:

$$P(\lambda) = \lambda^4 - \lambda^3(f_k^E - \delta + r) + \lambda^2(-Q_v Q_n + \frac{\theta}{c^D} r_k^E + r(f_k^E - \delta))$$
$$- \lambda(Q_v(f_n^E Q_k - (f_k^E - \delta)Q_n) + r\frac{\theta}{c^D} r_k^E) + Q_v(r_n^E Q_k - r_k^E Q_n).$$

Let $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ be the eigenvalues of the above system. Then $\lambda_1\lambda_2\lambda_3\lambda_4 = Q_v(r_n^E Q_k - r_k^E Q_n) > 0$. Lemma 3 implies that $r_n^E Q_k - r_k^E Q_n > 0$, so that an increase in n reduces $\kappa_N \pi_N - \kappa_I \phi(n)\pi_I$ near the steady state if capital adjusts immediately to keep the interest rate constant. The fact that $r_n^E Q_k - r_k^E Q_n > 0$ is the same force exploited in the stability of the balance growth path in the case where $\theta \to 0$. In particular, once again V_N/V_I is decreasing in n in the neighborhood of the balanced growth path. Thus, either two eigenvalues are positive and two are negative, or all have the same sign.

All eigenvalues cannot be negative either, since their sum is $f_k^E - \delta + r > 0$ (this is the trace of the system matrix). The last inequality follows by noting that $r > \delta$ (since $r = \rho + \delta + \theta g$).

Finally, all eigenvalues cannot be positive. This follows by noting that,

$$\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4 = -Q_v Q_n + \frac{\theta}{c^D} r_k^E + r(f_k^E - \delta),$$

and the right hand side is negative for $Q_v \to \infty$ (since $Q_n > 0$, capturing the economic force that n reduces incentives for creation of new tasks relative to automation holding capital constant).

Overall, this line of argument implies that the system has two positive and two negative eigenvalues, and since there are two state variables (k and n), it is locally saddle path stable.

Proof of Proposition 5

we start by providing formulas for F_N^P and F_I^P. These are given by

$$F_N^P = \frac{1}{\sigma - 1} Y c^u \left(\frac{w^P}{\gamma(n)}\right)^{\zeta - \sigma} \left(\sigma \left(\frac{w^P}{\gamma(n)}\right)^{1-\zeta} + (\mu\psi)^{1-\zeta} \left(\frac{\eta}{1-\eta}\right)^{\zeta}\right)$$
$$- \frac{1}{\sigma - 1} Y c^u \left(r^P\right)^{\zeta - \sigma} \left(\sigma r^{P 1-\zeta} + (\mu\psi)^{1-\zeta} \left(\frac{\eta}{1-\eta}\right)^{\zeta}\right)$$
$$F_I^P = \frac{1}{\sigma - 1} Y c^u \left(r^P\right)^{\zeta - \sigma} \left(\sigma r^{P 1-\zeta} + (\mu\psi)^{1-\zeta} \left(\frac{\eta}{1-\eta}\right)^{\zeta}\right)$$
$$- \frac{1}{\sigma - 1} Y c^u \left(w^P\right)^{\zeta - \sigma} \left(\sigma \left(w^P\right)^{1-\zeta} + (\mu\psi)^{1-\zeta} \left(\frac{\eta}{1-\eta}\right)^{\zeta}\right).$$

Using these formulas, we establish the decentralization result by construction.

First, assume the planner subsidizes a fraction $1-\mu$ to the price of intermediate goods, and sets a tax/subsidy to capital savings of $\tau_k = \omega^P L^s \frac{\partial \ln L^s}{\partial \ln \omega} \frac{\partial \ln \omega^P}{\partial \ln K}$. This guarantees households discount future income at a rate $r^P - \delta - r^P \omega^P L_\omega^s \omega_K^P$, which coincides with the planner's discount rate.

