COOPERATION, CREATIVITY AND CLOSURE IN SCIENTIFIC RESEARCH NETWORKS:

Modeling the Simpler Dynamics of Invisible Colleges

By

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SUMMARY

COOPERATION, CREATIVITY AND CLOSURE IN SCIENTIFIC RESEARCH NETWORKS

Scientific research communities or “invisible colleges” are conceptualized in this paper as social communications structures formed by the overlapping of more compact networks of personal associations among researchers in particular disciplinary fields of inquiry. Within these “colleges” circulate ideas and opinions regarding the “validity” of specific scientific propositions and research methods. Transmission of tacit knowledge (which is complementary to the codified information broadcast in archival publications) takes place through local social network connections, and is regarded to be critical in enabling individual researchers to participate in, and contribute to the collective epistemological tasks. Building on a model proposed by David (1998, 2003), this analysis posits a population of rational research agents engaged individually in continuous processes of experimental observation and Bayesian inference, whose interpretations of their current empirical observations are influenced by the prevailing distribution of opinion expressed within their respective local social networks Formulation of this structure in graph theoretic terms, and specification of a random “local opinion polling” process, permits the application of results from Markov random field theory to show how probabilistic micro-level knowledge communication behavior affects the capacity of the invisible college (as the ensemble of inter-linked social networks) to attain a collective cognitive state of “closure.” The latter state is represented as a configuration of correlated belief orientations in regard to the validity, or invalidity of particular scientific propositions. This stylized, highly simplified model’s key features resemble closely the stochastic process known as the “voter model,” a well-studied reversible spin system due to Clifford and Sudbury (1973) and Holley and Liggett (1975). It is shown that the ability of the entire epistemic community to arrive at “closure” regarding a given question depends critically upon the degree to which its members’ communication behaviors conform to the norms of cooperation, disclosure and universalism that Merton (1973) identifies with the institutionalized conduct of open science. But the nature of the mechanism posited is such that the resulting “knowledge consensus” will be an emergent and path dependent property of the network ensemble, as has been contended by more recent contributors to the sociology of scientific knowledge.

Further elaboration of this simple structure is focused upon characterizing the endogenous influences upon the communication behaviors of the agents, and deriving the macro-properties of the equivalent deterministic system corresponding to the stochastic “voter model.” Here the expected time for achieving closure and the expected rate of “collective creativity” attained through the recombinant generation of ideas are key macro-level performance features. Along with the size of the global ensemble, they are taken to influence behaviors governing the average rates of transmission of tacit knowledge and opinion through the structure of inter-linked local social networks. With the size of the global network being set exogenously, a wide range of size-related cognitive performance attributes for the invisible college may exist, each representing a (short-run) equilibrium. The larger among these systems generally are associated with greater average “openness” at the micro level, and higher rates of systemic creativity.

In the long run, however, it is supposed that those macro-properties, and the associated expected speed with which closure on new propositions is attained, would influence the network ensemble’s ability to grow by drawing in additional qualified researchers from the surrounding environment. Such a process of (lagged) adjustments in network size yields a simple deterministic dynamic model that is found to possess high-level and low-level equilibrium states, both of which are locally stable. An autonomously high average propensity on the part of researchers to disclose knowledge promotes dynamics that lead to the system’s high-level attractor. But, seemingly small policy interventions can adversely “shock” a communicative and creative scientific network, causing it to shrink until it can survive only as a much smaller, less open and less creative research community. The concluding section discusses the salient limitations of this heuristic model, and the respects in which its structure may be elaborated, and the resulting systems’ properties studied by means of stochastic simulation.

CONTENTS

1. Modeling the Workings of Open Science Communities: Motivation and Background

1.1 The pursuit of knowledge and the sources of technological change

Economists seeking to understand the sources of technological change have focused their attention upon the dynamics of the diffusion of innovations, and the generation and distribution of knowledge underpinning the development and commercial introduction of new products of production methods. Quite rightly, their quest for clearer vision of the insides of the "black boxes" of technology, and of innovation will continue to command the major share of the analytical and empirical attention devoted to providing firmer microeconomic foundations for the theories of endogenous economic growth.[1]

By comparison with what has been learned already concerning institutional arrangements and business strategies affecting corporate R&D investments, and the mechanisms enabling private appropriation of research benefits, it remains surprising that so much less is known about the institutional infrastructures and micro-motives that influence the allocation of economic resources within the domain of non-commercial, "academic" science. The "science base," as the publicly funded civilian R&D sector has come to be referred to in Britain, remains a sphere of activity that economic analysis tends to discuss more in terms of its *external effects* than its internal workings.[2] Research of an exploratory character, undertaken to discover new phenomena, or to explain the fundamental properties of physical systems, is cited as a source of directly useful innovations in instrumentation, or in generic techniques valued in applied research – such as synchrotron radiation, and restriction enzyme methods for "gene-splicing." Indirect cognitive contributions of a fundamental character also are seen to raise the expected marginal rate of return on investment in applied R&D, by establishing "possibilities" that may have practical application, such as the photo-electric effect described by Einstein's paper in *Annalen der Physik* (1915); and, also, in definitively excluding time-wasting traps as physical "impossibilities" – as in the case of machines of perpetual motion, and some among the myriad conceivable configurations for the bases in the structure of the DNA molecule.[3]

Despite the cognitive significance of the activities of "the science base," and despite its quantitative importance as the locus of employment and the training ground for expensively educated researchers, the discipline of economics still is in the early stages of a program of systematic inquiry into the ways that the pursuit of reliable knowledge is carried on within "the Republic of Science."[4] Although the latter domain forms a critical part of modern "social systems

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[1] See Rosenberg 1982, 1994 on the "black box of technology; Aghion and Howitt 1998, and Aghion and Tirole 1998 on "the black box of innovation." But see, also, David 1994 on "reopening another black box" – the economics of exploratory (academic) research.

[2] This focus upon the "externalities" created by fundamental research in science derives in large part from the preoccupation of the economics literature with arguments for public subsidies for such activities. See, e.g., David 2001, in exemplification.

[3] Crick 1988:pp. 139ff offers the case of DNA in illustration of his argument that physical science theory is of more help to biologists in establishing impossibilities than in guiding researchers to the particular solution that had been found "by Nature."

[4] One active part of this program is (self-) identified with "the new economics of science," following Dasgupta and David (1987, 1994), who took up Polanyi's (1962) conceptualization of "the Republic of Science" in describing the domain of interest. See also, for further explorations of this territory: Arora, David and Gambardella (1998); Arora and Gambardella (1994, 1998); Cowan, David and Foray (2000), Cowan and Jonard (2001), Dalle (2000), David (1994, 1995, 1996, 1998); David and Foray (1995); David, Geuna and Steinmueller (1995); David, Mowery and Steinmueller (1992); Gambardella (1994);

of innovation," it is one whose characteristic internal properties cannot simply be inferred from an understanding of the economics of industrial research and development.[5]

1.2 The logic of open science as a mode of organizing research

Within university-based research communities, especially, there are recognized a quite distinctive set of norms and conventions that constitute a well-delineated professional ethos to which scientists generally are disposed to publicly subscribe, whether or not their own behaviors always conform literally to its strictures governing the organization and conduct of research. The norms of the Republic of Science that have famously been articulated by the sociologist Robert K. Merton sometimes are conveniently summarized under the mnemonic CUDOS: communalism, universalism, disinterestedness, originality, skepticism.[6]

The "communal" ethos emphasizes the cooperative character of inquiry, stressing that the accumulation of reliable knowledge is an essentially social process, however much individuals may strive to contribute to it. The force of the universalist norm is to render entry into scientific work and discourse open to all persons of "competence" regardless of their personal and ascriptive attributes. A second aspect of "openness" concerns the disposition of knowledge: the full disclosure of findings, and methods, form a key aspect of the cooperative, communal program of inquiry. Full disclosure, in turn serves the ethos legitimating and, indeed, prescribing what Merton called "organized skepticism"; it supports the expectation that all claims to have contributed to the stock of reliable knowledge will be subjected to trials of replication and verification, without insult to the claimant. The "originality" of such intellectual contributions is the touchstone for the acknowledgment of individual scientific claims, upon which collegiate reputations and the material and non-pecuniary rewards attached to such peer evaluations are based.

By considering the economic logic of the organization of knowledge-producing activities, one may make a start towards grasping the connection between the existence of a social system distinguished by, and in some manner *regulated* by these norms, and the importance that has been attributed to non-commercially driven, exploratory science among the sources of technological progress. Indeed, it is possible in just such terms to give a complete functionalist account of the institutional complex that characterizes modern science.[7] In brief, the norm of "openness" is "incentive compatible" with a collegiate reputational reward system based upon accepted claims to priority; and it is conducive to individual strategy choices whose collective congruence reduces excess duplication of research efforts, and enlarges the domain of informational complementaries. This brings socially beneficial "spill-overs" among research programs, and abets rapid replication and swift validation of novel discoveries. The advantages of treating new findings as "public goods" in order to promote the faster growth of the stock of knowledge, thus, are contrasted with the requirements of secrecy for the purposes of securing a

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Trajtenberg, Henderson and Jaffee (1992). Some of the foregoing receive notice in the wider survey of the economics of science by Stephan (1996).

[5] On the concept of "social systems of innovation" see Amable, Barré and Boyer (1997). International differences in many dimensions of innovation activity, both as to its industrial organization and performance, are finely delineated in this work. Yet, apart from noting the tendency of scientific specialization to be aligned with the areas of concentration in patenting activity, very little notice is given to issues pertaining to corresponding similarities and differences in the structure and performance of "the science base;" actually, quantitative patterns of "scientific specialization" are inferred from those in patenting, rather than gauged from bibliometric analysis of scientific publications (see pp.4, 249-254).

[6] See Merton 1973: esp. Ch. 13; Merton 1986: Pt. III. On CUDOS, see Ziman 1994, p. 177.

[7] See, e.g., Dasgupta and David 1987, 1994; David 1993.

monopoly over the use of new information that may be directly or indirectly exploited in the production of goods and services.

This functionalist juxtaposition suggests a logical basis for the existence and perpetuation of institutional and cultural separations between two normatively differentiated communities of research practice, the open "Republic of Science" and the proprietary "Realm of Technology": the two distinctive organizational regimes serve different and potentially complementary societal purposes. Rather baldly stated, the first regime is well suited for the purpose of maximizing the rate of growth of stocks of reliable knowledge, whereas the second is better designed to maximize the flow of economic rent from existing knowledge. In the long run, neither can continue to function fruitfully in isolation from the other. This being the case, the challenge for science and technology policy may be construed as one of keeping these two sub-systems linked and in symbiotic balance, so that the performance of social systems of innovation as a whole do not become degraded. In preparation for such a task it will be important to try to redress the comparative deficiency in our knowledge of the workings of open science research communities. That is the larger purpose to which this paper is addressed.

1.3 Overview: from local micro-behaviors to macro-network dynamics

Scientific research communities may be studied as social networks within which ideas or statements circulate, acquire validity as reliable knowledge, and are recombined to generate further new ideas. Personal communications networks also form the locus for the transmission of tacit knowledge and skills requisite to the interpretation and operationalization of scientific statements. This paper builds upon an abstract, highly stylized account of the communications structure of larger ensembles of research scientists formed by the interconnections among more localized, interpersonal networks in which their professional activities are embedded. Such an account has been explored in previous work, employing graphic-theoretic apparatus to describe the structure of the social networks through which transactions in tacit knowledge are conducted, and results Markov random field theory to extract some implications of micro-level communications strategies for the ensemble's collective epistemological performance.[8]

Social networks have come to be modeled in many contexts involving strategic interdependence, where these structures are represented as conveying information, and forging mutual trust through repeated transactions – even though the connections among the players are highly localized and can be presumed to be effected without sophisticated technological supports. A number of lines of inquiry in game theory also have converged upon local network structures, as the terrain for analysing the equilibrium properties of games characterized by strategic complementaries and interactive learning on the part of players.[9] Interest there has ocused upon the strategic problem that arises when it is assumed that each player interacts directly with only some subset of the entire ensemble – those in the player's immediate "vicinity," but that the players are unable to adapt their behaviors to deal individualistically with each of their "neighbors". In such circumstances every player must select a strategy that is uniform with regard to all their neighbors. Within the social network context such a strategy choice could become generalized as a behavioral "norm." This has heightened interest in studying the conditions under which the dynamics of local interaction games of this kind will give rise to equilibria characterized by correlated beliefs or behaviors, in both deterministic decision frameworks and in dynamic stochastic settings. Models of the latter sort typically make use of results from Markov random field theory to show how local network externalities can

[9] See, among early examples Anderlini and Ianni (1993), Blume (1993), Ellison (1993), Bala and Goyal (1995), Morris (1996). For expositions and overviews of the social and economic networks literature, see Goyal (2007), Jackson (2008, 2011).

lead to *de facto* standardization in choices of production methods and the spontaneous formation of conventions.[10] As will be seen, the present paper is thus situated in that broader stream of the recent economics literature.

The main goal of the analysis developed in David (1998, 2002) was to show how behaviors regarding the disclosure of knowledge and current scientific opinion by the individual agents, and their receptivity to corresponding flows of information conveyed by members of their personal networks, would affect the capacity of the entire ensemble to attain a collective cognitive state of "scientific closure." What is meant by "closure" is simply the emergence of a preponderant "consensus." This epistemological condition is therefore represented as a configuration of correlated belief orientations among the members of a certain epistemic community identified with a particular scientific field, in regard to the validity or the invalidity of particular scientific propositions. The existence of a past record of matters on which informed opinion approached substantial unanimity, and the prospects of the ensemble being able to achieve comparable successes in the case of new propositions, constitute collective cognitive coherence; these conditions justify labelling as "an invisible college" what would otherwise be merely an aggregation of researchers.

Within the context of the model, the ability to reach "closure" in that sense can be seen to depend critically upon the degree to which the epistemic community's members conform in their communication behaviors to the norms of cooperation, disclosure and universalism that Merton (1973) identified with the institutionalized conduct of open, "modern" science. This conclusion follows as a formal proposition from the close resemblance between the properties of a particular stochastic process known as the "voter model," and the stylized account offered in David (1998, 2002, 2003) of the way that researchers' interpretations of their observational and experimental data are shaped (probabilistically) by the distribution of peer-opinion and related tacit knowledge in their local communication networks.

The structural underpinnings of the preceding analysis that is to be carried forward here are reviewed in the next two sections of the paper. Section 2 summarizes the basis for adopting that characterization of the probabilistic communication strategies of the individuals forming tacit knowledge networks. It also proposes an algorithmic representation of the role of local peer opinion in a Bayesian inferential procedure followed by empirical researchers, and suggests conditions on the structure of reputational rewards that would induce the individual agents to adopt strategies of "cognitive conformism." Section 3 briefly reviews the formal properties of the suggested probabilistic process of consensus formation, by showing its correspondence to the formal structure of the so-called "linear voter model." The latter is a well-known reversible spin system introduced by Clifford and Sudbury (1973) and Holley and Liggett (1975), the properties of which been thoroughly investigated both by mathematical and stochastic simulation methods (e.g., by Liggett 1985 and Durrett 1988, respectively).

A brief digression is undertaken in Section 4, back-tracking a bit in order to report on some more recent results that have been obtained for modifications and extensions of the classic Voter Model (discussed in section 2). One of these, the "non-linear voter model," allows for the probability that while an individual adopting a particular belief-orientation will be influenced positively by the preponderance current opinion in her local social network, the mapping from community to agent may not be strictly linear. A second extension introduces the possibility of "bias" favoring one of the binary alternatives in the probabilistic processes of individual opinion-reorientation. Doing so allows the stylized modeling framework to accommodate the position taken by the "realists" in recent the "culture wars" concerning the

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[10] See David (1988, 1992), Kirman (1992), David and Foray (1993, 1994), Dalle (1995), David, Foray and Dalle (1997), Ellison and Fudenberg (1995), Brock and Durlauf (1997).

nature and authority of scientific knowledge: namely, that the methodology of empirical research (an important hallmark of modernity) yields a palpable drift towards closer correspondence between objective realities of the world the cognitive content of those propositions upon which invisible colleges reach consensuses ("closures" of the collective epistemological evolution). In other words, embracing the "social constructivist" perceptions of the potent influence of consensus-reinforcing pressures within communities engaged in normal science, and allowing for the problems of ambiguity arising from experimental and observational error, does not undermine the claim that such communities are able to not only arrive at widespread agreements, but tend to approach reliable agreements about the actual ("true") workings of the physical world. A number of the properties of the linear voter model, especially the conditions for the existence of extremal equilibria where belief orientations are perfectly correlated, and the variation of the expected speed of achieving consensus with the number of nodes in the global network, are found to carry over from the classic Voter Model to these non-linear extensions, where there is both "noise" and feed-back from experimentation that results in persistent drift favoring one of the multiplicity of claims around which a consensus could be formed.[11]

Section 5 extends the Voter Model representation of a consensus formation process in a different direction, elaborating it by specifying feedback relationships whereby the macro-level performance characteristics of the global network affect the micro-level communication behaviors of the research-agents in their respective local social networks. In the most elementary formulation, the latter are described by the endogenous alterations that occur in the mean probabilities of sending messages and receiving messages, or "disclosing" and "reading (absorbing)" tacit knowledge in transactions with known personal correspondents.

Further results from Markov random field theory are introduced (in sub-section 5.1) in order to obtain specifications for the way that those probabilities and the size of the global network – measured as the number of researchers – affect the expected time for messages to "percolate" between random pairs of researchers at nodes in the connected graph envisaged by the linear Voter Model. For a community whose members are homogeneous in regard to their positive communication probabilities, and situated on a lattice of sufficiently low dimensionality, this provides a specification for the expected speed of "closure" with respect to a particular scientific proposition. A further elaboration (in sub-section 5.2) introduces the possibility that the exchange of tacit knowledge gives rise to new propositions, or conjectures, and suggests a specification for the homogeneous global network's expected rate of "creativity" through recombinant generation of ideas. This counterpart of the expected speed of closure is specified as a function of the network's size, and the mean probability that within the local social networks any agent having a new idea will share it freely when she is contacted.

The system is then closed (in Section 6.1) under the hypothesis that the micro-level communication probabilities of the (homogenous) agents will be endogenously determined in response to the expected performance properties of the global network, with regard to "closure speed and creativity." The resulting closed stochastic system is then reduced to its "equivalent deterministic system" for the purposes of analysis, and it is shown that parametric variation in network size generates a family of solutions for the endogenous variable describing the agents' communication behaviors – namely the mean probabilities of "sending" and "receiving" messages.

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[11] The properties remain subject, as will be seen, to certain important restrictions on the dimensionality of the social interaction landscape, as well as upon the parameters governing the expected signal-to-noise ratio of prevailing empirical methods, and the subjective relative weight that is accorded by the finding from their own application of those methods vis-à-vis the opinions prevailing among peers belonging to their immediate social network.

Although the underlying framework points to implementation in a fully agent-based stochastic simulation model, such as those which have recently become more widely employed in the social sciences, this is no small step and has been left for future research.[12] A fully dynamic *equivalent deterministic system* is obtained (in Section 6.2), by modeling the influence of the network's emergent performance properties upon its capacity to attract additional members from a surrounding eligible population. The latter is assumed to reflect the formation of expectations about the benefits that potential members would derive, given the expected rate of percolation of "ideas and opinions" and the rate of generation of "new ideas" within the network. Such benefits of membership must match those obtainable in competing employments to prevent net attrition from reducing the size of the network, and must exceed those opportunity costs in order for the network to grow. Hypothesizing that such adjustments in the net flow of researchers are effected with lags rather than instantaneously, yields a system that is shown to possess a multiplicity of equilibria in terms of size, macro-performance properties and microeconomic behaviors. Some suggestive features of the dynamic responses of the invisible college to "policy-perturbations" are examined (in subsection 6.3) by means of numerical simulations.

The concluding discussion (Section 7) comments on the salient limitations of this simple heuristic model, and the respects in which the properties exhibited by the equivalent deterministic model may mask important patterns of dynamic behavior that might characterize performance of the stochastic structure. Other aspects of the cognitive performance of research networks in which knowledge generation is localized and key information is similarly transmitted through pathways of social communication are no less important for an understanding of the workings of exploratory research communities than those upon which this paper focuses. "Small worlds"phenomena in the structure of social network communications is one of these which could substantially modify the findings presented here. Another, with similar potentialities, would consider the application of random graph theory to model endogenous forces affecting the formation of local social networks that channel "tacit" information exchanges – thereby relaxing the assumption of exogenously formed local social network relationships that underlies the approach taken here. Still a third line of extension would introduce one or more latent "invisible colleges" that could compete for the adherence of members of a growing (or dwindling) population of potential (i.e., qualified) researchers.

