The Empirical Merton Model

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Abstract

Although the Merton model of corporate debt as equivalent to safe debt minus a put option on the firm's assets fails to match observed credit spreads, we show that portfolios of long Treasuries and short *traded* put options ("pseudo bonds") closely match the properties of traded corporate bonds. Pseudo bonds display a credit spread puzzle that is stronger at short horizons, unexplained by standard risk factors, and unlikely to be solely due to illiquidity. We illustrate a novel, model-free benchmarking methodology to run data-based counterfactuals, with applications to credit spread biases, the impact of asset uncertainty, and bank-related rollover risk.

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1. Introduction

The Merton model for the valuation of defaultable corporate debt is the backbone of modern corporate bond valuation. The main insight of Merton (1974) is that the debt issued by a firm is economically equivalent to risk-free debt minus a put option on the assets owned by the firm. Despite its theoretical appeal, the ubiquitous Merton model produces implied credit spreads that are far smaller than estimates of credit spreads derived from actual, traded corporate bonds. A significant literature has emerged over the last several decades that aims to explain this "credit spread puzzle" and the sources of differences between theoretical credit spreads implied by the Merton model and spreads on actual traded bonds. Even with the insights from that literature, the practical applicability of the Merton model – in its original lognormal form – remains limited.

In this paper we propose a model-free methodology to provide empirical content to Merton's conceptual insight. In particular, we rely on the observed prices of traded options to extract the empirical properties of "pseudo bonds" issued by hypothetical firms whose assets are comprised solely of the assets underlying the options on which we rely, and we show how our option-based methodology can be used to provide empirically-based, counterfactual experiments in controlled environments. We begin by analyzing a pseudo firm whose only asset is the S&P 500 ("SPX") index. Pseudo bonds issued by this firm thus consist of risk-free Treasuries and short SPX put options. Because both Treasuries and SPX put prices are observable, we can compute observed market values of the pseudo bonds and analyze their properties empirically.

The implications of our analysis of SPX pseudo bonds are striking. First, as is the case for actual corporate bonds, the credit spread puzzle is also prominent in pseudo bond credit spreads. For example, the credit spreads of two-year pseudo bonds corresponding to the default probabilities for Aaa/Aa and A/Baa bonds are 0.54% and 1.31%, respectively. Those spreads are very similar to the average credit spreads observed for actual Aaa/Aa and A/Baa corporate bonds – *i.e.,* 0.68% and 1.33%, respectively. For high-yield ("HY") debt, pseudo bonds have relatively large credit spreads – *i.e.*, between 2.37% for Ba-rated bonds and 5.17% for Caa-rated bonds. Although these credit spreads are smaller than spreads on real corporate bonds (3.97% for Ba-rated bonds and 12.13% for Caa- rated bonds, respectively), they are nevertheless far greater than those implied by the lognormal Merton model, which are only 0.30% for Ba-rated bonds and 2.49% for Caa-rated bonds.

Second, like actual corporate bonds and in contrast with the lognormal Merton model, monthly returns on portfolios of pseudo bonds exhibit different Sharpe ratios across credit ratings, with the highest Sharpe ratios corresponding to bonds with intermediate credit ratings. The lognormal Merton model, moreover, implies that excess returns on bonds can be fully explained by excess returns on the firm's assets or equity. In contrast, our pseudo bonds show that a substantial alpha emerges both when we regress excess pseudo bond returns on excess returns of the fictitious firm's traded assets (*i.e.,* the SPX) and on the firm's pseudo equity (*i.e.,* traded call options on the SPX). Those estimated alphas are consistent with a model in which asset dynamics exhibit leptokurtic returns and in which investors demand a risk premium to compensate for the resulting tail risk. Notably, very similar results hold for actual corporate bonds.

Third, our empirical results hold not only for medium-term bonds (two years to maturity in our implementation) but also for short-term pseudo bonds. For example, investment-grade ("IG") pseudo bonds with 30 and 91 days to maturity have average credit spreads of 0.77% and 0.64%, respectively, as compared to average credit spreads of 0.62% and 0.60% for actual IG bonds and zero spreads implied by the lognormal Merton model. This result is especially important because even the majority of extensions to the original Merton model – not to mention the original model itself – typically cannot explain observed short-term credit spreads.

The explanation for the credit spread puzzle in our data is related to the equally notorious "put option overpricing puzzle" – *i.e.,* the well-established result in the equity options literature that put options are relatively overpriced *vis-a-vis* the theoretical prices implied by the Black-Scholes formula and lognormal distribution. The credit spread puzzle thus is plausibly due to the additional insurance premium that investors require to hold securities that are subject to tail risk.

Our empirical findings, moreover, suggest that the credit spread puzzle is unlikely to be solely attributable to theories of corporate behavior such as optimal default (*e.g.,* Leland and Toft (1996)), agency costs (*e.g.,* Leland (1998), Gamba, Aranda, and Saretto (2013)), strategic default (*e.g.,* Anderson and Sundaresan (1996)), asymmetric information, uncertainty and learning (*e.g.,* Duffie and Lando (2001) and David (2008)), corporate investment behavior (*e.g.,* Kuehn and Schmid (2014)), and the like. The reason is that our firm is a very simple one in which the asset value is observable, information is symmetric, managerial frictions do not exist (because there is nothing to be managed), and the leverage and default boundary are set mechanically. Yet, our pseudo bonds display properties that are surprisingly close – qualitatively and quantitatively – to those of real corporate bonds. Rather, our results provide an indirect argument that the underlying source of the large credit spread should be investigated in the dynamics of risk or investors' risk preferences (as in the habit models of Chen, Collin-Dufresne, and Goldstein (2009) or the long-run-risk models of Bhamra, Kuehn, and Strebulaev (2010) and Chen (2012)), as discount rate shocks simultaneously affect the market value of assets and the discount rate applied to value bonds.

One potential limitation of our analysis is that our results could be driven by the fact that our pseudo firm is highly systematic in the sense that its assets are the overall market. We thus extend our analysis of an SPX-only pseudo firm to form new pseudo firms whose assets are the individual equities of SPX constituent firms. For instance, a hypothetical firm may purchase Apple shares and finance it by issuing a zero-coupon bond and equity. The value of the zero-coupon bond is then equal to Treasuries minus Apple put options, and we can carry out a similar empirical analysis as the one described above for the SPX-based pseudo firm. Our results using individual options confirm the results we obtain using SPX options – *viz.*, credit spreads on individual fictitious firms' pseudo bonds are very high compared to their default probabilities and are similar to observed spreads on real corporate bonds. Also as is the case for actual corporate bonds, a strong alpha exists when we try to explain pseudo bond excess returns using the excess returns on our firms' pseudo equities.

We undertake a number of additional tests to investigate the similarity between pseudo bonds and real corporate bonds in more detail. In particular, we show that the large excess returns on pseudo bonds cannot be explained away by a host of risk factors, including traded risk factors to reflect term premium, corporate default, volatility, aggregate market liquidity, and tail risk. We do find, however, that pseudo bond returns load positively on the market, term premium, and corporate default factors, and load negatively on the volatility factor. Interestingly, real corporate bond returns' alphas are also not explained by the same factors, and we find that they also load positively on the term premium and corporate default factors, thereby confirming a strong similarity in the dynamic behavior of pseudo bonds and corporate bonds.

Our methodology also allows us to assess the impact of transactional liquidity on credit spreads. We compare the transactional liquidity of pseudo bonds to real corporate bonds by using the Roll (1984) bid-ask bounce measure of transactional illiquidity (see Bao, Pan, and Wang (2011)), which indicates that real corporate bonds are significantly less liquid than our pseudo bonds – especially pseudo bonds based on the SPX portfolio. This finding suggests that the credit spread puzzle is not likely to be solely (or even primarily) driven by the illiquidity of corporate bonds, given that pseudo bonds are more liquid than corporate bonds and yet still exhibit large credit spreads. By comparing the credit spreads of pseudo bonds to real corporate bonds, moreover, we provide a concrete estimate of the transactional illiquidity component of corporate bond spreads.

In the same spirit as the original lognormal Merton model, our empirical Merton model can be used as a benchmark to study counterfactual experiments that are difficult to implement in the real world. We provide examples of three such experiments. Our first experiment concerns the potential bias that may be introduced in average credit spreads and average returns by infrequent revisions in credit ratings. We show that if we assign credit ratings at the quarterly, semi-annual, or annual frequencies, average credit spreads for highly rated bonds do not change much, whereas average spreads on lower-rated bonds increase by as much as 60%. In contrast, average pseudo bond excess returns fall (by as much as 50% for lower-rated bonds) as the frequency of credit rating assignments declines. Different convexity effects result in varying impacts on average bond yields and returns.

As a second application of the empirical Merton model, we investigate the impact of asset value uncertainty on credit spreads. This relation is typically hard to estimate using real corporate bonds given the endogeneity of credit ratings (*i.e.*, firms with more uncertain assets should have lower credit ratings) and the difficulty of measuring the uncertainty of underlying asset values (which are generally unobservable). Our methodology overcomes both hurdles, and we find that, even taking into account the endogeneity of credit ratings, higher uncertainty translates into higher credit spreads and lower leverage. The impact of uncertainty on credit spreads is large, and similar in magnitude to the differential *across* credit ratings. Indeed, our empirical exercise demonstrates significant heterogeneity across pseudo bonds even conditioning on the same credit rating.

As a final application of our model, we examine the rollover risk and capital requirement of a hypothetical bank that extends loans to groups of individual pseudo firms. Because the pseudo bank has an asset portfolio comprised solely of a portfolio of pseudo bonds, we use empirical returns on individual firms' pseudo bonds to compute the empirical distribution of the assets of our pseudo bank. Assuming that the bank finances the purchase of those bonds by issuing equity and only short-term debt, we study the rollover risk of the pseudo bank and find the minimum capital required to avoid a default. Our empirical resuts suggest that common shocks to the individual firms' assets are amplified by the leveraged nature of loans, leading to negatively skewed and leptokurtic return distributions of our pseudo bank's assets that require higher levels of capital to support than would be needed for a loan portfolio with closer to normally distributed returns.

Our paper is clearly related to the large literature that sprang from the basic insight of Merton (1974). We do not attempt an exhaustive survey here, but instead refer readers to Lando (2004), Jarrow (2009) and Sundaresan (2013).[1] In addition, Huang and Huang

[1] In addition to the academic literature, numerous variants of the Merton mode are used in the industry

(2012) discuss the deficiencies of the Merton model and elaborate on the credit spread puzzle by showing that numerous structural models calibrated to match true default probabilities generate credit spreads that are still too small compared to the data. Most of these models have implications only for very long-term debt and do not explain short-term credit spreads. High short-term credit spreads are instead obtained by Zhou (2001) in a model in which asset values can jump, and by Duffie and Lando (2001) in a model of optimal default and uncertainty about the true value of assets. The approaches of all of these papers, however, are different from ours. We do not use any parametric model, but instead go straight to the data to evaluate the empirical relevance of Merton's insight.

A small number of papers link options to credit spreads. Cremers, Driessen, and Maenhout (2008) propose a structural jump-diffusion model for asset values for each firm in the S&P 100 and estimate the jump risk premium from S&P 100 index options. The calibrated model that takes into account the jump risk increases the credit spread to levels comparable to the data. Carr and Wu (2011) show theoretically and empirically that deep out-of-the-money put options are related to credit default swap spreads. The results in these papers are consistent with our empirical results, but our approach differs as we directly test the empirical implications of the Merton model using traded options. Finally, our approach is related to Coval et al. (2012) who study the valuation of collateralized debt obligations ("CDOs") and use traded SPX options as the basis for measuring the credit spread on put spreads (*i.e.*, long-short positions in put options with different strike prices that resemble tranches of CDOs). They show that the credit spreads in their SPX-based tranches are smaller than the spreads on corresponding CDO tranches. Although we also use options and the insight from the Merton model to study bonds, we focus on the empirical performance of the Merton model itself and in fact show that credit spreads from the empirical Merton model are very much in line with observed corporate credit spreads.[2]

The paper is organized as follows. Section 2. reviews the lognormal Merton model. Section 3. describes the empirical Merton model. Section 4. describes the data, as well as our estimation of *ex ante* default probabilities and *ex post* default frequencies of pseudo bonds. Section 5. contains our main empirical results, and Section 6. digs deeper into their sources. Section 7. offers some additional applications of our methodology, and Section 8. concludes. Several Appendices contain proofs and supplemental results.

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and by practitioners to evaluate the credit risk of individual firms (see e.g. Moody's KMV model) or of portfolios of credits (see e.g. Riskmetrics CreditMetrics).

[2]Our paper is also related to the literature that compares corporate bonds to "synthetic" corporate bonds, as given by risk free bonds plus credit default swaps (*e.g.* Duffie (1999), Longstaff, Mithal and Neis (2005)). Such synthetic bonds, however, do not facilitate the same kind of analysis of the Merton model that we undertake here using options on the underlying assets of the firm issuing the corporate debt.

2. The Merton Model

The original lognormal Merton (1974) model assumes that the market value of the assets of the firm A_t follows a lognormal process with mean drift rate μ_A and volatility σ_A:

$$dA_t = \mu_A A_t dt + \sigma_A A_t dW_{A,t} \tag{1}$$

where $dW_{A,t}$ is a Brownian motion. At time t, the firm issues a zero-coupon bond with face value K and maturity T. At maturity, if the assets of the firm exceed the face value of its debt ($A_T > K$), the firm can pay its debt in full – *i.e.*, debt holders receive K. If instead $A_T < K$, the firm defaults and debt holders receive A_T. The payoff to debt holders at T thus is

$$CF_T = K - \max(K - A_T, 0) \tag{2}$$

and the value of debt today $B_t(T, K)$ is given by

$$B_t(T, K) = KZ_t(T) - P_t(T, K) \tag{3}$$

where $Z_t(T)$ is the price of a zero-coupon bond at t with maturity T, and $P_t(T, K)$ is the price of a European put option at t with maturity T and strike price K. From the assumptions about A_t, the value of the put option $P_t(T, K)$ can be computed and the bond prices in equation (3) analyzed.[3] The corporate bond yield under the Merton model is given by

$$y_t(T, K) = \frac{1}{T-t} \log(K/B_t(T, K))$$

The following proposition is useful to frame some of our later discussion:

Proposition 1. Under the asset dynamics in equation (1), the bond price $B_t(T, K)$ in expression (3) has the following properties:

(a) The credit spread $y - r$ is positively related to leverage (K/A) and asset volatility (σ_A);

(b) The bond's excess return follows the process

$$\frac{dB_t}{B_t} = \mu_B \, dt + \sigma_B \, dW_t$$

where the expected excess return $\mu_B - r$ and volatility σ_B are given by

$$\mu_B - r = \beta (\mu_A - r); \text{ and } \sigma_B = \beta \sigma_A \tag{4}$$

with $\beta = \frac{Cov(dB/B, dA/A)}{\sigma_A^2} > 0$;

[3]The dynamics of assets in (1) is only convenient inasmuch as it provides a closed-form solution for the value of the put option in equation (3).

(c) The bond's expected excess return can be equivalently written as

$$\mu_B - r = \beta_E \left(\mu_E - r\right) \tag{5}$$

with $\beta_E = \frac{Cov(dB/B, dE/E)}{\sigma_E^2} > 0$.

(d) The bond's Sharpe ratio is equal to the Sharpe ratio of the firm's underlying assets:

$$\frac{\mu_B - r}{\sigma_B} = \frac{\mu_A - r}{\sigma_A}$$

Proof: See Appendix A.

Note, in particular, that in the lognormal Merton model the bond inherits the properties of expected excess returns from the firm's underlying assets through its beta β, and that the Sharpe ratio of corporate bonds is the same as for the firm's underlying assets. The Merton model thus implies that the Sharpe ratio for the firm's debt is independent of the bond's maturity or face value. Expression (5) for the bond's excess returns, moreover, is often convenient because, in analyzing real corporate bonds, we cannot observe the value of the firm's assets but do observe the value of its equity. For such securities, (5) thus has an empirical counterpart.[4]

Much of the literature that has expanded the original lognormal Merton model has focused on generalizing the asset dynamics in equation (1) – *e.g.*, by adding a jump process, incorporating stochastic volatility, stochastic interest rates, and endogenous default, allowing a firm to experience insolvency prior to maturity, etc. In this paper, we make no assumptions about A_t and instead use U.S. Treasuries and traded options to analyze the properties of bonds directly. In Appendix B, we discuss one specific modification of the Merton model in which the market value of the firm's assets A_t follows a jump-diffusion process with stochastic volatility. Although we do not estimate this model, the discussion and a related Proposition 2 in Appendix B shed light on some of our empirical results.

3. The Empirical Merton Model

Consider a hypothetical firm i that finances the purchase of its assets by issuing equity and zero-coupon debt. The firm is passive and engages in no discretionary investment or financing

[4]Note in this connection that we are *not* assuming that the CAPM has to hold under the lognormal Merton model. Indeed, under process (1) the normalized shock $dW_{A,t}$ could itself load on several pricing factors, which then would affect the level of the asset's expected return μ_A.

decisions. The only assets purchased by the firm are shares of an individual stock or shares of the portfolio underlying a traded index, such as the SPX. For illustrative purposes, we first consider the empirical Merton model when the firm simply purchases the SPX index. We return to the case of individual stocks in Section 5.4.

Let $K_{i,t}$ denote the face value of debt issued by the firm at time t, and let $t+\tau$ be the debt maturity. Using expression (3), the value of a τ-period zero-coupon defaultable bond is given by risk-free debt minus a put option on the assets of the firm. Because the firm's assets are given by the SPX portfolio, the put option is simply an option on the SPX index, which is observable. Thus, the value of debt is given by:

$$\widehat{B}_t\left(t+\tau, K_{i,t}\right) = K_{i,t}\widehat{Z}_t(t+\tau) - \widehat{P}_t(t+\tau, K_{i,t}) \tag{6}$$

where a "hat" indicates that the price is directly observable. In other words, we rely on the data and not on a parametric model for the values of variables with hats. We then compute the empirical properties of our pseudo bonds $\widehat{B}_t(t+\tau, K_{i,t})$ directly from observable option and U.S. Treasury prices. We refer to the ratio $L_{i,t} = K_{i,t}/A_t$ as the firm's leverage ratio, given by the face value of debt divided by the market value of assets.

Much of the existing literature on the credit spread puzzle categorizes corporate bonds by credit ratings. To compare our pseudo bonds to real corporate bonds and for consistency with the existing literature, we also assign a credit rating to each pseudo bond according to its *ex ante* probability of default.[5] At each time t and for each bond with maturity τ and face value $K_{i,t}$ we want to compute

$$p_t(L_{i,t}) = \Pr\left[A_{t+\tau} < K_{i,t}\,|\mathcal{F}_t\right] \tag{7}$$

where $\mathcal{F}_t$ denotes the information available at time t.[6] To avoid making explicit distributional assumptions about asset returns and to keep our approach as model-free as possible, we use the empirical distribution of the underlying assets to compute $p_t(L_{i,t})$. Still, we want to take into account time-varying market conditions, which may have a substantial impact on default probabilities for a given current leverage ratio $L_{i,t}$. As such, we first assume that the log asset value evolves as follows:

$$\ln A_{t+\tau} = \ln A_t + \mu_{t,\tau} - \frac{1}{2}\sigma_{t,\tau}^2 + \sigma_t \varepsilon_{t+\tau} \tag{8}$$

[5] We use nomenclature from Moody's Investors Service to describe the credit ratings we assign to our pseudo bonds. Nevertheless, our credit ratings are not intended to match the ratings that actually would be assigned by Moody's or any other rating agency to such bonds (if they existed) based on their own criteria. We rely solely on the methodology described herein – and not rating agency criteria – for this mapping exercise.