Absent the flow subsidies/taxes for successful innovators, the value of automating jobs or creating new tasks are given by a small modification of equations (22) and (23), which take into account that firms sell intermediates at a price ψ, but buyers perceive a price $\mu\psi$ because of the subsidy. These values also discount future profits as the same rate the planner does, because of the taxes/subsidies to capital accumulation. Thus:

$$V_I(t) =(1-\mu)\psi\left(\frac{\eta}{1-\eta}\right)^{\zeta}(\mu\psi)^{-\zeta}\int_t^{\infty} e^{-\int_t^{\tau}(r^P-\delta+W^P L^s_{\omega}\omega^P_K)ds}Y(\tau)\left(c^u\left(r^P(\tau)\right)^{\zeta-\sigma}-c^u\left(w^P(\tau)\frac{\gamma(I(\tau))}{\gamma(I(t))}\right)^{\zeta-\sigma}\right)d\tau,$$

$$V_N(t) =(1-\mu)\psi\left(\frac{\eta}{1-\eta}\right)^{\zeta}(\mu\psi)^{-\zeta}\int_t^{\infty} e^{-\int_t^{\tau}(r^P-\delta+W^P L^s_{\omega}\omega^P_K)ds}Y(\tau)\left(c^u\left(\frac{w^P(\tau)}{\gamma(n(t))}\frac{\gamma(I(\tau))}{\gamma(I(t))}\right)^{\zeta-\sigma}-c^u\left(r^P(\tau)\right)^{\zeta-\sigma}\right)d\tau.$$

Now, we can define the flow subsidies/taxes for successful innovators as follows. First, we have a component to adjust for the appropriability problems, $\tau^A_N(t),\tau^A_I(t)$. These are given by:

$$\begin{aligned}\tau^A_N(t) =& \frac{1}{\sigma-1}Yc^u\left(\frac{w^P}{\gamma(n)}\right)^{\zeta-\sigma}\left(\sigma\left(\frac{w^P}{\gamma(n)}\right)^{1-\zeta}+(\mu\psi)^{1-\zeta}\left(\frac{\eta}{1-\eta}\right)^{\zeta}\left(1-(\sigma-1)\frac{\mu}{1-\mu}\right)\right)\\ &-\frac{1}{\sigma-1}Yc^u\left(r^P\right)^{\zeta-\sigma}\left(\sigma r^{P1-\zeta}+(\mu\psi)^{1-\zeta}\left(\frac{\eta}{1-\eta}\right)^{\zeta}\left(1-(\sigma-1)\frac{\mu}{1-\mu}\right)\right)\\ \tau^A_I =& \frac{1}{\sigma-1}Yc^u\left(r^P\right)^{\zeta-\sigma}\left(\sigma r^{P1-\zeta}+(\mu\psi)^{1-\zeta}\left(\frac{\eta}{1-\eta}\right)^{\zeta}\left(1-(\sigma-1)\frac{\mu}{1-\mu}\right)\right)\\ &-\frac{1}{\sigma-1}Yc^u\left(w^P\right)^{\zeta-\sigma}\left(\sigma\left(w^P\right)^{1-\zeta}+(\mu\psi)^{1-\zeta}\left(\frac{\eta}{1-\eta}\right)^{\zeta}\left(1-(\sigma-1)\frac{\mu}{1-\mu}\right)\right).\end{aligned}$$

The wedge between the private and social values of innovation captured by $\tau^A_N(t),\tau^A_I(t)$ is well known, and arises because monopolists cannot extract the full value of introducing new tasks in models of expanding varieties. Here, this is the case for monopolists automating jobs or creating new tasks, so the taxes/subsidies $\tau^A_N(t),\tau^A_I(t)$ have ambiguous signs and orderings.

Second, $\tau^T_N(t_0,t),\tau^T_I(t_0,t)$, for $t\geq t_0$, are given by:

$$\begin{aligned}\tau^T_N(t_0,t) =&(1-\mu)\psi\left(\frac{\eta}{1-\eta}\right)^{\zeta}(\mu\psi)^{-\zeta}\left[c^u\left(\frac{w^S(t)}{\gamma(n(t))}\right)^{\zeta-\sigma}-c^u\left(\frac{w^S(t)}{\gamma(n(t))}\frac{\gamma(I(t))}{\gamma(I(t_0))}\right)^{\zeta-\sigma}\right]\geq 0\\ \tau^T_I(t_0,t) =&(1-\mu)\psi\left(\frac{\eta}{1-\eta}\right)^{\zeta}(\mu\psi)^{-\zeta}\left[c^u\left(w^S(t)\frac{\gamma(I(t))}{\gamma(I(t_0))}\right)^{\zeta-\sigma}-c^u\left(w^S(t)\right)^{\zeta-\sigma}\right]\leq 0.\end{aligned}$$

$\tau^T_N(t_0,t)\geq 0$ and $\tau^T_I(t_0,t)\leq 0$ correct for a *technological externality*: by inventing new tasks and increasing N, monopolists improve the quality of intermediates that future entrants will develop. The opposite occurs for automation: by automating task I, new entrants will be forced to automate more complex tasks, receiving fewer profits. These taxes/subsidies, depend on the time at which a task was introduced t_0— since they are a compensation (or charge) for all technologies built on top of them.

