Does the New Keynesian Model Have a Uniqueness Problem?

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Introduction

- New Keynesian (NK) model has played an important role in thinking about the causes of the Great Recession, as well as possible remedies.
  - Eggertsson and Woodford: collision between reduced spending and zero lower bound as cause of Great Recession.
  - Forward guidance.
  - Government spending multiplier.
- It has been discovered that the NK model has multiple rational expectations equilibria.
  - Benhabib, Schmitt-Grohe, Uribe (2000), Mertens and Ravn (2014), Braun, Körber, Yuichiro Waki (2014).
  - The different equilibria have very different implications for the Great Recession and for policy.

What We Do

- We study multiplicity properties of NK model.
  - Interested in ZLB and non-ZLB.
  - For today, we report results for ZLB.
- Describe the multiplicity problem in ZLB.
- Study usefulness of learnability as an equilibrium selection device.

Findings for Linearized Equilibrium Conditions in ZLB

- Analysis based on linearized equilibrium conditions
  - equilibrium unique and gov't spending multiplier big.

- Size of multiplier and drop in GDP in ZLB:
  - Bigger the more flexible are prices and the longer the expected duration of ZLB.

- Linearization appealing because results are analytic.
  - But, linearization may be misleading.

Findings Based on Actual Equilibrium Conditions in ZLB

- Two equilibria: **not-so-bad** and **really-bad**
- Not so bad: resembles equilibrium identified by linearization.
  - Government spending multipler big
  - Size of multiplier and drop in GDP in ZLB:
    - Bigger the more flexible are prices and the longer the expected duration of ZLB.
- Really bad:
  - *Huge* output drop.
  - Properties of equilibrium reversed.
  - Size of multiplier and drop in GDP in ZLB:
    - *Smaller* the more flexible are prices and the longer the expected duration of ZLB.

Learning

- Not-so-bad equilibrium: stable under learning.
- Really bad equilibrium: not stable under learning.

Outline

- Properties of Rational Expectation Equilibrium.
- Simple example to illustrate learning as an equilibrium selection device.
  - Laffer curve.
- Learning in the New Keynesian model.

Model

- Standard NK model
  - Representative household,
  - Monopolistically competitive firms face price-adjustment costs (Rotemberg, 1982),
  - Government.

- Results based on non-linear analysis of Calvo-pricing model.
  - Very similar conclusions.

Model

- A representative household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t\right) - \frac{\chi}{2} h_t^2 \right]$$

subject to

$$P_t C_t + B_t \leq \left(1 + R_{t-1}\right) B_{t-1} + W_t h_t + \Pi_t,$$

- Aggregate output, Y_t, is produced by representative, competitive final good producer using intermediate goods, Y_{jt}, $j \in [0, 1]$.

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \ \varepsilon \geq 1,$$

Model

- The monopolist that produces the j^{th} good has the following objective:

$$E_t \sum_{l=0}^{\infty} \beta^l \upsilon_{t+l} [(1+\nu) P_{j,t+l} Y_{j,t+l} - \overbrace{s_{t+l} P_{t+l} Y_{j,t+l}}^{\text{labor costs of production}}$$

$$- \overbrace{\frac{\phi}{2}\left(\frac{P_{j,t+l}}{P_{j,t+l-1}} - 1\right)^2 (C_{t+l} + G_{t+l})}^{\substack{\text{cost (in terms of final goods) of adjusting prices} \\ \text{related to aggregate level of output}}} \times P_{t+l}],$$

- υ_t: state and date-contingent value assigned to payments sent to households.

- ν: subsidy to firms to address distortions due to monopoly power.

