Optimal Pricing and Quality of Academic Journals and the Ambiguous Welfare Effects of Forced Open Access: A Two-Sided Model

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Abstract

We analyse optimal pricing and quality of a monopolistic journal and the optimality of open access in a two-sided model. The predominant aspect of the model that determines the quality levels at which open access is optimal is the nature of the relationship between readers and authors in a journal. In contrast to previous literature, we firstly show that there exist scenarios in which open access is a feature of high-quality journals. Second, we find that removal of copyright (and thus forced open access) decreases journal profits but has ambiguous social welfare effects.

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1 Introduction

Academic journals act as platforms upon which authors communicate their ideas to readers. As such, journals need to attract both authors and readers in order to be able to provide their service, which is beneficial to both readers and authors. However the interrelationship between authors and readers on the journal platform is more complex than a simple meeting place where ideas are exchanged. Readers attract authors to a journal, and authors attract readers to a journal, and both are attracted to higher quality journals.

The interesting part of the whole issue of academic publishing via journals is the fact that as the intermediary, the journal editors make decisions regarding readers and authors that are crucial to the final outcome of the quality that the journal achieves. Perhaps the most interesting model of a journal occurs when the journal acts in order to maximise profit.[1] In such a scenario, the journal must decide the subscription price for reader access, the author fee (submission and/or publication fees), and the overall quality of the journal, all with the objective of achieving maximal profit. It is by no means obvious, for example, that profits will be maximized by maximizing the quality of the journal. Neither is it clear how the reader subscription price should affect the author fee, and vice-versa.

In the present paper, a model of a journal as a two-sided platform is explored in order to consider some of the principal aspects of this complex market. The paper adds to a relatively young literature that considers academic journals in two-sided markets (Jeon and Rochet, 2010; McCabe and Snyder, 2007). The model is simplified to one in which a single monopolistic journal provides the service of publishing academic papers for both readers and au-

[1] Other objectives may also be considered – the journal might act in order to maximise its impact factor, or it might act in order to maximise readership (diffusion of ideas published).

thors. It is related to the monopoly platform model by Armstrong (2006). In contrast to Armstrong (2006), however, the platform's ultimate objective is to choose its "quality" and must also make an optimal choice of both the reader subscription price and the author fee. We also analyse the effects of a removal of copyright on journals, academics and social welfare. It is in this respect that we believe our analysis is different from existing works such as Jeon and Rochet (2010) and McCabe and Snyder (2007).

The assumption that the journal is fully monopolistic is crucial to the model. If there were other journals competing for both readers and authors, the results of the model would certainly be different. However bringing competition into the mix is excessively complex, so for now we have preferred to only look at a monopolistic journal in order to focus on the issue of open access and copyright. That said, our model can also be interpreted as a model of monopolistic competition, in a way that will be made clear in the paper. Second, we only study the case of an online journal, rather than a journal that publishes in hard-print format. This simplifies the analysis as it allows us to realistically assume that the marginal cost of supplying readers is zero.[2] Third, in order to get crisp theoretical results, it is necessary to make a large number of assumptions regarding the relationships between the different variables, and on the way the different variables affect the objective function. Rather than making all of the relevant assumptions and then putting forward theoretical results, we have preferred to carry out an analysis based upon numerical simulation. However, the main structural assumptions in the model, which are linear demand and either linear or concave production functions, are relatively standard. Given these structural assumptions the model only contains a single determining parameter, to which we give a specific value for our numerical simulations. Any number of other simulations can be generated by simply altering the values of this parameter.[3]

[2] For simplicity, we also assume the fixed cost of the journal to be zero.

[3] In fact, our single parameter (the effect of journal quality on the vertical intercept of the demand for journal services by both authors and readers) can also be split into two different parameters, one for each side of the market. We have elected to assume these two effects to be equal simply as we have no convincing evidence to the contrary.

We are interested in the following aspects of the journal management. Firstly, is it possible that one of the two prices (the reader price and the author price) can be optimally set to zero? Is it true that a journal that does optimally set the reader price to zero (i.e. an "open access" journal) is characterized by a lower quality level than a closed access journal (i.e. one with a strictly positive subscription price for readers) as is often thought to be true (McCabe and Snyder, 2005)? How would the removal of copyright in the papers published impact upon the optimal choices of a journal?[4] Above all, we calculate the welfare effects of the removal of copyright in our simulations.

We find several new results that add to the literature. Firstly, we find that there exist scenarios in which open access (i.e. an optimal reader price equal to zero) is a feature of lower quality journals, and others in which it is a feature of higher quality journals. The predominant aspect of the model that determines the quality levels at which open access is optimal is the nature of the relationship between readers and authors in a journal. Above all, we show that the conclusion of the model of McCabe and Snyder (2005), who find that open access is more likely to be a feature of lower quality journals, is not generally true. Second, we find that removal of copyright (and thus forced open access) will likely increase both readership and authorship, will decrease journal profits, and may increase social welfare.

2 Model

The journal chooses quality,[5] q, the price charged to readers, p_r, and the price charged to authors, p_a. We assume that the journal acts in order to maximise profits. Given the choice (q, p_r, p_a), the number of readers is endogenously given by $n_r(q, p_r, n_a)$, and the number of authors is endogenously given by

[4] We are interested in this aspect because of the recent, provocative, paper by Stephen Shavell (2010) that advocates abolishion of copyright in scientific publications.

[5] In this paper we treat quality rather lightly. It is a choice variable of the journal, but we do not specify or model how that choice is made. In reality, quality is controlled by the journal via the referee process. In the present paper, we simply assume that given a choice of a particular level of quality, the journal receives enough submissions of at least that level of quality to fill the journal with content.

$n_a(q, p_a, n_r)$. Both the number of readers and the number of authors is determined in part by the quality chosen. The number of readers (authors) has a direct dependence on the price charged to readers (authors). The dependence of number of readers on the price charged to authors, and the number of authors on the price charged to readers, is indirect. The number of readers is (partially) determined by the number of authors, and vice versa.

These assumptions reflect reality. Readers choose to read a journal depending on its content (which is given by the number of papers in it, n_a, and the quality of those papers, q), and the price charged to readers, p_r. Authors want to publish in a journal given the quality of the journal, q, the audience reached, n_r, and the cost involved in publishing, p_a.[6] The fact that the two functions $n_r(q, p_r, n_a)$ and $n_a(q, p_a, n_r)$ are interdependent with the value of each depending (in part) upon the value of the other, captures the two-sided market feature of academic journals as platforms for readers and authors.

In reality, we can understand the two functions $n_r(q, p_r, n_a)$ and $n_a(q, p_a, n_r)$ in two different ways, both of which will be exploited in the paper. First, for given values of q and n_a, say $\overline{q}$ and $\overline{n}_a$, we should understand $n_r(\overline{q}, p_r, \overline{n}_a)$ to be a demand function in the sense that it relates the price for reading to the number of readers who purchase. On the other hand, for given values of q and p_r, say $\overline{q}$ and $\overline{p}_r$, we should understand $n_r(\overline{q}, \overline{p}_r, n_a)$ to be a production function, in the sense that papers (here, authors) are what attract readers to a journal. In the same way, $n_a(\overline{q}, p_a, \overline{n}_r)$ is again a demand function, and $n_a(\overline{q}, \overline{p}_a, n_r)$ is a production function (this time, reflecting the dependence of the number of authors that are attracted to a journal on the number of readers of that journal). For $i, j = r, a$ and $i \neq j$, we make the following assumptions:

$$\frac{\partial n_i}{\partial p_i} < 0, \qquad \frac{\partial n_i}{\partial n_j} > 0, \qquad \frac{\partial^2 n_i}{\partial n_j^2} \leq 0.$$

Thus, the demand functions are negatively sloped, and the production func-

[6] For instance, p_a is the sum of submission fee and publication fee.

tions are positively sloped and (weakly) concave. We also assume that

$$\frac{\partial n_r}{\partial q} > 0, \qquad \frac{\partial n_a}{\partial q} > 0.$$

The first of these is a very natural assumption – readers prefer better quality papers. The second is not so obvious. Indeed, it will only hold locally, on a range of quality levels. The greater is the quality of a journal, the greater is the willingness of authors to supply papers to that journal (for the CV impact, and for the fact that higher quality journals are likely to reach a larger audience, and thus are more likely to be cited). But, the greater is the quality hurdle, the fewer will be the papers published from those submitted. Thus while a high quality journal will have a larger set of papers from which to choose, they are more selective in their choosing.[7] Almost certainly, the number of published papers is a non-monotone function of quality, since at some very high quality levels the quality filter will outweigh the effect of increased submissions.The assumption used in the present paper that for the range of levels of quality that is relevant, there are more published authors when quality increases. This assumption is based more than anything else upon observation. At least in the field of economics journals, those at the top of the quality ladder are typically able to publish many papers, while it is the journals of lower perceived quality that may struggle with finding papers to publish.

