Alternative Measures of OECD Output Growth and Inflation

W. Erwin Diewert,[1]
Discussion Paper 14-01,
School of Economics,
University of British Columbia,
Vancouver, B.C.,
Canada, V6T 1Z1.
Email: erwin.diewert@ubc.ca

Revised, July 5, 2014

Abstract

The paper addresses two problems: (i) how to measure aggregate real output and inflation for a group of countries and (ii) how to construct measures of real GDP for a group of countries where the country measures of real GDP are comparable across time and space. In order to address both problems, it is necessary that the group of countries construct at least one set of Purchasing Power Parities (PPPs). The OECD publishes PPPs for 34 of its member countries. These data are used for the years 2000-2012 to study the above two problems. The paper proposes a new method for harmonizing conflicting estimates of member country real GDP (that arise from the PPP generated cross sectional estimates of relative GDP) with estimates of country GDP that are consistent with national growth rates of GDP. A new measure of OECD inflation is also proposed.

Journal of Economic Literature Classification Numbers

C43, C82, E01.

Key Words

Purchasing Power Parities, PPPs, ICP, OECD country statistics, inflation, price and volume indexes, Fisher indexes, country competitiveness.

---

[1] W. Erwin Diewert: School of Economics, University of British Columbia, Vancouver B.C., Canada, V6T 1Z1 and the School of Economics, University of New South Wales, Sydney, Australia (email: erwin.diewert@ubc.ca). The author thanks Bert Balk, Yuri Dikhanov, Dennis Fixler, Kevin Fox, Anne-Sophie Fraisse, Robert Inklaar, Paul McCarthy, Prasada Rao and Paul Schreyer for helpful comments and gratefully acknowledges the financial support of the SSHRC of Canada and the World Bank. None of the above are responsible for the contents of this paper.

1. Introduction

Index number theory is used to decompose national nominal GDP into price and quantity (or volume) components. But the following quotation indicates that constructing estimates of inflation and real GDP growth for a group of countries that use different currencies is a more complicated operation:

"At the national level, current price (value) data can typically be de-composed into a volume (or quantity) series and price series. At the international level, a second 'price' component enters the picture in the form of a conversion rate from the domestic to a common currency. The implication is that values can be expressed at current market exchange rates (or current international prices, if purchasing power parities - *PPPs* - are used); and at constant exchange rates (or constant international prices)." OECD (2001; 6).

It should be noted that the OECD publishes estimates of Purchasing Power Parities on an annual basis and these PPPs can be used to generate estimates of real GDP for member countries that are comparable across countries for the given year. However, the resulting estimates of relative real GDP are not consistent with national estimates of real GDP growth for the member countries.

The present paper will use OECD data for the years 2000-2012 in order to study the following two measurement problems:

- How can estimates of OECD aggregate real GDP and associated measures of aggregate OECD inflation be constructed?
- How can the OECD annual PPP information be used in conjunction with member country data on real GDP growth to construct estimates of member country real GDP that are in principle comparable across space and time?

It should be noted that providing consistent (across time and space) estimates of real GDP across countries is important for many purposes; see Eurostat (2012) and the World Bank (2013).

In section 2, we first review some fundamentals of index number theory in order to justify our choice of index number formula. We then study the first measurement problem using just national data and exchange rate information. Sections 3-6 use the OECD PPP data and study the second measurement problem (and revisit the first problem). In section 3, we propose a method (the harmonized method) for constructing estimates of member country GDP volumes that are comparable across time and space. In sections 4 and 5, we compare our harmonized estimates with estimates of member country real GDP that are generated by using PPP data for only one year. These base year estimates are then projected to all other years using national growth rates of GDP. Section 4 uses the PPPs for 2000 and section 5 uses the PPPs for 2012. We find that the resulting two panel sets of real GDP estimates are very different from each other and they also are very different from our harmonized estimates developed in section 3. Section 6 uses PPP information for 2000 and for 2012 to provide (interpolated) estimates of

country volumes that are much closer to our preferred harmonized estimates of section 3. This interpolation method should be useful in a wider context.[2] Section 7 concludes.

2. OECD Growth and Inflation Using Market Exchange Rates

Our first measure of aggregate GDP growth over the member countries in the OECD during the years 2000-2012 uses national growth rates of GDP and domestic prices converted into US dollars at market exchange rates. The aggregation principle used to form OECD aggregate GDP volumes and prices in this section is the same that is used to aggregate prices and quantities across different regions in a country: each commodity in each region is regarded as a separate commodity in the index number formula.

Our first task is to use the OECD data base to form country GDP volumes. We will use the OECD ordering of countries, which is as follows:

- 1= Australia
- 2= Austria
- 3= Belgium
- 4= Canada
- 5= Chile
- 6= Czech Republic
- 7= Denmark
- 8= Estonia
- 9= Finland
- 10= France
- 11= Germany
- 12= Greece
- 13= Hungary
- 14= Iceland
- 15= Ireland
- 16= Israel
- 17= Italy
- 18= Japan
- 19= Korea
- 20= Luxembourg
- 21= Mexico
- 22= Netherlands
- 23= New Zealand
- 24= Norway
- 25= Poland
- 26= Portugal
- 27= Slovak Republic
- 28= Slovenia
- 29= Spain
- 30= Sweden
- 31= Switzerland

[2] For example, the World Bank provided PPPs for 155 countries for the year 2005 and has just provided a new set of PPPs for 2011. How can these two benchmark sets of PPPs be used in conjunction with national data in order to provide estimates of country real GDP that are comparable across all years from 2005 to 2012? The interpolation method explained in section 6 could be used in this context.

- 32= Turkey
- 33= United Kingdom
- 34= United States.

The country values for nominal GDP in the national currencies for the years 2000-2012 can be obtained from the OECD electronic data base, OECD.Stat. The relevant table is Table B1-GE: Gross domestic product (expenditure approach); National currency, current prices, millions, annual data. Convert these estimates into billions and denote the estimate for country n in year t by $V_n{}^t$ The corresponding volume estimates can be obtained from OECD.Stat TableB1-GE: Gross domestic product (GDP); National currency, constant prices, national base year, millions, annual data. Convert these estimates into billions and denote these volumes (or quantities) by $Q_n{}^t$ for n = 1,...,34 and t = 2000, ...,2012. The corresponding country price level for country n in year t is defined as $P_n{}^t \equiv V_n{}^t/Q_n{}^t$ for n = 1,...,34 and t = 2000,...,2012. These national price levels and volumes are listed in the Appendix; see Tables A1 and A2.[3]

Since the country volumes (or outputs) $Q_n{}^t$ are measured in domestic currency units (which are not comparable across countries), we need to convert the domestic nominal values of GDP into common currency units using the average exchange rates for each year. In principle, the numeraire country could be any of the 34 OECD countries but it seems reasonable to choose the largest country as the numeraire country. The OECD has conveniently done this for us, converting each country's nominal GDP into US dollars at the average market exchange rates for the given year. The relevant table is Table B1-GE: Gross domestic product (expenditure approach); US dollars, current prices, current exchange rates, millions, annual data. Convert these estimates into billions and denote the US dollar estimate for nominal GDP for country n in year t by $v_n{}^t$.

The year t, country n US dollar price level for GDP, $p_n{}^t$, is initially defined as $v_n{}^t/Q_n{}^t$ where the country volumes or real outputs $Q_n{}^t$ have already been defined using national data. The resulting $p_n{}^t$ were normalized so that $p_n{}^{2000} = 1$ for n = 1,...,34. The $Q_n{}^t$ were then normalized in the opposite direction so that US dollar values were preserved. Denote the resulting normalized $Q_n{}^t$ as $q_n{}^t$ for n = 1,...,34 and t = 2000,...,2012.[4] These US dollar price levels $p_n{}^t$ and the corresponding volumes $q_n{}^t$ are listed in Tables A3 and A4 in the Appendix.

We are now in a position to calculate aggregate OECD real output and the corresponding price level for the years 2000-2012 using the price and volume data, $p_n{}^t$ and $q_n{}^t$, as inputs into an index number formula. But which index number formulae should be chosen to do the aggregation?

---

[3] The country price series $P_n{}^t$ were normalized so that $P_n{}^{2000} = 1$ for n = 1,...,34. The units of the $Q_n{}^t$ were normalized in the opposite direction so that the product of the $P_n{}^t$ and $Q_n{}^t$ equalled $V_n{}^t$ for each n and t.

[4] Note that $v_n{}^t = p_n{}^t q_n{}^t$ for n = 1,...,34 and t = 2000,...,2012. For each n, the US dollar volumes $q_n{}^t$ are proportional to the national volumes $Q_n{}^t$; i.e., we have $q_n{}^t = \lambda_n Q_n{}^t$ for t = 2000,...,2012 for each country n where λ_n is the factor of proportionality for country n.

It will be useful to discuss the choice of index number formula in the context of providing index levels for two periods, say periods 0 and 1.[5] Suppose there are N commodities to be aggregated. Denote the price and quantity vectors for period t by $p^t \equiv [p_1^t,...,p_N^t]$ and $q^t \equiv [q_1^t,...,q_N^t]$ for t = 0,1. The value of transactions in the N commodities during period t is defined as $v^t \equiv \sum_{n=1}^N p_n^t q_n^t \equiv p^t \cdot q^t$.[6] The problem of choosing functional forms for the price and quantity indexes is usually phrased as follows: find two suitable functions of 4N variables, a price index function $P(p^0,p^1,q^0,q^1)$ and a quantity index function $Q(p^0,p^1,q^0,q^1)$, such that the product of these two functions is equal to the value ratio, v^1/v^0. Thus the functions P and Q are to satisfy the following equation:

(1) $p^1 \cdot q^1/p^0 \cdot q^0 = P(p^0,p^1,q^0,q^1)Q(p^0,p^1,q^0,q^1)$.

It can be seen that if the functional form for either the price or quantity index is determined, then the functional form of the corresponding quantity or price index is also determined using equation (1).[7]

Two natural choices for the functional form for the price index are the Laspeyres and Paasche price indexes, P_L and P_P, defined as follows:

(2) $P_L(p^0,p^1,q^0,q^1) \equiv p^1 \cdot q^0/p^0 \cdot q^0$;
(3) $P_P(p^0,p^1,q^0,q^1) \equiv p^1 \cdot q^1/p^0 \cdot q^1$.

It can be seen that the Laspeyres price index uses the “basket” of period 0 quantities, q^0, and prices out this basket at the prices of period 0 (in the denominator) and prices out the same basket at the prices of period 1 (in the numerator) and takes the ratio of these costs as the price index. The Paasche index is similar but uses the “basket” of period 1 quantities, q^1, as the common quantity vector in the numerator and denominator. Using (1), it can be seen that quantity indexes that match up with P_L and P_P are Q_P and Q_L defined as follows:

(4) $Q_P(p^0,p^1,q^0,q^1) \equiv p^1 \cdot q^1/p^1 \cdot q^0$;
(5) $Q_L(p^0,p^1,q^0,q^1) \equiv p^0 \cdot q^1/p^0 \cdot q^0$.

The Paasche and Laspeyres quantity indexes defined by (4) and (5) are also intuitively appealing. The Paasche index uses the prices of period 1, p^1, to value the outputs of periods 1 (numerator) and 0 (denominator) and takes the ratio of these two constant price aggregates as the quantity index. The Laspeyres quantity index is similar except it uses the prices of period 0 to value the quantity vectors q^0 and q^1.

---

[5] For materials on the historical development of index number theory, see Diewert (1993) and Balk (2008).
[6] The inner product of two vectors $x \equiv [x_1,...,x_N]$ and $y \equiv [y_1,...,y_N]$ of the same dimension N is defined as $x \cdot y \equiv \sum_{n=1}^N x_n y_n$.
[7] Once P and Q satisfying (1) have been chosen, the corresponding price levels for periods 0 and 1, say P^0 and P^1, and the corresponding quantity (or volume) levels for periods 0 and 1, say Q^0 and Q^1, are generally determined as follows: $P^0 \equiv 1$; $P^1 \equiv P(p^0,p^1,q^0,q^1)$; $Q^0 \equiv v^0 = p^0 \cdot q^0$ and $Q^1 \equiv v^0 Q(p^0,p^1,q^0,q^1) = v^1/P(p^0,p^1,q^0,q^1)$. Note that the price and quantity indexes can be interpreted as *ratios* of aggregate price and quantity *levels*; i.e., we have $P(p^0,p^1,q^0,q^1) = P^1/P^0$ and $Q(p^0,p^1,q^0,q^1) = Q^1/Q^0$.

It can be seen that the Paasche and Laspeyres price and quantity indexes are equally plausible. The problem is that they can generate quite different estimates of growth and inflation. Thus a natural solution to this problem is to take a symmetric average of these two equally plausible estimates. It turns out that taking the geometric mean of these two price indexes (and of the two corresponding quantity indexes) leads to indexes that have very good axiomatic properties.[8] This leads to the Fisher (1922) ideal price and quantity indexes, P_F and Q_F, defined as follows:[9]

(6) $P_F(p^0,p^1,q^0,q^1) \equiv [P_L(p^0,p^1,q^0,q^1)P_P(p^0,p^1,q^0,q^1)]^{1/2}$;
(7) $Q_F(p^0,p^1,q^0,q^1) \equiv [Q_L(p^0,p^1,q^0,q^1)Q_P(p^0,p^1,q^0,q^1)]^{1/2}$.

Having settled on the Fisher price and quantity indexes as suitable index number formulae when aggregating commodities over two periods, there is one more choice that needs some discussion: namely, should fixed base or chained Fisher indexes be used when aggregating over many periods?

The chain system[10] measures the change in prices going from one period to a subsequent period using a bilateral index number formula involving the prices and quantities pertaining to the two adjacent periods. These one period rates of change (the links in the chain) are then cumulated to yield the relative levels of prices over the entire period under consideration. Thus if the bilateral price index is P, the chain system generates the following sequence of price levels for the first three periods:

(8) $1, P(p^0,p^1,q^0,q^1), P(p^0,p^1,q^0,q^1)\, P(p^1,p^2,q^1,q^2)$.

On the other hand, the fixed base system of price levels using the same bilateral index number formula P simply computes the level of prices in period t relative to the base period 0 as $P(p^0,p^t,q^0,q^t)$. Thus the fixed base sequence of price levels for periods 0,1 and 2 is:

(9) $1, P(p^0,p^1,q^0,q^1), P(p^0,p^2,q^0,q^2)$.

There are two major problems associated with the use of fixed base indexes in the context of annual time series data:

- Over longer periods of time, it becomes more difficult to match up products in the current period with the corresponding products in a distant base period, leading to less accurate index numbers;

---

[8] See Fisher (1922) and Diewert (1992) (1997).
[9] It can be verified that $P_FQ_F = v^1/v^0$; i.e., the Fisher price and quantity indexes satisfy equation (1).
[10] The chain principle was introduced independently into the economics literature by Lehr (1885; 45-46) and Marshall (1887; 373). Both authors observed that the chain system would mitigate the difficulties due to the introduction of new commodities into the economy, a point also mentioned by Hill (1993; 388). Fisher (1911; 203) introduced the term "chain system".

- Fixed base indexes are subject to revisions (that can be substantial) when the base period is finally changed.

When using fixed base Paasche or Laspeyres indexes, the revision problem can become massive.[11] Thus a major advantage of the chain system in the context of aggregating annual data is that chaining will reduce the spread between the Paasche and Laspeyres indexes.[12] These two indexes each provide an asymmetric perspective on the amount of price change that has occurred between the two periods under consideration and it could be expected that a single point estimate of the aggregate price change should lie between these two estimates. Thus the use of either a chained Paasche or Laspeyres index will usually lead to a smaller difference between the two and hence to estimates that are closer to the "truth".[13]

It is not always best to use chained indexes. T.P. Hill (1993; 388), drawing on the earlier research of Szulc (1983) and T.P. Hill (1988; 136-137), noted that it is not appropriate to use the chain system when prices oscillate or "bounce" to use Szulc's (1983; 548) term. This bouncing phenomenon can occur when aggregating subannual data when there are seasonal fluctuations or periodic sales (deeply discounted prices).[14] However, in the context of more or less smoothly trending prices and quantities as is the usual case using annual data, Hill (1993; 389) recommended the use of chained symmetrically weighted indexes such as the Fisher ideal index. Thus in this paper, we will use chained Fisher indexes when aggregating over countries.

Recall that the country US dollar price levels p_n^t and the corresponding volumes q_n^t are listed in Tables A3 and A4 in the Appendix. Denote the chained Fisher aggregate OECD volume level for year t by Q^t and the corresponding US dollar year t price level by P^t for t = 2000,...,2012. For t = 2001,...,2012, define the year t OECD *volume growth rate* γ^t and the corresponding OECD *US dollar inflation rate* ρ^t in percentage points as follows:

(10) $\gamma^t \equiv 100[(Q^t/Q^{t-1}) - 1]$;
(11) $\rho^t \equiv 100[(P^t/P^{t-1}) - 1]$.

The chained Fisher OECD aggregate price and volume levels, P^t and Q^t, for the years 2000-2012 are listed in Table 1 along with the corresponding percentage point annual growth rates, ρ^t and γ^t, for the years 2001-2012. For comparison purposes, we also calculated the aggregate OECD chained Laspeyres and Paasche indexes over the same

[11] The US Bureau of Economic Analysis used to provide long term estimates of US GDP back to 1926 using fixed base Laspeyres volume indexes. When the base year was changed, the resulting Laspeyres estimates of real GDP growth changed massively and this fact led the BEA to switch to chained Fisher indexes in the early 1990s.

[12] See Diewert (1978; 895) and T.P. Hill (1988) (1993; 387-388).

[13] There is a more elaborate justification for chaining annual data that is based on aggregating over observations that have the most "similar" price structures; see R.J. Hill (2001), (2004) (2009) and Diewert (2009). Typically, adjacent annual observations will have more similar price structures than a pair of observations chosen from different decades.

[14] Constructing indexes for subannual data is a much more difficult problem; see Ivancic, Diewert and Fox (2011) for one solution to this problem.

period. The resulting Laspeyres and Paasche price levels, P_L^t and P_P^t, are also listed in Table 1. It can be seen that the chained Fisher, Laspeyres and Paasche price levels are all very close to each other so that for this particular application, the choice of index number formula does not matter very much.