[6] Under the assumptions of the lognormal Merton model, the default probability (7) can be readily computed from the normal cumulative distribution as $p_t^M(L_{i,t}) = N(-d_2)$ where $d_2 = \frac{-\ln(L_{i,t})+(\mu_A-\frac{1}{2}\sigma_A^2)\tau}{\sigma_A\sqrt{\tau}}$.

where $\varepsilon_{t+\tau}$ are standardized unexpected returns that have an unknown probability distribution. Because we do not impose any distributional assumption on $\varepsilon_{t+\tau}$, this is just a statement that $\ln A_{t+\tau}$ has some expected components and a scaling parameter $\sigma_{t,\tau}$. We estimate $\mu_{t,\tau}$ by running return forecasting regressions (excluding dividends) using the dividend-price ratio for τ horizons, and $\sigma_{t,\tau}$ by fitting a GARCH(1,1) process using monthly asset returns.[7] Given estimates of $\mu_{t,\tau}$ and $\sigma_{t,\tau}$, we collect the (overlapping) history of shocks

$$\varepsilon_{t+\tau} = \frac{\ln\left(A_{t+\tau}/A_t\right) - \left(\mu_{t,\tau} - \frac{1}{2}\sigma_{t,\tau}^2\right)}{\sigma_{t,\tau}}$$

and use the empirical distributions of these shocks to compute empirical default probabilities for each leverage ratio $L_{i,t}$ at any given time t.

In particular, we rewrite the probability $p_t(L_{i,t})$ in (7) as follows:

$$p_t(L_{i,t}) = \Pr\left[\varepsilon_{t+\tau} < X_{i,t} \middle| \mathcal{F}_t\right] \qquad \text{where} \qquad X_{i,t} = \frac{\ln\left(L_{i,t}\right) - \left(\mu_{t,\tau} - \frac{1}{2}\sigma_{t,\tau}^2\right)}{\sigma_{t,\tau}} \tag{9}$$

Thus, we can estimate such probability simply as:

$$\widehat{p}_t(L_{i,t}) = \frac{n(\varepsilon_{s+\tau} < X_{i,t})}{n(\varepsilon_{s+\tau})} \qquad \text{for all} \qquad s+\tau < t. \tag{10}$$

where $n(x)$ counts the number of events x. From this collection of default probabilities, we assign pseudo credit ratings to each of the pseudo bonds, as discussed in more detail in Section 4.2. We perform these computations on an expanding window so that at any time t we only use information available at time t to predict the default probability of a pseudo bond with leverage ratio $L_{i,t}$ at maturity $t+\tau$. Our approach is non-parametric and only relies on the empirical distribution of the shocks $\varepsilon_{t+\tau}$. If these shocks are not normally distributed, then the model-free default probabilities will be different from those implied by the lognormal Merton model. Panel A of Figure 1 presents the histogram of shocks $\{\varepsilon_{t+\tau}\}$ for maturity $\tau = 2$ and presents evidence of non-normality.[8]

So far we have assumed that options with the exact desired target maturity τ actually exist. In reality, at every given time t only certain maturities are available. For this reason, we take the Gaussian kernel-weighted average of all bonds with the same rating. Specifically, the weighting function has the form

$$w_t \propto \frac{1}{\sqrt{2\pi}s}\exp\left(-\frac{1}{2}\frac{\left(\hat{\tau}_t - \tau\right)^2}{s^2}\right)$$

[7]Specifically, we use monthly returns to estimate $\sigma_{t,1}^2$ and compute $\sigma_{t,\tau}^2$ for $\tau > 1$ from the properties of the fitted GARCH(1,1) model.

[8]The Kolmogorov-Smirnov test shows that normality is rejected at the 1% confidence level.

where $s = 30$ days. We use expression (6) with $\hat{\tau}_t$ instead of τ for all computations.

Given our empirical observations of pseudo bonds in expression (6), we compute the time series of their semi-annually compounded credit spreads as well as their monthly returns, which we use in our empirical tests.

4. Preliminaries: Data and Default Frequencies

4.1. Data

We use the OptionMetrics Ivy database for daily prices on SPX index options and options on individual stocks from January 4, 1996, through August 31, 2013. For SPX options, we generally follow Constantinides, Jackwerth and Savov (2013) to filter the data in order to minimize the effects of quotation errors. For individual equity options, we apply the same filters as Frazzini and Pedersen (2012). Stock prices are from the Center for Research in Security Prices ("CRSP").

We construct the panel data of corporate bond prices from the Lehman Brothers Fixed Income Database, TRACE, the Mergent FISD/NAIC Database, and DataStream, prioritized in this order when there are overlaps among the four databases. We exclude all bonds with floating-rate coupons and embedded options (*e.g.,* callable bonds) from our data set.

Risk-free rates and commercial paper rates (used to compute short-term credit spreads) are from Federal Reserve Economic Data ("FRED") database.

A more detailed description of the data is contained in Appendix C.

4.2. Default Frequencies and Probabilities of Pseudo Bonds

Our goal is to construct pseudo bonds that match the realized default frequencies of actual corporate bonds. To that end, we employ a large dataset of corporate defaults spanning the 44-year period from 1970 to 2013 obtained from Moody's Default Risk Service. For each credit rating assigned by Moody's to our universe of firms, we estimate *ex post* default frequencies at various horizons from 30 days up to two years. We use our own estimates rather than the original Moody's default frequencies for three main reasons. First, we are interested in the variation of default frequencies over the business cycle, whereas Moody's historical default frequencies are only available as unconditional averages. Second, we analyze default frequencies at horizons of below one year, which are not provided by Moody's. Third, because of lack of sufficient granularity of option strike prices, we are often unable to differentiate pseudo bonds with extremely low default probabilities and thus group IG bonds into two categories – Aaa/Aa and A/Baa – which we use to compute category-level default frequencies.[9] Appendix D further discusses the construction of these data. For reference, Table A1 in Appendix E shows that our annual estimates of default frequencies are very close to Moody's estimates, and further reports their disaggregation into different maturities and over the business cycle.

In this section, we discuss the methodology and the results by focusing on two-year bonds, which is the main case discussed throughout the paper. (Table A2 in Appendix E contains results for other frequencies.) Panel A of Table 1 shows the default frequencies estimated from Moody's dataset on corporate defaults for the credit ratings reported in the first column. In particular, the second column reports the estimated default frequencies, the third and fourth columns report the two-tailed 95% confidence intervals, and the last two columns report the default frequencies in booms and recessions.

For every month t and target maturity τ, we assign each pseudo bond to a credit rating category based on its estimated default probability $\widehat{p}_t(L_{i,t})$ by comparing it to the default frequencies in booms or recessions in Panel A. We use the midpoint of default probabilities across credit ratings to define the probability thresholds and automatically assign credit ratings to the pseudo bonds. For example, Panel A of Table 1 shows that the default frequencies during recessions of corporate bonds rated as Aaa/Aa, A/Baa, and Ba are 0.05%, 0.47%, and 3.76%, respectively. Thus, in a recession month t, we assign a pseudo bond to the Aaa/Aa pseudo credit rating if its probability satisfies $\widehat{p}_t(L_{i,t}) \leq 0.05\%/2{+}0.47\%/2 = 0.26\%$. Similarly, we assign a pseudo bond in recession month t to the A/Baa rating category if $0.26\% < \widehat{p}_t(L_{i,t}) \leq 0.47\%/2 + 3.76\%/2 = 2.11\%$. And so on.

Panel B of Table 1 reports the results of our pseudo bond credit rating assignment methodology. For each credit rating in the first column, the second and the third columns show the weighted average *ex ante* default probabilities for pseudo bonds in each rating category. According to the procedure, these probabilities should be close to the historical default frequencies reported in the last two columns of Panel A, and they are.

Column 4 of Table 1 reports *ex post* average default frequencies for pseudo bonds over the 1970 - 2013 sample.[10] The mean *ex post* default frequencies across credit ratings are

[9] Even with this slightly coarser definition of credit ratings, the Aaa/Aa category has 69 months of missing observations and the A/Baa group has six months of missing observations out of 212 months in our sample.

[10] We note that we do not need options to compute *ex post* default frequencies of pseudo bonds, as default

larger than the target *ex ante* probabilities in Columns 2 and 3, but they are not statistically distinguishable from one another, as the 95% confidence intervals reported in Columns 5 and 6 are rather wide. These wide confidence intervals underscore an important point about SPX-based pseudo bonds. Namely, because we construct pseudo bonds from a *single* fictitious firm that has only SPX shares as assets, we do not have a cross-section of firms over which to average default events. We only have one time series of assets (*i.e.*, the SPX) for our firm, and the difference across pseudo bond default frequencies only reflects different leverage ratios of that single fictitious firm and not different firms with different assets. So, the mean *ex post* default rate is noisy, and the confidence intervals are large.[11] Section 5.4. considers pseudo bonds issued by multiple, different pseudo firms and there we find *ex post* default frequencies and confidence intervals that are an order of magnitude smaller on the same sample thanks to the diversification effects across pseudo bonds (see Panel A, Table 5).

The second-to-last column in Panel B of Table 1 reports the average moneyness of the options used ($\overline{K/A}$). As is evident, the options used for highly rated pseudo bonds are deeply out-of-the-money to be consistent with a low probability of default. As noted and further discussed in the next section, we sometimes lack sufficient data to compute a default rate for the Aaa/Aa category at all because options so far out-of-the-money are excluded by our minimum liquidity filters (as discussed in Appendix C).

The last column of Panel B reports the average maturities $\overline{\tau}$ of the options used across credit ratings. These averages are between 580 and 650 days (1.59 and 1.78 years) and are a bit smaller than the two-year (730-day) target mainly due to lack of data in the early part of the historical sample. Even so, the lower average maturity would bias the empirical results against us, as shorter maturities imply lower probabilities for the put options to end up in-the-money at maturity. Notwithstanding the shorter average maturity, we continue to refer to our pseudo bonds as two-year bonds for simplicity.

5. The Credit Spread Puzzle in Pseudo Bonds

In this section, we analyze the empirical properties of our pseudo bonds and compare them to those of real corporate bonds and those implied by the original lognormal Merton model.

at $t+\tau$ only depends on whether $A_{t+\tau} < K_{i,t}$. Thus, for every month t and given estimates of $\mu_{t,\tau}$ and $\sigma_{t,\tau}$, we can back out for each credit rating the threshold $K_{i,t}$ so that the *ex ante* probability $p_t(L_{i,t})$ matches the corresponding target default frequency in Columns 2 and 3.

[11] Intuitively, out of our 44-year SPX sample we only have 22 independent observations over which we can compute default frequencies for two-year pseudo bonds. At this frequency, just one observation is sufficient to generate over 2% *average* default frequency, but with large standard errors.

We first illustrate our empirical results using only SPX-based pseudo bonds (Sections 5.1., 5.2. and 5.3.) and then turn to the empirical results using pseudo bonds based on individual stocks (Section 5.4.)

5.1. The Credit Spread Puzzle in Yields

Panel A of Table 2 reports summary statistics for our SPX two-year pseudo bonds in the 1996 to 2013 period. Each row corresponds to a different credit rating category (shown in the first column) based on the assignment approach discussed in Section 4.2.

Column two of Table 2 Panel A shows one of our main results – *viz.*, the credit spreads of two-year pseudo bonds display the same puzzle exhibited in real corporate bond spreads. The credit spreads of our pseudo bonds are large across all credit ratings (ranging from 0.54% for Aaa/Aa bonds to 5.17% for Caa- bonds) and are comparable to the credit spreads of actual corporate bonds shown in Panel B of Table 2. The match is especially close for bonds with high credit ratings. For instance, pseudo bond credit spreads for Aaa/Aa and A/Baa bonds are 0.54% and 1.31%, respectively, as compared to 0.68% and 1.33% for the same rating categories of real corporate bonds. By contrast, pseudo bonds exhibit somewhat lower credit spreads than real corporate bonds for lower credit ratings. For example, pseudo bonds with B and Caa- credit ratings show credit spreads of 3.67% and 5.17%, respectively, as compared to 6.20% and 12.13% for similarly rated actual corporate bonds. Section 6.2. shows that differential liquidity of pseudo and corporate bonds may help explain this difference.

The empirical credit spreads for both pseudo bonds and real corporate bonds are far higher than the credit spreads implied by the lognormal Merton model, whose values are reported in Panel C of Table 2. For Aaa/Aa and A/Baa bonds, the lognormal Merton model implies credit spreads of 0% and 0.04%, respectively, whereas for B and Caa- bonds the model implies spreads of 0.86% and 2.49%, respectively. These spreads stand in sharp contrast to those on both real and pseudo bonds. Table A3 in Appendix E shows that the same credit spread puzzle is apparent across the two sub-sample periods of 1996 – 2004 and 2005 – 2013.

Columns three and four of Table 2 report average credit spreads over the business cycle. Panel A indicates relatively large credit spreads for pseudo bonds both in booms and in recessions, showing that the high average credit spreads are not the result of occasional spikes during recessions but rather are a robust feature of the data. Similar results are visible for corporate bonds in Panel B, although average credit spreads during recessions are a bit larger than for pseudo bonds. In both cases, average credit spreads are far higher than those implied by the lognormal Merton model as shown in Panel C.

How can we interpret our results? First, given that our pseudo bonds are based solely on observed market prices of U.S. Treasuries and put options, our empirical results are model-free. One possibility, however, is that our "credit rating procedure" miss-allocated pseudo bonds in credit rating bins that have higher probabilities of default than the target probabilities. Although this is a possibility, we believe it is unlikely the source of our results. As discussed, although the *ex post* default frequencies are indeed higher than the *ex ante* target probabilities, they cannot be statistically distinguished from one another. We find the same results about credit spreads, moreover, when we apply a similar allocation procedure to the case of individual options in Section 5.4., for which *ex post* default frequencies are much closer to the *ex ante* target probabilities and for which the confidence intervals are tighter.[12] Finally, because the pseudo bonds' credit spreads are mechanically computed straight from U.S. Treasury and put option prices, our results do not hinge on theories of corporate behavior that relate credit spreads to corporate governance, funding constraints, investments, uncertainty about default threshold, and the like.

Instead, our results are consistent with the large literature documenting that put options (especially out-of-the-money puts) are overpriced compared to the Black-Scholes-Merton lognormal model. Our results so far do not shed any light on whether that overpricing is rational (*i.e.,* risk-based) or behavioral (*i.e.*, overpaying for insurance). The novelty of our approach, rather, is to document that such overpricing of put options is consistent with observed spreads on actual corporate bonds. Our results thus suggest that the source of the credit spread puzzle may be better explained by the same forces that explain why put options are expensive. More importantly, we show that the basic insight of Merton (1974) that corporate securities can be viewed as a portfolio of safe bonds plus a short put is quite accurate even if the exact model specification (*i.e.*, log-normal assets) is not.

Finally, we note that the results for the lognormal Merton model reported in Panel C of Table 2 correct for the influence of any bias generated by time-varying stock return volatility and/or the monthly sampling. In particular, all the statistics reported in Panel C are averages of the same statistics computed over 1,000 Monte Carlo simulations across 212

[12] As a robustness check, we also increased the volatility $\sigma_{t,\tau}$ used to scale the distribution of shocks $\epsilon_{t+\tau}$ by constant multiples $\phi = 1.05, .., 1.2$, and used the modified volatility for all of our calculations. The effect is to shift down the cutoff points of the various pseudo credit ratings (so that a Aaa/Aa bond, for instance, requires options even further OTM). The credit spread puzzle is still robust even when we consider a large 20% increase in volatility, which leads to average credit spreads of 0.44%, 0.84%, 1.82%, 3.07%, and 4.68% for credit ratings Aaa/Aa, A/Baa, Ba, B, Caa-, respectively.

months of asset values. Simulations are designed to replicate the GARCH(1,1) volatility and predictability found in the data. For each simulation of asset values, we use the Black and Scholes model (adjusted for a continuous dividend yield) to compute put and call prices across strike prices and then construct simulated bond values from these option prices. Employing simulations that feature time-varying volatility and predictability enable us to conclude that our empirical results in Panel A are not driven by our estimation of a GARCH(1,1) model, the fitting of predicting regressions, and/or the sampling of returns at the monthly frequency.

Figures 2 and 3 present graphical representations of the time series of credit spreads on pseudo bonds and actual corporate bonds, respectively.[13] The spreads of both pseudo bonds and corporate bonds skyrocketed during the 2008 financial crisis, and then returned to more normal levels by the end of the sample period. Both pseudo and corporate bond spreads also increased in 1998 around the time of the Asian and Russian macroeconomic crises. By contrast, pseudo bonds did not react significantly to the 2001 recession, whereas Ba-rated corporate bond spreads increased substantially during this period. Nonetheless, pseudo and corporate bond spreads still show significant comovement and have an average pairwise correlation of 35%.

5.2. The Credit Spread Puzzle in Excess Returns

In this section we take a different approach to the analysis of the credit spread puzzle and focus on pseudo bond excess returns instead of credit spreads. In particular, Proposition 1 (see Section 2.) provides us with testable hypotheses about the behavior of pseudo bond excess returns. We also compare our empirical results for pseudo bonds to the excess returns on actual corporate bonds.

5.2.1. Sharpe Ratios

We first examine the behavior of pseudo bond excess returns across credit ratings. Columns five to nine of Table 2 report summary statistics for monthly excess returns of pseudo bonds (Panel A), corporate bonds (Panel B), and the lognormal Merton model (Panel C). Consistent with the results discussed in the previous section, highly rated pseudo bonds display lower average excess returns (*e.g.*, 0.14% for Aaa/Aa) than lower-rated pseudo bonds (*e.g.*, 0.35% for Caa-). Similarly, highly rated pseudo bonds exhibit lower volatility (*e.g.*, 0.65%

[13]Figure 2 indicates that in the first part of the sample we do not have enough data to compute the spread of Aaa/Aa pseudo bonds. An insufficient number of deep out-of-the-money SPX options were available.

for Aaa/Aa) than lower-rated pseudo bonds (*e.g.*, 2.31% for Caa-). Both results are qualitatively consistent with the predictions of the Merton model (Proposition 1.b) because both average excess returns and volatility are increasing in leverage K/A.

Sharpe ratios for pseudo bonds, however, exhibit an inverted U-shaped pattern that is highest at the A/Baa rating category (Sharpe ratio equal 0.30) and lowest at the Aaa/Aa (0.22) and Caa- (0.15) categories. These differences in Sharpe ratios of pseudo bonds are in contrast with the testable implications of the lognormal Merton model, which implies that all zero-coupon corporate bonds should have the same Sharpe ratio (see Proposition 1).

Panel B of Table 2 shows that actual corporate bonds also display higher excess returns and volatility for lower ratings, consistent with the Merton model. Similar to pseudo bonds (Panel A) and in contrast with the lognormal Merton model, however, real corporate bonds also have Sharpe ratios that differ across credit ratings, with the highest Sharpe ratio occurring for bonds with an intermediate credit rating (*i.e.*, Ba-rated bonds with a Sharpe ratio of 0.26).

Panel C of Table 2 shows that even taking into account the influence of time-varying volatility on return series and monthly sampling of returns, the lognormal Merton model does not produce the kind of returns displayed in the first two panels. In particular, average returns and volatility estimates obtained for the lognormal Merton model with Monte Carlo simulations have much smaller magnitudes than are apparent in the data, and the simulated Sharpe ratios exhibit higher values for highly rated bonds than for lower-rated bonds.

The last two columns of Table 2 contain two other important statistics of excess bond returns – skewness and excess kurtosis.[14] For both pseudo bonds and real corporate bonds, excess returns are leptokurtic, albeit with no obvious pattern emerging across credit ratings. By contrast, the skewness of excess pseudo bond returns is negative for HY pseudo bonds, whereas no such pattern is visible in the skewness of actual HY corporate bonds.

Table A3 in Appendix E shows the same summary statistics discussed above for the 1996–2004 and 2005–2013 subsamples and demonstrates that the credit spread puzzle in pseudo bonds appears to be a robust phenomenon across time.

[14]Excess kurtosis refers to kurtosis in excess of three exhibited by the normal distribution.

5.2.2. Alphas and Betas

Our second set of return-based tests examine the determinants of excess bond returns in more detail. Specifically, the second and third columns of Table 3 report average excess returns and t-statistics by rating category. According to the lognormal Merton model, the average excess return on bonds should be explained by the firm's excess return on assets (Proposition 1). Because the market values of assets for actual firms are unobservable, we cannot analyze this relation empirically using real corporate bonds. But we can conduct such an analysis on our pseudo bonds, whose values are based on *observable* market values of our pseudo firm's assets. For both real corporate bonds and pseudo bonds, moreover, we can observe excess returns on *equity* and hence can perform the alternate test in Proposition 1.c and then compare results for corporate- and pseudo bond excess returns.