Our assumption that the number of published papers increases with quality is not innocuous to the results of our model. As we shall see, in the model we end up with profits being a strictly increasing function of quality, and thus each journal wants to increase quality as much as possible. However we should *not* interpret this result as implying that journals will set quality at an infinitely high level. We are only carrying out a local analysis in terms of quality.

[7]In the present paper we abstract away from modeling the referee process, under which papers are screened for quality. Here, all that is important is to recognise that n_a is the number of papers that end up being published, and that will be determined by the number of papers that are submitted (decreases with author price and increases with number of readers), and the quality of the journal. The assumption that the number of papers published increases with quality reflects the assumption that submissions of sufficient quality increase in quality.

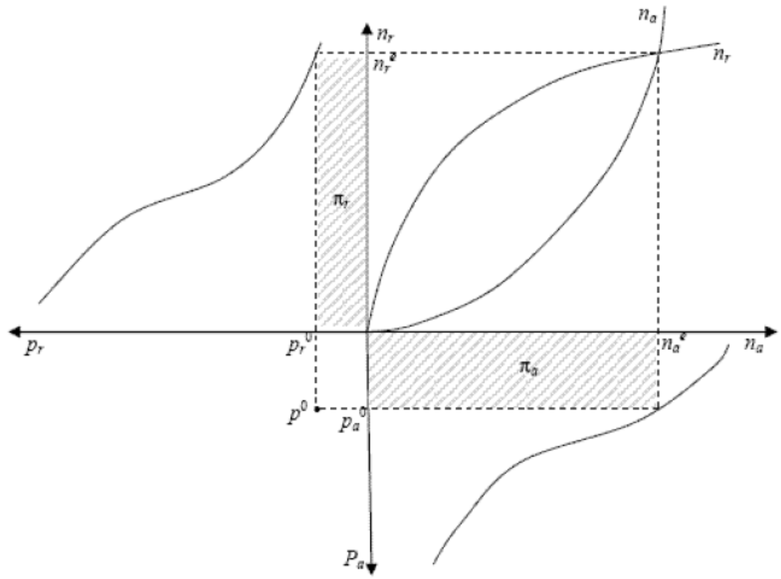


Figure 1: Two-sided market: demand and production functions and profit

If we were to carry out a full consideration of the non-monotone functional relationship between the number of papers published and the journal´s quality, then there would exist a sufficiently high level of quality such that profits end up decreasing with quality as it becomes extremely difficult to find papers of sufficient quality to publish. In such a model there would be a finite optimal level of quality. We are currently elaborating such a model.

In Figure 1 we show both the demand curve aspect, and the production function aspect of the journal platform, always taking quality to be fixed. The upper left-hand [lower right-hand] panel shows the demand curve aspect of $n_r(q, p_r, n_a)$ $[n_a(q, p_a, n_r)]$, and the upper right-hand panel shows the production function aspects. Figure 1 highlights a very important aspect of the journals market. It is two-sided, and so the choice of reader price (a determinant of the number of readers) cannot be taken independently from the choice of author price (which is a determinant of the number of authors). There is only one consistent choice in this graph, which is labeled as point p^0 in the south-west quadrant. Only with that choice of prices will the number of authors (n_a^e) be consistent with the number of readers (n_r^e), where the superscripts ^{e} refer to endogenous equilibrium values.

Imagine that, from the situation drawn in Figure 1, the journal decided to increase the reader price, p_r (leaving the author price unchanged). What would be the effect in the graph? The increase in reader price will cause a shift along the demand curve for the number of readers, thus reducing n_r. However, the production function for readers will itself shift, since it is parametised by the reader price. Since we assume that the number of readers is a decreasing function of the price for reading, the production function will shift downwards. There is a resulting shift along the production function for the number of authors. Next, the demand function for the number of authors is parametised by the number of readers. The number of readers has decreased, which will shift the demand function for the number of authors inwards. Finally, the number of authors has also been decreased, which will shift the demand function for the number of readers inwards. These shifts will continue until a new equilibrium point is attained. We assume throughout that the equilibrium process just described is stable, in the sense that for any (q, p_r, p_a), the curves adjust such that there is a pair (n_r, n_a) that are mutually compatible.

2.1 Profits

The profits that the journal makes can also be represented graphically, at least for the assumption that the journal is fully online only, and thus has no marginal costs. The profits earned by the journal are

$$\pi(q, p_r, p_a) = p_r \cdot n_r + p_a \cdot n_a = \pi_r + \pi_a.$$

In Figure 1 we can see the profits made from the reader side of the market (π_r) and the author side of the market (π_a). The sum of these two rectangular areas is the total profit. To illustrate, the effect of a unilateral increase of the reader price is to decrease the profit in the author market (since the author price stays constant and the number of authors decreases) and to change the profit in the reader market in such a way that it may increase or decrease (it goes from a tall thin rectangle to a shorter but wider one).

3 Profit maximising decisions

The journal chooses (q, p_r, p_a) in order to maximise profit. We model this recursively. First, hold quality at some fixed level, q, and given that quality, we analyse the optimal pricing policy of the journal, $p^*(q) = (p_r^*(q), p_a^*(q))$. Then, given the optimal prices for each quality level, we consider the optimal quality that the journal should choose.

Firstly, though, for any given (q, p_r, p_a) it is necessary to simultaneously solve the two equations $n_r(q, p_r, n_a)$ and $n_a(q, p_a, n_r)$ for the two equilibrium levels of readers and authors, $n_r^e(q, p_r, p_a)$ and $n_a^e(q, p_r, p_a)$. The profit of the journal (assuming that there are no marginal costs of supplying readers) is

$$\pi(q, p_r, p_a) = p_r \times n_r^e(q, p_r, p_a) + p_a \times n_a^e(q, p_a, p_r).$$

The derivatives of this with respect to the two prices are

$$\frac{\partial \pi}{\partial p_i} = n_i^e + p_i \frac{\partial n_i^e}{\partial p_i} + p_j \frac{\partial n_j^e}{\partial p_i},$$

where $i, j = r, a$ and $i \neq j$. Carrying out the implied second derivatives, it turns out that a sufficient condition for profits to be concave in the price p_i is $\frac{\partial^2 n_i}{\partial p_i^2} \leq 0$ and $\frac{\partial^2 n_i}{\partial n_j \partial p_i} \geq 0$. Assuming concavity, the two first-order conditions for optimal choices of the two prices are $\frac{\partial \pi}{\partial p_r} = 0$ and $\frac{\partial \pi}{\partial p_a} = 0$, the simultaneous solution of which give us the two optimal prices as functions of the quality, $p_r^*(q)$ and $p_a^*(q)$. The indirect profit function is then

$$\pi(q) = p_r^*(q) \times n_r^e(q, p_r^*(q), p_a^*(q)) + p_a^*(q) \times n_a^e(q, p_r^*(q), p_a^*(q)).$$

This is what must now be maximised with respect to q.