Table 1: OECD Annual Aggregate Volumes Q^t and Price Levels in US Dollars P^t, P_L^t and P_P^t, Price Levels in Euros P_{EU}^t, ICP Price Levels P_{ICP}^t and Percentage Point Changes, 2000-2012

Year t	Q^t	P^t	P_L^t	P_P^t	γ^t	ρ^t	ρ_{EU}^t	P_{EU}^t	P_{ICP}^t
2000	26694.3	1.0000	1.0000	1.0000				1.0000	1.0000
2001	27022.9	0.9794	0.9793	0.9795	1.23	-2.06	0.84	1.0084	1.0301
2002	27432.9	1.0081	1.0079	1.0082	1.52	2.92	-2.14	0.9868	1.0549
2003	28007.3	1.1079	1.1079	1.1078	2.09	9.90	-8.35	0.9044	1.0796
2004	28896.6	1.1933	1.1934	1.1932	3.18	7.71	-2.10	0.8854	1.1067
2005	29670.9	1.2271	1.2271	1.2272	2.68	2.84	2.68	0.9091	1.1326
2006	30566.7	1.2560	1.2558	1.2561	3.02	2.35	1.46	0.9224	1.1610
2007	31374.2	1.3392	1.3388	1.3396	2.64	6.63	-2.27	0.9015	1.1893
2008	31410.0	1.4109	1.4104	1.4114	0.11	5.36	-1.56	0.8874	1.2173
2009	30267.1	1.3729	1.3726	1.3732	-3.64	-2.69	2.60	0.9105	1.2305
2010	31138.6	1.4013	1.4007	1.4019	2.88	2.07	7.06	0.9748	1.2477
2011	31688.5	1.4776	1.4770	1.4782	1.77	5.44	0.46	0.9793	1.2697
2012	32162.6	1.4534	1.4525	1.4543	1.50	-1.63	6.42	1.0422	1.2888

The sample average of the year to year growth rates for OECD real GDP using US dollar weights, the γ^t, was 1.58% per year. It can be seen that there was only one year where OECD real growth was negative: 2009 (−3.64%). The sample average of the OECD inflation rates ρ^t (measured in US dollars at market exchange rates) was 3.24% per year. However, what is striking is the variability in these US dollar inflation rates.

We note that the principles used to construct our OECD aggregate measures of real GDP, Q^t, are the same principles used to construct country wide estimates of real GDP within a country. The country estimates of real GDP aggregate output growth over regions within the country use regional price levels as weights for the regional volumes. In constructing national estimates of real output, the national statistician does not assume that the quantities or volumes in each region are comparable across regions: all that is assumed is that whatever is being measured at the regional level *is comparable over time*. This is the same principle that is being used to construct the above OECD real output measures Q^t: there is no assumption that the country units of measurement are comparable across countries.

There is one difference in our suggested method for constructing OECD real GDP as opposed to methods used to construct national estimates of real GDP: in order to construct OECD real GDP, we needed to convert national values of GDP into a common currency using annual average market exchange rates. We chose to make this conversion using US dollars as the numeraire currency. In principle, we could have chosen the numeraire currency to be the currency of any one of the 34 member countries. What would happen if we chose another currency to be the numeraire currency? The unit of measurement would change, but the overall OECD *growth rates* for real GDP would remain the same; i.e., the γ^t listed in Table 1 *do not change* if we converted all country

nominal GDP estimates into a different common currency at annual average market exchange rates and then applied the same methodology to construct the overall OECD volume estimates.[15] On the other hand, switching to a different numeraire currency dramatically affects the inflation rates ρ^t. Thus the OECD aggregate price level estimates P^t and the resulting inflation rates ρ^t defined by (11) change with each choice of a numeraire currency.

In order to illustrate the dependence of the above OECD GDP inflation rates on the choice of the numeraire country, we computed the aggregate OECD price and volume levels, P_{EU}^t and Q_{EU}^t, using Germany (Country 11) as the numeraire country. Thus instead of using the US dollar estimates for nominal GDP for country n in year t defined earlier by v_n^t, we converted the v_n^t into Euros using the implied OECD exchange rates that can be obtained by dividing the national value estimates for GDP by their corresponding US dollar measures. The same Fisher index number methodology was then used to construct P_{EU}^t and Q_{EU}^t. Finally, the year t OECD *Euro volume growth rate* γ_{EU}^t and the corresponding OECD *Euro inflation rate* ρ_{EU}^t in percentage points were defined as $\gamma_{EU}^t \equiv 100[(Q_{EU}^t/Q_{EU}^{t-1}) - 1]$ and $\rho_{EU}^t \equiv 100[(P_{EU}^t/P_{EU}^{t-1}) - 1]$ for t = 2001,...,2012. The resulting Euro based price index P_{EU}^t and inflation growth rates ρ_{EU}^t in percentage points are listed in the last two columns of Table 1 above.[16] Comparing the inflation measures using the US and then Germany as the numeraire countries shows that the resulting price levels, P^t and P_{EU}^t, and inflation rates, ρ^t and ρ_{EU}^t, are *completely different*. P^t trends upward from 1.00 in 2000 to end up at 1.45 in 2012 whereas the Euro based OECD price level trends downward to 0.89 in 2008 and then trend upward to end up at 1.04 in 2012. The explanation for these diverging results is simple: they are driven by large exchange rate movements over the sample period.[17]

Our conclusion at this point is that our *first approach* to measuring OECD real output and inflation using national GDP data and market exchange rates between countries is (perhaps) satisfactory for measuring real output but that it is *not* satisfactory for measuring inflation. A satisfactory inflation measure will be introduced in the following section when we introduce our *second approach* to measuring aggregate OECD inflation.

The analysis presented in this section made no assumption that the goods and services produced in any country were comparable to the goods and services produced in any other country. In the following section, it will be assumed that the goods and services produced in each country *are comparable across countries* and different measures of OECD growth and inflation will be derived.

---

[15] In order for this statement to be true, we need our chosen bilateral index number formula to satisfy the following two tests: $Q(\lambda p^0,p^1,q^0,q^1) = Q(p^0,p^1,q^0,q^1)$ for all scalar $\lambda > 0$ and $Q(p^0,\lambda p^1,q^0,q^1) = Q(p^0,p^1,q^0,q^1)$ for all scalar $\lambda > 0$. The Fisher, Laspeyres and Paasche bilateral quantity indexes all satisfy these homogeneity in prices properties.

[16] The γ_{EU}^t are not listed in Table 1 because, as expected, $\gamma_{EU}^t = \gamma^t$ for each year t.

[17] Thus the number of Euros it took to buy one US dollar in 2000, 2008 and 2012 was 1.085, 0.683 and 0.778 respectively. Thus US prices in terms of Euros declined markedly from 2000 to 2008 and this explains the large number of negative ρ_{EU}^t over this period.

3. OECD Growth and Inflation Measurement Using Annual PPP Information

For many purposes, it is useful to be able to compare the GDP of one country with the GDP of another country in comparable units of measurement. Thus the OECD (in close cooperation with Eurostat) produces an annual series of price indexes that enable one to compare the real GDP of member countries with each other. These price indexes are called *Purchasing Power Parities* (PPPs).[18] For each OECD country n and each year t, PPP_n^t is an estimate of the number of units of the national currency of country n that is required to purchase one dollar of US GDP in year t. The relevant table of PPPs can be found in OECD.Stat, Table 4: PPPs and Exchange Rates; PPPGDP; Purchasing Power Parities for GDP; National currency per US dollar; Annual; 2000-2012. This Table is reproduced in the Appendix as Table A5.

Recall that the country n nominal value of GDP in year t in domestic currency was defined by V_n^t in the previous section. Divide these nominal values by the corresponding country n year t PPP in order to obtain an estimate, r_n^t, of country n's *real GDP* in year t in units that are comparable across countries for year t:[19]

$$(12)\ r_n^t \equiv V_n^t/PPP_n^t\ ; \qquad n = 1,...,34;\ t = 2000,...,2012.$$

Once the r_n^t have been calculated, they can be summed so that $r^t \equiv \sum_{n=1}^{34} r_n^t$ and then the *year t country n share of OECD real output* can be defined as follows:[20]

$$(13)\ s_n^t \equiv r_n^t/r^t\ ; \qquad n = 1,...,34;\ t = 2000,...,2012.$$

These country shares of OECD real GDP are listed in Table 2.[21] Note that country 34, the US, has the largest share (around 35-37%), followed by country 18, Japan, (10-11%) and country 11, Germany (7%). Table 2 enables analysts to compare GDP volumes across all OECD countries within each year.

Table 2: Country Shares of OECD Real GDP 2000-2012

s_n^{2000}	s_n^{2001}	s_n^{2002}	s_n^{2003}	s_n^{2004}	s_n^{2005}	s_n^{2006}	s_n^{2007}	s_n^{2008}	s_n^{2009}	s_n^{2010}	s_n^{2011}	s_n^{2012}
0.01885	0.01916	0.01950	0.01996	0.01996	0.01999	0.02002	0.02033	0.02027	0.02202	0.02203	0.02231	0.02261
0.00812	0.00788	0.00801	0.00798	0.00793	0.00772	0.00786	0.00778	0.00793	0.00797	0.00793	0.00799	0.00802
0.00994	0.00990	0.01010	0.00986	0.00960	0.00941	0.00938	0.00932	0.00948	0.00967	0.00968	0.00963	0.00962
0.03066	0.03071	0.03051	0.03103	0.03099	0.03158	0.03124	0.03110	0.03105	0.03122	0.03135	0.03153	0.03147
0.00518	0.00528	0.00530	0.00537	0.00556	0.00576	0.00666	0.00688	0.00661	0.00674	0.00750	0.00822	0.00848
0.00560	0.00581	0.00583	0.00601	0.00606	0.00607	0.00621	0.00647	0.00645	0.00659	0.00626	0.00622	0.00610

---

[18] The construction of these PPPs is explained in the Eurostat and OECD PPP Manual; see Eurostat (2012). The International Comparison Program (ICP) of the World Bank constructed PPPs for over 150 countries for 2005 and 2011. The ICP methodology is explained in World Bank (2013).

[19] These relative GDP volume measures for year t are not comparable across years.

[20] Note that the country shares s_n^t can be constructed without using country exchange rates (in principle). Thus the measures derived in this section are independent of country exchange rates.

[21] Row n+1 in the Table gives the shares for country n where we use the standard ordering of OECD countries listed in the previous section. Since the PPPs used by the OECD are invariant to the choice of the numeraire country (up to a scalar factor), it can be verified that the country shares listed in Table 2 are also invariant to the choice of numeraire country.

0.00540	**0.00532**	**0.00538**	**0.00515**	**0.00515**	**0.00502**	**0.00509**	**0.00507**	**0.00523**	**0.00518**	**0.00529**	**0.00516**	**0.00512**
0.00047	**0.00049**	**0.00053**	**0.00057**	**0.00059**	**0.00062**	**0.00067**	**0.00071**	**0.00071**	**0.00065**	**0.00064**	**0.00068**	**0.00069**
0.00466	**0.00465**	**0.00466**	**0.00452**	**0.00461**	**0.00449**	**0.00453**	**0.00471**	**0.00484**	**0.00465**	**0.00454**	**0.00455**	**0.00450**
0.05378	**0.05496**	**0.05547**	**0.05312**	**0.05204**	**0.05191**	**0.05176**	**0.05203**	**0.05238**	**0.05334**	**0.05240**	**0.05227**	**0.05150**
0.07428	**0.07426**	**0.07365**	**0.07343**	**0.07236**	**0.07158**	**0.07180**	**0.07199**	**0.07285**	**0.07145**	**0.07235**	**0.07346**	**0.07334**
0.00699	**0.00730**	**0.00765**	**0.00778**	**0.0078**	**0.00754**	**0.00775**	**0.00765**	**0.00795**	**0.00807**	**0.00732**	**0.00661**	**0.00620**
0.00426	**0.00461**	**0.00485**	**0.00488**	**0.00483**	**0.00478**	**0.00479**	**0.00469**	**0.00490**	**0.00497**	**0.00483**	**0.00479**	**0.00474**
0.00028	**0.00029**	**0.00029**	**0.00028**	**0.00029**	**0.00029**	**0.00028**	**0.00028**	**0.00030**	**0.00029**	**0.00027**	**0.00026**	**0.00026**
0.00385	**0.00399**	**0.00424**	**0.00435**	**0.00441**	**0.00450**	**0.00469**	**0.00487**	**0.00453**	**0.00443**	**0.00436**	**0.00440**	**0.00435**
0.00516	**0.00507**	**0.00502**	**0.00466**	**0.00474**	**0.00450**	**0.00439**	**0.00452**	**0.00447**	**0.00475**	**0.00485**	**0.00507**	**0.00512**
0.05144	**0.05248**	**0.05010**	**0.04930**	**0.04732**	**0.04624**	**0.04656**	**0.04684**	**0.04773**	**0.04748**	**0.04562**	**0.04488**	**0.04383**
0.11531	**0.11387**	**0.11295**	**0.11192**	**0.11094**	**0.10850**	**0.10557**	**0.10507**	**0.10253**	**0.09915**	**0.10088**	**0.09779**	**0.09745**
0.02836	**0.02902**	**0.03045**	**0.03030**	**0.03069**	**0.03059**	**0.03051**	**0.03124**	**0.03123**	**0.03213**	**0.03324**	**0.03352**	**0.03344**
0.00082	**0.00080**	**0.00084**	**0.00086**	**0.00088**	**0.00089**	**0.00096**	**0.00100**	**0.00099**	**0.00096**	**0.00100**	**0.00102**	**0.00102**
0.03463	**0.03406**	**0.03409**	**0.03478**	**0.03505**	**0.03609**	**0.03747**	**0.03772**	**0.03889**	**0.03931**	**0.04028**	**0.04308**	**0.04369**
0.01643	**0.01668**	**0.01678**	**0.01614**	**0.01596**	**0.01598**	**0.01617**	**0.01642**	**0.01687**	**0.01659**	**0.01623**	**0.01607**	**0.01569**
0.00288	**0.00292**	**0.00297**	**0.00300**	**0.00300**	**0.00294**	**0.00297**	**0.00301**	**0.00298**	**0.00320**	**0.00314**	**0.00315**	**0.00317**
0.00569	**0.00565**	**0.00547**	**0.00548**	**0.00576**	**0.00614**	**0.00652**	**0.00647**	**0.00699**	**0.00648**	**0.00660**	**0.00684**	**0.00715**
0.01418	**0.01414**	**0.01438**	**0.01437**	**0.01468**	**0.01468**	**0.01494**	**0.01572**	**0.01642**	**0.01756**	**0.01799**	**0.01841**	**0.01855**
0.00638	**0.00643**	**0.00646**	**0.00638**	**0.00616**	**0.00629**	**0.00632**	**0.00632**	**0.00633**	**0.00648**	**0.00639**	**0.00611**	**0.00580**
0.00208	**0.00219**	**0.00227**	**0.00230**	**0.00233**	**0.00243**	**0.00257**	**0.00277**	**0.00300**	**0.00299**	**0.00296**	**0.00294**	**0.00296**
0.00122	**0.00124**	**0.00128**	**0.00129**	**0.00131**	**0.00131**	**0.00133**	**0.00135**	**0.00140**	**0.00134**	**0.00128**	**0.00127**	**0.00123**
0.03010	**0.03105**	**0.03235**	**0.03263**	**0.03274**	**0.03316**	**0.03482**	**0.03560**	**0.03611**	**0.03593**	**0.03409**	**0.03296**	**0.03216**
0.00870	**0.00848**	**0.00850**	**0.00855**	**0.00864**	**0.00824**	**0.00842**	**0.00866**	**0.00873**	**0.00848**	**0.00865**	**0.00881**	**0.00889**
0.00819	**0.00813**	**0.00821**	**0.00796**	**0.00784**	**0.00767**	**0.00796**	**0.00832**	**0.00876**	**0.00889**	**0.00894**	**0.00914**	**0.00925**
0.02067	**0.01894**	**0.01861**	**0.01845**	**0.02034**	**0.02179**	**0.02326**	**0.02402**	**0.02553**	**0.02543**	**0.02708**	**0.02848**	**0.02946**
0.05445	**0.05557**	**0.05606**	**0.05622**	**0.05666**	**0.05599**	**0.05596**	**0.05448**	**0.05370**	**0.05262**	**0.05234**	**0.05119**	**0.05142**
0.36096	**0.35866**	**0.35724**	**0.36114**	**0.36276**	**0.36531**	**0.36054**	**0.35657**	**0.35186**	**0.35303**	**0.35168**	**0.35121**	**0.35272**

We now come to the following interesting questions that can be addressed when annual PPP data are available:

- How can we use the above information in order to construct estimates of overall real GDP growth and inflation across OECD countries?
- How can we use the above information in order to construct estimates of member country GDP volumes that are comparable over both time and countries?

A natural method for addressing the first question is to use the country shares in Table 2 as weights for national rates of growth of real GDP. The *year t growth factor for country n* can be defined as Q_n^t/Q_n^{t-1} where Q_n^t is country n's GDP volume in year t. Thus define the *OECD Laspeyres type growth factor* (or chain link) for year t, Γ_L^t, as the following weighted average of the national growth factors:[22]

$$(14)\ \Gamma_L^t \equiv \sum_{n=1}^{34} s_n^{t-1}(Q_n^t/Q_n^{t-1})\ ; \qquad t = 2001,...,2012.$$

The above measure of OECD GDP volume growth is the method used by the OECD to calculate their official measure of OECD volume growth. It certainly is a sensible measure, using country (one plus) growth rates going from year t−1 to year t, Q_n^t/Q_n^{t-1}, weighted by the country real volume shares s_n^{t-1} for year t−1, which were derived using PPPs. However, the above formula suffers from the same problem that the standard Laspeyres formula has: namely, it does not treat the periods in a symmetric fashion. Thus

[22] Suppose that there is only one homogeneous commodity in each country's GDP. Then the volume for country n in year t, Q_n^t, should be equal to the number of units of this homogeneous commodity. Under these conditions, it can be seen that Γ_L^t equals $\sum_{n=1}^{34} Q_n^t / \sum_{n=1}^{34} Q_n^{t-1}$.

the counterpart to the Laspeyres type formula defined by (14) is the following *Paasche type formula* (which applies the Laspeyres formula but reverses the role of time):[23]

(15) $\Gamma_P^t \equiv [\sum_{n=1}^{34} s_n^t(Q_n^t/Q_n^{t-1})^{-1}]^{-1}$; t = 2001,...,2012.

Since both indexes have the same logical foundation, it seems best to take a symmetric average of the two indexes, which leads to the following Fisher type formula for OECD volume growth for year t:[24]

(16) $\Gamma_F^t \equiv [\Gamma_L^t \Gamma_P^t]^{1/2}$; t = 2001,...,2012.

The growth factors (or chain link indexes) defined by (14)-(16) can be multiplied together to generate OECD aggregate volume levels. The growth factors can also be transformed into growth rates, γ_L^t, γ_P^t and γ_F^t (in percentage points), by using the following definitions for t = 2001,...,2012:

(17) $\gamma_L^t \equiv 100[\Gamma_L^t - 1]$; $\gamma_P^t \equiv 100[\Gamma_P^t - 1]$; $\gamma_F^t \equiv 100[\Gamma_F^t - 1]$.

The annual OECD volume growth measures defined by (17) as well as our earlier US dollar weighted measures γ^t are listed in Table 3.