The possibilities of thereby encompassing lines of inquiry opened by others, within an expansion of the basic framework considered here, suggest an extensive agenda for future investigations, and one that it seems could be readily pursued using stochastic simulation methods – until interesting conjectures emerged that could become the targets for more analytically oriented students of the new economics of science. Yet to do so properly (which is to say persuasively) would entail facing the challenge of developing a richer agent-based model that articulated a variety of actors having institutionalized functions, not all of which proceeded in lock-step with the iteration counter of the simulation program, so that the dynamics of the system would reflect complex interactions among processes moving in parallel through time with different time-constants. In such a construction the actions of the agents generating and filtering the codified research findings that form the evolving corpus of published communications would necessary occupy at least as much attention as the informal social communications, whereas the existence and epistemic role of the former have been blithely presupposed for the purposes of rendering manageable the present exploratory inquiries.

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[12] On the methodology and promise of agent-based modelling as a research strategy for the social sciences, see Epstein (2006). For stygmergic agent-based modelling applied to open source software development communities, see Dalle and David (2005, 2007, 2008), a topic related to but distinct from the present paper.

2. Micro-foundations: Social Networks, Tacit Knowledge, and the Influence of Peer Opinion

A brief discussion should suffice to highlight key features of the social channels of knowledge communication among research scientists that are captured, or, more properly "caricatured" in the micro-level model drawn from David (1998) and described formally in Section 3 (below). Starting from recognition of the distinct but complementary roles of tacit and codified knowledge in the conduct of research, we consider the nature of the cognitive transactions in which the members of the local network are engaged. This leads to an examination (in 2.1) the underlying incentives that reinforce cooperative exchanges of tacit knowledge among small, pre-existing networks of personal correspondents; and then to the identification (in 2.2) of special conditions under which the disclosure of provisional conclusions tending to conform with the consensus of local peer belief emerges as a rational reputation-building strategy. Extending grounds for attributing central importance to "conformity effects" in process of consensus formation, research activity can be viewed (in 2.3) as an iterative process of Bayesian belief that revision in individual researchers' *observations* and *interpretations* of their experimental results are being powerfully shaped by *a priori* expectations that reflect peer opinion concerning the subject under investigation.

2.1 Informal knowledge transactions inside "invisible colleges"

Analysis of the economic logic of the academic science reward system concurs with the functionalist sociology tradition in studies of the cultural ethos of modern science, by laying stress upon the centrality of the norm of public disclosure of knowledge among those who belong to the Republic of Science. Thus, Ziman (1984: p. 58) holds that "the fundamental social institution of science is thus its system of *communication*." Accordingly, much attention has been focused upon bibliometric studies of patterns of transmission of information via books, journals, other archival publications, in an effort to identify the participants, and map the respective cognitive domains of the "*invisible colleges"* in which those transactions arise. [13] The common features of invisible colleges in science are that they remain quite fluid as to membership and variable in size, generally do not become highly structured internally, and, in today's world of telecommunication technology and cheap air travel, have become less and less localized along institutional, geographical, and national lines.[14]

The existence of the "broadcast" modes of distributing *codified* knowledge forms an essential background condition for the personal, interactive transactions among the members of modern scientific research communities. Rewards structures for participants in open science are tied to publication in those media, as has been noted, and, correspondingly it is upon that objective that rivalries for priority within invisible colleges tend to be focused. Within these extensive communities whose membership numbers in the hundreds, however, there are rather smaller and communicatively more compact relational entities. These are referred to here as *local social research networks*, or simply as "local networks". The latter term is appropriate for the interactive, two-way communications flows among their members: characteristically, information is personally conveyed in conversations via telephone, fax and email messages; but

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[13] Price (1965), Narin (1976), van Raan (1988), and their followers in "scientometrics" apply bibliometric methods to the study of cognitive structures in science, as do some proponents of the sociology of scientific knowledge (SSK). The "translation" school of Callon et al. (1989) holds that social networks of research and knowledge dissemination have corresponding linkages in the cognitive domain; that they give rise there to counterpart "connected clusters of connected nodes" in co-citation networks formed among papers published in the scientific literature, patent applications, and other "inscriptions".

[14] The same tendency has been discerned recently from bibliometric studies of formal scientific collaborations. See, e.g., Katz (1994), Katz and Martin (1997), Hicks and Katz (1996).

also by visiting each other's laboratories, meeting for seminars and workshop presentations, and circulating pre-publication drafts for private comment.

Even when more tightly clustered, the latter social groupings are better described as research "cliques," than as organized "teams," and indeed, these social networks may encompass some members who belong also to other project teams. By comparison with the larger, invisible colleges formed on disciplinary and sub-disciplinary lines, the members of social networks in science tend to be rather more strongly localized in one or more dimensions of association: they may share personal histories of training, or an area of problem specialization, or geographical and institutional proximity; co-location is not essential, but it affords more frequent opportunities for face-to-face communications and informal collaborative activity.

Within the more restricted ambit of a researcher's local network will be circulating many bits of crucial knowledge, about experimental procedures, equipment functioning, data analysis algorithms, database codebooks – all of which often escape being fully codified and clearly revealed in published accounts of research procedures and findings. Although the development and circulation of codified knowledge traditionally was a matter of central, indeed, of exclusive interest in philosophical and sociological studies of science, the significance of non-codified, *tacit* forms of knowledge, and their role in the craft practice of science has come to be more generally appreciated. Tacit knowledge, as conceptualized by Michael Polanyi (1966), refers to a fact of common perception that we are aware of certain objects without being focused on them. Lying outside the zone of conscious attention does not make them the less important, however; they form the context that makes focused perception possible, understandable, and productive.

Tacit and codified knowledge should thus be viewed generally as complements, rather than substitutes in human cognitive processes. Both as a matter of formal logic, and in practical affairs, knowledge may be either disclosed to others or kept secret, regardless of whether it exists in codified form or remains tacit.[15] The view taken here is that for the ideas contained in scientific statements to be understood and rendered operational, researchers must possess the complementary tacit cognitive associations. This is the case because, like many other human pursuits, scientific inquiry draws upon sets of skills and techniques that are acquired experientially and transferred by demonstration, by personal instruction and the provision of expert services. Knowledge of this sort may be highly precise and intricate, but it is most typically conveyed as a *gestalt*, and referred to by language and signs that is idiosyncratic, rather than being reduced to constituent elements and operations denoted by standard codes from which might be assembled programs of implementation. The importance of this kind of "hands-on" experience in many laboratory- and facility-based research disciplines makes the problem of social communication of tacit knowledge especially germane for the cognitive work of those fields.[16]

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[15] See David and Foray (1995), where three distinct dimensions are recognized as defining a space in which knowledge products can be located: the codified-tacit axis, the disclosure-secrecy axis, and the public-private property axis. The implications of these dimensions are examined further in Cowan, David and Foray (2000).

[16] Many "craft" aspects of scientific practice must be learned in modes of instruction akin to an "apprenticeship" by being afforded opportunities for first-hand observation of how they are done, leading to trials under the guidance and supervision of experts. Otherwise, something like the original process of acquiring mastery of such knowledge has to be repeated *ab initio*, guided and encouraged only by the belief that others have found this to be possible. A striking instance of the "craft knowledge" deployed in science is documented by Harry Collins' (1974) detailed and influential study of the construction of the TEA laser. See also Latour and Woolgar (1979).

To simplify matters for the purpose of analysis, the model formulated in David (1998) assumes that codified knowledge alone can be *broadcast* effectively through a variety of public media that identify the authors of messages but are non-specific with respect to the identities of the recipients. On the other hand, it is assumed that messages whose cognitive content combines uncodified (or incompletely and idiosyncratically coded information) with some codified scientific statements to which such craft knowledge and informal judgments relate, are emitted locally in the first instance. Such mixed-content messages are thus held to diffuse first within the immediate social network neighborhoods in which they originate.

The local networks described here are not regarded as a strategic instrument whose primarily functional role is that of "capturing" and exploiting the benefits of tacit knowledge for its members. On the contrary, the benefits that it provides for individual scientists *qua* research workers are those of access to cooperative, reciprocated transactions in the otherwise undisclosed knowledge possessed by specialists; the sharing of that expertise enables their correspondence to increase their chances of solving complex, multi-step problems sooner than would be the case were they to work in isolation. In keeping with this, it is further supposed that the local networks are not autarkic: by having some members in common with other, similarly local social groups, they are rendered more or less inter-communicative. Thus, complementary packets of codified and tacit knowledge, along with explicit and implied conjectures about promising lines of scientific inquiry, eventually do percolate outward from particular local networks and so become diffused throughout the wider community of researchers that constitutes the "invisible college".

Cooperative behavior in the form of technical knowledge-sharing and the disclosure of provisional scientific judgments can emerge and be sustained within a limited social sphere, without requiring the prior perfect socialization of researchers to conform (altruistically) to the norm of full disclosure and cooperation. This is a rather straightforward instance in which insights from the theory of repeated games are helpful in accounting in rational terms for the patterns of reciprocated cooperative behavior among potentially rivalrous researchers.[17] Small cooperative "networks" of information sharing can be supported among researchers engaging in recurring problem-solving situations because pooling of information furthers the self-interest of the members in their respective races for priority against researchers situated outside their immediate "clique"; correspondingly, individuals who deviate persistently by withholding knowledge, or otherwise behaving in opportunistic ways to the detriment of others from whom they have drawn help, risk discovery and the future denial of access to pools of specialized knowledge, which would tend to place them at a considerable disadvantage in problem-solving.[18]

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[17] Arguments on this proposition, which invoke *inter alia* the "folk theorem" as applicable to the situation of researchers contemplating careers in academic science, are developed with some illustrative detail in David (1998: sect. 7.4). The so-called "folk theorem" of game theory holds that (if future payoffs are discounted by each player at a low rate) in the "super game" obtained by repeating a finite two-person game indefinitely, any outcome that is individually rational can be implemented by a suitable choice among the multiplicity of Nash equilibria that exist. See Rubinstein (1979, 1980), Fudenberg and Maskin (1984).

[18] Thus, "circles" or "networks" that informally facilitate the pooling of knowledge among distinct research entities on a restricted basis can exist as exceptions to both the dominant mode of "public knowledge" characterizing academic Science, and the dominant mode of "proprietory knowledge" characterizing industrial R&D organizations. Eric von Hippel (1988) and others have described how firms tacitly sanction covert reciprocal exchanges of information (otherwise treated as proprietary and protected under the law as trade secrets) among their respective engineer-employees. The existence of a "private professional network" upon whom the engineers can call for help is, in effect, a knowledge asset that can be valuable to her employer, even though exploiting it necessitates exposing the nature of the research problems upon which the firm is working. It is significant that for employees engaged in such

Such considerations, however, do not imply that the normative content of Merton's communalistic norm of disclosure plays no essential role in fostering cooperation among citizens of the Republic of Science. Quite the contrary. Networks of reciprocal information-sharing will be more likely to form spontaneously if the potential participants start by expecting others to cooperate, than if they expect "trust" to be betrayed; game theory also suggests that cooperative patterns of behavior will be sustained longer if participants have reason to expect to encounter refusals to cooperate only in retaliation for their own deviations from that norm.[19] Moreover, the detection of deviant behavior warranting punishment, and implementation of the retribution of ostracism from a particular network, will have more broadly damaging reputational consequences when the norms of behavior involved (i.e., the "custom" within the network in question) is common knowledge, and part of the shared socialization among all the potential members of networks. Therefore, even were the process of socialization among scientists to be weak and quite imperfect, the common "culture of Science" makes it much more likely that the rule of priority will not tempt individuals to engage in opportunistic withholding of knowledge, and instead, will engage the self-interest of researchers in reinforcing adherence to the norm of disclosure – at least among those restricted circles of colleagues that form his or her local social network.

Two sorts of cognitive communications flows are envisaged to take place within the local social network structures of this model. Information in the form of codified statements can be passed between agents by the act of one of them sending a message or "sharing" a piece of knowledge, and the other receiving or "reading" it. The substance of the generic message-transaction comes in two parts: the first component contains (or otherwise identifies) a particular scientific *statement* – a proposition asserted in regard to a phenomenon in nature, or about the design of a measurement instrument or other artifact, an experimental procedure, or, perhaps a logically connected "bundle" of such statements. The second part of the message conveys the sender's present state of belief as to the "reliability" or "unreliability" of the accompanying statements.

Extending that metaphor, one may imagine that the channels of communication are multiplexed, and thus capable of bundling (or "packaging") cognitively interrelated propositions, so that it would be possible for the human transmitters and receivers to handle a flow of numerous, more or less concurrent messages of the foregoing kind that pertain to many distinct, and cognitively independent scientific statements. Each of those problematics could be assigned to its own "layer" over the network, and the resulting information-processing architecture thus would be enabled to simultaneously execute multiple consensus building routines in parallel.[20] For analytical tractability, however, the model presented deals with only one such "layer" of discourse.

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knowledge-trading networks, expert help from peers outside the firm can be professionally evaluated and reciprocated in kind; one who accepted money rather than professional assistance in repayment of help which entailed disclosed knowledge gained in the course of her professional work, most probably would be dismissed by her employer and prosecuted for theft of trade secrets.

[19] See David (1998:p.130) on Axelrod's (1984) findings regarding the effectiveness of 'tit-for-tat' strategies in sustaining cooperative play in the repeated Prisoners' Dilemma.

[20] The condition of *independence* that qualifies the preceding formulation is not innocuous. It serves to eliminate the complications that can arise from inter-layer "cognitive spillover" effects, especially those of the competitive or "cancellative" sort rather than the complementary or "additive" kind; these occur where establishment of a consensus on the reliability of the statement(s) carried in Layer A are likely to prompt the disintegration of a previously form consensus regarding statement(s) carried by Layer B. For example, the initial establishment of scientific consensus on the reliability of propositions deriving from quantum mechanical calculations about the behavior of light could be viewed as "unsettling" prior consensuses regarding propositions about light that derived from wave mechanics.

It is important to emphasize a further simplification: the nature of the messages transmitted regarding "reliability" are not concerned with subjective probabilities as to the ultimate "truth" of a specified hypothesis, nor do they offer assessments of the "degree of reliability" adhering to particular statements. At any moment the researchers (acting either in a team organization or as a solo investigator) impose a binary classification upon whatever opinions they hold in such matters, and so they mark the cognitive statements they emit either as having attained a level of reliability that is "acceptable", or "unacceptable". But these are understood to be provisional judgments; their minds are open, in the sense that from moment to moment they can find cause to revise their labeling of the same statement(s).

The revision process is precisely where the cognitive content of the messages' second component (beliefs) comes into play. At least two caricature-accounts can be given about how scientific inquiry is conducted in the world of this model: in both the researchers revise their belief-orientations regarding the reliability or unreliability of propositions under discussion in ways that leave them open to being influenced, if not completely "persuaded" by the information they receive as to the beliefs held by correspondents in their immediate social network. The following sub-sections sketch the two accounts offered by David (1998) in this connection, one appealing to the taste of economists concerned to find rational grounds for individual behaviors, the other framed in terms of the epistemology and psychology of experimental and observational inquiry. Introducing further strategic considerations provides a direct rationale for the persuasive power that a prevailing consensus for, or against, a given proposition might exert in the formation of an individual researcher's reported beliefs about its validity. Unlike the foregoing arguments from the philosophy and history of experimental science, the following section is more generically "economic" in its appeal. Yet, it will be evident that two accounts are mutually compatible for present purposes, so that it is not necessary to choose between them

2.2 Conformity to local peer consensus as a reputational strategy

In the spirit of arguments suggested by Dasgupta and David (1994), one may suppose that a scientist working in a collegiate reputational reward system would consider the nearer-term reputational consequences of current actions (including expressions of scientific opinion), as well as longer-term payoff possibilities in the form of lasting fame for "having gotten it right". Whether a researcher will be found to have been "right" in the judgment of their peers depends upon the alignment of their recorded (or remembered) beliefs in relation to the consensus that existed at the time among members of the local network to whom those views were disclosed; and also on the relationship of those opinions to the global consensus that in time may emerge within the invisible college as to the "truth" or "falsity" of the proposition in question. It is then quite straightforward to envisage a structure of "expected reputational payoffs" that makes it a dominant strategy to be found to have had beliefs conforming with those presently held by most of one's local network, even when the ethos of the lone scientific hero would accord maximum kudos and immortal glory to a researcher who had not conformed with local peer opinion, and whose beliefs eventually came to be shared by an overwhelming segment of the discipline at large.

Consider the following example. Let ***c*** denotes conformity with the preponderance of scientific opinion in one's local network, and ***d*** denotes disagreement with that consensus**.** For expositional convenience we may examine the situation where the prevailing consensus holds a particular statement, ***S*** to be "reliable/true", denoted by *R*. (One can treat symmetrically the opposite case, in which the consensus among the agent's reference groups holds the statement to be "unacceptable/false", *not-**R*** or, equivalently *W*.) The consensus that can emerge *eventually* in the global network, i.e., the limiting configuration of beliefs among members of the invisible college, may either hold ***S*** to be "reliable knowledge", denoted by ***R***, or not, denoted by ***W*** ("Wrong"). But we suppose that this eventual determination cannot be known with certainty when the researcher is deciding which opinion to offer on the matter at hand. The incipient emergent consensus is that sense an unobserved "state of (social) nature". If the individual researcher treats the local network as the reference group whose esteem matters, we then have the following notation for possible states, to each of which there will correspond "reputational payoffs" for the representative researcher:

$\{c, R|\ S$ is $R\} = \mathbf{b_1}$: being right, with the crowd;
$\{c, R|\ S$ is $W\} = \mathbf{b_2}$: being wrong, with the crowd;
$\{d, R|\ S$ is $R\} = \mathbf{b_3}$: being in a minority and wrong;
$\{d, R|\ S$ is $W\} = \mathbf{b_4}$: being in a minority and right.

A suitable general payoff structure for an epistemic community is one that assigns greater value to an individual researcher who is found to be "in the right" than to one who had embraced the "wrong" view. But it is also plausible that being wrong in a crowd will be deemed not as bad (for her subsequent reputational standing) as being found to have been wrong more-or-less on one's own; whereas being "lonely yet right" is deemed to be more glorious (reputationally) than is having been correct among a crowd of one's peers. This scheme of valuation of the outcomes of the game against (social) nature corresponds to the condition: $\mathbf{b_4} > \mathbf{b_1} > \mathbf{b_2} > \mathbf{b_3}$.[21]

Now, let θ denote the subjective probability that the individual assigns to the outcome that the global consensus eventually will form on ***R*****,** i.e., holding ***S*** to be "reliable." Then the expected payoff, π_c, to an individual whose strategy was to conform to the preponderance of peer opinion would be

$$\pi_c = \{\theta \mathbf{b_1} - (1-\theta)\mathbf{b_2}\},$$

whereas the corresponding expected payoff for the strategy of dissenting is

$$\pi_d = \{(1-\theta)\mathbf{b_4} - \theta \mathbf{b_3}\}.$$

It follows that $\pi_c > \pi_d$ is sufficient for the pure strategy of "conformity" to maximize the individual researcher's expected reputational payoff; and where the inequality is reversed, the pure strategy of "dissent" will be dominant.

The sufficient condition for individual to "conform" to the prevailing preponderance of belief is readily obtained. By substitution we have

$$[(\mathbf{b_1} + \mathbf{b_3}) / (\mathbf{b_4} + \mathbf{b_2})] > [(1-\theta)/\theta],$$

which may be re-written as

$$\theta > (\mathbf{b_4} + \mathbf{b_2})[(\mathbf{b_1} + \mathbf{b_3}) + (\mathbf{b_4} + \mathbf{b_2})]^{-1} = \theta^*.$$

To put this more explicitly: there exists a critical value $0 < \theta^*$ such that the strategy of "conforming" is dominant when $\theta > \theta^*$.[22] Further, because the foregoing specifications on the structure of the payoffs imply the inequality $(\mathbf{b_1} + \mathbf{b_3}) < (\mathbf{b_4} + \mathbf{b_2})$, it follows that $\theta^* > ½$. Figure 1 depicts this proposition graphically.

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[21] A reasonable interpretation of this game would, in addition, specifiy that $\mathbf{b_1} \geq 0$.

[22] It is evident that in the illustrative case presented here, the restrictions $(\mathbf{b_4} > \mathbf{b_1} > \mathbf{b_2} > \mathbf{b_3})$ and $\mathbf{b_1} \geq 0$ guarantee that $0 < \theta^* < 1$, so that $\theta \in [0,1]$ can satisfy the sufficient condition for conformity to be dominant as a pure strategy.

