

Fluctuating Attention and Contagion: Theory and Evidence from the U.S. Equity Market*

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Abstract

Contagion in financial markets occurs when return and volatility transmit between fundamentally unrelated sectors. We develop an equilibrium model showing that contagion arises because investors pay fluctuating attention to news. As a negative shock hits one sector, investors pay more attention to it. This raises the volatility of equilibrium discount rates resulting in simultaneous spikes in cross-sector correlations and volatilities. We test our theory in the U.S. equity market from 1980 to 2009. Using comprehensive customer-supplier relationships data to identify unrelated firms, we find evidence consistent with the model's predictions.

Keywords: Asset Pricing, General Equilibrium, Learning, Attention, Contagion

*We would like to thank Daniel Andrei, Tony Berrada, Harjoat Bhamra, Pierre Collin-Dufresne, Julien Cujean, Giuliano Curatola, Jerome Detemple, Bernard Dumas, Michal Dzielinski, Rüdiger Fahlenbrach, Damir Filipovic, Hoang Ngoc Giang, Harrison Hong, Julien Hugonnier, Alexandre Jeanneret, Jan Kulak, Semyon Malamud, Lorian Mancini, Erwan Morellec, Rémy Praz, Michael Rockinger, Wei Xiong, Qunzi Zhang, and seminar participants at Boston University, EPFL Workshop in Lausanne, HEC Paris, INSEAD, McGill, NCCR workshop in Gerzensee, Ohio State University, University of Geneva, University of Maryland, University of North Carolina, University of Rochester, University of Toronto, and the University of Zürich for their comments. We thank Ling Cen for providing us with customer-supplier relationships data. We are responsible for any inadequacies. This paper was previously circulated under the title "Fluctuating attention to news and financial contagion." Financial support from the Swiss Finance Institute, NCCR FINRISK of the Swiss National Science Foundation, and the University of Toronto is gratefully acknowledged.

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1 Introduction

Evidence of contagion in financial markets has been extensively documented in the empirical literature (e.g., Hamao, Masulis, and Ng, 1990; Lin, Engle, and Ito, 1994). More precisely, there exists ample evidence of volatility and return spillovers between two seemingly unrelated assets.¹ Such phenomena have become increasingly important in light of the recent subprime and sovereign debt crises. This is because simultaneous spikes in asset volatilities and cross-asset correlations significantly alter risk management strategies, optimal portfolio choices, and the trading of derivatives.

In this paper, we provide theoretical and empirical evidence that investors' fluctuating attention to news is an important channel through which contagion arises in financial markets. We show that when investors' attention to a particular sector increases, risk-adjusted discount rates become more volatile. As a result, cross-sector correlations and volatilities increase simultaneously in the entire market, despite the fact that cash-flows and news associated to each sector are independent from one another.

We consider a pure-exchange economy *à la* Lucas (1978) with two risky assets—*sectors*—that are claims to two exogenous and independent dividend streams.² The economy is populated by a representative investor who needs to estimate both unobservable expected dividend growth rates (henceforth *fundamentals*). The investor has two different types of relevant information at hand: information provided by the observation of dividends, and information provided by the observation of news. The key innovation here is that the investor pays fluctuating attention to news, which is supported by empirical evidence in Da, Engelberg, and Gao (2011).³ In other words, there are periods when she is well focused and capable of processing many news sources, and periods when she is not. We assume that investor's attention to a given asset depends on the past performance of that asset's dividend growth. Importantly, we emphasize that attention to one asset is independent from attention to the other because dividends are independent from each other.

The main prediction of our model is that fluctuating attention implies return and volatility spillover effects among fundamentally unrelated market sectors. The intuition is as follows. As a negative shock hits one market sector, more attention is paid to news on that sector. Since the content of news is used to estimate the economic fundamental, a rise in attention implies a faster transmission of news, thereby increasing the volatility of that

¹See also King and Wadhvani (1990), Kaminsky and Reinhart (2000), Bae, Karolyi, and Stulz (2003), Bekaert, Harvey, and Ng (2005), Diebold and Yilmaz (2009), King, Sentana, and Wadhvani (1994), Kallberg and Pasquariello (2005), Barberis, Shleifer, and Wurgler (2005), and Boyer, Kumagai, and Yuan (2006).

²This assumption can be relaxed. It is imposed for the model to focus exclusively on an attention-based contagion instead of a fundamental-based contagion.

³See also, Vlastakis and Markellos (2012), and Andrei and Hasler (2015).

sector’s estimated fundamental. In equilibrium, a more volatile estimated fundamental endogenously generates more volatile equilibrium discount rates. This in turn implies increases in the volatility of the sector hit by the shock, the volatility of the sector that is unrelated to it, and the cross-sector return correlation. The key mechanism of how shocks propagate from one market sector to another is therefore through attention and discount rates.

The model reduces to a simple two-tree economy with investor learning when the fluctuating attention feature is shut down, i.e., when attention is constant. In this case, dividend share becomes the main driver of volatility and correlation. An increase in the dividend share of one market sector, however, mechanically decreases that of the other. As a result, market volatilities move in opposite directions and a simultaneous increase in market volatilities and cross-market correlations is inconceivable. Thus, fluctuating investor attention to news is necessary for observing the volatility and return spillover effects that are synonymous with contagion in financial markets.

We put our model to the test by examining contagion among unrelated sectors in the U.S. equity market from 1980 through 2009. We choose the U.S. market because contagion in our model arises via shocks to the economy’s equilibrium discount rates. Therefore, using a domestic market rather than international markets places less reliance on the assumption of a well-integrated economy. Our empirical design is to examine contagion between an industry and a group of firms that are fundamentally unrelated to that industry. We form industry portfolios following the Fama-French portfolio definitions. We identify a group of firms unrelated to each industry using a customer-supplier relationships database constructed from the COMPUSTAT Segment Customer File.⁴ The relationship status are updated annually. The database identifies 66,290 customer-supplier relationships during our sample period. We use a conservative approach to form a portfolio of firms that is unrelated to each industry. For firms to be considered fundamentally unrelated, they must belong to different industries and there must be at least six degrees of separation between them in the customer-supplier relationships database.

We examine contagion on eight pairs of unrelated portfolios that we construct: the industry portfolio and its unrelated-firm portfolio. Monthly volatilities and return correlations are estimated using GARCH and DCC-GARCH models (see [Engle \(2002\)](#)). Following [Gervais, Kaniel, and Mingelgrin \(2001\)](#), [Barber and Odean \(2008\)](#) among others, we use trading volume as a proxy for investor attention. We choose trading volume because it is easily measurable and as argued by [Hou, Xiong, and Peng \(2009\)](#), it should be highly correlated with attention because investors cannot actively trade a stock if they do not pay attention

⁴Our method of identifying related firms is similar to [Cohen and Frazzini \(2008\)](#) who find a predictable return pattern between economically linked firms

to it. Precisely, our monthly measure of investor attention for each portfolio is defined by its level of share turnover, i.e., trading volume during the month divided by number of common shares outstanding. Using the autoregressive moving-average model with exogenous variables (ARMAX), we examine how changes in attention to two unrelated portfolios affect changes in their volatilities and correlations.

We find clear evidence that supports the model's predictions. Estimation results show that a change in attention to the industry portfolio not only increases its portfolio volatility, but also the volatility of the portfolio that is unrelated to it. We obtain the exact same conclusions when we look at the effect of changes in attention on the correlation between two unrelated portfolios. As a robustness check, we estimate time-varying dynamics of attention and contagion using a multivariate VAR framework and reach the same conclusions. Overall, our empirical results provide strong evidence that an increase in attention in one market positively affects the volatility of the other market, as well as their cross-market return correlation, despite the two markets being fundamentally unrelated.

This paper contributes to two strands of literature. The first is the asset pricing literature examining how contagion arises in financial markets. It is generally difficult to explain both return and volatility spillover effects in a unified general equilibrium framework. For instance, in [Cochrane, Longstaff, and Santa-Clara \(2008\)](#) and [Martin \(2013\)](#), market volatilities and cross-market correlations are driven by dividend shares. An increase in the dividend share of one market mechanically decreases that of another causing volatilities to move in opposite directions. As a result, the model cannot generate a simultaneous increase in market volatilities and cross-market correlations. Our paper shows that such challenge can be resolved if one accounts for the observation that investors pay fluctuating attention to news.

Other studies that offer theoretical explanations for contagion include [Kyle and Xiong \(2001\)](#), [Kodres and Pritsker \(2002\)](#), [Dumas, Harvey, and Ruiz \(2003\)](#), [Yuan \(2005\)](#), and [Pasquariello \(2007\)](#) among others. [Kodres and Pritsker \(2002\)](#) show that when the differences between unobservable payoffs and their corresponding signals are cross-sectionally correlated, contagion arises through a portfolio re-balancing channel. [Yuan \(2005\)](#) shows that asymmetric information and financial constraints lead to contagion. In [Pasquariello \(2007\)](#), contagion is implied by asymmetric information and systemic risk. As shown by [Dumas, Harvey, and Ruiz \(2003\)](#), observed excess correlations can be explained by a high level of market integration. In these aforementioned studies, contagion is defined as the correlation in excess of a benchmark model. Such definition differs slightly from ours in that we require return volatilities and cross-return correlations of fundamentally unrelated firms to increase simultaneously. Using the same definition of contagion as ours, [Kyle and Xiong \(2001\)](#) show that contagion is implied by wealth effects. Our paper is complementary

because it shows both theoretically and empirically that investors' fluctuating attention to news is a key determinant of contagion in financial markets.

This paper also contributes to the growing literature examining how investor attention affects asset prices. Earlier evidence that new information can only influence prices if investors pay attention to it is shown in [Huberman and Regev \(2001\)](#). More recently, [Dellavigna and Pollet \(2009\)](#) find the post-earnings announcement drift on stock returns is stronger subsequent to earnings announced on Friday relative to other weekdays, suggesting that inattention associated with Friday announcements delays information from being reflected in stock prices.

The role of fluctuating attention on asset prices has also been documented by a number of recent studies. Using Google search frequencies on companies' names to measure investor attention, [Da, Engelberg, and Gao \(2011\)](#) find their attention proxy positively predicts short-term stock returns. [Garcia \(2013\)](#) finds that good and bad news recorded from the New York Times explain returns on the Dow Jones Industrial Average better in recessions than in expansions. This suggests that more attention is allocated to processing new information during down markets. [Andrei and Hasler \(2015\)](#) find that stock-return volatility and risk premia increase with both investor attention and uncertainty. In a static framework, [Mondria and Quintana-Domeque \(2013\)](#) examines the role of limited information processing capacity on two fundamentally unrelated markets and find that an increase in uncertainty of one market implies a price drop in the other. Our results add to this growing literature by showing the effect of fluctuating attention on the cross-section of returns, generating a new and complementary contribution.

The remainder of the paper is organized as follows. Section 2 describes the model and derives the equilibrium variables. Section 3 exposes the model's predictions. Section 4 discusses our empirical tests and results. Section 5 concludes. Derivations and computational considerations are provided in Appendix A.

2 Model

We consider an infinite horizon economy populated by a single investor who pays fluctuating attention to news. There are two output processes (henceforth the *dividends*) with unobservable expected growth rates (henceforth the *fundamentals*). The investor estimates the value of the fundamentals by observing the dividends and two signals. Naturally, the accuracies of the signals are positively related to the level of the investor's attention.

All quantities are expressed in units of a single perishable good with price equal to unity. The set of securities available for investment consists in one riskless asset in zero net supply

and two risky assets (stocks) in positive supply of one unit. The riskless asset is locally deterministic and pays a riskless rate r to be determined in equilibrium. The two stocks are claims to exogenous dividends δ_1 and δ_2 and have prices P_1 and P_2 , respectively. Dividends dynamics are written as

$$\frac{d\delta_{it}}{\delta_{it}} = f_{it}dt + \sigma_\delta dW_{it}^\delta, \quad i \in \{1, 2\} \quad (1)$$

where $(W_1^\delta, W_2^\delta)^\top$ is a standard Brownian motion.

Although the investor does not observe fundamentals f_1 and f_2 , she knows that these processes follow

$$df_{it} = \lambda(\bar{f} - f_{it})dt + \sigma_f dW_{it}^f, \quad i \in \{1, 2\} \quad (2)$$

where $(W_1^f, W_2^f)^\top$ is a standard Brownian motion. Hence fundamentals mean-revert to their long-term means \bar{f} at speed λ .

The investor has four pieces of information available to estimate the value of the fundamentals. The first two pieces consist in the dividend growth rates $\frac{d\delta_1}{\delta_1}$ and $\frac{d\delta_2}{\delta_2}$. Because fundamentals drive dividends, observing dividend growth rates provides valuable information about the level of fundamentals.

The remaining two pieces of information are signals denoted by s_1 and s_2 . Their dynamics are

$$ds_{it} = \Phi_{it}dW_{it}^f + \sqrt{1 - \Phi_{it}^2}dW_{it}^s, \quad i \in \{1, 2\} \quad (3)$$

where $\Phi_1, \Phi_2 \in [0, 1]$ represent the precisions of the signals. We interpret these two variables as *attention* paid to stock 1 and stock 2, respectively. The reason being simply that the more attention paid to a given sector, the more precise the news signal on that sector becomes. The 6-dimensional vector $(W_1^\delta, W_2^\delta, W_1^f, W_2^f, W_1^s, W_2^s)^\top$ is a standard Brownian motion. Therefore, markets are perfectly symmetric and fundamentally unrelated. This assumption allows us to precisely determine the mechanism leading to contagion.

We motivate the dynamic of the information signals in Equation (3) as follows. Assume the investor collects m_{it} , $i \in \{1, 2\}$ signals s_{it}^j , $j = 1, \dots, m_{it}$ at time t . s_{it}^j is the j -th noisy signal providing information on fundamental i . These publicly available sources of information represent, for instance, CNN Money, Financial Times, Bloomberg, Wall Street Journal, etc. For simplicity, let us assume that the accuracies of these individual signals are the same. That is, $ds_{it}^j = a dW_{it}^f + \sqrt{1 - a^2} dW_{it}^j$, where $0 < a < 1$ is the accuracy of the

individual signals and all Brownian motions are uncorrelated. By aggregating, the investor can summarize these m_i sources of information into two signals s_i whose dynamics are

$$ds_{it} = \Phi_{it}dW_{it}^f + \sqrt{1 - \Phi_{it}^2}dW_{it}^s,$$

where $\Phi_{it} = \frac{a}{\sqrt{\frac{1}{m_{it}}(1+(m_{it}-1)a^2)}}$. The above equation is equivalent to Equation (3). It shows that the investor controls the accuracy of information Φ_i by choosing the number of signals m_i she acquires. When the investor is very attentive to news, the number of individual signals collected is large and leads to high accuracy. When the investor is inattentive to news, the number of signals acquired is small and leads to low accuracy. For this reason, we call Φ_i the *attention* to news associated to fundamental i .

Our specification for the information signal in Equation (3) follows that in Scheinkman and Xiong (2003), Dumas, Kurshev, and Uppal (2009), and Xiong and Yan (2010). It shows that signals, s_1 and s_2 , provide information on the unexpected fluctuations driving fundamentals and *not* on the level of the fundamentals. This specification differs from, for instance, Detemple and Kihlstrom (1987), Veronesi (2000), Peng and Xiong (2006), and Huang and Liu (2007), which model the dynamic of a signal s about the fundamental f as $ds_t = f_t dt + \sigma dW_t^s$. Intuitively, the latter specification implies that the observed signal provides information on the level of the fundamental while in Equation (3), the signal provides information on unexpected changes in the fundamental. Although we adopt the former specification, our results would also hold under the alternative.

2.1 Definition of fluctuating attention

Following Andrei and Hasler (2015), attention to stock i satisfies

$$\Phi_{it} = \frac{\Psi}{\Psi + (1 - \Psi)e^{\Lambda\pi_{it}}}, \quad i \in \{1, 2\} \quad (4)$$

$$\pi_{it} = \int_0^t e^{-\omega(t-u)} \left(\frac{d\delta_{iu}}{\delta_{iu}} - \hat{f}_{iu} du \right), \quad (5)$$

where $\Psi > 0$, $\omega > 0$, $\Lambda > 0$, and $\Lambda \in \mathbb{R}$.