Table 3: Annual Percentage Point Changes in OECD PPP Based Laspeyres, Paasche and Fisher Volume Measures γ_L^t, γ_P^t and γ_F^t, US Dollar Weighted Volume Measures γ^t and Laspeyres, Paasche and Fisher PPP Based Inflation Measures, ρ_L^t, ρ_P^t and ρ_F^t: 2001-2012

Year t	γ_L^t	γ_P^t	γ_F^t	γ^t	ρ_L^t	ρ_P^t	ρ_F^t
2001	1.2908	1.2964	1.2936	1.2313	3.2162	2.7982	3.0070
2002	1.6832	1.6767	1.6799	1.5171	2.5138	2.3123	2.4130
2003	2.1670	2.1610	2.1640	2.0937	2.3856	2.2900	2.3378
2004	3.3269	3.3331	3.3300	3.1753	2.5261	2.5007	2.5134
2005	2.8318	2.8311	2.8314	2.6796	2.3409	2.3358	2.3384
2006	3.1525	3.1592	3.1558	3.0193	2.5117	2.4998	2.5057
2007	2.7065	2.7074	2.7070	2.6415	2.4516	2.4353	2.4434
2008	0.1912	0.1902	0.1907	0.1144	2.3586	2.3427	2.3506
2009	-3.5714	-3.5736	-3.5725	-3.6386	1.0917	1.0809	1.0863
2010	2.9946	3.0011	2.9978	2.8791	1.4082	1.3888	1.3985
2011	1.9562	1.9629	1.9595	1.7662	1.7626	1.7621	1.7623
2012	1.5428	1.5303	1.5365	1.4962	1.5068	1.4978	1.5023
Average	1.6893	1.6897	1.6895	1.5812	2.1728	2.1037	2.1382

The annual average growth rates for the γ_L^t, γ_P^t, γ_F^t and γ^t are listed in Table 3. It can be seen that the Laspeyres, Paasche and Fisher measures of OECD growth explained in this section are virtually identical so that moving from the OECD Laspeyres type measure of overall volume growth to the Fisher measure did not make much difference for this data

[23] In the one homogeneous commodity case, it can be verified that Γ_P^t also equals $\sum_{n=1}^{34} Q_n^t / \sum_{n=1}^{34} Q_n^{t-1}$.
[24] Taking the geometric mean of Γ_L^t and Γ_P^t leads to an index which satisfies the time reversal test, which is a useful property for an index number formula. If the PPPs are independent of the choice of the numeraire country (up to a scalar factor), then the growth factors, Γ_L^t Γ_P^t and Γ_F^t will not depend on the choice of the numeraire country.

set.[25] It can also be seen that our preferred Fisher measure of OECD growth in comparable units across countries, γ_F^t, grew on average about 1/10 of a percentage point more rapidly per year than our preferred measure of OECD GDP growth using US dollar weights (or more generally, using exchange rate weighted weights), γ^t, that was explained in the previous section. Although this is not a large difference in growth rates, it is significant and so users need to decide which measure, γ_F^t or γ^t, best suits their needs.

Which measure of group growth is better: the measure γ^t defined by (10) in the previous section (which did not use PPPs) or the measure γ_F^t defined by (17) in the present section? The measure γ^t can be defined using just national information on domestic price and quantity (or volume) indexes and exchange rates while the measure γ_F^t requires information on domestic values, domestic volume indexes and PPPs. The problem with the γ_F^t measure defined in this section is that it relies on the accuracy of the PPPs. Unfortunately, PPPs are not likely to be nearly as accurate as national measures of price and volume change due to the difficulties in matching products across countries. There are additional difficulties with the treatment of international trade in the construction of PPPs. But the γ^t measure has the problem that large fluctuations in exchange rates can lead to fluctuations in the γ^t [26] while the PPP based γ_F^t measures should be independent from exchange rate movements. Viewing the γ^t and γ_F^t estimates found in Table 3 above, it can be seen that the dependence of the γ^t estimates on exchange rates causes these estimates to fluctuate much more than the corresponding PPP based estimates for the γ_F^t. Thus if the PPPs are determined “reasonably” accurately, my own preference is to use the PPP based estimates for group volume growth since these estimates will not be influenced by somewhat random exchange rate fluctuations.

The OECD real output shares, s_n^t defined by (13), can be used as weights for national GDP inflation rates. Recall that the national currency GDP price deflator for country n in year t was defined as P_n^t. Recall also that (14)-(16) defined aggregate OECD Laspeyres, Paasche and Fisher volume link indexes, Γ_L^t, Γ_P^t and Γ_F^t. Use modifications of these definitions, with the national inflation factors (P_n^t/P_n^{t-1}) replacing the national volume growth factors (Q_n^t/Q_n^{t-1}) in order to define the *OECD Laspeyres, Paasche and Fisher PPP based chain link price indexes*, Π_L^t, Π_P^t and Π_F^t for t = 2001,...,2012:[27]

---

[25] Recall that the official OECD measure of real GDP growth is the Laspeyres measure, γ_L^t. Our estimates differ slightly from the official measures due to rounding errors. The exchange rate weighted growth rates γ^t should be somewhat lower than the PPP weighted growth rates γ_F^t due to the Samuelson-Balassa effect and this expectation is realized for the OECD data. We would expect the divergence to grow as less rich countries are added to the list of countries.

[26] Exchange rate movements do not directly affect the domestic rates of growth (the Q_n^t/Q_n^{t-1}) but they do affect the *weights* used to aggregate the country real growth rates into the overall OECD Laspeyres, Paasche and Fisher growth rates. Exchange rate fluctuations are large enough to materially affect the weights, which in turn lead to material fluctuations in the overall OECD volume growth rates.

[27] The official OECD measure of household inflation over member countries is the Laspeyres measure defined in (18) where household consumption replaces GDP; see the OECD (2014).

(18) $\Pi_L^t \equiv \sum_{n=1}^{34} s_n^{t-1}(P_n^t/P_n^{t-1})$; $\Pi_P^t \equiv [\sum_{n=1}^{34} s_n^t(P_n^t/P_n^{t-1})^{-1}]^{-1}$; $\Pi_F^t \equiv [\Pi_L^t \Pi_P^t]^{1/2}$.[28]

These chain link indexes can be multiplied together to generate the corresponding OECD aggregate price levels. The resulting Fisher OECD price level index for year t is denoted by P_{ICP}^t and it is listed in the last column of Table 1.[29] The inflation growth factors can also be transformed into *growth rates*, ρ_L^t, ρ_P^t and ρ_F^t in percentage points, by using the following definitions for t = 2001,...,2012:

(19) $\rho_L^t \equiv 100[\Pi_L^t - 1]$; $\rho_P^t \equiv 100[\Pi_P^t - 1]$; $\rho_F^t \equiv 100[\Pi_F^t - 1]$.

The above PPP based inflation rates (in percentage points) are listed in the last 3 columns of Table 3.

The sample averages of the ρ_L^t, ρ_P^t and ρ_F^t are 2.17, 2.10 and 2.14 percentage points. Viewing Table 3, it can be seen that all three PPP based price indexes are reasonably close but their are some significant differences between the three measures of OECD inflation that are PPP based.[30] Comparing the numbers in Tables 1 and 3, it can be seen that the PPP based estimates of OECD inflation are much more reasonable than the estimates that were based on country exchange rates that were listed in Table 1, the ρ^t and ρ_{EU}^t. Our conclusion is that the Fisher OECD Fisher price index P_{ICP}^t is a much better measure of OECD inflation than the indexes that used exchange rates instead of PPPs.

Now we come to the most difficult problem: how can we use PPP information and national growth rates to obtain estimates of member country GDP volumes that are comparable across time and space? The Eurostat (2012) *Manual* offers the following advice:

"To trace the evolution of relative GDP volume levels between countries over time, it is necessary to select one of the reference years as a base year and to extrapolate its relative GDP volume levels over the other years. Extrapolation is done by applying the relative rates of GDP volume growth observed in the different countries. This provides a time series of volume indices at a constant uniform price level that replicates exactly the relative movements of GDP volume growth of each country." Eurostat (2012; 18).

---

[28] The Laspeyres measure of OECD inflation going from period t−1 to period t, Π_L^t, uses the country share of OECD real output in period t−1, s_n^{t-1}, to weight the country n (one plus) domestic inflation rate, P_n^t/P_n^{t-1}, and sums over countries. The Paasche measure of OECD inflation going from period t−1 to period t, Π_P^t, uses the country share of OECD real output in period t, s_n^t, to weight the country n *reciprocal* (one plus) domestic inflation rate, P_n^{t-1}/P_n^t, and sums over countries and then takes the reciprocal of the resulting sum to convert a backwards looking estimate into a forward looking estimate of overall inflation. Thus both of these measures weight domestic inflation rates by the relative volume importance of the countries in the OECD and both measures are in principle, independent of exchange rate movements (as is the Fisher measure, Π_F^t).

[29] This price index satisfies the time reversal test whereas its Laspeyres and Paasche counterparts do not satisfy this important test. Thus the Fisher ICP based inflation index P_{ICP}^t is our preferred measure of OECD aggregate inflation.

[30] In view of these differences in the three indexes of OECD GDP inflation, it may be preferable for the OECD to replace their Laspeyres type indexes of OECD household inflation by their Fisher counterparts.

We will implement the strategy suggested by Eurostat in sections 4 and 5 below, where we choose the relative country GDP volumes given by the country shares of OECD aggregate GDP for 2000 (section 4) and for 2012 (section 5) and we use national growth rates for country GDP volumes to extrapolate these base shares to all time periods. However, it will be seen that the resulting comparable country volumes over time and space differ considerably, depending upon which base year is chosen. This is rather inconvenient: analysts studying competitiveness of OECD countries and living standards convergence across countries will want to use country volume series that are not subject to violent revision.[31]

Our suggested solution to the problem of harmonizing national growth rates of GDP with the country shares of OECD aggregate real GDP rests on two principles:

- The resulting harmonized estimates of country volumes must be consistent with the real annual cross country volume shares s_n^t listed in Table 2 above;
- OECD aggregate real GDP growth must be equal to the rates of aggregate growth generated by our recommended Fisher indexes Γ_F^t defined by (16).

Using the above two principles, the country GDP volumes are uniquely determined (up to a scalar units of measurement factor). To see this, first define the OECD volume index that chains the Γ_F^t defined by (16) into a time series index, Q_H^t. Define Q_H^t as follows:

(20) $Q_H^{2000} \equiv 1$; $Q_H^t \equiv Q_H^{t-1}\,\Gamma_F^t$; t = 2001,...,2012.

Now use the country shares of OECD real GDP s_n^t listed in Table 2 and the aggregate index Q_H^t to define the following preliminary *harmonized country volumes* for country n in year t, q_{Hn}^t, as follows:

(21) $q_{Hn}^t \equiv Q_H^t\, s_n^t$; n = 1,...,34 ; t = 2000,...,2012.

Note that for each year t, $\sum_{n=1}^{34} q_{Hn}^t = \sum_{n=1}^{34} Q_H^t\, s_n^t = Q_H^t\, (\sum_{n=1}^{34} s_n^t) = Q_H^t$ and so the harmonized volumes satisfy the two principles listed above. In principle, the country volumes defined by (21) are independent of country prices and exchange rates.[32]

[31] McCarthy (2013; 484-486) explains in some detail why estimates of real GDP based on national growth information do not match up exactly with relative GDP estimates based on PPP benchmark information. The PPP information is generally not as accurate as national price index information due to the difficulty of matching representative products across countries. However, country methodology for constructing national price indexes differs considerably across countries; e.g., some countries may use out of date reference expenditure baskets, some countries use Carli indexes at the elementary level while others use the Jevons or Dutot indexes which generate lower estimates of inflation at the elementary level and some countries may use quality adjustment methods more extensively than others. All of these methodological differences lead to inconsistencies between the time series and cross sectional estimates. Finally, the index number formulae used at the national levels and in the construction of the benchmark PPPs are in general not transitive and so it is impossible to achieve perfect consistency.

[32] However, in practice, the PPPs do not do a perfect job in eliminating exchange rate effects (since adjusted exchange rates are used in place of true PPPs to deflate international trade flows). If the relative PPPs are independent of the choice of the numeraire country, then the relative volumes defined by (20) will also be independent of this choice.

It is of interest to define US dollar prices for real GDP for each country. Recall that the value of country n's nominal GDP converted into US dollars at market exchange rates for year t was defined as v_n^t. Thus the corresponding *harmonized US dollar price* of a unit of (comparable across countries) real GDP for country n in year t is defined as follows:

$$(22)\ p_{Hn}^t \equiv v_n^t/q_{Hn}^t\ ; \qquad n = 1,...,34\ ;\ t = 2000,...,2012.$$

In order to make the harmonized volumes and prices defined by (21) and (22) comparable to the country prices and volumes expressed in US dollars that are listed in the Appendix in Tables A3 and A4, we impose a normalization on the prices defined by (22) that makes the price level for the US in 2000 equal to unity; i.e., we divide all prices defined by (22) by a constant that sets the resulting p_{H34}^{2000} equal to 1 and the quantities or volumes defined by (21) are all multiplied by this constant. The resulting normalized q_{Hn}^t and p_{Hn}^t are listed in Tables 4 and 5.

Table 4: Harmonized OECD Country GDP Volumes in Comparable US Dollar Units of Measurement q_{Hn}^t

n	q_{Hn}^{2000}	q_{Hn}^{2001}	q_{Hn}^{2002}	q_{Hn}^{2003}	q_{Hn}^{2004}	q_{Hn}^{2005}	q_{Hn}^{2006}	q_{Hn}^{2007}	q_{Hn}^{2008}	q_{Hn}^{2009}	q_{Hn}^{2010}	q_{Hn}^{2011}	q_{Hn}^{2012}
1	537.3	553.2	572.5	598.6	618.7	637.2	658.2	686.6	685.8	718.5	740.3	764.3	786.4
2	231.6	227.6	235.2	239.3	245.8	246.0	258.4	262.7	268.2	259.9	266.6	273.8	279.1
3	283.4	285.9	296.6	295.8	297.6	299.9	308.5	314.9	320.7	315.3	325.3	330.1	334.5
4	874.1	886.8	895.8	930.9	960.5	1006.5	1027.2	1050.1	1050.5	1018.4	1053.4	1080.3	1094.9
5	147.6	152.5	155.5	160.9	172.3	183.5	219.1	232.4	223.7	219.7	252.1	281.6	295.0
6	159.7	167.8	171.3	180.2	187.7	193.5	204.1	218.4	218.2	214.8	210.3	213.1	212.0
7	153.9	153.7	157.9	154.4	159.8	159.9	167.4	171.2	177.0	168.8	177.8	176.6	178.2
8	13.5	14.2	15.6	17.1	18.3	19.8	22.0	24.1	23.9	21.2	21.3	23.2	23.9
9	132.9	134.2	136.8	135.5	143.0	143.2	149.1	159.0	163.6	151.6	152.6	155.8	156.5
10	1533.0	1587.0	1628.6	1593.3	1613.0	1654.4	1701.9	1756.9	1772.1	1740.1	1760.9	1790.8	1791.5
11	2117.5	2144.2	2162.4	2202.5	2242.8	2281.5	2360.8	2431.0	2464.8	2330.8	2431.2	2516.6	2551.1
12	199.2	210.8	224.6	233.5	241.7	240.4	254.9	258.2	269.0	263.2	246.0	226.3	215.8
13	121.3	133.0	142.3	146.3	149.8	152.2	157.4	158.2	165.8	162.1	162.4	164.2	164.9
14	8.1	8.5	8.5	8.4	9.0	9.2	9.3	9.6	10.2	9.5	8.9	9.0	9.1
15	109.8	115.4	124.4	130.5	136.6	143.4	154.2	164.5	153.2	144.5	146.5	150.6	151.4
16	147.0	146.3	147.5	139.9	146.8	143.5	144.2	152.8	151.2	155.0	163.1	173.8	178.1
17	1466.5	1515.3	1471.0	1478.8	1466.6	1473.6	1530.6	1581.8	1614.8	1549.1	1533.1	1537.5	1524.6
18	3287.0	3288.1	3316.3	3357.2	3438.6	3458.3	3471.1	3548.1	3468.9	3234.7	3389.8	3350.2	3390.1
19	808.4	837.8	894.1	908.8	951.4	975.1	1003.2	1054.8	1056.5	1048.2	1116.9	1148.5	1163.3
20	23.4	23.2	24.5	25.8	27.2	28.2	31.7	33.7	33.3	31.2	33.4	34.9	35.4
21	987.1	983.6	1000.8	1043.2	1086.4	1150.3	1232.1	1273.9	1315.8	1282.3	1353.6	1475.9	1519.7
22	468.3	481.5	492.7	484.2	494.8	509.4	531.7	554.5	570.8	541.2	545.2	550.5	545.9
23	82.1	84.2	87.1	90.0	92.9	93.6	97.7	101.6	100.7	104.2	105.4	108	110.4
24	162.3	163.2	160.6	164.4	178.6	195.8	214.4	218.5	236.5	211.3	221.9	234.2	248.8
25	404.3	408.3	422.3	430.9	454.9	467.7	491.1	530.9	555.6	572.8	604.4	630.8	645.2
26	182.0	185.7	189.6	191.2	190.9	200.4	207.7	213.4	214.2	211.4	214.7	209.3	201.9
27	59.3	63.3	66.6	68.9	72.2	77.5	84.7	93.6	101.5	97.6	99.5	100.6	102.9
28	34.9	35.8	37.7	38.6	40.7	41.8	43.6	45.7	47.5	43.7	43.0	43.5	42.7
29	858.1	896.4	949.8	978.8	1014.8	1056.9	1144.6	1202.1	1221.6	1172.3	1145.6	1129.2	1118.6
30	248.0	244.8	249.6	256.5	267.6	262.5	277.0	292.5	295.4	276.7	290.8	301.7	309.2
31	233.6	234.8	241	238.7	243.0	244.4	261.7	280.9	296.5	290.0	300.3	313.1	321.8
32	589.3	546.9	546.5	553.4	630.4	694.6	764.7	811.3	863.7	829.6	909.8	975.7	1024.8
33	1552.1	1604.6	1646.0	1686.4	1756.3	1784.4	1839.8	1839.7	1816.7	1716.6	1758.8	1753.8	1788.8
34	10290	10356	10489	10833	11243	11643	11854	12040	11904	11517	11817	12032	12270