Specifically, we run the following monthly regressions and report the results in Table 3:

$$R^e_{B,t} = \alpha + \beta\, R^e_{i,t} + \varepsilon_t$$

where $R^e_{i,t}$ denotes the excess return of bonds ($i = B$), assets ($i = A$), or equity ($i = E$). For pseudo bonds, we observe both assets (*i.e.*, the SPX) and pseudo equity (*i.e.*, call options on the SPX). For actual corporate bonds, we only observe the firms' equity returns. The null hypothesis according to the lognormal Merton model is that $\alpha = 0$. We note that this null hypothesis holds only for instantaneous returns conditional on a given leverage ratio. To address that issue at least in part, we rebalance our portfolios monthly so that the leverage ratio K/A is relatively constant over our unit of observation. In addition, Panel C reports results from the simulated lognormal Merton model with time-varying volatility and predictability to analyze any potential bias in the average α due to time variation in β.

Panel A of Table 3 shows that pseudo bonds display a significantly positive α across credit ratings when excess returns are regressed on the pseudo firm's excess return on assets. The alphas are larger for lower credit ratings, ranging from 0.12% (Aaa/Aa) to 0.17% (Caa-) per month. The regression betas are also increasing with leverage and are strongly significant, with R^2 values ranging between 18% (Aaa/Aa pseudo bonds) to 83% (Caa- pseudo bonds). Panel B shows that similar results obtain when we regress pseudo bond excess returns on pseudo equity excess returns (given by returns on corresponding SPX call options). Once again, alphas are significantly positive and larger for lower credit ratings, and regression betas are increasing with leverage. Both the betas and the R^2 of the regressions on pseudo equity, however, are lower than the results of the regressions on assets shown in Panel A.

The strong positive alphas that appear in Panels A and B suggest that the likely source of the credit spread puzzle for pseudo bonds is the existence of jump and volatility risk premiums that together push average excess returns (and therefore credit spreads) higher. Indeed, Appendix B discusses a simple extension of the Merton model that adds stochastic volatility and jumps to the dynamics of assets, and the evidence in Panels A and B is consistent with Proposition 2 in Appendix B in that regard.

Panel C of Table 3 presents the results of excess returns on actual corporate bonds regressed on excess pseudo equity returns. The results are similar to those for the pseudo bond regressions shown in Panel B. In particular, the corporate bond alphas (like the pseudo bond alphas) are positive and increasing in credit quality. For the most highly rated Aaa/Aa corporate bonds, alpha is a relatively low 0.11% (as compared to 0.12% for pseudo bonds), but, unlike the pseudo bond alpha, is not statistically significant. Actual corporate bond excess return betas with respect to equity are also similar to their pseudo bond counterparts and are significant. R^2's are a bit smaller for real corporate bonds than for pseudo bonds, but, on average, are not small in magnitude. Overall, we see some strong similarities between the behavior of excess returns of corporate and pseudo bonds *vis-a-vis* excess equity returns.

Panels D and E of Table 3 show the same results as in Panels A and B, but for simulations of excess bond returns based on the lognormal Merton model (as discussed in Section 5.1.). When we run the same regressions based on simulated excess returns using the Merton model, the estimated alphas are much smaller than the alphas estimated using real and pseudo bonds and are not significantly different from zero. Betas are again increasing with leverage, but are much smaller than those estimated using the empirical observations.

Table A4 in Appendix E reports results from comparable excess return regressions on the two subsamples, 1996 – 2004 and 2005 – 2013. The results are generally similar to those for the full sample. One notable exception is that estimated alphas for both pseudo bonds and real corporate bonds are especially high and significant during the second subperiod (which includes the financial crisis), whereas they are not significantly different from zero in the first subperiod. This result makes sense in light of Proposition 2 from Appendix B – *i.e.*, the increase in the likelihood of a jump in the underlying assets reflected in the second subperiod seems to result in a correspondingly higher risk premium arising from heightened tail risk that manifests in the form of a higher estimated jump risk premium α_B (see equation (15) in Appendix B).

5.3. Short-Term Pseudo Bonds

The previous sections focused on two-year pseudo bonds and corporate bonds. In this section we examine the term structure of credit spreads of pseudo bonds. Unfortunately, this exercise is hindered by two data limitations. First, higher-rated bonds in the Aaa/Aa and A/Baa categories have negligible historical default frequencies over short time horizons (*i.e.*, 30 and 91 days). As a result, there is not enough granularity in available option strike prices to differentiate the default probabilities of Aaa/Aa pseudo bonds from A/Baa pseudo bonds. For such short maturities, we therefore combine all pseudo bonds with ratings of Baa or higher into a single IG credit rating bin. Second, we do not have reliable corporate bond data for maturities of less than 91 days. We thus rely instead on commercial paper ("CP") issued with original maturities of 270 days or less. Below-investment-grade CP, however, is not available in the data. As such, our actual corporate bond data includes no empirical observations for 30- and 91-day corporate debt with ratings of Ba or lower.

Panel A of Table 4 reports the term structure of credit spreads for pseudo bonds across credit ratings. Evidently, pseudo bonds display high credit spreads across maturities even for highly rated bonds. For example, IG pseudo bonds have credit spreads of 77, 64, and 69 bps at the 30-, 91-, and 183-day maturities, respectively. In addition, the term structure of credit spreads is mildly U-shaped across maturities, with higher credit spreads occurring for very short or long maturities and lower spreads for intermediate maturities.

Panel B of Table 4 reports similar results for corporate bonds. For comparison with pseudo bonds, we also report average credit spreads for IG bonds. Like pseudo bonds, highly rated corporate bonds display substantial credit spreads at short maturities and a mild U-shape across maturities. The magnitudes of the credit spreads, moreover, are similar to the pseudo bonds. Although we lack data on short-dated real corporate debt, the term structure nevertheless displays the same increasing pattern that we observe for pseudo bonds for maturities from six months to two years. The magnitudes are also comparable to the pseudo bonds except for very low-rated bonds (Caa-), which have much higher credit spreads than their pseudo bond counterparts.

Panel C of Table 4 displays the implied credit spreads from the simulated Merton model. Consistent with previous results in the literature and presented elsewhere in this paper, the simulated spreads implied by the lognormal Merton model differ dramatically from the empirical credit spreads for both actual and pseudo bonds. The effect is especially pronounced for short times to maturity, where the Merton-model-implied spreads are close to zero for all but the highest-risk credit rating category.

Table A5 in Appendix E shows that short-term pseudo bond and real corporate bond excess returns display similar properties as those documented in Table 3 for the two-year bonds.

5.4. The Empirical Merton Model for Individual Firms

Our analysis thus far has been restricted to a single pseudo firm holding a single asset portfolio (*i.e.*, the SPX). Apart from the limitations of using a single fictitious firm, another potential issue about our methodology and inferences is that our pseudo firm's assets are systematic in nature, given that they only comprise shares of the S&P 500 portfolio of companies. Therefore, a legitimate concern is that the size of the pseudo bond credit spreads may be due to the specific and systematic nature of the assets held by our pseudo firm. In this section we address that concern by creating additional pseudo firms whose assets are shares of individual firms in the SPX.

Specifically, for each company in the SPX, we form a pseudo firm whose only assets are shares of that company. Each fictitious firm purchases those shares by issuing a zero-coupon bond and equity. As was the case for our hypothetical SPX-based firm, the value of the defaultable zero-coupon bond issued by each such pseudo firm is economically equivalent to the value of a safe risk-free zero-coupon bond minus the value of a European put option on the pseudo firm's stock. Given data on Treasuries and put options on individual equities, we perform the same analysis that we did for the SPX-based pseudo firm.

Two practical difficulties arise when we use individual firms for our analysis. First, individual firms' listed equity options are American (as opposed to SPX options, which are European). In its original form, the Merton (1974) model requires the underlying put option be European. Because of a potential early exercise premium, American options have prices at least as high (and usually higher) than European options. Given that a defaultable bond is short a put option, using data based on American instead of European options thus could bias the implied credit spread upwards. Because the practical construction of pseudo bonds in our methodology involves the use of deep out-of-the-money options, however, any such bias is likely to be small – *i.e.*, the early exercise premium for deep out-of-the-money options is relatively low, especially for short-dated options.

To verify that the early exercise premium has a negligible impact on our empirical results (which are based on American options), we use implied volatilities provided by OptionMetrics to convert American option prices into corresponding European option prices through the

Black and Scholes formula and then use the converted European option prices to test all of our calculations. The results of this robustness check (see Table A6 in Appendix E) are identical to those based on American options data directly. The results reported below thus rely on unadjusted American option prices.

The second difficulty stems from the issue of survivorship bias in the computation of model-free default probabilities. Specifically, consider the same procedure described in Section 3. when applied to individual firms. For each stock at time t we would consider the idiosyncratic shocks of its equity and use the histogram of those shocks to back out implied default probabilities and rating classifications. Clearly, that approach is conditioned on firms that are part of the index at t, and, as such, on firms that have "survived" and done sufficiently well to remain or be included in the index. The procedure thus skews the shock distribution to the right.

To avoid survivorship bias in the case of individual pseudo firms, for every t we consider the full cross-section of all firms underlying the SPX index before t (including those that dropped out of the index.) For each firm i and $s < t$, we use its previous-year return volatility and its unconditional average return (before s) to compute its normalized return shock. We then use the full empirical distribution of all these normalized shocks across firms i for all $s < t$ to obtain the default probabilities for each bond of each individual pseudo firm j as of time t. In this computation, like before, for each firm j we scale the shocks by their unconditional means and previous-year volatilities. Given these individual default probabilities, we then assign credit ratings to each pseudo bond. Panel B of Figure 1 shows the histogram of resulting normalized shocks. Like the shocks in Panel A computed for SPX index, the shocks display fat tails.

Panel A of Table 5 reports historical default frequencies for the individual firms in our sample. The first column is the credit rating category, and the next three columns are the historical default frequencies of actual corporate bonds for the firms in our sample along with their two-tailed confidence intervals. The next three columns report the *ex post* default frequencies of our pseudo bonds at the two-year horizon, which are comparable to their *ex ante* targets. For the Aaa/Aa category, the *ex post* default probability is slightly higher than the *ex ante* results in the data, but the noise of the estimates is largely responsible for this difference, as can be seen from the large confidence interval.[15] The last two columns report the average leverage of pseudo firms in each of the credit rating categories ($\overline{K/A}$) and the average time to maturity $\overline{\tau}$.

[15]Indeed, because of the various filters, we only have 106 observations for Aaa/Aa bonds, 199 for A/Baa bonds, and the full sample of 211 observations for the other credit ratings.

Panel B of Table 5 reports the summary statistics of credit spreads and excess returns of pseudo bonds for each credit rating category. The average credit spreads of pseudo bonds calculated from individual pseudo firms are very similar to the credit spreads for pseudo bonds for the SPX-only pseudo firm. For example, Aaa/Aa and A/Baa-rated pseudo bonds issued by single-stock-specific pseudo firms have credit spreads of 0.89% and 1.23%, respectively, as compared to 0.54% and 1.31% for the SPX-only firm (as reported in Table 2). These credit spreads are all comparable to the real corporate bond credit spreads (0.68% and 1.33%, respectively, from Panel B of Table 2). In all cases, the pseudo bond credit spreads are far higher than those implied by the lognormal Merton model. The same result holds for other credit rating categories, as well.

Panel B also reports the summary statistics of the excess returns for each portfolio of pseudo bonds by credit rating category. Clearly, the properties of excess portfolio returns of pseudo bonds constructed from individual firms' securities are similar to those obtained based on SPX options as reported in Table 2. In particular, the average excess returns are higher for lower-rated pseudo bonds. With the exception of the Aaa/Aa category, volatilities are also higher for lower pseudo bond ratings. The Sharpe ratios, however, do not show any visible pattern (perhaps as a result of noise). The distribution of returns still appears to be leptokurtic and negatively skewed, as was shown for SPX-based pseudo bonds and actual corporate bonds.

Panel C of Table 5 reports the results of regressions of pseudo bond excess returns on average excess asset returns for each credit rating. In contrast to what we observed for pseudo bonds issued by the SPX-only pseudo firm, most of the returns on pseudo bonds issued by stock-specific pseudo firms can be explained by the average excess returns on assets – *i.e.*, the portfolio alphas are mostly statistically indistinguishable from zero. One possible reason for this difference may be the inherently less systematic nature of jumps or volatility in individual equities than in the SPX index itself (see Proposition 2 in Appendix B). Interestingly, however, in Panel D we see that the same regression of excess pseudo bond returns on excess pseudo equity returns generates a statistically significant alpha. This empirical finding suggests a potentially important role for discretization bias in inference about expected excess returns, possibly induced by the strong non-linearity of equity returns for levered firms. Panel C of Table 3 reported similar results for actual corporate bonds – *i.e.*, real corporate bond alphas cannot be explained by the same corporate equity excess returns. The parallel between real corporate bonds and pseudo bonds is noteworthy. In the next section, we delve deeper into the source of pseudo bond and corporate-bond excess returns.

6. Inspecting the Mechanism

The previous section documents a strong similarity between the behavior of pseudo bonds and real corporate bonds. In this section, we carry out additional tests on pseudo bonds and corporate bonds to highlight further similarities, and important differences, that can help us better understand the source of the credit spread puzzle.

6.1. Asset Pricing Tests

Given the results presented in Section 5., one question is whether or not the positive alphas for both pseudo bonds and corporate bonds may be explained by systematic risk factors. Accordingly, Table 6 examines whether a number of common risk factors help explain the positive estimated alphas in the pseudo bond and actual corporate bond portfolios.

We run the regression

$$R^e_{i,t} = \alpha_i + \beta_i\ RMRF_t + c_i\ TERM_t + d_i\ DEF_t + e_i\ dVIXSQ_t + f_i\ dTED_t + g_i\ Tail_t + \epsilon_{i,t},$$

where $R^e_{i,t}$ is the excess return on portfolio i, $RMRF_t$ is the excess return on the value-weighted stock market portfolio,[16] $TERM_t$ is the return on the long-term Treasury bonds in excess of T-bill rates, DEF_t is the return on the aggregate long-term corporate bond market portfolio from Ibbotson in excess of the return on long-term Treasury bonds, $dVIXSQ_t$ is the excess return on the option portfolio that underlies the VIX index, $dTED_t$ is the return on a portfolio that replicates the Treasury-Eurodollar ("TED") spread, and $Tail_t$ is the return on the tail-risk factor of Kelly and Jiang (2014). All of these factors are constructed to mimic traded portfolios, thereby enabling us to interpret alpha as an excess return.[17]

Panel A shows the results for pseudo bonds based on the SPX-based pseudo firm. Even controlling for these six systematic risk proxies, the alphas are significant across credit ratings. In other words, these six systematic factors do not explain the average excess return of pseudo bonds. In terms of factor loadings, pseudo bonds load significantly on the market excess return, the $TERM_t$ and the DEF_t factor, as well as the volatility factor $dVIXSQ_t$. The fact that the excess return on the aggregate corporate bond portfolio (DEF_t) is significant in explaining pseudo bonds computed from U.S. Treasuries and SPX put options further demonstrates the close connection between the underlying common risk premium for

[16] We initially also included Fama-French SMB and HML factors. They did not help explain the alphas of these regressions, and so we left them out of the table for parsimony.

[17] The VIX index is the square-root of the value of a portfolio of options. Thus, $VIXSQ = VIX^2$ is effectively the value of a traded portfolio.

pseudo bonds and corporate bonds. Somewhat surprisingly, the TED spread liquidity proxy does not seem to have much impact on pseudo bond returns.[18] One reason could be that the TED spread reflects variations in both liquidity and credit risk across corporate and government bonds, and, to the extent the TED spread is indicating credit risk over the sample period, the risk may already be reflected in other variables.[19] Tail risk, by contrast, enters significantly for some credit ratings, possibly due to the jump probability in the underlying SPX. Yet, the estimated alphas of pseudo bonds are still strongly significant, showing that there are other sources of risk not captured in the risk factors above.

Panel B documents the results for pseudo bonds created from options on individual stocks. The results are consistent with those in Panel A, except that now the tail risk factor is mostly statistically insignificant. This result is consistent with the earlier finding in Section 5.4. that jump risk seems to be less of a source of risk premiums for individual pseudo bonds. Still, alphas are strongly significant across credit ratings except for the Aaa/Aa credit rating. As shown in Panel C of Table 5, however, the pseudo bonds in the highest rating category do not display a significant average excess return to start as the category suffers from significant noise.

Panel C shows the results of similar regressions for real corporate bonds. Like Panels A and B, alphas are strongly significant across all credit ratings, showing that the proposed risk factors do not explain the corporate bonds' risk premia. The main explanatory variables for corporate bond excess returns are the term premium $TERM_t$ and (not surprisingly) the corporate default risk factor DEF_t. The volatility risk factor $dVIXSQ_t$ mostly enters negatively in the regressions (as in Panels A and B) but is not significant. The R^2s of the regressions, moreover, are far smaller for actual corporate bonds than for the pseudo bonds, perhaps due to the additional noise introduced by the lower liquidity of lower-rated corporate bonds.[20]

6.2. The Transactional Liquidity of Pseudo Bonds

Whereas the TED spread is generally considered a proxy for aggregate market liquidity, we can also follow Bao, Pan, and Wang (2011) and consider the Roll (1984) measure of transactional liquidity. Unlike the TED spread, the Roll "bid-ask bounce" is a measure of

[18]Using the LIBOR-OIS spread instead of the TED spread did not significantly change our results.

[19]On the other hand, we also used the Pastor and Stambaugh (2003) factor and found similar results.

[20]An interesting question is whether for each credit rating, our pseudo bond returns explain the real corporate bond returns. Except for the top credit rating Aaa/Aa, the slope coefficients of regressions of real excess bond return on SPX-based pseudo bond excess returns are significant. The R^2 of such regressions, however, are small and some of the alphas are significantly positive.

transactional liquidity that reflects the degree to which traded prices bounce up and down (the logic being that large reversals indicate relatively less transactional liquidity and higher sensitivities of bid and offer prices to large orders). To quantify the bid-ask bounce, the Roll measure uses the negative autocovariance of log price changes.

Following Roll (1984), we compute the transactional market illiquidity measure for pseudo bond i in month t as

$$Illiquidity_t = \sqrt{-Cov_t(\Delta p_{i,t,d}^{Bid \to Ask}, \Delta p_{i,t,d+1}^{Ask \to Bid})} \tag{11}$$

where $\Delta p_{i,t,d}^{Bid \to Ask} \equiv \log Ask_{i,t,d} - \log Bid_{i,t,d-1}$ and $\Delta p_{i,t,d}^{Ask \to Bid} \equiv \log Bid_{i,t,d} - \log Ask_{i,t,d-1}$.[21] We compute the Roll measure for all pseudo bonds that have more than 10 return observations in a month. The portfolio-level Roll measure is computed by the kernel-weighted average of the pseudo bonds for which we can compute the Roll measure, where we again use the Gaussian kernel for calculating weighted returns. In addition to the Roll measure, we also compute the bid-ask spreads, calculated as $(B_{i,t}^{Ask} - B_{i,t}^{Bid})/B_{i,t}^{Mid}$. The portfolio bid-ask spread is the kernel-weighted average across pseudo bonds.

For corporate bonds, bid and ask spreads are not available. Thus, we only compute the Roll measure. Using the daily price observations, the Roll measure for corporate bond i in month t is computed by

$$Illiquidity_t = 2\sqrt{-Cov_t(\Delta p_{i,t,d}^{Transaction}, \Delta p_{i,t,d+1}^{Transaction})} \tag{12}$$

where $p_{i,t,d}^{Transaction}$ is the log transaction price of corporate bond i on day d. We compute the Roll measure for all corporate bonds that have more than 10 return observations in a month.[22] As in credit spreads and excess returns, the Roll measure for a portfolio is the value-weighted average of the corporate bonds for which we can compute the Roll measure.