The parameter Ψ is the long-run level of attention paid to each stock. We assume the attention paid to the stock i at time t depends on stock i 's past dividend performance π_i . The process π_i measures the performance of past surprises in dividend i 's growth rate relative to \hat{f}_i , the investor's estimate of fundamental f_i .⁵ We refer to π_i as the *performance index*. In

⁵The dividend performance index is inspired by Koijen, Rodriguez, and Sbuelz (2009) who measure the past performance of stock returns to allow for mean reversion.

order to map the level of performance indices, π_i , to the level of investor attention $\Phi_i \in [0, 1]$, we use the logistic transformation shown in Equation (4).

The coefficient Λ indicates how the level of attention Φ_{it} changes in relation to the dividend performance index π_i . When Λ is positive, a positive shock to π_i , i.e., positive dividend growth surprises, would decrease the level of attention, implying that attention is counter-cyclical. On the other hand, when Λ is negative, attention is pro-cyclical as a positive shock to π_i would increase Φ_{it} . The magnitude of Λ also determines the range of attentions. A large magnitude of Λ pushes attentions to effectively belong to the entire interval $[0, 1]$, while a smaller magnitude would limit the range of attentions to be smaller. When Λ is zero, attentions become constant and equal to the long-run level Ψ .

The parameter ω in Equation (5) controls the importance of past dividend growth surprises relative to the current dividend growth surprise. If ω is small, then past dividend surprises matter in the determination of the current performance index. Conversely if ω is large, then past realizations of dividend surprises do not significantly alter the value of the performance index. Equation (5) shows that the performance indices have the following dynamics

$$d\pi_{it} = -\omega\pi_{it}dt + \sigma_\delta dW_{it}, \quad i \in \{1, 2\}, \quad (6)$$

where $dW_{it} = \frac{1}{\sigma_\delta} \left(\frac{d\delta_{it}}{\delta_{it}} - \widehat{f}_i dt \right)$ is a scaled surprise in dividend i at time t . The dynamic of the performance indices in Equation (6) shows that π_i reverts to 0 at speed ω . Since fundamentals belong to the real line, dividend performance indices belong to the real line, too.

Evidence of how attention fluctuates in relation to the dividend performance index is provided in [Andrei and Hasler \(2015\)](#). In their study, the parameters driving the attention dynamic are estimated using the U.S. GDP data from 1969 to 2012. Table 1 reports their parameters estimated using the Generalized Method of Moments. The parameter Λ is positive and significant, suggesting the investor is attentive and gathers accurate information when the dividend performance index is low (i.e., in bad macroeconomic episodes), and vice versa. This finding suggests that attention is counter-cyclical. In other words, investors build more accurate forecasts and react more aggressively to the incoming news in recessions than in expansions.

To summarize, Equations (4) and (5) provide a one-to-one mapping between attention and performance indices. Such mapping implies that attention is observable and depends on the vector of state variables that are conditionally Gaussian. This modeling feature makes it simple to apply standard Bayesian filtering techniques to the model.

2.2 Filtered state variables

The investor learns about the fundamental f_i , $i \in \{1, 2\}$ by observing two different sources of information: the dividend δ_i and the signal s_i . Proposition 1 describes the dynamics of the variables inferred using these two sources of information.

Proposition 1. *Following Liptser and Shiryaev (2001), the dynamics of the inferred vector of state variables satisfy*

$$\frac{d\delta_{1t}}{\delta_{1t}} = \widehat{f}_{1t}dt + \begin{pmatrix} \sigma_\delta & 0 & 0 & 0 \end{pmatrix} dW_t \quad (7)$$

$$\frac{d\delta_{2t}}{\delta_{2t}} = \widehat{f}_{2t}dt + \begin{pmatrix} 0 & \sigma_\delta & 0 & 0 \end{pmatrix} dW_t \quad (8)$$

$$d\widehat{f}_{1t} = \lambda(\bar{f} - \widehat{f}_{1t})dt + \begin{pmatrix} \frac{\gamma_{1t}}{\sigma_\delta} & 0 & \sigma_f\Phi_{1t} & 0 \end{pmatrix} dW_t \quad (9)$$

$$d\widehat{f}_{2t} = \lambda(\bar{f} - \widehat{f}_{2t})dt + \begin{pmatrix} 0 & \frac{\gamma_{2t}}{\sigma_\delta} & 0 & \sigma_f\Phi_{2t} \end{pmatrix} dW_t \quad (10)$$

$$d\pi_{1t} = -\omega\pi_{1t}dt + \begin{pmatrix} \sigma_\delta & 0 & 0 & 0 \end{pmatrix} dW_t$$

$$d\pi_{2t} = -\omega\pi_{2t}dt + \begin{pmatrix} 0 & \sigma_\delta & 0 & 0 \end{pmatrix} dW_t$$

$$d\gamma_{1t} = \left(-\frac{\gamma_{1t}^2}{\sigma_\delta^2} - 2\lambda\gamma_{1t} + \sigma_f^2(1 - \Phi_{1t}^2) \right) dt \quad (11)$$

$$d\gamma_{2t} = \left(-\frac{\gamma_{2t}^2}{\sigma_\delta^2} - 2\lambda\gamma_{2t} + \sigma_f^2(1 - \Phi_{2t}^2) \right) dt. \quad (12)$$

The innovation process W is a standard Brownian motion defined by

$$dW_t \equiv \begin{pmatrix} dW_{1t} \\ dW_{2t} \\ dW_{3t} \\ dW_{4t} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sigma_\delta} \left(\frac{d\delta_{1t}}{\delta_{1t}} - \widehat{f}_{1t}dt \right) \\ \frac{1}{\sigma_\delta} \left(\frac{d\delta_{2t}}{\delta_{2t}} - \widehat{f}_{2t}dt \right) \\ ds_{1t} \\ ds_{2t} \end{pmatrix}. \quad (13)$$

Proof. See Theorem 12.7 of Liptser and Shiryaev (2001).

The dynamic of filtered dividend processes in Equations (7) and (8) follows closely the modeling assumption in Equation (1), but with the filtered fundamental \widehat{f}_{it} appearing in the drift term. The dynamic of the filtered fundamental, however, differs from that in Equation (2). When the investor is able to learn about the fundamental by observing the dividend and the signal, Equations (9) and (10) show that the volatility of the filtered fundamentals

is stochastic and driven by two components. The first, γ_i , is defined as the *uncertainty* of the current value of the fundamental. The second is *attention* Φ_i .

The level of attention also impacts the level of uncertainty, γ_i . Equations (11) and (12) show that γ_i and Φ_i impacts the volatility of filtered fundamentals in opposite directions. As attention increases, the investor gathers more accurate information. Hence the learning procedure becomes more efficient and uncertainty decreases. Conversely, as attention drops investors acquire less accurate information and uncertainty rises. For sufficiently high (low) attention level, the third component in Equations (11) and (12) will decrease (increase) enough to generate lower (higher) uncertainty. Interestingly, there is a lag between a change in attention and a change in uncertainty because uncertainty is locally deterministic. Therefore, high attention implies low future uncertainty, whereas low attention is followed by high uncertainty.

Next we examine how the filtered fundamentals evolve. Equation (13) shows the vector of innovations that are used to estimate what the value of the fundamentals are: the scaled dividend surprise, $\frac{1}{\sigma_\delta} \left(\frac{d\delta_{it}}{\delta_{it}} - \widehat{f}_i dt \right)$, and the news signal, ds_{it} . From Equations (9) and (10), we see that uncertainty, γ_i , loads on the dividend innovation while attention, Φ_i , loads on the news innovation. As attention increases, the investor perceives the news source as more important relative to the dividend source. Conversely, reduced attention pushes the investor to weight the information content of the dividend performance more than that of the news signal. This naturally implies that an increase (decrease) in attention weakens (strengthens) the correlation between dividends and fundamentals.

2.3 Equilibrium

The representative investor has CRRA utility over consumption. Since the investment horizon is assumed to be infinite, the investor maximizes her expected lifetime utility of consumption subject to a budget constraint

$$\begin{aligned} & \sup_{C,n} \mathbb{E}_t \left(\int_t^\infty e^{-\Delta(s-t)} \frac{C_s^{1-\alpha}}{1-\alpha} ds \right) \\ & \text{s.t. } dV_t = (r_t V_t + h_t \text{diag}(P_t) (\mu_t - r_t \mathbb{1}_{2 \times 1}) - C_t) dt + h_t \text{diag}(P_t) D_t dW_t, \end{aligned}$$

where C is consumption, V wealth, μ the 2×1 vector of expected return, h the 1×2 vector of risky asset holdings, D the 2×4 matrix of diffusion, Δ the subjective discount rate, and α the coefficient of relative risk aversion. The risk-free rate r and the 2×1 vector of stock prices P are determined in equilibrium.

Solving the optimization problem and clearing markets yields the following state-price

density

$$\xi_t = e^{-\Delta t} \left(\frac{C_t}{C_0} \right)^{-\alpha} = e^{-\Delta t} \left(\frac{\delta_{1t} + \delta_{2t}}{\delta_{1,0} + \delta_{2,0}} \right)^{-\alpha}. \quad (14)$$

Equation (14) shows that the state-price density depends on dividend 1 and dividend 2. As either fundamental 1 or fundamental 2 increases, the expected value of discount factors decreases or, in other words, the expected discount rates increase. As will be explained further, the interactions between fundamentals and discount rates implied by fluctuating attention to news are key determinants of the contagion phenomenon.

Since the state-price density, ξ , prices future cash-flows, the price P_i^T of a security paying a single-dividend δ_{iT} at time T is defined by

$$P_{it}^T = e^{-\Delta(T-t)} \mathbb{E}_t \left(\left(\frac{\delta_{1T} + \delta_{2T}}{\delta_{1t} + \delta_{2t}} \right)^{-\alpha} \delta_{iT} \right).$$

Proposition 2 characterizes the price of the single-dividend paying securities.

Proposition 2. *At time t , the prices P_{1t}^T and P_{2t}^T of the securities paying the single-dividends δ_{1T} and δ_{2T} at time T satisfy*

$$P_{1t}^T = e^{-\Delta(T-t)} e^{\alpha(\zeta_{1t} - Q_t)} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + \alpha Q_T} \right) \quad (15)$$

$$P_{2t}^T = e^{-\Delta(T-t)} e^{\alpha(\zeta_{1t} - Q_t)} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + (\alpha-1)Q_T} \right) - P_{1t}^T, \quad (16)$$

where $\zeta_i \equiv \log \delta_i$ is the log-dividend, and $Q = \log \frac{\delta_1}{\delta_1 + \delta_2}$ the log-dividend share.

Proof. See Appendix A.3.

The stock price P_{it} , $i \in \{1, 2\}$ at current time t is defined as the sum of the single-dividend paying securities P_{it}^T over maturities T

$$P_{it} = \int_t^\infty P_{it}^T dT. \quad (17)$$

Equations (15) and (16) show the single-dividend paying securities are determined by moment-generating functions (henceforth *transforms*) of the vector $(\zeta_1, Q)^\top$. Computing these transforms is challenging, as the vector of state variables is not affine-quadratic. Appendix A.4 exposes a simple methodology that allows us to accurately approximate them.

σ_δ	\bar{f}	λ	σ_f	ω	Λ	Ψ	α	Δ
1.4%	2.8%	0.42	2.9%	4.74	286	0.368	3	1%

Table 1: Calibration.

This table reports parameters determining the dynamics of dividends and fundamentals obtained from [Andrei and Hasler \(2015\)](#). These parameters are calibrated to match several moments of the U.S. GDP growth rate and its corresponding forecast from 1969 to 2012.

ζ_1	Q	$\hat{f}_1 = \hat{f}_2$	$\pi_1 = \pi_2$	$\gamma_1 = \gamma_2$
0	$\log(0.5)$	\bar{f}	0	γ_{ss}

Table 2: Initial state values.

This table reports initial values of state variables that we use to generate the model implications. Initial dividend is set to unity, i.e., $\zeta_1 \equiv \log \delta_i = 0$. Dividends of stocks 1 and 2 are initially of the same size, while the other values are set to long-term levels. Long-term levels of the fundamentals and the performance indices are derived in [Appendix A.1](#). Long-term uncertainty, γ_{ss} , is defined and derived in [Appendix A.2](#).

3 Model implications

In this section, we investigate the implications of fluctuating attention on the dynamics of return volatilities and cross-return correlation of two fundamentally unrelated stocks. We show that increased attention to one stock raises return volatilities on both stocks, as well as their cross-return correlation. That is, fluctuating attention implies return and volatility spillover effects among fundamentally unrelated stocks. We first discuss the volatility spillover effects in relation to fluctuating attention. After, we examine the return spillover effects.

3.1 Volatility spillovers

In order to understand the process leading to contagion, let us first characterize the components in the stock-return diffusion. Recall that from [Equation \(13\)](#), the innovation process driving the filtered state variables consists of four components. Applying the Itô's lemma to stock prices in [Equation \(17\)](#), we can write the components in the stock-return diffusion matrix D as

$$D \equiv \begin{pmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \end{pmatrix}, \quad (18)$$

where D_{ij} is the j component in the return diffusion process of stock i . Expressions for the components of the diffusion matrix D are provided in Definition 1 below.

Definition 1. *The diffusion components of stock i satisfy*

$$\begin{aligned} D_{i1} &= \frac{P_{i\hat{f}_1}}{P_i} \frac{\gamma_1}{\sigma_\delta} + \sigma_\delta \left(1 + \frac{P_{i\pi_1}}{P_i} + \frac{P_{i\tilde{Q}}}{P_i} \left(3 - 2e^{-\tilde{Q}} - e^{\tilde{Q}} \right) \right) \\ D_{i2} &= \frac{P_{i\hat{f}_2}}{P_i} \frac{\gamma_2}{\sigma_\delta} + \sigma_\delta \left(\frac{P_{i\pi_2}}{P_i} + \frac{P_{i\tilde{Q}}}{P_i} \left(-3 + 2e^{-\tilde{Q}} + e^{\tilde{Q}} \right) \right) \\ D_{i3} &= \frac{P_{i\hat{f}_1}}{P_i} \sigma_f \Phi_1 \\ D_{i4} &= \frac{P_{i\hat{f}_2}}{P_i} \sigma_f \Phi_2, \end{aligned}$$

where P_{iy} stands for the derivative of stock i with respect to the state variable y and $\tilde{Q} \equiv \log \left(1 + \frac{\delta_1}{\delta_1 + \delta_2} \right)$.

For brevity, we drop the time t notation when writing the stock price P_i and its partial derivative P_{iy} with respect to state y . Nevertheless, we note that all notations regarding the stock price should be referenced against the current time period t .

Definition 2. *The variance of stock i , σ_i^2 , satisfies*

$$\sigma_i^2 = \sum_{j=1}^4 D_{ij}^2,$$

where D_{ij} is the component in the stock-return diffusion matrix D in Equation (18), and its expression is provided in Definition 1.

Definitions 1 and 2 show that a change in attention to stock 1 impacts the volatility of both stock 1 and stock 2 through the stock-return diffusion component D_{i3} . The significance of the impact is determined by the sensitivity of stock prices P_1 and P_2 to the shock in the filtered fundamental of stock 1, i.e., \hat{f}_1 . This is essentially the value of the derivative $P_{i\hat{f}_1}$; $i \in 1, 2$, which we discuss below.

A positive shock in the filtered fundamental \hat{f}_1 has two opposite effects on stock 1. First, dividend δ_1 is expected to increase. This is the direct channel. Second, discount rates rise (see Equation (14)), which represent the indirect channel. The direct channel pushes stock price P_1 up, while the indirect channel pushes stock price P_1 down. As explained in Veronesi (2000), the discounting effect is stronger than the dividend effect as long as risk aversion is sufficiently large. An increase in expected future consumption increases current consumption

because the investor smooths consumption over time. Hence savings (investments) decrease, as do the demands for stock 1, stock 2, and the riskless asset i.e. stock prices decline and the risk-free rate rises. The decline in stock price P_2 , however, can be more pronounced than the decline in stock price P_1 because stock 2 is only impacted by the discounting effect. Note that if the representative agent had either a small risk aversion or recursive utility (Epstein and Zin, 1989; Duffie and Epstein, 1992) and a preference for early resolution of uncertainty, then stock 1 would be more sensitive to a change in fundamental 1 than stock 2.

In order to see the effect of attention on volatilities visually, Figure 1 plots the relationship between attention to stock 1 and return volatilities of both stocks. The model parameters that we use are estimated from the U.S. GDP growth and forecast from 1969 to 2012; they are reported in Table 1. The initial values of the state variables are reported in Table 2. Most of the initial state values are set equal to their long-run means. We assume the initial dividend the two stocks are of equal size.