Table 5: Harmonized OECD Country GDP Price Levels in Comparable US Dollar Units of Measurement p_{Hn}^t

n	p_{Hn}^{2000}	p_{Hn}^{2001}	p_{Hn}^{2002}	p_{Hn}^{2003}	p_{Hn}^{2004}	p_{Hn}^{2005}	p_{Hn}^{2006}	p_{Hn}^{2007}	p_{Hn}^{2008}	p_{Hn}^{2009}	p_{Hn}^{2010}	p_{Hn}^{2011}	p_{Hn}^{2012}
1	**0.7628**	**0.7059**	**0.7602**	**0.9313**	**1.0947**	**1.1925**	**1.2394**	**1.4324**	**1.5341**	**1.4029**	**1.7396**	**1.9883**	**1.9982**
2	**0.8294**	**0.8422**	**0.8825**	**1.0610**	**1.1856**	**1.2398**	**1.2575**	**1.4279**	**1.5441**	**1.4763**	**1.4165**	**1.5192**	**1.4135**
3	**0.8209**	**0.8131**	**0.8525**	**1.0537**	**1.2155**	**1.2583**	**1.2965**	**1.4598**	**1.5822**	**1.5009**	**1.4484**	**1.5553**	**1.4436**
4	**0.8293**	**0.8068**	**0.8201**	**0.9302**	**1.0330**	**1.1265**	**1.2448**	**1.3562**	**1.4305**	**1.3134**	**1.4970**	**1.6087**	**1.6255**
5	**0.5284**	**0.4678**	**0.4511**	**0.4731**	**0.5764**	**0.6705**	**0.7061**	**0.7447**	**0.8030**	**0.7825**	**0.8630**	**0.8922**	**0.9096**
6	**0.3682**	**0.3836**	**0.4579**	**0.5287**	**0.6072**	**0.6721**	**0.7271**	**0.8264**	**1.0330**	**0.9178**	**0.9441**	**1.0140**	**0.9265**
7	**1.0403**	**1.0439**	**1.1009**	**1.3771**	**1.5317**	**1.6111**	**1.6388**	**1.8193**	**1.9434**	**1.8393**	**1.7604**	**1.8893**	**1.7683**
8	**0.4197**	**0.4379**	**0.4707**	**0.5768**	**0.6591**	**0.7020**	**0.7642**	**0.9133**	**0.9929**	**0.9176**	**0.8923**	**0.9713**	**0.9360**
9	**0.9166**	**0.9287**	**0.9885**	**1.2120**	**1.3223**	**1.3669**	**1.3948**	**1.5483**	**1.6621**	**1.5793**	**1.5512**	**1.6839**	**1.5806**
10	**0.8652**	**0.8433**	**0.8916**	**1.1248**	**1.2744**	**1.2915**	**1.3254**	**1.4699**	**1.5980**	**1.5055**	**1.4567**	**1.5536**	**1.4575**
11	**0.8909**	**0.8772**	**0.9280**	**1.1004**	**1.2156**	**1.2125**	**1.2296**	**1.3673**	**1.4702**	**1.4150**	**1.3592**	**1.4416**	**1.3429**
12	**0.6322**	**0.6159**	**0.6502**	**0.8261**	**0.9430**	**0.9987**	**1.0266**	**1.1829**	**1.2698**	**1.2198**	**1.1960**	**1.2808**	**1.1534**
13	**0.3823**	**0.3963**	**0.4663**	**0.5710**	**0.6802**	**0.7247**	**0.7148**	**0.8602**	**0.9299**	**0.7811**	**0.7853**	**0.8370**	**0.7555**
14	**1.0724**	**0.9365**	**1.0432**	**1.3090**	**1.4661**	**1.7693**	**1.7875**	**2.1236**	**1.6509**	**1.2727**	**1.4090**	**1.5558**	**1.4921**
15	**0.8862**	**0.9117**	**0.9891**	**1.2163**	**1.3640**	**1.4130**	**1.4451**	**1.5777**	**1.7235**	**1.5600**	**1.4297**	**1.5011**	**1.3912**
16	**0.8444**	**0.8359**	**0.7651**	**0.8469**	**0.8611**	**0.9315**	**1.0064**	**1.0889**	**1.3328**	**1.2566**	**1.3348**	**1.4017**	**1.3534**
17	**0.7528**	**0.7415**	**0.8329**	**1.0241**	**1.1833**	**1.2122**	**1.2237**	**1.3448**	**1.4289**	**1.3629**	**1.3407**	**1.4289**	**1.3206**
18	**1.4394**	**1.2651**	**1.2004**	**1.2817**	**1.3540**	**1.3220**	**1.2552**	**1.2278**	**1.3979**	**1.5566**	**1.6211**	**1.7602**	**1.7584**
19	**0.6598**	**0.6023**	**0.6441**	**0.7084**	**0.7589**	**0.8664**	**0.9487**	**0.9948**	**0.8816**	**0.7957**	**0.9087**	**0.9704**	**0.9710**
20	**0.8659**	**0.8706**	**0.9206**	**1.1297**	**1.2510**	**1.3328**	**1.3426**	**1.5218**	**1.6415**	**1.5861**	**1.5569**	**1.6623**	**1.5560**
21	**0.6450**	**0.6931**	**0.7105**	**0.6713**	**0.6983**	**0.7355**	**0.7703**	**0.8111**	**0.8299**	**0.6863**	**0.7618**	**0.7827**	**0.7708**
22	**0.8223**	**0.8321**	**0.8886**	**1.1118**	**1.2325**	**1.2534**	**1.2746**	**1.4114**	**1.5257**	**1.4714**	**1.4255**	**1.5128**	**1.4106**
23	**0.6549**	**0.6352**	**0.7112**	**0.9241**	**1.0931**	**1.2156**	**1.1269**	**1.3313**	**1.2956**	**1.1375**	**1.3612**	**1.5107**	**1.5510**
24	**1.0371**	**1.0474**	**1.1947**	**1.3677**	**1.4559**	**1.5531**	**1.5860**	**1.8006**	**1.9189**	**1.7928**	**1.8970**	**2.0949**	**2.0081**
25	**0.4236**	**0.4663**	**0.4693**	**0.5031**	**0.5557**	**0.6497**	**0.6955**	**0.8008**	**0.9529**	**0.7522**	**0.7773**	**0.8177**	**0.7593**
26	**0.6446**	**0.6478**	**0.6976**	**0.8468**	**0.9709**	**0.9572**	**0.9717**	**1.0861**	**1.1759**	**1.1074**	**1.0664**	**1.1365**	**1.0509**
27	**0.3440**	**0.3335**	**0.3672**	**0.4831**	**0.5839**	**0.6182**	**0.6591**	**0.8007**	**0.9289**	**0.8939**	**0.8769**	**0.9528**	**0.8880**
28	**0.5723**	**0.5724**	**0.6144**	**0.7562**	**0.8308**	**0.8554**	**0.8924**	**1.0359**	**1.1490**	**1.1270**	**1.0933**	**1.1564**	**1.0625**
29	**0.6763**	**0.6792**	**0.7226**	**0.9029**	**1.0293**	**1.0699**	**1.0801**	**1.1991**	**1.3044**	**1.2406**	**1.2088**	**1.2881**	**1.1820**
30	**0.9970**	**0.9286**	**1.0054**	**1.2269**	**1.3529**	**1.4115**	**1.4409**	**1.5811**	**1.6459**	**1.4663**	**1.5924**	**1.7764**	**1.6939**
31	**1.0963**	**1.1184**	**1.1896**	**1.4017**	**1.5402**	**1.5742**	**1.5482**	**1.6040**	**1.7681**	**1.7569**	**1.8285**	**2.1046**	**1.9612**
32	**0.4523**	**0.3584**	**0.4255**	**0.5476**	**0.6221**	**0.6953**	**0.6943**	**0.7977**	**0.8456**	**0.7408**	**0.8036**	**0.7941**	**0.7692**
33	**0.9623**	**0.9256**	**0.9847**	**1.1120**	**1.2645**	**1.3009**	**1.3496**	**1.5530**	**1.4795**	**1.2863**	**1.3052**	**1.4041**	**1.3817**
34	**1.0000**	**1.0260**	**1.0469**	**1.0627**	**1.0919**	**1.1247**	**1.1691**	**1.2026**	**1.2365**	**1.2519**	**1.2658**	**1.2910**	**1.3240**

Note that $q_{H34}^{2000} = q_{34}^{2000}$ and $p_{H34}^{2000} = p_{34}^{2000} = 1$ so that country GDP volumes are measured as multiples of a bundle of US GDP in the year 2000. Thus the price levels in Table 5 measure the US dollar value of constant bundle of GDP that is (in theory) comparable across countries. The price levels in Table 5 are comparable across space and time, whereas the price levels p_n^t listed in Table A3 of the Appendix are only comparable across time for each country.

From Table 5, it can be seen that the countries with the lowest price levels (in US dollars) in 2012 are countries 13, 21, 25 and 32 (Hungary, Mexico, Poland and Turkey) with price levels in the 0.76 to 0.77 range. Countries with relatively high price levels in 2012 are countries 1 (Australia, $p_{H1}^{2012} = 2.00$), 4 (Canada, $p_{H4}^{2012} = 1.62$), 7 (Denmark, 1.77), 9 (Finland, 1.58), 18 (Japan, 1.76), 20 (Luxembourg, 1.56), 23 (New Zealand, 1.55), 24 (Norway, 2.01), 30 (Sweden, 1.69) and 31 (Switzerland, 1.96). These price level estimates are (imperfect)[33] indicators of the competiveness of the country on international markets, with lower price levels indicating greater competiveness.

[33] The price levels p_{Hn}^t are imperfect indicators of competiveness because not all components of GDP are internationally traded. Moreover, these price levels are not independent of the choice of the numeraire currency (US dollars in this case). They are also imperfect because they depend heavily on the accuracy of the underlying PPPs and these PPPs are subject to considerable error variances due to the difficulties involved in matching product prices (and quantities) across countries.

A problem with the volume estimates listed in Table 4 is that they do not respect national growth rates of GDP by country; only the aggregate OECD growth rate is respected. In the following two sections, we will derive alternative country volume estimates that are comparable over time and space. These alternative estimates will respect country growth rates but they will not reproduce the real OECD country expenditure shares listed in Table 2 for all time periods.

4. OECD Growth and Inflation Using Country Annual GDP Volume Growth Rates and Base Period Shares of OECD Real GDP

In this section, we will generate comparable country GDP volume estimates for OECD countries covering the period 2000-2012 by using the real GDP country volume shares for 2000, the s_n^{2000} listed in Table 2 above, along with the national growth rates of country real GDP relative to 2000, the Q_n^t/Q_n^{2000} listed in Table A2 of the Appendix. This is a typical strategy in forming estimates of real GDP that rely on PPPs that are only produced infrequently. Our purpose in listing these estimates is to evaluate how different the resulting estimates are from our preferred harmonized volume estimates, q_{Hn}^t, listed in Table 4 above.

Define preliminary *base period estimates of country GDP volumes for year t and country* n, q_{Bn}^t, as follows:

$$(23)\ q_{Bn}^t \equiv s_n^{2000}\,(Q_n^t/Q_n^{2000})\ ; \qquad n = 1,...,34\ ;\ t = 2000,...,2012.$$

The above estimates are obviously based on the country shares of real OECD GDP that prevailed in 2000 (the s_n^{2000}) and the long term country growth rates of real GDP (the Q_n^t/Q_n^{2000}). The companion *country US dollar price levels* for country n and year t, p_{Bn}^t, are defined as follows:

$$(24)\ p_{Bn}^t \equiv v_n^t/q_{Bn}^t\ ; \qquad n = 1,...,34\ ;\ t = 2000,...,2012$$

where v_n^t is the nominal value of GDP for country n in year t converted into US dollars at market exchange rates for that year.

In order to make the volumes and prices defined by (23) and (24) comparable to the country prices and volumes expressed in US dollars that are listed Tables 4 and 5 in the previous section, we impose a normalization on the prices defined by (24) that makes the price level for the US in 2000 equal to unity; i.e., we divide all prices defined by (24) by a constant that sets the resulting p_{B34}^{2000} equal to 1 and the quantities or volumes defined by (23) are all multiplied by this constant. The resulting normalized p_{Bn}^t are listed in Table 6.[34]

[34] The entries in Tables 4 and 5 enable one to recover the US dollar values of GDP, equal to $v_n^t = p_{Hn}^t q_{Hn}^t$ for n = 1,...,34 and t = 2000,...,2012. Then the q_{Bn}^t can be recovered as v_n^t/p_{Bn}^t.

Table 6: OECD Country GDP Price Levels in Comparable US Dollar Units of Measurement p_{Bn}^t Based on Country Growth Rates of GDP Volumes and Year 2000 Country Shares of OECD Output

n	p_{Bn}^{2000}	p_{Bn}^{2001}	p_{Bn}^{2002}	p_{Bn}^{2003}	p_{Bn}^{2004}	p_{Bn}^{2005}	p_{Bn}^{2006}	p_{Bn}^{2007}	p_{Bn}^{2008}	p_{Bn}^{2009}	p_{Bn}^{2010}	p_{Bn}^{2011}	p_{Bn}^{2012}
1	0.7628	0.6994	0.7556	0.9295	1.0944	1.1914	1.2325	1.4319	1.5070	1.4141	1.7639	2.0137	2.0084
2	0.8294	0.8207	0.8738	1.0600	1.1858	1.2118	1.2455	1.3861	1.5090	1.4537	1.4059	1.5058	1.4155
3	0.8209	0.8136	0.8729	1.0675	1.1995	1.2300	1.2698	1.4183	1.5504	1.4878	1.4476	1.5497	1.4600
4	0.8293	0.8042	0.8023	0.9282	1.0314	1.1440	1.2547	1.3674	1.4330	1.3119	1.4986	1.6106	1.6217
5	0.5284	0.4678	0.4480	0.4686	0.5713	0.6665	0.7927	0.8435	0.8475	0.8198	0.9806	1.0697	1.0825
6	0.3682	0.3910	0.4663	0.5460	0.6235	0.6665	0.7105	0.8173	0.9902	0.9070	0.8911	0.9526	0.875
7	1.0403	1.0356	1.1169	1.3605	1.5308	1.5733	1.6203	1.8104	2.0149	1.9289	1.9172	2.0229	1.9172
8	0.4197	0.4339	0.4779	0.5961	0.6850	0.7272	0.7985	0.9718	1.0952	1.0422	0.9967	1.0768	1.0283
9	0.9166	0.9171	0.9768	1.1634	1.2861	1.2940	1.3164	1.4791	1.6297	1.5683	1.5003	1.6183	1.5391
10	0.8652	0.8572	0.9216	1.1273	1.2609	1.2870	1.3261	1.4842	1.6289	1.5559	1.4976	1.5921	1.4940
11	0.8909	0.8750	0.9334	1.1317	1.2584	1.2681	1.2832	1.4228	1.5346	1.4725	1.4184	1.5071	1.4134
12	0.6322	0.6256	0.6803	0.8478	0.9602	0.9888	1.0216	1.1515	1.2907	1.2522	1.2074	1.2806	1.1745
13	0.3823	0.4190	0.5048	0.6117	0.7122	0.7415	0.7280	0.8794	0.9878	0.8700	0.8668	0.9200	0.8481
14	1.0724	0.9401	1.0554	1.2689	1.4215	1.6292	1.5908	1.8414	1.4995	1.1551	1.2491	1.3599	1.2966
15	0.8862	0.9121	1.0121	1.2589	1.4179	1.4535	1.5150	1.6817	1.7484	1.5947	1.4970	1.5817	1.4717
16	0.8444	0.8332	0.7695	0.7959	0.8098	0.8163	0.8376	0.9065	1.0548	1.0084	1.0735	1.1486	1.1014
17	0.7528	0.7522	0.8165	1.0098	1.1375	1.1599	1.1901	1.3292	1.4586	1.4122	1.3516	1.4379	1.3520
18	1.4394	1.2611	1.2033	1.2791	1.3521	1.3106	1.2282	1.2017	1.3518	1.4857	1.5494	1.6722	1.6579
19	0.6598	0.6003	0.6395	0.6953	0.7454	0.8390	0.8987	0.9426	0.8179	0.7301	0.8356	0.8850	0.8790
20	0.8659	0.8417	0.9036	1.1477	1.2858	1.3494	1.4534	1.6448	1.7675	1.6895	1.7260	1.8876	1.7975
21	0.6450	0.6909	0.7151	0.6946	0.7230	0.7808	0.8336	0.8780	0.9168	0.7861	0.8744	0.9427	0.9211
22	0.8223	0.8394	0.9166	1.1232	1.2448	1.2770	1.3109	1.4567	1.5922	1.5115	1.4529	1.5422	1.4441
23	0.6549	0.6282	0.6925	0.8931	1.0525	1.1413	1.0867	1.2886	1.2651	1.1331	1.3694	1.5237	1.5496
24	1.0371	1.0328	1.1426	1.3257	1.4745	1.6807	1.8374	2.0712	2.3875	2.0259	2.2403	2.5799	2.5485
25	0.4236	0.4654	0.4774	0.5029	0.5565	0.6458	0.6833	0.7964	0.9434	0.7555	0.7930	0.8329	0.7760
26	0.6446	0.6485	0.7075	0.8740	0.9853	1.0117	1.0490	1.1768	1.2795	1.2246	1.1748	1.2362	1.1391
27	0.3440	0.3439	0.3811	0.4947	0.5970	0.6356	0.6834	0.8310	0.9881	0.9619	0.9216	0.9831	0.9200
28	0.5723	0.5703	0.6200	0.7589	0.8438	0.8563	0.8821	1.0026	1.1175	1.0949	1.0327	1.0965	1.0160
29	0.6763	0.6844	0.7512	0.9383	1.0740	1.1224	1.1791	1.3285	1.4556	1.3815	1.3181	1.3838	1.2788
30	0.9970	0.9054	0.9751	1.1949	1.3190	1.3085	1.3511	1.5156	1.6029	1.4088	1.5087	1.6966	1.6423
31	1.0963	1.1107	1.2100	1.4121	1.5420	1.5438	1.5670	1.6778	1.9112	1.8938	1.9826	2.3371	2.2156
32	0.4523	0.3527	0.3941	0.4879	0.5774	0.6560	0.6746	0.7856	0.8808	0.7787	0.8487	0.8268	0.8234
33	0.9623	0.9364	0.9990	1.1119	1.2763	1.2923	1.3452	1.4966	1.4188	1.2291	1.2570	1.3335	1.3368
34	1.0000	1.0229	1.0386	1.0594	1.0884	1.1233	1.1579	1.1886	1.2118	1.2211	1.2359	1.2602	1.2822

The differences between the entries in Tables 5 and 6 are very large. If we take each column in Table 5, subtract the corresponding entries in the same column of Table 6 and then take the absolute value of the differences, we find that the *average* absolute difference grows from 0 in 2000 to 9.4 percentage points in 2012.[35] The maximum absolute difference grows from 0 in 2000 to 54.0 percentage points in 2012. These are massive differences in price levels, which translate into massive differences in GDP levels. This problem of the inconsistency of the PPPs with national growth rates is well known[36] but most users of PPP adjusted country real volume estimates are not aware of how large these inconsistencies are.

[35] The sequence of *average* absolute differences in percentage points over the 13 years is as follows: 0, 0.8, 1.6, 2.0, 2.2, 3.8, 5.2, 6.4, 7.6, 6.8, 8.0, 9.3, 9.4. The sequence of *maximum* absolute differences in percentage points over the 13 years is as follows: 0, 2.9, 5.2, 6.0, 5.4, 14.0, 25.1, 28.2, 46.9, 24.8, 34.3, 48.5, 54.0.

[36] See the Eurostat-OECD Manual on this point; i.e., see Eurostat (2012; 18).

In the following section, we undertake a computation that is similar to the computations in the present section except that we use the real volume shares of 2012 as the benchmark shares instead of the shares of 2000.