Table 7 shows the results. Comparing Panels A and B, we see that the liquidity of pseudo bonds based on the SPX is far higher than the liquidity of pseudo bonds based on individual stocks. Both the bid-ask spreads and the Roll (1984) illiquidity measure of the SPX-based pseudo bonds are about one fifth the size of those same measures computed for pseudo bonds from individual stocks. This is not surprising given that SPX options are far more liquid than most individual equity options.[23]

[21]This formula slightly differs from Roll (1984) formula, which is used instead in equation (12) below, as for pseudo bonds we have available bid and ask prices. Thus, we can compute the round-trip trading cost without imputing a transaction to be performed at the bid or ask with 50-50 probability, a computational assumption in Roll (1984). We thank Jack Bao for pointing out the correct formula to us.

[22]Daily transaction prices are obtained from Mergent FISD/NAIC and TRACE.

[23]Panel A of Table 7 also shows that highly rated bonds are more liquid than lower rated bonds, which

Comparing Panels A and B to Panel C, it appears that pseudo bonds, and especially those based on SPX options, have far lower average execution costs and far greater transactional liquidity than real corporate bonds. The only exception is for the Aaa/Aa category in which transaction costs for corporate bonds are still higher but closer to those of pseudo bonds.[24] Pseudo bonds based on individual stocks have illiquidity measures that are somewhat closer to the ones computed for real corporate bonds, except for lower-rated bonds for which corporate bonds still show far higher execution costs (less liquidity). Interestingly, these lower-rated bonds also show the highest credit spreads compared to pseudo bonds. This difference between our benchmark option-based model and the observed credit spreads on HY debt may provide an indication of the illiquidity risk premium, which is about 6% on average for Caa- bonds. In other words, over half of the credit spread of HY corporate bonds may be attributable to transactional illiquidity.[25]

Overall, these results suggest that illiquidity alone is unlikely to be the source of the credit spread puzzle, given that especially our SPX-based pseudo bonds are far more liquid than corporate bonds and yet show similar credit spreads. Still, given that lower-rated corporate bonds are far more illiquid than comparable pseudo bonds, we can ascribe at least some of the difference in credit spreads for such bonds to a liquidity factor.

7. Applications of the Empirical Merton Model

The previous sections document that pseudo bonds and real corporate bonds are similar both in terms of credit spreads and excess returns. In the same spirit as the original Merton (1974) model, we can use our empirical Merton model as a laboratory to perform counterfactual experiments that would be hard or impossible to do in the real world. The benefit of our methodology is that our findings are extracted straight from the data without the filter of a parametric model. We offer here three applications for illustration, and leave more elaborate examples to future research.

may be surprising given that highly rated bonds use put options that are further out-of-the-money, and hence more illiquid. The reason for this result is that we follow Bao, Pan, and Wang (2011) and use log prices for our estimates of the Roll measure, and highly rated bonds have higher prices. Thus, highly rated bonds may have a lower "dollar" liquidity but a higher "percent" liquidity.

[24]Interestingly, Aaa/Aa bonds have the highest R^2 in our Table 6, and have loadings on $TERM$, DEF, $dVIXSQ$ and $dTED$ that are significant and with the intuitive signs.

[25]A future study might examine this issue further by using more liquid credit default swap spreads to construct implied bond prices and returns and comparing the results to those presented here for cash bonds.

7.1. Credit Spreads and Frequency of Credit Rating Revisions

In previous sections, we assign a credit rating to each of our pseudo bonds every month. We then sort bonds on those credit ratings and form portfolios. In reality, of course, credit rating agencies do not assign corporate credit ratings at exactly a monthly frequency. Given the apparent strong reliance that many investors place on published ratings and the importance of potential clientele effects resulting from institutional portfolio constraints involving minimum credit ratings, an important question is how the frequency of credit rating assessments may impact *ex post* average credit spreads and excess returns.[26] To examine this question, we now assume that credit ratings are assessed at a lower (and exogenous) frequency – specifically, every three, six, and 12 months. We continue computing average credit spreads and bond returns at the monthly frequency.

Table 8 reports average credit ratings and summary statistics for pseudo bonds using these three new, lower-frequency credit rating assignment intervals. We can see in columns three and four that as we decrease the ratings frequency, average credit spreads become smaller during booms and substantially larger during recessions. Although a negative bias from less frequent credit ratings assignments in booms and a positive one in recessions seems intuitive, the size of the effects is surprising, especially during recessions. Still, as shown in column two, because booms last longer than recessions, the grand average credit spreads across credit rating categories are similar to average spreads based on a monthly rating assignment frequency. The only noticeable difference is at the very lowest credit rating (Caa-) for which the average credit spread moves from 5.17% at the monthly assignment frequency (Table 2) to 5.53%, 7.32% and 8.37% at the quarterly, semi-annual, and annual assignment frequencies, respectively. As first pointed out by David (2008), this result is likely due to the convexity that exists between credit spreads and leverage (K/A) – *i.e.*, time variation in market values of underlying assets A_t over longer periods generate an increase in the average credit spread, which is more pronounced for pseudo bonds closer to at-the-money (high K/A).[27]

Table 8 also shows that average excess pseudo bond returns are smaller the more infrequent the credit-rating assignment. Again, this effect is likely due to a negative convexity effect – *i.e.*, the fact that bond prices are capped up when asset values A increase while they may decrease to zero when A decrease. Thus, over a longer period, the variation of asset

[26] Our analysis of the empirical implications of the frequency of ratings assignments is not intended to be a proscriptive commentary on how often ratings "should" be assigned or re-evaluated. Indeed, rating agencies typically assign ratings based on a variety of considerations, not all of which immediately imply a simple rule for frequency of evaluations.

[27] See also Federhutter and Schaefer (2014) for a discussion of this convexity issue and its impact on estimation of credit spreads.

value generates a negative convexity bias in average bond returns as the underlying asset moves away from the initial leverage ratio K/A that defines its credit rating at rebalancing time. The effect of such negative bias is large and affects all credit ratings, with the largest impact on relatively higher-risk bonds. For instance, the average excess return for Caa- bonds decreases from 0.35% when credit ratings are assigned at the monthly frequency (Panel A, Table 2) to 0.17% when credit ratings are assigned at the annual frequency (Panel C, Table 8). Indeed, the Sharpe ratio for bonds in that category drops to just 0.06 (despite the relatively high credit spread of 8.37%).

In sum, less frequent credit rating revisions generate two convexity biases that move in opposite directions: average credit spreads increase while average returns decrease, and these effects are especially large for the lowest credit rating categories.

7.2. Uncertainty about Asset Values and Credit Spreads

Our model-free approach can also be used to investigate the vexing issue of how uncertainty about asset values is related to credit spreads and bond returns (*e.g.,* Duffie and Lando (2001), Yu (2005), Polson and Korteweg (2010)). All else equal, put option values are higher for larger amounts of uncertainty about underlying asset values. In the Merton framework, bonds (which include short put options) thus have prices that are decreasing in underlying asset uncertainty.

The empirical question, however, is the extent to which higher underlying asset uncertainty gives rise to higher credit spreads across different credit ratings. The question is complicated by the fact that a bond's credit rating should already take into account (at least to some extent) uncertainty in asset values – *i.e.,* firms with more uncertain asset values should have lower credit ratings, *ceteris paribus*. Thus, it is not obvious that firms in the same credit rating category with higher underlying asset uncertainty should exhibit higher credit spreads. This endogeneity issue is hard to resolve using real corporate bond data because (*i*) natural experiments are rare in which everything stays constant except uncertainty about underlying asset values, and (*ii*) asset value uncertainty is difficult to measure. Our model-free approach allows us to overcome both issues: (*i*) we assign bonds to credit rating categories according to a specific rule that holds constant most confounding variables and allows us to focus more specifically on asset uncertainty, and (*ii*) our options-centric approach enables us to measure the uncertainty of our pseudo firms' asset values by analyzing volatilities of the assets underlying the options on which we rely.

Consider the pseudo bonds computed from individual stocks included in the SPX index, as in Section 5.4. For each time t, we sort individual firms' pseudo bonds according to their pseudo credit rating. For each credit rating category, we then sort pseudo bonds into low, medium, and high asset volatility categories. For each credit rating/volatility combination, Table 9 reports pseudo bonds' average credit spreads (Panel A), excess returns (Panel B), leverage K/A (Panel C), and underlying asset volatility (Panel D).

Panel A indicates that for all rating categories, credit spreads for pseudo bonds with high-volatility assets are higher than spreads on pseudo bonds with low-volatility assets. The net effect of higher underlying asset uncertainty thus is indeed a higher credit spread, even after taking into account the endogenous effect that higher uncertainty translates into lower average leverage K/A to qualify for a given credit rating (Panel C). The magnitudes are large, moreover, especially for lower-rated bonds. For instance, a Ba-rated pseudo bond has 1.65% spread in the low-volatility bin but a 2.37% spread in the high-volatility bin. These magnitudes are close to the difference in average credit spreads between A/Baa and Ba rated bonds, as shown in Table 2. The pattern of average excess returns mostly mimics the pattern of credit spreads, although noise in the data may at times generate different particular patterns.

Panel C shows the intuitive fact that, conditional on individual credit ratings, high underlying asset volatility corresponds to lower leverage. Panel D provides a sense of the difference in average asset volatility within credit ratings. For instance, the difference in volatility for Aaa/Aa credit rating is relatively small – only safe assets make it into the high credit quality bin. By contrast, the difference in asset volatility for lower-rated pseudo bonds can be substantial – *e.g.*, from 25% to 44% for Caa- bonds.

7.3. Pseudo Bank Rollover Risk and Capital Requirements

As a final application of our empirical approach to the Merton model, we study the rollover risk and capital requirement of a pseudo bank that lends money to the individual pseudo firms whose assets are based on the stocks of SPX constituent companies. Specifically, a hypothetical bank that issues short-term debt (*e.g.,* demand deposits and CP) to finance the extension of longer-dated zero-coupon commercial loans can be viewed as purchasing pseudo bonds from the firms to which it is extending credit. To analyze the impact of maturity transformation and rollover risk, we assume that the pseudo bank issues debt with only one month to maturity (see Figure 4 for a schematic representation of the pseudo bank). Given the empirical properties of monthly pseudo bond returns, we can evaluate the pseudo bank's own probability of default.

In particular, suppose that the pseudo bank defaults if the market value of its assets are below the face value of the bank's debt when that debt matures. For every t, default thus occurs if $A_t^{Bank} < K_{t-1}^{Bank}$, where K_{t-1}^{Bank} is the total face value of short-term debt issued by the pseudo bank in previous month $t-1$. Given that the bank's assets are made of a portfolio of pseudo bonds issued by the bank's pseudo firm borrowers, we have $A_t^{Bank} = A_{t-1}^{Bank}(1+R_{t-1,t}^{Port})$, where $R_{t-1,t}^{Port}$ is the return on the portfolio of bonds between $t-1$ and t. Therefore, the requirement for one-month survival for the bank is $R_{t-1,t}^{Port} > -(1 - \frac{K_{t-1}^{Bank}}{A_{t-1}^{Bank}}) = -(1 - L_{t-1})$, where L_{t-1} is the bank's leverage ratio at $t-1$. We want to find the minimum equity capital the bank has to have to keep the probability of failure at t small – *i.e.*, we want to find $\overline{L}$ such that $Pr(R_{t-1,t}^{Port} < -(1-\overline{L})) = \alpha$ for some small probability α.

We consider three types of pseudo bond portfolios that make up the assets of the pseudo bank. The first is an "All " portfolio consisting of a portfolio of pseudo bonds diversified by maturity and credit rating. In addition, we consider IG and HY portfolios that contain only pseudo bonds with credit ratings above (and equal to) or below Baa, respectively. Although the IG and HY portfolios are distinguished by credit quality, we assume that both portfolios are diversified across maturities. All of these pseudo bonds are issued by the individual hypothetical firms discussed in Section 5.4., and we assume the bank only extends one loan to each pseudo firm.

We construct our pseudo bank's loan portfolios to have approximately constant characteristics across the overall sample. We draw the maturities of our pseudo bonds from only three maturity bins – up to 273 days, 274 to 548 days, and 549 days or longer.[28] We also choose a minimum portfolio size $N = 30$ to ensure some diversification benefits for the pseudo bank. Specifically, for every month t, for each firm and for each rating, we randomly choose one maturity bin per firm/borrower and select one pseudo bond as the bank's loan to that firm. Some firms may have no pseudo bonds with the selected maturity/credit rating combination, in which case the firm is not part of the portfolio. For the IG and HY portfolios, if the number of firms with the selected pseudo bonds is more than N, we average them and record the portfolio returns. Otherwise we have missing data for that month. For the "All" portfolio, if the number of IG firms is more than $N/2$, then we randomly pick the same number of HY bonds as IG bonds and compute bond returns of the overall portfolio. This methodology ensures that the "All" portfolio has an equal representation of IG and HY pseudo bonds.[29] We repeat this procedure for the overall 1996 – 2013 sample. In addition,

[28]We choose these three maturity bins because they are equally well-populated across the overall sample.
[29]This procedure avoids sample selection issues in which the "All" portfolio may end up with over-

we simulate this procedure 1,000 times to compute representative portfolios. Note that the simulation only pertains to the choice of the portfolio at any t; the portfolio return itself is not simulated and is an actual market return on the chosen pseudo bonds.

Panels A to C of Figure 5 show the return distributions of our pseudo bond portfolios. For comparison, Panels D to E show the return distributions of the portfolios of assets underlying the pseudo bond portfolios. All distributions are all normalized to have a zero mean and unitary standard deviation for ease of comparison. Several results are apparent. First, the distributions of pseudo bonds (top row) are always more dispersed than the corresponding distributions of assets that underly the pseudo bonds (bottom row) – *i.e.*, the diversification benefit in a portfolio bonds is not as strong as for the portfolio of underlying assets inasmuch as diversification does not curtail the tails by the same amount. Second, although the difference in dispersion is mild for the IG portfolio, the difference is large for the HY portfolio. The underlying assets have a maximum negative return of about four standard deviations below the mean, whereas the underlying HY portfolio reaches eight standard deviations below the mean. High-risk pseudo bonds thus are especially prone to "Black Swans" (*i.e.*, low-frequency, high-severity events) even if the underlying asset portfolios do not show such risks. Third, the "All" portfolio has some observations that are over eight standard deviations below its mean, although their frequency is smaller than the HY portfolio as a result of the mixture of HY and IG pseudo bonds to make it a more balanced portfolio.

To gain further insights on the relation between the distribution of the pseudo bank assets and the portfolio of assets underlying those pseudo bonds, Figure 6 shows the scatterplot of the distributions contained in Figure 5. Panel A shows the interesting result that for the IG portfolio, the worst returns of pseudo bonds do not necessarily occur for dramatic negative returns on the underlying asset portfolio. Indeed, the four standard deviation declines of the IG portfolio occur for declines in the underlying assets of about two standard deviations. Significant negative pseudo bond returns thus may occur, for example, as a result of large increases in volatility or a sudden reduction in liquidity.

Panel B shows that for the HY portfolio, the worst returns on pseudo bonds occur around the worst returns on underlying asset values. The eight standard deviation decline in the HY bond portfolio occurs at the same time as a four standard deviation decline in the value underlying assets. The apparent concave relation between HY returns and underlying asset returns is attributable to leverage, but the magnitude of this effect is the interesting part in

representation of HY pseudo bonds simply because there may be more such pseudo bonds available in a given month. This is likely to happen as HY pseudo bonds use put options that are less out-of-the-money than IG pseudo bonds. A drawback, however, is that there are months with no observations, and thus the empirical distributions across panels in Figure 5 are not comparable, as they may include different samples.

this exercise.

Finally, Panel C shows that a balanced portfolio comprising 50% IG bonds and 50% HY bonds still shows a potentially devastating eight standard deviation drop in value due to a four standard deviation drop in asset values. So, even if the pseudo bank's loan portfolio is well-diversified across credit ratings, the leverage of the pseudo bond portfolio is still sufficient to generate a potential "Black Swan" scenario that could have a devastating effect on the bank itself.

We can use the return distribution of our pseudo bank's assets to obtain the amount of equity capital required to make the probability of default "small." For example, the minimum, 99.5%, and 99% percentiles of the (non-normalized) monthly return distributions for the "All" portfolio are -7.04%, -2.17%, and -2.02% respectively. If we want to ensure zero probability of default over a monthly horizon, the minimum equity capital requirement would have to be more than 8% of assets at a minimum. The same percentiles for the IG portfolio are -1.49%, -1.13%, and -1.07%, and, for the HY portfolio, -10.69%, -5.58%, and -4.45%. Based on this data, a pseudo bank that only lends to IG firms could ensure no default over a one-month time horizon by having an equity capital buffer of just 2%, whereas a pseudo bank that specializes in HY loans would need a much higher capital buffer of over 11% to absorb "maximum" possible default-related losses.

8. Conclusions and Discussion

In this paper we have introduced the empirical Merton model – a primarily model-free methodology that utilizes traded options to quantify the implications of the original Merton (1974) model for the valuation of defaultable corporate debt. The main insight of the Merton model is that the value of defaultable debt can be computed as the value of risk-free zero-coupon debt minus the value of a put option on the firm's assets. By imagining that hypothetical pseudo firms issue debt and equity securities to finance their purchases of underlying traded assets such as the SPX index portfolio or individual firms' stocks, we can study the empirical properties of the pseudo bonds issued by such firms.

The empirical results are striking. We find that the credit spreads generated by pseudo bonds (whose values are directly observable and involve no parametric assumptions) are surprisingly comparable to the credit spreads observed for real corporate bonds, especially for bonds with high credit qualities. Such credit spreads are orders of magnitudes higher than those implied by the original Merton model, which assumes that the value of the assets underlying defaultable corporate debt are lognormally distributed. Our empirical investigation of such pseudo bonds demonstrates numerous similarities between the properties of pseudo bonds and the empirical properties of real corporate bonds, thus potentially calling into question elaborate theories of the credit spread puzzle that rely on agency costs, asymmetric information, learning, uncertainty, and the like. Our results instead point at a genuine risk premium required by investors to hold securities that could suffer following the occurrence of tail events in the underlying asset distribution.

We have also shown how our model-free approach to bond valuation offers a benchmark to conduct counterfactual experiments that are grounded in the data but that would be otherwise hard or impossible to perform with actual corporate bond data. For instance, we have shown the type of biases we should expect in average credit spreads and bond returns when credit ratings are not updated with sufficient regularity. We also study how uncertainty about underlying asset values affects credit spreads once we take into account the endogenous effect of asset uncertainty on credit ratings. Finally, we presented an application to banking and capital requirements by looking at the empirical distribution of several simulated loan portfolios. Such experiments are important because they capture the full extent of the variation in debt valuations arising from discount rate movements, as opposed to just shocks to cash flows. Those variations in discount rates generate significant changes in the mark-to-market values of assets that impact the market values of debt in a systematic fashion. This has important implications for debt valuation, as well as capital requirements.

A potential criticism of our approach is that our results are driven by the special nature of the assets held by our pseudo firms – namely stock indices or individual stocks, which may be too volatile and prone to market crashes or run-ups compared to the real assets in which other (especially non-financial) firms invest. We believe the opposite is true and that the observability of the market values and volatilities of our pseudo firms' underlying assets is a virtue of our approach rather than a limitation. Indeed, even if unobservable, the *market* values of assets underlying real firms are likely to be quite volatile and prone to crashes, as well. In fact, recall that stocks are just claims on future dividends, which are relatively smooth and not too volatile (*e.g.,* Shiller (1981)). In spite of the low volatility of dividends, stock prices themselves are highly volatile. As is well known from the work of Campbell and Shiller (1988), Vuolteenaho (2002), and others (*e.g.,* Cochrane (2005, 2008)), discount rate shocks are critical determinants of the the volatility of market values. It is only logical to conclude that a similar channel – the discount rate channel – affects the market value of firms' underlying assets and thus that such unobservable market values of assets are in fact highly volatile and leptokurtic. Our empirical results offer indirect evidence that market values of real firms' assets likely have similar empirical characteristics.