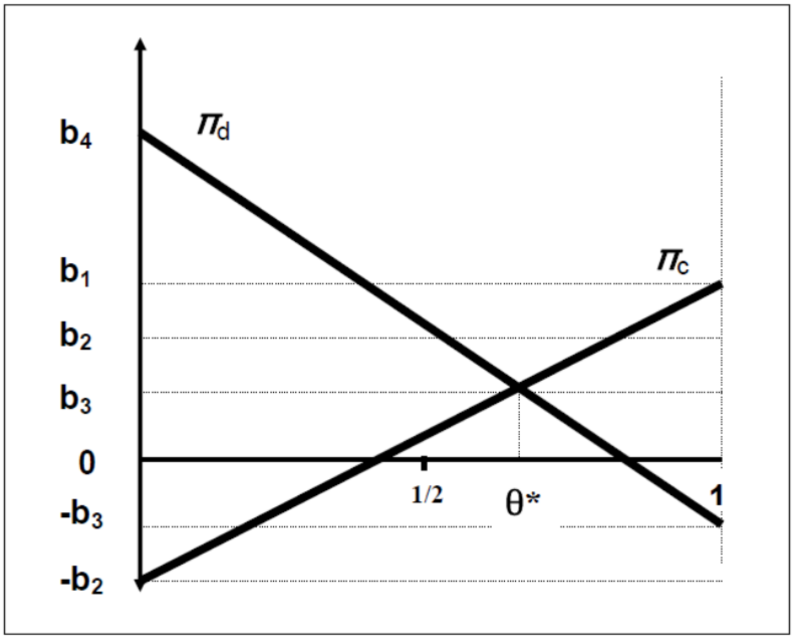

Figure 1. ***Conditions for dominance of conformity as a reputational strategy***

The intuition here is clear enough: as the critical value of the *a priori* probability that ***S*** will turn out to be false is approached, i.e., $\theta \rightarrow \theta \underline{\leq} ½$, the prospect of receiving the large payoff $\{\boldsymbol{d}, R|\boldsymbol{W}\} = \mathbf{b_4}$ for being "right in a minority" renders the non-conformist strategy increasingly attractive for an expected utility-maximizing agent. Conversely, when $(-\mathbf{b_4} \rightarrow |\mathbf{b_2}|) \rightarrow 0$, the pure strategy of conformity becomes "locked in". There are special circumstances that would "lock-in" the agent to invariant dissent from the majority position on the truth (and symmetrically, the falsity) of ***S***: where the gain from being right when conforming with majority opinion is matched, or over-matched by the absolute magnitude of the "penalty" incurred for being wrong in dissent, $(-\mathbf{b_1} \rightarrow |\mathbf{b_3}|) \rightarrow 0$ and $\theta^* \rightarrow 1$; consequently, the critical value for dominance of dissent $(1- \theta^*) \rightarrow 0$ and can always be satisfied. But, so long as all the payoffs remain positive, neither of the pure strategies (***c, d***) can be dominant without reference to the researcher's subjective probability evaluation of ***S***.

It will then be rational for the individual researcher to follow a "mixed strategy," by forming a provisional opinion on the question under consideration by choosing between the available (binary) options on a probabilistic basis. To see how a mixed-strategy of that kind might be implemented, one may start by asking how the subjective estimate of probability θ would be formed. Although this point has not surfaced explicitly in the foregoing discussion, it is now relevant to notice that there is a variety of stochastic processes representing dynamic consensus formation through the revision of opinions induced by the interactions among members of a finite population of agents. For some processes of that kind, the configuration of opinions in the entire ensemble eventually will become perfectly correlated on one or the other of the possible binary orientations – on either *R*, or *W* in the present example. Furthermore, as will be seen from Section 3.2, below, there is one such process in which the distribution of binary opinion-orientations prevailing at the start of the process constitutes the best *a priori* probability estimate that the limiting outcome will be the establishment of such correlation ("closure") on, say, the option ***R*.** This is the so-called "Simple Voter Model," a reversible spin system that is well-known in the literature on interacting particle systems.[23]

Allowing, for the moment that there is sufficient warrant to work with that model of the process of the tacit knowledge transactions among the members of an invisible college, the preceding discussions of the role of local social networks implies that the initial state of (provisional) opinions throughout the whole of the "invisible college" hardly would be known to any of its members. The individual researchers' respective information fields on such a matter would restrict them, at best, to knowing something about the current distribution of opinion among members of their immediate social network. Nonetheless, the proportion among the latter who presently say "R" when canvassed would provide the individual's estimate (θ**)** of the probability that the invisible college as a whole eventually would achieve "closure" by accepting the "*R*-ness" (i.e., "reliablity") of the proposition in question. Let us see what this would imply.

2.3 Cognitive communications: beliefs, Bayesian learning, and conformity to consensus

Postulating that the conditions just described hold for mixed-strategy micro-level behaviors, it is useful to adopt a simple characterization of the process of provisional opinion formation and revision, by invoking a well-studied probabilistic model of cellular automata. The latter represent the collection of "researcher-agents" who are symmetrically situated in their respective local networks.

Suppose these agents execute the following algorithm to select a binary orientation with regard to the reliability of a pre-specified scientific proposition, ***S***. At a random interval in time each agent polls the opinion-messages emitted by other agents belonging to the same local network. If there is unanimity among them (either on the reliability or the unreliability of a particular proposition) the polling agent accepts the local consensus and accordingly adapts the messages it sends to neighbors concerning the validity of the proposition in question. When a disagreement is found within the local network, an agent will follow this quasi-Bayesian procedure: he/she probabilistically selects an opinion on the given question by a procedure that is equivalent to tossing a coin whose loading mirrors the division of opinion among the set of network-neighbors. By so doing, implicitly, equal influence is accorded to the opinions of the local reference network members, and, in effect, the agent selects an orientation from the binary options with probabilities that are proportional to the currently observed frequency of that orientation within the agent's local network. The procedure described corresponds immediately to the mechanism postulated by the so-called "Voter Model," of which more will be said below (in Section 3).

How plausible is such an obviously artificial representation? By way of an answer that endeavors to address at least some of the "realistic" concerns that the discussion of the previous section (2.2) is likely to raise for students of scientific communities, an alternative, "non-strategic" account may be given of the influence that information about the distribution of local peer opinion exerts upon the orientation of researchers' judgments in regard to the "reliability" of a particular cognitive propositions.

The researchers in this story are depicted as engaging in a bounded form of Bayesian information-processing, drawing inferences from the observations generated in their own

[23] Strictly speaking, as in David (1998, 2002), the form known as the "Simple" or "Linear" Voter Model is examined in Section 3.2 Despite the anthropomorphic allusion in the name, this structure originally was developed in a quite different (particle physics) context by Clifford and Sudbury (1973), and Holley and Liggett (1975). See Liggett (1985), Ch.5 for an overview and discussion of its relationship to the class of stochastic reversible spin systems.

experiments and, possibly also from reports of the inferences arrived at by others they know to have been similarly engaged. It is assumed that they are all following a common epistemological strategy, and thus refer to the same subjective probability thresholds when declaring some particular conjecture about the underlying "state of nature" to be "acceptably reliable," or alternatively "not reliable." Furthermore, the existence of that shared strategy itself would constitute a subject of common knowledge within the community – indeed, it might be said to be one of the procedural rules that characterize the epistemic community in question. Consequently, every researcher views the *a priori* beliefs conveyed by their correspondents to have been shaped by a Bayesian revision process similar to their own, and therefore to contain data worth taking into account.

Therefore, the distribution of priors underlying the announced binary orientations among the researchers with regard to the scientific proposition in question would be subject to revisions that were generated in two ways. One would be the periodic routine of Bayesian "updating," based upon their own calculations of the likelihoods of the results observed in their own experimental work and data analysis. The other would reflect the (presumed *a posteriori* judgments) gathered from the distribution of evaluative opinions within their own local network, which, in turn, would reflect the pooling of categorical expressions of belief communicated by the members of their correspondents' networks. Inasmuch as the individual scientist's conduct of experiments and the taking of observations are likely to occur with less frequency than the arrival of messages reporting the state of opinions held by other researchers in her network, the effects of the latter might well be expected to overwhelm those of the former. This could well be the case even were the findings of her own research to be given greater weight than the informally communicated views held by members of her peer group.

That considerable weight is accorded, *de facto* to peer-opinion in the interpretation of observational data, is suggested by the doubts that sociological, philosophical, and psychological studies have raised concerning traditional views of the nature of experimental science. These critiques call into question the degree to which scientific progress actually occurs through the experimental refutation or "invalidation" of conjectural propositions, as was proposed in Popper's (1959) account of the scientific discovery process.[24] Historians of science have contributed to the present skepticism regarding the supposedly central role played since the 17th century by "crucial experiments", and the power of unalloyed "observation" to dislodge an established consensus view. Indeed, the very occurrence of the famous Tower of Pisa experiment – whereby Galileo's finding of essentially the same rate of fall of two unequal weights dropped from the Tower, supposedly undermined the authority of Aristotelian mechanics – is now suspect; Alder and Coulter's (1978) modern replication study revealed that the observable difference in the speeds of the objects over their 200-foot descent would have been too large to justify Galileo's reporting that it was negligibly small.[25]

Similarly, through the work of Worrall (1976), revisionist history of science now instructs us that the experiments of Thomas Young could not have overturned Newton's

[24] See David (1998: pp. 134-138) for further discussion of the critiques advanced by Kuhn (1962/1970), Lakatos (1970) and Feyerabend (1975), which reinvigorated the epistemological problems posed for Popper's (1959) account in the writings of P. Duhem, and by W. V. O. Quine (1953). See Harding's (1976) discussion of the so-called "Duhem-Quine problem" regarding the possibility of scientific refutation. According to Franklin's (1986:4, 106) reading of the modern skepticist position, whereas the theory-laden nature of experiments and "observation" has the effect of opening the whole edifice of scientific theory to the risk of empirical refutation, particular theories or hypotheses could thus escape experimental falsification.

[25] Franklin (1986: p. 2, n. 7), citing the replication study by Alder and Coulter (1978), goes on to point out that an Aristotelian could readily have modified the theory to accommodate the experimental data.

corpuscular theory of light and so established the wave theory, if only because corpuscular explanations were available for both interference and diffraction.[26] There are, to be sure, striking counter-examples of instances in which experiments did prove "crucial" in overturning a prevailing theoretical model. But, even in these cases careful historical re-examination sometimes serves mainly to expose their atypicality, and highlights the special nature of the circumstances that would make it likely for experimental results to prove decisive in rapidly altering a scientific consensus.[27] More generally, close examination of the ways in which runs of results are generated in modern experimental physics, such as has been undertaken by Franklin (1986), reveals how the latter can sometimes appear to imply that parameter magnitudes are either conditioned by theoretical expectations, or that when initial values were obtained in contradiction of received theory, extended replications were undertaken until significant alterations caused the magnitudes to converge to the theoretical expectations.

Considerations of the foregoing kind support the view of behaviors in conformity with peer-opinion as having powerful short-run impacts in the process of scientific consensus formation.[28] The interpretation this suggests for the toss of the (local opinion-weighted) coin envisaged in the Voter Model algorithm (described at the end of the previous sub-section, 2.2), is that this routine mimics the conduct of an inherently ambiguous experiment or observational procedure. In such a situation the "reading" of the results generated would be strongly shaped by the "prior's" held by the individual experimenter or observer as to the validity of the hypothesis under examination. Only when opinion in the local peer group is quite evenly balanced would the testimony of the experiment exercise potent leverage upon the experimenter's reported belief, but then the interpretation placed upon a "face value" reading would still be subject to some stochastic influences. Of course, there is a question as to whether such influences are properly represented by the toss of a "fair" coin, as the simple algorithm of the voter model suggests. trivalof the results remain uncertain.

But, even the foregoing simplistic (and for some, undoubtedly rather troubling) view of the power that the currently prevailing consensus of local peer opinions exercises in the work of individual scientists should not be dismissed as having ignoring element of built-in correction that operates in open science research processes over the longer run. Where the magnitudes at issue in a particular scientific theory are "important" and relied upon widely in drawing out its implications, advances in experimental technique create opportunities to score scientific "coups" by establishing a new and different value from the ones previously accepted for the parameters in question. The greater is the resolution of observational equipment, and the less error prone are the experimental techniques and methods of data analysis, the higher is the signal-to-noise ratio from this source of information, given greater power for the "signals from stationary properties of Nature" to push the social process of consensus formation in one direction. Furthermore, so long as there is some persisting and acknowledged discrepancy between the theoretical expectation and the previous experimental findings, there is hope for

[26] Moreover, the early (pre-Fresnel) wave model could not account for the rectilinear propagation of light, which was as troublesome for that theory as interference was for the corpuscular model. See Worrall (1976), discussed by Franklin (1986: pp. 2, n. 8).

[27] See, e.g., Franklin (1986: Ch.1) on the experimental discovery of the non-conservation of parity in the weak interactions within the atom – which supported Lee and Yang's (1957) famous theoretical paper questioning the theory of parity conservation (mirror symmetry) that the physics community had accepted as universal, on the basis of its successful characterization of the strong and electromagnetic interactions.

[28] The implication is that the mechanism of consensus formation in science considered as a social system would be "neutral" with respect to the objective Truth of the proposition under discussion. This will be recognized as a central proposition asserted by adherents to the so-called Edinburgh "Strong Programme" in the sociology of scientific knowledge, following Bloor's (1976) seminal formulation.

individual researchers to achieve peer-recognition and enhanced professional status by successfully reconciling the two, in some novel way or another. This process may be seen to bear a resemblance to (Lamarkian) evolutionary selection, as the distribution of opinion is the counterpart of the distribution of (non-genetic) “traits” in a given population whose members are interacting with an environment fixed by the “reality” of the objective physical relations they are studying. Obviously, the features latter must display substantial stationarity in order for the force of “selection” to have the effects envisaged by proponents of a evolutionary epistemological view of the way science works.

These aspects of the science reward structure thus function to set some bounds upon unintended tendencies that might otherwise push the reading of empirical data into conformity with currently prevailing theoretical expectations. The force of their operation imparts an evolutionary drift of the dominant scientific consensus towards closer and closer concordance with, and hence a more reliable representation of underlying “physical realities.” This is a compromise position in the “culture war” between the “social constructionist” and “scientific realist” camps, whose implications for formal modeling can be examined more explicitly after considering (in Section 3) what can be said simply on the basis of the unadorned Linear Voter model of consensus formation.

3. From Stochastic Social Communications towards a Model of the Global Network

The strands of the preceding arguments can be drawn together now, in order to examine the properties of the stochastic communications model to which they lead. For this purpose, the apparatus of graph-theoretic representation of connected local networks of research units forming an “invisible college” is briefly introduced (in 3.1). A correspondence is then asserted between the micro-level network interactions specified by the preceding sections and the Markov random field model known as the Voter Model.” The latter’s basic properties are reviewed (in 3.2) for the cases of networks that can be represented as one- or two-dimensional connected graphs. Some additional properties of the dynamics of consensus formation in variant formulations of the Voter Model are commented upon (in 3.3), along with the broader significance of these and related theoretical results pertaining to critical properties of other stochastic structures – specifically those deriving from the branch of probability known as *percolation theory*.

3.1 Graph-theoretic representations of social networks, and random Markov fields

For analytical and expositional simplicity we may begin with a schematic representation of the social space in which are located the agents constituting an invisible college of a finite and fixed size. This population is envisaged as situated on a two-dimensional regular lattice. Its particular spatial configuration is described by a non-directed graph G, of the kind encountered above (in 2.2): there is a total of N nodes in the lattice, representing population of researchers, and there are in all N-1 ‘connections’ or ‘channels’ that run between pairs of nodes.

Every node has a set of 4 communication channels, each providing a direct connection with a single agent-node. These “correspondents” are situated respectively at the 4 quarters of the compass in relation to the index-node. The channels connecting the nodes of this sub-graph, can be made of equal length, *l*, so that a circle centered on the index agent, *i* , having radius *l* , can be drawn to pass through all of the agent-nodes that can be reached directly by *i*’s personal hub-and-spoke communication network. The 5 agents enscribed within the *i*-th circle in this fashion, form the local social network associated with its hub-member; alternatively, this 5-agent configuration sometimes is referred to as the index-agent's "von Neuman neighborhood."

Another, similar 5-agent von Neuman neighborhood may be formed for hub-agent *j*, who is one of the 4 nodes positioned on the perimeter of the *i*-th circle. The *i*-th and the *j*-th circles therefore intersect, because their respective hub-agents are located in the other's neighborhood. By continuing to add neighborhoods in this modular way, the entire square lattice arrangement of the invisible college may be constructed. To keep everything perfectly symmetrical and leave no nodes in boundary positions, the resulting two-dimensional lattice array can be wrapped around in both the horizontal and vertical directions, connecting the right side edges to nodes on the left, and those on the top side to the nodes on the bottom – thereby forming a two-dimensional *torus*.

The foregoing spatial representation of an invisible college as *a network of localized social networks* abstracts from many realistic complications. Choosing this particular graphical form makes the neighborhoods, or local social networks of each researcher symmetrical with those of all the others, and holds them to be fixed for the purposes of the analysis.[29] Both assumptions prove to be convenient as a point of departure in this line of investigation, which perhaps is the most that can be said for making them. In being grounded upon a static network configuration, the resulting model of local network interactions examined here is enormously simplified, and it must be hoped that the gains in terms of analytical tractability compensate for inability to address phenomena that arise in ensembles formed from social networks that are neither symmetric nor constituted of homogenous agents.[30]

3.2 The Linear Voter Model and its properties

The undeniable attraction of the probabilistic routine for opinion formation set out in the preceding sections is that it corresponds directly with the well studied linear "voter model," a reversible spin system was introduced in different contexts by Clifford and Sudbury (1973) and Holley and Liggett (1975), and is best known in the form elaborated by Harris (1978).[31] Leaving aside technicalities, this framework can be set out schematically as a representation of scientific communication and consensus formation in inter-linked local networks. Following the notation by Kinderman and Snell (1980), we begin with the basic definitions relating to Markov random fields.

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[29] It would appear feasible to treat the local social communication networks explicitly as coalitions, and, following the lead of Kirman, Oddou and Weber (1986), to model their endogenous formation, and possibly also their ramifying interconnections. This would entail application of concepts and analytical techniques from the branch of probability known as random graph theory. See Bollobás (1979). Although this approach has not been attempted as an extension of the consensus formation framework employed in the Voter Model, the discussion in Section 6 below notices the interesting use which Carayol and Dalle (2000) make of random graph theory to model the stochastic process of problem choice in science that gives rise to "knowledge trees."

[30] Morris (1996), using mathematical tools other than those employed below, has shown that a number of the key properties of local interaction games, concerning the dynamic propagation of strategies chosen in particular locations, and the existence of correlated equilibria, hold generally for a wide class of local (spatial) structures. On the other hand, the assumptions of symmetry and homogeneity are not wholly innocuous. For example, Bala and Goyal (1995) show that greater symmetry increases the speed of information diffusion in a local interactive learning game.

[31] Based upon Markov random field theory, this model has lent itself to a variety of applications in the study of human and machine networks, for which a good introductory discussion is provided by Kinderman and Snell (1980). More recently, it has been extended to the analysis of the dynamics of technological competitions in economic contexts that are characterized by the existence of local network externalities. See, *inter alia*, David (1988, 1993b); David and Foray (1993).

Let **G** = (**O**,**T**) be a non-directed graph, with vertices **O** = ($o_1, o_2,...,o_n$) being the set of nodes representing research organizations, or simply "researchers," and channels **T**=(t_1, t_2,...,t_m) representing the set of information transmission channels. For the moment, we restrict the discussion to connected graphs of social networks that are defined in one or two dimensions. A *configuration* **x** is an assignment of an element of the finite set **S** to each point of **O**. We denote this configuration by **x** = (x_o) where x_o is the element of **S** assigned to vertex o. If we let **S** = [*u*, *a*] represent assignments of the two possible opinion orientations regarding the reliability of a given scientific statement (*a* standing for "acceptably reliable", *u* for "unreliable"), a configuration would be an assignment of either o_u or o_a to each of the points in **O**. A *random field* p is a probability measure p(**x**) assigned to the set **X** of all configurations, such that p(**x**)>0 for all **x**. By the "neighbors" **N**(o) of the point o we shall mean the set of all points o' in **O** such that (o'o) is an edge. A random field p is called a *Markov random field* if:

$$p\{\mathbf{x}_o = \mathbf{s}|\mathbf{x}_{O\text{-}o}\} = p\{\mathbf{x}_o = \mathbf{s}|\mathbf{x}_{N(o)}\}.$$

That is, given the values at all other points of **O,** the value at o (either *u* or *a* in the example) can be predicted from the sub-set consisting only of the values assigned to the neighbors of o.

Assume now, following the "voter model", that associated with each point of a graph we have a researcher or research unit, and that with every such unit there is a reference set comprised of other units; this constitutes the neighborhood (i.e., the local social network) available for polling. At random moments in exponential time, each research unit, having polled its local network, reassessed its orientation in regard to the statement in question, *u* or *a*. At these times it will commit to the choice *u* with a probability equal to the proportion of *u*-oriented research units in its reference set, or, correspondingly select the other of the binary options. This procedure may be seen to be equivalent to random, equi-probable polling of the agent's neighborhood, and the mimicry of the orientation of the selected member.[32]

The global dynamic process of migration between the alternative orientations of opinion as to the reliability or unreliability of a given scientific proposition is therefore represented as a finite state continuous time Markov chain, with states being configurations of the form**:**

x = (*u, a, u, u, a,..., u, a, u*), where x(i) is the choice of research unit i.