Equations Defining a RE Equilibrium

$$
\begin{aligned}
R_t &= \max\left\{1, \frac{1}{\beta} + \alpha(\pi_t - 1)\right\} \\
\frac{1}{R_t} &= \frac{1}{1+r} E_t \frac{C_t}{C_{t+1}\pi_{t+1}} \text{ where } \beta = 1/(1+r) \\
(\pi_t - 1)\pi_t &= \frac{1}{\phi}\epsilon(s_t - 1)\frac{Y_t}{C_t + G_t} \\
&\quad + \frac{1}{1+r} E_t(\pi_{t+1} - 1)\pi_{t+1}\frac{C_{t+1} + G_{t+1}}{C_{t+1}}\frac{C_t}{C_t + G_t} \\
Y_t &= h_t = C_t + G_t + \frac{\phi}{2}(\pi_t - 1)^2(C_t + G_t)
\end{aligned}
$$

The ZLB

- As in Eggertsson and Woodford (2003), we assume:
  - $r = r^\ell \leq r^h$ at time zero
  - r jumps to $r^h > 0$ with probability $1 - p$
  - r^h is an absorbing state.

- We assume that agents expect equilibrium C and π return to zero-inflation SS when $r = r^h$.

- As in EW, focus on equilibria in which

$$C_t = C^\ell,\ \pi_t = \pi^\ell,$$

for all t while ZLB lasts.

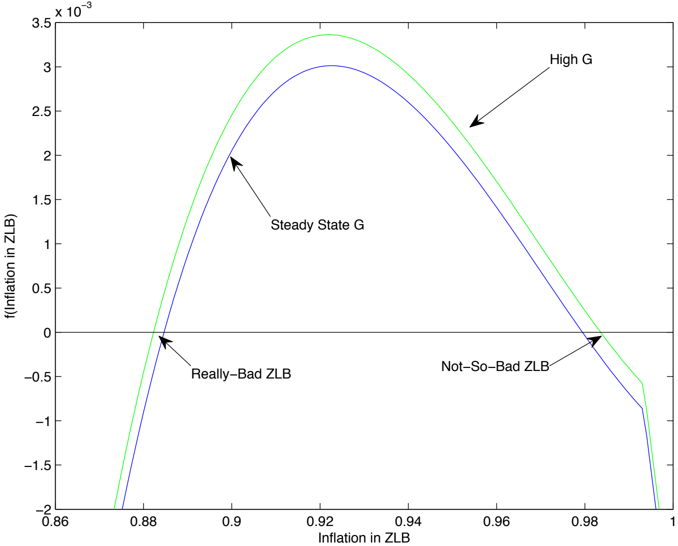
The ZLB

- Phillips curve, resource constraint and intertemporal Euler equation collapse into one equation in one unknown, π^ℓ :

$$f(\pi^\ell) = 0$$

- Function, f, has inverted U shape on set of potential equilibrium values of π^ℓ.
  - Implication: generically, either there is no equilibrium, or two.

Inflation at the ZLB



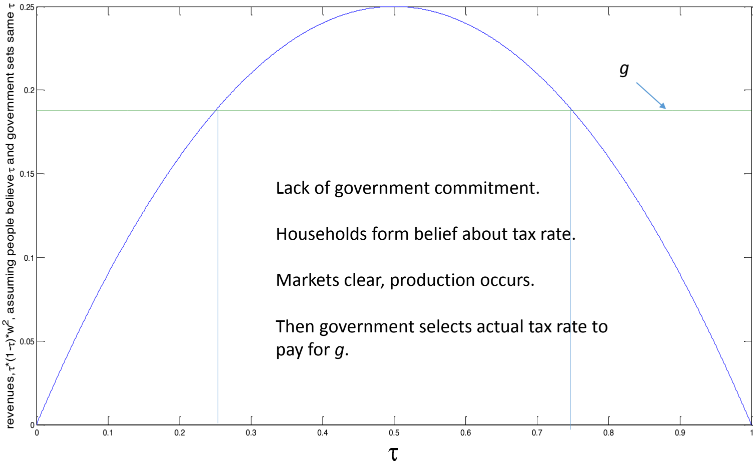
Numerical Results for Two Equilibria

	Really-Bad ZLB	Not-So-Bad ZLB	Log-Linear
Multiplier	0.26	2.36	2.77
% Drop in GDP	37.55	5.38	5.99
Drop in Inflation Rate	11.77	1.64	1.90