4 Simplified models

In order to see how the model works, we assume three different, but similar, versions of it. Each of the three models is characterized by linear demand functions for both readers and authors, and they differ with respect to the degree of concavity of the two production functions. Specifically, in model 1 we assume that both production functions are affected by diminishing returns (i.e. they are both concave functions). In model 2 the production of readers (taking authors as an input) has diminishing returns (i.e. is concave) while the production of authors (taking readers as the input) is assumed linear. In model 3 the reader production function is linear and the author production function is concave.

In each of the three models, the demand formulation is given by a linear form, with vertical intercept (i.e. maximum feasible price) equal to αq. Thus, greater levels of quality correspond to parallel shifts of the two demand curves. We have no particular reason to assume that the effect of a marginal change in quality upon the demand curve of readers is any different to the same effect for authors. So in the interests of keeping our model as uncluttered as possible, we assume that this effect is equal for both sides of the market (α).[8]

[8] The assumption of linear demand is, of course, only intended as a first approximation to any real life scenario. Non-linear forms increase the complexity of the model enormously, with no real change in the results that are obtained. Basically the linear form is the least complex way in which we can assure that when there are no readers, $n_r = 0$, then no authors are attracted to the journal, so that $n_a = 0$ also. Likewize, no authors implies no readers. This feature can also be incorporated into non-linear demand forms, but as stated above, this leads to significant analitical complexity with no real gain in what the models output.

4.1 Model 1: Diminishing returns on both sides

In this model, we assume[9]

$$
\begin{aligned}
n_r &= \sqrt{n_a}\left(\alpha q - p_r\right), \\
n_a &= \sqrt{n_r}\left(\alpha q - p_a\right).
\end{aligned}
$$

Notice that these two equations can be written as

$$
n_r = \sqrt{n_a}\beta_r, \tag{1}
$$

$$
n_a = \sqrt{n_r}\beta_a, \tag{2}
$$

where $\beta_i \equiv \alpha q - p_i$ for $i = r, a$. Recall that both of n_r and n_a are constrained to be positive, so we are restricted to parameter values such that $\beta_i > 0$ for $i = r, a$, that is, we can only consider prices that satisfy $p_i < \alpha q$ for $i = r, a$.

It is easy to show that the solution to the two equations (1) and (2), outside of the trivial solution at $(0, 0)$, is at

$$
n_r = \left(\beta_r^4 \beta_a^2\right)^{\frac{1}{3}}, \quad n_a = \left(\beta_r^2 \beta_a^4\right)^{\frac{1}{3}}.
$$

The profits of the journal are given by

$$
\pi = p_r n_r + p_a n_a = p_r \left(\beta_r^4 \beta_a^2\right)^{\frac{1}{3}} + p_a \left(\beta_r^2 \beta_a^4\right)^{\frac{1}{3}}.
$$

The profit function is perfectly symmetric in the two prices. That is, the

[9] When understood as production functions (that is, holding the two prices constant), both are concave production functions of their respective input. When understood as demand functions (that is, holding constant the number of authors in the equation for the number of readers, and vice versa), both are linear. The linear assumption on demand is intended to be understood as a first-order approximation, a simplification that is only in place to ease the complexity of the model. When demand is linear, the maximum price that readers are willing to pay for the first unit of quality is independent of the number of authors present, and vice versa. While not a particularly realistic assumption, it does significantly ease the solution to the model as compared to an assumption in which α is understood as a function of the number of readers and authors.

function is of the form

$$\pi = f(p_r, p_a) + f(p_a, p_r)$$

where $f(x, y) \equiv x\left((\alpha q - x)^4 (\alpha q - y)^2\right)^{\frac{1}{3}}$. Thus, it makes no difference to the problem how we label our price variables. In the optimal solution it must be true that $p_r^* = p_a^*$. We can use this insight to help us solve the maximization problem. We add the restriction $p = p_r = p_a$ to the existing restrictions $p_i < \alpha q$ for $i = r, a$. Substituting this first restriction into the objective function gives

$$\pi = 2p\left(\beta^4\beta^2\right)^{\frac{1}{3}} = 2p\left(\beta^6\right)^{\frac{1}{3}} = 2p\beta^2.$$

Here, $\beta = \alpha q - p$, so that $\frac{\partial\beta}{\partial p} = -1$. The first-order condition for an optimal solution is

$$\frac{\partial\pi}{\partial p} = 0 \qquad \Rightarrow \qquad 2\beta^{*2} - 4p^*\beta^* = 0$$

where $\beta^* = (\alpha q - p^*) > 0$. This equation can be expressed as $p^* = \frac{\alpha q}{3}$. The second order condition on this maximization problem is $-8\beta + 4p < 0$ which is $-8\alpha q + 12p < 0$. At the stationary point (which is unique on the range $p < \alpha q$), we have $-8\alpha q + 12p^* = -8\alpha q + 4\alpha q = -4\alpha q < 0$. Thus, the second order condition is satisfied at the optimal solution. In short, the two optimal prices for model 1 are identical linear functions of quality;

$$p_r^* = p_a^* = \frac{\alpha q}{3}.$$

4.2 Model 2: Diminishing returns to authors

We now assume

$$n_r = \sqrt{n_a}\left(\alpha q - p_r\right), \tag{3}$$

$$n_a = n_r\left(\alpha q - p_a\right). \tag{4}$$

In Appendix A, we show that the optimal prices in this model are:

$$p_a^* = \frac{5\alpha q - \sqrt{4\alpha^2 q^2 + 7\alpha q}}{7}, \tag{5}$$

$$p_r^* = \frac{21\alpha q - 4\alpha^2 q^2 - 2\alpha q\sqrt{4\alpha^2 q^2 + 7\alpha q}}{49}. \tag{6}$$

The two optimal prices are graphed in Figure 2.[10]

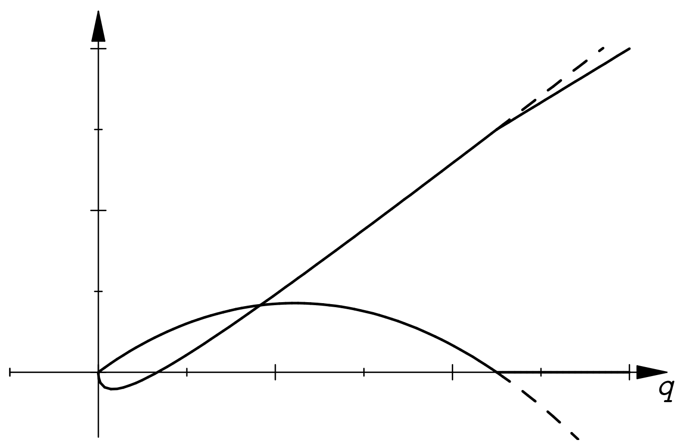


Figure 2: Model 2 optimal prices

In Figure 2 we can see that we are assuming that it is not possible for the journal to pay readers, that is, the reader price cannot be negative. In reality, the optimal reader price equation (5) dictates negative reader prices for all quality levels above the quality level q_0, which is the strictly positive solution to $p_r^*(q_0) = 0$. Figure 2 shows these negative prices as a dashed curve. Since it is not realistically feasible to pay readers, on that range of quality levels the journal would be restricted to the corner solution with $p_r^* = 0$, which is indicated by the continuation of the solid curve along the axis. Thus, the optimal reader price is a piecewise function.

This also affects the optimal author price. When the reader price is re-

[10] In the simulations that we have done, we took $\alpha = 1$, although it is relatively simple to see that taking any other (positive) value would not alter the shapes of the graphs obtained, only their values.

stricted to 0, the optimal author price is no longer given by equation (6). As it happens, above q_0, the optimal author price is linear and equal to $\frac{\alpha q}{3}$.[11] Thus, the author price graph in Figure 2 is also piecewise, as can be seen by the kink in the optimal author price graph as drawn solid (the dashed curve is the continuation of the optimal author price, which would assume negative reader prices are feasible).