A number of important properties of this well-studied process may now be briefly summarized:

Property (1): It is evident on even the briefest consideration that the extremal states $\mathbf{x}^u$ = (*u, u, u, ..., u, u, u*) and $\mathbf{x}^a$ = (*a, a, a,..., a, a*), in which there is a perfect correlation of beliefs throughout the population, constitute absorbing states for this system. Once such a state is entered, there can be no further change. The existence of a multiplicity (two) of absorbing states tells us plainly that *this process is essentially historical, in the sense of being non-ergodic* – it cannot invariably shake loose from all initial configurations.

Property (2)*:* A somewhat less obvious proposition, also true, is that for any starting state **x** the chain eventually will end up in either $\mathbf{x}^u$ or $\mathbf{x}^a$. Thus, in the limit, *the process must become "locked-in" to one of its extremal solutions.* The system invariably does produce eventual "closure" on the scientific issues submitted to it.

Property (3): There exists a limiting probability distribution over the macrostates (opinion configurations) of the system which is non-continuous, such that, starting in **x**, the probability that the chain will end in $\mathbf{x}^u$ is equal to the proportion of *u* in the initial configuration

[32] Although the intuition for this is quite transparent, David (1998: pp. 140-142) may be consulted for illustrative examples, for a variety of *local* network sizes and the corresponding connected graphs.

x (without regard to their position in the array); and the probability that it will end up in $\mathbf{x}^a$ is equal to the proportion of *a* in the initial configuration **x**. Therefore, although subject to random influences, the nature of the asymptotic macrostate consensus in this system can be predicted (not with certainty, but probabilistically) from information on the initial configurations of opinions.

The most immediately salient implication of this model is that a formal connection can be established between the social organization of science affecting the communications behaviors of the micro-level agents, and an important performance attribute of science communities in the cognitive domain, namely, the ability to achieve "closure". Another direct result is the support provided for the view "the details of history may matter" for the cognitive development of a scientific field. Further, in this light, the propensity of scientific communities to remark especially on instances in which new ideas have won eventual acceptance in the face of an initial consensus opposing them, is entirely understandable, because at least in the near term, such cases would constitute the rarer contingencies.

Several technical qualifications should be noticed in regard to the foregoing properties, especially as these also admit of some interesting interpretations in the present context. First, the property of complete closure, in the sense of perfect unanimity, does not survive extension of the model to graphs of higher dimensionality. From simulation studies it is found substantial but less-than-perfect correlations in orientation emerge in the case of lattices on a three-dimensional *torus* (see Kinderman and Snell 1980b). One may surmise, plausibly enough, that as social networks become "less compact" by extending into still higher dimensional spaces, clusters of minority opinion are less likely to be surrounded by neighborhoods of countervailing consensus and so tend to persist. Perhaps the recurring formation of disciplinary sub-specialities in science serves as a "social compacting process", the latent function of which is to preserve network performance in terms of the achievement of substantially strong degrees of consensus, approaching unanimity among groups self-identified as "experts." On the other side of the coin, as was just suggested, higher dimensional social networks tend to increase the likelihood of "heterodox" opinions being able to survive within small clusters of researchers who, in effect, shield one another from the conformity-inducing pressure of exposure to the preponderance of opinion throughout the epistemic community at large.[33]

A second point of qualification is that the properties of lock-in to closure, and predictability of the nature of the resolution, are ones that strictly hold only for *finite* populations. If the population of the network were to be constantly growing at a comparatively rapid rate - strictly, at a pace rapid enough to cause the introduction of newcomers (who are entering the field with randomly distributed beliefs about the scientific issues of the day) to overwhelm the pace of the process of random polling in the local social networks, then closure would no longer be assured. Under those conditions the nature of the cognitive outcome would cease to be predictable on the basis of the system's initial configuration.[34]

This suggests a further respect in which the cognitive performance of scientific communities may be seen to depend on their organizational dimensions and dynamic attributes. Those characteristics would certainly have to include the rate of entry of new members in relation to the speed of informational transactions affecting the revision of scientific judgements within local social networks. Another factor to be considered in the same connnection is the "pre-entry orientation" of new recruits – particularly in reference to the

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[33] The broader significance of this will be further remarked upon below, in sub-section 3.3

[34] Kinderman and Snell (1980) report that probability theorists surmise that the dynamics of convergence to one or the other extremal (uniform consensus) configurations in a "large" finite system would approximate those of the infinite population case. Such systems continue to migrate back-and-forth between the extremal states, albeit with very prolonged transit times.

prevailing distribution of scientific opinions held by those who currently constitute "the field". Of course, once an invisible college "stabilizes" demographically – in the sense that its growth rate slows to the point that it is exceeded by the average rate of internal opinion-polling, a substantial consensus can be expected to emerge even in the absence of strong pre-orientation as a criterion of eligibility for entry.

It is there that enhanced communications technology may prove of particular importance in supporting the rapid growth of research communities; a speed-up of the effective "polling rate" will permit the mobilization of additional (human) resources at a research frontier to proceed more quickly, without jeopardizing the network's ability to reach closure on the new questions that it has taken up for investigation. Moreover, if improved communications technology can accelerate the pace of knowledge exchanges and opinion revision within interlinked local networks, it becomes a functional substitute for pre-orientation training of new citizens of the Republic of Science and thus may reduce the sort of disciplinary training that tends to curtail heterodoxy of opinion and the susceptibility of fields to radical reorientations in the nature of consensus thought.[35]

Yet another, and quite important class of qualifications arises from closer consideration of the assumptions of the basic voter model with regard to the uniformity of communication behaviors on the part of the research-agents. These can be brought out more clearly, however, by turning to consider the properties of a somewhat different stochastic communications structure, one that does not assume that all the actors are following the same policy of openness in their knowledge transactions with other members of the community.

3.3 On impaired communications: percolation theory and norms supporting openness

The population of researchers is portrayed by the basic voter model to be homogeneous, in three distinct respects: (a) the structure of communication links among them is completely symmetrical; (b) their interactions are assumed to take a rather special form that is tantamount to assuming that transmission of influence in dyadic transactions is deterministic, even though the identity of the dyadic pairing is probabilistic (being established by the random polling of a single member).[36] All of the researchers are always sharing their opinions with all who ask, as all others in their social network stand ready to do. Putting aside the possibility of entry, the source of randomness in the revisions of beliefs within the population has to do not with whether or not particular researchers might be open to the influence of particular neighbors, but rather with the direction of the re-orientation of beliefs that such influences would bring about. The concepts and terminology of *percolation theory* provides a precise way of describing these specifications, and showing their relationship to a more general specification of the model.

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[35] For increases in the density and bandwidth of communication channels to achieve such an effect, of course, it must be supposed that the availability of information from external correspondence constitutes the binding constraint upon the revision of beliefs. Historically, that may well have been so, and the hypothesized effects would appear to be well worth empirical investigation. But, as Herbert Simon and many others have pointed out, the super-abundance of information in more recent times has made human "attention" the scarce resource.

[36] This interpretation is not the only one possible. The equivalent alternative construction of the Voter Model would admit full canvassing of the index-agent's social network, but selection of an orientation (opinion) using probability weights that reflect the observed distribution of opinions. In this formulation the homogeneity assumptions appear in the symmetry of the connected graphs describing every agent's local networks, and in the linear mapping from observed local frequencies to probability weights, whereby equal influence is accorded to the opinions held by every one of the agent's "neighbors".

The term *percolation* refers to the dual of a diffusion process (see Grimmet 1988). “Diffusion,” to speak strictly, refers to the random movements of particles through an ordered, non-random medium – as in the case of the diffusion of molecules of salt in water. By contrast, the term “percolation” conjures up the image of droplets of water moving under the deterministic pull of gravity through a disordered, random medium – such as a filtration tank filled with sand and pebbles of different sizes. When the water, entering at some source sites, eventually finds its way into enough open channels to pass throughout, wetting the entirety of the interior surfaces, *complete percolation* is said to have taken place. It is from this that the mathematical statistics describing the properties of analogous processes have acquired the label “percolation” theory.

Adapting the notation of Hammersley and Welsh (1980) to the Markov random field framework, let G be a graph in which some, none, or all of the connections (channels between nodes) may be directed. Thus, as before, G consists of a set of research units (corresponding to the graph’s vertices or nodes), $O = (o_1, o_2,...,o_n)$. These are connected by a set of (possibly directed) edges representing channels of social communications, $T = (t_1, t_2,...,t_m)$. An *operative path* in G from a research unit o_1 to another research unit, o_n, is a finite sequence of this form:

$$\{t_{12}\ o_2\ t_{23}\ o_3 \ldots t_{[n-1]n}\ o_n\},$$

where t_{ij} denotes a relational path connecting o_i to o_j. The graph G is *connected* if for each pair of researchers o_i and o_j, there is a path in G from o_i to o_j.

Now construct a *random maze* on G, as follows. Let each research node o of G be *open*, or ready with probability p_s to transmit messages that can influence any of its neighbors’ opinions on the reliability of the statement at issue. Alternatively, it will be *closed* (unwilling to share its present knowledge on the question) with probability $q_s = 1- p_s$. Similarly, each line of inter-personal or inter-organizational communications t_{ij} may be thought of as potentially carrying messages that will be actually “read” with probability p_r, or fail to do so with probability $q_r = 1-p_r$. Furthermore, we shall assume all these events are to occur independently of each other. An operative path, $D = \{\ t_{12}\ o_2\ t_{23}\ o_3 \ldots t_{[n-1]n}\ o_n\}$ from o_1 to o_n is said to be “open” if all its communication links are functioning and all its research nodes are ready to “share” their knowledge-conclusions. Thus, the probability that the particular path D is operational in that sense is given by $(\ p_r\, p_s\ )^{n-1}$.

Let Z be some given set of “source” research units, from whom a particular “idea” or scientific statement emerges into G. The decisions to adopt that statement as reliable (or not) can flow along any open path from a source research unit and will then similarly reorient the other units on such a path (“wetting” them, to use the natural percolation metaphor). The *percolation probability* $P(p_r,p_s|Z,G)$ is the probability that Z can thus reorient some infinite set of nodes in G. In the present application context it is natural to label the parameters p_r, and p_s, the mean probabilities of “reading,” and of “sending” or “sharing” information, respectively. In other words, in a large population, it can be expected that a proportion p_r are receptive to their neighbors’ opinion on the reliability of a statement, whereas a proportion $1-p_r$ are unreceptive. The transactional lines (edges) of G connect pairs of (nodes) research-neighbors researchers and the model supposes that a researcher already committed to disclosing a given scientific position has a chance p_s of “infecting” her neighbor, conditional on the latter being receptive or open to receiving that communication. Then $P(p_r,p_s|Z,G)$ is the probability that a provisional scientific opinion initially established in the “source” research units of Z can propagate through the random maze on G and thereby become adopted universally.

Suppose that Z and G are fixed, that G is an infinite graph, and adopt the abbreviated notation: $\boldsymbol{P}(p_r,p_s|Z,G) = \boldsymbol{P}\ (p_r,\ p_s) = \boldsymbol{P}$. Clearly, the mixed percolation probability P is a non-decreasing function of p_r and p_s, and it follows that: $\boldsymbol{P}\ (0,0) = \boldsymbol{P}\ (1,0) = \boldsymbol{P}\ (0,1) = 0$, whereas

P (1,1) = 1. Consequently, $\boldsymbol{P}_s(p) = \boldsymbol{P}(p_s, 1)$, and $\boldsymbol{P}_r(p) = \boldsymbol{P}(1, p_r)$, will denote the *node percolation* and *connection percolation* probabilities of this system, respectively.

A fundamental mathematical property of the percolation process is that there exists some critical values of $p_r > p^*_r$ and $p_s > p^*_s$ beyond which there will be a positive probability that percolation occurs, but below which the percolation probability is zero.[37] In other words, the system undergoes a "phase transition" when these underlying critical probabilities are attained. There are corresponding critical values at which the node-percolation and edge-percolation probabilities, respectively, become positive. These define the endpoints of a region above which a "mixed-percolation process" (one for which it is not certain that either all nodes or all edges of the graph are open), will have positive probability of achieving complete percolation. This is depicted below, in Figure 2.

What these results from percolation theory tell us in the present context is that there is a minimum level of *persistently* communicative behavior that a finite size science network must maintain if ideas are to percolate within it, so that closure can be obtained. Considerable significance therefore can be attached to this fundamental property of percolation processes. For a community of scientists to exist as a cognitively functioning entity, it has just been seen that there is a formal necessity of attaining some critical measures of "expected connectedness," that depend upon the communication behaviors of its representative constituent. A second pertinent result from percolation theory is that there is an important asymmetry between the effects upon network performance of reducing the representative agent's probability of sending, and of receiving messages.

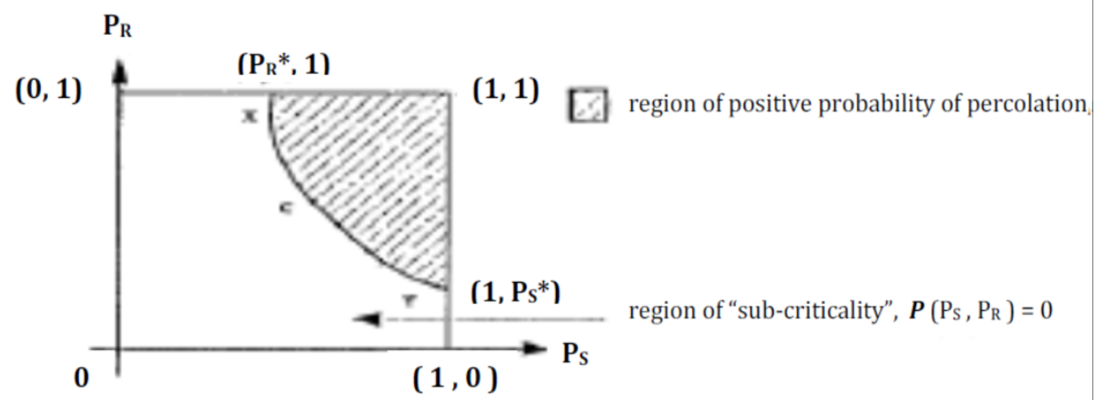

Figure 2. ***Critical boundary for percolation in an infinite graph, G***

[37] See Hammersley and Welsh (1980); Grimmett (1988).

Thus, a given proportional reduction of the mean probability of sending messages (node openness) has a larger effect in degrading the percolation performance of the system than with the equi-proportional reduction of the mean probability of a communication channel being open (edge openness).[38] In view of this, the invisible colleges' first condition for functionality, in the sense of its most exacting requirement, is that the network must maintain at least the critical level of openness in regard to the behavior of a "representative node", i.e., in the expected proportional composition of member "types" with respect to disclosure of their scientific knowledge.

The representative researcher, of course, is a purely statistical construct in the percolation model framework – an average of nodes that are *permanently* open and those that are *permanently* closed. The fraction of those who, being closed, will never "share" (or never "write") what they have learned, therefore, must not be allowed to exceed the critical level ($1-p_s$*) if the invisible college is to retain a positive probability of reaching closure on scientific questions. Thus, the "mix" of persisting behaviors is a critical matter for the system. David (1998: section 7.7) finds considerable significance in the fact that the qualitative performance of this communication system undergoes this critical, discontinuous degradation when the mixing fraction passes below a specific level, especially as its precise magnitude is not likely to be known in advance. In such circumstances it would be sensible to protect the cognitive functionality of the global network by maintaining a "safety-first" policy of selectivity in regard to the recruitment of researchers to the college; in other words, impose some prior test of intrinsic propensity towards "openness" concerning what they will find and conclude in the course of their researches.

By the same token, the existence of strong and universal norms requiring cooperative behavior on the part of researchers, especially in disclosing what they learn, and a reward system that elicits such disclosures as the basis for establishing a collegial reputation, would serve as important bulwark protecting the invisible college's ability to deliver a clear consensus on the questions brought before it. Furthermore, inasmuch as $p_s^* > p_r^*$, in designing the incentives for individual behavior in such a system, it is reasonable from the viewpoint of assuring "connectivity" that assuring the consideration by others of one's own "findings" should take some measure of precedence over concerns about the arrangements and facilities that affect the average propensity of network members to attend to each others' messages. Support for norms (and reward systems) that maintain critical levels of social disclosure ("sharing" and "publication") have are in this sense more potent in keep the invisible college's global network of communications sufficiently open to achieve consensus, than are inducements to receptively receive ("read") the opinions of local network of peers.

The other side of this interpretation is that there would seem to have been a serious failure of understanding among the sociological "relativists," and others who similarly have been inclined dismiss the Mertonian "norms" as a self-serving ideology promulgated by scientists to support their claims to special status and authority. No less mistaken is the argument that the irrelevance of the supposed ethos of open, academic science is transparent, because its norms are repeatedly transgressed due to fallible practitioners of science who are pursuing their material self-interests, or gripped by ego-driven compulsions in rivalries for

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[38] See Hammersley and Welsh (1980) for proof of the generalized asymmetry theorem. Figure 2 [adapted here from David (1998; p.151)] provides a graphical depiction of the shape of the continuous concave boundary of critical values – i.e., the pairs (P_S*, P_R*) below which the percolation probability ***P***(P_S*, P_R*) = 0, labelled the 'sub-critical region. Notice that the node-percolation intercept on the vertical line at (1.0) lies farther below (1,1) than the channel-percolation intercept on the horizontal line at (0,1) lies to the left of (1,1). David and Foray (1993, 1994) discuss this theorem's application in the context of technology diffusion. Other applications can be found in Ligget (

fame. It is an evident sociological error to suppose that the essential features of the qualitative performance of a mode of social organization will be lost if any one among its "norms" is violated by some members at some points in time. Deviance is a phenomenon that is found in all institutionalized social relations. Any system of behavioral norms that is so rigid or non-robust as to be incapable of tolerating some degree of deviant action is not likely to survive for very long. Furthermore, as can be seen from the properties of the Voter Model, some measure of intermittent (random) suspension of communications on the part of individual agents is not destructive of the collective's ability to arrive at "closure". Still more apposite is the point underscored by reference to the percolation model results. Even the presence among the entire population of research workers of some proportion who remain persistently non-communicative, does not necessarily vitiate the possibility that knowledge and provisional judgements can percolate throughout the imperfect communications system. Thus, among those who are only transiently reticent in disclosing their knowledge and provision judgments, or transiently non-receptive to the messages of particular colleagues, it remains possible for the process described by the Voter Model eventually to effect substantial "closure."[39]

These observations point to a formal, communications-theoretic rationale for the emphasis that Merton (1973) and his followers place on the functional importance of the behavioral norm of openness among scientists; and also for the corresponding tendency of that literature to de-emphasize the effects of particular macro-institutional arrangements and technological communications capabilities upon the qualitative performance of scientific communities in the epistemological domain. Still another implication of the Mertonian "norms" for the conduct of (non-proprietary) research is brought into sharper focus by the stochastic models presented here. *Disinterestedness*, *universality*, and *disclosure* can be seen to be crucial in their joint effects, precisely because they reinforce micro-level behaviors that permit the tendency for more "objectively reliable" consensuses to emerge in evolutionary fashion over the long run. They do so by enjoining members of the community to accept dissenting claims as worthy of examination, without regard for the economic, social, political or nationality status of the claimants; by insisting on disclosure as the condition for successful claims to the reputation-based rewards that are attached to priority of discovery; and by preventing secrecy and selective disclosure of knowledge being employed to protect a provisionally established consensus from reasoned challenges.

Thus, the joint effect of the norms characterizing open science is to render it more likely that signs of the collisions between the constructs of social communication and the constraints implied by the structures of the material world will be registered, and circulated within the invisible college. Further, the ethos and reward system of the Republic of Science encourages the perception on the part of its citizens that such signs are to be read as indicating the existence of opportunities for achieving greater recognition and reward, rather than areas where one risks stumbling into heresies that will bring exclusion from future access to the pooled knowledge-resources of fellow scientists. As was suggested previously, an evolutionary selection process in the cognitive domain has a higher chance of discarding a socially influenced consensus that recurrently is found to fit awkwardly with empirical observations. Moreover, so long as some substantial measure of diversity, or disparity of considered opinion is be preserved among researchers who continue in open communication with their scientific peers, such "de-selections" will occur sooner, and the construction of increasingly reliable bodies of knowledge pertaining to the natural and made worlds therefore tends to proceed more swiftly.

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[39] Trust in that capability, of course, is what has been presented (in section 2, above) as underpinning the rational micro-level strategies of the agents engaged in polling their respective networks under the conditions stipulated by the basic Voter Model.