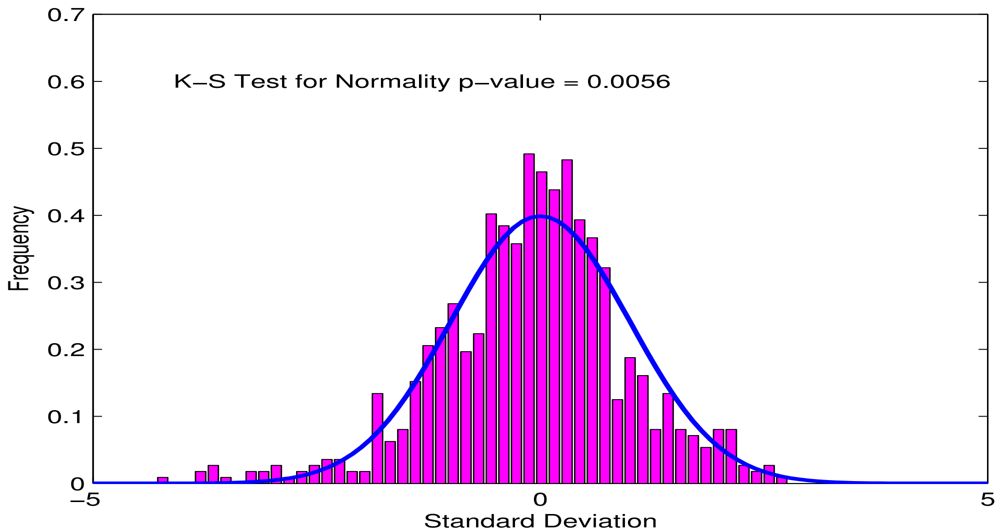
Our empirical approach can be generalized and extended in multiple directions. For instance, future research could investigate hypothetical firms with different types of traded assets, such as commodities, currencies, Treasury bonds, swaps, exchange-traded funds, and the like. As long as traded options exist on the underlying assets, our empirical Merton model can be used as a benchmark for investigating the relation between the risk characteristics of underlying assets (which are observable for pseudo firms) and the risk characteristics of pseudo bonds issued by those pseudo firms. Such empirical research could shed further light on the cross-sectional and time series determinants of credit spreads.

Future research might also extend our framework to deal with coupon-bearing pseudo bonds, pseudo bonds with embedded options, and the like. Indeed, one could use options with various maturities to extract assets' risk-neutral distributions and then use the risk-neutral methodology to value defaultable bonds with more realistic features than just zero-coupon bonds. One could then investigate the empirical properties of such bonds and shed additional light on related issues like optimal prepayment and redemption decisions, the design of structured hedges embedded into debt instruments, and more.

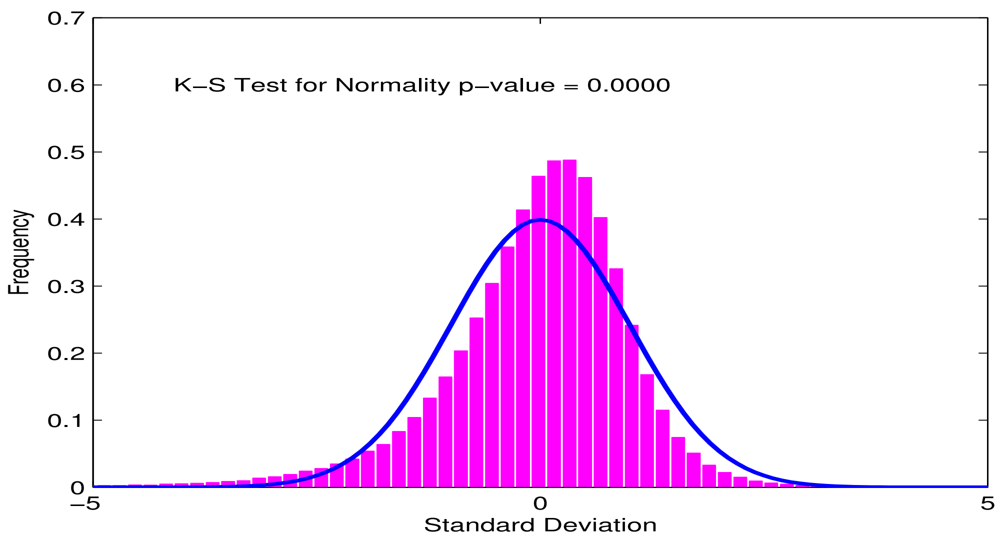
Subsequent research might also consider additional counterfactual experiments. For example, it would be interesting to extend our simple banking example to more elaborate cases, assess the appropriateness of various parametric modifications to the lognormal Merton model currently used in academia and industry, analyze the implications of legal and institutional issues like solvency tests (ability-to-pay vs. balance-sheet), and the like. One could also adopt our model-free methodology to investigate issues in corporate finance, such as the trade-off theory of capital structure in which the tax benefits of debt are traded for additional costs of financial distress. By using our pseudo firms as a laboratory, one could obtain implications that naturally take into account the true risk premia required by investors to hold pseudo bonds, and thus obtain quantitative implications of optimal capital structure in a controlled environment.

Figure 1: Normalized Monthly Shocks to Two-Year Pseudo Bonds

Panel A: S&P500 Index as Assets



Panel B: Individual Firms as Assets



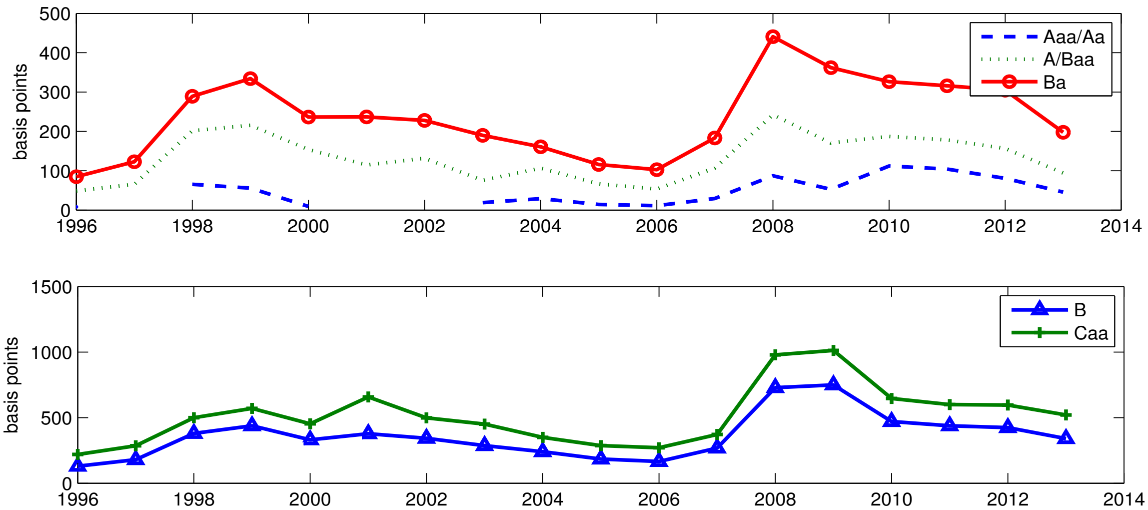
Notes: Histograms of residuals computed as

$$\epsilon^i_{t,t+\tau} = \frac{\log\left(A^i_{t+\tau}/A^i_t\right) - \left(\mu_{i,t,\tau} - \frac{1}{2}\sigma^2_{i,t,\tau}\right)}{\sigma_{i,t,\tau}}$$

In Panel A, A^i_t is the S&P 500 index, $\mu_{i,t,\tau}$ is computed from a predictive regression of future two-year returns using the dividend yield as predictors, and $\sigma_{i,t,\tau}$ is obtained from fitting a GARCH(1,1) model to monthly stock returns. All computations are made on an expanding window.

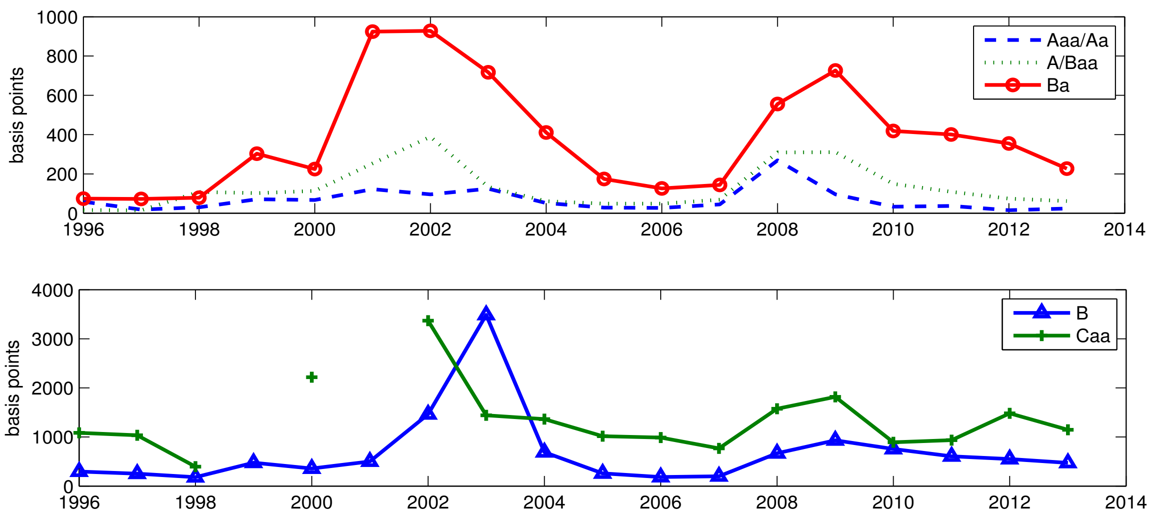
In Panel B, A^i_t are the individual stocks in the S&P 500 index, where $\mu_{i,t,\tau}$ is the average two-year stock return until t, and $\sigma_{i,t,\tau}$ is the realized volatility the previous year. For every t, all the stocks in the S&P 500 index are used to compute shocks before t to avoid survivorship bias.

Figure 2: Credit Spreads of Two-Year Pseudo Bonds



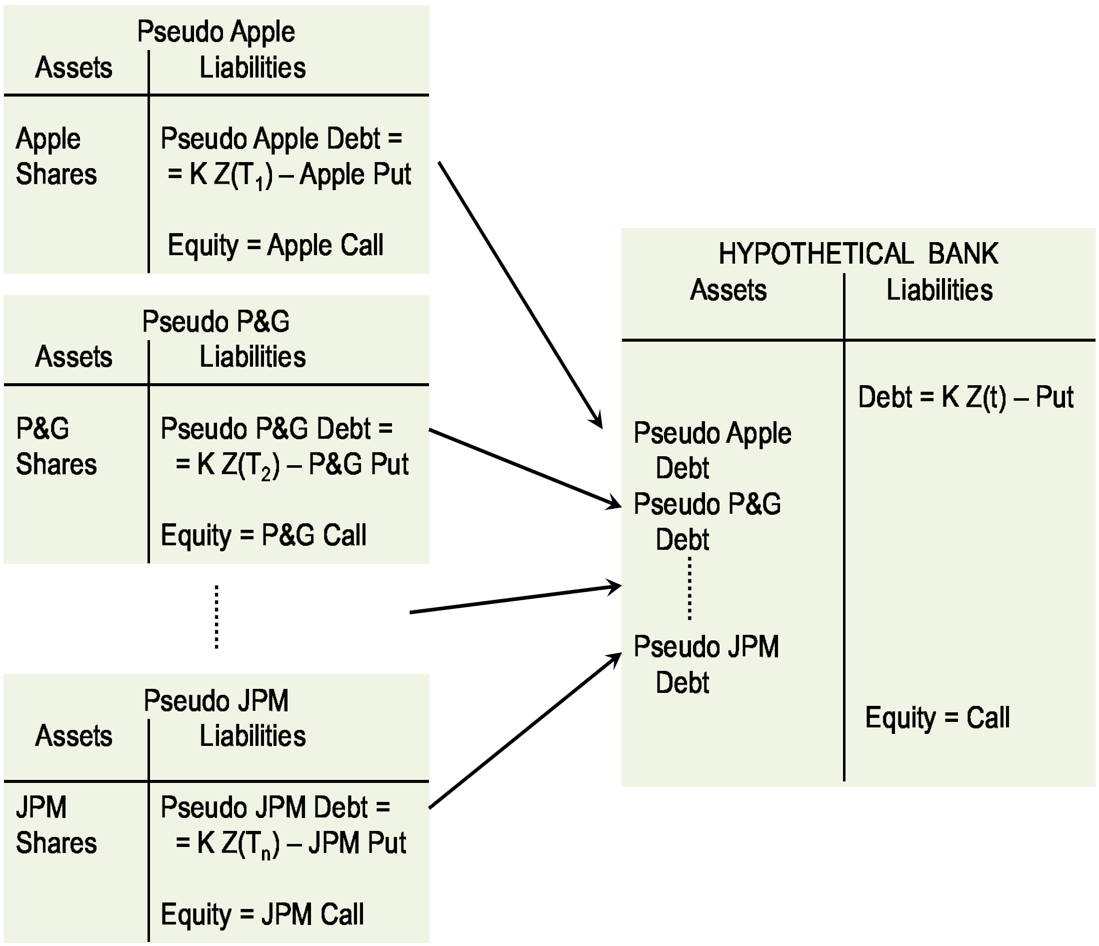
Notes: Credit spreads of two-year pseudo bonds. Pseudo bonds are constructed from a portfolio of risk free debt minus put options on the SPX index. Pseudo credit ratings of pseudo bonds are assigned based on the pseudo bond *ex ante* default probability (i.e. the probability the put option is in the money at maturity) during booms and recessions. The *ex ante* default probabilities of pseudo bonds are computed by inverting the empirical distribution of the residuals from a simple GARCH(1,1) model of asset returns with expected growth obtained from predictive regressions.

Figure 3: Credit Spreads of Two-Year Corporate Bonds



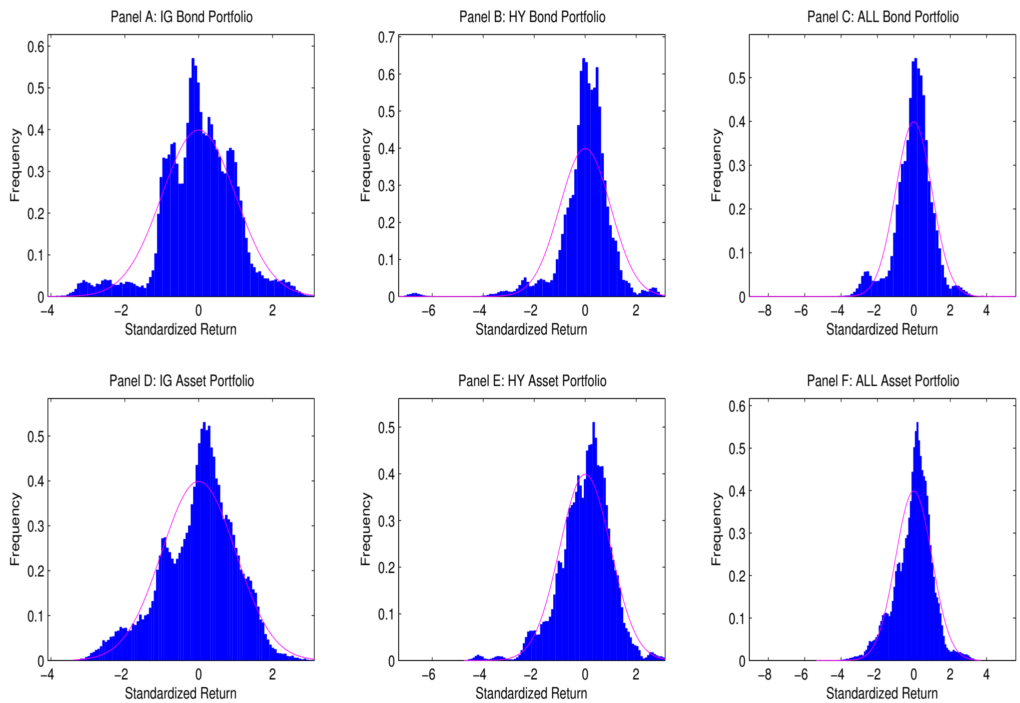
Notes: Credit spreads of two-year corporate bonds. Corporate bond data are from the Lehman Brothers Fixed Income Database, the Mergent FISD/NAIC Database, TRACE and DataStream. The sample is monthly between 1996 to 2013. The credit spread is computed as the difference between the semi-annual yield-to-maturity and the corresponding Treasury yield.

Figure 4: The Assets of a Pseudo Bank



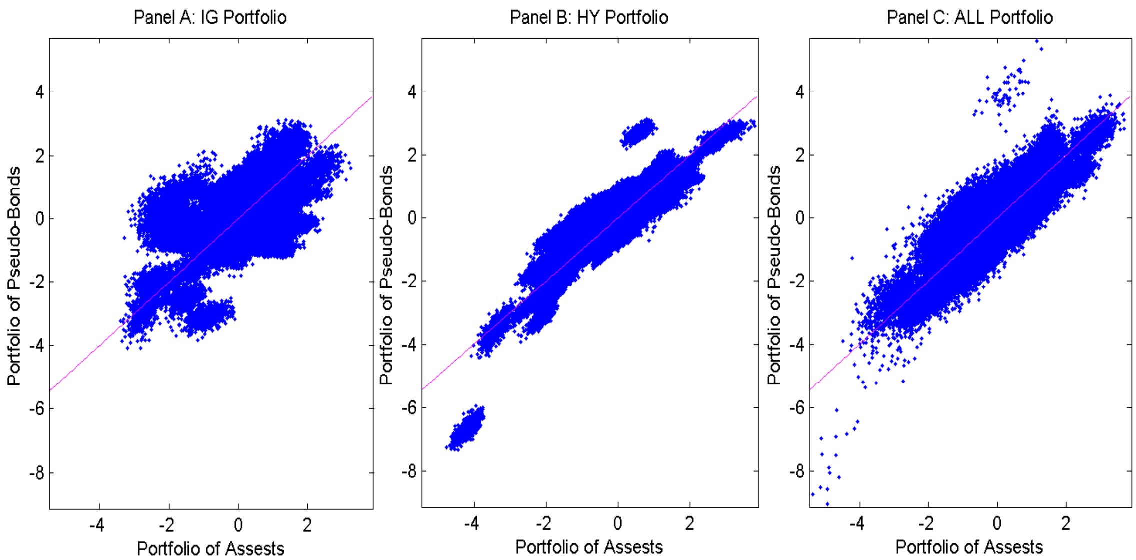
Notes: This diagram represents the assets of a fictitious pseudo bank that lends money to the pseudo firms in our sample. Pseudo firms are hypothetical firms that purchase shares of underlying traded firms, and that finance those purchases by selling equity and zero-coupon bonds. The values of these zero-coupon bonds are given by safe U.S. Treasury zero-coupon bonds minus traded put options on the underlying firms. In the figure, the pseudo bank purchases the pseudo bonds, which then form its loan asset portfolio, and finances the acquisition of its portfolio by issuing equity and short-term zero-coupon debt.

Figure 5: Return Distribution of Pseudo Bond Portfolios



Notes: Panels A, B, and C show the return distributions of random portfolios of pseudo bonds over the sample 1996 – 2013, while panel D shows the return distribution of portfolio of stocks underlying the "All Bond" portfolio. The distributions have been normalized to have zero mean and unit standard deviations. The random portfolios in each panel are constructed as follows: For every month t, we consider all potential available pseudo bonds for all the 500 firms in the S&P 500 index. We group such bonds in credit rating / maturity bins. We consider only two credit ratings: Investment Grade (i.e. Aaa/Aa and A/Baa) or High Yield (i.e. Ba, B, Caa-) and only three maturity ranges (0,273), (274,548), (549, ∞). For each firm and for each rating, we randomly choose one maturity bin per firm, when available. For the IG and HY portfolios, if the number of firms is more than 30, then we average them and record the portfolio returns. If not, we record a missing observation for the portfolio return in the month. For "All" portfolio, if the number of IG firms is more than 15, then we randomly pick the same number of HY bonds as the IG bonds, and then compute the average across all the bonds. This procedure is performed for every month t in the sample, and repeated 1,000 times to obtain return distributions.