4.3 Model 3: Diminishing returns to readers

Our third model is the opposite of model 2. Specifically, in model 3 we assume

$$\begin{aligned} n_r &= n_a \left(\alpha q - p_r\right), \\ n_a &= \sqrt{n_r} \left(\alpha q - p_a\right). \end{aligned}$$

Given the symmetry between models 2 and 3, it is straight forward to see that the solution will be exactly the opposite as in model 2, that is

$$\begin{aligned} p_a^* &= \frac{21\alpha q - 4\alpha^2 q^2 - 2\alpha q\sqrt{4q^2 + 7\alpha q}}{49}, \\ p_r^* &= \frac{5\alpha q - \sqrt{4\alpha^2 q^2 + 7\alpha q}}{7}. \end{aligned}$$

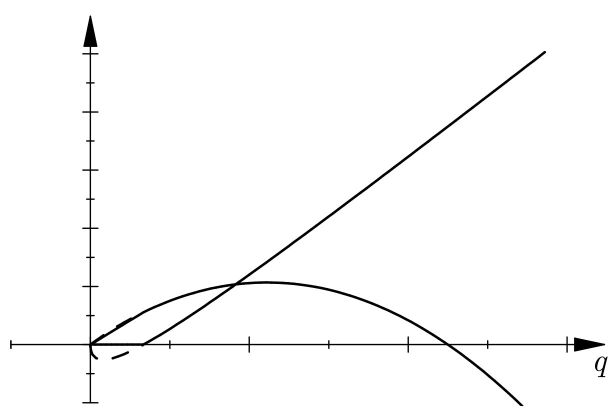


Figure 3: Model 3 optimal prices

[11] See Appendix B.

The same comments as for Figure 2 apply, but now the zone of qualities for which the reader price is set to 0 as a corner solution is $q < q_0$, where q_0 is the positive solution to $p_r^*(q) = 0$. On this zone, again $p_a^*(q) = \frac{\alpha q}{3}$.[12]

4.4 Discussion

Our simulations serve to show a couple of important points as regards pricing. It is crucial to the results which of the production technologies has the decreasing returns. When both the production of readers using authors as an input and the production of authors using readers as an input are concave production processes, then our simulation points to there being no quality levels for which either price goes to zero. Thus, in that model, there is no scope at all for open access as an optimal pricing strategy. On the other hand, when the production of readers has decreasing returns to the addition of authors, but the production of authors is linear in readers (model 2), then our simulation reveals that it becomes optimal for the journal to be open access (i.e. to charge readers a price of zero) when the quality of the journal is relatively high. Thus, in this model, open access is a feature of high, rather than low, quality journals. Thirdly, when it is the author production process that has decreasing returns to the addition of readers, and the reader production function is linear, then we get the opposite result; open access is a feature of optimal journal pricing only for very low quality journals. These results point to it not being generally true that open access journals are of lower quality, as has been argued in the previous literature (McCabe and Snyder, 2005).

It is also interesting that our simulations reveal that there is scope for negative author prices in two of our scenarios, something that is rather rare to find in the real-world of journal management. In model 2, we get very low quality journals having to pay authors in order to attract them to publish in the journal, while in model 3, it is very high quality journals that pay their authors.

[12] See Appendix B.

5 Comparative analysis

We now compare each of our three models graphically, looking at the values of a series of important endogenous variables. We look at the level of profit obtained, the level of social welfare, and the share of total social welfare that is retained by academics (readers and authors), all as functions of q.

We have already determined above the optimal prices in each of the three models. The other graphs are then all derived from those optimal prices. The easiest way to show the actual equations involved is to recall that the equilibrium numbers of both authors and readers, n_r^* and n_a^* are both functions of the two optimal prices. And since the two optimal prices are both functions of quality q, then so are both n_r^* and n_a^* functions of quality. Then, whatever is the model involved, the equilibrium level of profits is just

$$\pi(q) = p_r^*(q)n_r^*(q) + p_a^*(q)n_a^*(q).$$

To calculate welfare, we look at the surplus retained by academics (the set of readers and authors) plus profits. To consider the welfare of academics, we use the concept of consumer surplus. Our demand curves for the journal's services by both readers and authors are linear, thus "consumer surplus" on each side of the market is a triangle. Since our demand curves are $n_i = g(n_j)(\alpha q - p_i)$, for $i, j = 1, 2$ and $i \neq j$, where $g(n)$ is either $\sqrt{n}$ or n, depending on the model, the vertical intercept (i.e. the price at which quantity goes to 0) is αq. The area of the triangle on side i of the market is

$$CS_i(q) = \frac{1}{2}n_i^*(q)\left(\alpha q - p_i^*(q)\right), \quad i = r, a.$$

Given this, total welfare is given by

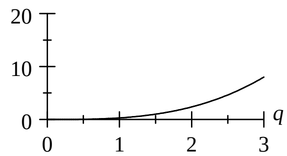
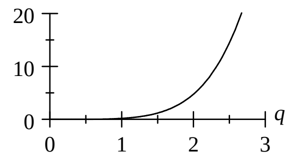
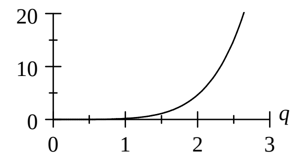
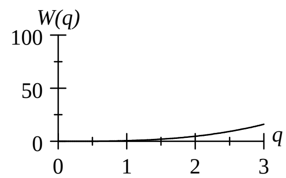
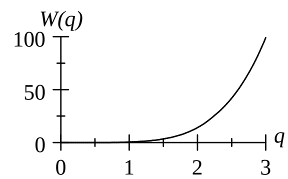
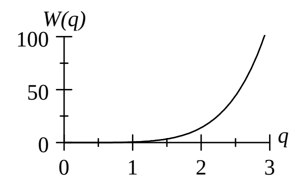
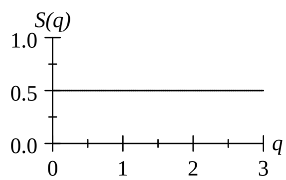
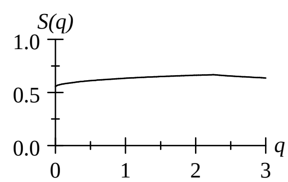
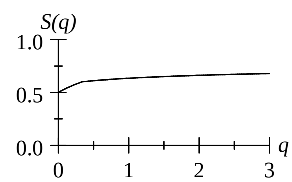
$$W(q) = CS_r(q) + CS_a(q) + \pi(q)$$

and the share of welfare that is retained by academics is given by

$$S(q) = \frac{CS_r(q) + CS_a(q)}{W(q)}.$$

The graphs of the principal variables of the three models are given in Table 1.[13] Recall that in models 2 and 3, an unrestricted analysis would set negative reader prices for some ranges of quality. This is not realistically feasible, and so in reality, on those ranges of quality, the reader price would be set at 0. This has been taken into account in all of the graphs that appear in Table 1, that is, the graphs for models 2 and 3 are actually piecewise functions. For all of the simulations from here on, we have used $\alpha = 1$.

[13] All of the graphs have been generated using the MuPAD 3 package in Scientific Workplace, and they have also all been independently checked using Mathematica. All of the working behind the actual graphs was also done by hand. Details are available from the authors upon request.

Table 1: Comparison of models 1, 2 and 3			
	model 1	model 2	model 3
profit			
welfare			
share			

In all of our models, the level of profit that the journal earns is always strictly increasing in quality. Thus journal managers will always strive to increase the perceived quality of their publication. In essence, journals will, at some point, run into a capacity constraint on either authors or readers, that will determine the exact level of quality that their journal attains. In that way, our model can also be interpreted as one of monopolistic competition, where the entire population of, say, authors is divided into mutually exclusive subsets, one for each journal. The quality of the journal is then determined by when their allocated number of authors is reached. We have not modelled the details of this process here, but rather we limit ourselves to a more informal discussion below.