Panels A and B of Figure 1 plot the impact of increasing attention on volatilities of stock 1 and stock 2, holding all other variables constant. We see that an increase in attention to stock 1, i.e., Φ_1 , increases both return volatilities. That is, fluctuating attention implies volatility spillover effects. The volatility of stock 2 increases more than the volatility of stock 1 because stock 2 is influenced by the discounting effect only. Regarding stock 1, the effect of dividend δ_1 dampens the discounting effect and therefore implies a weaker increase in its volatility. Symmetrically, the volatility of stock 1 increases more than the volatility of stock 2 as attention to stock 2, i.e., Φ_2 , increases.

The joint effect of increasing attention paid to each stock 1 and stock 2, are illustrated in Panels C and D of Figure 1. The results show that an increase in attention to any stock, i.e., either Φ_1 and Φ_2 , will increase both stock-return volatilities. Importantly, increasing attention to both stock 1 and stock 2 appear to work together by pushing stock-return volatilities higher. In other words, when total attention, i.e., $\Phi_1 + \Phi_2$, paid to the stocks increases, the volatility spillover effects become most evident. Overall, Figure 1 shows that an increase in attention to any stock leads to contagion in the form of increased stock-return volatilities in the entire market. However, the effect is largest when the total attention paid to the whole market increases.

While Figure 1 describes the static relationship between attention and volatilities, we next examine how stock-return volatilities change relative to attention in a dynamic setting. Such analysis is useful for understanding the economic relevance of fluctuating attention on volatilities under the presence of noises generated by other state variables. We simulate the model at the monthly frequency for 20 years, using the parameters and initial variables reported in Tables 1 and 2. Figure 2 reports the results. Stock return volatilities are

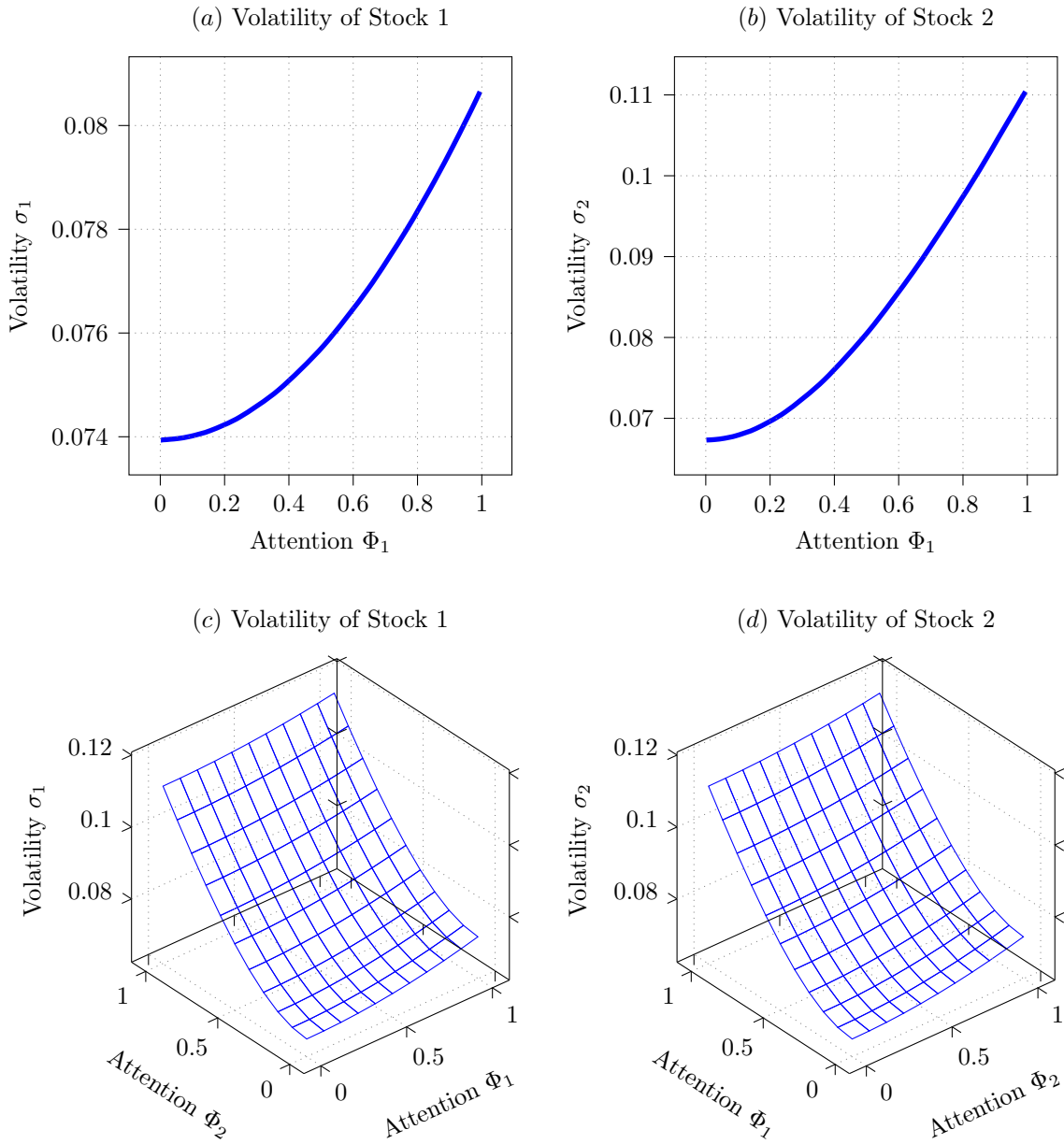


Figure 1: Stock-return volatilities vs. attention.

This figure provides a static analysis of our model's implications on the volatility spillover effects. Panels A and B plot the relationship between attention paid to stock 1 on the return volatilities of stock 1 and 2, respectively. In Panels B and C, we plot the joint relationship of attention paid to stock 1 and stock 2 on the return volatilities of stock 1 and stock 2, respectively. The model parameters used to generate the results are reported in Table 1. Initial values of the state variables are reported in Table 2.

plotted against aggregate attention paid to stocks 1 and 2. The figure shows a strong positive relationship between increasing aggregate attention and volatilities of both stocks. This finding is consistent with our previous results which show that stock-return volatilities

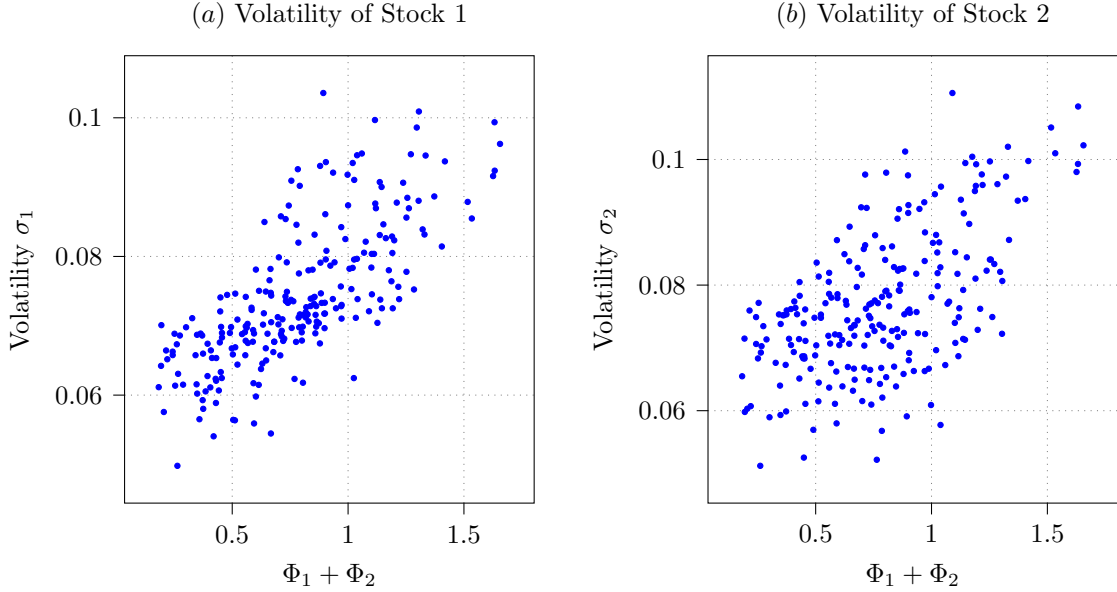


Figure 2: Volatilities vs. aggregate attention.

This figure plots simulation results showing the dynamic relationship between volatilities and aggregate attention. We simulate 20-year of data at the monthly frequency using the model introduced in Section 2. The model parameters used to generate the results are reported in Table 1. Initial values of the state variables are reported in Table 2. In both panels, the x-axis indicates the aggregate attention paid to stocks 1 and 2. The left panel plots the stock-return volatility for stock 1, while the right panel plots the stock-return volatility for stock 2. Each dot represents one simulated monthly observation.

increase with attention to stocks 1 and 2 separately. Therefore, we expect volatilities of both stocks to increase with aggregate attention as depicted in Figure 2. Overall, the simulation results show a strong dependence between volatilities and aggregate attention. They also confirm that the dynamics of the other state variables have very little influence on the dynamics of stock-return volatilities.

In order to compare the dynamics of stock-return volatilities in a fluctuating attention model to an otherwise equivalent constant attention model, Figure 3 plots results from a 20-year simulation study for a model with fluctuating attention (Panel A), and for a model with constant attention (Panel B). For the constant attention model, we turn off the fluctuating attention feature by setting $\Lambda = 0$ in Equation (4), resulting in an economy with constant investor attention, i.e., $\Phi_1 = \Phi_2 = 0.368$. Figure 3 shows the volatility dynamics significantly differ between the fluctuating attention model and the constant attention model. In the fluctuating attention model, volatilities swing rapidly from month-to-month and simultaneous spikes in volatilities of stock 1 and stock 2 are often observed. On the other hand, results from the constant attention model show that volatilities of the two stocks move smoothly

and, on average, in opposite directions. In order to quantify the average correlation between volatilities of stocks 1 and 2, we repeat this simulation study 10,000 times. We find the volatility of volatility in the fluctuating attention model is about 1% for both stocks, while the correlation between the volatilities of stock 1 and stock 2 is around 0.19. In the constant attention model, however, volatility of volatility falls to 0.4%, while the correlation between volatilities of stock 1 and stock 2 is -0.95 .

Results from the above simulation exercise confirm that the volatility dynamics significantly differ between the fluctuating attention model and the constant attention model. The economic intuition for such difference is as follows. When attention is constant, volatilities are principally driven by dividend shares. As the dividend share of stock 1 increases, the dividend share of stock 2 mechanically decreases. As a result, volatilities move in opposite directions. On the other hand, in our fluctuating attention model, variations in dividend shares are outweighed by variations in aggregate attention. This implies an amplification of the fluctuations in stock-return volatilities and a positive co-movement between them. To summarize, the model with investor learning alone (e.g. the constant attention model) does not help understand the dynamics of stock-return volatilities in the cross-section. However, learning with fluctuating attention does. This explains why volatilities among fundamentally unrelated sectors fluctuate strongly and co-vary positively, consistent with empirical findings in Fleming, Kirby, and Ostdiek (1998).

3.2 Return spillovers

We now turn to the relationship between attention and the co-movement of stock returns. The covariance and correlation between returns of stock 1 and stock 2 are described below.

Definition 3. *The cross-return covariance, σ_{12} , satisfies*

$$\sigma_{12} = \sum_{j=1}^4 D_{1j} D_{2j},$$

where D_{ij} is the component in the stock-return diffusion matrix D in Equation (18), and its expression is provided in Definition 1.

Definition 4. *The cross-return correlation, ρ_{12} , satisfies*

$$\rho_{12} = \frac{\sigma_{12}}{\sigma_1 \sigma_2} = \frac{\sum_{j=1}^4 D_{1j} D_{2j}}{\sqrt{\sum_{j=1}^4 D_{1j}^2} \sqrt{\sum_{j=1}^4 D_{2j}^2}},$$

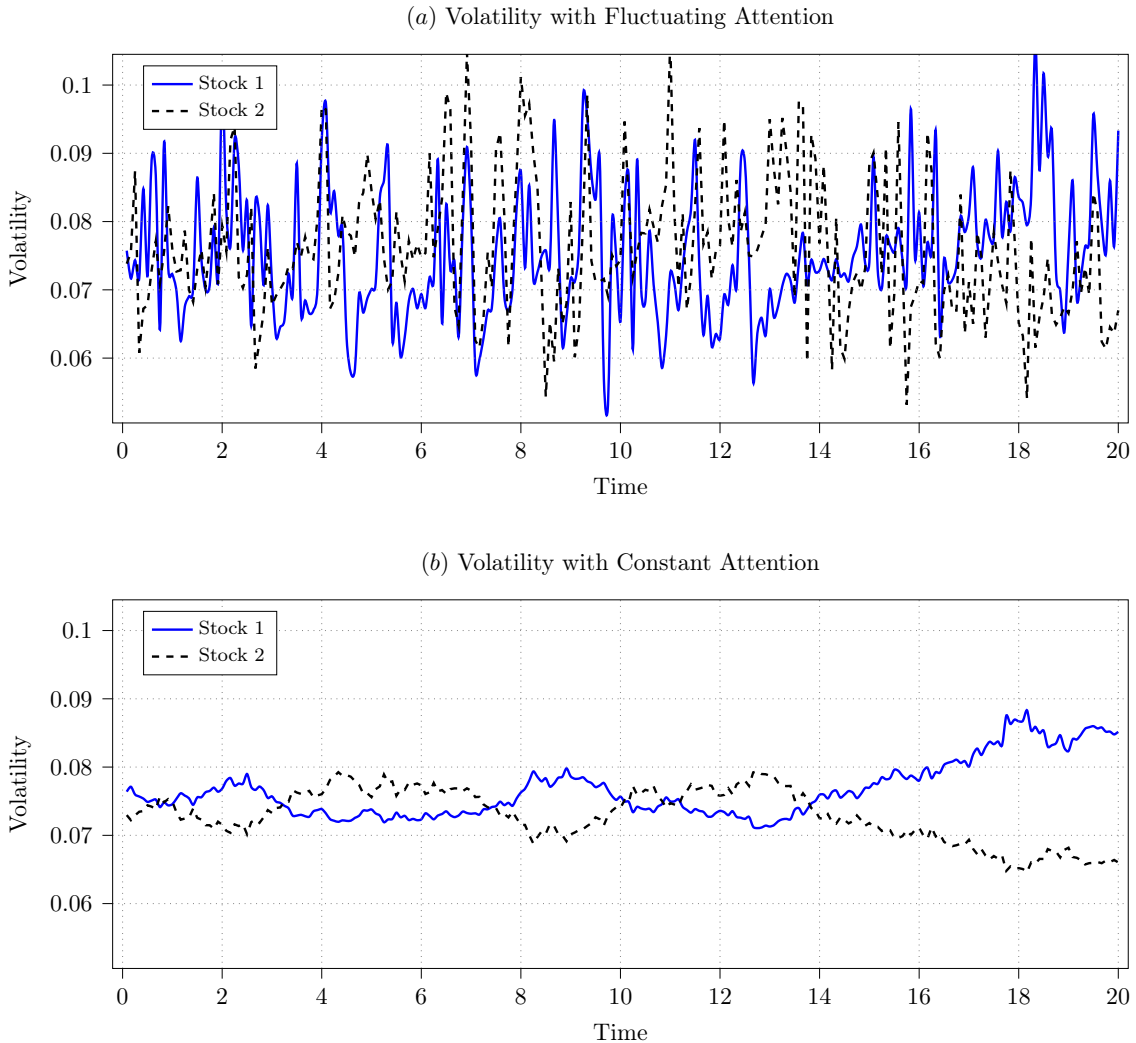


Figure 3: Time-series of cross-market volatilities: A simulation study

We plot time-series results of monthly volatilities from a 20-year simulation of the economy introduced in Section 2. Monthly volatilities are calculated following Definition 2, and reported in annualized terms. Panel A plots monthly volatilities of stock 1 and stock 2 for an economy with fluctuating investor attention. In Panel B, we shut off the fluctuating attention feature of the model by setting $\Lambda = 0$ in Equation (4), and plot the monthly volatilities for an economy with constant investor attention ($\Phi_1 = \Phi_2 = 0.368$). The model parameters and their initial values are reported in Tables 1 and 2, respectively. In each panel, the blue line indicates volatilities of stock 1, while the dashed black line indicates volatilities of stock 2.

where D_{ij} is the component in the stock-return diffusion matrix D in Equation (18), and its expression is provided in Definition 1.