Third, the planner should add subsidies $\Psi_I \phi'(N-I) \geq 0$ to monopolists introducing new tasks and a tax $\Psi_I \phi'(N-I) \leq 0$ to monopolists automating tasks. These subsidies and taxes simply correct for the spillovers captured by the function ϕ.

Finally, τ_N^W and τ_I^W correct for the fact that technology monopolists do not take into account the effect of technologies on the quasi-supply of labor.

It is straightforward to verify that once we add these flow subsidies/taxes to the private profits from developing new technologies, we obtain these become Ψ_N and Ψ_I, establishing the decentralization result.

Notice that the scientist allocation can be decentralized in many ways. In particular, since there is a fixed supply of scientists, we only need to get the *relative* expected profits from each type of innovation right. The particular decentralization outlined here guarantees the level of innovators' profits also matches the social value of innovation. Even if both types of technology end up being subsidized in equilibrium, this does not matter because the money can be recovered by taxing scientists.

Proof of Proposition 6

Let $S_I^D(t)$ and $S_N^D(t)$ denote the allocation of scientists, and consider the allocation obtained by a small deviation $S_N^P(t) = \min\{S_N^D(t) + \epsilon, 0\}$ and $S_I^P(t) = \max\{S_I^D(t) - \epsilon, 0\}$ if $S_I^D < 1$, and $S_N^P(t) = S_N^D(t)$, $S_I^P(t) = S_I^D(t)$ otherwise. We prove in the appendix that for a small $\epsilon > 0$, such deviation increases welfare and reduces the extent of automation.

Clearly, the new allocation satisfies $n^P(t) \geq n^D(t)$ as wanted. For ϵ small enough, we have that the above allocation changes welfare by $\epsilon(\kappa_N \mu_N - \kappa_I \mu_I)$, whenever $S_I^D(t), S_N^D(t) \in (0,1)$. Moreover, in these cases $\kappa_N V_N(t) = \kappa_I V_I(t)$.

Thus, to prove our deviation increases welfare, it is enough to verify $\kappa_N \mu_N - \kappa_I \mu_I > 0$ whenever $\kappa_N V_N(t) = \kappa_I V_I(t)$. In fact, we prove the stronger statement, that at all points in time $\frac{\Psi_N(t)}{\Psi_I(t)} > \frac{V_N(t)}{V_I(t)}$.

Notice that, as $\epsilon \to 0$, we are along the market allocation. Thus, we can compute Ψ_N and Ψ_I as:

$$\Psi_N(t) = \int_t^\infty e^{-\int_0^\tau (r^P(n^D(s),k^D(s))-\delta)ds} \left(\frac{\sigma + \frac{\eta}{1-\eta}}{\sigma - 1} Y(\tau) \left[c^u \left(\frac{w^P(\tau)}{\gamma(n(\tau))} \right)^{\zeta-\sigma} - c^u \left(r^P(\tau) \right)^{\zeta-\sigma} \right] + W^P L_\omega^S \omega_N^P + \Psi_I \phi'^D(\tau)) \right) d\tau,$$

$$\Psi_I(t) = \int_t^\infty e^{-\int_0^\tau (r^P(n^D(s),k^D(s))-\delta)ds} \left(\frac{\sigma + \frac{\eta}{1-\eta}}{\sigma - 1} Y(\tau) \left[c^u \left(r^P(\tau) \right)^{\zeta-\sigma} - c^u \left(w^P(\tau) \right)^{\zeta-\sigma} \right] + W^P L_\omega^S \omega_I^E - \Psi_I \phi'(\tau)) \right) d\tau.$$

