User-Generated Capital and Firm Value: Theory and Evidence from Internet Firms

Erik Brynjolfsson (MIT Center for Digital Business and NBER)

Seon Tae Kim (ITAM Business School)

JooHee Oh (MIT Center for Digital Business)

Abstract

This paper studies the mechanism of users' online activities and their implications for the firm value, and quantifies them for Internet firms that have been recently growing fast. Users collectively spend billions of person-hours creating user-generated content which is then becomes an asset that produces a stream of value for fellow users, and of course, the host site itself, labeled *user-generated capital*. Using the firm-level characteristics and data on users' online activities, we find that an Internet firm's value is significantly positively associated with users' time spent on the Internet firm's own website(s). Building and studying a dynamic general equilibrium model of users' time allocation among competing websites, we find that improvements in either the website quality—i.e., the firm's platform capital—or the efficiency of user-capital investment immediately increase the Internet firm's value by shifting up users' time spent on the website in the future. Using the calibrated model, we carry out a counterfactual experiment of completely removing user-generated capital now and temporarily keeping it to a low level in the near future; we find that in this case, user-generated capital account for about 60 percent of the Internet firm's value.

Keywords: User-generated content, Firm value, Intangibles, User capital, Internet firms.

I. Introduction

Recent years have witnessed rapid growth in household time spent online. At the same time, many Internet business firms have exhibited very high growth. For instance, the annual revenue of Facebook grew from $0.78 billion in 2009 to $3.71 billion in 2011. Interestingly, during the same period 2009-2011, such high growth of Facebook's revenue came without much growth in the expenditures on tangible assets and employees. Instead, it was associated with the increased user contributions on Facebook, of which annual growth rate is about 200 percent. As exemplified by the Facebook case, growth opportunities of many Internet firms are mainly determined by the user activities, in particular, the users' contribution of time and content.[1] Users of websites collectively spend billions of person-hours creating "user-generated capital" which then becomes an asset that produces a stream of value for fellow users, and of course, the host site itself. We are living unique time in history where more than 2 billion people are not only connected by digital structure. This gives them access to a wealth of information. But what is less often noted is that it also has created a platform for billions of people to contribute to the world's stock of knowledge and entertainment. Users do this by creating and uploading photos, videos, written posts and other content, creating a set of assets that are valuable not only to fellow users, but also to the host internet firms which typically find ways to monetize their traffic. Given the amount of time users spend consuming content created by other users, and the prodigious valuations of Internet companies despite having few employees, or physical assets, it is plausible that the economic and business value of user-generated capital is quite high. Until now, no research has studied this contribution made by users and its impact on firm value. This paper examines these issues both empirically and theoretically to address the questions as follows: What is the mechanism of the formation of user-generated capital? How much does user-generated capital contribute to the value of an Internet firm? How can an Internet firm effectively elicit users' investment of time so as to increase the firm's own value?

This paper contributes to the literature that studies intangibles and firm value. Compared to the traditional physical-asset-intensive firms, a new type of firms such as an Internet firm is affected more by intangible assets for their core engines of growth (Zingales 2000, Jorgenson and Stiroh 2000, Brynjolfsson et al. 2002, Jorgenson et al. 2002, Corrado et al. 2009). Unfortunately, intangibles are difficult to measure: both their quantities and returns, which is a main stumbling block for the study of the formation process and valuation mechanism of intangibles.[2] Accordingly, there has been recently new attempts to quantify particular classes of intangibles, e.g., customer, R&D, organization, and brand capital (Gupta et al. 2004, McGrattan and Prescott 2010, Lin 2012, Eisfeldt and Papanikolaou 2013, Belo et al.

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[1] Note that almost all of content generation on websites such as Facebook, Youtube and Wikipedia comes from the efforts of their users rather than from employees of Internet firms owning such websites.
[2] See the 'intellectual capital' literature for categorizing and measuring various components of intangibles.

2013). We extend this approach by defining and measuring a new category of intangibles: *user-generated capital*, which measures online user engagement and is believed by many experts to be crucial for an Internet firm's business and its value (Ahn and Dabbish 2008, Shirky 2011). Moreover, we also propose an integrated equilibrium framework to quantify the contribution of user-generated capital to the market value of an Internet firm. More specifically, we document stylized facts on the relationship between users' online activities and market value of an Internet firm and study it, both analytically and quantitatively, in a dynamic general equilibrium model. In particular, we discuss incentives and constraints related to users' investment as well as consumption of content on Internet so that rich implications of dynamics of user-generated capital are fully understood.

We begin by analyzing the relationship between a firm's market value and users' online activities for a number of Internet and media companies. Using the COMPUSTAT annual files over the period 2000-2012 and data on users' time spent on websites, we find that for the case of Internet-exclusive media firms, the firm value—both in terms of the level of market value and Tobin's *q* ratio—is significantly positively associated with hours spent by users on the firm's own website. For instance, one-percentage increase in users' hours spent on an Internet firm's website is associated with an increase in the firm's market value by about 0.12 percent.

We, in turn, develop a model of the user's time allocation among a number of competing websites. In the model, the household makes the decision on consumption of Internet vs. non-Internet good where the Internet good is a constant-elasticity-of-substitution (CES) aggregate of a continuum of varieties, i.e., differentiated websites. Moreover, the household also makes the decision on investment of Internet good e.g., uploading pictures, so as to increase the future amount of online content. Thus, the household determines how much time to spend on three types of activities: working, online content consumption, and online content investment. An Internet firm owns a unique website and monopolistically competes with other Internet firms (Dixit and Stiglitz 1977) to attract users' attention/time by improving its intangible capital, i.e., the quality of its website: a user-friendly and high-quality website can attract a large number of users that actively generate content. An Internet firm's production function is Cobb-Douglas of two inputs: tangible capital and the household consumption-purpose time spent on the firm's own website, capturing the fact that an Internet firm's revenue source is largely advertising. Two key features are introduced: (i) the household utility of per-hour Internet consumption is increasing in the user-generated capital stock (i.e., the effective amount of content) on a given website and (ii) the website-level next-period user-generated capital stock is increasing jointly in the website-owning firm's intangible capital (i.e., the website quality) and the household investment-purpose hours spent on the website.

In the model, the household consumption-purpose hours spent on a website are, in equilibrium, increasing in the level of user-generated capital available on the website, and the household investment-purpose hours are (in equilibrium) increasing in the level of the Internet firm's intangible capital. Taking the household decision rules as given, an Internet firm optimally accumulates tangible and intangible capital. For a given Internet firm, improvements in either intangible capital or efficiency of user content generation immediately increase the Internet firm's value by shifting up the future expected household time spent on the firm's own website. The key mechanism is dynamic and as follows: Consider an increased efficiency of investment in user-generated capital: the firm's intangible capital can generate the larger effective amount of content, holding all else constant. In this case, the firm's intangible return shifts up overall, and hence the firm increases intangible investment now; then user-generated capital increases in the future, and so does the household consumption-purpose time. Thus, the firm's current value immediately jumps up greatly, consistent with the recent observations for many Internet firms.

We let the model confront the data on firm-level financial variables and user engagement. Results of our empirical analysis provide suggestive evidence supporting the key mechanism of the model. First, we find that indeed, the market value of an Internet-exclusive media firm is higher than non-Internet media firm of the same size, and more so for Internet firms of the higher user engagement than those of the lower user engagement. Second, by decomposing user engagement, we find that the higher is investment-related user engagement, e.g., the number of posts in the company's Facebook page, the greater is consumption-related user engagement, e.g., the viewership of the company's own website. That is, user engagement is significantly related to an Internet firm's online engagement.

We then calibrate the model to the U.S. data over the period 2000-2002: (i) Internet firms' balance sheets and income statements and (ii) household time allocation on Internet. Quantitative properties of the calibrated model are consistent with analytic results of the model. Such a calibrated model is then used in counterfactual experiments to quantify the importance of user-generated capital in accounting for the Internet firm's observed market value. More specifically, we carry out a counterfactual experiment in which an Internet firm's user-generated capital is completely removed now and kept to stay in a low level temporarily in the near future. We find that in this case, the Internet firm's value drops down substantially, by about 60 percent relative to the steady-state firm value. Moreover, we also consider the case in which the efficiency of investment in user-generated capital permanently worsens by 20 percent so that user-generated capital is permanently reduced by 35 percent: in this case, the firm value greatly decreases, by about 48 percent. To the best of our knowledge, this paper is the first to quantify the contribution of user-generated capital to the Internet firm's value by using a dynamic general equilibrium model. By doing this, we provide a tool to measure the value of free goods/services on Internet beyond measuring consumer surplus: essentially we take into consideration producer surplus. Calculating the contribution of user-generated capital to the Internet firm's value, as done in this paper, would enable researchers to measure the present value of producer surplus in the future as well as in the current period beyond computing the year-base levels of (or changes in) consumer surplus.

Literature Review Our work is related to several strands of literature. Previous studies have investigated the link between intangible assets and firm value. It is shown that intangible capital is a dominant source of firm performance and rapid growth in the economy. (See, e.g., Brynjolfsson et al. 2002, Corrado et al. 2009, Hall 2001, Jorgenson and Stiroh 2000, and Jorgenson et al. 2002, McGrattan and Prescott 2000). For instance, Brynjolfsson, Hitt, and Yang (2002) provide evidence that firms' investments in IT-related intangible capital are significantly associated with these firms' market values.

Many studies are based on the approach that the market value represents the true intrinsic value of the firm at any time—an *efficient market* argument—. Discrepancy between the book value and market value of a firm's total assets is often referred to as compelling evidence supporting the existence of an intangible asset. More recently, Saunders and Brynjolfsson (2011) examined the value of intangible assets in the firm level and found that intangible assets are associated with significantly higher market values, about 30-55% premium. Tambe, Hitt and Brynjolfsson (2011) analyze both price fluctuations and quantity in measuring the value of IT-related intangible capital and found that by 2006, IT-related assets had grown to about one-third of the level of physical assets.

The aforementioned equity market approach, quantifying the amount of intangible assets and relates their influence to the market value, leaves a number of relevant questions unanswered. In principle, the difference between the book and market values of an asset indicates the difference between the purchased price of capital in the past when installed and the resale price of capital prevailing currently in the market. Not only the quantity/quality of capital but also the price, market value of a capital, could explain this difference. (See, e.g., Tambe et al. (2011).) For instance, in an extreme case, even in the absence of any intangible capital, the starkly higher market value of any capital—due to any reason that would shift up the demand whereas the supply is fixed—can result in the Tobin's Q ratio being higher than one.