We plot the static relationship between the level of attention paid to stock 1, i.e., Φ_1 , and cross-return correlation in Panel A of Figure 4. We note that by symmetry, the relationship

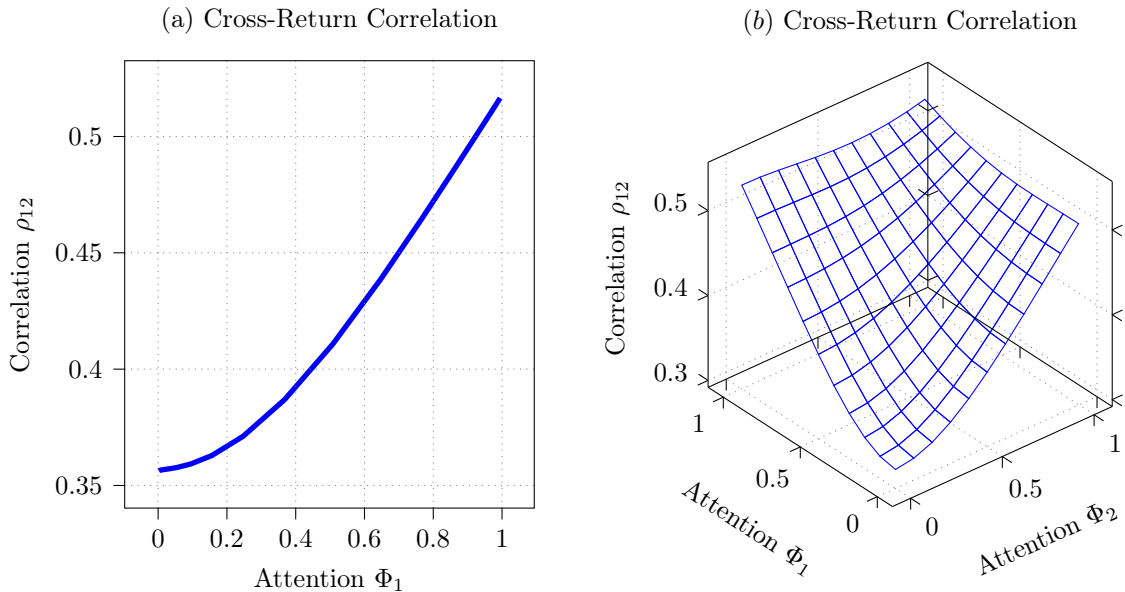


Figure 4: Cross-return correlation vs. attention.

This figure provides a static analysis of our model’s implications on the return spillover effect. Panel A plots the relationship between attention paid to stock 1 and the level of cross-return correlation between stock 1 and stock 2, holding other variables fixed. In Panel B, we plot the joint relationship between attention paid to stock 1 and stock 2 of the level of their cross-return correlation, while holding other variables fixed. The model parameters used to generate the results are reported in Table 1. Initial values of the state variables are reported in Table 2.

plot between Φ_2 and ρ_{12} would look identical. Panel A shows that the cross-return correlation between stock 1 and stock 2 is positive and increases with attention even though dividends, fundamentals, and signals are uncorrelated. In Panel B, we plot the static relationship showing the joint impact of attentions Φ_1 and Φ_2 on the cross-return correlation ρ_{12} . Similar to our results for volatilities, we find that an increase in attention to stock 1 and stock 2, together, further strengthens the return spillover effects. Thus, contagion arises most prominently when the attention paid to the whole market increases.

We now turn to discuss the economic reasons leading to a positive cross-return correlation in our model. First, we explain how cross-return correlation arises in the standard equilibrium model, and after, discuss how the fluctuating attention feature produces the positive cross-return correlation between two stocks. Following the exposition in [Cochrane, Longstaff, and Santa-Clara \(2008\)](#), the cross-return correlation between stock 1 and stock 2 can arise due to co-movements in one of the following four relationship pairs. The first is the co-movement between price-dividend ratio $\frac{P_1}{\delta_1}$ and dividend δ_2 . The second is the co-movement between price-dividend ratio $\frac{P_2}{\delta_2}$ and dividend δ_1 . The third is the co-movement between

price-dividend ratios $\frac{P_1}{\delta_1}$ and $\frac{P_2}{\delta_2}$. Finally, the fourth, is the co-movement between dividends δ_1 and δ_2 . Because our model assumes that dividend δ_1 and dividend δ_2 are uncorrelated, the fourth relationship source can be eliminated.

The positive correlation between returns of stock 1 and stock 2 in our model is observed through the discount rates. Consider a negative shock in the dividend of stock 1, i.e., the Brownian shock dW_{1t} is negative in Equation (7). This shock reduces the performance of dividend δ_1 thereby raising the attention paid to it. In the meantime, Equation (9) shows that this dividend shock pushes the investor to decrease her estimation of its fundamental \hat{f}_1 . The decrease in the expectation of future dividend growth causes the discount rates to fall and triggers an increase in price-dividend ratio $\frac{P_2}{\delta_2}$. As for stock 1, because risk aversion is relatively large, the discounting effect outweighs its dividend effect. This also results in an increase in price-dividend ratio $\frac{P_1}{\delta_1}$. The end result is a positive co-movement between price-dividend ratios $\frac{P_1}{\delta_1}$ and $\frac{P_2}{\delta_2}$, while the co-movement between δ_1 and $\frac{P_2}{\delta_2}$ is negative. The former effect dominates the latter and implies a positive cross-return correlation because the discounting channel is the strongest.

In the previous paragraph, we explain how given a sufficient risk aversion level, returns between two unrelated stocks co-move due to the the discount rates effect. However, an important result from our model is that the magnitude of cross-return correlation depends on the level of investor attention (see Figure 4). This result can also be directly seen from Definition 1, which shows that when attention paid to stock 1 increases, the cross-return correlation rises because the diffusion components D_{13} and D_{23} increase (in absolute value). The intuition to why attention impacts the magnitude of return spillover effects is discussed below.

Recall that the diffusion process of the filtered fundamentals reflects two pieces of information: dividend innovations (dW_{1t}, dW_{2t}) and signal innovations (dW_{3t}, dW_{4t}). It follows from Equations (9) and (10) that the uncertainty γ_i loads on dividend innovations, while attention Φ_i loads on signal innovations. As a negative shock hits dividend δ_1 , attention to stock 1 increases but uncertainty γ_1 remains constant because it is deterministic. In other words, the weight assigned to signal innovations rises while the weight assigned to dividend innovations remains unchanged. Consequently, the variance of the filtered fundamental increases while the covariance between the dividend and the filtered fundamental, i.e., $E_t \left[\frac{d\delta_{1t}}{\delta_{1t}}, d\hat{f}_{1t} \right]$, remains constant. This means that an increase in attention disconnects the dividend dynamic from the filtered fundamental dynamic. Since the filtered fundamental drives discount rates, the (positive) correlation between discount rates and dividend of stock 1 is reduced. As a result, the negative co-movement between price-dividend ratio of stock 2 and dividend of stock 1 is less pronounced when attention to stock 1 is high rather than low.

The mechanism above explains why the cross-return correlation increases with attention to stock 1, and by symmetry, with attention to stock 2. As a result, the cross-return correlation rises, exactly like stock-return volatilities, with aggregate attention. Figure 5 plots cross-return correlation results from a simulation study over 20-year horizon. In Panel A, monthly return correlations are plotted against aggregate total attention. We observe a strong positive relationship between correlation ρ_{12} and aggregate attention $\Phi_1 + \Phi_2$, similar to the finding for volatilities.

Panel B of Figure 5 plots the monthly time-series of cross-return correlations for the fluctuating attention model, as well as for the constant attention model. The correlation in the constant attention model is positive. This finding is consistent with the prediction that correlations between asset returns naturally arise due to the discount rate channel. However, monthly cross-return correlations for the constant attention model do not fluctuate significantly from month-to-month, suggesting that fluctuations in dividend shares alone cannot generate sudden cross-return spikes that are distinctive of financial contagion. However, in the fluctuating attention model, monthly cross-return correlations swing more intensely. This finding suggests that fluctuating attention is an important feature for generating the return spillover effects observed in financial markets.

Performing 10,000 simulations of our economy over 20 years, we find that fluctuating attention significantly amplifies the variations in cross-return correlation compared to an economy in which the investor pays constant attention. Simulation results show that correlation ranges from 0.38 to 0.47 in the constant attention model, and from 0.30 to 0.65 in the fluctuating attention model. In addition, volatilities and correlation co-move positively in the fluctuating attention model because those quantities are mainly driven by aggregate attention. This result, however, does not hold when attention is constant because, in that case, the dividend share is the main driver of volatility and correlation.

4 Empirical evidence

We test the model's predictions described in Section 3 empirically. The objective is to show that the transmission of return and volatility from one industry sector to other fundamentally unrelated firms is associated with changes in investor attention. Our empirical setup is based on the U.S. equity market during the period 1980-2009. We first describe the data and method used to form groups of firms that are fundamentally unrelated. We then discuss the estimation of return volatilities, cross-return correlations, and attention for each pair of fundamentally unrelated portfolios. Next, we outline testable hypotheses based on the model's predictions. After, we discuss the empirical findings.

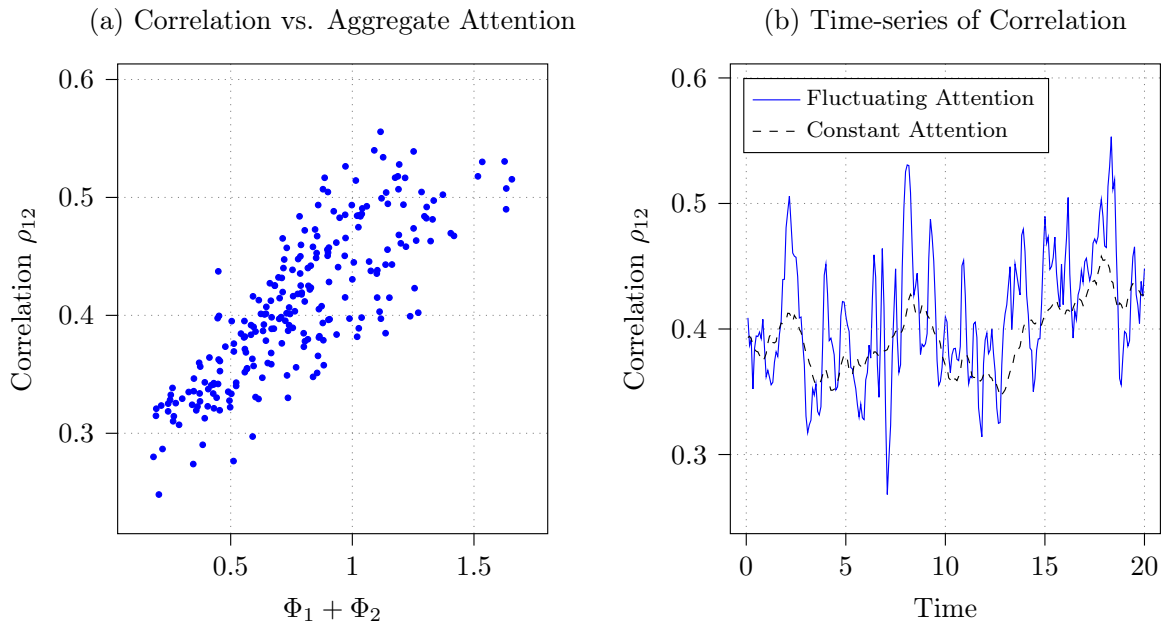


Figure 5: Cross-return correlation and attention: A simulation study

We plot cross-return correlations results generated from a 20-year simulation. The model parameters that we use are reported in Table 1. Initial values of the state variables are reported in Table 2. Panel A plots simulation results showing the dynamic relationship between correlation and aggregate attention $\Phi_1 + \Phi_2$. In Panel B, we plot the time-series of monthly return correlations between stock 1 and stock 2 for an economy with fluctuating attention (blue line) and constant attention (dashed black line). Monthly return correlations are calculated following Definition 4.

4.1 Data construction

Our sample consists of U.S. incorporated firms that traded on the NYSE/AMEX and NASDAQ between 1979 and 2009. We require that firms must have information available on both CRSP and COMPUSTAT databases. Only firms with share code equal to 11 or 12 are retained. We group firms into different industries following the Fama-French's 17 industry portfolios. Industry definitions are obtained from Kenneth French's website.⁶ We choose the 17-industry classification because each industry's size is sufficiently large to capture shocks to its sector, and also because it is more refined than the Fama-French's 12 portfolio definition. Table 3 displays the names of industry portfolios that we study. There are 15 industries listed in Table 3 because we do not consider firms in the financial industry, as well as firms classified under "others" according to industry portfolio definitions.

For each industry, we identify firms that are not fundamentally related to it using customer-supplier relationships data constructed from the COMPUSTAT Segment Customer

⁶http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

File.⁷ Statement of Financial and Account Standards (SFAS) No. 14 requires that each firm discloses the existence and sales to individual customers (public or private entities) accounting for more than 10% of its revenue.⁸ In practice, we find that several customers who do not represent more than 10% of a supplier firms' total revenue are voluntarily reported. We retain all relationships reported as identifications of relevant cash flow relationships. The names of corporate customer are manually matched with their COMPUSTAT identifiers, i.e., GVKEYS, following the approach used in Banerjee, Dasgupta, and Kim (2008) and Cohen and Frazzini (2008). The procedure identifies 66,290 customer-supplier-year relationships between 1979 and 2009.

Using customer-supplier relationships data, we identify a set of firms unrelated to each industry using the following procedure. For each calendar year, we identify firms in the CRSP universe that are a customer of or a supplier to firms in the industry as reported in the relationships database in the current year, the past year, as well as the next year. We use the three-year window for identification in order to account for relationships that are emerging, and those may have been delayed in the reporting. A direct customer-supplier relationship between two firms is referred to as the first-level relationship. When two firms are connected indirectly via their connections with a common firm, we refer to the link as the second-level relationship. Similarly, when two firms are indirectly connected through a firm in their second-level relationship, i.e., there is three degrees of separation between them, we refer to the link as the third-level relationship. Following this definition, we say that a firm is fundamentally related to an industry if it is connected with any firms in that industry up to the sixth relationship level. Our relationships identification using up to six degrees of separation is conservative. It accounts for direct and indirect relationships that could cause firms to be fundamentally connected.

We apply additional filters to firms in the sample. We require that firms have been public for more than one year, and have non-zero trading volume for all months in the current calendar year. We eliminate firms that have prices below \$5, firms that have market capitalization less than \$25 million, and firms that have a trading volume less than 4,000 per year. These filters ensure our results are not driven by small-firm effects and effects related to initial public offerings. Finally, we remove financial firms from the sample as they may have lending and underwriting relationships that cannot be identified using the customer-supplier relationships database. Following the above requirements, the final firm sample contains an average of 2670 eligible firms per year between 1980 and 2009. Year 1979 is excluded from

⁷We thank Ling Cen for sharing with us the customer-supplier relationships database

⁸This requirement is relaxed after 1998 (See SFAS No. 131). However, most firms continue to report the names of their customers in COMPUSTAT as well as in their 10-K filing.

the final sample to eliminate potential biases associated with sample initiation. The number of eligible firms in the final sample is relatively stable and has the inter-quartile range of 870.

We refer to a portfolio of firms classified under one of the Fama-French industries as the *industry portfolio*, and refer to a portfolio of firms that is fundamentally unrelated to it as the *unrelated-firm portfolio*. Firms in the unrelated-firm portfolio are from industries other than the industry portfolio and must not have any links with firms in the industry portfolio as defined by our relationships identification method described above. Table 3 summarizes the number of firms that are in each pair of industry and unrelated-firm portfolios. The number of firms in each portfolio changes annually as new relationships are formed and severed. Panel A reports the median, minimum, and maximum number of firms entering into each portfolio between 1980 and 2009. In all cases, the number of firms in an industry portfolio is less than that of its corresponding unrelated-firm portfolio. Among the fifteen industries, *Machinery* has the largest number of firms, with a median of 245 firms per year, while its unrelated-firm portfolio has a median of 890 firms per year.

Panel B of Table 3 examines aggregate characteristics of each portfolio. We look at three dimensions: size, earnings, and dividend. The values reported in Panel B represent their time-series averages in units of one million U.S. dollars. We obtain firm-level information from the COMPUSTAT Annual Fundamental File. *Size* is calculated annually as the total market capitalization of each portfolio. *Earnings* of each portfolio are calculated annually by multiplying annual earnings per share of each firm with its number of common shares, and aggregating it across all firms in each portfolio. The calculation for *Dividend* is similar to that for earnings. On average, the industry portfolio has smaller size, earnings, and dividend payout relative to its unrelated-firm portfolio.