However, this implies the inequalities:

$$
\begin{aligned}
\frac{\Psi_N(t)}{\Psi_I(t)} &= \frac{\int_t^\infty e^{-\int_0^\tau (r^P(n^D(s),k^D(s))-\delta)ds}\left(\frac{\sigma+\frac{\eta}{1-\eta}}{\sigma-1}Y(\tau)\left[c^u\left(\frac{w^P(\tau)}{\gamma(n(\tau))}\right)^{\zeta-\sigma}-c^u\left(r^P(\tau)\right)^{\zeta-\sigma}\right]+W^P L_\omega^S \omega_N^P+\Psi_I\phi'\right)d\tau}{\int_t^\infty e^{-\int_0^\tau (r^P(n^D(s),k^D(s))-\delta)ds}\left(\frac{\sigma+\frac{\eta}{1-\eta}}{\sigma-1}Y(\tau)\left[c^u\left(r^P(\tau)\right)^{\zeta-\sigma}-c^u\left(w^P(\tau)\right)^{\zeta-\sigma}\right]+W^P L_\omega^S \omega_I^E-\Psi_I\phi'\right)d\tau} \\
&> \frac{\int_t^\infty e^{-\int_0^\tau (r^P(n^D(s),k^D(s))-\delta)ds}\left(\frac{\sigma+\frac{\eta}{1-\eta}}{\sigma-1}Y(\tau)\left[c^u\left(\frac{w^P(\tau)}{\gamma(n(\tau))}\right)^{\zeta-\sigma}-c^u\left(r^P(\tau)\right)^{\zeta-\sigma}\right]+W^P L_\omega^S \omega_N^P\right)d\tau}{\int_t^\infty e^{-\int_0^\tau (r^P(n^D(s),k^D(s))-\delta)ds}\left(\frac{\sigma+\frac{\eta}{1-\eta}}{\sigma-1}Y(\tau)\left[c^u\left(r^P(\tau)\right)^{\zeta-\sigma}-c^u\left(w^P(\tau)\right)^{\zeta-\sigma}\right]+W^P L_\omega^S \omega_I^E\right)d\tau} \\
&> \frac{\int_t^\infty e^{-\int_0^\tau (r^P(n^D(s),k^D(s))-\delta)ds}\left(\frac{\sigma+\frac{\eta}{1-\eta}}{\sigma-1}Y(\tau)\left[c^u\left(\frac{w^P(\tau)}{\gamma(n(\tau))}\gamma(I(\tau-t))\right)^{\zeta-\sigma}-c^u\left(r^P(\tau)\right)^{\zeta-\sigma}\right]+W^P L_\omega^S \omega_N^P\right)d\tau}{\int_t^\infty e^{-\int_0^\tau (r^P(n^D(s),k^D(s))-\delta)ds}\left(\frac{\sigma+\frac{\eta}{1-\eta}}{\sigma-1}Y(\tau)\left[c^u\left(r^P(\tau)\right)^{\zeta-\sigma}-c^u\left(w^P(\tau)\gamma(I(\tau-t))\right)^{\zeta-\sigma}\right]+W^P L_\omega^S \omega_I^E\right)d\tau} \\
&\geq \frac{\int_t^\infty e^{-\int_0^\tau (r^P(n^D(s),k^D(s))-\delta)ds}\frac{\sigma+\frac{\eta}{1-\eta}}{\sigma-1}Y(\tau)\left[c^u\left(\frac{w^P(\tau)}{\gamma(n(\tau))\gamma(I(\tau-t))}\right)^{\zeta-\sigma}-c^u\left(r^P(\tau)\right)^{\zeta-\sigma}\right]d\tau}{\int_t^\infty e^{-\int_0^\tau (r^P(n^D(s),k^D(s))-\delta)ds}\frac{\sigma+\frac{\eta}{1-\eta}}{\sigma-1}Y(\tau)\left[c^u\left(r^P(\tau)\right)^{\zeta-\sigma}-c^u\left(w^P(\tau)\gamma(I(\tau-t))\right)^{\zeta-\sigma}\right]d\tau} \\
&= \frac{V_N(t)}{V_I(t)},
\end{aligned}
$$

as wanted.

The first inequality results from the spillovers. The second from the technological externality; which as explained above pushes towards the underprovision of new tasks. The third inequality results from the novel inefficiency underscored in this paper: the fact that labor gets rents in equilibrium pushes towards the underprovision of new tasks and excessive automation. This inequality is strict whenever $L_\omega^s > 0$— that is, labor gets rents.