5. OECD Growth and Inflation Using Country Annual GDP Volume Growth Rates and Current Period Shares of OECD Real GDP

In this section, we will generate comparable country GDP volume estimates for OECD countries covering the period 2000-2012 by using the real GDP country volume shares for 2012, the s_n^{20012} listed in Table 2 above, along with the national growth rates of country real GDP relative to 2000, the Q_n^t/Q_n^{2000} listed in Table A2 of the Appendix. This method for forming comparable country GDP volumes is used by the World Bank[37] when the International Comparisons Project produces a new set of PPPs. The methodology is straightforward and follows the approach used in the previous section except that the 2012 country volume shares are used in place of the 2000 shares.

Define preliminary *end of sample period estimates of country GDP volumes for year t and country* n, q_{En}^t, as follows:

$$(25)\ q_{En}^t \equiv s_n^{2012}\,(Q_n^t/Q_n^{2012})\ ; \qquad n = 1,\ldots,34\ ;\ t = 2000,\ldots,2012.$$

The above estimates are obviously based on the country shares of real OECD GDP that prevailed in 2012 (the s_n^{2012}) and the levels of real GDP in year t relative to the corresponding country n level in 2012 (the Q_n^t/Q_n^{2012}). The companion *country US dollar price levels* for country n and year t, p_{En}^t, are defined as follows:

$$(26)\ p_{En}^t \equiv v_n^t/q_{En}^t\ ; \qquad n = 1,\ldots,34\ ;\ t = 2000,\ldots,2012$$

where v_n^t is the nominal value of GDP for country n in year t converted into US dollars at market exchange rates for that year.

In order to make the volumes and prices defined by (25) and (26) comparable to the harmonized country prices and volumes expressed in US dollars that are listed Tables 4 and 5 in section 3, we impose a normalization on the prices defined by (26) that makes the price level for the US in 2000 equal to unity; i.e., we divide all prices defined by (26) by a constant that sets the resulting p_{E34}^{2000} equal to 1 and the quantities or volumes defined by (25) are all multiplied by this constant. The resulting normalized p_{En}^t are listed in Table 7.[38]

[37] See Chapter 18 in the World Bank (2013). The Penn World Tables use the extrapolation methodology described in this section and the previous section to construct estimates of comparable real GDP for periods subsequent to the last available ICP round and prior to the first available ICP round; see Summers and Heston (1991) and Feenstra, Inklaar and Timmer (2013).

[38] As in the previous section, the q_{En}^t can be recovered as v_n^t/p_{En}^t.

Table 7: OECD Country GDP Price Levels in Comparable US Dollar Units of Measurement p_{En}^{t} Based on Country Growth Rates of GDP Volumes and Year 2012 Country Shares of OECD Output

n	p_{En}^{2000}	p_{En}^{2001}	p_{En}^{2002}	p_{En}^{2003}	p_{En}^{2004}	p_{En}^{2005}	p_{En}^{2006}	p_{En}^{2007}	p_{En}^{2008}	p_{En}^{2009}	p_{En}^{2010}	p_{En}^{2011}	p_{En}^{2012}
1	0.7350	0.6739	0.7281	0.8957	1.0545	1.1481	1.1876	1.3797	1.4521	1.3626	1.6997	1.9404	1.9352
2	0.8021	0.7937	0.8451	1.0251	1.1467	1.1719	1.2045	1.3404	1.4593	1.4058	1.3596	1.4562	1.3689
3	0.7861	0.7791	0.8359	1.0223	1.1487	1.1778	1.2160	1.3582	1.4847	1.4247	1.3862	1.4840	1.3981
4	0.8051	0.7806	0.7788	0.9010	1.0012	1.1105	1.2180	1.3274	1.3911	1.2735	1.4547	1.5635	1.5742
5	0.4300	0.3807	0.3646	0.3814	0.4649	0.5424	0.6451	0.6864	0.6897	0.6671	0.7980	0.8705	0.8809
6	0.3776	0.4009	0.4782	0.5599	0.6394	0.6835	0.7285	0.8381	1.0154	0.9301	0.9137	0.9768	0.8973
7	0.9293	0.9251	0.9977	1.2153	1.3674	1.4054	1.4474	1.6171	1.7998	1.7230	1.7126	1.8070	1.7126
8	0.3700	0.3825	0.4213	0.5254	0.6038	0.6410	0.7039	0.8567	0.9654	0.9187	0.8786	0.9492	0.9065
9	0.9117	0.9122	0.9715	1.1571	1.2791	1.2870	1.3093	1.4712	1.6209	1.5599	1.4922	1.6096	1.5308
10	0.8174	0.8099	0.8707	1.0651	1.1914	1.2160	1.2529	1.4023	1.5390	1.4700	1.4149	1.5042	1.4116
11	0.8198	0.8052	0.8590	1.0414	1.1580	1.1669	1.1808	1.3093	1.4122	1.3550	1.3052	1.3868	1.3006
12	0.6013	0.5950	0.6470	0.8063	0.9133	0.9404	0.9716	1.0952	1.2275	1.1909	1.1483	1.2179	1.1171
13	0.3299	0.3615	0.4356	0.5278	0.6145	0.6397	0.6281	0.7588	0.8522	0.7507	0.7478	0.7937	0.7317
14	1.1953	1.0478	1.1763	1.4143	1.5843	1.8158	1.7730	2.0523	1.6712	1.2874	1.3922	1.5157	1.4451
15	0.8114	0.8350	0.9266	1.1526	1.2981	1.3308	1.3870	1.5397	1.6007	1.4600	1.3706	1.4482	1.3474
16	1.0048	0.9915	0.9157	0.9471	0.9637	0.9714	0.9968	1.0788	1.2552	1.2000	1.2775	1.3669	1.3108
17	0.7122	0.7117	0.7725	0.9553	1.0761	1.0974	1.1259	1.2575	1.3799	1.3360	1.2787	1.3603	1.2790
18	1.4785	1.2953	1.2360	1.3139	1.3888	1.3462	1.2615	1.2344	1.3885	1.5261	1.5915	1.7176	1.7030
19	0.7059	0.6423	0.6842	0.7439	0.7974	0.8976	0.9614	1.0084	0.8751	0.7811	0.8940	0.9468	0.9404
20	0.7260	0.7057	0.7576	0.9622	1.0780	1.1314	1.2185	1.3790	1.4819	1.4165	1.4471	1.5825	1.5070
21	0.5228	0.5600	0.5796	0.5630	0.5860	0.6328	0.6757	0.7116	0.7431	0.6371	0.7087	0.7640	0.7466
22	0.7779	0.7941	0.8671	1.0625	1.1775	1.2080	1.2401	1.3780	1.5062	1.4298	1.3744	1.4589	1.3661
23	0.6349	0.6089	0.6713	0.8658	1.0202	1.1063	1.0534	1.2491	1.2263	1.0983	1.3275	1.4770	1.5021
24	0.7914	0.7881	0.8719	1.0117	1.1252	1.2826	1.4021	1.5805	1.8219	1.5460	1.7096	1.9688	1.9448
25	0.4014	0.4410	0.4524	0.4765	0.5274	0.6119	0.6475	0.7546	0.8939	0.7159	0.7514	0.7892	0.7353
26	0.5760	0.5794	0.6322	0.7809	0.8803	0.9040	0.9372	1.0515	1.1432	1.0942	1.0496	1.1045	1.0178
27	0.3215	0.3215	0.3562	0.4624	0.5580	0.5941	0.6388	0.7767	0.9236	0.8991	0.8614	0.9189	0.8600
28	0.5796	0.5776	0.6279	0.7686	0.8546	0.8673	0.8934	1.0155	1.1318	1.1089	1.0459	1.1105	1.0290
29	0.6054	0.6127	0.6724	0.8399	0.9614	1.0047	1.0555	1.1892	1.3030	1.2367	1.1799	1.2387	1.1447
30	0.9959	0.9044	0.9740	1.1936	1.3175	1.3071	1.3496	1.5139	1.6012	1.4072	1.5070	1.6947	1.6405
31	0.9398	0.9522	1.0373	1.2105	1.3219	1.3235	1.3433	1.4384	1.6384	1.6235	1.6996	2.0035	1.8994
32	0.4092	0.3191	0.3566	0.4414	0.5224	0.5935	0.6104	0.7108	0.7969	0.7046	0.7679	0.7481	0.7450
33	0.9633	0.9375	1.0001	1.1131	1.2777	1.2937	1.3467	1.4982	1.4204	1.2304	1.2583	1.3349	1.3382
34	1.0000	1.0229	1.0386	1.0594	1.0884	1.1233	1.1579	1.1886	1.2118	1.2211	1.2359	1.2602	1.2822

It can be seen that there are substantial differences between the price levels listed in Table 7 as compared to the price levels listed in Table 6 and the harmonized price levels listed in Table 5. If we take each column in Table 5, subtract the corresponding entries in the same column of Table 7 and then take the absolute value of the differences, we find that the *average* absolute difference for 2000 over the 34 countries is 6.0 percentage points, which increases to 7.9 percentage points for 2005 and then gradually decreases to 4.2 percentage points in 2012. Over all observations, the *maximum* absolute deviation is 35.6 percentage points.[39] Again these are large differences in price levels, which translate into large differences in GDP levels.

[39] The sequence of *average* absolute differences over the 34 countries in percentage points over the 13 years is as follows: 6.0, 6.1, 6.2, 7.0, 7.8, 7.9, 6.2, 6.8, 5.9, 5.4, 5.1, 5.2, 4.2. The sequence of *maximum* absolute differences in percentage points over the 13 years is as follows: 24.6, 25.9, 32.3, 35.6, 33.1, 27.1, 20.5, 22.0, 16.0, 24.7, 18.7, 12.6, 6.3. Recall that we normalized the price level of the US to be 1 in 2000 for the p_{Hn}^{t} and the p_{En}^{t}. If instead of using the normalizations $p_{H34}^{2000} = p_{E34}^{2000} = 1$ when constructing Tables 5 and 8, we used the normalizations $p_{H34}^{2012} = p_{E34}^{2012} = 1$, we would find that the absolute differences between the resulting p_{Hn}^{t} and p_{En}^{t} would equal 0 for all countries n for t = 2012. Thus the choice of normalization (and hence of the units of measurement) can affect the results.

For analysts who are interested in comparing real GDP levels across time and space, the results presented in this section indicate that the strategy of using national growth rates and a single cross country comparison of real GDP levels will not lead to stable comparisons. The harmonization strategy suggested in section 3 will lead to stable comparisons and if the accuracy of the annual sequence of PPPs is roughly constant, the resulting harmonized estimates seem to be preferable to the consistent national growth rate estimates that are based on a single cross country comparison.

6. OECD Growth and Inflation Using Adjusted Country Annual GDP Volume Growth Rates and OECD Shares of Real GDP for Two Benchmark Years

The OECD provides annual PPPs so that estimates of relative GDP volumes can be constructed for all member countries for each year. However, the World Bank's ICP PPPs are only available at infrequent intervals.[40] The question arises: how exactly should two widely separated benchmark real GDP shares be used along with annual growth rate information to interpolate between the benchmark years? We will model this situation using our OECD data base but will use only the benchmark GDP shares for the years 2000 and 2012 along with information on national GDP growth rates in order to interpolate between the benchmark years.

We will propose an interpolation method that leads to country shares of real GDP that are exactly consistent with the shares s_n^{2000} for the year 2000 and the shares s_n^{2012} for the year 2012.[41]

Our first task is to use the same methodology that we used in section 3 above to construct country measures of real GDP that jump from the year 2000 to the year 2012. The *long term growth factor for country n* can be defined as Q_n^{2012}/Q_n^{2000} where Q_n^t is country n's GDP volume in year t.[42] Now use these long term growth factors along with the year 2000 country shares of OECD real GDP, s_n^{2000}, in order to define the *OECD Laspeyres type long term growth factor*, Γ_L, as the following weighted average of the national long term growth factors:

$$(27)\ \Gamma_L \equiv \sum_{n=1}^{34} s_n^{2000}(Q_n^{2012}/Q_n^{2000}) ; \qquad t = 2001,...,2012.$$

The counterpart to the Laspeyres type formula defined by (27) is the following *Paasche type formula* that uses the shares of 2012 and reciprocal long term growth rates:

$$(28)\ \Gamma_P \equiv [\sum_{n=1}^{34} s_n^{2012}(Q_n^{2012}/Q_n^{2000})^{-1}]^{-1} ; \qquad t = 2001,...,2012.$$

[40] The World Bank has produced benchmark PPPs for over 150 countries for 2005 and 2011.

[41] Recall that these shares are listed in Table 2 above. For an alternative interpolation method that has the property that the resulting estimates are exactly consistent with the PPPs at the beginning and end of a sample period, see Feenstra, Timmer and Inklaar (2013). The econometric method developed by Rao, Rambaldi and Doran (2010) (2011) can also be adapted to have this property.

[42] These long term country growth factors are conveniently listed in the last column of Table A2 in the Appendix.

Since both indexes have the same logical foundation, it seems best to take a symmetric average of the two indexes, which leads to the following Fisher type formula for *OECD long term volume growth* going from the year 2000 to the year 2012:

(29) $\Gamma_F \equiv [\Gamma_L \Gamma_P]^{1/2}$; t = 2001,...,2012.

The long term indexes defined by (27)-(29) turn out to be 1.2207 1.2209 1.2208 respectively, so that there is practically no difference in the three indexes for this data set.[43] We will use the Fisher measure as our preferred measure of OECD volume growth between 2000 and 2012. We use this measure in order to define country volumes for 2012.

Preliminary estimates of country GDP volumes in comparable units for the years 2000 and 2012, q_{In}^{2000} and q_{In}^{2012} (the index I indicates that these are interpolated estimates), are defined as follows:

(30) $q_{In}^{2000} \equiv s_n^{2000}$; $q_{In}^{2012} \equiv \Gamma_F\, s_n^{2012}$; n = 1,...,34.

The volumes defined by (30) will be imposed as constraints on our interpolation scheme. Define the *implied long term growth factor* over the years 2000-2012 for country n, g_n, that is implied by the estimates of country levels given by equations (30):

(31) $g_n \equiv q_{In}^{2012}/q_{In}^{2000}$; n = 1,...,34.

These growth factors are not necessarily equal to the national growth factors G_n that are implied by the national growth rates listed in Table A2 of the Appendix:

(32) $G_n \equiv Q_n^{2012}/Q_n^{2000}$; n = 1,...,34.

Thus for each country n, there is an "error" factor or discrepancy, $E_n \equiv g_n/G_n$ between the implied growth rates g_n defined by (31) and the national growth rates between 2000 and 2012, G_n defined by (32). We will distribute these errors in a proportional manner and use the resulting adjusted national growth rates to interpolate between the two benchmark observations. Thus define the *country n proportional annualized discrepancy factor*, α_n, as follows:[44]

(33) $\alpha_n \equiv [g_n/G_n]^{1/12}$; n = 1,...,34.

[43] Note that (29) defines a direct comparison of the data of 2000 with the data of 2012 whereas in section 3 above, we used chained Fisher type indexes to go from 2000 to 2012. The chained Fisher index for 2012 relative to 2000 is equal to 1.2203 (see the Q_{UV}^t column in Table 6), which is very close to its direct counterpart, 1.2208.

[44] The average of the g_n/G_n was 1.03. The maximum ratio was 1.27 (Norway) and the minimum ratio was 0.81 (Israel). The PPP based growth rates treat changes in the terms of trade differently than the nationally based growth rates and so fluctuations in the price of oil probably explain the Norwegian divergence. For the three largest countries, the ratio was 1.05 (Germany), 0.94 (Japan) and 0.97 (US).

The q_{In}^t for nonbenchmark years t can now be defined as follows:[45]

(34) $q_{In}^t \equiv q_{In}^{t-1}(Q_n^t/Q_n^{t-1})\alpha_n$; n = 1,...,34 ; t = 2001,....,2011.

Once the q_{In}^t have been defined, the corresponding US dollar price levels p_{In}^t are defined in the usual way:

(35) $p_{In}^t \equiv v_n^t/q_{In}^t$; n = 1,...,34 ; t = 2001,....,2011.

In order to make the volumes and prices defined by (34) and (35) comparable to the harmonized country prices and volumes expressed in US dollars that are listed Tables 4 and 5 in section 3, we impose a normalization on the prices defined by (35) that makes the price level for the US in 2000 equal to unity; i.e., we divide all prices defined by (35) by a constant that sets the resulting p_{I34}^{2000} equal to 1 and the quantities or volumes defined by (34) are all multiplied by this constant. The resulting normalized p_{In}^t are listed in Table 8.[46]