The general equilibrium model that we introduced in Section 2 provides predictions of contagion between two firms with similar market share. In order to keep our empirical design comparable to the model’s assumption, we make additional requirements for industry pairs that we use to test the model’s predictions. We require that the average size of the industry portfolio must be at least 10 percent of the size of its unrelated-firm portfolio. There are nine industry pairs that meet this requirement. We denote them with an asterisk next to the industry name in Table 3.

4.2 Time-varying volatilities, correlations, and attention

We measure contagion between the industry portfolio and its unrelated-firm portfolio by looking at the dynamics of their return volatilities and cross-return correlation. The monthly return for each portfolio is calculated as the weighted-average return across firms. We use

firms' ex-dividend returns in the calculation and use their market capitalizations calculated on June 30th of each year as portfolio weights. The portfolio is formed in the beginning of July and held through June of the next year. We do not re-balance the portfolio monthly in order to avoid excess volatility generated through the mechanic of portfolio re-balancing.

We estimate the volatility and correlation dynamics for 9 industry and unrelated-firm portfolios marked with an asterisk in Table 3. Each portfolio is estimated individually using monthly returns from January 1980 to December 2009. There are 380 observations in each estimation. We assume the return of each portfolio follows an AR(1) process and model its volatility using a GARCH(1,1) specification. The choice of GARCH(1,1) is motivated by its parsimony which is appropriate for the short time series that we have. Figure 6 plots conditional volatilities, in annualized terms, for eight portfolio pairs estimated using GARCH(1,1). Although we fit GARCH to nine portfolio pairs, we do not plot volatility dynamics for the *Utilities* industry pair because maximum likelihood estimation of its unrelated-firm portfolio does not converge. As a result, we drop *Utilities* from our analysis. From this point onward, eight portfolio pairs shown in Figure 6 will be used in the empirical tests.

We follow Engle (2002) and estimate the correlation dynamics for each pair of portfolio returns using the Dynamic Conditional Correlation (DCC) model (see also Tse and Tsui (2002) for a similar model). We choose the DCC model instead of the multivariate GARCH model because it is parsimonious. We consider the GARCH(1,1)-type specification for the DCC model. Figure 7 plots monthly conditional correlations between the industry portfolio and its unrelated-firm portfolio for eight portfolio pairs. In all cases, we find strong correlations between the two fundamentally unrelated portfolios. This shows that significant correlations between two groups of firms can arise even though they do not have cash flow connections. Importantly, Figure 7 shows strong time-varying dynamics in monthly return correlations between most portfolio pairs.⁹ For all portfolio pairs, we find that correlations are highest during the month of October 1987 crash when the S&P 500 index dropped by more than 23% in one day.

Our empirical objective is to show that time-varying volatilities and correlations that we observe in Figures 6 and 7 are associated with changes in investor attention. It is generally difficult to measure direct investor attention and the existing literature is inconclusive on the preferred proxy. Following Gervais, Kaniel, and Mingelgrin (2001) and Barber and Odean (2008) among others, we use trading volume as a proxy for investor attention. We choose trading volume because it is easily measurable for all stocks traded in the CRSP universe. As argued by Hou, Xiong, and Peng (2009), trading volume should be highly correlated with

⁹For brevity, we do not report parameter estimates from the GARCH and DCC models. All parameters are statistically significant and can be made available upon requests.

attention because investors cannot actively trade a stock if they do not pay attention to it. The evidence that trading volume is informative of investor attention is provided by [Chordia and Swaminathan \(2000\)](#), who find that high volume stocks tend to respond more quickly to information than do low volume stocks.

We construct a monthly measure of investor attention to each stock by dividing its total trading volume during the month by its number of common shares outstanding. This measure is commonly referred to as *share turnover*. The scaling of trading volume by the number of shares controls for the size differences between firms. Share turnover is always greater than zero, and in most cases, its value is below one.¹⁰ Therefore, our empirical proxy for attention generally lies in the interval $[0,1]$, which closely resembles the definition of attention used in our theoretical model (see Section 2.1). Monthly investor attention to the industry and unrelated-firm portfolios is calculated as the weighted-average share turnover of firms in their portfolio. Similar to the monthly return calculation, we use firms' market capitalizations on June 30th of each year as portfolio weights. The weights are applied to the portfolios starting in the following month, i.e., July, and held through June of next year.

Figure 8 plots monthly share turnover of eight portfolio pairs that we study from 1980 through 2009. We find a strong increasing trend in monthly share turnover in all portfolios as financial markets become more liberalized and the trading is moved to electronic trading platforms. The level of share turnover also changes from month-to-month for each portfolio, though the magnitude of changes are less noticeable than their conditional volatilities and correlations. Overall, Figures 6 to 8 show that the variables we use to measure contagion and attention vary substantially through time and that their dynamics are quite persistent. In our empirical tests, we will focus on their monthly changes, and carefully control for any lag effects associated with their dynamics.

4.3 Hypotheses and empirical tests

This section outlines four hypotheses that we test based on the model's predictions. For ease of reference, we first describe the notations that we use. We refer to the volatility of the industry portfolio in month t as $\sigma_{1,t}$, and the volatility of the unrelated-firm portfolio in month t as $\sigma_{2,t}$. The correlation between the two portfolios in month t is denoted by $\rho_{12,t}$. Finally, we let $\Phi_{1,t}$ and $\Phi_{2,t}$ be the share turnover, i.e., proxy for investor attention to the industry portfolio and the unrelated-firm portfolio in month t , respectively. To keep notations simple, we do not add subscripts identifying different industry pairs to our monthly variables. Nevertheless, we emphasize that these time-series notations apply at the portfolio-pair level.

¹⁰In exceptional cases, monthly trading volume of a heavily traded stock can be higher than its number of common shares.

The first hypothesis that we test is how changes in attention to the industry portfolio and the unrelated-firm portfolio are associated with contemporaneous changes in their volatilities. It can be stated as follows.

Hypothesis 1. *Change in volatility of the industry portfolio, i.e. $\Delta\sigma_{1,t}$, is positively related to a contemporaneous change in its investor attention, $\Delta\Phi_{1,t}$, as well as a contemporaneous change in investor attention to its unrelated-firm portfolio, $\Delta\Phi_{2,t}$ (see Figure 1).*

We empirically test the above hypothesis by estimating the following model:

$$\Delta\sigma_{1,t} = \alpha + \sum_{i=0}^1 \beta'_i \Delta\Phi_{t-i} + \sum_{j=1}^p \gamma_j \Delta\sigma_{1,t-j} + \sum_{k=1}^q \theta_k \varepsilon_{t-k} + \varepsilon_t, \quad (19)$$

where $\Delta\Phi_t = [\Delta\Phi_{1,t}, \Delta\Phi_{2,t}]'$, and β_0 and β_1 are 2×1 vectors. The residual of the regression model is denoted by ε_t . The regression model in Equation (19) is an autoregressive moving-average model with exogenous variables (ARMAX). The exogenous variables here are the monthly change in share turnover of the two unrelated portfolios. We allow up to one lag of $\Delta\Phi_t$ in the regression to control for persistence in the monthly change in share turnover.¹¹ The number of lags in the autoregressive term and in the moving-average term is denoted by p and q , respectively.

An important implication of the first hypothesis is that an increase in attention to one sector can increase the volatility of the other sector, even though the two are fundamentally unrelated. Although the statement in Hypothesis 1 focuses on the change in volatility of the industry portfolio, $\Delta\sigma_{1,t}$, its prediction must symmetrically apply to the change in volatility of the unrelated-firm portfolio, $\Delta\sigma_{2,t}$. That is, we expect $\Delta\sigma_{2,t}$ to be positively related to $\Delta\Phi_{1,t}$, as well as $\Delta\Phi_{2,t}$. To test this, we estimate the ARMAX model similar to Equation (19), but with $\Delta\sigma_{2,t}$ as the dependent variable.

The next hypothesis examines the relationship between *total* attention to the portfolio pairs and their volatilities.

Hypothesis 2. *Change in volatility of the industry portfolio, i.e. $\Delta\sigma_{1,t}$, is positively related to a contemporaneous change in total investor attention to the industry portfolio and its unrelated-firm portfolio, $\Delta\Phi_{1,t} + \Delta\Phi_{2,t}$ (see Figure 2).*

We empirically test the above hypothesis by estimating the following model:

$$\Delta\sigma_{1,t} = \alpha + \sum_{i=0}^1 \beta_i \Delta(\Phi_{1,t-i} + \Phi_{2,t-i}) + \sum_{j=1}^p \gamma_j \Delta\sigma_{1,t-j} + \sum_{k=1}^q \theta_k \varepsilon_{t-k} + \varepsilon_t, \quad (20)$$

¹¹Our main results remain qualitatively the same when longer lags of exogenous variables are added to the regression.

In Equation (20), the coefficients β_0 and β_1 are scalar because there is only one exogenous variable, which is the monthly change in total share turnover of the two portfolios. The prediction of the above hypothesis must also apply symmetrically to the change in volatility of the unrelated-firm portfolio. Therefore, we expect that $\Delta\sigma_{2,t}$ is positively related to $\Delta\Phi_{1,t} + \Delta\Phi_{2,t}$. We test this complementary hypothesis by estimating the ARMAX model similar to Equation (20), but with $\Delta\sigma_{2,t}$ as the dependent variable of interest.

In the next two hypotheses, we examine how fluctuating attention affects return correlations between two fundamentally unrelated portfolios.

Hypothesis 3. *Change in correlation between the industry and its unrelated-firm portfolios, i.e. $\Delta\rho_{12,t}$, is positively related to a contemporaneous change in investor attention, $\Delta\Phi_{1,t}$ and $\Delta\Phi_{2,t}$ (see Figure 4).*

We test the above hypothesis by estimating the following regression model:

$$\Delta\rho_{12,t} = \alpha + \sum_{i=0}^1 \beta'_i \Delta\Phi_{t-i} + \sum_{j=1}^p \gamma_j \Delta\rho_{12,t} + \sum_{k=1}^q \theta_k \varepsilon_{t-k} + \varepsilon_t, \quad (21)$$

where $\Delta\Phi_t = [\Delta\Phi_{1,t}, \Delta\Phi_{2,t}]'$. Next, we examine how contemporaneous changes in total attention to the two portfolios affects their return correlations.

Hypothesis 4. *Change in correlation between the industry and its unrelated-firm portfolios, i.e. $\Delta\rho_{12,t}$, is positively related to a contemporaneous change in their total investor attention, $\Delta\Phi_{1,t} + \Delta\Phi_{2,t}$ (see Figure 5).*

We verify the above hypothesis by estimating the following model:

$$\Delta\rho_{12,t} = \alpha + \sum_{i=0}^1 \beta'_i \Delta(\Phi_{1,t-i} + \Phi_{2,t-i}) + \sum_{j=1}^p \gamma_j \Delta\rho_{12,t} + \sum_{k=1}^q \theta_k \varepsilon_{t-k} + \varepsilon_t. \quad (22)$$

In all regression models, the optimal lags in the AR(p) and MA(q) representation are determined using the Akaike information criterion (AICC), together with the Portmanteau test for cross-autocorrelations between residuals. The model selection procedure that we use follows that in Burnham and Anderson (2002).

An important remark regarding the notations used in Equations (19) through (22) is that the ARMAX coefficients are identically named. We emphasize the coefficient names are reused when writing the regression models simply to avoid notations overload. In practice, their estimates will be different because the models are separately estimated. All regression models are estimated in a panel on eight portfolio pairs using maximum likelihood. There are 2864 monthly observations available in each estimation. Tables 4 and 5 reports their results.

4.4 Results

Table 4 reports three sets of estimation results based on the regression models in Equations (19) and (21). In the first and second regression specifications, the dependent variables are $\Delta\sigma_{1,t}$ and $\Delta\sigma_{2,t}$, respectively. The dependent variable in the third regression specification is $\Delta\rho_{12,t}$. Looking at the first regression specification, we find the coefficient estimates on $\Delta\Phi_{1,t}$ and $\Delta\Phi_{2,t}$ are positive and significant. Such results are in line with the prediction of Hypothesis 1. A positive and statistically significant coefficient on $\Delta\Phi_{1,t}$ shows that the levels of volatility and investor attention to one sector tend to move in the same direction. This finding is also consistent with Lamoureux and Lastrapes (1990) who find that the trading volume of a stock has a positive and significant explanatory power on its volatility. Importantly, we find that $\Delta\Phi_{2,t}$ is positive and strongly significant, although its magnitude is smaller compared to the estimate on $\Delta\Phi_{1,t}$. This finding suggests that changes in attention to one sector can positively affect changes in volatility of the other sector, despite the two sectors being fundamentally unrelated.

We obtain a similar conclusion when looking at the impact of changes in attention on the volatility of the unrelated-firm portfolio, i.e. regression specification (II). An increase in share turnover in each of the two portfolios significantly pushes the volatility of the unrelated-firm portfolio, $\sigma_{2,t}$, higher. Further, the positive relation between attention and volatilities appear to be only in the contemporaneous month. This is because we find that estimates on the lagged attention variables, i.e. $\Delta\Phi_{1,t-1}$ and $\Delta\Phi_{2,t-1}$, are not positive and statistically significant. Overall, results obtained from regression specifications (I) and (II) in Table 4 provide a strong validation for our first hypothesis.

Next, we look at estimation results from the third regression specification in Table 4. Similar to the results for volatilities, we find that changes in monthly correlations between returns of two fundamentally unrelated portfolios are positively associated with changes in investor attention. The coefficient estimates on both $\Delta\Phi_{1,t}$ and $\Delta\Phi_{2,t}$ are positive and significant at the five and one percent levels, respectively. Overall, our empirical results here are consistent with the prediction of Hypothesis 3.

In the next set of results, we look at how changes in total attention to the two fundamentally unrelated portfolios affect their volatilities as well as their return correlations. Table 5 reports estimation results based on the regression models in Equations (20) and (22). Overall, we find a positive and highly significant coefficient on $\Delta\Phi_{1,t} + \Delta\Phi_{2,t}$ for all three regression specifications that we examined. The lagged variable for the change in total attention, however, does not appear to have a positive influence on volatilities and correlations. Thus, the positive effect of fluctuating attention is limited to the contemporaneous month, which is consistent with our earlier finding in Table 4. Overall, the results in Table

5 strongly support the model's predictions stated in Hypotheses 2 and 4.

Our results in Tables 4 and 5 are based on the univariate ARMAX model. That is, monthly changes in volatilities of the two portfolios, i.e., $\sigma_{1,t}$ and $\sigma_{2,t}$, and their correlations, $\rho_{12,t}$, are separately modeled. We further examine the robustness of this assumption by estimating the dynamics of these three variables in a multivariate framework. The model that we use is the vector autoregressive moving-average with exogenous variable (VARMAX). We first examine the predictions of Hypotheses 1 and 3. We estimate the following model

$$\Delta Y_t = \Lambda + \beta_0 \Delta \Phi_t + \beta_1 \Delta \Phi_{t-1} + \Gamma_1' \Delta Y_{t-1} + \Theta_1' \epsilon_{t-1} + \epsilon_t, \quad (23)$$

where

$$\Delta Y_t = [\Delta \sigma_{1,t}, \Delta \sigma_{2,t}, \Delta \rho_{12,t}]' \quad \text{and} \quad \Delta \Phi_t = [\Delta \Phi_{1,t}, \Delta \Phi_{2,t}]'.$$

The vector of residual terms is represented by $\epsilon_t = [u_{1,t}, u_{2,t}, u_{3,t}]'$. Similar to our previous estimations, we allow for one lag in the exogenous variable, $\Delta \Phi_t$. In Equation (23), β_0 and β_1 are 3×2 matrices that capture the relationships between ΔY_t and changes in investor attention during the current and previous months, i.e., $\Delta \Phi_t$ and $\Delta \Phi_{t-1}$. We estimate the model with one lag in the autoregressive and moving-average processes. The 2×1 vectors Γ_1 and Θ_1 are coefficient estimates on the autoregressive and moving-average terms. Panel A of Table 6 reports estimation results of the model in Equation (23) on the eight portfolio pairs.