Figure 6: Pseudo Bond Portfolio Returns versus Underlying Asset Portfolio Returns



Notes: Panels A, B, and C show the scatter-plot of pseudo bond portfolio returns versus underlying asset portfolio returns. The distributions have been normalized to have unit standard deviations. The random portfolios in Panel A are constructed as follows: For every month t, we consider all potential available pseudo bonds for all the 500 firms in the S&P 500 index. We group such bonds in credit rating / maturity bins. We consider only two credit ratings: Investment Grade (i.e. Aaa/Aa and A/Baa) or High Yield (i.e. Ba, B, Caa-) and only three maturity ranges (0,273), (274,548), (549, ∞). For each firm and for each rating, we randomly choose one maturity bin per firm, when available. For the IG and HY portfolios, if the number of firms is more than 30, then we average them and record the portfolio returns. If not, we record a missing observation for the portfolio return in the month. For "All" portfolio, if the number of IG firms is more than 15, then we randomly pick the same number of HY bonds as the IG bonds, and then compute the average across all the bonds. This procedure is performed for every month t in the sample, and repeated 1,000 times to obtain return distributions.

Table 1: Default Frequencies of Two-Year Corporate Bonds and Pseudo Bonds

Panel A of this table reports *ex post* default frequencies of corporate bonds by credit rating category (shown in the first column.) The mean is the aggregate average, and confidence intervals (C.I.) are the 2.5% and 97.5% bands of the computed frequencies. The last two columns report default frequencies during NBER booms and recessions, respectively. Panel B reports the results of our credit rating system for pseudo bonds. Pseudo bonds are constructed from a portfolio of risk free debt minus SPX put options. Pseudo credit ratings of pseudo bonds are assigned based on the pseudo bond *ex ante* default probability (i.e. the probability the put option is in the money at maturity) during booms and recessions. The *ex ante* default probabilities of pseudo bonds are computed by inverting the empirical distribution of the residuals from a simple GARCH(1,1) model of asset returns with expected growth obtained from predictive regressions. The first two columns of Panel B reports the *ex ante* average default probabilities for bonds in each pseudo credit rating category. The next three columns show the actual *ex post* default frequencies of the pseudo bonds across the pseudo credit ratings and their confidence intervals. The *ex post* default frequency is computed as the fraction of times that the two-year return (excluding dividends) on SPX index falls below the given moneyness of the pseudo bonds in each portfolio. The last two columns report the average moneyness of the options $\overline{K/A}$, and the average maturity $\overline{\tau}$ in days. The sample is 1970 to 2013.

Panel A: Corporate Bonds

Credit	Historical Default Frequencies				
Rating	Mean	C.I.(2.5%)	C.I.(97.5%)	Boom	Recession
Aaa/Aa	0.03	0.02	0.03	0.02	0.05
A/Baa	0.31	0.30	0.32	0.28	0.47
Ba	3.23	2.84	3.63	3.15	3.76
B	9.16	5.90	12.42	8.67	12.81
Caa-	25.18	0.00	82.93	21.93	41.37

Panel B: Pseudo Bonds

Credit	*Ex ante* Default Probability		*Ex post* Default Frequency				
Rating	Boom	Recession	Mean	C.I.(2.5%)	C.I.(97.5%)	$\overline{K/A}$	$\overline{\tau}$
Aaa/Aa	0.02	0.09	1.98	0.00	4.75	0.43	590
A/Baa	0.99	1.49	2.18	0.00	5.30	0.60	580
Ba	3.59	4.98	7.14	0.10	14.19	0.72	617
B	9.74	18.88	12.90	1.33	24.46	0.83	645
Caa-	23.77	45.41	20.04	5.57	34.51	0.93	650

Table 2: Two-Year Pseudo and Corporate Bonds: 1996 - 2013

Credit spreads and summary statistics are shown for pseudo bonds (Panel A), corporate bonds (Panel B), and the lognormal Merton model (Panel C). Pseudo bonds are constructed from a portfolio of risk free debt minus put options on the SPX index. Pseudo credit ratings of pseudo bonds are assigned based on the pseudo bond *ex ante* default probability (i.e. the probability the put option is in the money at maturity). The default probabilities for pseudo bonds are computed by inverting the empirical distribution of residuals from a simple GARCH(1,1) model of asset returns with expected growth obtained from predictive regressions. Corporate bonds are non-callable corporate bonds with time to maturity between 1.5 and 2.5 years. The sample period is January 1996 to August 2013. The lognormal Merton model's statistics are averages over 1,000 Monte Carlo simulations of 212 months of asset values. Simulations are designed to replicate the time-variation in volatility and predictability found in the data.

Credit	Credit Spreads			Monthly Returns in Excess of T-bill (%)				
Rating	Average	Boom	Recession	Mean	Std	Sharpe Ratio	Skew	Excess Kurtosis
			Panel A: Pseudo Bonds					
Aaa/Aa	54	51	83	0.14	0.65	0.22	0.41	5.60
A/Baa	131	121	207	0.25	0.86	0.30	0.09	4.82
Ba	237	216	375	0.27	1.41	0.19	-2.06	15.17
B	367	313	718	0.34	1.80	0.19	-1.27	7.87
Caa-	517	450	957	0.35	2.31	0.15	-1.20	6.01
			Panel B: Corporate Bonds					
Aaa/A	68	49	192	0.11	0.88	0.12	-1.01	26.34
A/Baa	133	107	305	0.11	1.39	0.08	-7.19	78.35
Ba	397	352	679	0.58	2.22	0.26	0.04	17.96
B	620	598	794	0.57	2.99	0.19	-3.00	28.31
Caa-	1213	1123	1808	0.83	3.96	0.21	0.93	7.21
			Panel C: Lognormal Merton Model					
Aaa/Aa	0	0	1	0.07	0.48	0.15	0.37	1.55
A/Baa	4	3	10	0.07	0.48	0.14	0.31	1.50
Ba	30	26	61	0.08	0.62	0.12	-0.61	3.80
B	86	72	184	0.09	0.94	0.10	-0.92	5.15
Caa-	249	195	603	0.14	1.68	0.09	-0.52	4.03

Table 3: Returns on Two-Year Pseudo Bonds and Corporate Bonds

This table reports the results of the regression specification

$$R^e_{B,t} = \alpha + \beta\, R^e_{i,t} + \epsilon_t$$

where $R^e_{B,t}$ is the excess return of pseudo bonds (Panels A and B), corporate bonds (Panel C), and simulated bonds from the lognormal Merton model (Panels D and E). The explanatory variable $R^e_{i,t}$ is the excess return on assets ($i = A$, Panels A and D) or equity ($i = E$, Panels B, C, and E). In all cases, bonds are sorted monthly into credit rating categories, and portfolio returns in excess of the U.S. Treasury bill rate are computed over the following month. The sample is January 1996 to August 2013. Statistics for the lognormal Merton model in Panels D and E are averages of 1,000 Monte Carlo simulations of 212 months of underlying asset values. Simulations are designed to replicate the time-variation in volatility and predictability found in the data.

Credit Rating	Mean (%)	$t(Mean)$	α	$t(\alpha)$	β	$t(\beta)$	R^2
			Panel A: Pseudo Bonds on Assets				
Aaa/Aa	0.14	(2.80)	0.12	(2.61)	0.07	(4.00)	0.18
A/Baa	0.25	(4.17)	0.13	(2.38)	0.18	(8.99)	0.53
Ba	0.27	(2.70)	0.16	(2.60)	0.27	(13.41)	0.68
B	0.34	(2.83)	0.18	(2.72)	0.37	(18.16)	0.76
Caa-	0.35	(2.19)	0.17	(2.46)	0.49	(23.07)	0.83
			Panel B: Pseudo Bonds on Equities				
Aaa/Aa	0.14	(2.80)	0.13	(2.67)	0.03	(3.11)	0.12
A/Baa	0.25	(4.17)	0.14	(2.15)	0.07	(7.18)	0.42
Ba	0.27	(2.70)	0.19	(2.38)	0.09	(9.14)	0.52
B	0.34	(2.83)	0.23	(2.32)	0.10	(10.30)	0.54
Caa-	0.35	(2.19)	0.23	(1.88)	0.10	(10.99)	0.56
			Panel C: Corporate Bonds on Equities				
Aaa/Aa	0.11	(1.76)	0.11	(1.03)	0.02	(0.67)	0.02
A/Baa	0.11	(1.12)	0.19	(3.11)	0.05	(2.91)	0.15
Ba	0.58	(3.69)	0.44	(4.49)	0.09	(3.57)	0.28
B	0.57	(2.43)	0.59	(3.85)	0.05	(3.59)	0.11
Caa-	0.83	(2.32)	0.60	(2.16)	0.14	(5.42)	0.40
			Panel D: Lognormal Merton Model's Bonds on Assets				
Aaa/Aa	0.07	(2.24)	0.07	(1.85)	0.00	(0.24)	0.00
A/Baa	0.07	(2.08)	0.07	(1.74)	0.01	(1.43)	0.01
Ba	0.08	(1.81)	0.06	(1.61)	0.07	(5.57)	0.21
B	0.09	(1.49)	0.06	(1.44)	0.15	(7.65)	0.49
Caa-	0.14	(1.23)	0.07	(1.29)	0.33	(9.40)	0.73
			Panel E: Lognormal Merton Model's Bonds on Equities				
Aaa/Aa	0.07	(2.24)	0.07	(1.85)	0.00	(-0.68)	0.01
A/Baa	0.07	(2.08)	0.07	(1.77)	0.00	(0.38)	0.01
Ba	0.08	(1.81)	0.06	(1.63)	0.02	(4.73)	0.15
B	0.09	(1.49)	0.07	(1.41)	0.03	(7.89)	0.37
Caa-	0.14	(1.23)	0.06	(1.00)	0.05	(9.93)	0.54

Table 4: The Term Structure of Credit Spreads

This table reports the term structure of credit spreads for pseudo bonds (Panel A), corporate bonds (Panel B), and the lognormal Merton model (Panel C). Pseudo bonds are constructed from a portfolio of risk free debt minus put options on the SPX index. Pseudo credit ratings of pseudo bonds are assigned based on the pseudo bond *ex ante* default probability (i.e. the probability the put option is in the money at maturity). The pseudo bonds default probabilities are computed by inverting the empirical distribution of residuals from a simple GARCH(1,1) model of asset returns with expected growth obtained from predictive regressions. For very short maturities there is not enough granularity in strike prices to compute pseudo bonds for high credit ratings and thus we only report "investment grade" (IG) pseudo bonds. Corporate bonds' credit spreads for maturities between 30 and 91 days are based on commercial paper rates. Corporate bonds' credit spreads for maturities between 183 and 730 days are based non-callable corporate bonds. The Merton model's statistics are based on Monte Carlo simulations to replicate the time-variation in volatility and in predictability. Credit spreads are in basis points. The sample is January 1996 to August 2013.

Credit	Days to Maturity				
Rating	30	91	183	365	730
			Panel A: Pseudo Bonds		
IG	77	64	69	75	108
Aaa/Aa			50	42	54
A/Baa			106	97	131
Ba	165	133	169	186	237
B	286	262	287	311	367
Caa-	503	495	471	469	517
			Panel B: Corporate Bonds		
IG	62	60	83	127	123
Aaa/Aa	32	30	42	68	68
A/Baa	69	67	92	139	133
Ba			232	329	397
B			276	517	620
Caa-			1225	1264	1213
			Panel C: Lognormal Merton Model		
IG	0	0	0	0	0
Aaa/Aa	0	0	0	0	0
A/Baa	0	0	0	1	4
Ba	1	2	5	11	30
B	4	10	19	39	86
Caa-	41	77	113	166	249

Table 5: Assets as Common Stock of Individual Firms in the S&P500 Index

This table reports our empirical results when pseudo firms have assets that comprise shares of individual stocks in the S&P 500 index. For each credit rating, the first three columns report the target default frequencies of real corporate bonds and the confidence intervals. The next three columns report the *ex post* default frequencies of pseudo bonds and their confidence interval. The *ex post* default frequency is computed as the fraction of times that the two-year return of individual stocks falls below the average moneyness for each credit rating between 1970 and 2013. The credit rating of each pseudo bond is assigned every month according to its *ex ante* default probability over the next two years. The latter is computed non-parametrically for each firm and for each time t by exploiting the empirical distribution of normalized shocks of all the firms in the S&P500 index prior to t to avoid survivorship bias. Confidence intervals use standard errors that are corrected for the series correlation induced by overlapping observations. The last two columns report the average moneyness $\overline{K/A}$ and the average maturity $\overline{\tau}$ of the pseudo bonds.
Panel B reports summary statistics of the pseudo bond portfolios. Column 5 reports the equal weighted average credit spread of pseudo bonds in each credit rating category, while the next several columns report summary statistics of excess returns. For each credit rating, Panel C reports the time-series regression of the pseudo bond portfolio excess returns on the average excess returns of pseudo assets (i.e. stocks of underlying individual firms). For each credit rating, Panel D reports the time-series regression of the pseudo bond portfolio excess returns on the average excess returns of pseudo equity (i.e. call options of the underlying individual firms).

Panel A: Summary Statistics

Credit Rating	Target Probability of Default: Historical Defaults 1970-2013			*Ex post* Probability of Default: Stock Returns 1970-2013			$\overline{K/A}$	$\overline{\tau}$
	Target PD	C.I. (2.5%)	C.I. (97.5%)	PD	C.I. (2.5%)	C.I. (97.5%)		
Aaa/Aa	0.03	0.02	0.03	0.20	0.00	0.45	0.33	641
A/Baa	0.31	0.30	0.32	0.75	0.00	1.58	0.38	608
Ba	3.23	2.84	3.63	3.85	0.69	7.01	0.48	631
B	9.16	5.90	12.42	8.71	3.54	13.88	0.63	658
Caa-	25.18	0.00	82.93	22.55	15.93	29.16	0.83	682

Panel B: Average Credit Spreads and Monthly Returns' Summary Statistics

	Credit Spreads			Monthly Returns in Excess of T-bill (%)				
	Average	Boom	Recession	Mean	Std	SR	Skew	Ex. Kurt
Aaa/Aa	89	89	82	-0.08	1.42	-0.06	-5.80	40.56
A/Baa	123	118	155	0.12	0.82	0.14	-0.63	5.47
Ba	190	184	231	0.22	1.08	0.21	-0.77	7.79
B	336	312	497	0.27	1.51	0.18	-1.83	10.30
Caa-	656	588	1103	0.50	2.07	0.24	-0.88	3.40

Panel C: Regression of Pseudo Bonds Excess Returns on Assets' Excess Returns

	Mean (%)	$t(Mean)$	α	$t(\alpha)$	β	$t(\beta)$	R^2
Aaa/Aa	-0.08	(-0.80)	-0.17	(-1.13)	0.11	(2.42)	0.26
A/Baa	0.12	(2.00)	0.08	(1.89)	0.08	(6.48)	0.35
Ba	0.22	(3.14)	0.10	(1.59)	0.14	(8.68)	0.62
B	0.27	(2.70)	0.07	(0.89)	0.26	(8.07)	0.69
Caa-	0.50	(3.57)	0.10	(1.16)	0.43	(11.94)	0.79

Panel D: Regression of Pseudo Bonds Excess Returns on Pseudo Equities' Excess Returns

	Mean (%)	$t(Mean)$	α	$t(\alpha)$	β	$t(\beta)$	R^2
Aaa/Aa	-0.08	(-0.80)	-0.07	(-0.46)	0.03	(1.34)	0.08
A/Baa	0.12	(2.00)	0.11	(2.24)	0.04	(5.29)	0.29
Ba	0.22	(3.14)	0.26	(4.56)	0.07	(6.43)	0.35
B	0.27	(2.70)	0.29	(4.39)	0.13	(9.68)	0.59
Caa-	0.50	(3.57)	0.47	(5.34)	0.17	(14.77)	0.62

Table 6: Time Series Regression on Risk Factors

This table reports the result of the following time-series regression for each bond portfolio:

$$R^e_{i,t} = \alpha_i + \beta_i\ RMRF_t + c_i\ TERM_t + d_i\ DEF_t + e_i\ dVIXSQ_t + f_i\ dTED_t + g_i\ Tail_t + \epsilon_{i,t},$$

where $R^e_{i,t}$ is the excess return on portfolio i, $RMRF_t$ is the excess return on the value-weighted stock market portfolio, $TERM_t$ is the return on the long-term Treasury bonds in excess of T-bill rates, DEF_t is the return on the aggregate long-term corporate bond market portfolio from Ibbotson in excess of the return on the long-term Treasury bonds, $dVIXSQ_t$ is the return on the square of the VIX index in excess of risk free rate, and $dTED_t$ is the change in the TED spread. $Tail_t$ is the "tail" risk factor of Jiang and Kelly (2014). $\bar{R}^2$ is adjusted R-squared and t-statistics are in parenthesis. The sample is monthly from January 1996 to August 2013.

	α_i	$RMRF_t$	$TERM_t$	DEF_t	$dVIXSQ_t$	$dTED_t$	$Tail$	$\bar{R}^2$
				Panel A: Pseudo Bonds (SPX)				
Aaa/Aa	0.18	0.07	0.03	0.01	-0.07	0.39	0.01	0.25
	(2.77)	(3.63)	(1.99)	(0.35)	(-0.55)	(1.91)	(0.57)	
A/Baa	0.27	0.09	0.03	0.09	-0.50	-0.12	0.03	0.51
	(4.65)	(7.15)	(2.28)	(2.66)	(-3.31)	(-0.54)	(2.91)	
Ba	0.30	0.18	0.03	0.08	-0.76	0.52	0.02	0.65
	(4.29)	(7.14)	(1.49)	(2.24)	(-3.93)	(1.22)	(2.25)	
B	0.39	0.25	0.06	0.16	-0.86	0.46	0.03	0.77
	(5.67)	(9.94)	(3.15)	(3.38)	(-4.26)	(1.13)	(2.61)	
Caa-	0.33	0.36	0.05	0.18	-0.85	0.62	0.03	0.82
	(4.14)	(12.08)	(2.25)	(3.20)	(-3.59)	(1.44)	(1.79)	
				Panel B: Pseudo Bonds (Individual Stocks)				
Aaa/Aa	-0.13	0.17	0.04	0.02	0.59	0.63	-0.02	0.04
	(-0.40)	(1.74)	(0.85)	(0.13)	(1.14)	(0.49)	(-0.28)	
A/Baa	0.23	0.04	0.06	0.06	-0.46	0.08	0.04	0.26
	(3.23)	(1.62)	(2.56)	(1.51)	(-2.42)	(0.26)	(1.41)	
Ba	0.30	0.12	0.05	0.07	-0.43	-0.16	-0.01	0.48
	(4.23)	(5.42)	(2.59)	(2.08)	(-2.45)	(-0.47)	(-0.88)	
B	0.37	0.18	0.07	0.17	-0.81	0.31	-0.01	0.71
	(4.65)	(7.12)	(2.80)	(4.16)	(-4.27)	(0.79)	(-0.43)	
Caa-	0.55	0.28	0.07	0.24	-0.96	0.57	0.01	0.79
	(6.10)	(10.44)	(2.68)	(4.41)	(-4.86)	(1.65)	(0.54)	
				Panel C: Corporate Bonds				
Aaa/Aa	0.33	-0.05	0.08	0.13	-0.52	-1.28	0.02	0.41
	(4.36)	(-2.37)	(4.19)	(2.54)	(-2.24)	(-3.02)	(1.72)	
A/Baa	0.27	0.05	0.10	0.14	0.22	-0.72	0.00	0.10
	(2.44)	(1.01)	(3.19)	(4.44)	(0.41)	(-0.92)	(-0.14)	
Ba	0.78	0.03	0.07	0.17	-0.75	0.28	0.01	0.04
	(4.31)	(0.44)	(2.08)	(3.42)	(-1.89)	(0.40)	(0.11)	
B	0.78	0.03	0.12	0.26	-0.17	0.35	0.19	0.03
	(3.03)	(0.70)	(2.48)	(2.82)	(-0.43)	(0.66)	(1.54)	
Caa/C	0.86	0.23	0.14	0.42	-1.02	2.23	-0.44	0.20
	(2.01)	(3.19)	(1.72)	(3.60)	(-1.23)	(1.97)	(-2.21)	

Table 7: The Transactional Liquidity of Pseudo Bonds and Corporate Bonds

Panels A and B show credit spreads, average monthly returns in excess of T-bills, and transalctional liquidity measures of pseudo bonds based on the SPX and individual stocks, respectively. The bid-ask spread for pseudo bond i in month t is computed by $(B_{i,t}^{Ask} - B_{i,t}^{Bid})/B_{i,t}^{Mid}$. The portfolio bid-ask spread is the kernel-weighted average of pseudo bonds, where the kernel is the same as the one used for returns. The Roll (1984) measure for pseudo bond i in month t is computed by $\sqrt{-Cov_t(\Delta p_{i,t,d}^{Bid \to Ask}, \Delta p_{i,t,d+1}^{Ask \to Bid})}$ using the daily price observations. We compute the Roll measure for all pseudo bonds that have more than 10 return observations in a month. The portfolio-level Roll measure is computed by the kernel-weighted average of the pseudo bonds for which we can compute the Roll measure.