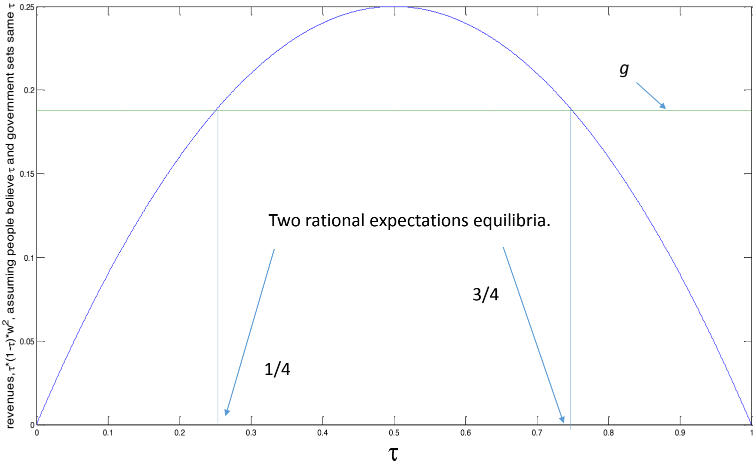
The Not-So-Bad ZLB looks like the log-linear approximation

The Really-Bad ZLB is completely different!

Laffer Curve Model



Laffer Curve Model



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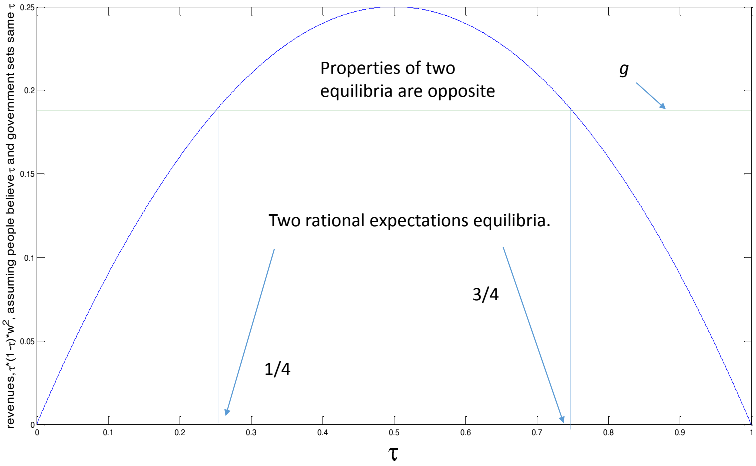

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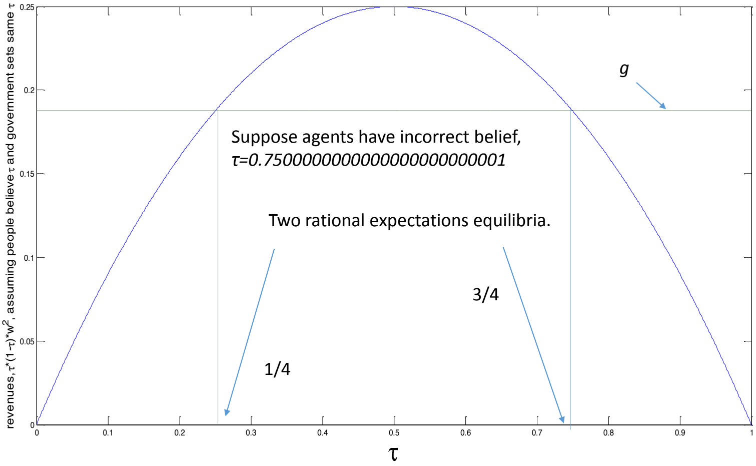