Importantly, intangible capital is a broad concept and evolves over time, making it challenging to correctly measure the amount of intangibles. Many scholars have struggled and contributed to measuring different aspects of intangible assets. Corrado, Hulten, and Sichel (2006) denote the components of intangible assets as R&D expenditures—both scientific and non-scientific—, software, worker-training, brand equity, and organizational development. Hall (2000) proposed a concept of e-capital, to characterize as a body of technical and organizational know-how; e-capital includes business methods and systems such as speed and accuracy in handling enormous flows of transactions based on computers instead of counting computers itself. Cummins (2003) argues that intangible capital is not a distinct input to production like physical capital or labor but rather it is the glue that creates value from other inputs, which can be defined in terms of adjustment costs.

This paper is also related to the literature that studies the recent phenomenon that the more new companies depends less on physical assets as their core engines of growth. Human capital—broadly defined—of employees' and customers' is emerging as the most crucial sources of the firm's growth. Accordingly, more and more researchers in these studies recognize companies' non-financial stakeholders as an important and growing component of a firm's intangible asset. For example, Gupta et al. (2004) show that the long-term value of customers can be a strong and stable determinant of firm value. Belo et al. (2012) examined the role of brand capital for firm valuation and risk in the cross-section of publicly traded firms. Gourio and Rudanko (2011) study the implications of customer capital for firm value and investment dynamics. Combined together, all these studies point that in the current new era of economy, intangibles are more important in determining the firm value than before and hence call for deepening our understanding of causes and consequences of intangibles, to which this paper contributes.

The rest of paper proceeds as follows. Section II develops the model and presents analytical results. Key implications of the model are empirically examined in Section III. Section IV calibrates the model and discusses the simulated results. Section V concludes.

II. **Model**

This section lays out an economic environment in which user-generated capital is a key factor in generating the cash flow of an Internet firm. The model is a dynamic general equilibrium model of the household time allocation among competing websites owned by Internet firms. In the model, the household allocates time on three types of activities: (i) consumption of user-generated capital, i.e., viewing content that is already available on Internet, (ii) investment in user-generated capital, i.e., creating and uploading pictures, and (iii) earning labor income. An internet firm owns its unique website differentiated from others and generates the cash flow by using two input factors: the firm's own tangible capital and the household time spent on consuming user-generated capital on the firm's own website. Importantly, an Internet firm can indirectly improve the next-period user-generated capital stock, and hence its future cash flow, by investing in its own intangible capital, i.e., the website quality.

Environment

There are two goods: composite and Internet goods. The composite good is homogeneous and can be consumed and invested in the formation of tangible and intangible capital, while Internet goods are heterogeneous, differentiated among each other, i.e., a number of competing varieties of websites, and consumed by the household. The composite good is produced by the stand-in firm as well as by a number of Internet firms, while a particular variety of an Internet good is produced only by one corresponding Internet firm. Thus, an Internet firm serves dual role: providing a differentiated Internet good via its own unique website as well as producing the composite good. In return to the supply of the Internet good, an Internet firm earns its revenue indirectly by using the household time spent on the firm's own website for producing the composite good rather than directly charging service fees for the usage of the Internet good.

Household A representative household has preferences written as:

$$\underset{n_t,\{c_t(i),h_t^C(i),h_t^I(i)\}}{Max}\left\{E\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{(n_t)^{1-\sigma}}{1-\sigma}+\varphi\frac{(v_t)^{1-\sigma}}{1-\sigma}\right)\right]\right\} \quad where \;\; v_t=\left[\int_0^1\omega(i)\cdot[c_t(i)]^{(\eta-1)/\eta}di\right]^{\eta/(\eta-1)}$$

and $\beta\in(0,1)$ refers to discount factor, and $\sigma>0$ the risk-aversion—at the same time, the inverse of the inter-temporal elasticity of substitution—. n_t refers to consumption of the composite good, v_t consumption, in a CES aggregate form, of a continuum of measure one of Internet good varieties $c_t(i)$ for $i\in[0,1]$, $\omega(i)$ the variety-level weight for preferences, and φ the preferences factor for the utility of the Internet good consumption relative to that of the composite good. And $\eta\geq 0$ refers to the (constant) elasticity of substitution between the internet good varieties $c_t(i)$ and $c_t(i')$ for $i\neq i'$ where consumption of the internet good variety i is written as:

$$c_t(i)=k_t^U(i)h_t^C(i) \tag{1}$$

and $k_t^U(i)$ refers to user-generated capital for the internet good variety i , of which law of motion will be discussed soon. Note that the household time spent on consumption of the internet good variety $h_t^C(i)$ can be thought of as the utilization rate of the given user-capital stock in determining the *service flow* consumed, similar to the variable utilization of physical capital in production (Greenwood et al. 1988).

User-generated capital $k_t^U(i)$ evolves as follows:

$$k_{t+1}^U(i)=[1-\delta^U]k_t^U(i)+z_t^U(i)G\left(k_t^P(i),A_th_t^I(i)\right),\quad \delta^U\in[0,1] \tag{2}$$

where δ^U refers to the depreciation rate of user-generated capital, $z_t^U(i)$ the efficiency of investment in user-generated capital, and A_t the labor-augmenting technology growing at the constant rate of $\gamma > 0$: $A_t = (1+\gamma)^t$, $\forall t \geq 0$. And $G\left(k_t^P(i), A_t h_t^I(i)\right)$ refers to the user-capital formation function written as:

$$G\left(k_t^P(i), A_t h_t^I(i)\right) = \left(k_t^P(i)\right)^{\alpha^P} \left(A_t h_t^I(i)\right)^{1-\alpha^P}, \ \alpha^P \in (0,1) \tag{3}$$

where $k_t^P(i)$ refers to the firm i's intangible capital stock, $h_t^I(i)$ the household time spent on accumulating user-generated capital, and α^P the share of $k_t^P(i)$ in the user-capital formation function.

The household allocates one unit of time for three purposes every period: consumption of the internet good $h_t^C(i)$, investment in user-generated capital $h_t^I(i)$, and earning labor income. And the household owns all firms and hence receives a total of dividend payments of these firms $\int_0^1 d_t(i)di$ every period. Thus, given the wage rate w_t and the price of the composite good normalized to one, the household per-period budget constraint is written as:

$$n_t = w_t \left[1 - \int_0^1 h_t^C(i) + h_t^I(i)di\right] + \int_0^1 d_t(i)di \tag{4}$$

which says that the household uses labor and dividend incomes in consuming the composite good n_t.

Technology The stand-in firm produces the composite good using the technology linear in labor[3]: $o_t = A_t h_t$ where o_t refers to output of the composite good produced by hiring labor h_t given the labor-augmenting technology A_t. By contrast, each Internet firm produces the composite good by using the household hours spent on consumption of content on the firm's own website rather than by hiring labor. That is, differently from the stand-in firm, Internet firms have their own know-hows in attracting and utilizing the household attention/time in running their businesses, which is discussed in detail below.

Each Internet firm owns its unique website and provides the household with online content, which has been labeled 'Internet-good variety' earlier, differentiated from other Internet firms'. Rather than charging fees for the usage of such online content, an Internet firm indirectly generates its revenue by utilizing the household attention/time spent on consumption of content on the firm's own website $h_t^C(i)$ and combining

[3] For simplicity, the representative firm's technology is assumed to be linear in labor. Alternatively, we could model the case in which the representative firm's production function is of constant returns to scale for two inputs, tangible capital and labor, which would make no difference to the model results. Given the representative firm's technology is of constant returns to scale, the wage rate is in equilibrium determined so that the representative firm's profit is zero.

it with tangible capital $k_t^T(i)$ in producing the composite good. More specifically, letting $y_t(i)$ denote firm i's output of the composite good, measured as the after-tax cash flow before investment expenditures in the data, we write the firm i's production function of the composite good as:

$$y_t(i) = z_t(i)F(k_t^T(i), A_t h_t^C(i)) - \tau \tag{5}$$

where $z_t(i)$ refers to firm i's cash-flow productivity, τ the fixed cost of operation, and $F(\cdot,\cdot)$ the firm i's *variable* production function, which satisfies the standard properties.[4] Firm i's dividend payment $d_t(i)$, which could be either negative or positive, is then written as: $d_t(i) = y_t(i) - x_t^P(i) - x_t^T(i)$ where $x_t^T(i)$ refers to firm i's investment in tangible capital, and $x_t^P(i)$ the expenditures on accumulating the firm's own intangible capital $k_t^P(i)$. Note that in this paper, intangible capital $k_t^P(i)$ specifically refers to intangible assets that are related and restricted to the firm's website quality/efficiency, such as usability, friendliness of access, visibility of content to other users and so on, so that the household/user's one hour input can create the greater incremental amount, in the effective terms, of content. Thus, $x_t^P(i)$ also refers to the expenditures to improve the website quality, e.g., costs to develop the user-friendly online platform.

Firm i's tangible capital stock evolves as:

$$k_{t+1}^T(i) = [1-\delta^T]k_t^T(i) + x_t^T(i), \quad \delta^T(i) \in [0,1] \tag{6}$$

where δ^T refers to the depreciation rate of tangible capital. Similarly, intangible capital evolves as:

$$k_{t+1}^P(i) = [1-\delta^P]k_t^P(i) + x_t^P(i), \quad \delta^P(i) \in [0,1] \tag{7}$$

where δ^P refers to the depreciation rate of intangible capital that is, as discussed earlier, used in the formation of user capital $k_{t+1}^U(i)$ as: $k_{t+1}^U(i) = [1-\delta^U]k_t^U(i) + z_t^U(i)G\left(k_t^P(i), A_t h_t^I(i)\right)$.