Social welfare as defined by the total sum of consumer surplus on both sides of the market plus journal profits, is strictly increasing in journal quality. Thus, the greater is the level of quality that journals can attain, the greater is the level of social welfare. However, the way that welfare is shared among the market participants is again critically dependent upon the modelling assumptions. In our model 1 (diminishing returns on both production functions), the academics and the journal share welfare equally regardless of the quality of the journal. In the other two models, the share of total surplus that is retained by readers and authors falls between limits, both upper and lower. As quality increases the share of welfare retained by individuals increases, but is never greater than 0.67 in model 2 and 0.75 in model 3. It also never falls below 0.5 in both model 2 and model 3, that is, in those two models the readers and authors in aggregate always retain a strictly larger share of total surplus than does the journal (so long as quality is strictly positive).

The piecewise nature of the graphs in Table 1 deserve comment. The graphs where the piecewise element has the greatest effect are the graphs of the share of academics' welfare in total welfare. In Figures 4 (model 2) and 5 (model 3) we show larger versions of these two graphs. Notice that, in model 2, the share of academic welfare in total welfare is increasing up to the point at which the reader price goes to zero, and is decreasing after that (the dashed line indicates where this share would go if it were feasible to pay readers). In Figure 5 we can see the detailed graph of academic welfare as a fraction of total welfare in model 3. In model 3, the share of academic welfare in total welfare is always increasing, but it is lower than it would be if readers could be paid on the section of the graph for which the journal is open access.

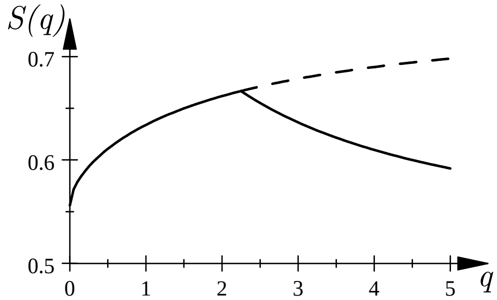


Figure 4: Share of academic in total welfare, model 2

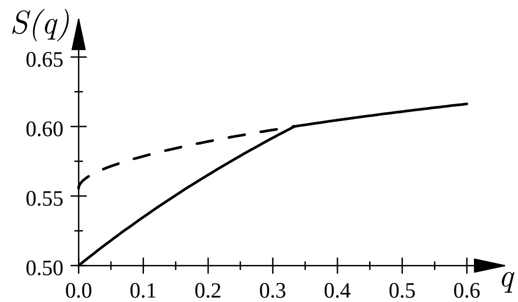


Figure 5: Share of academic in total welfare, model 3

6 The effects of removal of copyright

We can analyse the issue of copyright by simply noting that when there is copyright protection in place, the journal can act in the market for readers as a monopolist, while if there is no copyright, then the journal is far more open to competition from other publishers (including author's own websites etc.). Thus, assume that the models analysed above are those corresponding to the existence of copyright protection, and that when copyright protection is lifted (Shavell, 2010), then the journal no longer gets to choose the reader price, which is fixed at 0. This simplifies the model significantly.

Now, the profit that the journal earns is equal only to what it can earn from authors. In Appendix B, we show that the optimal author prices when copyright is removed are:

Table 2: Author Prices under No-Copyright

Model 1	Model 2	Model 3
$p_a^* = \frac{3\alpha q}{7}$	$p_a^* = \frac{\alpha q}{3}$	$p_a^* = \frac{\alpha q}{3}$

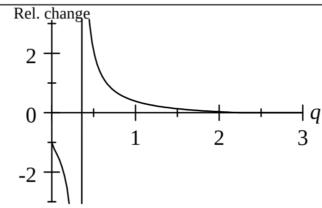
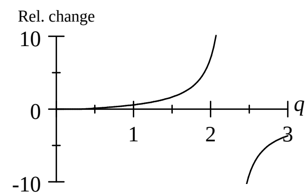
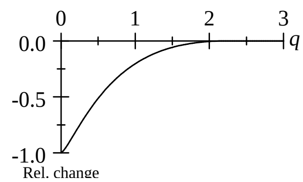
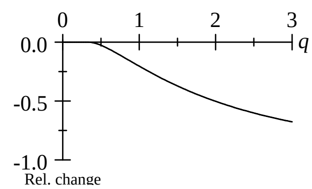
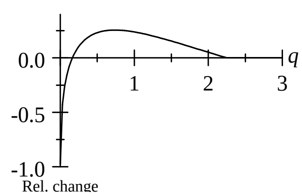
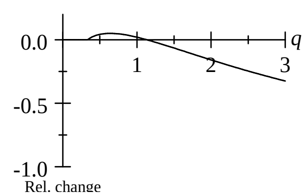
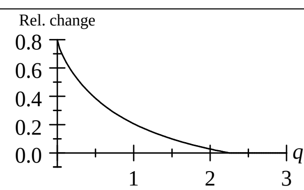
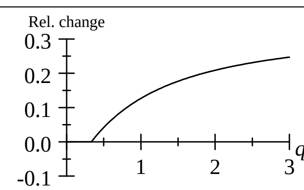
By comparing these prices with the optimal author prices under copyright we can see that the removal of copyright serves to increase the optimal author price in all three models. In contrast to the case of copyright protection, now the optimal author prices in models 2 and 3 are strictly positive, and linear, for all levels of quality.

We now compare the three models both with and without copyright. We present some results both in absolute values, and in relative values.[14] We firstly look at the relative comparisons, that is, say we are interested in the variable $z(q)$, where z can represent the optimal author price, profits, welfare, or share of welfare. Let $z(q)_c$ be the value of z under a regime of copyright protection, and let $z(q)_{nc}$ be its value when copyright is removed. Then we are interested in the relative change in z from the removal of copyright:

$$\frac{z(q)_{nc} - z(q)_c}{z(q)_c}.$$

It turns out that in model 1, all of the relative changes are independent of the level of quality, and thus can be given as a specific percentage change. In the other models, the relative effect from removal of copyright differs as quality changes. In Table 3, all of the graphs shown are piecewise, since even under copyright, the inability to pay readers implies that for the ranges of quality when it would be optimal to pay readers, the reader price must be set at 0. Thus, the removal of copyright has no effects at all on those zones of quality. We can now see that there are some significant differences between models 2 and 3. While in all of the models, the journal loses profit when copyright is removed (on the zone for which they would like to charge a positive reader price), but the percentage loss in profit is decreasing in model 2 and increasing in model 3. That is, in model 2, the higher is the level of quality of the journal, the smaller is the percentage loss in profits when copyright is removed, while in model 3 the opposite is true.

[14] The absolute values of our variables would be altered by simply changing, for example, our assumption on the value of α. However, as we shall see, the absolute value comparison, given α, is still interesting.

Table 3: Effect of removal of copyright, relative change			
	model 1	model 2	model 3
p_a^*	28.57%		
profit	−31.41%		
welfare	15.26%		
share	40.49%		

The relative analysis of the effect of removal of copyright on profits is interesting, but more enlightening is the analysis of the absolute loss in profits in models 2 and 3. In Figures 6 and 7 we show the absolute change in profits for these two models. The important thing to notice about Figures 6 and 7 is the huge difference in the scale of the vertical axis. While in both models, under copyright, the levels of profit attained are the same (see Table 1, row 1), the removal of copyright in model 2 results in a relatively small absolute loss in profits at **all** quality levels (outside of those for which the reader price under copyright would be set at 0), while in model 3 it results in a similarly small loss for small levels of quality (below about $q = 1.2$), but very large absolute loss in profits for high quality journals.