4. Accommodating Realism : A Non-Linear Voter Model with Evolutionary Drift

4.1 Recognizing "reality": an evolutionary resolution for the culture wars

Perhaps too optimistically, David (1998: sect.7.5.3) proposed to effect a compromise between the positions of the contending camps in the recent "culture wars" along the lines of an evolutionary epistemological synthesis such as the one just suggested. Under the proposed terms of peace, both sides should agree that a scientific community can arrive via generic "social processes" at a consensus on the acceptability of certain statements about the material world. Also, there are some rules governing the way those statements are presented and treated by members of the community, the effect of which preserves the possibility that such a socially constructed "truth" will nevertheless remain open to revision, and even to rejection. Indeed, there is a long-run expectation that such social constructions will be discarded should they be found repeatedly to be difficult to square with other "truths" – especially those which possess a higher measure of "fit" with the logical implications and inferences that can be drawn from the available body of empirical observations.

Yet the foregoing discussion has simply pointed to one way in which the door might be left open to such a possibility -- namely, by the likely inability of a invisible college whose social substructure was sufficiently "diverse" (i.e., having sufficiently high dimensionality) to spontaneously achieve perfect correlation of its members opinions on the scientific question(s) under consideration. But, for this to have the proposed effect, more is required than simply preventing the consensus formation process envisaged by the voter model from turning an (objectively) wrong conjecture into a dogma. There also must be some mechanism that eventually steers the process toward the set of consensuses that are found to be "stable" in a evolutionary sense: being consistent with, and hence sustained under repeated exposure to challenges. The minimal mechanism to effect this would rely only on the proposition that an "objective reality" (even it is external to *direct* perception by the members of the research community) would be more likely to generate empirical observations whose interpretation would support a randomly chosen individual's belief in its "truth" than in the opposite conviction, so long as there was a balance of peer opinion on the question. Hence, where an individual researcher entertained the reigning dominant opinion, and the latter was aligned correctly with the underlying (objective) state of nature, exposure to observational evidence would reinforce the individual in her acceptance of the globally dominant belief even though the opposing views happened to be more-or-less equally represented within her immediate local network. Conversely, where the same balance of local opinion prevailed, exposure to observational evidence would work to reverse the belief of an individual that was not correctly aligned with the underlying (objective) state of nature. In this way, although in a probabilistic fashion, over the long run the expected motion of the orientation of the preponderance of belief among the members of the global network would be towards alignment with the underlying "reality."

At least two lingering doubts should be noted, concerning the efficacy of this mechanism. If one entertains the idea that observational evidence is employed by researchers within the framework of Bayesian inference, it is important to stipulate that the objectively "true" state of nature be included among the admissible states. This requires not only that it not be excluded as a matter of "dogma," but also that the research paradigm (or "program") within which the members of the community are working is sufficiently comprehensive, or "fruitful" to allow for it among the operational possibilities entertained. There are times, however, when practical considerations limiting the degree of resolution of observational instrument, or the computational constraints on the processing of captured data, exclude practical consideration of some possibilities. Recognition that such constraints are binding might well induce the abandonment of a subject of inquiry accompanied by declarations of agnosticism, pending advances in the needed scientific apparatus. But studies in the history of science do not provide a warrant for assuming that extended and inconclusive inquiries would automatically be truncated until more powerful empirical techniques were made available.[40]

The second occasion for doubts recapitulates the worrisome problem posed by the existence of experimental or observational errors that may arise even when the theory underlying the experimental design and the instruments employed to capture the data are correct. There are implementation failures, or technical imperfections that may introduce "noise," and worse, "bias" into the observations. Hence, the supposition that there will be a persistent direction to the evolutionary "drift" – under the pressure of inconsistencies between beliefs and the data that are generated in experimental and observational encounters with (objective) reality – is implicitly optimistic about the secular perfection of empirical methods. On this score, studies in the history of science are rather more supportive of a progresssive, "Whiggish" reading of the long-run record.

This would appear to redound at least as much to the credit of the forces making for greater technological sophistication as to the advancement of scientific understanding *per se*. Needless to say, the two do not always proceed hand-in-hand and we might well consider how seriously the evolutionary epistemic drift toward "truth" can be compromised by the persistence of an invariant margin of error in the available experimental or observational evidence. To do this, it will be useful to formally represent a collective belief formation process in which scientific peer opinion within local networks tempers – or, alternatively, is tempered by -- individual researchers' reading of imperfect evidence which is subject to *a given* margin of experimental or observational error.

Note: The text for section 4, subsections 4.2 and 4.3, and sections 5-7, is not available, but extracts of key material appears (below) in the accompanying presentation that has been appended following the References.

[40] In the absence of this, i.e., where the true state is not among the set of admissible priors, there is nothing to guarantee that in the limit a Bayesian updating of beliefs would the a posteriori probablity distribution to converge on the "true" state. For further discussion see David, 2000, where it is shown that the absence of Bayesian learning in the usual "cognitive" sense does not exclude the possibility of strongly adaptive behavior in the modal behavior of a population whose individual member's sequential actions were guided by a strictly Bayesian process of inference.

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COOPERATION, CREATIVITY AND CLOSURE IN SCIENTIFIC RESEARCH NETWORKS:

Modeling the Simpler Dynamics of Invisible Colleges

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Scholarly Communication: Open Science and Its Impact

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Cooperation, Creativity and Closure in Scientific Research Networks: Modeling the Simpler Dynamics of Invisible Colleges

CONTENTS

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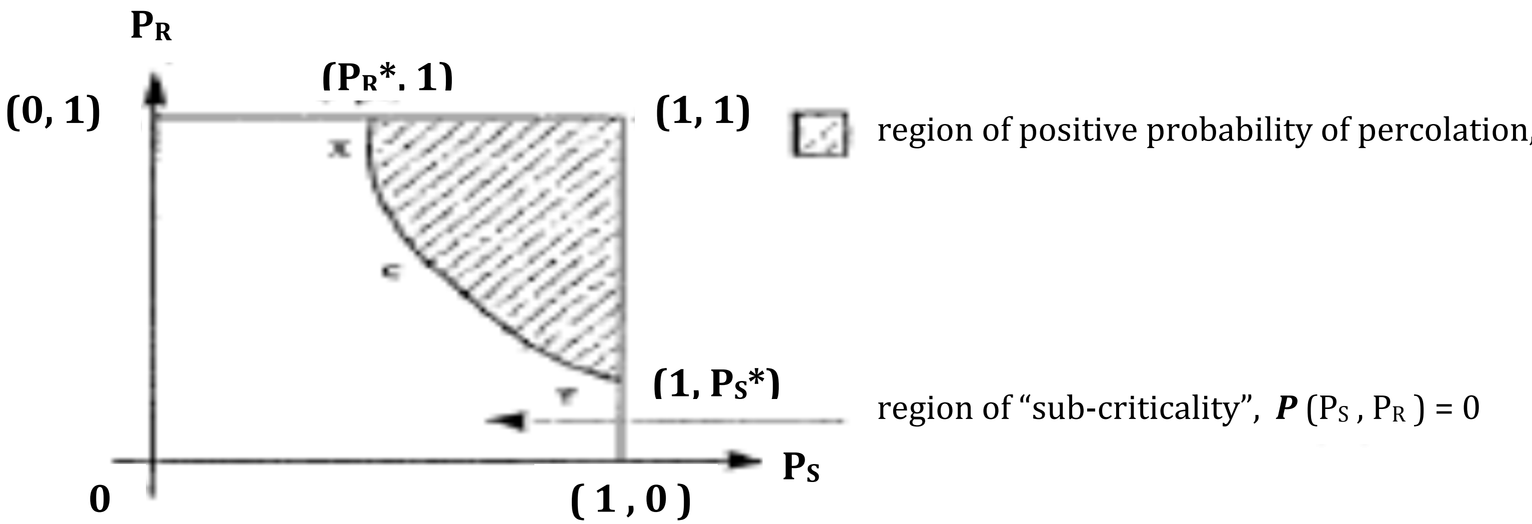


Critical boundary for percolation in an infinite graph, G

PERCOLATION IN SOCIAL COMMUNICATION SYSTEMS

"Percolation" processes are dual to diffusion – as they involve systematic motion in a disordered medium -- akin to moving from one side to the other of a region on which a random maze has been imposed. The maze could represent a connected graph (forming a lattice) in which some nodes or channels have been randomly blocked, preventing messages from passing. If the mean probability of a randomly selected node, or a channel, being permanently blocked is above some critical level, percolation does not occur.

The simulation model of local social communications presented here specifies a "transient maze" (rather than one that is fixed): randomly drawn nodes and connecting channels remain closed with mean probabilities P_S and P_R, respectively, for only the iteration concerned. The effect is to slow the formation of consensus, never to completely block it.

CONFORMITY AS A RATIONAL STRATEGY: A RATIONALE FOR THE VOTER MODEL

SPECIFYING CONDITIONS FOR "CONFORMITY" TO BE A DOMINANT STRATEGY

Notation:

c denotes conformity with the preponderance of scientific opinion in the research-agent's local social network;

d denotes disagreement with that consensus ;

R denotes a state of (social) nature in which a particular statement, ***S*** eventually is found by global consensus to be "reliable/true";

W (equivalently, **not-R**) denotes a state in which ***S*** eventually is found by global consensus to be "unreliable/false"

θ denotes the individual's subjective probability assigned to the outcome that the global consensus eventually forms on **R,** i.e., holding ***S*** to be "reliable"

The representative research-agent treats the local network as the reference group whose esteem matters, and the corresponding "reputational payoffs" for the strategies are:

{c, R**|** ***S*** **is R}= b_1** : being right, with the crowd;

{c, R**|** ***S*** **is W}**=**b_2** : being wrong, with the crowd;

{d, R**|** ***S*** **is R}**= **b_3**: being in a minority and wrong;

{d, R**|** ***S*** **is W}**=**b_4**: being in a minority and right.

Restrictions on the structure of reputational rewards:

$$\mathbf{b_4} > \mathbf{b_1} > \mathbf{b_2} > \mathbf{b_3} \quad , \qquad \text{where } \mathbf{b_1} \geq 0.$$

***Remark*:** The restrictions assign greater value to an individual researcher who is found to be "in the right" than to one who embraced the "wrong" view, and requires that a "penalty" cannot be incurred for having been "in the right." But, being wrong in a crowd is not as bad (for subsequent reputational standing) as being found to have been "in the wrong" more-or-less on one's own; being "lonely yet right" is reputationally more glorious than being correct among a crowd of one's peers.

Expected reputational pay-offs, conditional on strategy choices in the “game against nature”:

The agent’s expected payoffs conditional on the strategy of conforming with/ dissenting from with the preponderance of local peer opinion, are respectively

$\pi_c = \{\theta \mathbf{b_1} - (1-\theta)\mathbf{b_2}\}$,

and

$$\pi_d = \{(1-\theta)\mathbf{b_4} - \theta \mathbf{b_3}\} .$$

Sufficient conditions for the strategy of “conformity”:

$\pi_c > \pi_d$ is sufficient for the strategy of “conformity” to maximize the individual researcher’s expected reputational payoff (whereas the pure strategy of “dissent” will be dominant when $\pi_c < \pi_d$).

The sufficient condition is readily obtained by substitution:

$$[(\mathbf{b_1} + \mathbf{b_3}) / (\mathbf{b_4} + \mathbf{b_2})] > [(1-\theta) / \theta] ,$$

which also may be expressed as

$$\theta > (\mathbf{b_4} + \mathbf{b_2}) [(\mathbf{b_1} + \mathbf{b_3}) + (\mathbf{b_4} + \mathbf{b_2})]^{-1} = \theta^* .$$

Remarks: (i)There exists a critical value $0 < \theta^*$ s.t. “conforming” is the dominant strategy when $\theta > \theta^*$.

(ii) Restrictions ($\mathbf{b_4} > \mathbf{b_1} > \mathbf{b_2} > \mathbf{b_3}$) and $\mathbf{b_1} \geq 0$ guarantee that $0 < \theta^* < 1$, and hence the sufficient condition for conformity can be met by $\theta \in [0,1]$ . .

(iii) The restriction also imply $(\mathbf{b_1} + \mathbf{b_3}) < (\mathbf{b_4} + \mathbf{b_2})$, from which it follows that $\theta^* > ½$.

CONDITIONS FOR “CONFORMITY” TO BE A DOMINANT STRATEGY – continued

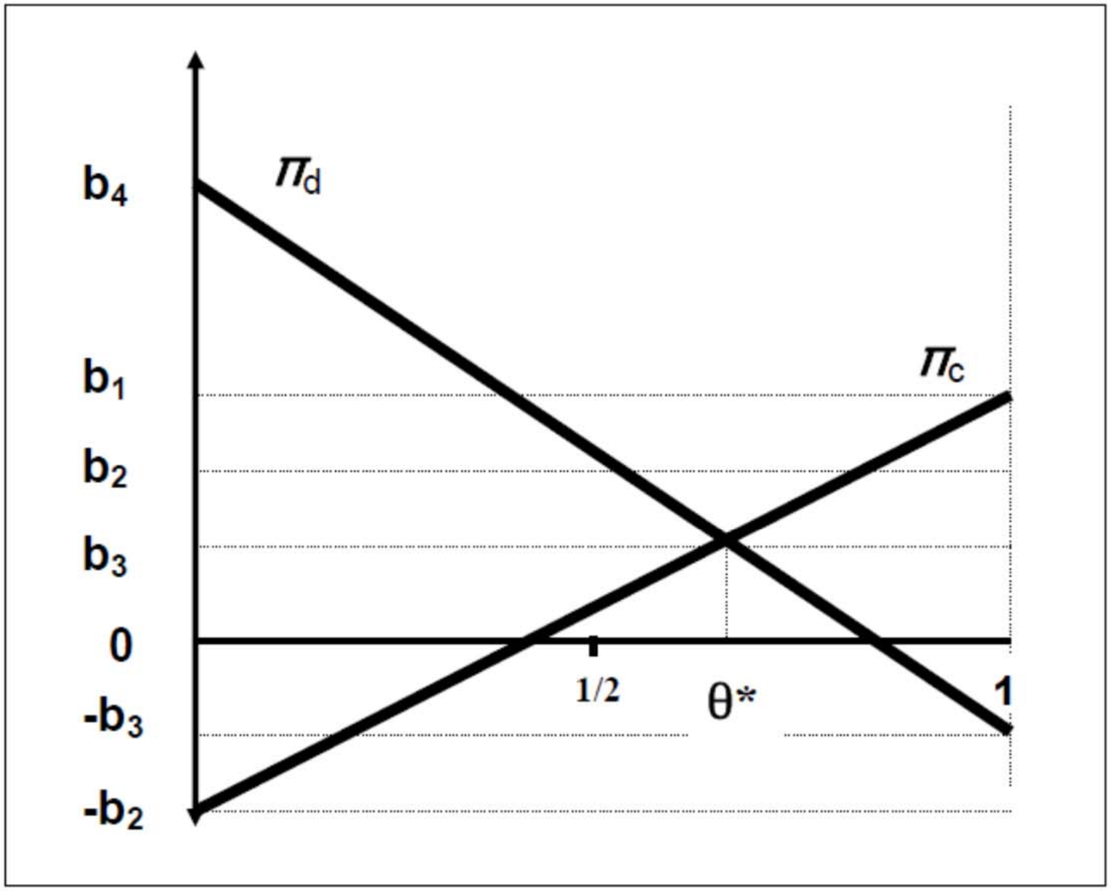


Figure 1.Conditions for dominance of conformity as a reputational strategy

Notes on Accommodating ‘Realism’ (section 4):

A Non-Linear Voter Model with Evolutionary Drift

4.1 Recognizing “reality”: an evolutionary resolution for the culture wars

4.2 Modeling evolutionary “drift” in micro-level Bayesian belief formation shaped by imperfect experimental evidence and the pull of peer conformity

4.3 Stochastic simulation results for the non-linear voter model with noisy experiment-driven drift

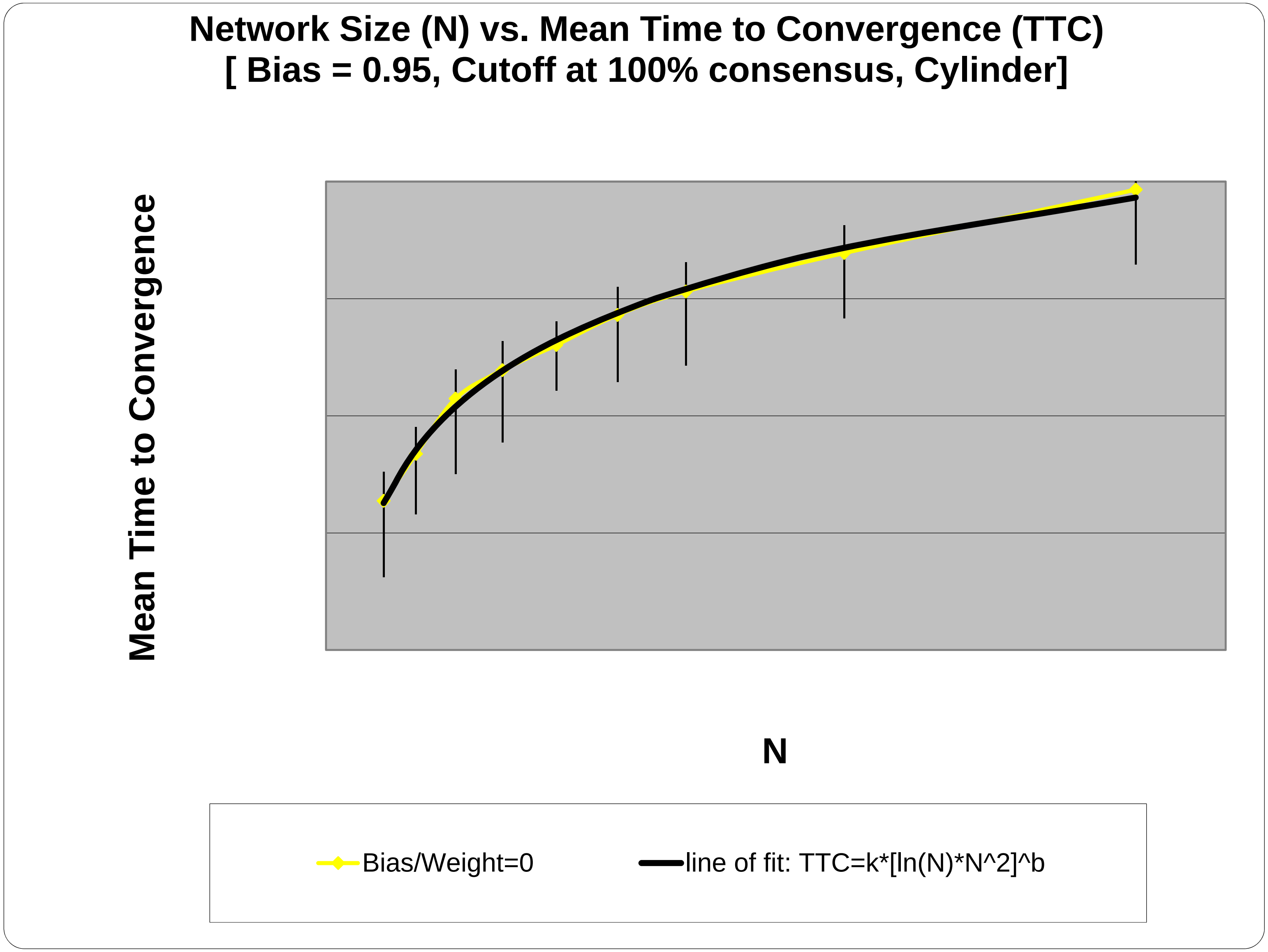
ALLOWING “REALITY” TO AFFECT CONSENSUS IN A MODIFIED “VOTER MODEL”

The David-Waterman “Science Simulator” (Version 2, 2001) models the scientific community as a network of researchers (or research groups) interconnected by various paths of communication. Each researcher is a node in the network connected by arcs to some set of other researchers' nodes. A researcher need not be in communication with all other researchers in the community, just some group of scientists whose opinions matter.