Firm i is exposed to exogenous shocks to two fundamentals: (i) the cash-flow productivity $z_t(i)$ and (ii) the efficiency of user-capital investment $z_t^U(i)$. The stochastic process of $(z_t(i), z_t^U(i))$ is as follows: The cash-flow productivity $z_t(i)$ and the efficiency of user-capital investment $z_t^U(i)$ are independent of each other. Moreover, each of $z_t(i)$ and $z_t^U(i)$ follows its own first-order Markov process and independent across i. And distributions of both $z_t(i)$ and $z_t^U(i)$ are constant over time so that this

[4] $F(k^T, h^C)$ is homogeneous of degree one w.r.t. $(k^T, h^C) \in R_{++}^2$: $F(\lambda \cdot k^T, \lambda \cdot h^C) = \lambda \cdot F(k^T, h^C), \forall \lambda > 0, \forall (k^T, h^C) \in R_{++}^2$, concave, twice continuously differentiable, and satisfies that the marginal product of k^T and h^C is positive and decreasing, respectively.

economy is on the balanced growth path, even though an individual Internet firm is still subject to shocks to $(z_t(i), z_t^U(i))$.[5]

Firm Value For simplicity, we assume that Internet firms are financed entirely by equity but not by debt, which is not restrictive so much given the fact that in the data, Internet firms' leverage ratio is often low, about five percent or below. The Internet firm i's beginning-of-period equity value $V_t(i)$ is the sum of the current and discounted expected future dividend payments and written as:

$$V_t(i) = d_t(i) + \frac{1}{1+r} E_t\left[\sum_{\tau=t+1}^{\infty} d_\tau(i)\right] \tag{8}$$

which is referred to as the (current) value of firm i throughout this paper. The constant discount rate $r > 0$ is written as: $r = r_f + \pi$ where r_f refers to the constant risk-free rate and π the constant risk premium.[6]

Market Structure Below we discuss the market structure for the two goods, composite and Internet goods. For the case of the composite good, the market is of perfect competition (homogeneity of the composite good). By contrast, the market for the Internet good is monopolistically competitive, mainly due to the differentiated websites, in the sense of Dixit and Stiglitz (1977): Taking as given decision rules of the household and other firms, an Internet firm optimally controls its tangible and intangibles.

Resource Constraint The resource constraint for the composite good is written as:

$$n_t + \int_0^1 x_t^T(i) + x_t^P(i)di = A_t\left[1 - \int_0^1 \left(h_t^C(i) + h_t^I(i)\right)di\right] + \int_0^1 \left[z_t^U(i)F(k_t^T(i), A_t h_t^C(i)) - \tau\right]di \tag{9}$$

which says that the composite good produced is used for consumption and investment.

Discussion: User Capital vs. Intangible Capital User-generated capital indirectly increases the value of an Internet firm as follows: In consuming the Internet goods/services, the household/user pays his/her attention/time to the website in spite of no monetary payment to the website-owning firm (Brynjolfsson and Oh 2012). Importantly, the household attention/time paid to the website results in, even if not intended by the household, generating cash flow to the website-owning firm, e.g., the advertising revenue. Thus, the higher the level of user-generated capital available on an Internet firm's website, the larger the viewership on the firm's own website, and hence the greater the firm's value.

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[5] This property would be formally derived, due to the law of large numbers, from the assumption that the initial cash-flow productivity $z_0(i)$ is independently drawn from a common distribution and that the initial user-capital investment efficiency $z_0^U(i)$ is also independently drawn from another common distribution.

[6] This paper's primary goal is the valuation of an Internet firm rather than the explanation of determinants of the cost of capital for an Internet firm. Thus, in the model, the cost of capital is exogenously given rather than endogenously determined.

Thus, user-generated capital can be broadly thought of as an intangible asset in the sense that it increases the firm's value in spite of no increase in the firm's own tangible asset. Importantly, user-generated capital differs from other intangibles that have been discussed in the literature—e.g., human, organization, customer, and brand capital—in the sense that investment in user-generated capital is mainly expensed by, and hence under control of, users rather than by the firm itself. An Internet firm can at most indirectly manage and control the formation of user-generated capital by increasing the quality/efficiency of the firm's own website. The key is that an online user wants to consume content that is voluntarily created either by the user itself or by other users but not by people hired by the firm.

Nonetheless, an Internet firm can still control, even though in an indirect way, user-generated capital by improving the quality/efficiency of its website. The better quality of an Internet firm's website leads to the higher level of user-generated capital and hence the larger viewership and the greater firm value. Thus, the website quality of an Internet firm can be also thought of as another type of intangibles and labeled *platform capital* in the sense that it contributes to the improvement in the firm value mainly by, and to the extent of, promoting users' online activities to take place on the firm's own website.

The key idea that user-generated capital can be broadly thought of as an intangible asset also has important implications for measuring the value of Internet. The traditional approach measures the value of Internet based on consumer surplus (Brynjolfsson and Oh 2012, Greenstein and McDevitt 2011), which this paper extends by measuring the contribution of free goods/services on Internet to an Internet firm's value. Note that the firm value can be thought of as the discounted sum of producer surplus in the future as well as that in the current period; thus, calculating an Internet firm's value, as done in this paper, would enable researchers to measure an important part of the value of free goods/services on Internet that would be otherwise overlooked by the traditional method of calculating consumer surplus alone.

Equilibrium

We rewrite variables in order for the model economy to be stationary; let the hatted variable denote the ratio of a variable to the labor-augmenting technology $A_t = (1+\gamma)^t$:

$$\hat{n}_t \equiv \frac{n_t}{A_t}, \hat{v}_t \equiv \frac{v_t}{A_t}, \hat{k}_t^U(i) \equiv \frac{k_t^U(i)}{A_t}, \hat{y}_t(i) \equiv \frac{y_t(i)}{A_t}, \hat{d}_t(i) \equiv \frac{d_t(i)}{A_t}, \hat{V}_t(i) \equiv \frac{V_t(i)}{A_t}, \hat{k}_t^T(i) \equiv \frac{k_t^T(i)}{A_t},$$

$$\hat{k}_t^P(i) \equiv \frac{k_t^P(i)}{A_t}, \hat{x}_t^T(i) \equiv \frac{x_t^T(i)}{A_t}, \hat{x}_t^P(i) \equiv \frac{x_t^P(i)}{A_t}, \hat{w}_t \equiv \frac{w_t}{A_t}. \quad (10)$$

Thus, hatted variables are stationary. Let $\tilde{z}(i)$ denote the vector of firm i's cash-flow productivity and user-capital investment efficiency: $\tilde{z}(i) \equiv (z(i), z^U(i))$. Then we write the internet firm i's value function $\hat{V}(\cdot)$, which refers to the firm's beginning-of-period equity value relative to A_t, in recursive form as:

$$\hat{V}(\hat{k}^P(i), \hat{k}^U(i), \hat{k}^T(i), \tilde{z}(i)) =$$

$$\underset{\hat{k}^T{}'(i), \hat{k}^U{}'(i)}{Max} \left\{ z(i) \cdot F(\hat{k}^T(i).h^C(i,\cdot)) - \hat{x}^P(i) - \hat{x}^T(i) + \frac{1+\gamma}{1+r} E\left[\hat{V}(\hat{k}^P{}'(i), \hat{k}^U{}'(i), \hat{k}^T{}'(i), \tilde{z}'(i)) \mid \tilde{z}(i)\right] \right\} \tag{11}$$

subject to the laws of motion for tangible, intangible and user capital written as:

$$[1+\gamma]\hat{k}^T(i) = [1-\delta^T]\hat{k}^T(i) + \hat{x}^T(i) \;, \tag{12}$$

$$[1+\gamma]\hat{k}^P(i) = [1-\delta^P]\hat{k}^P(i) + \hat{x}^P(i) \;, \tag{13}$$

$$[1+\gamma]\hat{k}^U(i) = [1-\delta^U]\hat{k}^U(i) + z^U(i) G\left(\hat{k}^P(i), h^I(i,\cdot)\right), \tag{14}$$

and the household (equilibrium) policy functions of $h_t^C(i,\cdot)$ and $h_t^I(i,\cdot)$ taken as given.

We study a stationary recursive equilibrium (Prescott and Mehra 1980) that satisfies: (i) both the household and firms take into consideration all available information in making their decisions, (ii) the household and stand-in firm take prices and law of motion for aggregate state variables as given, (iii) an internet firm monopolistically competes with other Internet firms by taking as given the policy functions of the household and other firms, (iv) aggregate variables grow along the balanced growth path so that the hatted aggregate variables are constant, and (v) decision rules of firms and the household are recursive.[7]

A Firm's Decision Rules In this section, we purposefully focus on the case in which volatilities of cash-flow productivity $z(i)$ and efficiency of investment in user capital $z^U(i)$ are of magnitude small enough for an Internet firm to continue business every period.[8] Given the firm i's state of $(\hat{k}^P(i), \hat{k}^U(i), \hat{k}^T(i), \tilde{z}(i))$, firm i's equilibrium decision rules are characterized as follows. First, the firm's choice of the next-period tangible capital stock $k^T{}'(i)$ is written as—the usual firm-side Euler equation—:

[7] The first three conditions (i)-(iii) define the substantive property of the equilibrium, while the last two conditions (iv)-(v) essentially say that the equilibrium is stationary and in recursive form. More specifically, by the stationary equilibrium outcome, we mean that the distribution of key variables across the internet good variety is constant even though individual firm is still subject to uncertainty, which is similar to the (stationary) industry equilibrium studied in the literature (Hopenhayn 1992, Miao 2005).

[8] In general, the sufficiently large volatility either of $z(i)$ or of $z^U(i)$ can cause an Internet firm to stop operating business and to exit the market if it is hit by extremely bad shocks either to $z(i)$ or to $z^U(i)$, which is not of interest in this paper.

$$1+\gamma = \frac{1+\gamma}{1+r} E\left[1-\delta^U + z'(i) F_1(\hat{k}^T{}'(i), h^C{}'(i,\cdot))\right] \tag{15}$$

where $F_1(\hat{k}^T,\cdot) \equiv \partial F(\hat{k}^T,\cdot)/\partial \hat{k}^T$ for $\hat{k}^T > 0$ refers to the first-order partial derivative of $F(\hat{k}^T, h^C)$ with respect to $\hat{k}^T > 0$, i.e., the next-period marginal product of tangible capital. The optimality condition for $\hat{k}^T{}'(i)$ above says that the firm chooses its $\hat{k}^T{}'(i)$ such that the present value of the next-period (expected) marginal product of tangible capital net depreciation—the marginal benefit of $\hat{k}^T{}'(i)$ — should equal the reduction in the current dividend payment—the marginal cost of $\hat{k}^T{}'(i)$ —.