The results in Panel A confirm that our previous conclusions are robust after controlling for potential cross-dynamics between volatilities and correlations. All coefficients on $\Delta \Phi_{1,t}$ are positive and statistically significant, confirming that an increase in attention to the industry portfolio raises the volatility of its portfolio, the portfolio that it is unrelated to, as well as their returns correlation. We obtain a similar conclusion when looking at the coefficients on $\Delta \Phi_{2,t}$. Importantly, we find the magnitude of the coefficient estimates on $\Delta \Phi_{1,t}$ and $\Delta \Phi_{2,t}$ are very similar between Table 4 and Panel A of Table 6. This finding suggests that our parameters estimates are robust to the model specification. Finally, we examine the predictions of Hypotheses 2 and 4 in a multivariate framework. These two hypotheses predict that changes in total attention to the portfolio pairs should raise their portfolio volatilities as well as their correlations. We estimate the following model:

$$\Delta Y_t = \Lambda + \beta_0 \Delta (\Phi_{1,t} + \Phi_{2,t}) + \beta_1 \Delta (\Phi_{1,t-1} + \Phi_{2,t-1}) + \Gamma_1' \Delta Y_{t-1} + \Theta_1' \epsilon_{t-1} + \epsilon_t, \quad (24)$$

where ΔY_t and ϵ_t are 3×1 vectors of dependent variables and residual terms, respectively. The exogenous variable in Equation (24) is the total change in attention to the two portfolios,

$\Phi_{1,t} + \Phi_{2,t}$. β_0 and β_1 are 3×1 vectors that capture the relationship between ΔY_t and changes in total attention during the current and previous months. Estimation results of the model in Equation (24) are reported in Panel B of Table 6. Clearly, the coefficient estimates on $\Phi_{1,t} + \Phi_{2,t}$ are positive and significant, confirming our results in Table 5. We also note that the coefficient estimates in Panel B and those in Table 5 are very comparable in magnitude. Overall, using both univariate and multivariate regression models, we find strong evidence supporting our four hypotheses.

5 Conclusion

Recent empirical studies shed light on how investors focus on public information. These studies document that attention to news is fluctuating and counter-cyclical. This paper shows both theoretically and empirically that fluctuating attention implies return and volatility spillover effects among fundamentally unrelated sectors. Indeed, a bad shock affecting one sector propagates to other sectors through attention, simultaneously raising each sector's volatility and cross-sector correlation. We verify the claims of our model's predictions in the U.S. equity market using customer-supplier relationships data to identify unrelated groups of firms. Through a series of empirical tests, we find strong evidence supporting fluctuating attention as a channel through which contagion arises.

Our paper contributes to the existing literature by offering an explanation for financial contagion through the well-documented observation that investors pay fluctuating attention to news. That is, return and volatility transmissions between fundamentally unrelated sectors are natural outcomes of investors learning about news with counter-cyclical attention. Possible extensions of our work include incorporating more flexible preferences, e.g., Epstein-Zin preferences, as well as allowing for jumps in the dividend dynamics.

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A Appendix

A.1 Long-term means and variances

Let us consider the 4-dimensional vector $Y = \left(\hat{f}_1 \quad \hat{f}_2 \quad \pi_1 \quad \pi_2 \right)^\top$. The dynamics of Y in vector notation is

$$dY_t = (A - BY_t) dt + CdW_t$$

where

$$\begin{aligned}
A &= \begin{pmatrix} \lambda \bar{f} & \lambda \bar{f} & 0 & 0 \end{pmatrix}^\top \\
B &= \begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \omega & 0 \\ 0 & 0 & 0 & \omega \end{pmatrix} \\
C &= \begin{pmatrix} \frac{\gamma_{1t}}{\sigma_\delta} & 0 & \sigma_f \Phi_{1t} & 0 \\ 0 & \frac{\gamma_{2t}}{\sigma_\delta} & 0 & \sigma_f \Phi_{2t} \\ \sigma_\delta & 0 & 0 & 0 \\ 0 & \sigma_\delta & 0 & 0 \end{pmatrix}.
\end{aligned}$$

Applying Itô's lemma on $F \equiv e^{Bt}Y$ yields

$$dF = \begin{pmatrix} \frac{e^{\lambda t}(\lambda \bar{f} \sigma_\delta dt + \gamma_{1t} dW_{1t} + \sigma_f \sigma_\delta \Phi_{1t} dW_{3t})}{\sigma_\delta} \\ \frac{e^{t\lambda}(\lambda \bar{f} \sigma_\delta dt + \gamma_{2t} dW_{2t} + \sigma_f \sigma_\delta \Phi_{2t} dW_{4t})}{\sigma_\delta} \\ e^{\omega t} \sigma_\delta dW_{1t} \\ e^{\omega t} \sigma_\delta dW_{2t} \end{pmatrix}.$$

Integrating from 0 to t and taking expectation yields

$$\begin{pmatrix} e^{\lambda t} \mathbb{E} \left(\widehat{f}_{1t} \right) - \widehat{f}_{10} \\ e^{\lambda t} \mathbb{E} \left(\widehat{f}_{2t} \right) - \widehat{f}_{20} \\ e^{\omega t} \mathbb{E} \left(\pi_{1t} \right) - \pi_{10} \\ e^{\omega t} \mathbb{E} \left(\pi_{2t} \right) - \pi_{20} \end{pmatrix} = \begin{pmatrix} (e^{\lambda t} - 1) \bar{f} \\ (e^{\lambda t} - 1) \bar{f} \\ 0 \\ 0 \end{pmatrix}.$$

Therefore, the long-term means satisfy

$$\lim_{t \rightarrow +\infty} \begin{pmatrix} \mathbb{E} \left(\widehat{f}_{1t} \right) \\ \mathbb{E} \left(\widehat{f}_{2t} \right) \\ \mathbb{E} \left(\pi_{1t} \right) \\ \mathbb{E} \left(\pi_{2t} \right) \end{pmatrix} = \begin{pmatrix} \bar{f} \\ \bar{f} \\ 0 \\ 0 \end{pmatrix}.$$

Similar computations yield the long-term variances

$$\lim_{t \rightarrow +\infty} \begin{pmatrix} \text{Var} \left(\widehat{f}_{1t} \right) \\ \text{Var} \left(\widehat{f}_{2t} \right) \\ \text{Var} \left(\pi_{1t} \right) \\ \text{Var} \left(\pi_{2t} \right) \end{pmatrix} = \begin{pmatrix} \frac{\sigma_f^2}{2\lambda} \\ \frac{\sigma_f^2}{2\lambda} \\ \frac{\sigma_\delta^2}{2\omega} \\ \frac{\sigma_\delta^2}{2\omega} \end{pmatrix}.$$

A.2 Long-term uncertainty

The dynamics of the uncertainty γ_i conditional on $\pi_i = 0$ is

$$d\gamma_{it} = \left(-\frac{\gamma_{it}^2}{\sigma_\delta^2} - 2\lambda\gamma_{it} + \sigma_f^2 (1 - \Psi^2) \right) dt.$$

The dynamics of the uncertainty at the “steady-state” is

$$\frac{d\gamma_{ss}}{dt} = 0.$$

Solving yields

$$\gamma_{ss} = \sigma_\delta \sqrt{\sigma_f^2 (1 - \Psi^2) + \lambda^2 \sigma_\delta^2} - \lambda \sigma_\delta^2.$$

A.3 Proof of Proposition 2

The price of the single-dividend paying securities S_1^T is defined by

$$P_{1t}^T = \mathbb{E}_t \left(\frac{\xi_T}{\xi_t} \delta_{1T} \right). \quad (25)$$

Substituting Equation (14) in Equation (25) yields

$$\begin{aligned} P_{1t}^T &= e^{-\Delta(T-t)} (\delta_{1t} + \delta_{2t})^\alpha \mathbb{E}_t \left(\delta_{1T} \left(\frac{1}{\delta_{1T} + \delta_{2T}} \right)^\alpha \right) \\ &= e^{-\Delta(T-t)} (\delta_{1t} + \delta_{2t})^\alpha \mathbb{E}_t \left(\delta_{1T}^{1-\alpha} \left(\frac{\delta_{1T}}{\delta_{1T} + \delta_{2T}} \right)^\alpha \right) \\ &= e^{-\Delta(T-t)} e^{\alpha(\zeta_{1t} - Q_t)} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + \alpha Q_T} \right) \end{aligned}$$

where $\zeta_i \equiv \log \delta_i$ is the log-dividend and $Q = \log \frac{\delta_1}{\delta_1 + \delta_2}$ the log-dividend share. Similarly, the price of the single-dividend paying security P_2^T satisfies

$$\begin{aligned}
P_{2t}^T &= \mathbb{E}_t \left(\frac{\xi_T}{\xi_t} \delta_{2T} \right) \\
&= e^{-\Delta(T-t)} (\delta_{1t} + \delta_{2t})^\alpha \mathbb{E}_t \left(\delta_{2T} \left(\frac{1}{\delta_{1T} + \delta_{2T}} \right)^\alpha \right) \\
&= e^{-\Delta(T-t)} (\delta_{1t} + \delta_{2t})^\alpha \mathbb{E}_t \left(\delta_{1T}^{-\alpha} \delta_{2T} \left(\frac{\delta_{1T}}{\delta_{1T} + \delta_{2T}} \right)^\alpha \right) \\
&= e^{-\Delta(T-t)} e^{\alpha(\zeta_{1t} - Q_t)} \mathbb{E}_t \left(e^{-\alpha\zeta_{1T}} (e^{-Q_T} - 1) e^{\zeta_{1T}} e^{\alpha Q_T} \right) \\
&= e^{-\Delta(T-t)} e^{\alpha(\zeta_{1t} - Q_t)} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + (\alpha-1)Q_T} - e^{(1-\alpha)\zeta_{1T} + \alpha Q_T} \right) \\
&= e^{-\Delta(T-t)} e^{\alpha(\zeta_{1t} - Q_t)} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + (\alpha-1)Q_T} \right) - P_{1t}^T
\end{aligned}$$

□

A.4 Approximation of the transforms

The idea consists in approximating the dynamics of the state-vector, and then computing the transforms appearing in Equations (15) and (16) by applying the theory on affine processes (e.g., Duffie, Pan, and Singleton, 2000). An accurate approximation of the dynamics includes second-order terms. Consequently, before approximating we augment the state-vector by these second order terms (Cheng and Scaillet, 2007). Then, we compute the drift and variance-covariance matrix of the augmented state-vector. Finally, we approximate the augmented drift and variance-covariance matrix by performing a Taylor expansion.

Because the dividend share belongs to the interval $(0, 1)$, the log-dividend share Q belongs to $(-\infty, 0)$. Therefore, the dynamics of Q cannot be accurately approximated by performing Taylor expansions. To overcome this problem we perform the following change of variable¹²

$$\tilde{Q} \equiv \log \left(1 + \frac{\delta_1}{\delta_1 + \delta_2} \right) = \log (1 + e^Q)$$

where $\tilde{Q} \in]0, \log(2)[$. The dynamics of \tilde{Q} are

$$\begin{aligned}
d\tilde{Q}_t &= e^{-2\tilde{Q}_t} \left(e^{\tilde{Q}_t} - 2 \right) \left(e^{\tilde{Q}_t} - 1 \right) \left(\left(e^{2\tilde{Q}_t} - 2 \right) \sigma_\delta^2 + e^{\tilde{Q}_t} \left(\hat{f}_{2t} - \hat{f}_{1t} \right) \right) dt \\
&\quad + \left(\sigma_\delta \left(3 - 2e^{-\tilde{Q}_t} - e^{\tilde{Q}_t} \right) \quad \sigma_\delta \left(-3 + 2e^{-\tilde{Q}_t} + e^{\tilde{Q}_t} \right) \quad 0 \quad 0 \right) dW_t.
\end{aligned}$$

¹²Note that this change of variable could be omitted. If it was, then the approximation of the transforms would be slightly less accurate.

Proposition 3. *Under the change of variable stated above and the assumption that the coefficient of relative risk aversion α is an integer, the single-dividend paying securities appearing in Equations (15) and (16) satisfy*

$$P_{1t}^T = e^{-\Delta(T-t)} \left(\frac{e^{\zeta_{1t}}}{e^{\tilde{Q}_t} - 1} \right)^\alpha \sum_{j=0}^{\alpha} \binom{\alpha}{j} (-1)^{\alpha-j} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + j\tilde{Q}_T} \right) \quad (26)$$

$$P_{2t}^T = e^{-\Delta(T-t)} \left(\frac{e^{\zeta_{1t}}}{e^{\tilde{Q}_t} - 1} \right)^\alpha \sum_{j=0}^{\alpha-1} \binom{\alpha-1}{j} (-1)^{\alpha-1-j} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + j\tilde{Q}_T} \right) - S_{1t}^T. \quad (27)$$

Proof. The single-dividend paying securities price P_1^T satisfies

$$\begin{aligned} P_{1t}^T &= e^{-\Delta(T-t)} e^{\alpha(\zeta_{1t} - Q_t)} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + \alpha Q_T} \right) \\ &= e^{-\Delta(T-t)} \left(\frac{e^{\zeta_{1t}}}{e^{\tilde{Q}_t} - 1} \right)^\alpha \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T}} \left(e^{\tilde{Q}_T} - 1 \right)^\alpha \right) \\ &= e^{-\Delta(T-t)} \left(\frac{e^{\zeta_{1t}}}{e^{\tilde{Q}_t} - 1} \right)^\alpha \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T}} \sum_{j=0}^{\alpha} \binom{\alpha}{j} (-1)^{\alpha-j} e^{j\tilde{Q}_T} \right) \\ &= e^{-\Delta(T-t)} \left(\frac{e^{\zeta_{1t}}}{e^{\tilde{Q}_t} - 1} \right)^\alpha \sum_{j=0}^{\alpha} \binom{\alpha}{j} (-1)^{\alpha-j} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + j\tilde{Q}_T} \right). \end{aligned} \quad (28)$$

Similarly, P_2^T satisfies

$$\begin{aligned} P_{2t}^T &= e^{-\Delta(T-t)} e^{\alpha(\zeta_{1t} - Q_t)} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + (\alpha-1)Q_T} \right) - P_{1t}^T \\ &= e^{-\Delta(T-t)} \left(\frac{e^{\zeta_{1t}}}{e^{\tilde{Q}_t} - 1} \right)^\alpha \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T}} \left(e^{\tilde{Q}_T} - 1 \right)^{\alpha-1} \right) - P_{1t}^T \\ &= e^{-\Delta(T-t)} \left(\frac{e^{\zeta_{1t}}}{e^{\tilde{Q}_t} - 1} \right)^\alpha \sum_{j=0}^{\alpha-1} \binom{\alpha-1}{j} (-1)^{\alpha-1-j} \mathbb{E}_t \left(e^{(1-\alpha)\zeta_{1T} + j\tilde{Q}_T} \right) - P_{1t}^T. \end{aligned} \quad (29)$$

□

We now proceed with the approximation method that allows us to compute the transforms appearing in Equations (28) and (29). Let the state-vector x be defined by

$$\begin{aligned} x &\equiv (x_i)_{i=1}^8 \\ &= \left(\zeta_1 \quad \tilde{Q} \quad \hat{f}_1 \quad \hat{f}_2 \quad \pi_1 \quad \pi_2 \quad \gamma_1 \quad \gamma_2 \right)^\top, \end{aligned}$$

with the dynamic

$$dx_t \equiv \mu(x_t) + \sigma(x_t)dW_t. \quad (30)$$

In Equation (30), the state-vector x has a non-affine dynamic with a non-affine drift $\mu(x)$ and a non-affine variance-covariance matrix $\sigma(x)\sigma(x)^\top$. Given the structure of $\mu(x)$ and $\sigma(x)\sigma(x)^\top$, the augmented state-vector X is chosen to be

$$\begin{aligned} X &\equiv (X_i)_{i=1}^{17} \\ &= \begin{pmatrix} \zeta_1 & \tilde{Q} & \hat{f}_1 & \hat{f}_2 & \pi_1 & \pi_2 & \gamma_1 & \gamma_2 & \dots \\ \dots & \tilde{Q}^2 & \tilde{Q}\hat{f}_1 & \tilde{Q}\hat{f}_2 & \tilde{Q}\gamma_1 & \tilde{Q}\gamma_2 & \pi_1^2 & \pi_2^2 & \gamma_1^2 & \gamma_2^2 \end{pmatrix}^\top \\ dX_t &\equiv \mu(X_t) + \sigma(X_t)dW_t. \end{aligned}$$

Approximated expressions for the augmented drift $\mu(X)$ and the variance-covariance matrix $\Sigma(X) \equiv \sigma(X)\sigma(X)^\top$ are derived using a Taylor expansion around the reference vector x_0 . We discuss the procedure below in Definition 5.