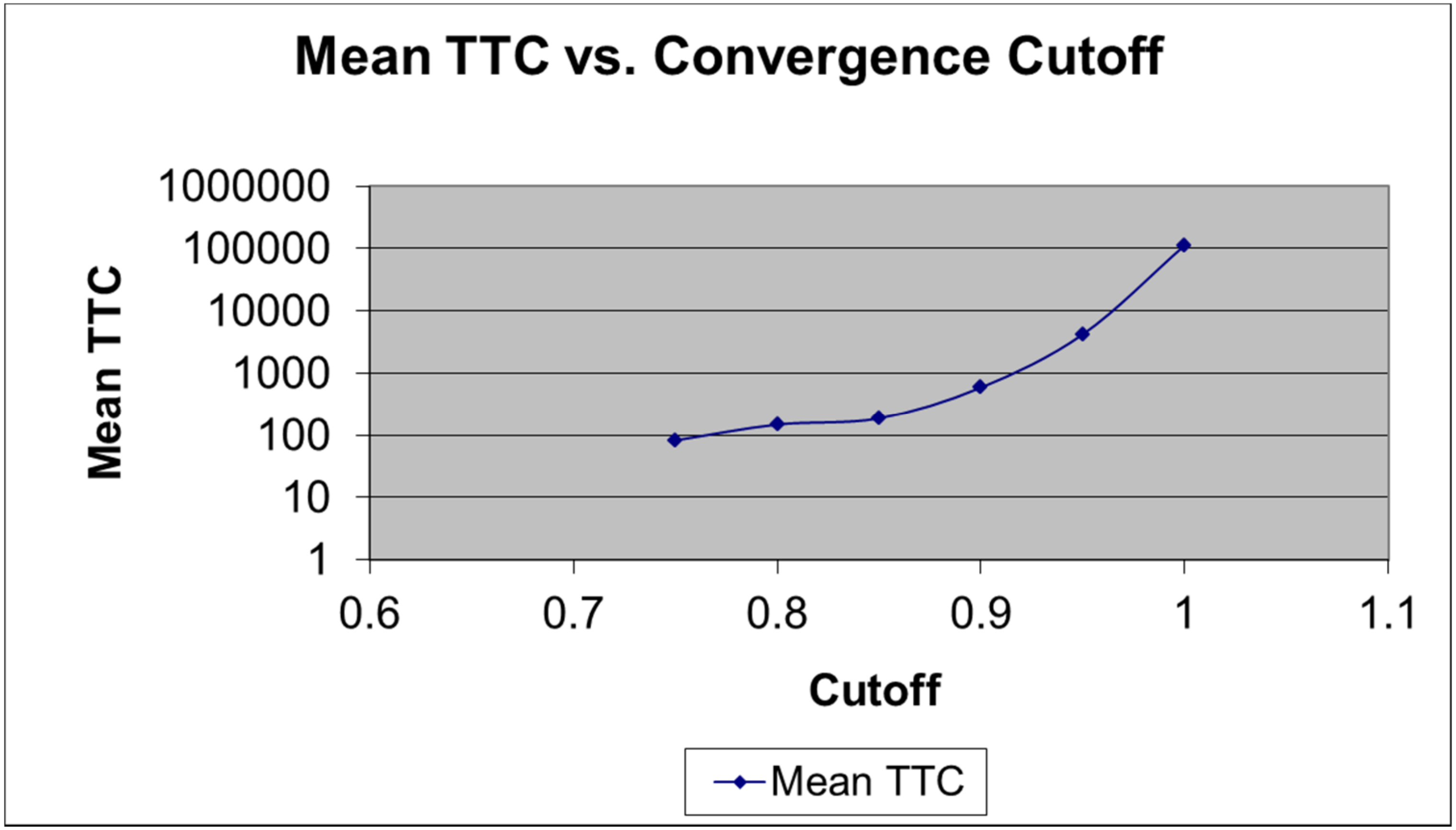
Every researcher is assumed to favor – i.e., believe in the greater reliability of one of two competing “scientific propositions” (called her "theory choice"). These can take the simple form such as “the model (M) is correct” vs. “model M is not correct”. All researchers begin with a randomly selected belief about M’s correctness. The simulation generates the evolution of the configuration of the agents’ beliefs over a fixed period of time, as the outcome of a stochastic process that is a variant of reversible “spin system” known as “the linear voter model.”

At each instance in time, a researcher is selected (randomly) to revise her “theory orientation” as follows: the researcher polls the theory choices of those researchers in her local social network, and selects a theory-orientation with probability equal to the portion of polled researchers who hold that theory to be correct. But rather than giving full weight to the “consensus” of her (polled) social group, the randomly selected “orientation” can be given a positive weight ($0 \leq w \leq 1$). This is equivalent to allowing an stochastic term to “perturb” the consensus message, and the distribution of that shock can have a mean bias reflecting the “reality” – as if it were the feedback from uncertain experimental observations.

All nodes and channels in the “community” graph are “open” with finite probability, and the question of interest is the speed and extent of complete percolation, convergence to “closure”, and its relationship to network size. This is evaluated for a system where nodes and channels are open ($p_s = p_r = 1$).



Remark: **Yellow curve is fitted to Simple Voter Model (simulated by D-W_SCI-SIMI, without experimental shocks**



Means generated by varying the cut-off for the cutoff degree of Consenus in simulation runs of the Simple Voter Model with a finite 13x13 square lattice (d=2) on a cylinder: N =169.

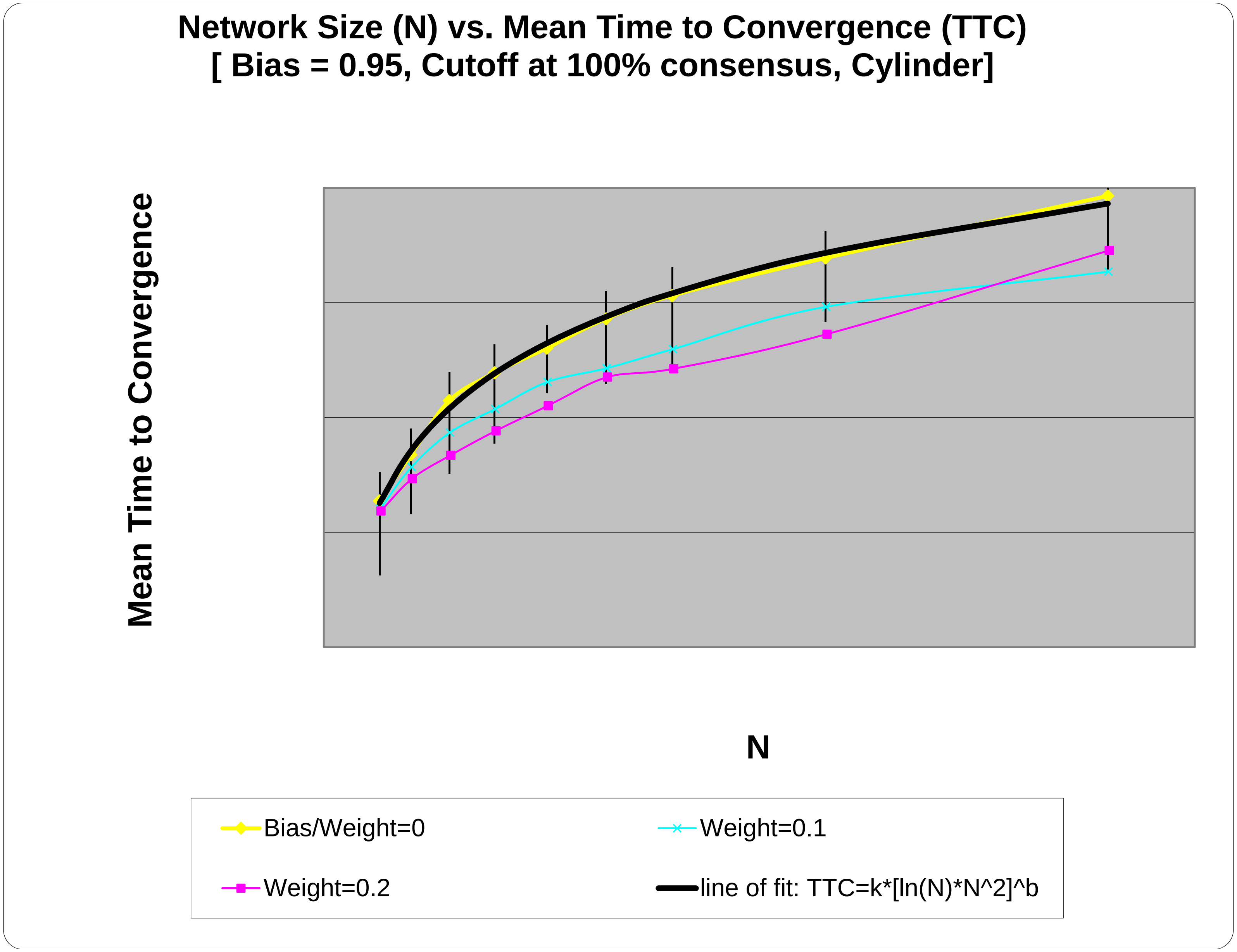
SIMPLE VOTER MODEL: MOMENTS OF THE DISTRIBUTION OF DURATION TO CONSENSUS

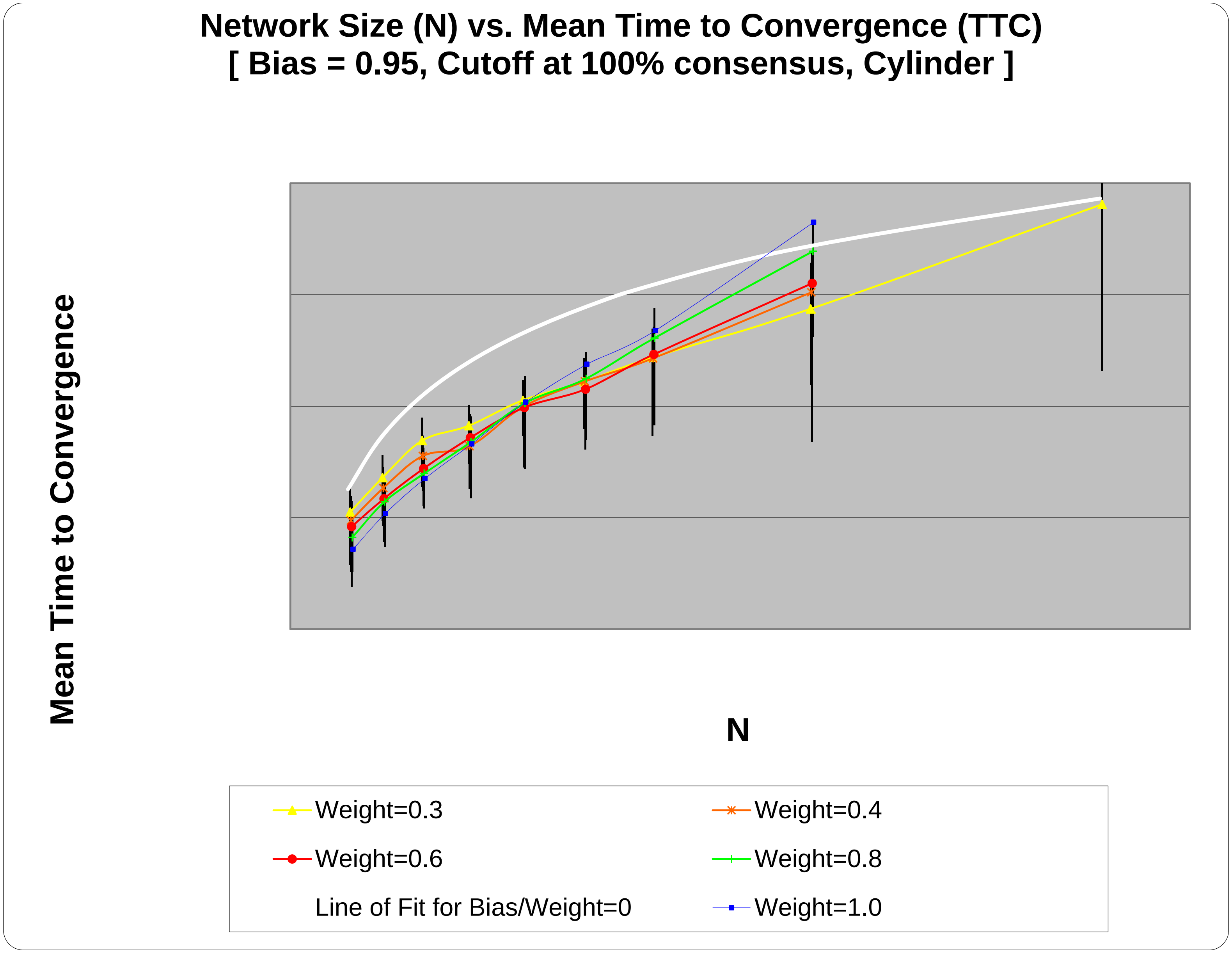
(TIME IN ITERATION COUNTS)

Network Size: N	25	81	144	196
			for 100% Consensus	
Mean Consensus Time	500.6061	8575.758	278375.4	1948848
Sample Std. Error	446.5855	7920.057	1671.4	8875856
Relative Variance	0.7877855	0.8443111	.0036	20.53313
			for 98% Consensus	
Mean Duration Time	498.1818	8500	302979.4	33954.55
Sample Std. Error	392.6899	6196.551	2752901.0	31312.22
Relative Variance	0.6150557	0.5260815	82.5619	0.8418268

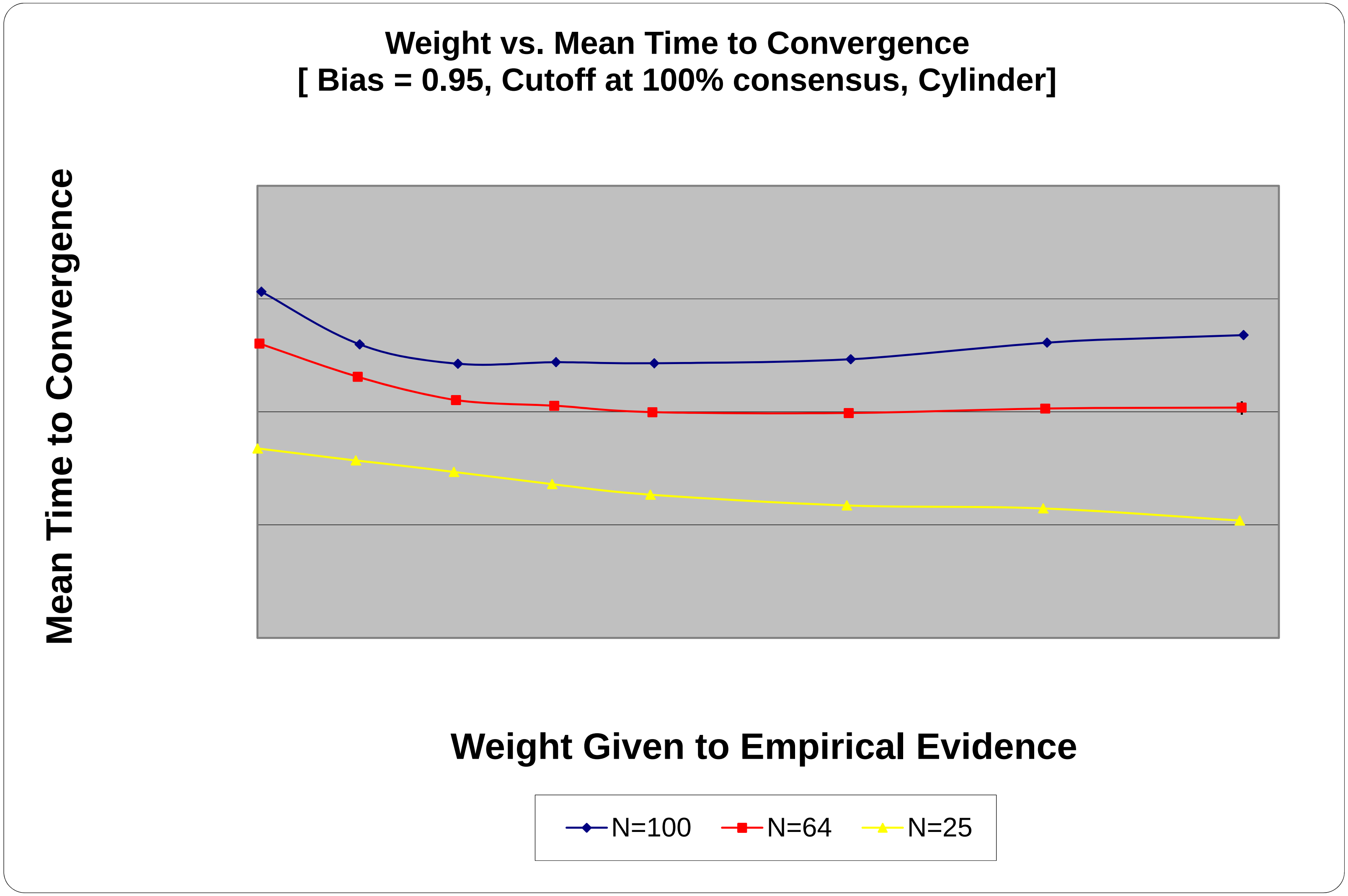
Notes: Sample means and variances shown for the distributions of the convergence time (iteration count) for a simulation run of the simple voter model (of node size N) were computed for samples of 100 runs of the percolation process for a square lattice on a cylinder (torus of d=2), with the mean node percolation and channel percolation probabilities set at $(P_n, P_c) = (P_S, P_R) = (1,1)$. Convergence to "100 % consensus" was determined by persistence of perfect agreement of the node orientations on acceptance of Theory One (orientation 1) or on the alternative (orientation 0) for 2 iterations following the first appearance of that configuration during the run. The same criterion was used to establish the iteration at which a 98% consensus was reached. The square matrix of size N (nodes) representing the "invisible college" was randomly populated before each run by equi-probable assignment of orientations (1,0). Consequently, with node orientation switching at each iteration driven solely by random polling of the von Neumann neighborhood of the node that is selected randomly at that iteration, the expected outcome of the run is an equally likely consensus on Theory One, or on Theory Two.

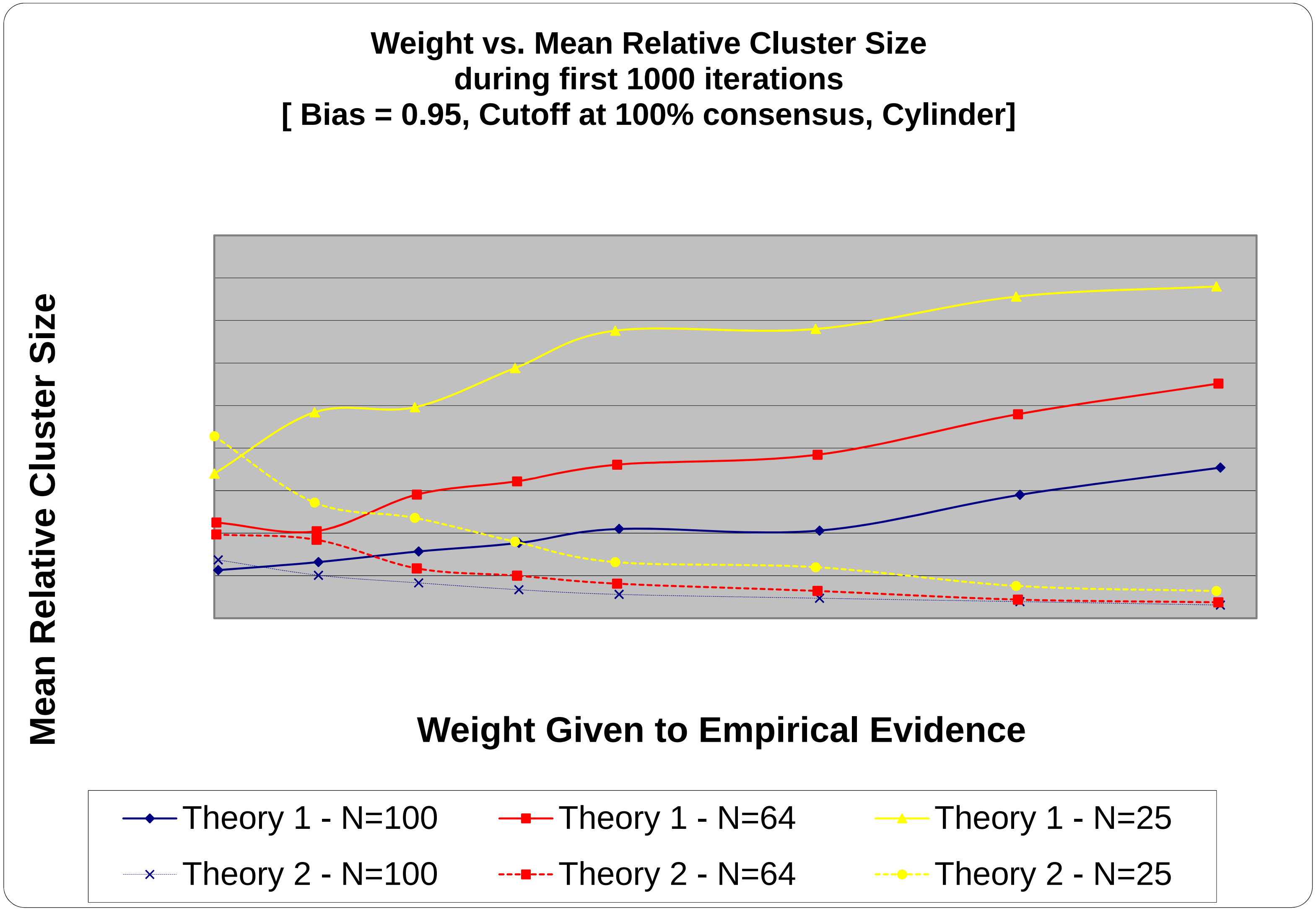
***Caution in re* entries for N= 144. The algorithm that retrieves and process the D&W_SCI-SIM output is suspect.**