Panel C shows the same statistics for corporate bonds. The Roll measure for corporate bond i in month t is computed by $2\sqrt{-Cov_t(\Delta p_{i,t,d}^{Transaction}, \Delta p_{i,t,d+1}^{Transaction})}$ using the daily price observations. We compute the Roll measure for all corporate bonds that have more than 10 return observations in a month. As in credit spreads and excess returns, the Roll measure for a portfolio is the value-weighted average of the corporate bonds for which we can compute the Roll measure.

Credit Rating	Credit Spread (bps)	Mean Returns (%)	Bid-Ask Spread (%)	Roll Measure (%)
Panel A: Pseudo Bonds (SPX)				
Aaa/Aa	54	0.14	0.25	0.08
A/Baa	131	0.25	0.25	0.08
Ba	237	0.27	0.28	0.12
B	367	0.34	0.28	0.14
Caa-	517	0.35	0.28	0.18
Panel B: Pseudo Bonds (Individual Stocks)				
Aaa/Aa	89	-0.08	1.36	0.32
A/Baa	123	0.12	1.20	0.53
Ba	190	0.22	1.26	0.50
B	336	0.27	1.30	0.48
Caa-	656	0.50	1.37	0.51
Panel C: Corporate Bonds				
Aaa/Aa	68	0.11		0.51
A/Baa	133	0.11		1.03
Ba	397	0.58		1.78
B	620	0.57		2.04
Caa-	1213	0.83		3.00

Table 8: Sorting Frequency and Pseudo Bond Returns

Credit spreads and excess return summary statistics are shown for pseudo bonds (Panel A), corporate bonds (Panel B), and the lognormal Merton model (Panel C). Pseudo bonds are constructed from a portfolio of risk free debt minus SPX put options. Pseudo credit ratings of pseudo bonds are assigned based on the pseudo bond *ex ante* default probability (i.e. the probability the put option is in the money at maturity). The pseudo bond default probabilities are computed by inverting the empirical distribution of residuals from a simple GARCH(1,1) model of asset returns with expected growth obtained from predictive regressions. The sample is January 1996 to August 2013. Credit spreads are expressed in basis points.

Credit Rating	Credit Spread			Monthly Returns in Excess of T-bill (%)				
	Average	Boom	Recession	Mean	Std	Sharpe Ratio	Skew	Excess Kurtosis
				Panel A: Sort Every 3 Months				
Aaa/Aa	57	53	91	0.11	0.69	0.16	0.07	6.16
A/Baa	132	118	228	0.18	1.12	0.16	-3.63	35.46
Ba	246	214	452	0.25	1.51	0.17	-2.31	18.33
B	385	312	880	0.35	1.94	0.18	-1.95	14.53
Caa-	553	449	1233	0.35	2.45	0.14	-1.43	8.84
				Panel B: Sort Every 6 Months				
Aaa/Aa	67	55	164	0.10	0.73	0.14	0.07	5.73
A/Baa	142	110	347	0.14	1.14	0.12	-3.77	34.96
Ba	249	204	541	0.21	1.53	0.14	-2.46	17.82
B	371	299	834	0.26	1.96	0.13	-2.08	13.68
Caa-	732	441	2623	0.30	2.53	0.12	-1.53	8.37
				Panel C: Sort Every 12 Months				
Aaa/Aa	53	39	165	0.10	0.51	0.20	2.00	10.07
A/Baa	127	93	335	0.16	0.75	0.21	0.65	6.94
Ba	273	200	707	0.12	1.61	0.07	-2.46	17.37
B	372	285	916	0.15	1.97	0.08	-1.87	12.55
Caa-	837	452	3348	0.17	2.64	0.06	-1.74	10.15

Table 9: Asset Uncertainty and Credit Spreads of Pseudo Bonds

This table shows the impact of asset volatility on pseudo bond' credit spreads and returns. The sample is the pseudo bonds of pseudo firms whose assets are the stock of individual firms that are in the S&P 500 index. Pseudo bonds are portfolios of risk-free debt minus put options on the underlying assets (i.e. stock) of individual firms. Pseudo credit ratings are assigned using a model-free methodology that computes the probability of default at maturity. For each time t, we first sort pseudo bonds according to their pseudo credit rating, and then according to the volatility of their pseudo firm's assets (individual stocks). Panel A reports the average credit spreads for each credit rating / volatility bin, and Panel B reports the corresponding average excess returns. Panels C and D report the average leverage K/A and the average asset volatility for each credit rating / volatility combination.

Credit Rating	Volatility Low	Volatility Medium	Volatility High	Credit Rating	Volatility Low	Volatility Medium	Volatility High
Panel A: Credit Spread				Panel B: Average Excess Returns			
Aaa/Aa	71	90	134	Aaa/Aa	-0.15	0.00	0.20
A/Baa	117	113	152	A/Baa	0.21	0.14	0.09
Ba	165	177	237	Ba	0.20	0.20	0.18
B	335	341	429	B	0.26	0.29	0.44
Caa-	651	691	792	Caa-	0.50	0.56	0.77
Panel C: Average Leverage $\overline{K/A}$				Panel D: Volatility			
Aaa/Aa	0.39	0.30	0.24	Aaa/Aa	0.20	0.22	0.28
A/Baa	0.50	0.43	0.38	A/Baa	0.22	0.26	0.34
Ba	0.58	0.51	0.46	Ba	0.24	0.30	0.40
B	0.73	0.65	0.59	B	0.25	0.32	0.43
Caa-	0.90	0.84	0.78	Caa-	0.25	0.33	0.44

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Appendix A. Proof of Proposition 1.

Proof of Proposition 1. (a) Immediate from the properties of the Black and Scholes formula.

(b) From Ito's lemma:

$$dB = rKe^{-r(T-t)}dt - \left(\frac{\partial P}{\partial t} + \frac{\partial P}{\partial A}\mu_A A + \frac{1}{2}\frac{\partial^2 P}{\partial A^2}A^2\sigma_A^2\right)dt - \frac{\partial P}{\partial A}A\sigma_A dW$$

The Black and Scholes pricing Partial Differential Equation has

$$\frac{\partial P}{\partial t} + \frac{1}{2}\frac{\partial^2 P}{\partial A^2}A^2\sigma_A^2 = rP - \frac{\partial P}{\partial A}Ar$$

Substitution into the previous equation proves the claim, with

$$\beta = \frac{-\frac{\partial P}{\partial A}A}{B} = \frac{\sigma_B\sigma_A}{\sigma_A^2} = \frac{Cov(dA/A, dB/B)}{Var(dA/A)}$$

and where $\sigma_B = -\frac{1}{B}\frac{\partial P}{\partial A}A\sigma_A$.

The proof of part (c) follows from the same steps as in part (b) but applied to a call option.

Part (d) also follows from the excess return expression above, once we divide by σ_B the expected return equation. Q.E.D.

Appendix B. Jumps and Stochastic Volatility in the Merton Framework.

Some of the empirical results in the paper can be better understood if we examine the specific implications for relaxing the original Merton lognormality assumption and assume instead that the market value of the firm's assets A_t follows a jump-diffusion process with stochastic volatility:

$$dA_t = [\mu_A - \lambda E(J_A - 1)] A_t dt + \sigma_{A,t} A_t dW_{A,t} + (J_A - 1) A_t dQ_t \quad (13)$$

$$d\sigma_{A,t} = \mu_\sigma(\sigma_{A,t})\, dt + s(\sigma_{A,t})\, dW_{\sigma,t} \quad (14)$$

where dQ_t is the increment of a Poisson process with intensity λ, J_A is a random variable determining the size of the jump (*see, e.g.,* Zhou (2001)), and $\mu_\sigma(.)$ and $s(.)$ are a drift and diffusion that satisfy the usual regularity conditions. Following the analysis of Broadie, Chernov, Johannes (2009), we then obtain the following:

Proposition 2. Under the asset dynamics in Equations (13) and (14), the bond price $B_t(T, K)$ in expression (3) has a risk premium given by

$$\mu_B - r = [\alpha_B - \beta_A\, \alpha_A + \beta_\sigma \xi s(\sigma_{A,t})] + \beta_A\, (\mu_A - r) \quad (15)$$

where $\beta_A = \frac{\partial \ln(B(t,A,\sigma_A))}{\partial \ln A}$ is the loading on the "asset risk", $\beta_\sigma = \frac{\partial \ln(B(t,A,\sigma_A))}{\partial \sigma_A}$ is the loading on volatility risk, α_B and α_A are the jump risk premia on bonds and on assets, respectively, and ξ is the market price of volatility risk.

Expression (15) illustrates how the violations of Merton's lognormality assumption manifest themselves in the risk premium. Because generally $\alpha_B \neq \beta\, \alpha_A$, we should expect a non-zero estimated intercept in a regression of excess bond returns on excess asset returns if jumps reflect an important component of the bond's excess returns and/or volatility dynamics are priced.[30]

Proof of Proposition 2. From standard arguments, the pricing partial differential equation of $B_t = B(t, A, \sigma)$ when A follows a jump-diffusion process with stochastic volatility is

$$\begin{aligned} & \frac{\partial B}{\partial t} + \frac{1}{2}\frac{\partial^2 B}{\partial A^2} A^2 \sigma_A^2 + \frac{1}{2}\frac{\partial^2 B}{\partial \sigma_A^2} s(\sigma_A)^2 + \frac{\partial^2 B}{\partial A \partial \sigma_A} A \sigma_{A,t} s(\sigma_{A,t})\, \rho_{A,\sigma} \\ = \;& rB - \frac{\partial B}{\partial A} A \{r - \lambda^* E^*[J_A - 1]\} - \frac{\partial B}{\partial \sigma_A}[\mu_\sigma(\sigma_A) - \xi s(\sigma_A)] - \lambda^* E^*[B(AJ_A, t) - B(A, t)] \end{aligned}$$

[30] As discussed in Broadie et al. (2009, Appendix B), additional alpha may result from discretization bias and the covariance between asset value and volatility.

where λ^* is the risk neutral jump probability, and $E^*[]$ are the risk neutral expectations of the jump J_A, and ξ is the market price of volatility risk. From Ito's lemma, the process for B under the physical measure is

$$\begin{aligned} dB &= \left\{ \frac{\partial B}{\partial t} + \frac{\partial B}{\partial A} A \left[\mu_A - \lambda E(J_A - 1) \right] + \frac{\partial B}{\partial \sigma_A} \mu_\sigma \left(\sigma_A \right) + \frac{1}{2} \frac{\partial^2 B}{\partial A^2} \sigma_A^2 A^2 + \frac{1}{2} \frac{\partial^2 B}{\partial \sigma_{A,t}^2} s \left(\sigma_A \right)^2 \right. \\ &\quad \left. + \frac{\partial^2 B}{\partial A \partial \sigma_{A,t}} \sigma_A A s \left(\sigma_{A,t} \right) \rho_{A,\sigma} \right\} dt + \frac{\partial B}{\partial A} \sigma_A A dW_{A,t} + \frac{\partial B}{\partial \sigma_A} s \left(\sigma_A \right) dW_{\sigma,t} \\ &\quad + \left[B(AJ_A, t) - B(A,t) \right] dQ \end{aligned}$$

Taking the expectation under the physical measure, and using the PDE above, we obtain

$$\begin{aligned} E[dB]/dt &= rB - \frac{\partial B}{\partial A} A \left\{ r - \lambda^* E^*[J_A - 1] \right\} - \lambda^* E^* \left[B(AJ_A, t) - B(A,t) \right] \\ &\quad + \frac{\partial B}{\partial A} A \left[\mu_A - \lambda E(J_A - 1) \right] + \frac{\partial B}{\partial \sigma_A} \xi s \left(\sigma_A \right) + \lambda E \left[B(AJ_A, t) - B(A,t) \right] \end{aligned}$$

or

$$\begin{aligned} E\left[\frac{dB}{B} \right] /dt - r &= \frac{1}{B} \frac{\partial B}{\partial A} A \left[\mu_A - r - \left[\lambda E(J_A - 1) - \lambda^* E^*[J_A - 1] \right] \right] + \frac{1}{B} \frac{\partial B}{\partial \sigma_A} \xi s \left(\sigma_A \right) \\ &\quad + \lambda E \left[\frac{B(AJ_A, t)}{B} - 1 \right] - \lambda^* E^* \left[\frac{B(AJ_A, t)}{B} - 1 \right] \\ &= \alpha_B - \beta_A \alpha_A + \beta_\sigma \xi s \left(\sigma_A \right) + \beta_A [\mu_A - r] \end{aligned}$$

where

$$\begin{aligned} \beta_A &= \frac{1}{B} \frac{\partial B}{\partial A} A; \quad \beta_\sigma = \frac{1}{B} \frac{\partial B}{\partial \sigma_A} \\ \alpha_A &= \lambda E(J_A - 1) - \lambda^* E^*[J_A - 1] = \text{jump risk premium of assets} \\ \alpha_B &= \lambda E \left[\frac{B(AJ_A, t)}{B} - 1 \right] - \lambda^* E^* \left[\frac{B(AJ_A, t)}{B} - 1 \right] = \text{jump risk premium of } B \end{aligned}$$

Q.E.D.

Appendix C. Data.

We use the OptionMetrics Ivy DB database for daily prices on SPX index options and options on individual stocks from January 4, 1996, through August 31, 2013. To minimize the effects of quotation errors in SPX options, we generally follow Constantinides, Jackwerth and Savov (2013) ("CJS") to filter the data. As in CJS, we apply the filters only to the prices to buy – not to the prices to sell – so that our portfolio formation strategy is feasible for real-time investors. As in CJS, we apply the following specific filters:

1. *Level 1 Filters:* We remove all but one of any duplicate observations. If there are quotes with identical contract terms but different prices, we pick the quote with the implied volatility ("IV") closest to that of the moneyness of its neighbors and remove the others. We also remove the quotes with bids of zero.

2. *Level 2 and Level 3 Filters:* Because we need quotes for long-term, deep out-of-the-money puts and deep in-the-money calls, we do not apply filters based on moneyness or maturity, but we remove all options with zero open interest. Following CJS, we also remove options with less than seven days to maturity. We also apply "implied interest rate < 0," "unable to compute IV," "IV," and "put-call parity" filters.[31]

For individual equity options, as put-call parity only holds with inequality for American options, we apply a different set of filters. We follow Frazzini and Pedersen (2012) to detect likely data errors. Specifically, we drop all observations for which the ask price is lower than the bid price and the bid price is equal to zero. In addition, we require options to have positive open interest, and non-missing delta, implied volatility, and spot price. We also drop options violating the basic arbitrage bound of a non-negative time value P-V where V is the option intrinsic value equal to $\max(K - S, 0)$ for puts. We then drop equity options with a time value $(P - V)/P$ (in percentage of option value) below 5%, as the low time value tends to lead to early exercise. Finally, to mitigate the effect of the outliers, we drop options with embedded leverage, $\frac{\partial P}{\partial S}\frac{S}{P}$, in the top or bottom 1% of the distribution.

We obtain stock prices and accounting information from the Center for Research in Security Prices ("CRSP"). We use SPX returns in the postwar period (1946 - 2013) to compute asset returns and *ex ante* default probabilities for our pseudo firms.

We construct the risk-free zero coupon bonds from 1-, 3-, and 6-month T-bill rates and 1-, 2-, and 3-year constant maturity Treasury yields obtained from the Federal Reserve Economic Data ("FRED") database. We convert constant maturity yields into zero-coupon yields and linearly interpolate to match option maturities. We also obtain commercial paper rates from FRED, which we use to compute credit spreads for short-term debt.

We construct the panel data of corporate bond prices from the Lehman Brothers Fixed Income Database, TRACE, the Mergent FISD/NAIC Database, and DataStream, prioritized in this order when there are overlaps among the four databases. Detailed descriptions of these databases and the effects of prioritization are discussed in Nozawa (2014). In addition, we remove bonds with floating coupon rates and embedded option features. We also apply several filters to remove observations that may be subject to erroneous recording. Following Duffee (1998), we remove bonds with buy-in prices greater than twice and less than 1/100 of their par amounts. We also remove observations for bonds that show large bounce-backs. Specifically, we compute the product of adjacent monthly returns and remove both observations if the product is less than -0.04. For example, if the price of a given bond jumps up by more than 20 percent in one month and then comes down by more than 20 percent in the following month, we assume that the price observation in the middle is recorded with errors and exclude that observation.

Appendix D. Default Frequencies.

Our goal is to construct pseudo bonds that match the realized default frequencies of the actual corporate bonds used as our main empirical benchmark. To that end, we employ a

[31] The "implied interest rate <0" filter removes the options with negative interest rates implied by put-call parity. The "unable to compute IV" filter removes options that imply negative time value. The "IV" filter removes options for which implied volatility is one standard deviation away from the average among the peers. In this case, the peer group is defined by the bins of moneyness with a width of 0.05. The "put-call parity" filter removes options for which the put-call parity implied interest rate is more than one standard deviation away from the average among the peers.

large dataset of corporate defaults spanning the 44-year period from 1970 to 2013 obtained from Moody's Default Risk Service. For each credit rating assigned by Moody's to our universe of firms, we estimate *ex post* default frequencies at various horizons from 30 days up to two years. We use our own estimates rather than the original Moody's default frequencies for two main reasons. First, we are interested in the variation of default frequencies over the business cycle, whereas Moody's historical default frequencies are only available as unconditional averages. Second, we are interested in the default frequencies at horizons of below one year, and default frequencies are not provided by Moody's for such short time horizons.

Table A1 reports historical default rates from 1970 through 2013 from our sample of firms across credit rating categories and time horizons. We compute historical default frequencies separately for international and U.S. firms. Our results are directly comparable to Moody's historical default rates (reported in Moody's (2014)) for one- and two-year horizons. As Table A1 shows, our estimated default rates closely match the Moody's global default rates for those horizons.

The last two columns of Table A1 report default rates for U.S. firms in NBER-dated booms and recessions. Predictably, we find that default frequencies are higher in recessions than in booms across all credit ratings. At the 1-year horizon, for instance, A-rated bonds have a default frequency of only 0.02% in booms but 0.13% in recessions (as compared to an unconditional U.S. average of 0.04%). Default frequencies for speculative-grade bonds also show large variations over the business cycle. For example, a B-rated bond has a 3.57% default rate at the 1-year horizon during booms but more than twice that in recessions (as compared to an unconditional average of 4.01%).