Definition 5. *The reference vector x_0 satisfies*

$$\begin{aligned} x_{02} &= \log(1.5) & x_{03} &= x_{04} = \bar{f} \\ x_{05} &= x_{06} = 0 & x_{07} &= x_{08} \equiv \gamma_{ss}. \end{aligned}$$

Note that x_{01} is not defined because ζ_1 neither shows up in the drift $\mu(X)$ nor in the variance-covariance matrix $\Sigma(X)$. \bar{f} is the long-term mean of \hat{f}_1 and \hat{f}_2 , 0 is the long-term mean of π_1 and π_2 , and $\gamma_{ss} = \sigma_\delta \sqrt{\sigma_f^2(1 - \Psi^2) + \lambda^2\sigma_\delta^2 - \lambda\sigma_\delta^2}$ is the uncertainty conditional on $\pi_i = 0$. The derivations of the long-term means are provided in Appendix A.1. The long-term uncertainty γ_{ss} is computed in Appendix A.2.

The drift $\mu(X)$ and the variance-covariance matrix $\Sigma(X)$ are expanded around the reference vector x_0 defined in 5. More precisely, $\mu(X)$ and $\Sigma(X)$ are written

$$\begin{aligned} \mu(X) &\approx K_0 + K_1 X \\ \Sigma(X) &\approx H_0 + \sum_{i=1}^{17} H_i X_i, \end{aligned}$$

where K_1 and H_i , $i = 0, \dots, 17$ are 17-dimensional squared matrices and K_0 a 17-dimensional vector. K_0 , K_1 , and H_i , $i = 0, \dots, 17$ are available upon request.

Using the approximation, the theory on affine processes applies. Following Duffie, Pan, and Singleton (2000), the transforms defined in Equations (26) and (27) are approximated by

$$\mathbb{E}_t \left(e^{\epsilon \zeta_{1T} + \chi \tilde{Q}_T} \right) \approx e^{\bar{\alpha}(T-t) + \sum_{i=1}^{17} \bar{\beta}_i(T-t) X_i}, \quad (31)$$

where the functions $\bar{\alpha}(\cdot)$ and $\bar{\beta}_i(\cdot)$, $i = 1, \dots, 17$, solve a set of 18 Riccati equations subject to $\bar{\alpha}(0) = 0$, $\bar{\beta}_1(0) = \epsilon$, $\bar{\beta}_2(0) = \chi$, and $\bar{\beta}_i(0) = 0$, $i = 3, \dots, 17$.

The system of Riccati equations is

$$\begin{aligned} \bar{\beta}'(\tau) &= K_1^\top \bar{\beta}(\tau) + \frac{1}{2} \bar{\beta}(\tau)^\top H_+ \bar{\beta}(\tau) \\ \bar{\alpha}'(\tau) &= K_0^\top \bar{\beta}(\tau) + \frac{1}{2} \bar{\beta}(\tau)^\top H_0 \bar{\beta}(\tau), \end{aligned}$$

where $\tau = T - t$.¹³ The set of Riccati equations is solved numerically. Then, substituting Equation (31) in Equations (26) and (27) determines the single-dividend paying securities prices P_1^T and P_2^T . As described in Equation (17), stock prices are obtained by numerically integrating over the single-dividend paying securities.

¹³Note that the matrix H_+ is 3-dimensional. It consists in the concatenation of the matrices H_i , $i = 1, \dots, 17$. This notation is used to avoid writing an equation for each $\bar{\beta}$.

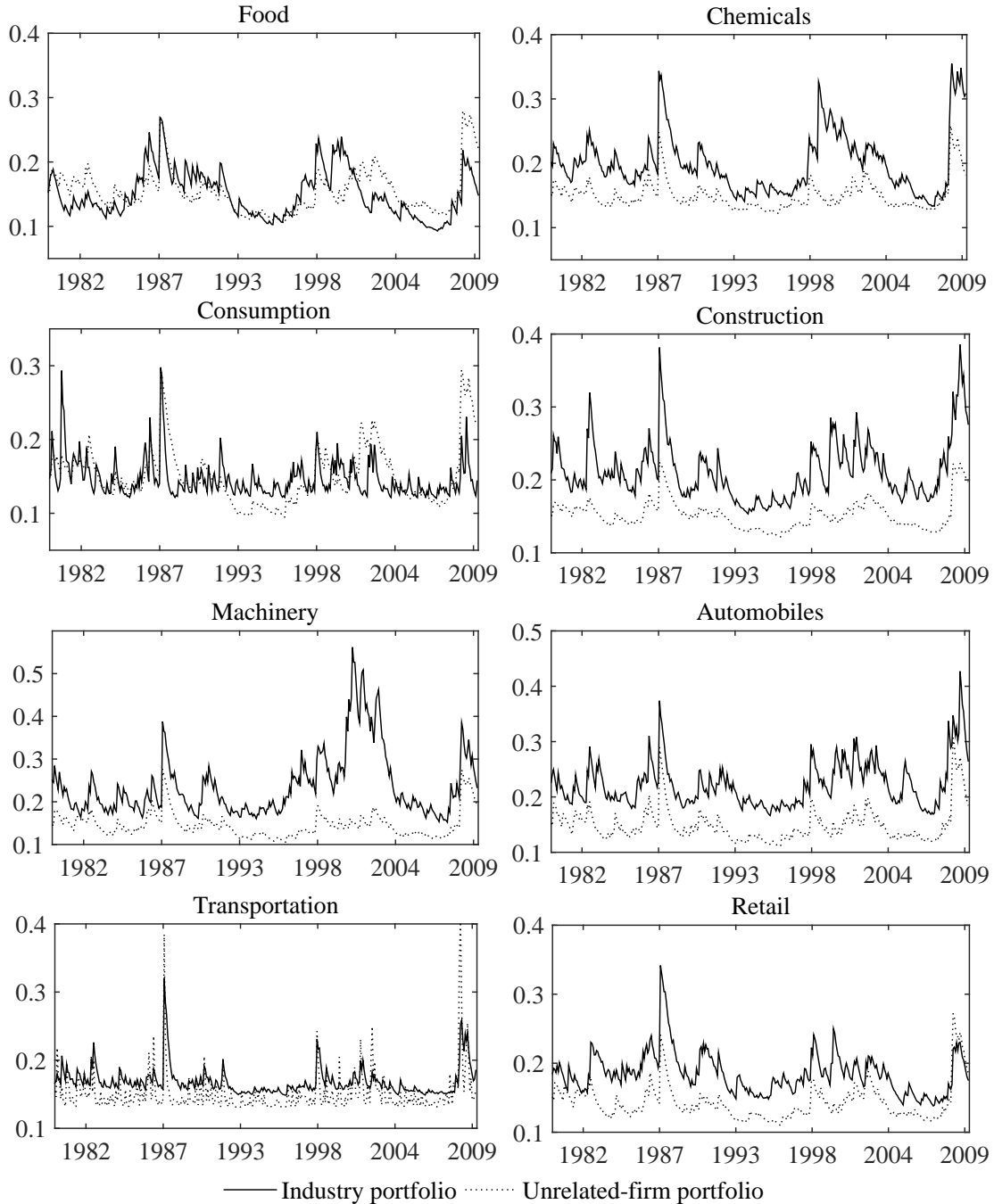


Figure 6: Conditional volatility

We plot monthly conditional volatilities, in annualized terms, of eight industry portfolios and their corresponding unrelated-firm portfolios. We assume that monthly portfolio return follows an AR(1) process and model its volatilities using GARCH(1,1). We define industry sectors following the Fama-French classifications (see Table 3). The title of each panel indicates the name of the industry portfolio. A portfolio of firms unrelated to each industry, i.e. *Unrelated portfolio*, is identified using U.S. supplier-customer relationships data from 1980 through 2009. We use size-weighted averages to calculate monthly returns of firms in each portfolio. Portfolio weights are re-balanced once per year. In each panel, the solid line plots conditional volatilities of the industry portfolio, while the dotted line plots conditional volatilities of the unrelated-firm portfolio.

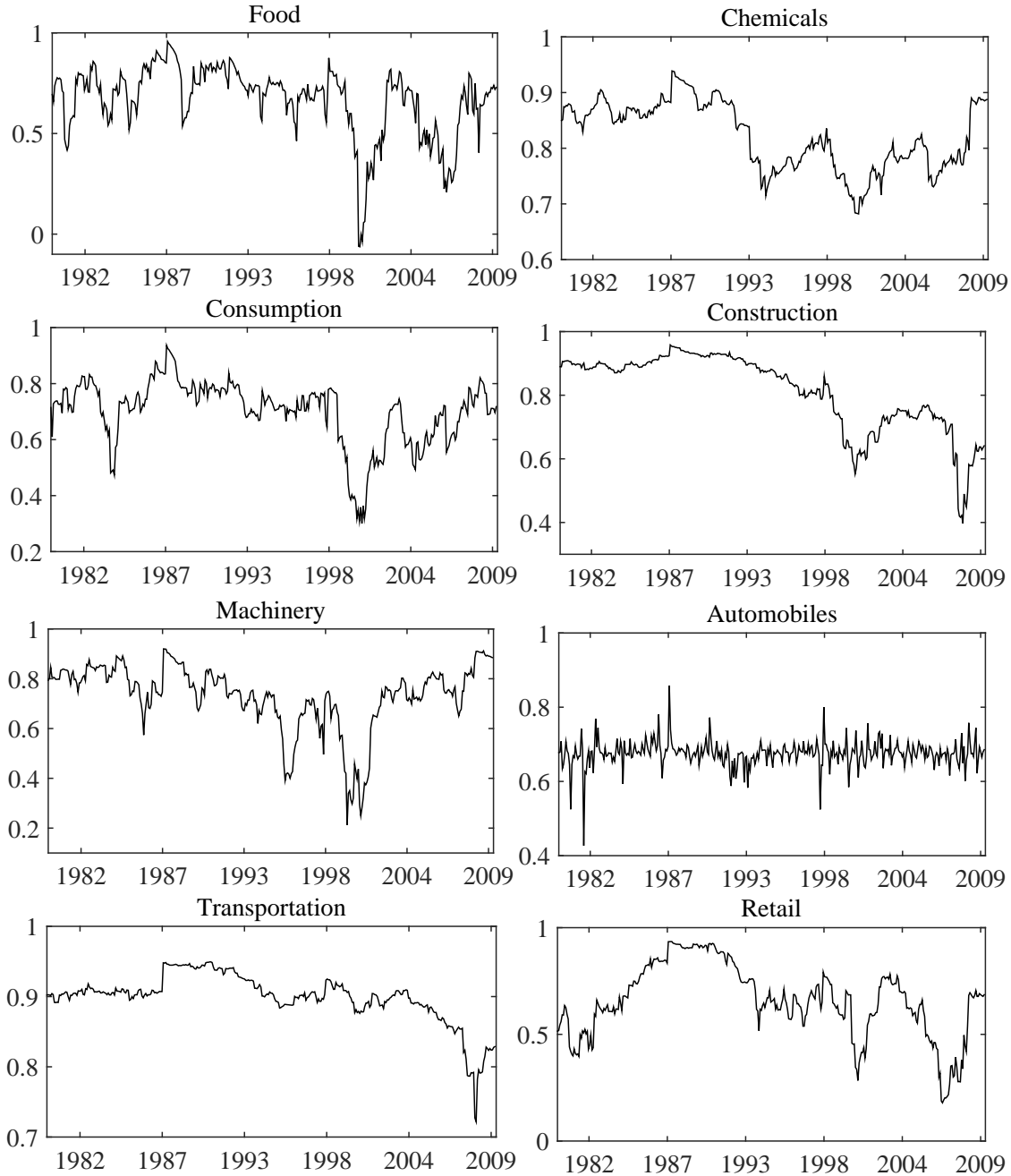


Figure 7: Conditional correlation

We plot monthly conditional correlations between eight industry portfolios and their corresponding unrelated-firm portfolios. Conditional correlations for each portfolio pair are estimated using the Dynamic Conditional Correlation (DCC) model (Engle (2002)). We define industry sectors following the Fama-French classifications (see Table 3). The title of each panel indicates the name of the industry portfolio. A portfolio of firms unrelated to each industry, i.e. *Unrelated-firm portfolio*, is identified using U.S. supplier-customer relationships data from 1980 through 2009. Monthly portfolio returns are calculated using size-weighted averages. Portfolio weights are re-balanced once per year.

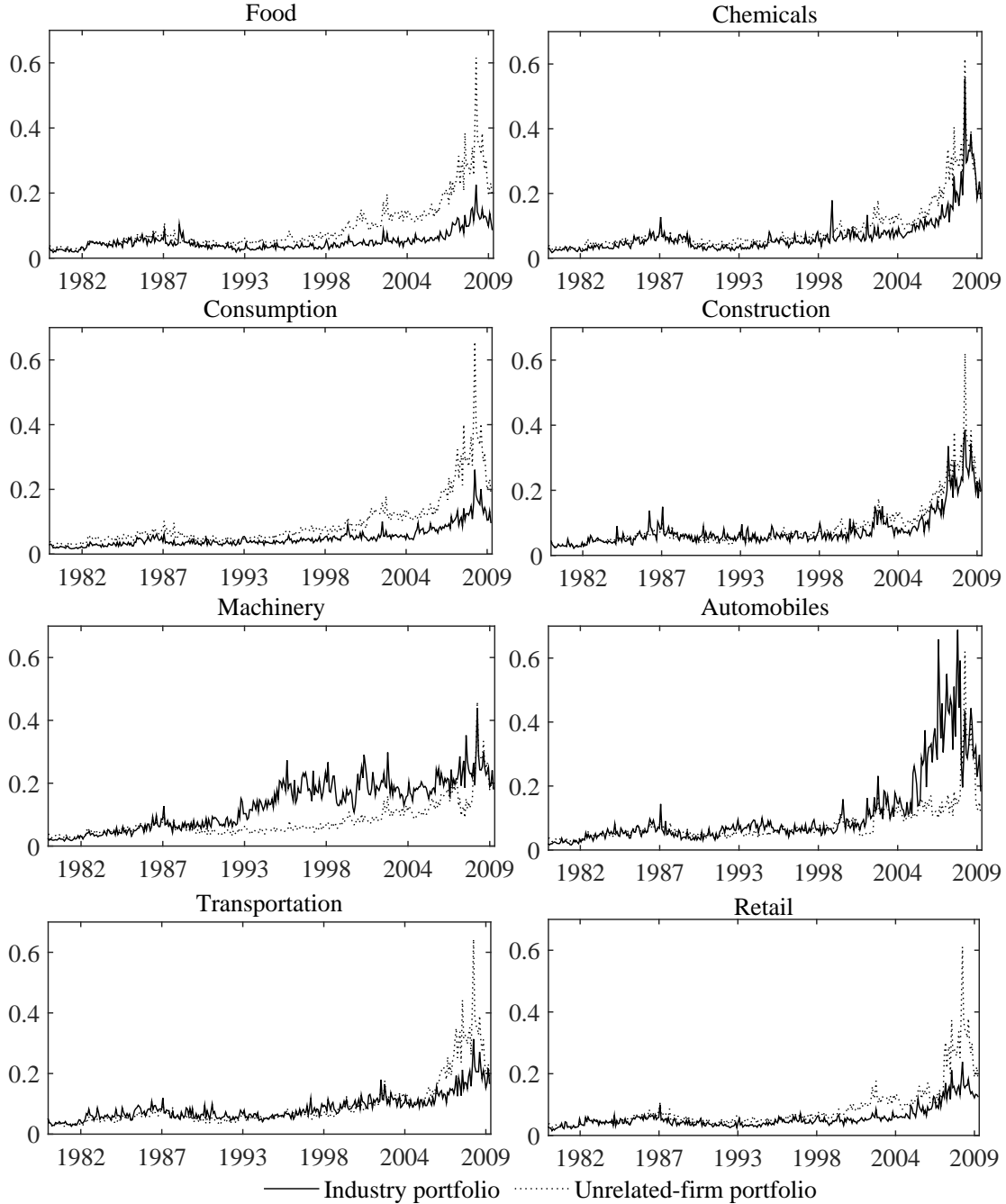


Figure 8: Time-varying attention

We plot monthly share turnover of eight industry portfolios and their corresponding unrelated-firm portfolios. Monthly share turnover for a firm is calculated by dividing its monthly trading volume by its number of common shares outstanding. Portfolio share turnover is calculated by size-weighted averaging share turnover across firms. Portfolio weights are re-balanced once per year. The title of each panel indicates the name of the industry portfolio. We define industry sectors following the Fama-French classifications (see Table 3). A portfolio of firms unrelated to each industry, i.e. *Unrelated-firm portfolio*, is identified using U.S. supplier-customer relationships data from 1980 through 2009. In each panel, the solid line plots share turnover of the industry portfolio, while the dotted line plots share turnover of the unrelated-firm portfolio.