Second, the firm i's decision for the next-period intangible capital stock $\hat{k}^P{}'(i)$ is written as:

$$1+\gamma = \frac{1+\gamma}{1+r}[1-\delta^U] + \frac{[1+\gamma]^2}{[1+r]^2} E\left[z''(i) F_2(\hat{k}^T{}''(i), h^C{}''(i,\cdot)) \frac{\partial h^C{}''(i,\cdot)}{\partial \hat{k}^U{}''(i,\cdot)} \left(\frac{\partial \hat{k}^U{}''(i,\cdot)}{\partial \hat{k}^P{}'(i)} + \frac{\partial \hat{k}^U{}''(i,\cdot)}{\partial h^I{}'(i,\cdot)} \frac{\partial h^I{}'(i,\cdot)}{\partial \hat{k}^P{}'(i)} \right) \right] \tag{16}$$

where double-primed variables refer to the variables in two periods later from now. The optimality condition for $\hat{k}^P{}'(i)$ above says that the marginal cost of $\hat{k}^P{}'(i)$, which is a reduction in the current dividend payment, must equal its marginal benefit that consists of the two components: first, the direct benefit of intangibles net depreciation in the next period, and second, the indirect benefit of the increased cash flow in two periods later from now via the increased user capital stock $\hat{k}^U{}''(i,\cdot)$ that would induce the household consumption-purpose hours $h^C{}''(i,\cdot)$ to increase.

The Household Decision Rules We turn to discussing the household equilibrium decision rules. First, the household consumption-purpose time allocation across internet good varieties is written as:

$$(\hat{n})^{-\sigma} = \varphi(\hat{v})^{-\sigma} (\hat{v})^{1/\eta} \omega(i) \left[\hat{k}^U(i)\right]^{\frac{\eta-1}{\eta}} \left[h^C(i)\right]^{-\frac{1}{\eta}} \tag{17}$$

which is simplified to:

$$h^C(i)/h^C(\tilde{i}) = \left(\omega(i)/\omega(\tilde{i})\right)^{\eta} \left(\hat{k}^U(i)/\hat{k}^U(\tilde{i})\right)^{\eta-1}. \tag{18}$$

We impose the restriction that relative to the reference variety $\tilde{i}$, the household consumption-purpose time spent on a given variety i must be *strictly increasing* in the relative user-capital stock $k^U(i)/k^U(\tilde{i})$, and importantly, at the *decreasing rate*, which is essentially governed by the elasticity-of-substitution parameter η as follows:

Assumption 1: $1 < \eta < 2$.

From now on, we assume that as in Assumption 1, η is larger than one and smaller than two.

Second, the household investment-purpose time allocation across internet good varieties $h^I(i)$ is written as:

$$(\hat{n})^{-\sigma} = \beta[1+\gamma]^{1-\sigma} \cdot \frac{1}{1+\gamma} \frac{z^U(i)\partial G(\hat{k}^P(i), h^I(i))}{\partial h^I(i)} \cdot E\left[(\hat{n}')^{-\sigma} \frac{h^C{}'(i)}{\hat{k}^U{}'(i)}\right] \quad (19)$$

which says that at the optimum, the marginal cost of the investment-purpose hours—measured as the marginal utility of the composite-good consumption today—must equal its marginal benefit that is measured as the (discounted) marginal utility of the composite-good consumption in the next period multiplied by the marginal product of the investment-purpose hours in increasing the next-period user capital stock and utilization rate of such user capital.

Analytic Results In this section, we discuss analytic results of the model to understand the key mechanism. Formal proofs are provided in the online appendix. From now on, for the purpose of exposition, we consider the simple case as follows: (i) no fixed cost of operation $\tau = 0$ and (ii) no uncertainty such that $(z(i), z^U(i))$ is deterministic for every variety i, and (iii) $1 < \eta < 2$.

Proposition 1: *The household allocation of investment-purpose hours* $h^I(i)$ *is increasing in the level of corresponding (concurrent) intangible capital* $\hat{k}^P(i)$*:* $dh^I(i)/d\hat{k}^P(i) > 0$.

As shown by Proposition 1, the higher the level of an Internet firm's intangible capital—i.e., the better the website quality—the larger the amount of household investment-purpose time spent on that particular website. The main reason is the complementarity between the Internet firm's own intangible capital and the household investment-purpose hours in accumulating user-generated capital. The increased current level of an Internet good firm's intangible capital shifts up the marginal product of the household investment-purpose hours in creating user-generated capital, which leads to an increase in the household equilibrium investment-purpose hours via the *inter-temporal substitution* channel: moving the marginal hours from working to creating user-generated capital in response to the currently increased efficiency of user-capital investment, the household can improve his/her welfare by increasing the future utility high enough to compensate for more than the decreased current utility.

Note that results in Proposition 1 do not necessarily imply that an Internet good firm can be always better off by simply investing more in its intangible capital because such investment is costly. The optimal firm-side investment in intangible capital would be determined by equating the present value of an increase in its future cash flow to the current cost at the margin.

Lemma 1: *For a given variety* i*, an increase in the next-period user-generated capital stock* $\hat{k}^{U}{}'(i)$ *causes increases in the concurrent consumption-purpose hours, tangible capital stock and cash flow in the next period:* $dh^{C}{}'(i)/d\hat{k}^{U}{}'(i)>0, d\hat{k}^{T}{}'(i)/d\hat{k}^{U}{}'(i)>0, d\hat{y}'(i)/d\hat{k}^{U}{}'(i)>0$.

Results of Lemma 1 essentially say that the household consumption-purpose time allocation is increasing in the concurrent level of user-generated capital, which, combined with the indirect effect on the concurrent level of tangible capital, also applies to the Internet firm's concurrent cash flow.

Proposition 2: *For a given variety* i*, an increase in the current intangible capital stock* $\hat{k}^{P}(i)$ *results in an immediate increase in the current firm value as well as subsequent increases in the user-generated capital stock, consumption-purpose hours, tangible capital stock and cash flow in the next period:* $d\hat{V}(i)/d\hat{k}^{P}(i)>0, d\hat{k}^{U}{}'(i)/d\hat{k}^{P}(i)>0, dh^{C}{}'(i)/d\hat{k}^{P}(i)>0, d\hat{k}^{T}{}'(i)/d\hat{k}^{P}(i)>0, d\hat{y}'(i)/d\hat{k}^{P}(i)>0$.

Results of Proposition 2 essentially say that an increase in the current level of an Internet firm's intangible capital boosts up the future user-generated capital stock $d\hat{k}^{U}{}'(i)/d\hat{k}^{P}(i)>0$ and hence the next-period household consumption-purpose hours spent on the firm's own website, which would then increase the firm's future cash flow—i.e., more advertising revenue in the future—and hence the current firm value. Moreover, the increased household consumption-purpose hours also shifts up the marginal return to the firm's tangible investment—complementarity in production function—, and hence the firm's equilibrium tangible investment increases, which further increases the firm's future cash flow.

Proposition 3: *For a given variety* i*, the increased efficiency of user-capital investment* $z^{U}(i)$ *results in an immediate increase in the current firm value as well as subsequent increases in the user-generated capital stock, consumption-purpose hours, tangible capital stock and cash flow in the next period:* $d\hat{V}(i)/dz^{U}(i)>0, d\hat{k}^{U}{}'(i)/dz^{U}(i)>0, dh^{C}{}'(i)/dz^{U}(i)>0, d\hat{k}^{T}{}'(i)/dz^{U}(i)>0, d\hat{y}'(i)/dz^{U}(i)>0$.

Results of Proposition 3 are essentially the same with those of Proposition 2 except that the increased efficiency of user-capital investment now replaces the role of the increased intangible capital stock in Proposition 2. The reason is as follows. The improved efficiency of user-capital investment effectively shifts up the marginal products of both the firm's own intangible capital and the household investment-purpose hours in accumulating user-generated capital. These high returns induce both the firm- and household-side investment in user-generated capital to increase. Thus, the increased future user-generated capital stock makes both the household consumption-purpose hours and firm's cash flow increase in the future. Therefore, the firm value immediately increases now.

III. Empirical Analysis

Data We collect data both in the company- and website-level. First, company-level data is from the COMPUSTAT annual industrial files that provide information on financial/accounting variables of publicly traded US companies. Systematic relationship between the market value of an Internet firm and user engagement is of our interest. Thus, Internet-exclusive media companies are of our consideration, while Internet retailers such as *Amazon.com* are not. All other COMPUSTAT companies and non-Internet media COMPUSTAT companies, if needed, are compared to Internet-exclusive media COMPUSTAT companies. Our unbalanced panel data covers the period 2000-2012 annually and have a total of 1,400 observations for the 196 Internet-exclusive media companies (hereafter, referred to as Internet firms). All COMPUSTAT variables are deflated, so as to be in real terms, by the U.S. Consumer Price Index (CPI) provided by the BLS website.[9] Second, website-level data is from *Alexa.com* and authors' collection. For both groups of Internet-exclusive and other media companies, each company can own multiple websites to interact with their users. For companies that exist in 2012, we collect 459 website-URLs for a total of 87 Internet-exclusive media companies and 1,011 website-URLs for a total of 79 other media companies. For each website URL, data on the Internet viewership—e.g., time spent by visitors—is provided by *Alexa.com*. We also collect data on user activities for a given media company's *Facebook* page: the amount of posts, likes and comments generated by the company and those by users.

Below we document facts so as to understand how the recent increase in users' time spent on Internet has affected the market value of an Internet firm. It will be shown that the cash-flow growth aspect of Interment firms starkly differs from that of all other firms, in particular, the growth rate of the market value and either the Tobin's Q ratio or the market value-to-tangible assets ratio. We then examine factors that are systematically related to such stylized facts via the regression analysis.

Table 1 presents key financial variables of Internet firms in comparison to all other firms over the sample period 2000-2012. For a robustness check, we also control for the possible *size effects*: we sort

[9] The "base-years" period of CPI is during the period 1982-1984 when a nominal variable is on average of the same level with the real.

firms by size—measured as the book value of tangible assets—and select all other firms—labeled *all other small firms*—such that their median firm size is equal to the median firm size of Internet firms.

