Discussion of "Information Aggregation in a DSGE Model" by Tarek Hassan and Thomas Mertens

Guido Lorenzoni

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Agenda

- Models in which asset prices aggregate information (e.g., Grossman, Hellwig)
- Can we study their quantitative implications in dynamic general equilibrium models?
- Problem interesting both for its asset pricing side and for its business cycle side
- Computational challenge: once we move away from CARA-normal world, how do we handle signal-extraction problem?
- Paper proposes a new approach to deal with computational challenge and obtains some results

A simple model without CARA

- Two period economy, OLG structure
- Continuum of young consumers $i \in [0, 1]$ at date 1
- Supply K is random (same role as noise traders)

1.

Young work

Young observe private signal and price, buy k_i

Old sell K and consume

2.

Productivity A realized

Young (now old) consume Ak_i

Optimization

- Productivity

$$\log A = \eta$$

- Private signals

$$s_i = \eta + \nu_i$$

- Conjecture price monotone function of

$$q = \eta + \psi \tau$$

- So individual demand is

$$K_d(\beta_1 s + \beta_2 q, Q) = \arg\max_k E_i[u(Ak) - v(Qk) | s, q]$$

Market clearing

- Across agents s is distributed normally with mean η, so given η and q we can compute aggregate demand

$$\overline{K}_d(\beta_1\eta + \beta_2 q, Q) = \int_{-\infty}^{\infty} K_d(\beta_1\eta + \beta_2 q + \beta_1 \nu, Q)\, d\Phi(\nu)$$

- Market clearing requires

$$\overline{K}_d(\beta_1\eta + \beta_2 q, Q) = K$$

- Can we make assumptions on random supply K which ensure our conjecture is correct?
- Yes, by reverse engineering

Reverse engineering

- Suppose all shock distributions are given, ψ (in $q = \eta + \psi\tau$) is given and

$$Q = H(q)$$

for some given strictly monotone function H

- Then we can find a function $T(q,\tau)$ such that if $K = T(q,\tau)$, Q is the equilibrium price in our model
- Just choose

$$T(q,\tau) = \overline{K}_d(\beta_1(q - \psi\tau) + \beta_2 q, H(q))$$

- That's too much freedom...

Restrictions

- ...so the paper imposes restrictions
- Represent T and H as Taylor series

$$T(q,\tau) = T_0 + T_q q + T_\tau \tau + \frac{1}{2}\left(T_{qq}q^2 + 2T_{q\tau}q\tau + T_{\tau\tau}\tau^2\right) + ...$$

- Impose restrictions on coefficients $T_0, T_q, ...$
- Use them to compute $H_0, H_1, H_2, ...$ and ψ
- Setting $T_0 = 0$ gives H_0
- Setting $T_q = 0$ and $T_\tau = ...$ gives H_1 and ψ (here βsare involved too)
- 2nd order and up we only have one unknown H_j, so I guess we can only impose one restriction

A simple case I can solve by hand

- $u(c)$ CRRA with coeff. γ
- $v(n)$ linear
- Then demand for capital is

$$k_i = \left\{ E_i \left[A^{1-\gamma} \right] / Q \right\}^{1/\gamma}$$

- If we assume

$$K = T(q, \tau) = e^{-T_\tau \tau}$$

  we can find equilibrium in closed form
- In particular

$$\psi \beta_1 = \frac{\gamma}{1-\gamma} T_\tau$$

  where

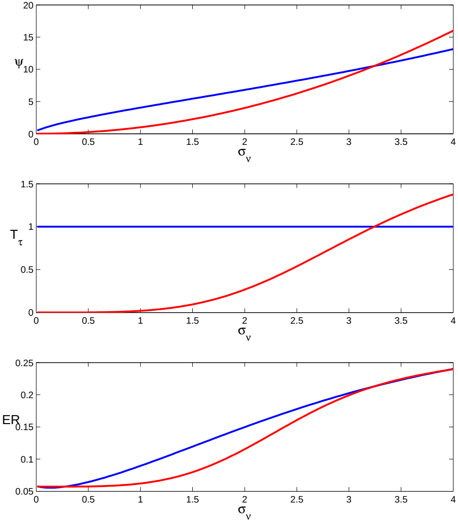
$$\beta_1 = \frac{1/\sigma_v^2}{1/\sigma_\eta^2 + 1/\sigma_v^2 + 1/(\psi^2 \sigma_\tau^2)}$$

Comment 1

- There is a mapping between:
  - T_τ, first-order response of noise trading to shock τ
  - and ψ, price signal response to τ
- My natural choice would be to set $T_\tau = 1$ and find ψ endogenously
- The paper does the opposite, sets $\psi = \sigma_v^2 / \sigma_\eta^2$ and derives T_τ endogenously

Comment 1 (continued)

- Does it matter? Yes



Comment 2

- Where is the risk-free rate coming from?
- I think paper looks at interim period before private info and Q observed
- That allows authors to avoid trading of bonds after private info received
- Avoids information revelation through the interest rate
- Does it matter? Yes. E.g., when $\gamma = 1$ no info revelation through Q, but info revelation through interest rate

Limitations

- Limitations of this approach is endogeneity of higher order terms like $T_{\tau\tau}$
- These terms may matter for asset pricing purposes
- The choice of restrictions is not obvious and may matter too
- Other important limitation is all info is shared at end of each period t (Rondina-Walker show this can matter)
- But do we have alternatives?

Bounded rationality

- Make assumptions on T
- Given β's solve for optimal demand

$$\overline{K}_d(\beta_1\eta + \beta_2 q, Q) = T(\tau)$$

- Find $Q(\eta, \tau)$
- Replace expectation with linear projection

$$P[\eta | Q, s] = \beta_1 s + \beta_2 \log Q$$

  (only requires covariances, does not require normality of $\log Q$, normality of v_i can still be used for aggregation)
- Look for fixed point in β_1, β_2
- Can be extended to environments where agents info is not perfectly revealed after one period
- Higher order terms could be added to the projection step