Network Size (N) vs. Mean Time to Convergence (TTC)
[Bias = 0.95, Cutoff at 100% consensus, Cylinder]
Mean Time to Convergence
N
Bias/Weight=0
Weight=0.1
Weight=0.2
line of fit: TTC=k*[ln(N)*N^2]^b



Network Size (N) vs. Mean Time to Convergence (TTC)
[Bias = 0.95, Cutoff at 100% consensus, Cylinder]
Mean Time to Convergence
N
Weight=0.3
Weight=0.4
Weight=0.6
Weight=0.8
Line of Fit for Bias/Weight=0
Weight=1.0


Weight vs. Mean Time to Convergence
[Bias = 0.95, Cutoff at 100% consensus, Cylinder]
Mean Time to Convergence
Weight Given to Empirical Evidence
N=100
N=64
N=25


Weight vs. Mean Relative Cluster Size
during first 1000 iterations
[Bias = 0.95, Cutoff at 100% consensus, Cylinder]
Mean Relative Cluster Size
Weight Given to Empirical Evidence
Theory 1 - N=100
Theory 1 - N=64
Theory 1 - N=25
Theory 2 - N=100
Theory 2 - N=64
Theory 2 - N=25

Towards Specification of the Dynamic Behavior of a Simple and Complete Knowledge-Generating System

COMMUNICATION TRANSACTIONS AND GROWTH IN A SCIENTIFIC COMMUNITIES

Structural Assumptions on the Communication System

i) The system is a connected graph with agents (at nodes) linked (by channels) to neighboring agents.

ii) Every agent has two functional modes in regard to current messages: "writing" and "reading".

iii) An agent in either mode can be in one of two states with regard to that mode:

"sending" or "not sending" messages if in writing mode; **"receiving" or "not receiving"** messages if in reading mode**.**

INTUITIONS FOR AN ELEMENTARY DYNAMICAL SYSTEM

- **More regularly open pathways for communications between randomly selected agents increase the expected speed of "news" percolation.**
- **Faster expected arrival of "news" makes readers monitor channels more frequently; *cet. par.*, it makes the network more attractive to readers.**
- **Larger network audiences (having more, and more attentive readers) encourage more frequent transmissions of messages from writers; *cet. par.*, it makes the network more attractive to writers.**
- **In larger networks, *cet. par.*, the expected time taken for complete percolation will be greater.**
- **In larger networks, *cet. par.*, there will be a higher expected rate of emergence of novel ideas, produced by recombination of ideas transmitted by the agents.**
- **A higher expected rate of arrival of "new news" – reaching one randomly located agent from another randomly located originating agent – makes the network more attractive to join, for both readers and writers.**

Remark: **Dynamical systems with strong positive feedbacks are potentially unstable. Small shocks can trigger growth, or collapse.**

Structure of the Equivalent Deterministic System --
With N Research-Agents on a Torus of Low Dimensionality

Rate of percolation, or "closure" speed:

$$S = S(N, p_s, p_r)$$

Rate of "creativity" (generation of new ideas):

$$K = F(N, p_s)$$

Rate of arrival of new "reliable" knowledge at a random node of the network:

$$S \cdot K = F(N, p_s, p_r)$$

Homogeneous agents' induced knowledge-sharing propensity:

$$p_s - p_s^0 = G(N, p_r)$$

Homogeneous agents' induced knowledge-monitoring and -absorbing propensity:

$$P_r - p_r^0 = H([F(N, p_s, p_r)])$$

Equilibrium (consistent) communication propensities for a network of size N:

$$Q(p_s^*, p_r^* \mid N) = 0$$

Endogenous network size adjustments:

$$\Delta N \equiv [N_{t+1} - N_t] = Z(N, p_s, p_r \mid \mathrm{P}_t, \omega_0), \quad \text{for } \mathrm{P}_{\mathbf{t}} = \boldsymbol{P}.$$

Specifications for the Equivalent Deterministic System

Expected "Consensus" or "Closure" Speed of Network N

$$S(N, p_r, p_s) = \frac{k p_r p_s}{N^3} \qquad k > 0$$

Expected "Creativity" of Network N

$$K(p_s, N) = 2^{(\eta p_s N)^{\mu}} \qquad 0 < \mu < 1, \quad 0 < \eta \leq 1$$

Expected Rate of Arrival of New Ideas at Random Site in Network N

$$F(N | p_s, p_r) = [K(\cdot)][S(\cdot)]$$

Determination of Homogeneous Probabilities of Knowledge Communication

-- for "Writing and Disclosing":

$$p_s = min\left[\left(b\{p_r N\}^{\beta} + p_s^0\right), 1\right], \qquad b > 0, \ \beta > 0$$

-- for "Receiving and Reading":

$$p_r = min\left[\left(a\{F(N | p_s, p_r)\}^{\alpha} + p_r^0\right), 1\right], \qquad a > 0, \ 0 < \alpha < 1$$

Network Growth: Specifications of the Distributed Lag "Stock Adjustment"Model

$$\dot{N} \approx [\, N_{t+1} - N_t \,] = (\, N^{*}_{t+1} \,)^{\lambda} \,(\, N_t \,)^{1-\lambda}$$

where

$$N^{*}_{t+1} = \{\, \Omega \,[\, N_t \,,\, p_s(t\,) \,,\, p_r(t) \,] \,\} \, P_t \;,$$

and

$$\Omega_t(\,\bullet\,) = 1/(1 + M \;\; exp\{-\omega(t)\}) \;,\;\; 0 < M$$

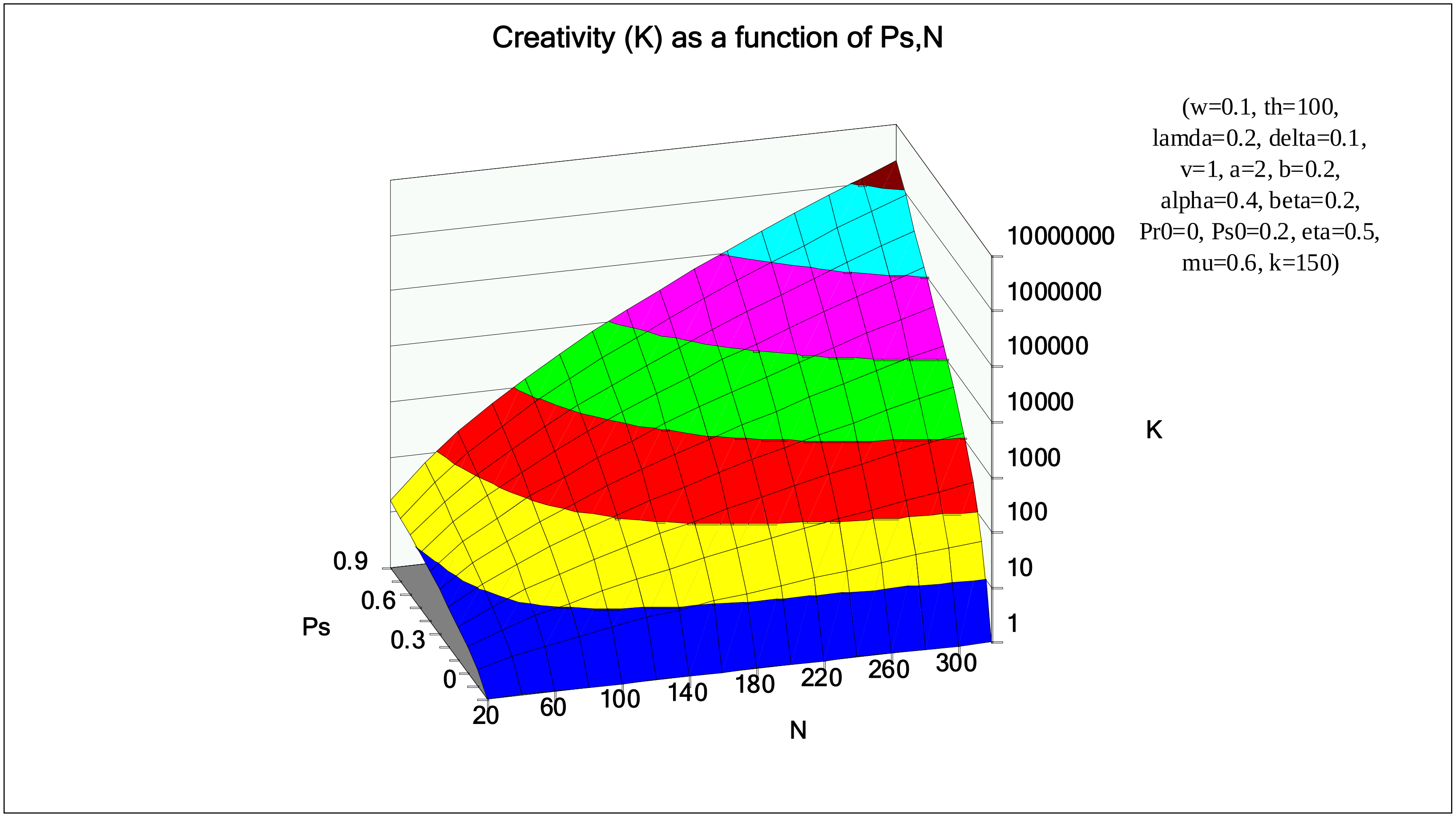
and

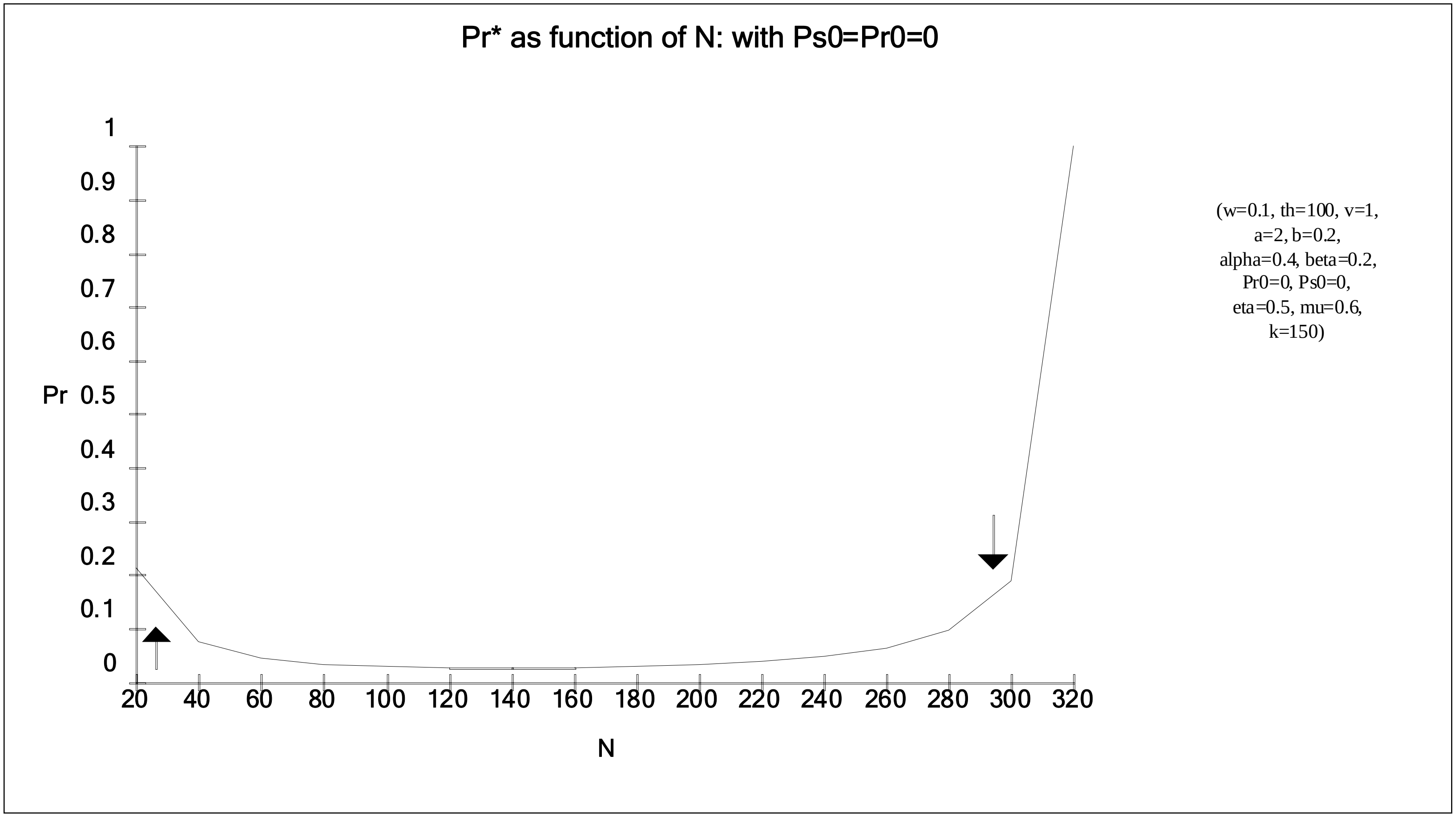
$$\omega(t) = \{[\; \theta_r(\; p_r(t) + \nu \;\; p_s(t)) \;] \bullet F(\; N_t \,,\, p_s(t) \,,\;\; p_r(t))\}$$

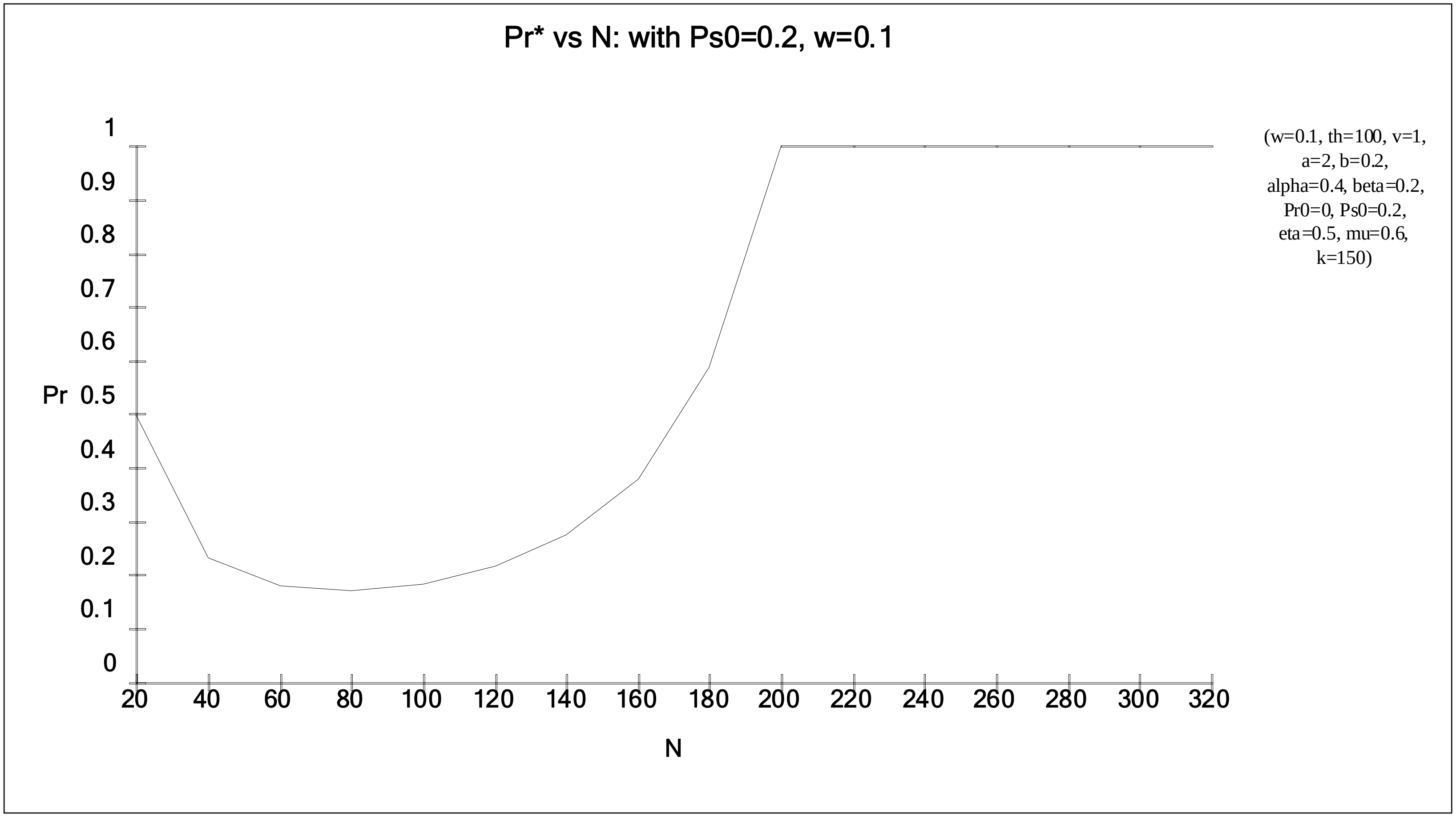
For the stationary population case $P_t = \boldsymbol{P}$ for all t, there is a solution set:

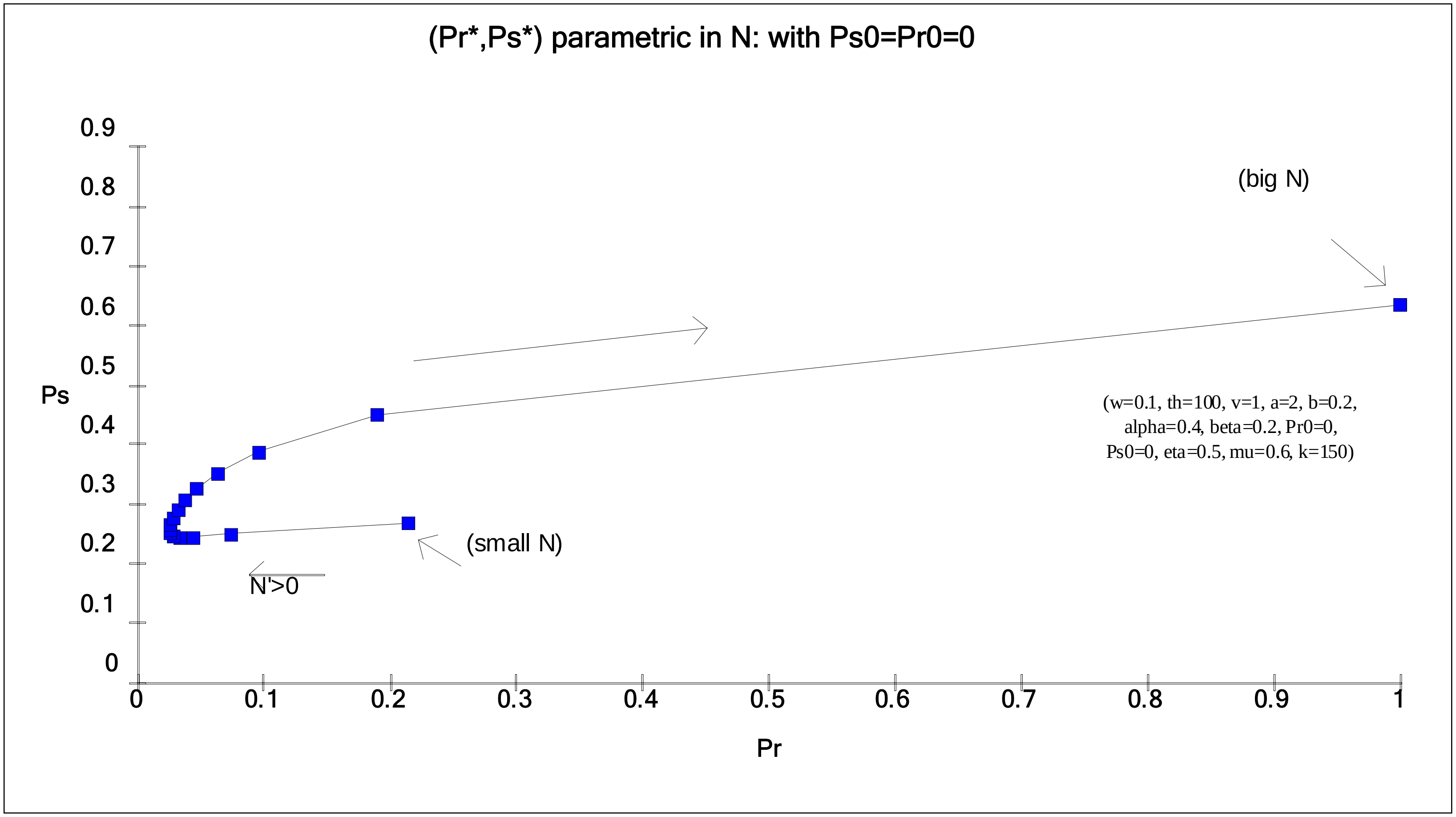
$$N^{**} = \Omega\,(\, N^{**},\, p_r^{**},\, p_s^{**} \;)\, P$$

, that will satisfy $\dot{N} = 0$.