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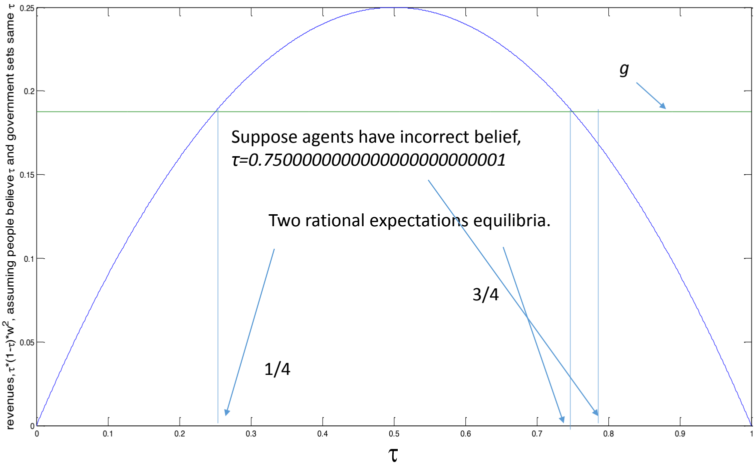
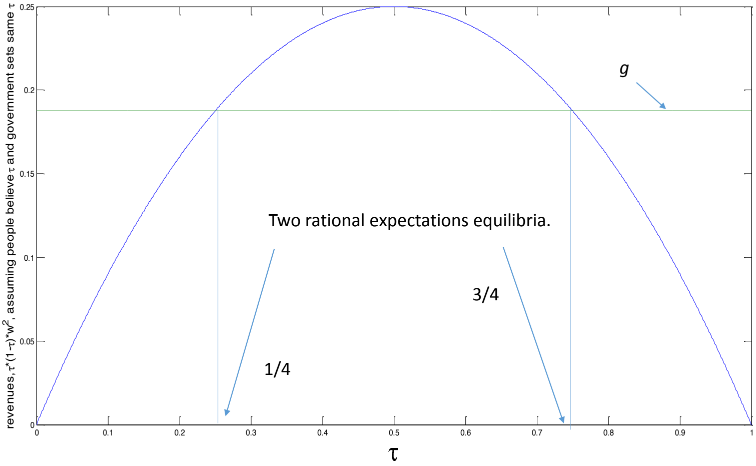


Laffer Curve Model



Learning in the New Keynesian Model

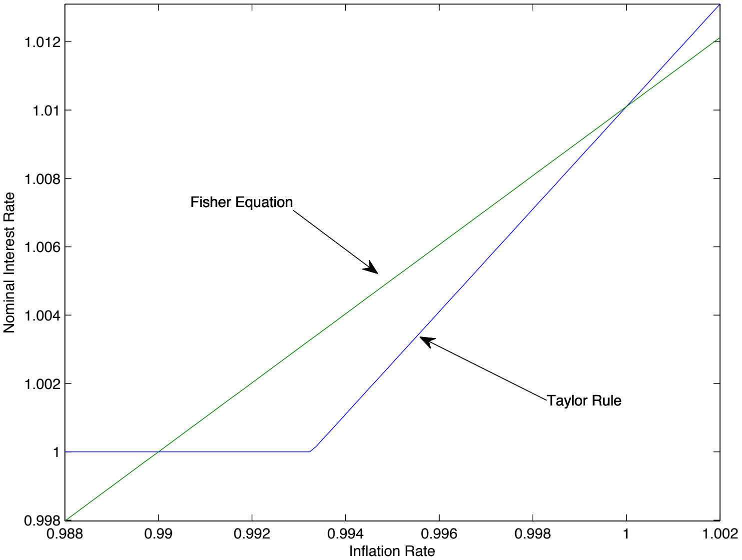
- Firms need to have expectations about variables beyond their control.
- Current period variables.
  - Firms do not observe actual aggregate price index at the time they choose their price.
    - 'Our learning' versus 'Evans-Honkapohja' learning.
  - Current aggregate output, consumption and price (inflation):

$$\begin{aligned} x_t^e &= \omega x_{t-1} + (1-\omega)\, x_{t-1}^e \\ &\omega \text{ ~ gain parameter} \end{aligned}$$