Table 1: Summary Statistics: Internet Firms vs. All Other Firms

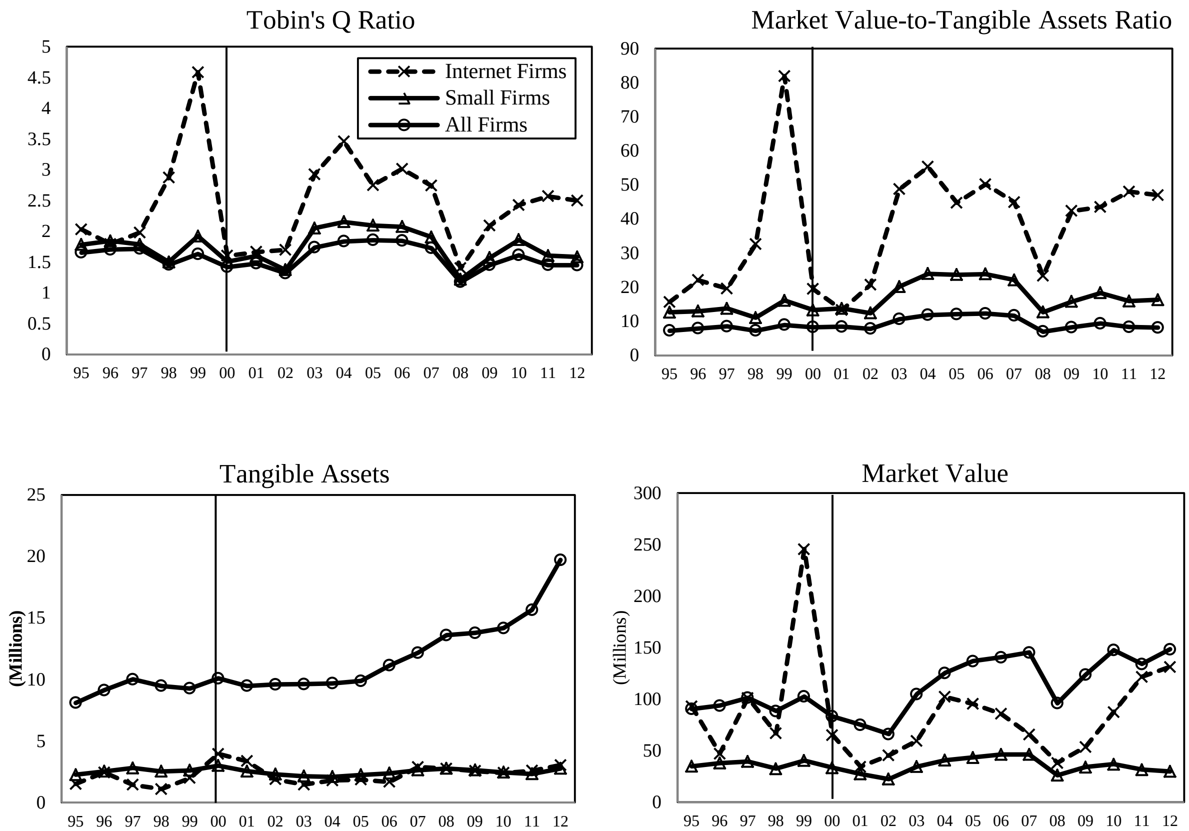
	Internet-exclusive Media Firms	All Other Small Firms	All Other Firms
Tangible Assets: Level (SD)	2.48 (188.8)	2.48 (10.38)	12.20 (2,817.9)
Growth rate	0.30	0.06	0.25
Total Assets: Level (SD)	35.44 (1,777.5)	20.31 (176.6)	73.17 (8,231.3)
Growth rate	0.51	0.13	0.28
Market Value: Level (SD)	75.87 (6,350.0)	34.91 (403.1)	117.53 (20,701.4)
Growth rate	0.29	0.15	0.21
Operating Cash Flow: Level (SD)	117.27 (1,031.9)	99.40 (146.6)	137.86 (3,154.6)
Growth rate	0.037	0.004	0.010
Number of Employees: Level (SD)	0.308 (5.53)	0.176 (4.90)	0.546 (40.94)
Growth rate	0.15	0.03	0.09
Tobin's Q ratio: Level (SD)	2.37 (666.7)	1.74 (1,149.3)	1.57 (934.7)
Market Value/Tangibles: Level (SD)	38.58 (23,668.7)	17.86 (3,673.6)	9.53 (5,647.1)
Total Assets/Tangibles: Level (SD)	13.98 (97.3)	8.54 (1,086.8)	5.11 (854.3)
Capital Expenditures/Tangibles: Level (SD)	0.44 (0.32)	0.26 (18.10)	0.21 (13.96)
Operating Cash Flow/Tangibles: Level (SD)	29.98 (4,338.6)	31.52 (8,183.6)	11.83 (6,612.2)
Salary expenditure/Tangibles: Level (SD)	3.86 (31.5)	1.08 (111.5)	0.56 (79.0)
Number of firms	196	10,241	13,565
Number of observations	1,400	60,090	94,003

Note: this table presents key financial variables of Internet-exclusive media firms in comparison to all other firms, which do not belong to Internet exclusive media firms, over the sample period 2000-2012. All variables are deflated by the U.S. Consumer Price Index, measured as their cross-sectional median values for a given year and then averaged over the sample period. Similarly, the standard deviation of a variable across firms for a given year is calculated and then averaged over the sample period, which is reported inside the parenthesis next to the variable's median value. Growth rate of a variable is calculated in the same way and reported below the level. 'All Other Small Firms' refers to firms other than Internet-exclusive media firms that belong to the bottom of size-sorted distribution so that their median size, measured as the book value of tangible assets, is the same with that of Internet-exclusive media firms, while 'All Other Firms' refers to all firms other than Internet-exclusive media firms. 'Operating Cash Flow (OCF)' refers to the after-tax cash flow from operation before tangible and intangible investment and is measured as sales net the cost of goods sold, staff expenses and taxes before depreciation, advertising and selling expenses.

In Table 1, for a given group of firms, the level of a variable is calculated as its cross-sectional median value for a given year and then averaged over the sample period where its standard deviation is reported inside the parenthesis and its average growth rate is below the level. Most notable, the growth rate of the market value of an Internet firm is as about twice high as that of all other firms—both conditional and unconditional on the firm size—. Moreover, for Internet firms, the market value-to-book value ratio, denoted as the Tobin's Q ratio, is also much higher, by about 33%, than for all other firms, suggesting that for Internet firms, the amount of *unmeasured* intangibles is larger than for all other firms.

It is interesting to ask what factors are behind such a fact about the larger amount of *unmeasured* intangibles for Internet firms relative to that for all other firms. We consider the main hypothesis that for Internet firms, user activities—which are not recorded as book value of assets according to the current accounting practice—are more important as a value-creation engine and contribute to a larger part of the firm value than for all other firms. The reason is that an Internet firm starkly differs from all other firms in its unique business model such that user activities taking place on the Internet firm's website are the main source of revenue, e.g., cash generation by users' clicking the advertising banners. We proceed to measuring the importance of users' online activities in explaining the Internet firm's value below.

Figure 1: Tobin's Q Ratio, Market Value, and Tangible Assets



Note: this figure plots time-series of key financial variables for three groups of firms included in the COMPUSTAT database during the period 1995-2012 by expanding the sample of our consideration dating back to 1995 so that the overall trend, if any, can be more easily seen than for the sample period only. All variables are median values across firms belonging to a given group of firms for a given year. 'Internet firms' refer to the Internet-exclusive media firms, and 'Small firms' the firms other than Internet-exclusive media firms that belong to the bottom of size-sorted distribution so that their median size, measured as the book value of assets, is the same with that of Internet-exclusive media firms. 'All firms' refer to the firms other than Internet-exclusive media firms. 'Market Value' refers to the market value of a firm and is measured as the sum of market value of equity and book value of debt, and 'Tangible Assets' the book value of (net) property, plant and equipment.

Figure 1 plots time-series of key financial variables for three groups of firms by expanding the sample dating back to 1995 so that the overall trend, if any, can be more easily seen than for the sample period only. Note that the sample period of 2000-2012 is still of main interest in the sense that it has not been until 2000 that users actively engage in online activities, at least in terms of their contribution to the value creation of Internet firms, by spending considerable amount of hours and hence generating content available online.

As shown by Figure 1, all of key variables exhibit significant differences between the two groups, Internet firm vs. all other small firms of the same size—again in terms of the book value of tangible assets—. The market value-to-tangible assets ratio clearly shows the 1999-2000 dotcom bubble incident as well as a jump to a high level since 2003, which guides us to use the period of 2000-2002 as the sample period for calibrating the model as discussed later. Below we proceed to documenting key factors—e.g., the amount of users' online activities—that are likely to have driven such dramatic changes in the market value of Internet firms relative to that of all other firms.

Table 2: User Activities on Websites Owned by Internet Firms in 2013

Variable	Obs.	Mean	SD	Median	Min	Max
Total hours per site (hours)	303	481,604	4,763,542	522	0.397	71,190,567
Total page views (thousands)	331	21,409	207,495	20.7	0.005	2,692,080
Time spent on site per user (min.)	303	4.03	3.35	2.97	0.32	27.77
Page views per user	331	3.61	3.0	2.6	1	18.43
Number of users (thousands)	331	1,716	13,019	7.57	0.005	157,710

Note: this table presents statistics of several dimensions of users' online activities on websites owned by Internet-exclusive media firms in 2013. Data source is *Alexa.com*. Hours spent by users and page views, both in terms of total and per-user, and number of users are calculated for a given website in 2013 and then their cross-sectional statistics are reported. 'Obs.' refers to the number of websites, and 'SD' the standard deviation.

Table 2 provides summary statistics of several dimensions of users' online activities on websites owned by Internet-exclusive media companies in 2013.[10] For causes and consequences of user activities, we examine two hypotheses, discussed in the analytical section earlier, as follows: First, user activities greatly contribute to the Internet firm's value: the household consumption-purpose online hours (in Table 2) directly increases the Internet firm's current cash flow, while the household investment-purpose online hours indirectly and dynamically increases the Internet firm's future cash flow by increasing the next-period user-generated capital stock. Second, an Internet firm can affect user engagement to creating and uploading content to the firm's own website by investing in the firm's own intangible capital so as to improve its website quality. We examine these two hypotheses by collecting data on both user- and company-side content generation behaviors.

Table 3 summarizes statistics of company- and user-activities on *Facebook* pages of Internet firms. We want to empirically analyze the interplay between the user- and firm-side online activities. Thus, we collect data on a broad range of user- and firm-side activities, e.g., posting, commenting, and clicking the

[10] We adjust the scale of total page views by taking into consideration that *at least* total 36 billion pages are available online on average during the period from July 2012 to June 2013 for web pages indexed by the Google search engine (http://www.worldwidewebsize.com/).