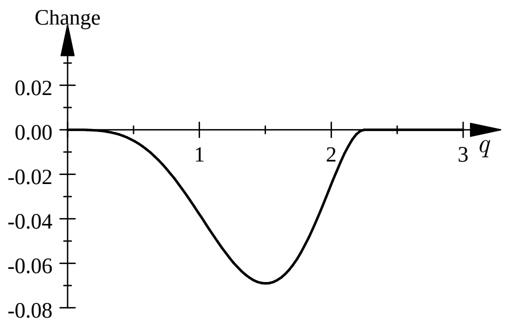


Figure 6: Absolute change in profit from removal of copyright; model 2

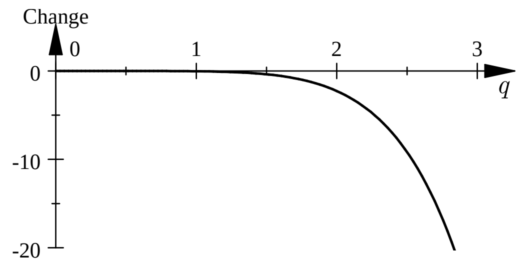


Figure 7: Absolute change in profit from removal of copyright; model 3

While the relative effect upon journal profit in model 2 is seemingly large for lower levels of quality, these losses are for very low levels of profit anyway. Removal of copyright in model 2 hardly affects the profits of journals at any quality levels. However, removal of copyright leads to large profit losses when profits are large in model 3, a much more devastating result. If, for example, journals did have some fixed costs of operation (as is likely in the real world), then removal of copyright would lead to the closure of only very low quality journals in model 2, but it can lead to the closure of high quality journals in model 3. The removal of copyright as suggested by Shavell (2010) may be a rather dangerous strategy in a scenario like that of model 3.

In the welfare analysis, in model 2 there is a rather large zone of positive welfare gains in percentage terms, whereas in model 3 the zone of welfare gains is much smaller, and the relative gains are also smaller. Thus, assuming that social welfare is the policy objective, it would appear that removal of copyright might be a reasonable policy in model 2, but not in model 3. This intuition can again be confirmed by looking at the absolute changes in welfare from removal of copyright in Figures 8 and 9. Again we need to look at the scale of the vertical axis. In model 2 (Figure 8), while there is a very small negative part of the graph at levels of quality below about 0.2, the scale of these losses is totally insignificant compared to the gains at larger quality levels.[15] In short, in model 2 removal of copyright leads to hardly any danger of welfare loss, and relatively interesting (upwards of about 20%) welfare gains for almost all levels of quality.

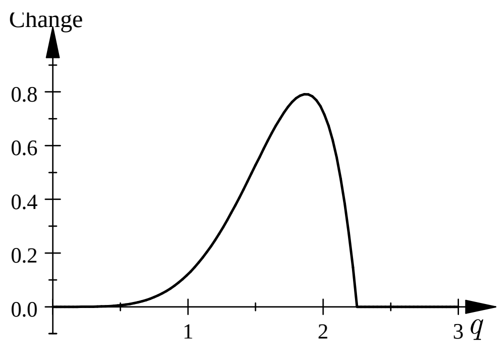


Figure 8: Absolute change in social welfare from removal of copyright; model 2

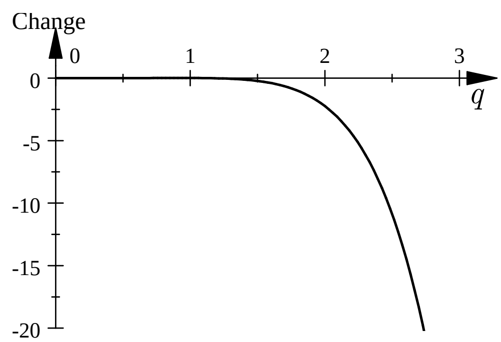


Figure 9: Absolute change in social welfare from removal of copyright; model 3

On the other hand, consider the absolute welfare change in model 3 (Figure 9). In this graph there is a positive section between levels of quality of $\frac{1}{3}$ and about 1.1.[16] All of the rest of the graph lies below the horizontal axis, and at relatively large numbers, which implies that removal of copyright leads to large welfare losses for those levels of quality. Thus, in model 3 the removal of copyright can improve welfare for low levels of quality but the improvement is miniscule, whereas for higher levels of quality, the change in social welfare is negative and significant.

Finally, we comment on the last row of Table 3. The relative change in the share of welfare that goes to academics is decreasing in quality in model 2 and increasing in quality in model 3. That is, when copyright is removed,

[15] Indeed, the negative section of the graph cannot even be discerned unless the vertical scale is changed by a factor of about $\frac{1}{100}$.

[16] Again this positive part cannot be discerned in the graph, unless we change the vertical scale by a factor of about $\frac{1}{100}$.

if we are in model 2, while total welfare is much more likely to go up, the share of this welfare that accrues to academics drops. If we are in model 3, the share of academic welfare in total welfare rises when copyright is removed, but it is more likely that total welfare drops. We are also able to perform a welfare analysis for readers and authors separately. If that is done, then it turns out that removal of copyright in either model leads to less author welfare and more reader welfare, and the gain in reader welfare outweighs the loss in author welfare. However, since in reality readers and authors are generally the same people (academics), it is probably more interesting to consider the sum of welfare going to the readers and authors.[17]

7 A consideration of capacity constraints

Above, we have noted that in order to consider some degree of competition in our model, it would be relevant to impose capacity constraints on both of the two sides of the market. In this way, the journals market can be thought of as operating in an environment of monopolistic competition. A full consideration of capacity constraints in the simulations that we have done of the model would add quite a large number of new scenarios to consider. We feel that it is best to leave a detailed analysis of it to future research, although it is worthwhile to mention here how things would play out.

Under a capacity constraint, the journal could count on a certain maximum number of both readers and authors. The number of readers and authors are both increasing functions of quality in all of the model configurations that we have used. Thus, although the journal's profit is also increasing in quality, the journal would not be able to set quality arbitrarily high, as at some point it would run out of either readers or authors. In this way, the capacity constraints

[17] However, if we were to consider academics at different universities, and since the authorship at some universities is significantly higher than at others (high ranked universities versus low ranked ones on a scale of publications), then we might want to calculate reader and author welfare separately. At universities with low publication outputs, the academics are mainly readers. These universities would apparently gain significantly from removal of copyright. The same may not be true in universities with a high number of publications. See Mueller-Langer and Watt (2010) for further details.

would determine the final quality achieved in the model.

The introduction of capacity constraints would have important effects when the removal of copyright is considered. Unless the removal of copyright somehow were to alter the binding constraint (something that would seem not to be logical), then the capacity constraints have the potential to intervene in the welfare analysis of the previous section.

Take for example our model 1. When copyright is removed, social welfare increases by around 15% regardless of the level of quality. However, in that model, the removal of copyright will also increase the numbers of both readers and authors at each level of quality. This in turn implies that the capacity constraint must now bind at a lower level of quality, and so in the end the final quality that is actually achieved is decreased. Since social welfare is an increasing function of quality, there is an off-setting effect on social welfare that may or may not counterbalance the 15% gain that is initially found by removing copyright.

Model 2 works in a similar way to model 1 in respect of this capacity constraint effect. Removal of copyright will increase social welfare at almost all levels of quality for which open access was not optimal with copyright in place, but it will also increase the numbers of both readers and authors at each quality level. Thus the capacity constraint will bind at a lower level of quality, and so final quality achieved will go down.[18] The social welfare gains are, at least partially, off-set by the welfare loss of a lower quality level. On the other hand, in model 3, the opposite occurs. Removal of copyright is likely to increase social welfare, but in that model the numbers of readers and authors are decreased at each level of quality when copyright is removed.[19] The capacity constraint then would bind at a higher level of quality than before, implying a welfare gain that (at least partially) off-sets the losses from removal

[18] Again, this is assuming that the optimal reader price with copyright was not set at 0. If open access were optimal under copyright, then of course no effect at all happens when copyright is removed.

[19] In relality this only happens in model 3 when quality is above a certain threshold. However, the threshold is at a low level of quality, and below this threshold although the numbers of academics served actually increase, the change is rather infinitesimal.

of copyright.