- Future Variables while in ZLB:

$$x_{t+1}^e = x_t^e.$$

Future Variables out of ZLB



Learning in the New Keynesian Model

- Problem:

$$E_t \sum_{l=0}^{\infty} \beta^l v_{t+l}[(1+\nu)\, P_{j,t+l} Y_{j,t+l} - s_{t+l} P_{t+l} Y_{j,t+l}$$

$$-\frac{\phi}{2}\left(\frac{P_{j,t+l}}{P_{j,t+l-1}} - 1\right)^2 (C_{t+l} + G_{t+l}) \times P_{t+l}],$$

- First order condition:

$$(1+\nu)\,\frac{P_{j,t}}{P_t^e} = \frac{\varepsilon}{\varepsilon-1}\chi h_t^e C_t^e +$$

$$\phi\frac{1}{\varepsilon-1}\left(\frac{P_{j,t}}{P_t^e}\right)^{\varepsilon}\frac{C_t^e}{Y_t^e}[-\left(\frac{P_{j,t}}{P_{j,t-1}} - 1\right)\frac{P_{j,t}}{P_{j,t-1}}\frac{(C_t^e + \psi G_t^e)}{C_t^e}$$

$$+\frac{p}{1+r^l}\left(\left(\frac{P_{j,t+1}}{P_{j,t}}\right)^e - 1\right)\left(\frac{P_{j,t+1}}{P_{j,t}}\right)^e\left(\frac{C_{t+1}^e + \psi G_{t+1}}{C_{t+1}^e}\right)]$$

Equilibrium Conditions, Learning

- Phillips curve

$$0 = \left[(1+\nu)(1-\varepsilon) \left(\frac{\pi_t^l}{\pi_t^e} \right)^{-1-\varepsilon} + \varepsilon \chi h_t^e C_t^e \left(\frac{\pi_t^l}{\pi_t^e} \right)^{-\varepsilon-2} \right] \frac{Y_t^e}{C_t^e}$$

$$-\phi \left(\pi_t^l - 1 \right) \pi_t^e \frac{(C_t^e + G_t^e)}{C_t^e} + \frac{p \phi \pi_t^e}{1+r^l} \left(\pi_t^l - 1 \right) \left(\frac{C_{t+1}^e + G_{t+1}}{C_{t+1}^e} \right)$$

- Household intertemporal Euler equation:

$$1 = \frac{1}{1+r^l} \left[p \frac{C_t^l}{\left(C_{t+1}^l\right)^e \left(\pi_{t+1}^l\right)^e} + (1-p) \frac{C_t^l}{C^h} \right]$$

- Resource constraint

$$h_t^l = C_t^l + G_t^l + \frac{\phi}{2} \left(\pi_t^l - 1 \right)^2$$

Compact Representation of Equilibrium Conditions and Learning

$$z_t = \begin{pmatrix} C_t^l \\ h_t^l \\ \pi_t^l \end{pmatrix},\ z_t^e = \begin{pmatrix} \left(C_t^l\right)^e \\ \left(h_t^l\right)^e \\ \left(\pi_t^l\right)^e \end{pmatrix}$$

$$\begin{pmatrix} z_t \\ z_t^e \end{pmatrix} = f \begin{pmatrix} z_{t-1} \\ z_{t-1}^e \end{pmatrix}.$$