Like button, on the Internet firm's *Facebook* page. By doing so, we aim to infer the strategy by which an Internet firm can effectively increase user engagement to creating content on the firm's *Facebook* page; here we assume that such a strategy can be also effectively applied to increasing user engagement to creating content on the firm's own website, too.

Table 3: User- and Company-generated Content on the Internet Firms' Facebook Pages

User-side	Obs.	Mean	S.D	Median	Min.	Max.
Likes #	136	1,888	8,125	43	0	60,388
Posts#	136	411	1,037	55	1	6,593
Comments#	136	609	3,032	35	0	32,805
Shares#	136	48	297	0	0	2,274
Length of Posts#	136	68,743	192,727	7,794	0	1,581,674
Length of Comments#	136	88,775	503,907	3,299	0	5,561,240
Unique posters and commenters#	136	466	1,187	67	0	9,237
Company-side	Obs.	Mean	(S.D)	Median	Min.	Max.
Likes #	206	20,337	117,620	291	0	1,284,713
Posts#	206	275	410	149	1	3,035
Comments#	206	1,906	8,218	38	0	91,445
Shares#	206	4,058	23,243	13	0	246,060
Length of Posts#	206	32,671	46,900	15,828	0	347,939
Length of Comments#	206	117,898	453,558	3,094	0	4,583,558
Unique posters and commenters#	206	912	3,393	24	0	29,673

Note: this table presents summary statistics of user- and company-generated content on the company's Facebook page for Internet firms in 2012. Content available on a given Internet firm's Facebook page is categorized into like, post, comment, share, and photo, which is then further categorized into either user- vs. company-generated content; number and length of content and unique users creating such content are computed for each of these categories of content for a given firm, and then their statistics across different Facebook pages are calculated. 'Obs.' refers to the number of Facebook pages that are one-to-one mapped to URLs owned by a given Internet firm where one Internet firm can own multiple websites.

Table 3 shows that, as for the case of the number of characters in the posts, the amount of content generated by users is larger than that by the firm. That is, the Internet firm has a high leverage in terms of inducing user-generated content per company-generated content as long as the company-generated content has some positive causal effect on user-generated content; moreover, in such a case, it could be the case that the firm also has a high leverage in terms of the firm's own value per company-generated content as long as high user engagement increases the firm's future cash flow.

Naturally, we investigate how the difference in the amount of users' involvement/activities is related to the difference in the firm value for Internet firms. For this purpose, we compare a number of financial variables between two groups of Internet firms: small vs. large amount of users' engagement/activities. Thus, we start by documenting an Internet firm's market value and such a firm's asset-side balance sheet components conditional on the amount of users' engagement/activities, e.g., their time spending, on the website(s) owned by such an Internet firm. See Table 4.

Table 4 provides key statistics of firm value and the related variables for Internet firms: market value of the firm and the firm's asset-side components are calculated conditional on the amount of users' engagement/activities on the website(s) owned by the firm. More specifically, a total of 87 Internet-exclusive media firms are categorized into the two groups: small—below median—vs. large—above median—amount of users' engagement/activities that are measured as users' time spent on the firm's own website(s). Then, within-group averages of key variables related to the firm value are computed.

Table 4: Users' Activities and Firm Value: Internet-exclusive Media Companies

Amount of Users' Activities	Small	Large
Tobin's Q	1.932 (58.5)	2.774 (2.85)
Market Value	$95.77 mil. ($1,688.5 mil.)	$657.82 mil. (17,662.6 mil.)
Market Value/Tangible Assets	40.01	46.98
Total Assets/Tangible Assets	19.163	16.248
Capital Expenditures/Tangible Assets	0.488	0.5
Number of Firms	43	44

Note: This table compares key statistics in 2012 between two groups of Internet-exclusive media companies that differ in the amount of users' online activities measured as the amount of time spent by users on the company's own website. Market Value refers to the company's total assets, in million (real) U.S. dollars, at market prices, Tangible Assets the book value of tangible assets, and Capital Expenditures the expenditures to formation of tangible assets. Tobin's Q refers to the company's market-to-book value ratio of total assets. All variables refer to their median values in 2012 for a given group of Internet-exclusive media companies, while standard deviations of these variables are inside parentheses.

From Table 4, we can see that for the high user-engagement group of firms, the firm value—in terms of both the level of the market value and the Tobin's Q ratio—is considerably higher than for the low user-engagement group of firms. Note that such a difference in the market value of the firm between these two groups of Internet firms is not simply the results of asset-side difference: all asset-side balance sheet components—e.g., intangible vs. tangible mixture and investment rate—are almost identical between the two groups! For instance, the market value-to-tangible assets ratio is, similar to the case of the Tobin's *q*-ratio, higher for the high user-engagement Internet firms than for the low user-engagement Internet firms, and importantly this fact is not simply caused by the difference in the size of intangibles between these two groups of firms: Total assets-to-tangible assets ratio is higher for the low user-engagement group than for the high user-engagement group! Thus, we can say that for the high user-engagement Internet firm, the return to assets is greater than for the low user-engagement Internet firm. Put differently, the high user-engagement Internet firm has the greater amount of unmeasured intangibles than the low user-engagement Internet firm does. This difference-in-difference statistic suggests evidence supporting our main hypothesis that user-generated content can account for the large part of the market value of Internet firms, which is consistent with the dominant view among the Internet industry experts that user engagement is one of the most crucial determinants of an Internet firm's value.

Thus, it is important to understand the most effective strategies to improve online user engagement so as to maximize an Internet firm's value. Thus, we need to understand what Internet firms actually do online and how such Internet firm's online activities interact with and involve users' online activities. Because of the data limitation, it is almost impossible to track and collect data on user- and firm-side online activities for a sufficiently long period. Following the recent popular approach in the literature, we circumvent this problem by using data on user- and firm-side activities on the firm's own *Facebook* page by assuming that user- and firm-side activities and interaction between them would be the same both on the firm's *Facebook* page and on the firm's own website.

Table 5: Users' Investment- and Consumption-Purpose Activities: Internet-exclusive Media Companies

Investment-Purpose Activities	Low Level	High Level
Company's Own Website		
1.Mean (SD): Hours per site	7,576 (32,133)	144,623 (455,074)
1.Median: Hours per site	334	6,828
Company's Page on Facebook		
2.Mean (SD): Number of likers	492 (1,101)	64,183 (208,554)
2.Median : Number of likers	123	5,505

Note: this table presents statistics of the amount of users' consumption-purpose online activities in 2012 for Internet-exclusive media companies conditional on the amount of users' investment-purpose online activities measured as the amount of user-generated content on the company's Facebook page. Users' consumption-purpose activities are proxied/measured in two ways: (i) total hours spent by users on the company's own website and (ii) number of likers for the company's Facebook page. These statistics are again calculated separately for the two groups of companies: low- and high-level of UGC on the company's Facebook page.

As shown by Table 5, we can say that the larger the amount of users' investment-purpose online activities, the greater the amount of users' consumption-purpose online activities. That is, the positive association between the engagement of users in creating content and the viewership on the Internet firm's own website suggests that inducing users to be actively engaged in creating content can be crucial in increasing the user base that is important for generating an Internet company's cash flows.

In the end, we are interested in the question: how can an Internet firm increase user engagement—in terms either of consumption- or of investment-purpose activities—? Naturally, we proceed to investigating whether and how the firm-side online activities would affect user-side online activities.

Table 6: User- and Company-generated Content: Internet-exclusive Media Companies

Amount of Company-Generated Content	Small	Large
1. Mean (SD): User UGC	40,449 (95,783)	251,427 (781,403)
1. Median: User UGC	4,860	26,166
2. Mean (SD): Number of likers	1,498 (5,275)	52,626 (191,288)
2. Median : Number of likers	86	3,476

Note: this table presents users' activities on the firm's Facebook page conditional on the amount of content generated by the firm on its Facebook page. All variables are measured in 2012. SD refers to the standard deviation of a variable.

Table 6 presents the amount of user-generated content on the firm's *Facebook* page conditional on the amount of company-generated content on the given company's *Facebook* page. We can see that for the Internet firm more actively engaging in generating/sharing content, the amount of user-generated content is of magnitude greater than for the less actively engaging firm. This suggests evidence supporting the hypothesis that indeed, users do respond to the message that an Internet firm shares with users on the firm's *Facebook* page, i.e., users are likely to spend more time and to pay more attention to the company-generate content of the higher quality than to that of the lower quality.

So far, we have provided suggestive, but not formal, evidence on the relationships among company- and user-side online activities. We proceed to investigate them formally via the regression analysis below. More specifically, we test the hypotheses (i) whether viewership of an Internet firm's website is closely related to the amount of user-generated content on the company's *Facebook* page and (ii) whether user- and company-generated content on the company's *Facebook* page are closely related to each other—which have been discussed earlier in Table 5 and 6—by employing either Seeming Unrelated Regression (SUR) or system GMM estimation methods where the system of regression equations is written as:

$$\log(Site\_Hours_i) = \beta_0 + \beta_1 \log(User\_UGC_i) + \varepsilon_i \tag{20}$$

$$\log(User\_UGC_i) = \beta_2 + \beta_3 \log(Company\_UGC_i) + e_i \tag{21}$$

where $Site\_Hours_i$ refers to users' hours spent on website i, $User\_UGC_i$ the amount of user-generated content on the *Facebook* page corresponding to website i, $Company\_UGC_i$ the amount of company-generated content on the *Facebook* page corresponding to website i, and ε_i and e_i are the error terms.

Table 7: Determinants of Online User Engagement

	SUR		System GMM	
Dependent variable	Site Hours	User UGC	Site Hours	User UGC
User UGC	0.598*** (0.000)		0.672*** (0.000)	
Company UGC		0.939*** (0.000)		0.919*** (0.000)
R-square	0.332	0.221		
Number of Websites	74	74	74	74

Note: this table presents estimation results of the system of regression equations (19) and (20). Site Hours refers to the hours spent by users on the given website owned by a given firm, User UGC the amount of user-generated content on the *Facebook* page corresponding to the given website, and Company UGC the amount of company-generated content on the same *Facebook* page. *** indicates significance at the 1 percent level.