It is impossible to know which of the two effects (the directy welfare effect at each level of quality from removal of copyright, or the indirect welfare effect of the change in quality due to the capacity constraints) is the larger. However, studying this effect would make an interesting extension to the present paper.

8 Conclusions

All of our conclusions are based upon numerical simulation and particular functional forms, and so should be read with due care. However, the only variable in our model is the effect of an increased level of quality upon the number of readers and authors, that is, the vertical intercept of our demand curves. Different values for this vertical intercept would change the numbers we get, but not the structure of the models.

Our conclusions are the following. Firstly, with regard to the relationship between journal quality and open access in a copyright protected regime (i.e. the status quo), our model suggests that it is NOT true that open access journals will necessarily have lower quality than closed access journals as has been suggested in the literature. Indeed, we find that under appropriate profit maximisation on both sides of the journal market, there exist configurations under which it is the higher quality journals that will have the open access format (our model 2). We also have a model (model 1) in which open access is never a feature of an optimally priced journal.

Second, with regard the hypothesized removal of copyright, as suggested by Shavell (2010), we find that removal of copyright will have a different effect depending upon the configuration of the market. We find scenarios in which removal of copyright will have hardly any effect on profits, but will increase social welfare for almost all quality measures (model 2), and other scenarios in which removal of copyright will have a serious negative effect on the profits of high quality journals, and that will reduce social welfare (model 3). Thus, again we cannot unambiguously support removal of copyright, but nor can we unambiguously support its continued retention. In our model 1, we find that removal of copyright is unambiguously social welfare improving, but it will also have a serious negative effect on journal profits. If the real state of the world is something like model 1, then removal of copyright is likely to be a beneficial social policy, but it may have to be accompanied by an alternative business model for publication of scientific work.

This paper suggests several directions in which future research could be directed. Firstly, it would be most interesting to verify empirically which, if any, of our three models is most likely to be real-world relevant. Models 1 and 2 provide support for removal of copyright, while model 3 does not. The critical issue is where the diminishing returns lie; is it the production of readers with authors as an input that suffers diminishing returns, or rather is diminishing returns a feature of the production of authors with readers as an input? We can think of logical reasons to support either argument.[20] Perhaps an empirical examination could throw some light on this issue. Second, our model has been calibrated with a single parameter for the effect of increased quality upon the demand for journal space by both authors and readers. While considering different values of this parameter will not alter our model in any significant manner, it would certainly be of interest to consider that the effect is different for authors as for readers. Doing so would unbalance the model, and would certainly have the potential to alter some of our conclusions. However, again it is very hard to think of convincing reasons why an increase in journal quality will attract new readers in a notably different way to how it attracts new authors. Third, the model generates specific formulas for the numbers of readers and the numbers of authors for each quality level. The ratio between these two gives us the number of readers per published paper, something that we may associate with the "impact" of the journal. Further, the impact factor

[20] On the one hand, the addition of readers into the author production function is the addition of new units of perfectly substitutable inputs, which traditionally would be thought to involve diminishing returns. For this reason, perhaps model 3 is realistic. However, on the other hand, even though additonal authors are non-perfect substitutes for producing readers, surely each reader has a strict time budget constraint for reading papers, and so adding papers is the addition of a variable (all-be-it non perfectly substitutable) input to a fixed time input, which again we might think would imply diminishing returns, making model 2 realistic.

that is habitually used (e.g. by ISI), which is cites per paper published, can be seen as nothing more than readers per paper times the probability that any given reader will end up citing the paper he or she reads in a follow-up paper. It would be of great interest to attempt to identify an appropriate function for the probability of citing (as a function of the quality of the journal article read), so that our model may then be applied directly to an analysis of the validity of the ISI impact factor as an indicator of journal quality. Fourth, the journal that we have modelled is an online product only. This simplifies things as regards the costs of running the journal, and thus the journal's profit function. We would, however, be interested in a version of this model being applied to journals with both hard-print and online formats, and above all, a journal with a hybrid-open access policy (a policy in which the author can decide, and pay a corresponding fee to the journal, in order to have the article priced at zero to readers). Finally, our model has paid scant attention to competition over journals (our model is really a monopolistic journal, or at most, a monopolistically competitive journal), and scant attention to the precise manner in which quality is chosen (i.e. the referee process). Accounting for either or both of these features would greatly improve the model, although we hypothesize at a significant increase in complexity.

References

Armstrong, M. (2006), "Competition in Two-sided Markets", *RAND Journal of Economics*, 37(3), 668-91.

Jeon, D.S. and J.C. Rochet (2010), "The Pricing of Academic Journals: A Two-Sided Market Perspective", *American Economic Journal: Microeconomics*, 2; 222-55.

McCabe, M.J. and C.M. Snyder (2005), "Open Access and Academic Journal Quality", *American Economic Review*, 95(2); 453-8.

McCabe, M.J. and C.M. Snyder (2007), "Academic Journal Prices in a Digital Age: A Two-Sided Market Model", *The B.E. Journal of Economic Analysis and Policy*, 7(1) (Contributions), Article 2.

Mueller-Langer, F. and R. Watt (2010), "Copyright and Open Access for Scientific Works", *Review of Economic Research on Copyright Issues*, 7(1); 45-65.

Shavell, S. (2010), "Should Copyright of Academic Works be Abolished?", *Journal of Legal Analysis*, 2(1); 301-58.

Appendix A: Optimal prices in model 2 under copyright protection

The two simultaneous equations (3) and (4) can be written as

$$n_r = \sqrt{n_a}\beta_r, \tag{7}$$

$$n_a = n_r\beta_a, \tag{8}$$

where $\beta_i \equiv \alpha q - p_i$ for $i = r, a$. Both of n_r and n_a are constrained to be positive. We are restricted to parameter values such that $\beta_i > 0$ for $i = r, a$, that is, we can only consider prices that satisfy $p_i < \alpha q$ for $i = r, a$. It is easy to show that the solution to the two equations (7) and (8), outside of the trivial solution at $(0, 0)$, is at $n_r = \beta_r^2\beta_a$; $n_a = \beta_r^2\beta_a^2$. The profits of the journal are given by

$$\pi = p_r n_r + p_a n_a = p_r\beta_r^2\beta_a + p_a\beta_r^2\beta_a^2.$$

From the definitions of the two β_i functions, we can see that profit is now a third-order function of each price.

Consider first the optimal reader price. The two derivatives of the profit function with respect to p_r are:

$$\frac{\partial\pi}{\partial p_r} = \beta_r^2\beta_a - p_r 2\beta_r\beta_a - p_a 2\beta_r\beta_a^2,$$

$$\frac{\partial^2\pi}{\partial p_r^2} = -4\beta_r\beta_a + p_r 2\beta_a + p_a 2\beta_a^2.$$

The first-order condition for a maximum is

$$\frac{\partial\pi}{\partial p_r^*} = 0 \quad \Rightarrow \quad \beta_r^{*2}\beta_a - p_r^* 2\beta_r^*\beta_a - p_a 2\beta_r^*\beta_a^2 = 0$$

where $\beta_r^* = \alpha q - p_r^*$. Extracting the common factor, we have $\beta_r^* \beta_a \left(\beta_r^* - 2p_r^* - 2p_a \beta_a \right) = 0$. Since $\beta_r^* \beta_a > 0$, we have

$$\beta_r^* - 2p_r^* - 2p_a \beta_a = 0. \tag{9}$$

Substituting for β_r^*, this reads $\alpha q - p_r - 2p_r - 2p_a(\alpha q - p_a) = 0$. The final solution is given by

$$p_r^* = \frac{\alpha q - 2p_a(\alpha q - p_a)}{3}. \tag{10}$$

This solution is unique on the range $p_r < \alpha q$. Since our solution (10) is unique, in order to ensure that it is a maximum, we need to show that the second-order condition holds at that solution:

$$\frac{\partial^2 \pi}{\partial p_r^2} = -4\beta_r^* \beta_a + 2p_r^* \beta_a + 2p_a \beta_a^2 < 0 \Rightarrow -4\beta_r^* + 2p_r^* + 2p_a \beta_a < 0.$$

Equation (9) is $2p_a \beta_a = \beta_r^* - 2p_r^*$. Substituting this into our second-order condition we get

$$-4\beta_r^* + 2p_r^* + \beta_r^* - 2p_r^* < 0 \Rightarrow -3\beta_r^* < 0,$$

which holds for any $p_r^* < \alpha q$. Thus, (10) is indeed a maximum.