Panel A. Number of firms: Industry portfolio versus unrelated-firm portfolio

Industry name	Number of firms in the portfolio each month					
	Industry portfolio			Unrelated-firm portfolio		
	Median	Min	Max	Median	Min	Max
Food*	84	62	109	977	732	1436
Mining	24	16	35	1079	791	1641
Oil & petroleum	73	34	116	1018	746	1457
Clothing & textiles	49	27	73	1045	762	1469
Consumer durables	38	17	61	1030	785	1460
Chemicals	51	37	70	993	729	1421
Consumption*	126	73	179	950	719	1385
Construction*	94	68	124	954	716	1360
Steel	43	27	68	1035	752	1467
Fabricated products	25	15	35	1036	783	1483
Machinery*	245	132	386	890	636	1235
Automobiles*	42	19	66	1037	789	1478
Transportation*	223	119	330	831	605	1185
Utilities*	153	106	177	933	658	1418
Retail*	161	82	219	912	672	1320

Panel B. Aggregate portfolio characteristics: Industry portfolio versus unrelated-firm portfolio

Industry name	Total portfolio characteristics (annualized average in millions \$)					
	Industry portfolio			Unrelated-firm portfolio		
	Size	Earnings	Dividend	Size	Earnings	Dividend
Food*	133.26	6.41	2.77	639.38	38.53	19.49
Mining	13.31	0.53	0.33	836.12	47.19	22.50
Oil & petroleum	44.06	1.97	1.08	702.35	41.32	20.12
Clothing & textiles	13.12	0.90	0.24	709.13	42.61	21.27
Durables	11.71	0.61	0.25	705.68	42.56	21.15
Chemicals	71.53	4.73	2.33	635.77	38.27	19.28
Consumption*	424.36	24.52	12.35	549.36	33.05	16.62
Construction*	70.60	3.86	1.28	636.74	38.16	19.31
Steel	23.18	1.36	0.52	697.64	41.77	20.93
Fabricated products	8.50	0.48	0.18	680.80	40.91	20.69
Machinery*	217.84	9.70	3.28	597.76	37.12	19.03
Automobiles*	72.15	6.26	2.33	702.04	42.16	21.14
Transportation*	177.89	8.79	3.00	554.38	34.56	18.10
Utilities*	189.60	14.27	10.51	547.87	29.48	12.04
Retail*	220.09	10.63	2.85	661.68	40.48	20.77

Table 3: Summary statistics of portfolio pairs

This table presents summary statistics of the 15 industry portfolios and their corresponding unrelated-firm portfolios. The sample consists of U.S. firms in the CRSP database from 1980 through 2009 with non-missing information in COMPUSTAT. See text in Section 4.1 for additional filters applied to the data. We classify firms into different industries following the Fama-French 17-industry classifications. Industries labeled under "Financial" and "Others" are excluded. We refer to a portfolio of firms classified under one of the Fama-French industries as the *industry portfolio*. We use U.S. customer-supplier relationships data to identify a portfolio of firms that have unrelated business connections with firms in the industry portfolio; we refer to it as the *unrelated-firm portfolio*. For firms to be unrelated, we require that they have at least six degrees of separations between them in the relationships database. The U.S. customer-supplier relationships data are updated annually. Industry portfolios and their unrelated-firm portfolios are formed annually. (continued on next page...)

Table 3: (Continued) Panel A summarizes the median, minimum, and maximum number of firms that enter each portfolio from 1980 through 2009. In Panel B, we report time-series means of aggregate portfolio characteristics. For each portfolio, we calculate its total market capitalization (i.e. size), total earnings, and total dividend payout. These characteristics are obtained from the COMPUSTAT Annual Fundamentals database, and aggregated across firms in each portfolio. All values are reported in units of one million U.S. dollars. The model setup proposed in Section 2 provides predictions of contagion between two entities that have similar magnitude of cash flows. In order to make our empirical design similar to the model, we exclude portfolio pairs that greatly differ in size and earnings cash flow. We require that the average size of the industry portfolio is at least 10 percent relative to the average size of its unrelated-firm portfolio. There are nine portfolio pairs that meet the requirement. We denote them with an asterisk next to their industry name. We estimate conditional volatilities and correlations of these nine portfolio pairs using the GARCH(1,1) model and the DCC-GARCH(1,1) model, respectively. Each estimation consists of 380 monthly observations. Maximum likelihood estimation for the *Utilities* industry pair does not converge and are therefore removed from further analyses. As a result, eight portfolio pairs shown in Figure 6 are used in the empirical tests.

	Dependent variables		
	$\Delta\sigma_{1,t}$	$\Delta\sigma_{2,t}$	$\Delta\rho_{12,t}$
	(I)	(II)	(III)
<i>Time-varying attentions</i>			
$\Delta\Phi_{1,t}$	0.045*** (10.40)	0.015*** (4.53)	0.074** (2.23)
$\Delta\Phi_{2,t}$	0.021*** (5.07)	0.052*** (15.79)	0.130*** (4.08)
$\Delta\Phi_{1,t-1}$	-0.024*** (-3.58)	-0.010*** (-2.74)	-0.032 (-0.89)
$\Delta\Phi_{2,t-1}$	-0.016*** (-3.31)	0.004 (0.85)	0.067* (1.80)
<i>ARMA coefficients</i>			
AR(1)	0.961*** (4.82)	0.477*** (10.59)	0.196* (1.67)
AR(2)	-0.348*** (-4.00)	-0.079*** (-4.32)	
MA(1)	1.299*** (10.57)	0.659*** (16.09)	0.303** (2.33)
MA(2)	-0.420*** (-4.47)		
Model	ARMAX(2,2,1)	ARMAX(2,1,1)	ARMAX(1,1,1)
AICC	-10.62	-11.11	-6.53

Table 4: Fluctuating attention and contagion

This table reports maximum likelihood estimates of the autoregressive moving-average model with exogenous variables (ARMAX). The general regression model is:

$$\Delta y_t = \alpha + \beta_0' \Delta\Phi_t + \beta_1' \Delta\Phi_{t-1} + \sum_{j=1}^p \gamma_j \Delta y_{t-j} + \sum_{k=1}^q \theta_k \varepsilon_{t-k} + \varepsilon_t,$$

where Δy_t represents the dependent variable at time t , and $\Delta\Phi_t = [\Delta\Phi_{1,t}, \Delta\Phi_{2,t}]'$ is a vector of exogenous variables that proxy for monthly change in investor attention to the industry portfolio, $\Delta\Phi_{1,t}$, and its unrelated-firm portfolio, $\Delta\Phi_{2,t}$. β_0 and β_1 are 2×1 vectors. We estimate the model in a panel on eight pairs of industry portfolios and unrelated-firm portfolios. Figures 6 and 7 plot their time-series dynamics. The sample period is from 1980 through 2009, and the estimation frequency is monthly. A change in investor attention for each portfolio in month t is measured as the difference between portfolio share turnover levels in the current and previous months. Monthly share turnover for a firm is calculated by dividing its monthly trading volume by its number of common shares outstanding. Portfolio share turnover is calculated as the size-weighted average share turnover across firms. Portfolio weights are re-balanced once per year. The dependent variables in regression specifications (I) and (II) are monthly changes in conditional volatilities of the industry portfolio, $\Delta\sigma_{1,t}$, and the unrelated-firm portfolio, $\Delta\sigma_{2,t}$, respectively. In regression specification (III), the dependent variable is the monthly change in conditional correlations between the industry portfolio and its unrelated-firm portfolio, $\rho_{12,t}$. The number of lags in the ARMA representation is determined using the Akaike information criterion (AICC) and the Portmanteau test. The intercepts, α , is estimated but not reported. T-statistic is reported in brackets below each estimate. ***, **, * denote statistical significance at the confidence levels of 1, 5, and 10 percent, respectively.

	Dependent variables		
	$\Delta\sigma_{1,t}$	$\Delta\sigma_{2,t}$	$\Delta\rho_{12,t}$
	(I)	(II)	(III)
<i>Time-varying attentions</i>			
$\Delta(\Phi_{1,t} + \Phi_{2,t})$	0.032*** (15.71)	0.032*** (20.58)	0.103*** (6.66)
$\Delta(\Phi_{1,t-1} + \Phi_{2,t-1})$	-0.019*** (-4.56)	-0.003 (-1.39)	0.019 (0.85)
<i>ARMA coefficients</i>			
AR(1)	1.175*** (9.46)	0.526*** (11.45)	0.198* (1.66)
AR(2)	-0.341*** (-3.99)	-0.081*** (-4.36)	
MA(1)	1.282*** (10.34)	0.689*** (16.44)	0.305** (2.30)
MA(2)	-0.412*** (-4.46)	-0.096 (-0.98)	
Model	ARMAX(2,1,1)	ARMAX(2,1,1)	ARMAX(1,1,1)
AICC	-11.11	-11.09	-6.53

Table 5: Fluctuating *total* attention and contagion

This table reports maximum likelihood estimates of the autoregressive moving-average model with exogenous variable (ARMAX). The general regression model is:

$$\Delta y_t = \alpha + \beta_0 \Delta(\Phi_{1,t} + \Phi_{2,t}) + \beta_1 \Delta(\Phi_{1,t-1} + \Phi_{2,t-1}) + \sum_{j=1}^p \gamma_j \Delta y_{t-j} + \sum_{k=1}^q \theta_k \varepsilon_{t-k} + \varepsilon_t,$$

where Δy_t represents the dependent variable at time t , and $\Delta(\Phi_{1,t} + \Phi_{2,t})$ is the monthly change in total investor attention to the industry portfolio, $\Delta\Phi_{1,t}$, and its unrelated-firm portfolio, $\Delta\Phi_{2,t}$. β_0 and β_1 are scalars. We estimate the model in a panel on eight pairs of industry portfolios and unrelated-firm portfolios. Figures 6 and 7 plot their time-series dynamics. The sample period is from 1980 through 2009, and the estimation frequency is monthly. A change in investor attention to each portfolio in month t is measured as the difference between portfolio share turnover levels in the current and previous months. Monthly share turnover for a firm is calculated by dividing its monthly trading volume by its number of common shares outstanding. Portfolio share turnover is calculated as the size-weighted average share turnover across firms. Portfolio weights are re-balanced once per year. The dependent variables in regression specifications (I) and (II) are monthly changes in conditional volatilities of the industry portfolio, $\Delta\sigma_{1,t}$, and the unrelated-firm portfolio, $\Delta\sigma_{2,t}$, respectively. In regression specification (III), the dependent variable is the monthly change in conditional correlations between the industry portfolio and its unrelated-firm portfolio, $\rho_{12,t}$. The number of lags in the ARMA representation is determined using the Akaike information criterion (AICC) and the Portmanteau test. The intercept, α , is estimated but not reported. T-statistic is reported in brackets below each estimate. ***, **, * denote statistical significance at the confidence levels of 1, 5, and 10 percent, respectively.

	Panel A			Panel B		
	Dependent variables			Dependent variables		
	$\Delta\sigma_{1,t}$	$\Delta\sigma_{2,t}$	$\Delta\rho_{12,t}$	$\Delta\sigma_{1,t}$	$\Delta\sigma_{2,t}$	$\Delta\rho_{12,t}$
(I)	(II)	(III)	(IV)	(V)	(VI)	
<i>Time-varying attentions</i>						
$\Delta\Phi_{1,t}$	0.044*** (10.40)	0.016*** (4.69)	0.062* (1.86)			
$\Delta\Phi_{2,t}$	0.019*** (4.55)	0.052*** (15.68)	0.126*** (3.91)			
$\Delta\Phi_{1,t-1}$	-0.002 (-0.32)	-0.004 (-1.11)	-0.062* (-1.67)			
$\Delta\Phi_{2,t-1}$	-0.006 (-0.83)	-0.005 (-0.94)	0.035 (0.73)			
$\Delta(\Phi_{1,t} + \Phi_{2,t})$				0.031*** (14.36)	0.035*** (21.55)	0.089*** (5.73)
$\Delta(\Phi_{1,t-1} + \Phi_{2,t-1})$				-0.005 (-1.11)	-0.007** (-2.23)	-0.037 (-1.61)
<i>ARMA coefficients</i>						
$\Delta\sigma_{1,t-1}$	0.616*** (10.40)	-0.127*** (-2.73)	-0.065 (0.38)	0.450*** (2.69)	-0.235* (-1.78)	0.424 (0.39)
$\Delta\sigma_{2,t-1}$	-0.012 (-0.16)	0.568*** (11.31)	-1.512 (-0.17)	0.094 (0.77)	0.710*** (7.16)	-1.154 (-1.56)
$\Delta\rho_{12,t-1}$	0.042*** (3.11)	0.016 (1.35)	0.775*** (8.77)	0.073** (2.25)	0.039 (1.31)	0.705*** (3.19)
$u_{1,t-1}$	0.721*** (12.30)	-0.139*** (-2.97)	0.228 (0.62)	0.550*** (3.25)	-0.232* (-1.75)	0.802 (0.75)
$u_{2,t-1}$	-0.004 (-0.07)	0.797*** (18.11)	-1.397*** (-3.57)	0.104 (0.94)	0.902*** (10.24)	-1.228* (-1.87)
$u_{3,t-1}$	0.040*** (3.16)	0.014 (1.27)	0.824*** (10.80)	0.071** (2.21)	0.036 (1.25)	0.750*** (3.65)
Model	VARMAX(1,1,1)			VARMAX(1,1,1)		
AICC	-28.68			-28.64		

Table 6: Fluctuating attention and contagion: Multivariate model

This table reports maximum likelihood estimates of the vector autoregressive moving-average model with exogenous variables (VARMAX). The model is estimated in panel on eight pairs of industry portfolio and its unrelated-firm portfolio. The sample period is from 1980 through 2009. Figures 6 and 7 plot their time-series dynamics. We estimate the VARMAX model with lags $p = 1$ and $q = 1$. We allow for one lag of exogenous variable. Specifically, the regression model is:

$$\Delta Y_t = \Lambda + \beta_0 \Delta \Phi_t + \beta_1 \Delta \Phi_{t-1} + \Gamma_1' \Delta Y_{t-1} + \Theta_1' \epsilon_{t-1} + \epsilon_t,$$

where $\Delta Y_t = [\Delta\sigma_{1,t}, \Delta\sigma_{2,t}, \Delta\rho_{12,t}]'$ is a vector of monthly changes in volatilities and correlations for a portfolio pair: the industry portfolio, its unrelated-firm portfolio. The coefficients β_0 and β_1 are 3×2 matrix that capture the relationships between ΔY_t and changes in investor attention during the current and previous months, i.e., $\Delta\Phi_t$ and $\Delta\Phi_{t-1}$. (continued on next page...)

Table 6: (Continued) The vector of residual is represented by $\epsilon_t = [u_{1,t}, u_{2,t}, u_{3,t}]'$. The 2×1 vectors Γ_1 and Θ_1 are coefficient estimates on autoregressive and moving-average terms. In Panel A, we report estimation results from a VARMAX(1,1,1) model where the exogenous variable is a vector of monthly changes in investor attention to eight pairs of industry portfolios and unrelated-firm portfolios. That is, i.e., $\Delta\Phi_t = [\Delta\Phi_{1,t}, \Delta\Phi_{2,t}]'$, where $\Delta\Phi_{1,t}$ is the monthly change in attention to the industry portfolio, and $\Delta\Phi_{2,t}$ is the monthly change in attention to the unrelated-firm portfolio. In Panel B, we estimate the VARMAX(1,1,1) model where the exogenous variable is the monthly change in total attention to the portfolio pair, i.e., $\Delta\Phi_t = \Delta\Phi_{1,t} + \Delta\Phi_{2,t}$. A change in investor attention to each portfolio in month t is measured as the difference between portfolio share turnover levels in the current and previous months. Monthly share turnover for a firm is calculated by dividing its monthly trading volume by its number of common shares outstanding. Portfolio share turnover is calculated as the size-weighted average share turnover across firms. Portfolio weights are re-balanced once per year. The intercept, Λ , is estimated but not reported. T-statistic is reported in brackets below each estimate. ***, **, * denote statistical significance at the confidence levels of 1, 5, and 10 percent, respectively.