Table 7 summarizes the estimation results of the system of regression equations (20) and (21). Both the relationship between viewership on the Internet media firm's own website and the amount of user-generated content on the firm's *Facebook* page and that between user- and company-generated content on the firm's *Facebook* page are significant at the one percentage level. For instance, according to the system GMM estimation results, one percentage increase in the amount of user-generated content is significantly associated with an increase, by about 0.7 percent, in the amount of users' hours spent on the Internet firm's own website. This provides suggestive evidence that user engagement on the company's own website is significantly associated with user engagement to content generation on the company's *Facebook* page, suggesting that the company's *Facebook* page can be a good proxy of the company's own website in terms of users' online behaviors. And the regression results of the equation (21) show that one percentage increase in the amount of company-generated content on the company's *Facebook* page is significantly related to an increase, by about 0.9 percent, in the amount of user-generated content on the company's *Facebook* page; that is, the company- and user-side engagement to content generation is significantly correlated with each other, consistent with the model prediction. [11]

Regression Analysis of Firm Value This section examines the influence of users' online activities on the observed market value of an Internet firm. We carry out cross-section regression analysis of market values of Internet-exclusive media firms as of the end of 2013. It is of our interest to estimate how sensitively the Internet-exclusive media firm's market value is related to the amount of users' online activities on the firm's own website. More specifically, the regression equation of the Internet-exclusive media firm i 's market value MV_i is written as:

$$\log(MV_i) = \beta_0 + \beta_1 \log(Tang_i) + \beta_2 \log(\text{Intang}_i) + \beta_3 X_i + \varepsilon_i \tag{22}$$

where ε_i refers to the error term, $Tang_i$ the concurrent book value of tangible assets, Intang_i the concurrent intangibles, and X_i the vector of different measures of one-year lagged online user engagement for one year in 2012: either (i) $TotHours_i$ the log of total hours spent by users on the firm's own website or (ii) $PageViews_i$ the log of per-user page views and $User_i$ the logged number of users participating in the firm's own website. Both dependent and independent variables are logged.

Table 8 summarizes the estimation results of the regression equations (22). In general, both tangibles and intangibles are significant in their relationships with the market value of an Internet-exclusive media firm. Moreover, all different measures of online user engagement are significantly positively associated with the Internet-exclusive media firm's market value.

[11] Our goal is to examine the statistical relationship between company- and user-side engagement to content generation rather than to establish causal effect of one on the other.

Table 8: Determinants of the Market Value: Internet-exclusive Media Firms

Dependent Variable	Market Value	Market Value
Tangible Assets	0.559*** (0.000)	0.547***(0.000)
Intangible Assets	0.227*** (0.001)	0.231*** (0.000)
Total Hours Per site	0.124** (0.001)	
Page Views per User		0.481** (0.038)
Number of Users		0.106** (0.012)
Constant	2.276*** (0.000)	2.021*** (0.000)
Adjusted R-square	0.825	0.837
Number of Firms	78	78
Number of Websites	446	446

Note: this table presents the OLS estimation results of regressing the market value of an Internet-exclusive media firm on a number of asset-side COMPUSTAT variables and several measures of user activities taking place on the firm's website. All dependent and independent variables are logged. Standard errors are inside parentheses. * indicates significance at the 10 percent level, ** at the five percent level, and *** at the one percent level. All observations are in 2013.

Table 9: Decomposition of the Predicted Market-Value Variation: Internet-exclusive Media Firms

Dependent Variable	Market Value	Market Value
Tangible Assets	0.616	0.574
Intangible Assets	0.230	0.223
Total Hours Per Site	0.154	
Page Views Per User		0.085
Number of Users		0.117

Note: this table presents relative importance of independent variables in predicting the dependent variable, the market value of a firm. For a given independent variable, its one standard deviation is multiplied by the coefficient of the variable, which is then expressed as a ratio relative to the sum of it over all independent variables.

Table 9 presents the results of decomposing the variation of the firm's predicted market value into the variations and coefficients of independent variables. More specifically, we multiply the one standard deviation of an individual independent variable with its estimated coefficient, calculate its value relative to the sum of such values across all independent variables, and take it as the measure of the importance of the individual independent variable in accounting for the firm's predicted market value. We can see that the amount of online user activities accounts for a substantial part, about 15-20 percent, of the variation in the predicted market value of Internet-exclusive media firms. For comparison, tangible assets account for about 60 percent of the variation in the predicted market value of Internet-exclusive media firms, and intangible assets does about 23 percent. Thus, we conclude that for Internet-exclusive media firms, the amount of online user activities is crucial in determining the firm value, and hence a number of interesting questions arise as follows. What are the Internet firm's optimal strategies to manage, at least indirectly, users' activities? How large the *quantitative* importance of the amount of users' activities in terms of the contribution to the Internet firm's observed value? These questions call for the quantitative study of the model and will be addressed in the quantitative analysis Section IV.

IV. Quantitative Analysis

Calibration We calibrate parameters of the model such that in the deterministic steady state, the model matches the long-run averages of key variables in the data.[12] Table 10 presents calibration results, and Table 11 lists key statistics both in the model and in the data. As shown by Table 11, the model matches well the targeted as well as non-targeted statistics.

Table 10: Benchmark Parameter Values

Parameter	Symbol	Value
Risk-free real interest rate	r_f	3%
Risk premium on stock returns for Internet firms	π	5.2%
Growth rate of per-capita real GDP	γ	2%
Depreciation rate of tangible capital	δ^T	10%
Depreciation rate of intangible capital	δ^P	15%
Depreciation rate of user-generated capital	δ^U	15%
Preferences: discount factor	β	0.9709
Preferences: risk aversion	σ	2.0
Preferences: elasticity of substitution among websites	η	1.5
Preferences: weight to Internet good consumption	φ	0.0039
Tangible capital share in output	α^T	0.0808
Intangible capital share in user-generated capital investment	α^P	0.7479
Fixed cost of operation	τ	0.0451
Efficiency of user-generated capital investment	$\bar{z}^U$	3.0
Cash-flow productivity	z	1.0

Table 11: Key Statistics

Statistics: Targeted	**Data**	**Model**
Consumption-purpose Internet hours-to-total hours ratio	10.30%	10.30%
Investment-purpose Internet hours-to-total hours ratio	0.41%	0.41%
Operating cash flow-to-tangible capital ratio	2.25	2.25
Growth option-to-tangible capital ratio	7.62	7.50
Fixed cost-to-operating cash flow	0.53	0.53
Statistics: Non-targeted	**Data**	**Model**
Equity value-to-tangible capital ratio	17.91	16.67
Intangibles-to-tangible capital ratio	0.87	0.70

Note: this table presents key statistics in the data and in the model. Total hours refer to Internet hours plus working hours. Internet hours are measured as the household leisure-purpose hours spent on Internet in 2002 based on the *Consumer Technographics Survey* provided by *Forrester Rsearch*. Internet hours are further decomposed into consumption- vs. investment-purpose hours, which is according to the authors' calculation by using the statistics provided by *YouTube*. Firm-level financial variables are based on the *Compustat* annual industrial files. Ratios relative to tangible capital are median values across a number of Internet firms on average during the period 2000-2012. Operating cash flow refers to profits from business before investment, advertising and selling expenses in the data and is measured as output before fixed cost and investment expenses in the model. Auto correlation coefficients of operating cash flow and equity value are measured as their median values across Internet firms and then averaged over the period 1991-2012.

[12] Note that in the steady state, all of the key variables are homogeneous across the internet-good variety firms essentially due to productivities $(z(i), z^U(i))$ being assumed homogeneous $z(i) = z, z^U(i) = \bar{z}^U$ and constant for every Internet firm i.

Simulated Results In this section, the calibrated model is used as a laboratory to implement an experiment of imposing various shocks to the model economy. In particular, importance of user-generated capital for the firm value is discussed by conducting a counterfactual experiment of reducing the amount of user-generated capital. The equilibrium of the model is solved numerically so that effects of such an experiment are quantified.

More specifically, we aim to quantify the contribution of user-generated capital to the Internet firm's value by carrying out a counterfactual experiment of reducing the amount of user-generated capital in place either permanently or temporarily as follows: First, we consider the case in which the efficiency of investment in user-generated capital is permanently shifted to below the calibrated level so that the user-generated capital stock is kept permanently to below the calibrated level. In this case, the reduction in the user-generated capital stock is accompanied by drops down in the firm's intangible capital—specific to the website quality/efficiency—and tangible capital, i.e., general equilibrium effects, which would amplify the effects of such a reduction in the user-generated capital stock on the firm's value. Second, we also consider the case in which the user-generated capital stock is reduced below the calibrated level both now and in the near future, while the firm's tangible and intangible capital stocks are not necessarily reduced now. In contrast to the earlier first experiment, the second experiment is intended to emphasize the effect of reduction in user-generated capital by mitigating the general-equilibrium-effect channel—even though we are not able to completely shut down it—through which the firm's tangible and intangible capital stocks are in equilibrium affected by the reduction in the user-generated capital stock.

Effects of Permanent Reduction in User-Generated Capital In this section, we discuss the effects of imposing permanent shocks to the efficiency of investment in user capital for a given Internet firm. By the very nature, these permanent shocks affect all aspects of the firm of our consideration in the long run, i.e., shift in the steady-state values of the firm's user-generated, intangible and tangible capital stocks. The analysis in this section can be understood as comparative statics w.r.t. the efficiency of user-generated capital investment $z^U(i)$, even though the model results are still fully dynamic, in the sense that we focus on the changes in the steady-state key variables due to permanent changes in the efficiency of user-capital investment $z^U(i)$. More specifically, as in the calibration analysis, for Internet firm i of our consideration, we assume no uncertainty so that $z_t^U(i) = \bar{z}^U, \forall t$ and calculate again the steady-state values of the firm's key variables for different values of $\bar{z}^U$. Thus, this section discusses business implications of long-run changes in the environment specific to an Internet firm, in particular, the efficiency of investment in user capital specific to and serviced by the firm's own website.