Second, consider the optimal author price. The first two derivatives of the profit function with respect to p_a are:

$$\begin{aligned} \frac{\partial \pi}{\partial p_a} &= -p_r \beta_r^2 + \beta_r^2 \beta_a^2 - 2p_a \beta_r^2 \beta_a, \\ \frac{\partial^2 \pi}{\partial p_a^2} &= -4\beta_r^2 \beta_a + 2p_a \beta_r^2. \end{aligned}$$

The first-order condition is

$$-p_r \beta_r^2 + \beta_r^2 \beta_a^{2*} - 2p_a^* \beta_r^2 \beta_a^* = 0 \Rightarrow -p_r + \beta_a^{2*} - 2p_a^* \beta_a^* = 0. \tag{11}$$

The second order condition is

$$-4\beta_a^* + 2p_a^* < 0$$

which, upon substituting for β_a^* reduces to

$$p_a^* < \frac{2\alpha q}{3}. \tag{12}$$

Now, note that (11) is just

$$-p_r + (\alpha q - p_a^*)^2 - 2p_a^* (\alpha q - p_a^*) = 0$$

or

$$3p_a^{*2} - 4\alpha q p_a^* - p_r + (\alpha q)^2 = 0.$$

Using the quadratic formula, we know that the two roots of this equation satisfy

$$\frac{4\alpha q \pm \sqrt{16\alpha^2 q^2 - 12 (\alpha^2 q^2 - p_r)}}{6}.$$

Simplifying, we get

$$\frac{2\alpha q \pm \sqrt{\alpha^2 q^2 + 3p_r}}{3} = \frac{2\alpha q}{3} \pm \frac{\sqrt{\alpha^2 q^2 + 3p_r}}{3}.$$

We can see from the second-order condition that the higher of these two roots is a minimum, and the lower is the maximum. Thus, the optimal author price is given by

$$p_a^* = \frac{2\alpha q - \sqrt{\alpha^2 q^2 + 3p_r}}{3}. \tag{13}$$

In order to find the exact optimal prices for readers and authors, both as functions of only the journal quality q, we simultaneously solve the two first-order equations (10) and (13). To that end, substitute (10) into (13):

$$\begin{aligned} p_a^* &= \frac{2\alpha q - \sqrt{\alpha^2 q^2 + 3\left(\frac{\alpha q - 2p_a^*(\alpha q - p_a^*)}{3}\right)}}{3} \\ &= \frac{2\alpha q - \sqrt{\alpha^2 q^2 + \alpha q - 2p_a^*(\alpha q - p_a^*)}}{3}. \end{aligned}$$

Simple steps then give

$$\begin{aligned} 2\alpha q - 3p_a^* &= \sqrt{\alpha^2 q^2 + \alpha q - 2p_a^*(\alpha q - p_a^*)} \\ &\Rightarrow 4\alpha^2 q^2 - 12\alpha q p_a^* + 9p_a^{*2} = \alpha^2 q^2 + \alpha q - 2p_a^*(\alpha q - p_a^*). \end{aligned}$$

We get the following second-order equation:

$$7p_a^{*2} - 10\alpha q p_a^* + 3\alpha^2 q^2 - \alpha q = 0.$$

Applying the quadratic formula, we get

$$\begin{aligned} p_a^* &= \frac{10\alpha q \pm \sqrt{100\alpha^2 q^2 - 28(3\alpha^2 q^2 - \alpha q)}}{14} \\ &= \frac{5\alpha q \pm \sqrt{4\alpha^2 q^2 + 7\alpha q)}}{7}. \end{aligned}$$

The upper root of this is greater than[21] αq. So the unique value of p_a^* is

$$p_a^* = \frac{5\alpha q - \sqrt{4\alpha^2 q^2 + 7\alpha q)}}{7}. \tag{14}$$

Finally then, we need to substitute this back in to the equation for the

[21] The upper root is $\frac{5\alpha q}{7} + \frac{1}{7}\sqrt{4\alpha^2 q^2 + 7\alpha q} > \frac{5\alpha q}{7} + \frac{1}{7}\sqrt{4\alpha^2 q^2} = \frac{5\alpha q}{7} + \frac{2\alpha q}{7} = \alpha q$.

optimal reader price (10):[22]

$$\begin{aligned} p_r^* &= \frac{\alpha q - 2\left(\frac{5\alpha q - \sqrt{4\alpha^2 q^2 + 7\alpha q)}}{7}\right)\left(\alpha q - \left(\frac{5\alpha q - \sqrt{4\alpha^2 q^2 + 7\alpha q)}}{7}\right)\right)}{3} \\ &= \frac{21\alpha q - 4\alpha^2 q^2 - 2\alpha q\sqrt{q\left(4\alpha^2 q + 7\alpha\right)}}{49}. \end{aligned} \tag{15}$$

Appendix B: Optimal author prices when copyright is removed

Model 1 When copyright is removed, and the reader price is constrained to be equal to 0, the profit of the journal is given by $\pi = p_a n_a = p_a \left(\beta_r^2 \beta_a^4\right)^{\frac{1}{3}}$. We have $\beta_r = \alpha q$, so the profit function can be written as

$$\pi = p_a \left(\left(\alpha q\right)^2 \beta_a^4\right)^{\frac{1}{3}} = \left(\alpha q\right)^{\frac{2}{3}} p_a \beta_a^{\frac{4}{3}}.$$

The first-order condition[23] for an optimal choice of p_a is

$$\left(\alpha q\right)^{\frac{2}{3}} \left(\beta_a^{*\frac{4}{3}} - \frac{4}{3} p_a^* \beta_a^{*\frac{1}{3}}\right) = 0 \Rightarrow \beta_a^* = \frac{4}{3} p_a^*$$

which, since $\beta_a^* = (\alpha q - p_a^*)$, is the same as

$$p_a^* = \frac{3\alpha q}{7}.$$

Recall that under copyright, the optimal author price was $\frac{\alpha q}{3}$, thus aside from reducing the reader price to 0, the removal of copyright serves to increase the optimal author price by $\frac{3\alpha q}{7} - \frac{\alpha q}{3} = \frac{2\alpha q}{21}$.

Model 2 There is no need to re-do the optimisation under the restriction that $p_r = 0$. We only need to use that value of reader price in the equation

[22] The simplification for this was carried out using the package Mathematica.

[23] The second-order condition is $-\frac{8}{3}\beta_a^{*\frac{1}{3}} + \frac{4}{9} p_a^* \beta_a^{*-\frac{2}{3}} < 0$. This is satisfied if $p_a^* < 6\beta_a^*$. Using the definition of β_a^*, the second order condition can be written as $p_a^* < \frac{6\alpha q}{7}$. The solution to the first-order condition satisfies this, and so we can be assured that p_a^* is indeed a maximum.

(13) in Appendix A. Substituting in $p_r = 0$, and simplifying, we see that the optimal price without copyright is

$$p_a^* = \frac{\alpha q}{3}.$$

Again, the optimal author price increases with the removal of copyright. In contrast to the case of copyright protection, now the optimal author price is strictly positive, and linear, for all levels of quality.

Model 3 The relevant equation from Appendix A (with the subscripts switched to capture the modelling change) is $p_a^* = \frac{\alpha q - 2p_r(\alpha q - p_r)}{3}$. Clearly, setting $p_r = 0$ give us exactly the same author price as in model 2, namely

$$p_a^* = \frac{\alpha q}{3}.$$