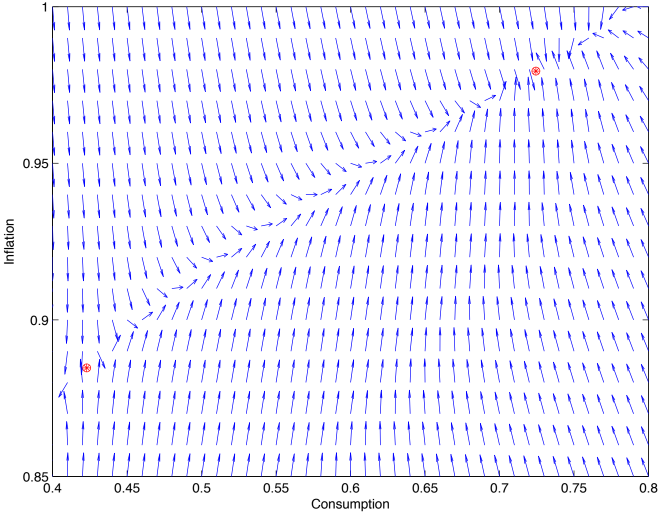
Parameterizing the model

$$
\begin{aligned}
G &= 0.20,\ \beta = 0.99,\ \frac{\varepsilon - 1}{\phi} = 0.02,\ \phi = 100, \\
\chi &= 1.25,\ \omega = 0.75,\ Y = h = 1.
\end{aligned}
$$

Stability and the ZLB

- If we start near the not-so-bad ZLB, we *converge* back to it.
  - There exist expectational points far from not-so-bad ZLB from which we diverge to negative consumption.
- Not-so-bad ZLB is stable.
- If we start near the really-bad ZLB, we either converge to the not-so-bad ZLB (generic case) or we diverge to negative consumption.
- Conclude: really-bad ZLB is not stable, not-so-bad ZLB is stable.

Alternative initial expectations



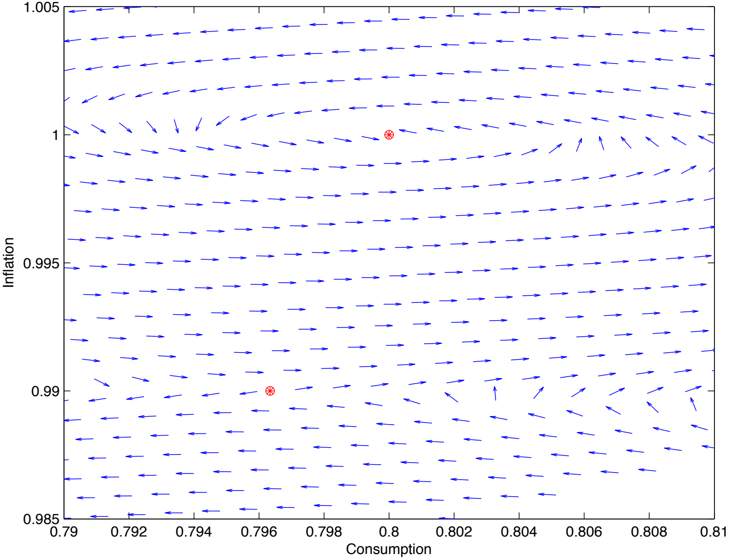
Benhabib, Schmitt-Grohe and Uribe

- Previous computations assume people expect to go to zero inflation steady state.

- What if they are slightly wrong about the equilibrium quantities in the steady state?

- Next, show:
  - zero inflation steady state stable under learning,
  - BSGU zero interest rate steady state not stable under learning.
  - See also Evans, Guse and Honkapohja (EER, 2008) and Benhabib, Evans and Honkapohja (2014).

Stability analysis



Robustness

- None of our results regarding stability are affected by
  - whether we do E-H or our learning,
  - which values of $\omega > 0$ that we use.

Learning Dynamics

- We have established that learning allows one to select a rational expectation equilibrium for ZLB analysis.
  - So, if you're into rational expectations, we're done.

- But, in the Great Recession, learning dynamics may be more interesting than rational expectations dynamics.

- Next we turn to government spending multipliers and convergence to rational expectations in learning equilibria.
  - Message: properties of learning equilibria sensitive to details.

The Government Spending Multiplier in the ZLB

- Multiplier in not-so-bad equilibrium is large, even under learning.
- Multiplier in really-bad equilibrium under learning is also large.
  - We will explain why M-R obtain a different result.
- As we saw, economy converges to not-so-bad ZLB.
- Fiscal multiplier converges to the large multiplier associated with the not-so-bad ZLB.