Table 8: OECD Country GDP Price Levels in Comparable US Dollar Units of Measurement p_{In}^t Based on Adjusted Country Growth Rates of GDP Volumes and Year 2000 and 2012 Country Shares of OECD Output

n	p_{In}^{2000}	p_{In}^{2001}	p_{In}^{2002}	p_{In}^{2003}	p_{In}^{2004}	p_{In}^{2005}	p_{In}^{2006}	p_{In}^{2007}	p_{In}^{2008}	p_{In}^{2009}	p_{n}^{2010}	p_{In}^{2011}	p_{In}^{2012}
1	0.7628	0.6990	0.7549	0.9282	1.0924	1.1887	1.2291	1.4273	1.5015	1.4083	1.7558	2.0036	1.9974
2	0.8294	0.8206	0.8736	1.0595	1.1850	1.2109	1.2443	1.3845	1.5071	1.4516	1.4037	1.5032	1.4129
3	0.8209	0.8128	0.8712	1.0644	1.1949	1.2240	1.2624	1.4087	1.5384	1.4748	1.4335	1.5332	1.4430
4	0.8293	0.8043	0.8025	0.9286	1.0320	1.1449	1.2559	1.3689	1.4348	1.3138	1.5009	1.6134	1.6248
5	0.5284	0.4611	0.4352	0.4486	0.5390	0.6198	0.7264	0.7619	0.7544	0.7192	0.8480	0.9116	0.9092
6	0.3682	0.3929	0.4707	0.5538	0.6354	0.6825	0.7309	0.8448	1.0284	0.9465	0.9342	1.0034	0.9261
7	1.0403	1.0286	1.1019	1.3332	1.4899	1.5209	1.5558	1.7265	1.9086	1.8148	1.7917	1.8777	1.7675
8	0.4197	0.4305	0.4704	0.5821	0.6637	0.6991	0.7616	0.9197	1.0283	0.9709	0.9212	0.9875	0.9356
9	0.9166	0.9191	0.9811	1.1710	1.2973	1.3082	1.3337	1.5019	1.6584	1.5994	1.5334	1.6577	1.5800
10	0.8652	0.8555	0.9177	1.1202	1.2504	1.2736	1.3095	1.4626	1.6018	1.5268	1.4665	1.5558	1.4569
11	0.8909	0.8713	0.9255	1.1172	1.2369	1.2412	1.2506	1.3807	1.4827	1.4167	1.3588	1.4375	1.3424
12	0.6322	0.6246	0.6782	0.8439	0.9543	0.9811	1.0122	1.1392	1.2748	1.2349	1.1888	1.2590	1.1530
13	0.3823	0.4150	0.4952	0.5942	0.6852	0.7065	0.6870	0.8219	0.9143	0.7976	0.7869	0.8272	0.7552
14	1.0724	0.9511	1.0803	1.3141	1.4894	1.7271	1.7062	1.9982	1.6462	1.2830	1.4037	1.5462	1.4915
15	0.8862	0.9078	1.0026	1.2412	1.3914	1.4196	1.4727	1.6271	1.6836	1.5284	1.4280	1.5017	1.3906
16	0.8444	0.8476	0.7963	0.8378	0.8672	0.8893	0.9283	1.0220	1.2097	1.1765	1.2741	1.3869	1.3528
17	0.7528	0.7508	0.8133	1.0037	1.1285	1.1485	1.1760	1.3108	1.4356	1.3872	1.3250	1.4068	1.3201
18	1.4394	1.2672	1.2151	1.2979	1.3787	1.3429	1.2646	1.2434	1.4054	1.5522	1.6267	1.7642	1.7576
19	0.6598	0.6053	0.6501	0.7128	0.7704	0.8744	0.9443	0.9987	0.8738	0.7864	0.9075	0.9692	0.9706
20	0.8659	0.8316	0.8821	1.1069	1.2252	1.2705	1.3520	1.5117	1.6050	1.5158	1.5300	1.6532	1.5554
21	0.6450	0.6807	0.6941	0.6643	0.6812	0.7248	0.7625	0.7912	0.8139	0.6876	0.7535	0.8004	0.7705
22	0.8223	0.8378	0.9130	1.1165	1.2349	1.2643	1.2953	1.4365	1.5670	1.4846	1.4242	1.5088	1.4100
23	0.6549	0.6282	0.6925	0.8932	1.0527	1.1415	1.0869	1.2889	1.2655	1.1335	1.3700	1.5244	1.5504
24	1.0371	1.0125	1.0980	1.2489	1.3617	1.5216	1.6306	1.8019	2.0362	1.6938	1.8361	2.0728	2.0072
25	0.4236	0.4645	0.4757	0.5001	0.5524	0.6398	0.6757	0.7861	0.9295	0.7430	0.7784	0.8161	0.7589
26	0.6446	0.6441	0.6980	0.8565	0.9590	0.9782	1.0073	1.1225	1.2122	1.1524	1.0981	1.1477	1.0504
27	0.3440	0.3429	0.3788	0.4903	0.5899	0.6262	0.6712	0.8138	0.9648	0.9363	0.8944	0.9513	0.8876
28	0.5723	0.5724	0.6246	0.7674	0.8563	0.8723	0.9019	1.0289	1.1510	1.1319	1.0716	1.1420	1.0620
29	0.6763	0.6799	0.7413	0.9199	1.0461	1.0860	1.1334	1.2686	1.3808	1.3019	1.2340	1.2869	1.1815
30	0.9970	0.9077	0.9801	1.2041	1.3324	1.3253	1.3718	1.5428	1.6359	1.4413	1.5475	1.7447	1.6931

45 It can be verified that if we apply definitions (34) for t = 2012, we obtain the q_{In}^{2012} defined by (30).
46 As usual, the q_{In}^t can be recovered as v_n^t/p_{In}^t.

31	**1.0963**	**1.0995**	**1.1856**	**1.3695**	**1.4804**	**1.4670**	**1.4740**	**1.5622**	**1.7614**	**1.7277**	**1.7904**	**2.0890**	**1.9604**
32	**0.4523**	**0.3507**	**0.3896**	**0.4796**	**0.5643**	**0.6375**	**0.6519**	**0.7548**	**0.8415**	**0.7398**	**0.8017**	**0.7766**	**0.7689**
33	**0.9623**	**0.9390**	**1.0045**	**1.1210**	**1.2903**	**1.3100**	**1.3674**	**1.5254**	**1.4501**	**1.2596**	**1.2917**	**1.3740**	**1.3812**
34	**1.0000**	**1.0256**	**1.0441**	**1.0678**	**1.1000**	**1.1382**	**1.1763**	**1.2107**	**1.2376**	**1.2504**	**1.2689**	**1.2972**	**1.3234**

As usual, it can be seen that there are some substantial differences between the price levels listed in Table 8 as compared to the price levels listed in Table 5 but the discrepancies are much reduced as compared to the discrepancies when only one benchmark set of PPPs is used. The overall sample average absolute discrepancy is now only 1.9 percentage points. The average absolute difference for 2000 over the 34 countries is 0, which increases to 3.2 percentage points for 2005 and 2007 and then gradually decreases to 0.3 percentage points in 2012. Over all observations, the *maximum* absolute deviation is 12.5 percentage points.[47]

Some tentative conclusions can be drawn from the Tables in this section and the previous sections:

- The interpolation method which is consistent with benchmark expenditure shares for two widely separated years seems to work pretty well and if the benchmark PPPs are of equal quality, the interpolation method is much better[48] than simply projecting the country shares from a single benchmark using national growth rates.
- If it is too expensive to prepare annual PPPs for a group of countries, then the interpolation method will probably generate comparable country real GDP volumes that are close to our preferred harmonized volumes described in section 3, provided that benchmark PPPs are calculated every 3-5 years.

The interpolation method that we described in this section is not the only possible method that could be used to calculate comparable real GDP series over time and space when benchmark PPPs are only available infrequently. In particular, econometricians may prefer to use an interpolation method that is based on the Kalman filter; see Rao, Rambaldi and Doran (2010) (2011) for the description of such a method.[49] However, statistical agencies are generally reluctant to adopt methods that rely heavily on econometrics so the simple method of interpolation described here is proposed as an alternative interpolation method.

Feenstra, Inklaar and Timmer (2013; 17-19) use a simple interpolation method to harmonize their new Penn World Table estimates of real GDP growth (in comparable

---

[47] The sequence of *average* absolute differences over the 34 countries in percentage points over the 13 years is as follows: 0, 1.0, 1.8, 2.0, 2.3, 3.2, 3.0, 3.2, 2.8, 2.6, 1.7, 1.2, 0.06. The sequence of *maximum* absolute differences in percentage points over the 13 years is as follows: 0, 3.9, 9.7, 11.9, 9.4, 10.7, 8.3, 12.5, 12.3, 9.9, 6.1, 3.2, 0.08. The reason why the differences are not all equal to 0 for 2012 is that the direct aggregate Fisher index going from 2000 to 2012 differs slightly from its chained counterpart defined in section 3.

[48] "Better" means "more consistent" with our preferred harmonized volumes that could be calculated if annual PPPs were available.

[49] For additional methods for harmonizing cross sectional and time series estimates of real GDP, see Rao, Rambaldi and Balk (2013). Summers and Heston (1991; 340-341) also used an econometric method to reconcile the differences between national growth rates and ICP generated estimates of relative GDP levels.

units of measurement) using both ICP information between two benchmarks and national information on GDP growth. Their interpolation method is similar to our suggested method, in that their interpolated estimates are consistent with the relative GDP levels for the PPP benchmark years. The key to their interpolation method is the construction of interpolated PPPs between ICP benchmarks. We explain their method using our notation and adapting their analysis to the problem of constructing PPPs for the years 2001-2011, given that we have PPPs for the benchmark years 2000 and 2012. Recall that the domestic price level for country n in year t was defined as $P_n^t \equiv V_n^t/Q_n^t$ for n = 1,...,34 and t = 2000,...,2012 (and these price levels are listed in Appendix Table A1). Recall also that PPP_n^t was defined as the number of units of the national currency of country n that is required to purchase one dollar of US (real) GDP in year t (and these OECD PPPs are listed in Appendix Table A5). The key to the Feenstra, Timmer and Inklaar (2013; 18) interpolation method is their method for constructing interpolated PPPs. Using our notation, their interpolated PPP for country n in year t, PPP_{In}^t, is defined as follows:

$$(36)\ PPP_{FTIn}^t \equiv (1 - w^t)(PPP_n^{2000})(P_n^t/P_n^{2000}) + w^t(PPP_n^{2012})(P_n^t/P_n^{2012})\ ; \quad n = 1,...,34;\ t = 2000, 2001, ...,2012$$

where the weight function w^t is defined as follows:

$$(37)\ w^t \equiv (t - 2000)/12\ ; \qquad t = 2000, 2001, ...,2012.$$

Thus w^t grows linearly in t with $w^{2000} = 0$ and $w^{2012} = 1$. Note that $PPP_{FTIn}^{2000} = PPP_n^{2000}$ and $PPP_{FTIn}^{2012} = PPP_n^{2012}$ so that the interpolated PPPs coincide with the actual PPPs for the two benchmark years, 2000 and 2012. Thus the interpolated PPP for country n in year t, PPP_{FTIn}^t, is a simple weighted average of two extrapolated PPPs for country n. The first index in the weighted average uses the PPPs for 2000, PPP_n^{2000}, and pushes these PPPs forward using the normalized domestic inflation rates P_n^t/P_n^{2000} and the second index uses the PPPs for 2012, PPP_n^{2012}, and pushes these PPPs backwards using the normalized domestic inflation rates P_n^t/P_n^{2012}.

We will not attempt to construct OECD real volumes for our sample period using the complete Feenstra, Timmer and Inklaar methodology but we did experiment with their method of weighting. Recall that Tables 6 and 7 listed the OECD country US dollar prices (in comparable units across time and space), p_{Bn}^t and p_{En}^t, where the prices p_{Bn}^t (p_{En}^t) were based on country growth rates of GDP volumes and year 2000 (2012) country shares of OECD output. Using the weights w^t defined by (37), define the FTI type US dollar price levels, p_{FTIn}^t, as the following weighted averages of the 2000 US dollar prices p_{Bn}^{2000} and 2012 US dollar prices p_{En}^{2012}:

$$(38)\ p_{FTIn}^t \equiv (1 - w^t)p_{Bn}^{2000} + w^t p_{En}^{2012}\ ; \qquad n = 1,...,34;\ t = 2000, 2001, ...,2012.$$

It can be seen that $p_{FTIn}^{2000} = p_{Bn}^{2000}$ and $p_{FTIn}^{2012} = p_{En}^{2012}$ for n = 1,...,34. Now compare the prices p_{FTIn}^t to our preferred Harmonized US dollar price levels p_{Hn}^t listed in Table 5. Take the absolute value of the differences, $p_{Hn}^t - p_{FTIn}^t$. The sample average absolute difference (over time periods t and countries n) is 12.2 percentage points. The within year

absolute difference grows from 0 in 2000 to 25.3 percentage points in 2008.[50] The maximum absolute difference grows from 0 in 2000 to 83.4 percentage points in 2007. These are large differences in price levels, which translate into large differences in real GDP levels.

The relative volumes generated by dividing the US dollar GDP values by the corresponding US dollar prices defined by (38) are no longer independent of the choice of the numeraire country. Thus instead of taking the weighted arithmetic means of the prices p_{Bn}^{2000} and p_{En}^{2012}, take the corresponding weighted geometric means and denote the resulting prices by p_{FTIGn}^{t}.[51] Compare the prices p_{FTIGn}^{t} to our preferred Harmonized US dollar price levels p_{Hn}^{t} listed in Table 5 and take the absolute value of the differences, $p_{Hn}^{t} - p_{FTIGn}^{t}$. The sample average absolute difference is now 13.6 percentage points, which is larger than the average differences using the weighted arithmetic means. The within year absolute difference grows from 0 in 2000 to 28.6 percentage points in 2008.[52] The maximum absolute difference grows from 0 in 2000 to 84.7 percentage points in 2007.

Why do the above variants of the interpolation method suggested by Feenstra, Timmer and Inklaar generate US dollar price levels (and the corresponding country real GDP levels) that are so different from the Harmonized country US dollar price levels that are listed in Table 5? The reason is that the interpolated PPPs defined by (36) (and their geometric counterparts) depend on country inflation rates, which are quite variable.[53] In order to eliminate the effects of country inflation rates, we tried the following variant of the FTI interpolation method: instead of using equations (36) to interpolate the PPPs between the years 2000 and 2012, use the following equations to define the interpolated PPP_{In}^{t} for country n and year t:

$$(39)\ PPP_{In}^{t} \equiv (1 - w^{t})PPP_{n}^{2000} + w^{t}\ PPP_{n}^{2012}\ ; \qquad n = 1,...,34;\ t = 2000, 2001, ...,2012$$

[50] The sequence of within year average absolute differences in percentage points over the 13 years is as follows: 0, 6.2, 6.2, 9.3, 14.5, 15.8, 14.7, 23.0, 25.3, 14.9, 9.4, 14.8, 4.2. The differences are nonzero in 2013 even though the corresponding PPPs for 2012 are exactly consistent with the OECD PPPs for 2012. Thus while the US dollar prices p_{Hn}^{2012} equal λp_{FTIn}^{2012} for n = 1,...,34, the factor of proportionality λ is not equal to one and thus the differences are nonzero in 2013.

[51] Usually, taking geometric means (rather than arithmetic means) of two indexes leads to indexes that have better invariance and homogeneity properties. For examples of this phenomenon, see Diewert (1997) and Hill and Fox (1997).

[52] The sequence of within year average absolute differences in percentage points over the 13 years is as follows: 0, 5.2, 5.0, 10.8, 16.7, 18.6, 17.5, 26.3, 28.6, 16.2, 11.6, 16.0, 4.2.

[53] More precisely, the FTI method interacts country inflation rates with the linear in time weights in equations (36) and *these weights are independent of the magnitude of economic price and quantity data that pertain to the countries* whereas our interpolation method depends only on country volume indexes over the sample period and the relative volumes generated by the PPPs at the beginning and end of the sample period. Thus if the α_n defined by (33) were all equal to unity, then the matrix of country real volumes generated by extrapolating the base period relative GDP volumes forward by national growth rates would be equal to the matrix of country real volumes generated by extrapolating the final period relative GDP volumes backwards (after normalization to a common base) and our interpolation method would generate this common matrix of comparable over time and space real GDP volumes. Under the same conditions, the FTI method would not generate the same matrix (except by chance).

where the weight w^t is still defined by (37). Now return to our description for the construction of the harmonized country estimates for GDP and US dollar price levels that is in section 3 but replace PPP_n^t in equations (12) by their interpolated counterparts PPP_{In}^t defined by (39). Denote the resulting US dollar price levels by p_{In}^t. Compare the prices p_{In}^t to our preferred US dollar price levels p_{Hn}^t listed in Table 5 and as usual, take the absolute value of the differences, $p_{Hn}^t - p_{In}^t$. The sample average absolute difference (over time periods t and countries n) is only 2.48 percentage points. The within year average absolute difference grows from 0 in 2000 to 4.3 percentage points in 2005.[54] The maximum absolute difference is 20.6 percentage points in 2003. The performance of this interpolation method is much better than the previous interpolation method but still not quite as good as our suggested interpolation method that was described at the beginning of this section (which generated an average absolute difference of only 1.92 percentage points).

The above numerical experiments with interpolation methods that are similar in spirit to the method used by Feenstra, Timmer and Inklaar are not conclusive since it assumes that the "truth" is best defined by the harmonized parities p_{Hn}^t defined earlier in section 3. However, the numerical experiments do show that the method of interpolation between benchmark Purchasing Power Parity rounds does matter. Additional research into alternative methods of interpolation is required.[55]

7. Conclusion

A number of interesting points emerged from our investigations:

- If our focus is on measuring overall OECD GDP growth and PPP information is not available, then the method that is explained in section 2 may be used. These section 2 overall OECD growth measures, γ^t, do not depend on PPPs or the choice of the numeraire currency but exchange rate fluctuations cause this measure of group GDP growth to fluctuate much more than the indexes of OECD GDP growth defined in section 2.
- We computed γ^t in section 2 using the US and then Germany as the numeraire country and found that while the Fisher index of OECD real GDP growth remained invariant, the accompanying Fisher price indexes, P^t and P_{EU}^t, exhibited wildly different rates of growth. Thus these *Approach 1* price indexes are useless as indicators of OECD inflation.
- Three alternative measures of overall OECD GDP growth were defined in section 3: the Laspeyres, Paasche and Fisher measures, γ_L^t, γ_P^t and γ_F^t. These *Approach 2* measures depended on the annual OECD PPP information. The Laspeyres measure is the official OECD measure for overall OECD growth but the Fisher

[54] The sequence of within year average absolute differences in percentage points over the 13 years is as follows: 0, 1.1, 1.9, 2.9, 4.0, 4.3, 3.7, 3.7, 3.6 2.3, 2.4, 1.9, 0.3. The sequence of within year maximum absolute differences in percentage points over the 13 years is as follows: 0, 6.7, 13.8, 20.6, 20.3, 18.6, 15.0, 13.3, 15.8, 8.2, 6.8, 5.8, 0.5.
[55] The OECD data base is a useful one for testing out different interpolation methods.

measure seems preferable on conceptual grounds. However, for our data set, all three measures were very close to each other.

- The section 3 Fisher measure of OECD growth, γ_F^t, (which used real GDP share weights constructed using PPPs) grew on average about 1/10 of a percentage point more rapidly per year over the period 2000-2012 than our section 2 measure of OECD GDP growth γ^t, (which used exchange rate based share weights). Since it is likely that the PPP information for OECD countries is reasonably accurate, we prefer the PPP based estimates for OECD real GDP growth, γ_F^t, over the corresponding section 2 estimates, γ^t, since the former measure is smoother and not affected by exchange rate movements.
- Section 3 also introduced 3 measures of OECD aggregate GDP price inflation, the Laspeyres, Paasche and Fisher measures ρ_L^t, ρ_P^t and ρ_F^t defined by (19). These inflation measures used PPP based country share weights to weight the country inflation rates and were much more satisfactory than the section 2 measures of OECD aggregate inflation. The three measures differed somewhat so the choice of index matters. Our preference is for the Fisher measure ρ_F^t since it satisfies a time reversal test whereas the other two indexes do not.
- We used two principles in section 3 to generate our *harmonized estimates of real GDP for OECD countries*: (i) The resulting harmonized estimates of country volumes q_{Hn}^t must be consistent with the real volume shares s_n^t listed in Table 2 and (ii) OECD aggregate real GDP growth must be equal to the rates of aggregate growth generated by our recommended Fisher indexes γ_F^t.
- Once the harmonized estimates of real GDP q_{Hn}^t have been generated, companion *US dollar country price levels* p_{Hn}^t can be generated as $p_{Hn}^t \equiv v_n^t/q_{Hn}^t$ where v_n^t is the exchange rate converted US dollar nominal value of GDP for country n in year t. These country price levels are useful (but imperfect) indicators of a country's competitiveness in year t.
- In sections 4 and 5, alternative measures of comparable levels of real GDP and the accompanying US dollar price levels were constructed. The measures constructed in section 4 used the PPP information for 2000 and national growth rates for real GDP by country whereas the estimates constructed in section 5 substituted the PPP information for 2012. We found tremendous discrepancies in these estimates as compared to the harmonized estimates constructed in section 3.
- The results listed in sections 3-5 show that it is very hazardous for analysts interested in comparative levels of GDP across countries to use national growth rates and a *single* cross country comparison of real GDP levels. Eventually, the single cross country comparison is replaced by another single cross country comparison but the new set of comparable GDP levels across time and space can be vastly different from the earlier set of GDP levels, particularly for small countries. These results reinforce the case for using the harmonized series that were defined in section 3. Using the section 3 methodology, the previously constructed harmonized estimates of relative GDP levels remain unchanged as another year of data is added.
- If PPP computations for a group of countries are only done on an infrequent basis (rather than on an annual basis as is the case for the OECD), then the

interpolation method explained in section 6 may prove to be a useful method for obtaining comparable GDP levels that are consistent with the GDP relative levels for the two benchmark years. The results in section 6 also indicate that different interpolation methods can generate very different results.