Table 12: Effects of Permanent Shocks to the Efficiency of User-Capital Investment $\bar{z}^U$

$\bar{z}^U$	$\hat{k}^T{}'(i)$	$\hat{k}^P{}'(i)$	$\hat{k}^U{}'(i)$	$h^I(i)$	$h^C(i)$	$\hat{V}(i)$
60%	61.6%	61.6%	37.9%	59.5%	61.6%	4.6%
80%	80.8%	80.8%	65.3%	79.7%	80.8%	52.4%
90%	90.4%	90.4%	81.7%	89.9%	90.4%	76.1%
110%	109.7%	109.7%	120.3%	110.1%	109.7%	124.0%
120%	119.4%	119.4%	142.5%	120.2%	119.4%	148.1%
140%	138.9%	138.9%	192.9%	140.4%	138.9%	196.5%

Note: this table presents the results of counterfactual experiment of imposing permanent shocks to $z^U(i)$: we present steady-state values of key variables corresponding to different values of the long-run average of $z^U(i)$ that is denoted by $\bar{z}^U$. The column headed '$\hat{k}^T{}'(i)$' refers to the firm's choice of the next-period tangible capital stock, $\hat{k}^P{}'(i)$ the next-period intangible capital stock, '$\hat{k}^U{}'(i)$' the next-period state of user capital, '$\hat{V}(i)$' the current-period equity value of the firm, and '$h^I(i)$' and '$h^C(i)$' the current-period response of the household's investment- and consumption-purpose hours, respectively. Every variable is in percentage terms to its corresponding value under the benchmark parameter setting.

Table 12 presents the results of permanently changing the efficiency of user-capital investment on key variables for a given Internet firm. As expected, negative (permanent) shocks to the long-run level of the efficiency of user-capital investment $\bar{z}^U$ reduce the steady-state user-generated capital stock because both the user- and company-side investment in user-generated capital decrease due to their low returns in the new degrading environment. Moreover, such effects are quantitatively substantial. For instance, consider the case of $\bar{z}^U$ being about 80 percent relative to the pre-shock value—that is, a permanent drop in $\bar{z}^U$ by 20 percent relative to the pre-shock state—: in this case, user-generated capital is about 65 percent relative to its pre-shock steady-state level, and the firm value is about 52 percent relative to the pre-shock steady-state firm value. Even for the mild decrease in $\bar{z}^U$ by 10 percent—that is, the line for $\bar{z}^U$ of 90% relative to its pre-shock value—, user-generated capital decreases by about 18 percent and the firm value drops down by about 24 percent.

Effects of Temporary Reduction in User-Generated Capital In this section, we discuss the effects of imposing *temporary* shocks to the efficiency of user-capital investment $z^U(i)$ so that user-generated capital is temporarily reduced, while the firm's tangible and intangible capital stocks are not necessarily reduced at the same time. Note that in the previous case of the permanent shocks to $z^U(i)$, all of firm's tangible and intangible capital stocks and user-generated capital stock are affected at the same time by the very nature of such permanent shocks to $z^U(i)$. Examining the effects of temporary shocks $z^U(i)$, we need to solve for the transition path, rather than simply new steady-state point in the state space, along which the economy moves after being hit by the temporary shock. For this purpose, we need to calibrate the process of temporary shocks to the efficiency of investment in user-generated capital and to solve for the equilibrium policy functions for every possible state. First, we calibrate the shock process for one individual firm i of our consideration; the cash-flow productivity is kept constant, while the efficiency of user-capital investment $z_t^U(i)$ is stochastic and follows the log-AR(1) process written as:

$$\log(z_{t+1}^U(i)) = (1-\rho)\cdot\log(\bar{z}^U) + \rho\cdot\log(z_t^U(i)) + \sigma^\varepsilon(i)\cdot\varepsilon_{t+1}(i), \quad 0<\rho<1,\ \sigma^\varepsilon(i)\geq 0 \tag{23}$$

where $(1-\rho)$ refers to the weight to the mean-reversion relative to the previous state and $\sigma^\varepsilon(i)$ the standard deviation of the i.i.d. error term. We choose $\rho = 0.868$ by targeting the autocorrelation of the market value of an Internet firm, which is 0.87 in the data and in the model, and $\sigma^\varepsilon(i) = 0.02$ as benchmark: in this case, the simulated firm value's autocorrelation is 0.88 and the firm value's (annual) volatility is 3.6 percentage points. Second, the state space of $(\hat{k}_t^P(i), \hat{k}_t^U(i), \hat{k}_t^T(i), z_t^U(i))$ is discretized, over which the equilibrium policy functions are approximated by using the first-order finite element method (FEM) as illustrated in McGrattan (1996) and then solved by the iteration method.[13]

Table 13: Effects of Temporary Shocks to Efficiency and Level of User-Generated Capital

	$\hat{k}^T{}'(i)$	$\hat{k}^P{}'(i)$	$\hat{k}^U{}'(i)$	$h^I(i)$	$h^C(i)$	$\hat{V}(i)$
Steady state: $\hat{k}^U(i) = 0.318, z^U(i) = 3.0$	0.043	0.030	0.320	0.004	0.103	0.711
Benchmark case: $\hat{k}^U(i) = 0, z^U(i) = 1.5$	0.013	0.072	0.031	0.008	0.000	0.287
Case: $\hat{k}^U(i) = 0.318, z^U(i) = 1.5$	0.040	0.001	0.288	0.002	0.103	0.519

Note: this table presents the results of counterfactual experiment of setting $\hat{k}^U(i)$ to zero together with imposing shocks to $z^U(i)$. All variables are in their levels. The line headed 'Steady state' refers to the calibrated model, i.e., no shocks to $\hat{k}^U(i)$ and no shocks to $z^U(i)$, while the two cases of imposing shocks are as follows: the line headed 'Benchmark case: $\hat{k}^U(i) = 0, z^U(i) = 1.5$' refers to the benchmark case of $\hat{k}^U(i)$ being set to zero and $z^U(i)$ being set to 1.5, i.e., 50 percent of the steady state of $z^U(i) = 3.0$, and the line headed 'Case: $\hat{k}^U(i) = 0.318, z^U(i) = 1.5$' the case of $z^U(i)$ being set to 1.5 and no shocks to $\hat{k}^U(i)$. $\hat{k}^T{}'(i)$ refers to the firm's choice of the next-period tangible capital stock, $\hat{k}^P{}'(i)$ the next-period intangible capital stock, $\hat{k}^U{}'(i)$ the next-period state of user-generated capital, $\hat{V}(i)$ the current-period equity value of the firm, and $h^I(i)$ and $h^C(i)$ the current-period response of the household investment- and consumption-purpose hours, respectively.

Table 13 presents results of the experiment of completely removing the current user-generated capital stock in place by lowering at the same time the efficiency of user-capital investment so that the user capital stock would be below the steady-state level in the near future as well as in the current period. In particular, we consider the benchmark case in which we set $\hat{k}^U(i)$ to zero and $z^U(i)$ to 50 percent of its steady-state level of $z^U(i) = 3.0$: we set $z^U(i)$ temporarily to such a low level—50 percent relative to the steady-state level—essentially in order to let user-generated capital be depressed in a low level for a time being; otherwise, user-generated capital would be restored to the steady-state level quickly via the firm's highly increased investment in intangible capital. We can see that for the benchmark case of completely

[13] The numerical errors between the guessed and updated policy functions are, except for the firm's own value function, about 0.00000001 percentage points relative to the steady-state values uniformly over the state space, while the numerical error between the guessed and updated firm's value function is about 0.1 percentage points relative to the steady-state value uniformly over the state space. See Mathematical Appendix for the equilibrium policy functions that are numerically solved for.

removing user-generated capital in place and setting $z^U(i)$ to 50 percent of its steady-state level, the firm value decreases substantially, by about 60 percent relative to the steady-state firm value. Here the key is that a large-scale of both firm- and household-side investment in user-generated capital would be, even though greatly needed, too expensive due to the lower efficiency of investment in user-generated capital $z^U(i)$, implying that the next-period user-generated capital stock also stays in a low level, about 10 percent of its steady-state level. As a result, both the household hours spent on viewing content on the firm's own website and the firm's cash flow stay in lower levels in the future as well as in the current period. Thus, the firm's current value immediately drops down greatly. Interestingly, we can also see that the firm's tangible investment, even though not directly intended, decreases greatly, which is mainly due to the fact that users' consumption-purpose hours are of low level and hence the next-period return to tangible capital drops down greatly.

For comparison, consider the case in which the efficiency of user-capital investment $z^U(i)$ is temporarily set to the 50 percent of the steady-state level and however, the current user capital stock stays at the steady-state level: see the line headed 'Case: $\hat{k}^U(i) = 0.318, z^U(i) = 1.5$' in Table 13. In this case, the firm value drops mildly, by about 27 percent, relative to the steady-state firm value essentially because the next-period user-generated capital stock $\hat{k}^{U\prime}(i)$ drops down slightly, by about 10 percent of the steady-state level. As a result, users' consumption-purpose hours is also below the steady-state level slightly in the near future, and hence so is the firm's cash flow, which makes the firm value drop down slightly.

V. Conclusion

This paper studies the effect of online user activities on the Internet firm's value. Using the data on firm-level financial variables and household time spent on websites, we find that the amount of *user-generated capital* on an Internet firm's website is significantly positively associated with the Internet firm's market value. This suggests that online user activities are an important part of an Internet firm's assets but are missing in the firm's balance sheet. Moreover, we also find that for an Internet firm, the firm's market-to-book value ratio is much higher than for other firms. All of these suggest that user-generated capital is quantitatively important in accounting for the Internet firm's market value.

We study the theoretical mechanism by which user-generated capital contributes to the Internet firm's value by building and calibrating a dynamic general equilibrium model of the household time allocation among competing websites. The model predicts that improvements in either an Internet firm's website quality or the efficiency of investment to user-generated capital increase the Internet firm's future cash flows. In turn, this will increase the current value of the firm: the main reason is that the expected users' hours spent on the firm's own website in the near future greatly increase. Calibrating the model to the U.S. data on Internet firms' financials and the household time spending, we quantify the contribution of user-generated capital to the value of an Internet firm by conducting a counterfactual experiment: we completely remove the user-generated capital stock in place now and keep it in a low level temporarily in the near future. We find that in this case, user-generated capital accounts for about 60 percent of the Internet firm's value. Moreover, we also consider the case in which the efficiency of investment in user-generated capital permanently worsens by 20 percent so that user-generated capital is permanently reduced by 35 percent: in this case, the firm value greatly decreases, by about 48 percent.

In this paper, the competition among Internet firms and these firms' strategic behaviors are modeled in a simple way of the monopolistic competition among a continuum of firms. It would be interesting to study the case in which a finite number of competing Internet firms strategically behave to poach the dominant online platform, which we leave for a future work.

VI. **References**

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