Table A1 also shows default frequencies at short horizons of 30, 91, and 183 days. At the 30-day horizon, all investment-grade bonds have essentially zero historical default frequencies (although, in recessions, the historical default rate ticks up 0.01% for bonds rated A- and Baa). Some more action for these bonds is observable at the 91- and 183-day horizons, especially during recessions. For example, Baa-rated bonds have defaulted with 0.04% and 0.12% frequencies at the 91- and 183-day horizons (respectively) during recessions, which are much higher than the corresponding unconditional default frequencies of 0.02% and 0.05%. High-yield bonds, by contrast, exhibit relatively substantial historical default activity even at short horizons. For instance, B-rated bonds have 0.22%, 0.75%, and 1.69% unconditional default frequencies over 30, 91, and 183 days, respectively, which increase to 0.43%, 1.48%, and 3.33%, respectively, during recessions.

Appendix E. Additional Tables.

Table A1: Corporate Bonds' Historical Default Rates: 1970 — 2013

This table reports the historical cumulative default rates (in percent) of corporate bonds in our sample of firms from 1970 - 2013 and compares them with Moody's default frequencies, when available. The "Global" sample is an international sample of firms. The "US" sample only focuses on US firms. Booms and recessions are determined by NBER business cycle dates, and default rates are computed using US firms.

Moody's		Our Sample			
Rating	Global	Global	US	Boom	Recession
		Horizon: 30 days			
Aaa-Aa	-	0.00	0.00	0.00	0.00
A	-	0.00	0.00	0.00	0.01
Baa	-	0.00	0.00	0.00	0.01
Ba	-	0.04	0.05	0.04	0.11
B	-	0.19	0.22	0.19	0.43
Caa-C	-	1.91	1.89	1.61	3.47
		Horizon: 91 days			
Aaa-Aa	-	0.00	0.00	0.00	0.01
A	-	0.01	0.01	0.00	0.03
Baa	-	0.02	0.02	0.01	0.04
Ba	-	0.17	0.19	0.16	0.38
B	-	0.67	0.75	0.65	1.48
Caa-C	-	4.99	4.90	4.07	9.51
		Horizon: 183 days			
Aaa-Aa	-	0.00	0.00	0.00	0.03
A	-	0.02	0.01	0.01	0.05
Baa	-	0.05	0.05	0.04	0.12
Ba	-	0.42	0.47	0.40	0.91
B	-	1.55	1.69	1.47	3.33
Caa-C	-	9.04	8.88	7.25	17.73
		Horizon: 365 days			
Aaa-Aa	0.01	0.01	0.01	0.00	0.05
A	0.06	0.06	0.04	0.02	0.13
Baa	0.17	0.16	0.16	0.13	0.34
Ba	1.11	1.08	1.19	1.08	1.91
B	3.90	3.78	4.01	3.57	7.31
Caa-C	15.89	15.46	15.37	12.63	29.49
		Horizon: 730 days			
Aaa-Aa	0.04	0.04	0.03	0.02	0.05
A	0.20	0.19	0.16	0.14	0.25
Baa	0.50	0.47	0.47	0.43	0.66
Ba	3.07	2.94	3.23	3.15	3.76
B	9.27	8.72	9.16	8.67	12.81
Caa-C	27.00	25.13	25.18	21.93	41.37

Table A2: Default Frequencies of Short-horizon Corporate Bonds and Pseudo Bonds

The left-hand-side of this table reports *ex post* default frequencies of corporate bonds with Moody's credit ratings reported in the first column across maturities. The mean is the aggregate average, and confidence intervals (C.I.) are the 2.5% and 97.5% bands of the computed frequencies. Columns 5 and 6 report default frequencies during NBER booms and recessions, respectively. The righ-hand-side of this table reports the results of our credit rating methodology for pseudo bonds. Pseudo bonds are constructed from a portfolio of risk free debt minus put options on the S&P500 index. Pseudo credit ratings of pseudo bonds are assigned based on the pseudo bonds *ex ante* default probability (i.e. the probability the put option is in the money at maturity) during booms and recession. The pseudo bonds *ex ante* default probability is computed by inverting the empirical distribution of the residuals from a simple GARCH(1,1) model of asset returns with expected growth obtained from predictive regressions. The first two columns on the right-hand-side of the table report the *ex ante* average default probabilities for bonds in booms and recessions, respectively, for each pseudo credit rating. The next three columns show the actual *ex post* default frequency of the pseudo bonds across the pseudo credit ratings, and their confidence intervals. The *ex post* default frequency is computed as the fraction of times the S&P500 return (exclusing dividends) drop below the portfolio moneyness in the sample. The last two columns collect the average leverage K/A of pseudo bonds, and their average time to maturity (days). The sample is 1970 to 2013.

	Corporate Bonds					Pseudo Bonds						
	Historical Default Frequencies					*Ex ante* Def. Prob.		*Ex post* Default Frequency			$\overline{K/A}$	$\overline{\tau}$
	Mean	C.I. (2.5%)	C.I. (97.5%)	Boom	Bust	Boom	Bust	Mean	C.I. (2.5%)	C.I. (97.5%)		
					Target Maturity: 30 days							
IG	0.00	0.00	0.01	0.00	0.01	0.01	0.02	0.38	0.00	0.92	0.74	55
Ba	0.05	0.00	0.12	0.04	0.11	0.10	0.19	0.38	0.00	0.92	0.80	45
B	0.22	0.00	0.44	0.19	0.43	0.33	0.97	0.57	0.00	1.23	0.85	39
Caa-	1.89	0.00	3.84	1.61	3.47	1.59	3.80	2.47	1.11	3.82	0.90	39
					Target Maturity: 91 days							
IG	0.01	0.00	0.02	0.01	0.03	0.58	0.08	0.57	0.00	1.22	0.65	118
Ba	0.19	0.11	0.27	0.16	0.38	0.22	0.71	0.57	0.00	1.22	0.73	115
B	0.75	0.50	1.00	0.65	1.48	1.18	3.05	1.71	0.00	3.51	0.81	84
Caa-	4.90	0.00	10.64	4.07	9.51	4.22	9.93	7.43	3.60	11.25	0.88	79
					Target Maturity: 183 days							
Aaa/Aa	0.00	0.00	0.01	0.00	0.03	0.01	0.02	0.77	0.00	1.96	0.57	194
A/Baa	0.03	0.02	0.05	0.02	0.09	0.15	0.39	0.96	0.00	2.51	0.67	182
Ba	0.47	0.39	0.55	0.40	0.91	0.48	1.46	2.11	0.00	4.83	0.72	184
B	1.69	1.25	2.12	1.47	3.33	2.32	6.26	2.30	0.00	5.24	0.79	180
Caa-	8.88	0.00	22.46	7.25	17.73	7.51	18.08	8.43	2.63	14.23	0.86	178
					Target Maturity: 365 days							
Aaa/Aa	0.01	0.00	0.02	0.00	0.05	0.01	0.06	1.16	0.00	3.08	0.46	356
A/Baa	0.10	0.09	0.11	0.08	0.24	0.30	0.78	2.13	0.00	5.02	0.59	340
Ba	1.19	1.08	1.31	1.08	1.91	1.35	2.71	3.29	0.00	7.61	0.70	350
B	4.01	2.88	5.14	3.57	7.31	4.97	11.03	6.98	0.42	15.53	0.79	347
Caa-	15.37	0.00	44.28	12.63	29.49	13.23	29.15	13.76	3.37	24.15	0.86	346

Table A3: Two-Year Pseudo and Corporate Bonds: Subsamples

Credit spreads and summary statistics of pseudo bonds (Panels A and B), and corporate bonds (Panels C and D). Pseudo bonds are constructed from a portfolio of risk free debt minus put options on the S&P500 index. Pseudo credit ratings of pseudo bonds are assigned based on the pseudo bonds *ex ante* default probability (i.e. the probability the put option is in the money at maturity). The default probability for pseudo bonds are computed by inverting the empirical distribution of residuals from a simple GARCH(1,1) model of asset returns with expected growth obtained from predictive regressions. Corporate bonds are non-callable corporate bonds with time to maturity between 1.5 and 2.5 years.

Credit	Prob.	Avg	Credit	Monthly Returns in Excess of T-bill (%)				
Rating	Of Default	K/A	Spread	Mean	Std	Sharpe Ratio	Skew	Excess Kurtosis
Panel A: Pseudo Bonds: January 1996 - December 2004								
Aaa/Aa	0.03	0.45	38	0.13	0.51	0.24	3.28	13.86
A/Baa	0.31	0.63	121	0.23	0.79	0.29	-0.66	8.85
Ba	3.23	0.73	211	0.30	1.11	0.27	-0.75	4.29
B	9.16	0.82	301	0.32	1.43	0.22	-0.78	3.25
Caa-	25.18	0.92	445	0.36	1.89	0.19	-0.70	1.92
Panel B: Pseudo Bonds: January 2005 - August 2013								
Aaa/Aa	0.03	0.42	62	0.15	0.71	0.22	-0.12	3.97
A/Baa	0.31	0.58	141	0.28	0.94	0.3	0.53	2.29
Ba	3.23	0.72	265	0.24	1.67	0.15	-2.28	14.32
B	9.16	0.84	436	0.35	2.13	0.17	-1.35	7.25
Caa-	25.18	0.95	593	0.35	2.70	0.13	-1.30	5.82
Panel C: Corporate Bonds: January 1996 - December 2004								
Aaa/Aa	0.03		71	0.11	0.56	0.19	-0.29	1.78
A/Baa	0.31		132	0.00	1.85	0.00	-5.97	47.25
Ba	3.23		446	0.60	2.91	0.21	-0.12	11.13
B	9.16		749	0.53	4.68	0.11	-2.17	11.30
Caa-	25.18		1292	0.64	5.36	0.12	2.07	7.74
Panel D: Corporate Bonds: January 2005 - August 2013								
Aaa/Aa	0.03		65	0.11	1.11	0.10	-0.97	19.88
A/Baa	0.31		134	0.22	0.64	0.34	0.70	8.95
Ba	3.23		352	0.56	1.35	0.42	1.27	19.55
B	9.16		516	0.60	1.22	0.49	1.49	7.94
Caa-	25.18		1182	0.89	3.48	0.26	-0.33	2.80

Table A4: Returns on Two-Year Pseudo Bonds and Corporate Bonds: Subsamples

This table reports the results of the regression specification

$$R^e_{B,t} = \alpha + \beta R^e_{i,t} + \epsilon_t; \qquad i = A,\ E$$

where $R^e_{B,t}$ is the excess return of pseudo bonds (Panels A -D), or corporate bonds (Panel E and F). The explanatory variable $R^e_{i,t}$ is the excess return on assets (Panels A and B) or equity (Panels C - F). Bonds are sorted monthly into credit rating portfolios, and portfolio returns in excess of the U.S. Treasury bill rate are computed over the following month.

Credit Rating	Mean (%)	$t(Mean)$	α	$t(\alpha)$	β	$t(\beta)$	R^2
Panel A: Pseudo Bonds on Assets: January 1996 - December 2004							
Aaa/Aa	0.13	(1.63)	0.09	(1.61)	0.06	(2.41)	0.20
A/Baa	0.23	(2.88)	0.07	(0.80)	0.19	(5.45)	0.62
Ba	0.30	(2.73)	0.16	(1.71)	0.26	(8.75)	0.74
B	0.32	(2.29)	0.14	(1.35)	0.33	(9.47)	0.78
Caa-	0.36	(2.00)	0.20	(1.92)	0.41	(12.72)	0.84
Panel B: Pseudo Bonds on Equities: January 1996 - December 2004							
Aaa/Aa	0.13	(1.63)	0.09	(1.57)	0.03	(2.45)	0.17
A/Baa	0.23	(2.88)	0.07	(0.75)	0.08	(4.75)	0.56
Ba	0.30	(2.73)	0.18	(1.56)	0.09	(6.15)	0.65
B	0.32	(2.29)	0.13	(0.99)	0.09	(6.44)	0.67
Caa-	0.36	(2.00)	0.21	(1.20)	0.10	(6.02)	0.64
Panel C: Pseudo Bonds on Assets: January 2005 - August 2013							
Aaa/Aa	0.15	(2.14)	0.14	(2.14)	0.07	(3.26)	0.18
A/Baa	0.28	(3.11)	0.16	(2.34)	0.17	(7.55)	0.48
Ba	0.24	(1.50)	0.16	(2.01)	0.28	(10.20)	0.66
B	0.35	(1.67)	0.20	(2.33)	0.40	(14.88)	0.77
Caa-	0.35	(1.30)	0.15	(1.75)	0.53	(19.88)	0.85
Panel D: Pseudo Bonds on Equities: January 2005 - August 2013							
Aaa/Aa	0.15	(2.14)	0.15	(2.20)	0.04	(2.26)	0.11
A/Baa	0.28	(3.11)	0.17	(2.09)	0.07	(5.74)	0.35
Ba	0.24	(1.50)	0.19	(1.85)	0.09	(6.86)	0.46
B	0.35	(1.67)	0.27	(2.09)	0.10	(8.08)	0.49
Caa-	0.35	(1.30)	0.23	(1.49)	0.11	(9.15)	0.54
Panel E: Corporate Bonds on Equities: January 1996 - December 2004							
Aaa/Aa	0.11	(1.94)	0.13	(1.09)	0.01	(0.51)	0.01
A/Baa	0.00	(0.00)	0.26	(1.68)	0.00	(-0.13)	0.00
Ba	0.60	(2.01)	0.21	(0.92)	0.15	(2.63)	0.22
B	0.53	(0.87)	1.03	(1.59)	-0.01	(-0.26)	0.00
Caa-	0.64	(0.63)	0.69	(1.05)	0.22	(3.85)	0.64
Panel F: Corporate Bonds on Equities: January 2005 - August 2013							
Aaa/Aa	0.11	(0.99)	0.11	(0.87)	0.02	(0.64)	0.02
A/Baa	0.22	(3.42)	0.19	(2.95)	0.06	(3.77)	0.23
Ba	0.56	(4.26)	0.49	(4.91)	0.08	(2.96)	0.33
B	0.60	(4.94)	0.51	(5.15)	0.06	(4.28)	0.34
Caa-	0.89	(2.49)	0.64	(2.07)	0.12	(5.88)	0.32

Table A5: Credit Spreads and Returns of Short-Horizon Pseudo and Corporate Bonds

Credit spreads and excess returns summary statistics are shown for short-term pseudo bonds (Panel A to D), and corporate bonds (Panel E and F). Pseudo bonds are constructed from a portfolio of risk free debt minus SPX put options. Pseudo credit ratings of pseudo bonds are assigned based on the pseudo bond *ex ante* default probability (i.e. the probability the put option is in the money at maturity). The pseudo bonds default probabilities are computed by inverting the empirical distribution of residuals from a simple GARCH(1,1) model of asset returns with expected growth obtained from predictive regressions. Corporate bonds are non-callable corporate bonds with time to maturity close to the one reported in the Panel's heading.

Credit Rating	Prob. Of Default	Avg K/A	Credit Spread	Monthly Returns in Excess of T-bill (%)				
				Mean	Std	Sharpe Ratio	Skew	Excess Kurtosis
				Panel A: 30 Days Pseudo Bonds				
IG	0.00	0.74	77	0.05	0.13	0.38	1.24	6.48
Ba	0.05	0.80	165	0.09	0.32	0.28	-1.95	25.53
B	0.22	0.85	286	0.11	0.69	0.16	-8.35	100.01
Caa-	1.89	0.90	503	0.22	0.91	0.24	-5.00	53.90
				Panel B: 91 Days Pseudo Bonds				
IG	0.01	0.65	64	0.08	0.26	0.30	-3.56	31.68
Ba	0.19	0.73	133	0.10	0.71	0.15	-7.82	95.83
B	0.75	0.81	262	0.19	0.66	0.28	-2.82	23.88
Caa-	4.90	0.88	495	0.26	1.19	0.22	-4.33	33.71
				Panel C: 183 Days Pseudo Bonds				
Aaa/Aa	0.00	0.57	50	0.07	0.29	0.24	-2.16	25.79
A/Baa	0.03	0.67	106	0.13	0.45	0.28	-0.62	17.45
Ba	0.47	0.72	169	0.12	0.83	0.15	-5.10	51.00
B	1.69	0.79	287	0.22	1.00	0.22	-2.43	17.05
Caa-	8.88	0.86	471	0.29	1.45	0.20	-2.33	14.92
				Panel D: 365 Days Pseudo Bonds				
Aaa/Aa	0.01	0.46	42	0.07	0.42	0.17	1.20	16.06
A/Baa	0.10	0.59	97	0.16	0.61	0.26	0.97	14.49
Ba	1.19	0.70	186	0.17	1.03	0.17	-2.78	22.70
B	4.01	0.79	311	0.28	1.29	0.22	-1.15	7.22
Caa-	15.37	0.86	469	0.32	1.75	0.18	-1.49	8.48
				Panel E: 183 Days Corporate Bonds				
Aaa/Aa	0.00		42	0.01	0.85	0.01	-4.21	34.16
A/Baa	0.03		92	0.18	0.47	0.39	4.46	32.19
Ba	0.47		232	0.31	0.95	0.32	-2.46	15.90
B	1.69		276	0.27	1.71	0.16	-3.30	18.49
Caa-	8.88		1225	0.71	3.10	0.23	1.79	10.79
				Panel F: 365 Days Corporate Bonds				
Aaa/Aa	0.01		68	0.18	0.60	0.30	2.96	17.63
A/Baa	0.10		139	0.19	0.69	0.27	-0.31	20.75
Ba	1.19		329	0.44	0.85	0.52	2.10	9.74
B	4.01		517	0.75	2.30	0.32	3.94	31.20
Caa-	15.37		1264	1.20	3.62	0.33	1.03	6.21

Table A6: Assets as Shares of Individual Firms in the S&P500: Equivalent European Options

This table is the equivalent of Table 5, except that pseudo bonds are computed out of European equivalent put options. European-equivalent put options are obtained from the implied volatilities reported from OptionsMetrics. Panel A reports summary statistics of the pseudo bond portfolios. Column 5 reports the equal weighted average credit spread of pseudo bonds in each credit rating category, while the next several columns report summary statistics of portfolio bond returns. For each credit rating, Panel B reports the time-series regression of the pseudo bond portfolio excess returns on the average excess returns of pseudo assets (i.e. stocks of underlying individual firms). For each credit rating, Panel C reports the time-series regression of the pseudo bond portfolio excess returns on the average excess returns of pseudo equity (i.e. call options of the underlying individual firms).

Panel A: Average Credit Spreads and Monthly Returns' Summary Statistics

	Prob. of Default		Credit Spread	Monthly Returns in Excess of T-bill (%) Mean	Std	SR	Skew	Ex. Kurt
Aaa/Aa	79	79	74	-0.09	1.21	-0.07	-5.51	37.85
A/Baa	123	119	154	0.09	0.85	0.10	-0.98	6.79
Ba	189	183	227	0.18	1.11	0.17	-0.59	7.14
B	323	297	492	0.19	1.51	0.13	-1.61	8.03
Caa-	614	544	1074	0.31	2.10	0.15	-0.86	2.55

Panel B: Regression of Pseudo Bonds Excess Returns on Assets' Excess Returns

	Mean (%)	$t(Mean)$	α	$t(\alpha)$	β	$t(\beta)$	R^2
Aaa/Aa	-0.09	(-1.13)	-0.18	(-1.43)	0.08	(2.36)	0.23
A/Baa	0.09	(1.50)	0.05	(1.10)	0.08	(5.94)	0.35
Ba	0.18	(2.25)	0.05	(0.84)	0.15	(9.32)	0.62
B	0.19	(1.90)	-0.01	(-0.14)	0.25	(8.52)	0.66
Caa-	0.31	(2.21)	-0.07	(-0.67)	0.41	(12.04)	0.71

Panel C: Regression of Pseudo Bonds Excess Returns on Pseudo Equities' Excess Returns

	Mean (%)	$t(Mean)$	α	$t(\alpha)$	β	$t(\beta)$	R^2
Aaa/Aa	-0.09	(-1.13)	-0.09	(-0.65)	0.02	(1.25)	0.08
A/Baa	0.09	(1.50)	0.08	(1.60)	0.05	(5.22)	0.30
Ba	0.18	(2.25)	0.22	(3.78)	0.07	(7.04)	0.36
B	0.19	(1.90)	0.18	(2.57)	0.12	(9.44)	0.54
Caa-	0.31	(2.21)	0.19	(1.74)	0.14	(10.53)	0.45