Of course, the harmonization methods that have been suggested in this paper can be applied to any other value aggregate, such as consumption, investment or domestic absorption.

The results in this paper show that if countries want to compare the size of their economies or measure expenditure growth or price inflation for a group of countries, it is absolutely essential that those countries undertake regular cross country comparisons of prices.

Appendix

This Appendix lists the underlying OECD data and some supplementary Tables. The source for all of the data listed in this Appendix is OECD.Stat.

The country price levels P_n^t using domestic currencies (normalized to equal unity in 2000) are listed in Table 1 and the corresponding volumes are listed in Table 2.

Table A1: OECD Country Price Levels in National Currencies P_n^t

n	P_n^{2000}	P_n^{2001}	P_n^{2002}	P_n^{2003}	P_n^{2004}	P_n^{2005}	P_n^{2006}	P_n^{2007}	P_n^{2008}	P_n^{2009}	P_n^{2010}	P_n^{2011}	P_n^{2012}
1	1.0000	1.0278	1.0571	1.0893	1.1310	1.1858	1.2440	1.3006	1.3655	1.3781	1.4616	1.4838	1.4743
2	1.0000	1.0187	1.0314	1.0432	1.0608	1.0824	1.1028	1.1249	1.1443	1.1624	1.1791	1.2032	1.2238
3	1.0000	1.0205	1.0410	1.0616	1.0843	1.1101	1.1361	1.1631	1.1880	1.2021	1.2267	1.2512	1.2754
4	1.0000	1.0112	1.0222	1.0558	1.0895	1.1255	1.1556	1.1924	1.2414	1.2175	1.2534	1.2940	1.3156
5	1.0000	1.0418	1.0826	1.1364	1.2213	1.3087	1.4743	1.5457	1.5530	1.6126	1.7550	1.8146	1.8470
6	1.0000	1.0464	1.0742	1.0838	1.1275	1.1236	1.1296	1.1671	1.1894	1.2166	1.1974	1.1861	1.2054
7	1.0000	1.0250	1.0486	1.0658	1.0906	1.1220	1.1458	1.1720	1.2215	1.2297	1.2822	1.2915	1.3206
8	1.0000	1.0648	1.1147	1.1597	1.2114	1.2849	1.3976	1.5602	1.6446	1.6474	1.6523	1.7018	1.7584
9	1.0000	1.0301	1.0432	1.0361	1.0411	1.0459	1.0547	1.0863	1.1183	1.1347	1.1386	1.1701	1.2041
10	1.0000	1.0201	1.0428	1.0636	1.0814	1.1021	1.1257	1.1548	1.1842	1.1926	1.2041	1.2196	1.2382
11	1.0000	1.0113	1.0257	1.0370	1.0481	1.0546	1.0579	1.0751	1.0834	1.0962	1.1075	1.1212	1.1376
12	1.0000	1.0312	1.0662	1.1081	1.1407	1.1728	1.2012	1.2411	1.2997	1.3296	1.3447	1.3588	1.3484
13	1.0000	1.1126	1.2067	1.2719	1.3384	1.3717	1.4196	1.4968	1.5758	1.6318	1.6707	1.7144	1.7695
14	1.0000	1.0863	1.1474	1.1545	1.1834	1.2170	1.3242	1.3990	1.5641	1.6939	1.8111	1.8703	1.9236
15	1.0000	1.0596	1.1180	1.1596	1.1872	1.2151	1.2555	1.2774	1.2409	1.1934	1.1751	1.1829	1.1908
16	1.0000	1.0178	1.0589	1.0528	1.0542	1.0641	1.0841	1.0817	1.0993	1.1518	1.1658	1.1938	1.2336
17	1.0000	1.0288	1.0618	1.0949	1.1211	1.1415	1.1610	1.1885	1.2186	1.2441	1.2489	1.2658	1.2877
18	1.0000	0.9880	0.9727	0.9560	0.9431	0.9313	0.9209	0.9123	0.9007	0.8962	0.8768	0.8604	0.8528
19	1.0000	1.0386	1.0721	1.1103	1.1440	1.1515	1.1499	1.1738	1.2079	1.2493	1.2945	1.3144	1.3270
20	1.0000	1.0008	1.0216	1.0820	1.1018	1.1546	1.2327	1.2787	1.2839	1.2941	1.3866	1.4448	1.4885
21	1.0000	1.0582	1.1321	1.2287	1.3378	1.3951	1.4897	1.5731	1.6730	1.7416	1.8115	1.9201	1.9889
22	1.0000	1.0510	1.0912	1.1150	1.1231	1.1504	1.1707	1.1924	1.2178	1.2190	1.2290	1.2429	1.2592
23	1.0000	1.0366	1.0386	1.0669	1.1015	1.1244	1.1624	1.2163	1.2485	1.2577	1.3180	1.3379	1.3267
24	1.0000	1.0173	0.9993	1.0283	1.0888	1.1862	1.2909	1.3299	1.4751	1.3956	1.4834	1.5840	1.6241
25	1.0000	1.0348	1.0580	1.0622	1.1056	1.1349	1.1517	1.1973	1.2345	1.2804	1.2987	1.3404	1.3726
26	1.0000	1.0357	1.0745	1.1068	1.1341	1.1628	1.1951	1.2289	1.2484	1.2599	1.2677	1.2709	1.2671
27	1.0000	1.0502	1.0909	1.1488	1.2160	1.2449	1.2816	1.2958	1.3329	1.3172	1.3237	1.3454	1.3622
28	1.0000	1.0865	1.1689	1.2336	1.2739	1.2951	1.3225	1.3777	1.4348	1.4822	1.4665	1.4835	1.4872
29	1.0000	1.0419	1.0873	1.1325	1.1783	1.2295	1.2804	1.3223	1.3537	1.3547	1.3558	1.3560	1.3558
30	1.0000	1.0237	1.0394	1.0577	1.0611	1.0705	1.0913	1.1214	1.1566	1.1803	1.1904	1.2060	1.2180
31	1.0000	1.0125	1.0187	1.0271	1.0357	1.0383	1.0612	1.0878	1.1181	1.1131	1.1168	1.1210	1.1222

32	1.0000	1.5285	2.1005	2.5893	2.9104	3.1166	3.4074	3.6194	4.0535	4.2681	4.5104	4.8972	5.2293
33	1.0000	1.0228	1.0481	1.0708	1.0961	1.1176	1.1496	1.1760	1.2135	1.2406	1.2791	1.3086	1.3306
34	1.0000	1.0229	1.0386	1.0594	1.0884	1.1233	1.1579	1.1886	1.2118	1.2211	1.2359	1.2602	1.2822

It can be seen that Country 18, Japan, had the lowest rate of domestic inflation, which was actually a deflation. Country 32, Turkey, had the highest rate of domestic inflation, which was 533% over the sample period.

Table A2: OECD Country GDP Volumes Relative to the Corresponding 2000 Volumes, Q_n^t/Q_n^{2000}

n	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
1	1.0000	1.0391	1.0719	1.1163	1.1519	1.1870	1.2319	1.2783	1.2994	1.3265	1.3588	1.4045	1.4562
2	1.0000	1.0086	1.0257	1.0345	1.0613	1.0868	1.1267	1.1684	1.1852	1.1399	1.1601	1.1930	1.2034
3	1.0000	1.0081	1.0218	1.0300	1.0638	1.0824	1.1112	1.1433	1.1545	1.1222	1.1483	1.1686	1.1670
4	1.0000	1.0178	1.0476	1.0673	1.1006	1.1338	1.1658	1.1915	1.1997	1.1665	1.204	1.2344	1.2555
5	1.0000	1.0327	1.0605	1.1005	1.1777	1.2504	1.3216	1.3898	1.4356	1.4207	1.5026	1.5905	1.6788
6	1.0000	1.0310	1.0531	1.0928	1.1446	1.2219	1.3077	1.3827	1.4255	1.3613	1.3949	1.4203	1.4058
7	1.0000	1.0070	1.0117	1.0156	1.0389	1.0644	1.1005	1.1179	1.1091	1.0463	1.0608	1.0722	1.0683
8	1.0000	1.0628	1.1325	1.2205	1.2979	1.4128	1.5555	1.6720	1.6026	1.3767	1.4120	1.5469	1.6079
9	1.0000	1.0228	1.0416	1.0626	1.1064	1.1387	1.1889	1.2523	1.2560	1.1487	1.1874	1.2197	1.2097
10	1.0000	1.0184	1.0278	1.0371	1.0635	1.0829	1.1096	1.1349	1.1340	1.0983	1.1173	1.1399	1.1401
11	1.0000	1.0151	1.0152	1.0114	1.0232	1.0302	1.0683	1.1032	1.1152	1.0578	1.1002	1.1369	1.1447
12	1.0000	1.0420	1.0778	1.1419	1.1917	1.2189	1.2861	1.3315	1.3287	1.287	1.2234	1.1364	1.0640
13	1.0000	1.0371	1.0839	1.1256	1.1796	1.2263	1.2741	1.2755	1.2869	1.1998	1.2125	1.2315	1.2110
14	1.0000	1.0392	1.0407	1.0660	1.1495	1.2326	1.2907	1.3679	1.3842	1.2934	1.2403	1.2736	1.2914
15	1.0000	1.0499	1.1067	1.1480	1.1962	1.2689	1.3388	1.4053	1.3750	1.2872	1.2735	1.3011	1.3032
16	1.0000	0.9982	0.9975	1.0122	1.0618	1.1140	1.1786	1.2481	1.2994	1.3138	1.3793	1.4428	1.4887
17	1.0000	1.0186	1.0232	1.0227	1.0404	1.0501	1.0732	1.0913	1.0787	1.0194	1.0370	1.0419	1.0155
18	1.0000	1.0036	1.0065	1.0234	1.0476	1.0612	1.0792	1.1029	1.0914	1.0310	1.0790	1.0729	1.0938
19	1.0000	1.0397	1.1141	1.1453	1.1982	1.2456	1.3101	1.3770	1.4087	1.4132	1.5025	1.5578	1.5896
20	1.0000	1.0252	1.0671	1.0849	1.1323	1.1918	1.2506	1.3330	1.3232	1.2497	1.2884	1.3130	1.3107
21	1.0000	0.9997	1.0074	1.0214	1.0630	1.0978	1.1533	1.1921	1.2066	1.1343	1.1946	1.2414	1.2884
22	1.0000	1.0193	1.0200	1.0235	1.0463	1.0678	1.1040	1.1473	1.1680	1.1251	1.1423	1.1531	1.1387
23	1.0000	1.0373	1.0891	1.1334	1.1749	1.2144	1.2345	1.2780	1.2555	1.2743	1.2762	1.3041	1.3459
24	1.0000	1.0199	1.0352	1.0454	1.0868	1.1149	1.1405	1.1708	1.1716	1.1524	1.1580	1.1721	1.2083
25	1.0000	1.0121	1.0267	1.0664	1.1234	1.1640	1.2365	1.3204	1.3881	1.4107	1.4653	1.5316	1.5613
26	1.0000	1.0197	1.0275	1.0182	1.0341	1.0421	1.0572	1.0822	1.0821	1.0506	1.0710	1.0576	1.0235
27	1.0000	1.0348	1.0822	1.1339	1.1913	1.2706	1.3766	1.5210	1.6085	1.5291	1.5968	1.6444	1.6741
28	1.0000	1.0294	1.0688	1.1001	1.1485	1.1946	1.2644	1.3524	1.3982	1.2871	1.3033	1.3126	1.2792
29	1.0000	1.0367	1.0648	1.0977	1.1335	1.1741	1.2219	1.2645	1.2757	1.2268	1.2244	1.2250	1.2049
30	1.0000	1.0126	1.0378	1.0620	1.1070	1.1420	1.1910	1.2305	1.2230	1.1615	1.2376	1.2739	1.2861
31	1.0000	1.0124	1.0143	1.0145	1.0391	1.0671	1.1071	1.1497	1.1746	1.1518	1.1858	1.2071	1.2197
32	1.0000	0.9430	1.0012	1.0539	1.1525	1.2494	1.3355	1.3978	1.4071	1.3391	1.4618	1.5900	1.6245
33	1.0000	1.0218	1.0453	1.0866	1.1211	1.1573	1.1892	1.2300	1.2205	1.1574	1.1766	1.1898	1.1912
34	1.0000	1.0095	1.0274	1.0561	1.0962	1.1329	1.1631	1.1840	1.1805	1.1474	1.1762	1.1979	1.2312

The OECD countries that exhibited the fastest rates of real GDP growth over the sample period were countries 5 (Chile), 27 (Slovak Republic), 32 (Turkey), 19 (Korea) and 25 (Poland) with growth rates equal to 168%, 167%, 162%, 159% and 156% respectively.

In order to obtain the country volume levels Q_n^t that match up with the price levels P_n^t in Table 1, the entries in the rows labeled 1-34 need to be multiplied by the country volume levels for 2000, the Q_n^{2000} for n = 1,...,34. These year 2000 levels are as follows: 706.89, 208.47, 252.54, 1076.58, 42094.99, 2269.70, 1293.96, 6.16, 132.19, 1439.60, 2047.50, 135.04, 13089.05, 683.75, 105.64, 506.17, 1198.29, 509860.00, 603236.00, 22.00, 6020.65, 417.96, 118.38, 1481.24, 744.38, 127.32, 31.18, 18.57, 629.91, 2265.45,

432.41, 166.66, 987.14, 10289.70. The units are in billions of year 2000 domestic currency units.

We turn now to the US dollar OECD data. Table A3 lists the country n, year t US dollar price levels for GDP, p_n^t, and Table A4 lists the corresponding volume levels, q_n^t. Note that $p_n^t q_n^t$ equals v_n^t, the year t value of country n's GDP in current US dollars.

Table A3: OECD Country Price Levels in US Dollars at Market Exchange Rates p_n^t

n	p_n^{2000}	p_n^{2001}	p_n^{2002}	p_n^{2003}	p_n^{2004}	p_n^{2005}	p_n^{2006}	p_n^{2007}	p_n^{2008}	p_n^{2009}	p_n^{2010}	p_n^{2011}	p_n^{2012}
1	1.00	0.9169	0.9906	1.2186	1.4347	1.5620	1.6157	1.8772	1.9756	1.8539	2.3124	2.6400	2.6330
2	1.00	0.9895	1.0535	1.2780	1.4296	1.4610	1.5016	1.6711	1.8194	1.7526	1.6950	1.8154	1.7066
3	1.00	0.9912	1.0634	1.3005	1.4613	1.4984	1.5469	1.7278	1.8888	1.8125	1.7635	1.8879	1.7786
4	1.00	0.9696	0.9674	1.1192	1.2436	1.3794	1.5129	1.6487	1.7278	1.5818	1.8069	1.9420	1.9554
5	1.00	0.8854	0.8479	0.8869	1.0812	1.2615	1.5002	1.5963	1.6039	1.5515	1.8559	2.0244	2.0486
6	1.00	1.0619	1.2664	1.4829	1.6934	1.8102	1.9295	2.2197	2.6892	2.4634	2.4199	2.5871	2.3765
7	1.00	0.9954	1.0736	1.3078	1.4715	1.5123	1.5575	1.7402	1.9368	1.8541	1.8429	1.9445	1.8429
8	1.00	1.0337	1.1386	1.4202	1.6320	1.7326	1.9024	2.3155	2.6095	2.4831	2.3747	2.5657	2.4502
9	1.00	1.0005	1.0657	1.2692	1.4031	1.4117	1.4361	1.6137	1.7780	1.7110	1.6368	1.7656	1.6792
10	1.00	0.9908	1.0652	1.3029	1.4574	1.4876	1.5327	1.7155	1.8827	1.7983	1.7309	1.8402	1.7268
11	1.00	0.9822	1.0478	1.2703	1.4125	1.4235	1.4404	1.5971	1.7226	1.6529	1.5921	1.6917	1.5865
12	1.00	0.9895	1.0761	1.3410	1.5189	1.5640	1.6159	1.8215	2.0415	1.9807	1.9098	2.0255	1.8578
13	1.00	1.0959	1.3204	1.6000	1.8628	1.9394	1.9041	2.3002	2.5835	2.2757	2.2671	2.4062	2.2182
14	1.00	0.8766	0.9841	1.1832	1.3254	1.5192	1.4834	1.7171	1.3982	1.0771	1.1647	1.2681	1.2090
15	1.00	1.0292	1.1421	1.4205	1.6000	1.6402	1.7095	1.8977	1.9729	1.7994	1.6892	1.7848	1.6607
16	1.00	0.9868	0.9113	0.9426	0.9590	0.9668	0.9920	1.0736	1.2492	1.1943	1.2713	1.3603	1.3044
17	1.00	0.9992	1.0846	1.3413	1.5109	1.5407	1.5808	1.7656	1.9375	1.8758	1.7953	1.9099	1.7958
18	1.00	0.8761	0.8360	0.8887	0.9394	0.9106	0.8533	0.8349	0.9391	1.0322	1.0765	1.1618	1.1518
19	1.00	0.9099	0.9692	1.0538	1.1297	1.2716	1.3620	1.4285	1.2396	1.1065	1.2664	1.3413	1.3322
20	1.00	0.9720	1.0436	1.3254	1.4849	1.5584	1.6785	1.8996	2.0413	1.9512	1.9933	2.1799	2.0759
21	1.00	1.0711	1.1086	1.0768	1.1208	1.2105	1.2924	1.3611	1.4213	1.2186	1.3555	1.4614	1.4280
22	1.00	1.0208	1.1147	1.3658	1.5137	1.5528	1.5941	1.7713	1.9362	1.8380	1.7667	1.8754	1.7561
23	1.00	0.9592	1.0573	1.3637	1.6070	1.7426	1.6592	1.9675	1.9317	1.7301	2.0910	2.3265	2.3661
24	1.00	0.9958	1.1017	1.2783	1.4218	1.6206	1.7716	1.9970	2.3021	1.9534	2.1601	2.4876	2.4573
25	1.00	1.0985	1.1270	1.1870	1.3137	1.5244	1.6130	1.8799	2.2269	1.7835	1.8719	1.9661	1.8319
26	1.00	1.0060	1.0976	1.3558	1.5285	1.5695	1.6273	1.8256	1.9848	1.8997	1.8224	1.9176	1.7671
27	1.00	0.9998	1.1079	1.4381	1.7354	1.8476	1.9866	2.4157	2.8724	2.7961	2.6789	2.8579	2.6745
28	1.00	0.9966	1.0833	1.3262	1.4744	1.4964	1.5414	1.7520	1.9527	1.9132	1.8046	1.9160	1.7754
29	1.00	1.0120	1.1107	1.3874	1.5880	1.6596	1.7434	1.9643	2.1522	2.0426	1.9490	2.0460	1.8908
30	1.00	0.9081	0.9780	1.1985	1.3229	1.3124	1.3551	1.5201	1.6077	1.4130	1.5132	1.7016	1.6472
31	1.00	1.0132	1.1038	1.2881	1.4066	1.4083	1.4294	1.5305	1.7434	1.7275	1.8085	2.1319	2.0211
32	1.00	0.7797	0.8713	1.0786	1.2765	1.4503	1.4914	1.7368	1.9472	1.7217	1.8764	1.8280	1.8204
33	1.00	0.9732	1.0382	1.1555	1.3264	1.3430	1.3980	1.5553	1.4745	1.2773	1.3063	1.3858	1.3892
34	1.00	1.0229	1.0386	1.0594	1.0884	1.1233	1.1579	1.1886	1.2118	1.2211	1.2359	1.2602	1.2822