Two Experiments

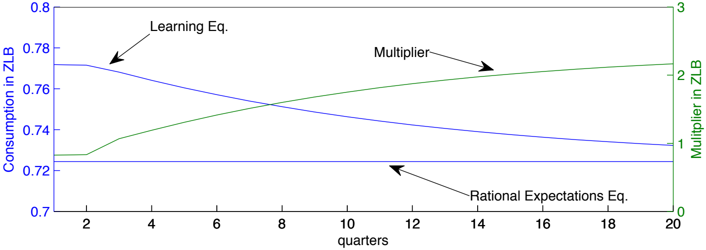
- Simulate:

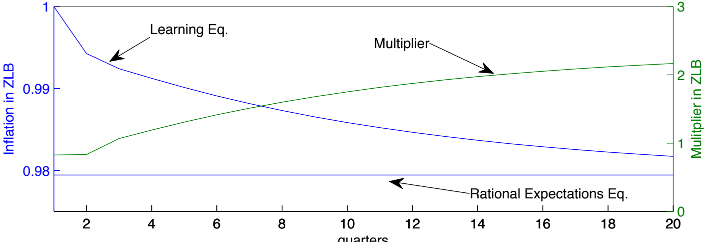
$$\begin{pmatrix} z_t \\ z_t^e \end{pmatrix} = f \begin{pmatrix} z_{t-1} \\ z_{t-1}^e \end{pmatrix}.$$

- Experiment #1: z_{t-1}, z_{t-1}^e correspond to old steady state.

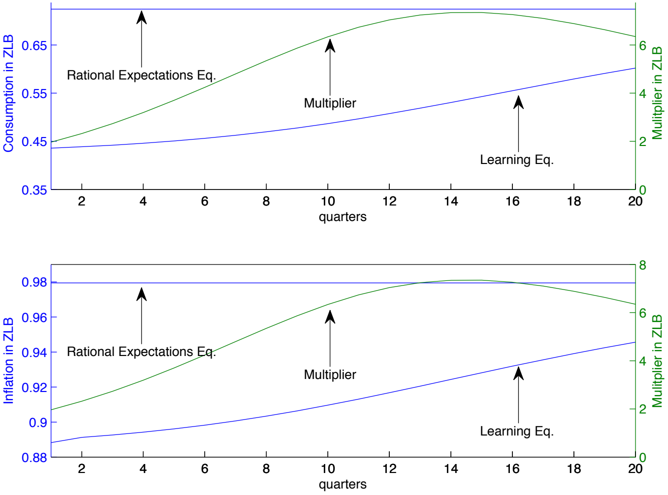
- Experiment #2: z_{t-1}, z_{t-1}^e near really bad ZLB.

Experiment #1: Initial Conditions Equal to Old Steady State





Experiment #2: Initial Conditions Near Really-Bad ZLB



Comparison with MR

- MR get very small multipliers when they start near to the really-bad ZLB.
  - We get big multipliers.

- Reason for difference
  - MR force inflation expectations in really-bad equilibrium to respond to government spending in the way they respond in the corresponding rational expectations equilibrium.
    - In rational expectations equilibrium, government spending drives inflation down.
    - Makes real interest rate high and reduces spending.
  - We follow our learning rule.
    - In our simulations, inflation rises a little with increase in G.
    - Makes real interest rate low and increases spending.

Conclusion

- If we require that RE equilibria are robust to small deviations from RE, then NK model does not have an interesting uniqueness problem in the ZLB.

- The qualitative conclusions from analysis based on linear approximations correspond closely to those of the learnable RE equilibrium.

- The quantitiative conclusions based on linear approximations must be handled with care.
  - When expected duration of ZLB is long and prices relatively flexible, linearization has some 'crazy' implications, like enormous multipliers (> 400).
  - These implications represent approximation error.

- Our analysis complements all the other evidence we have which suggests that how agents actually form beliefs matters.