Table A4: OECD Country GDP Volumes in US Dollar Units of Measurement q_n^t

q_n^{2000}	q_n^{2001}	q_n^{2002}	q_n^{2003}	q_n^{2004}	q_n^{2005}	q_n^{2006}	q_n^{2007}	q_n^{2008}	q_n^{2009}	q_n^{2010}	q_n^{2011}	q_n^{2012}
409.8	425.9	439.3	457.5	472.1	486.5	504.9	523.9	532.5	543.7	556.9	575.6	596.8
192.1	193.7	197.0	198.7	203.9	208.7	216.4	224.4	227.6	218.9	222.8	229.1	231.1
232.7	234.6	237.7	239.7	247.5	251.8	258.6	266.0	268.6	261.1	267.2	271.9	271.5
724.9	737.8	759.4	773.7	797.8	821.9	845.1	863.7	869.7	845.6	872.8	894.8	910.1
78.0	80.6	82.7	85.9	91.9	97.5	103.1	108.4	112.0	110.8	117.2	124.1	131.0
58.8	60.6	61.9	64.3	67.3	71.9	76.9	81.3	83.8	80.0	82.0	83.5	82.7
160.1	161.2	162.0	162.6	166.3	170.4	176.2	179.0	177.6	167.5	169.8	171.6	171
5.7	6.0	6.4	6.9	7.4	8.0	8.8	9.5	9.1	7.8	8.0	8.8	9.1
121.8	124.6	126.9	129.4	134.8	138.7	144.8	152.5	153	139.9	144.6	148.6	147.3
1326.3	1350.7	1363.2	1375.5	1410.5	1436.3	1471.7	1505.3	1504.1	1456.8	1481.9	1511.9	1512.1
1886.4	1915	1915.2	1908	1930.1	1943.3	2015.2	2081.1	2103.7	1995.4	2075.5	2144.7	2159.4
125.9	131.2	135.7	143.8	150.1	153.5	162.0	167.7	167.3	162.1	154.1	143.1	134.0
46.4	48.1	50.3	52.2	54.7	56.9	59.1	59.2	59.7	55.7	56.2	57.1	56.2
8.7	9.0	9.1	9.3	10.0	10.7	11.2	11.9	12.0	11.2	10.8	11.1	11.2

97.3	**102.2**	**107.7**	**111.7**	**116.4**	**123.5**	**130.3**	**136.8**	**133.8**	**125.3**	**124.0**	**126.6**	**126.8**
124.1	**123.9**	**123.8**	**125.7**	**131.8**	**138.3**	**146.3**	**154.9**	**161.3**	**163.1**	**171.2**	**179.1**	**184.8**
1104.0	**1124.6**	**1129.6**	**1129.1**	**1148.7**	**1159.4**	**1184.9**	**1204.8**	**1190.9**	**1125.4**	**1144.8**	**1150.3**	**1121.2**
4731.2	**4748.0**	**4761.8**	**4842**	**4956.3**	**5020.9**	**5105.9**	**5217.8**	**5163.5**	**4878.1**	**5105.0**	**5075.9**	**5175.2**
533.4	**554.6**	**594.2**	**610.9**	**639.1**	**664.4**	**698.8**	**734.5**	**751.4**	**753.8**	**801.4**	**830.9**	**847.9**
20.3	**20.8**	**21.6**	**22.0**	**22.9**	**24.2**	**25.3**	**27.0**	**26.8**	**25.3**	**26.1**	**26.6**	**26.6**
636.7	**636.5**	**641.4**	**650.4**	**676.8**	**699.0**	**734.3**	**759.0**	**768.3**	**722.2**	**760.7**	**790.5**	**820.4**
385.1	**392.5**	**392.8**	**394.1**	**402.9**	**411.2**	**425.1**	**441.8**	**449.8**	**433.3**	**439.9**	**444.0**	**438.5**
53.8	**55.8**	**58.6**	**61.0**	**63.2**	**65.3**	**66.4**	**68.7**	**67.5**	**68.5**	**68.6**	**70.1**	**72.4**
168.3	**171.6**	**174.2**	**175.9**	**182.9**	**187.6**	**191.9**	**197.0**	**197.2**	**193.9**	**194.9**	**197.2**	**203.3**
171.3	**173.3**	**175.8**	**182.6**	**192.4**	**199.4**	**211.8**	**226.1**	**237.7**	**241.6**	**251.0**	**262.3**	**267.4**
117.3	**119.6**	**120.5**	**119.4**	**121.3**	**122.2**	**124.0**	**126.9**	**126.9**	**123.2**	**125.6**	**124.1**	**120.1**
20.4	**21.1**	**22.1**	**23.1**	**24.3**	**25.9**	**28.1**	**31.0**	**32.8**	**31.2**	**32.6**	**33.6**	**34.2**
20.0	**20.6**	**21.4**	**22.0**	**22.9**	**23.9**	**25.3**	**27.0**	**27.9**	**25.7**	**26.0**	**26.2**	**25.6**
580.3	**601.6**	**617.9**	**637.0**	**657.8**	**681.4**	**709.1**	**733.8**	**740.4**	**712**	**710.6**	**710.9**	**699.2**
247.3	**250.4**	**256.6**	**262.6**	**273.7**	**282.4**	**294.5**	**304.3**	**302.4**	**287.2**	**306.0**	**315.0**	**318.0**
256.0	**259.2**	**259.7**	**259.8**	**266.0**	**273.2**	**283.5**	**294.4**	**300.7**	**294.9**	**303.6**	**309.1**	**312.3**
266.6	**251.4**	**266.9**	**280.9**	**307.2**	**333.0**	**356.0**	**372.6**	**375.1**	**357.0**	**389.7**	**423.8**	**433.0**
1493.6	**1526.2**	**1561.2**	**1622.9**	**1674.4**	**1728.5**	**1776.2**	**1837.0**	**1822.9**	**1728.6**	**1757.3**	**1777**	**1779.2**
10289.7	**10387.3**	**10571.8**	**10866.9**	**11279.6**	**11657.6**	**11968.5**	**12182.7**	**12147.3**	**11806.9**	**12102.9**	**12326.5**	**12669.0**

In order to save space, a column listing the country numbers has been dropped. We use the standard ordering of OECD countries in the above table (and in the following table).

Table A5: Annual Purchasing Power Parities PPP_n^t for OECD Countries 2000-2012, National Currencies per US Dollar

PPP_n^{00}	PPP_n^{01}	PPP_n^{02}	PPP_n^{03}	PPP_n^{04}	PPP_n^{05}	PPP_n^{06}	PPP_n^{07}	PPP_n^{08}	PPP_n^{09}	PPP_n^{10}	PPP_n^{11}	PPP_n^{12}
1.316	**1.330**	**1.336**	**1.351**	**1.363**	**1.388**	**1.408**	**1.423**	**1.479**	**1.437**	**1.498**	**1.493**	**1.458**
0.900	**0.917**	**0.896**	**0.885**	**0.874**	**0.886**	**0.857**	**0.868**	**0.852**	**0.849**	**0.845**	**0.846**	**0.831**
0.891	**0.886**	**0.865**	**0.879**	**0.897**	**0.900**	**0.884**	**0.887**	**0.874**	**0.863**	**0.864**	**0.867**	**0.849**
1.232	**1.218**	**1.229**	**1.226**	**1.231**	**1.214**	**1.208**	**1.211**	**1.234**	**1.199**	**1.218**	**1.233**	**1.227**
285.108	**289.501**	**296.891**	**307.807**	**321.76**	**333.69**	**320.257**	**323.512**	**339.271**	**350.588**	**347.852**	**334.241**	**334.206**
14.212	**14.222**	**14.319**	**14.034**	**14.291**	**14.316**	**14.053**	**13.945**	**14.262**	**13.977**	**14.243**	**13.899**	**13.700**
8.409	**8.468**	**8.302**	**8.537**	**8.404**	**8.59**	**8.336**	**8.235**	**8.012**	**7.877**	**7.821**	**7.857**	**7.736**
0.455	**0.477**	**0.477**	**0.481**	**0.486**	**0.502**	**0.521**	**0.555**	**0.549**	**0.527**	**0.532**	**0.541**	**0.550**
0.995	**1.012**	**1.003**	**1.011**	**0.975**	**0.977**	**0.951**	**0.941**	**0.918**	**0.908**	**0.925**	**0.938**	**0.929**
0.939	**0.919**	**0.905**	**0.938**	**0.940**	**0.923**	**0.904**	**0.893**	**0.882**	**0.866**	**0.869**	**0.866**	**0.857**
0.967	**0.955**	**0.942**	**0.917**	**0.897**	**0.867**	**0.838**	**0.831**	**0.812**	**0.814**	**0.811**	**0.803**	**0.789**
0.678	**0.671**	**0.660**	**0.689**	**0.696**	**0.714**	**0.700**	**0.719**	**0.701**	**0.701**	**0.713**	**0.714**	**0.678**
107.885	**110.652**	**114.88**	**120.516**	**126.307**	**128.594**	**128.637**	**131.336**	**129.429**	**126.256**	**129.005**	**130.345**	**128.453**
84.311	**88.930**	**91.342**	**94.484**	**94.248**	**99.078**	**107.307**	**113.108**	**117.421**	**125.692**	**136.066**	**139.737**	**140.967**
0.962	**0.993**	**1.004**	**1.014**	**1.006**	**1.01**	**0.985**	**0.958**	**0.952**	**0.897**	**0.853**	**0.836**	**0.818**
3.443	**3.426**	**3.463**	**3.629**	**3.535**	**3.717**	**3.836**	**3.720**	**3.867**	**3.947**	**3.943**	**3.885**	**3.942**
0.817	**0.808**	**0.845**	**0.854**	**0.873**	**0.867**	**0.834**	**0.817**	**0.789**	**0.784**	**0.800**	**0.796**	**0.776**
155.113	**149.857**	**143.774**	**139.824**	**134.161**	**129.552**	**124.864**	**120.216**	**116.846**	**116.348**	**112.418**	**108.812**	**105.972**
746.206	**757.829**	**769.772**	**794.282**	**795.998**	**788.92**	**774.815**	**768.65**	**785.718**	**811.664**	**829.897**	**833.034**	**826.191**
0.940	**0.948**	**0.934**	**0.942**	**0.923**	**0.953**	**0.915**	**0.925**	**0.906**	**0.912**	**0.929**	**0.926**	**0.915**
6.099	**6.311**	**6.554**	**6.815**	**7.217**	**7.127**	**7.181**	**7.370**	**7.470**	**7.409**	**7.604**	**7.532**	**7.668**
0.893	**0.906**	**0.902**	**0.927**	**0.909**	**0.896**	**0.869**	**0.857**	**0.842**	**0.846**	**0.850**	**0.843**	**0.829**
1.442	**1.473**	**1.469**	**1.497**	**1.510**	**1.535**	**1.486**	**1.506**	**1.491**	**1.454**	**1.492**	**1.481**	**1.446**
9.129	**9.180**	**9.111**	**9.112**	**8.988**	**8.896**	**8.701**	**8.776**	**8.752**	**9.006**	**9.058**	**9.095**	**8.824**
1.841	**1.861**	**1.829**	**1.841**	**1.861**	**1.869**	**1.846**	**1.843**	**1.857**	**1.875**	**1.852**	**1.877**	**1.868**
0.700	**0.706**	**0.708**	**0.706**	**0.716**	**0.684**	**0.663**	**0.660**	**0.649**	**0.637**	**0.636**	**0.633**	**0.618**
0.526	**0.522**	**0.528**	**0.555**	**0.573**	**0.566**	**0.556**	**0.546**	**0.533**	**0.514**	**0.523**	**0.531**	**0.522**
0.532	**0.565**	**0.588**	**0.615**	**0.611**	**0.612**	**0.608**	**0.629**	**0.634**	**0.648**	**0.652**	**0.644**	**0.625**
0.734	**0.740**	**0.733**	**0.753**	**0.759**	**0.765**	**0.736**	**0.728**	**0.720**	**0.713**	**0.721**	**0.718**	**0.695**
9.135	**9.349**	**9.352**	**9.335**	**9.105**	**9.378**	**9.094**	**8.886**	**8.773**	**8.965**	**9.067**	**8.935**	**8.668**
1.851	**1.840**	**1.771**	**1.776**	**1.754**	**1.743**	**1.660**	**1.601**	**1.549**	**1.527**	**1.506**	**1.448**	**1.389**
0.283	**0.428**	**0.613**	**0.773**	**0.812**	**0.831**	**0.848**	**0.864**	**0.890**	**0.917**	**0.954**	**1.030**	**1.044**
0.636	**0.627**	**0.628**	**0.641**	**0.633**	**0.636**	**0.627**	**0.645**	**0.651**	**0.660**	**0.667**	**0.679**	**0.661**
1.000	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**

References

Balk, B.M.(2008), *Price and Quantity Index Numbers*, New York: Cambridge University Press.

Rao, D.S.P., A. Rambaldi and B.M. Balk (2013), "Purchasing Power Parities, Price Levels and Measures of Regional and Global Inflation",

Diewert, W.E. (1978), "Superlative Index Numbers and Consistency in Aggregation", *Econometrica* 46, 883-900.

Diewert, W.E. (1992), "Fisher Ideal Output, Input and Productivity Indexes Revisited", *Journal of Productivity Analysis* 3, 211-248.

Diewert, W.E. (1993), "The Early History of Price Index Research", pp. 33-65 in *Essays in Index Number Theory,* Volume 1, W.E. Diewert and A.O. Nakamura (eds.), Amsterdam: North-Holland.

Diewert, W.E. (1997), "Commentary on Mathew D. Shapiro and David W. Wilcox: Alternative Strategies for Aggregating Prices in the CPI", *The Federal Reserve Bank of St. Louis Review*, Vol. 79:3, 127-137.

Diewert, W.E. (2009), "Similarity Indexes and Criteria for Spatial Linking", pp. 183-216 in *Purchasing Power Parities of Currencies: Recent Advances in Methods and Applications*, D.S. Prasada Rao (ed.), Cheltenham UK: Edward Elgar.

Eurostat (2012), *Eurostat-OECD Methodological Manual on Purchasing Power Parities*, 2012 Edition, Luxembourg: Publications Office of the European Union.

Feenstra, R.C., A. Heston, M.P. Timmer and H. Deng (2009), "Estimating Real Production and Expenditures Across Countries: A Proposal for Improving the Penn World Tables", *Review of Economics and Statistics*, 91:1, 201-212.

Feenstra, R.C., R. Inklaar and M.P. Timmer (2013), "The Next Generation of the Penn World Table", NBER Working Paper No. 19255, Cambridge MA: NBER.

Fisher, I. (1911), *The Purchasing Power of Money*, London: Macmillan.

Fisher, I. (1922), *The Making of Index Numbers*, Boston: Houghton-Mifflin.

Hill, R.J. (2001), "Measuring Inflation and Growth Using Spanning Trees", *International Economic Review* 42, 167-185.

Hill, R.J. (2004), "Constructing Price Indexes Across Space and Time: The Case of the European Union", *American Economic Review* 94, 1379-1410.

Hill, R.J. (2009), "Comparing Per Capita Income Levels Across Countries Using Spanning Trees: Robustness, Prior Restrictions, Hybrids and Hierarchies", pp. 217-244 in *Purchasing Power Parities of Currencies: Recent Advances in Methods and Applications*, D.S. Prasada Rao (ed.), Cheltenham UK: Edward Elgar.

Hill, T.P. (1988), "Recent Developments in Index Number Theory and Practice", *OECD Economic Studies* 10, 123-148.

Hill, T.P. (1993), "Price and Volume Measures", pp. 379-406 in *System of National Accounts 1993*, Eurostat, IMF, OECD, UN and World Bank, Luxembourg, Washington, D.C., Paris, New York, and Washington, D.C.

Hill, R.J. and K.J. Fox (1997), "Splicing Index Numbers", *Journal of Business and Economic Statistics* 15:3 (Jul., 1997), 387-389.

Ivancic, L., W.E. Diewert and K.J. Fox (2011), "Scanner Data, Time Aggregation and the Construction of Price Indexes", *Journal of Econometrics* 161, 24-35.

Lehr, J. (1885), *Beitrage zur Statistik der Preise*, Frankfurt: J.D. Sauerlander.

Marshall, A. (1887), "Remedies for Fluctuations of General Prices', *Contemporary Review* 51, 355-375.

McCarthy, P. (2013), "Extrapolating PPPs and Comparing ICP Benchmark Results", pp. 473-505 in *Measuring the Real Size of the World Economy*, Washington D.C.: World Bank.

OECD (2001), "Introduction of the Euro in OECD Statistics", Statistics Directorate, National Accounts, STD/NA(2001)8, Paris: OECD, September 14.

OECD (2014), "Methodological Notes: Compilation of G20 Consumer Price Index", Paris: OECD.

Rao, D.S.P., A. Rambaldi and B.M. Balk (2013), "Purchasing Power Parities, Price Levels and Measures of Regional and Global Inflation", preliminary paper presented at the Economic Measurement Group Workshop 2013, November 28, University of New South Wales, Sydney, Australia.

Rao, D.P., A.N. Rambaldi, and H.E. Doran (2010), "Extrapolation of Purchasing Power Parities using Multiple Benchmarks and Auxiliary Information: A New Approach", *The Review of Income and Wealth* 56: Supplement, S59-S98.

Rao, D.S.P., A.N. Rambaldi, and H.E. Doran (2011), "An Econometric Approach to the Construction of Complete Panels of Purchasing Power Parities: Analytical

Properties and Empirical Results", School of Economics, The University of Queensland, St Lucia, QLD 4072, Australia.

Summers, R. and A. Heston (1991), "The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950-1988", *Quarterly Journal of Economics* 106, 327-368.

Szulc, B.J. (1983), "Linking Price Index Numbers," pp. 537-566 in *Price Level Measurement*, W.E. Diewert and C. Montmarquette (eds.), Ottawa: Statistics Canada.

World Bank (2013), *Measuring the Real Size of the World Economy*, Washington D.C.: World Bank.