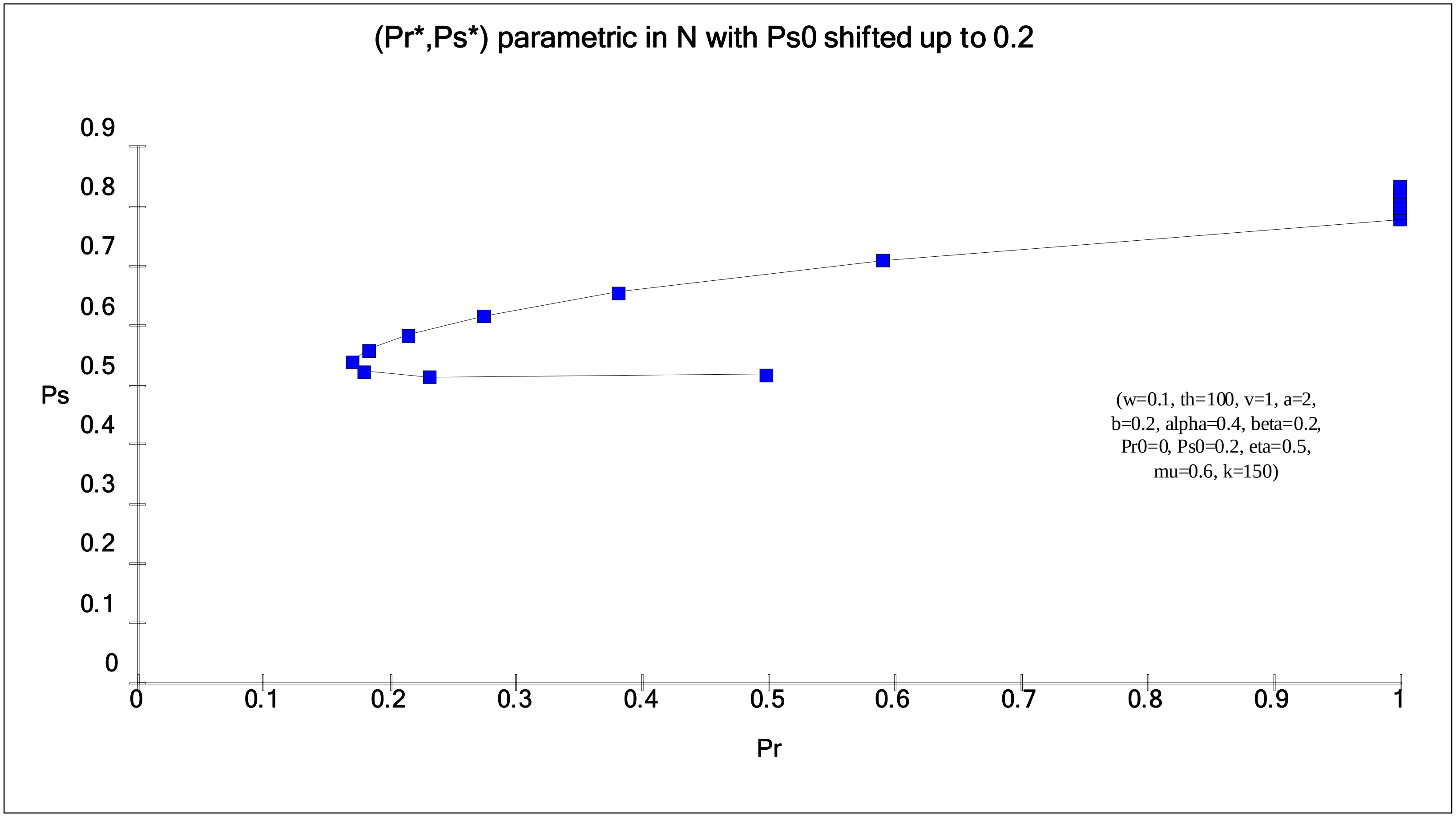

Creativity (K) as a function of Ps,N
(w=0.1, th=100,
lamda=0.2, delta=0.1,
v=1, a=2, b=0.2,
alpha=0.4, beta=0.2,
Pr0=0, Ps0=0.2, eta=0.5,
mu=0.6, k=150)
10000000
1000000
100000
10000
1000
100
10
1
K
0.9
0.6
0.3
0
Ps
20
60
100
140
180
220
260
300
N

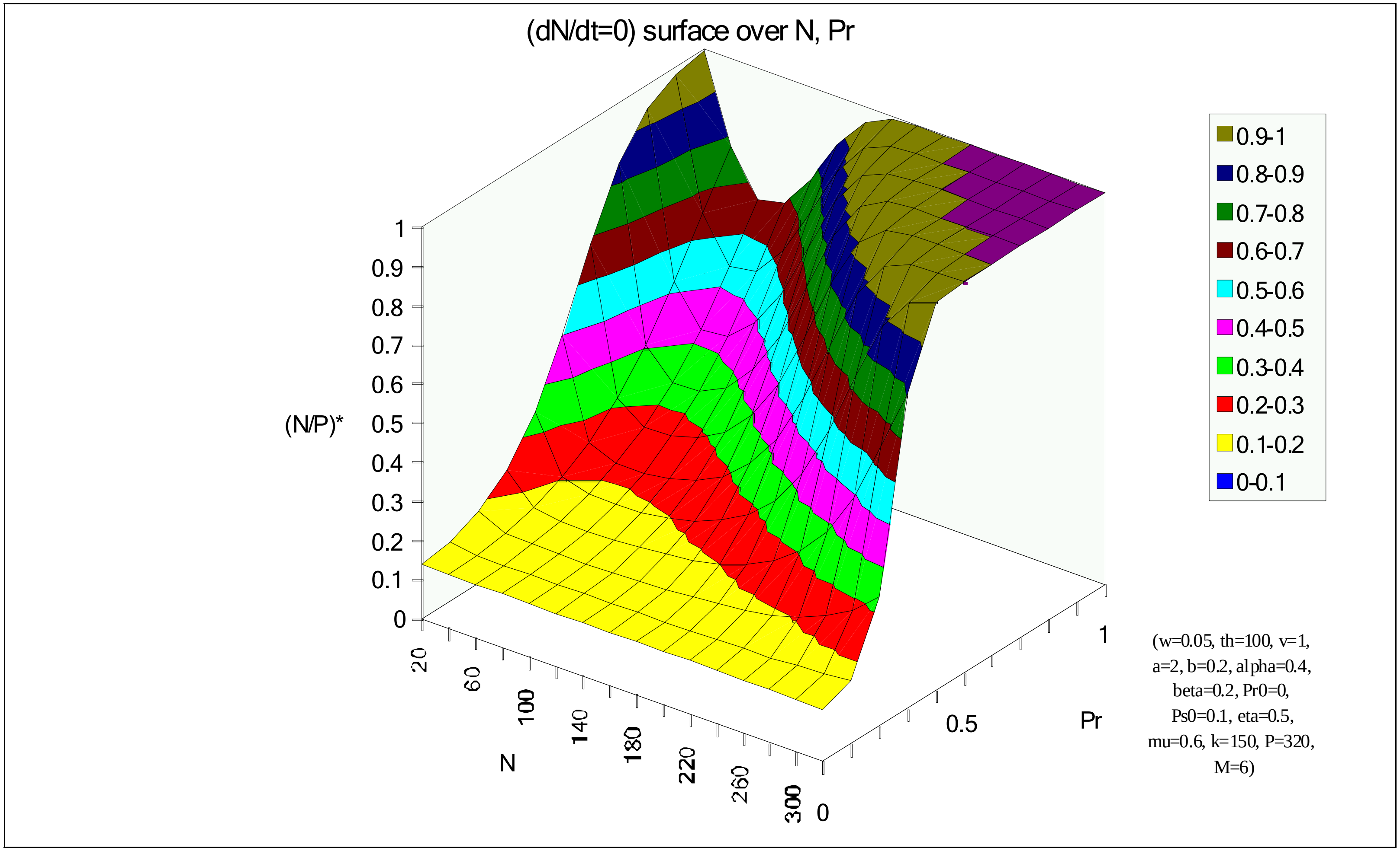

Pr* as function of N: with Ps0=Pr0=0
1
0.9
0.8
0.7
0.6
Pr 0.5
0.4
0.3
0.2
0.1
0
20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320
N
(w=0.1, th=100, v=1,
a=2, b=0.2,
alpha=0.4, beta=0.2,
Pr0=0, Ps0=0,
eta=0.5, mu=0.6,
k=150)

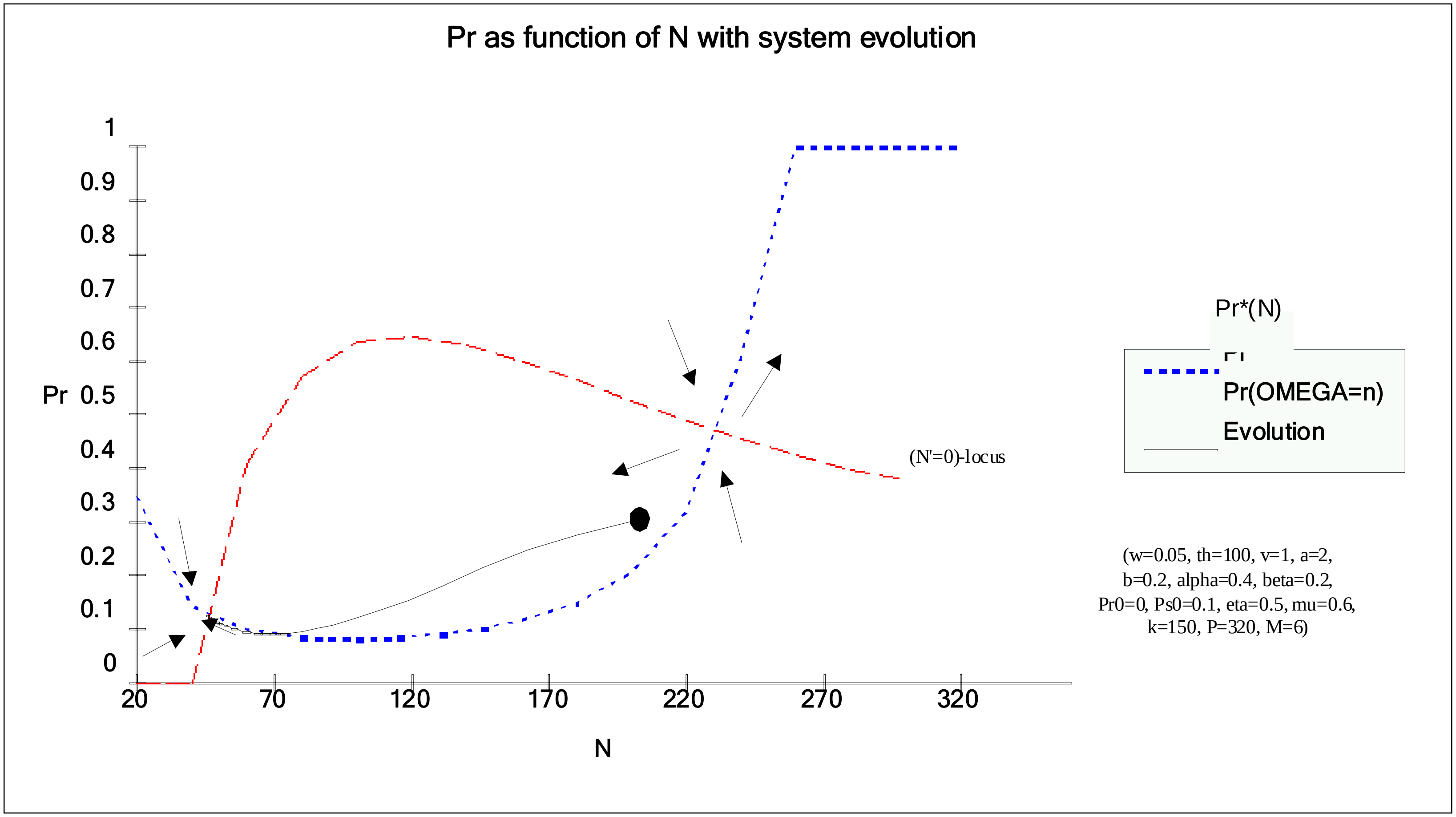

Pr* vs N: with Ps0=0.2, w=0.1
Pr
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
N
(w=0.1, th=100, v=1,
a=2, b=0.2,
alpha=0.4, beta=0.2,
Pr0=0, Ps0=0.2,
eta=0.5, mu=0.6,
k=150)

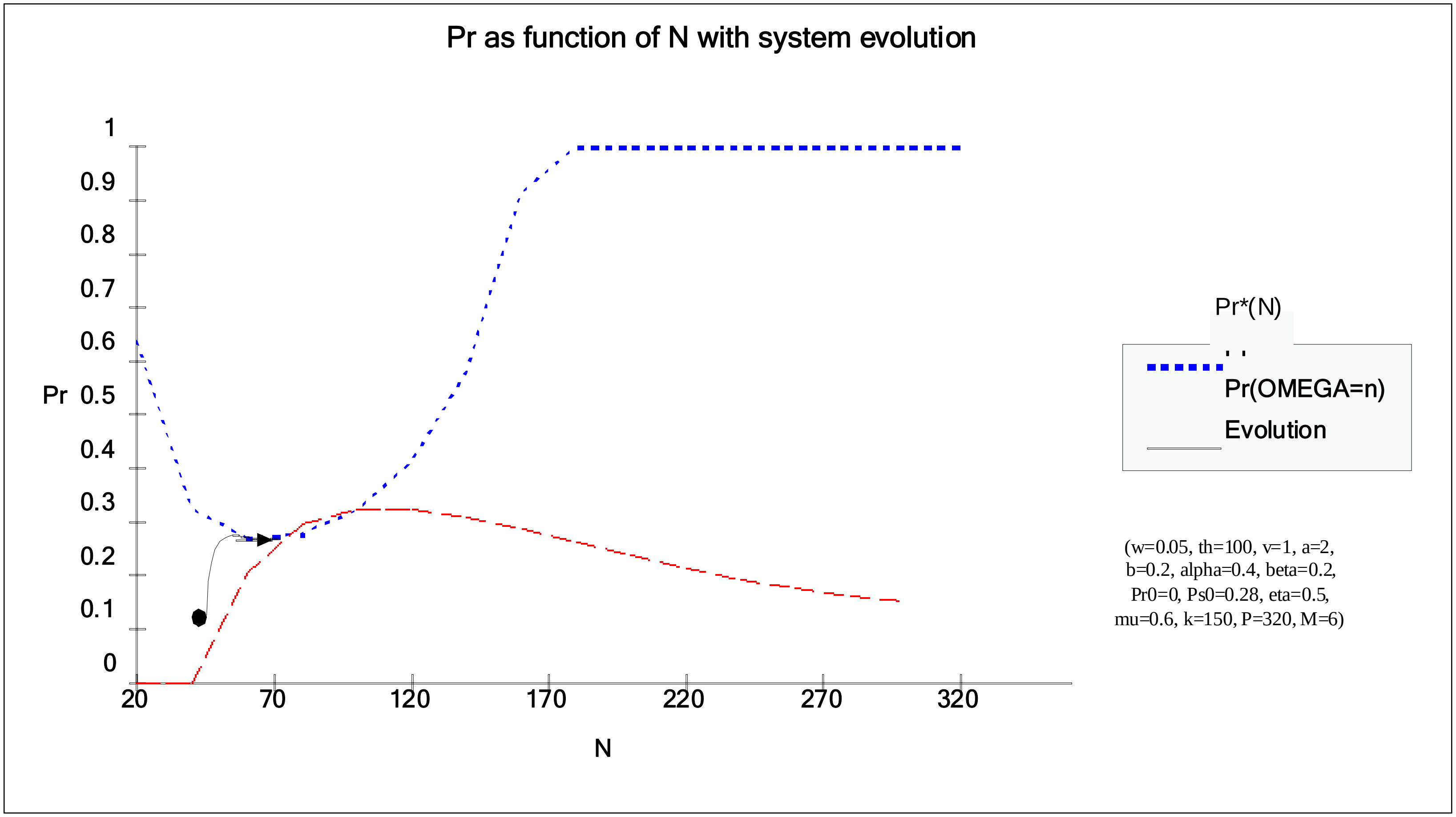


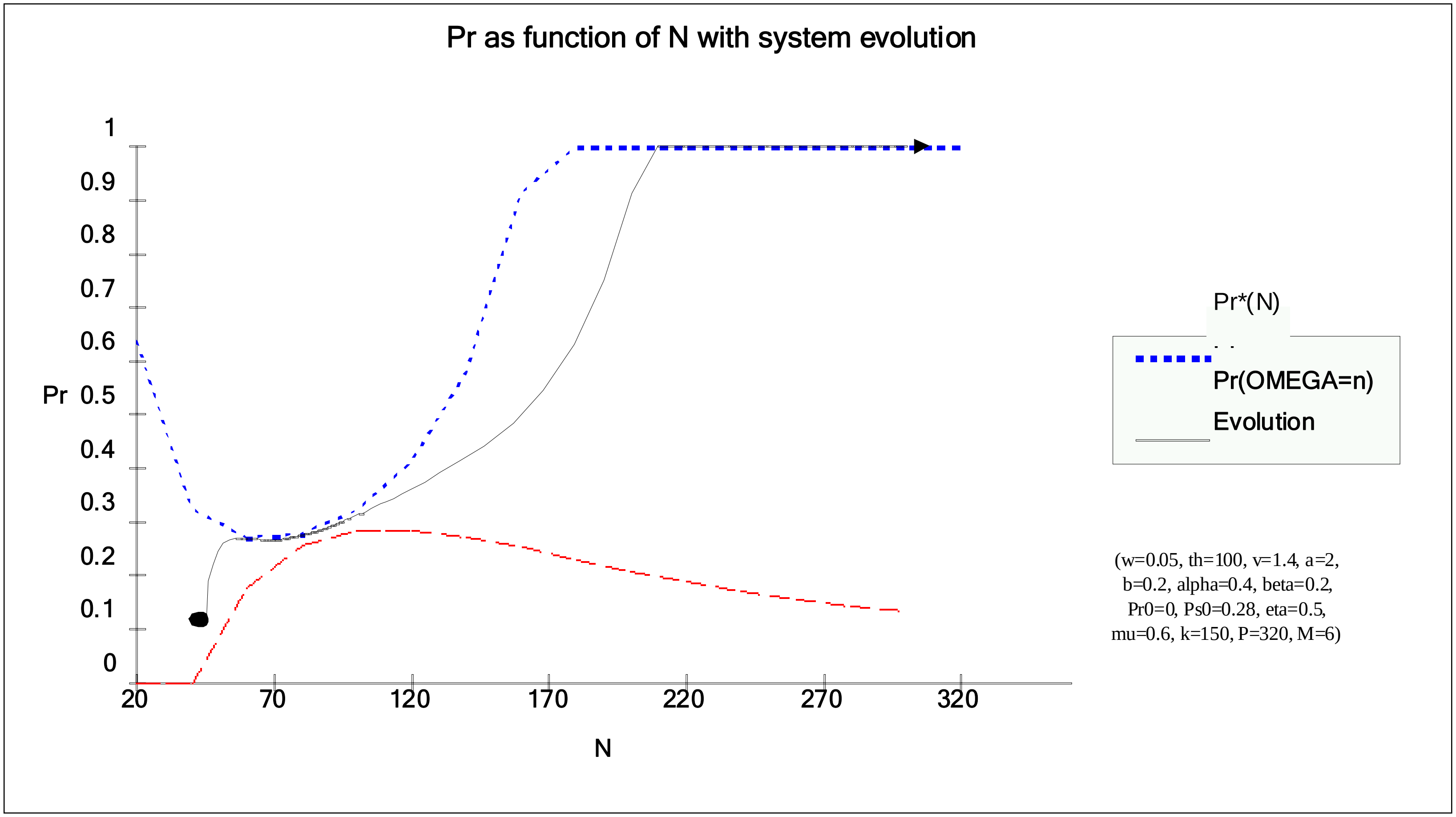
(Pr*,Ps*) parametric in N: with Ps0=Pr0=0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Ps
(big N)
(w=0.1, th=100, v=1, a=2, b=0.2,
alpha=0.4, beta=0.2, Pr0=0,
Ps0=0, eta=0.5, mu=0.6, k=150)
(small N)
N'>0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Pr

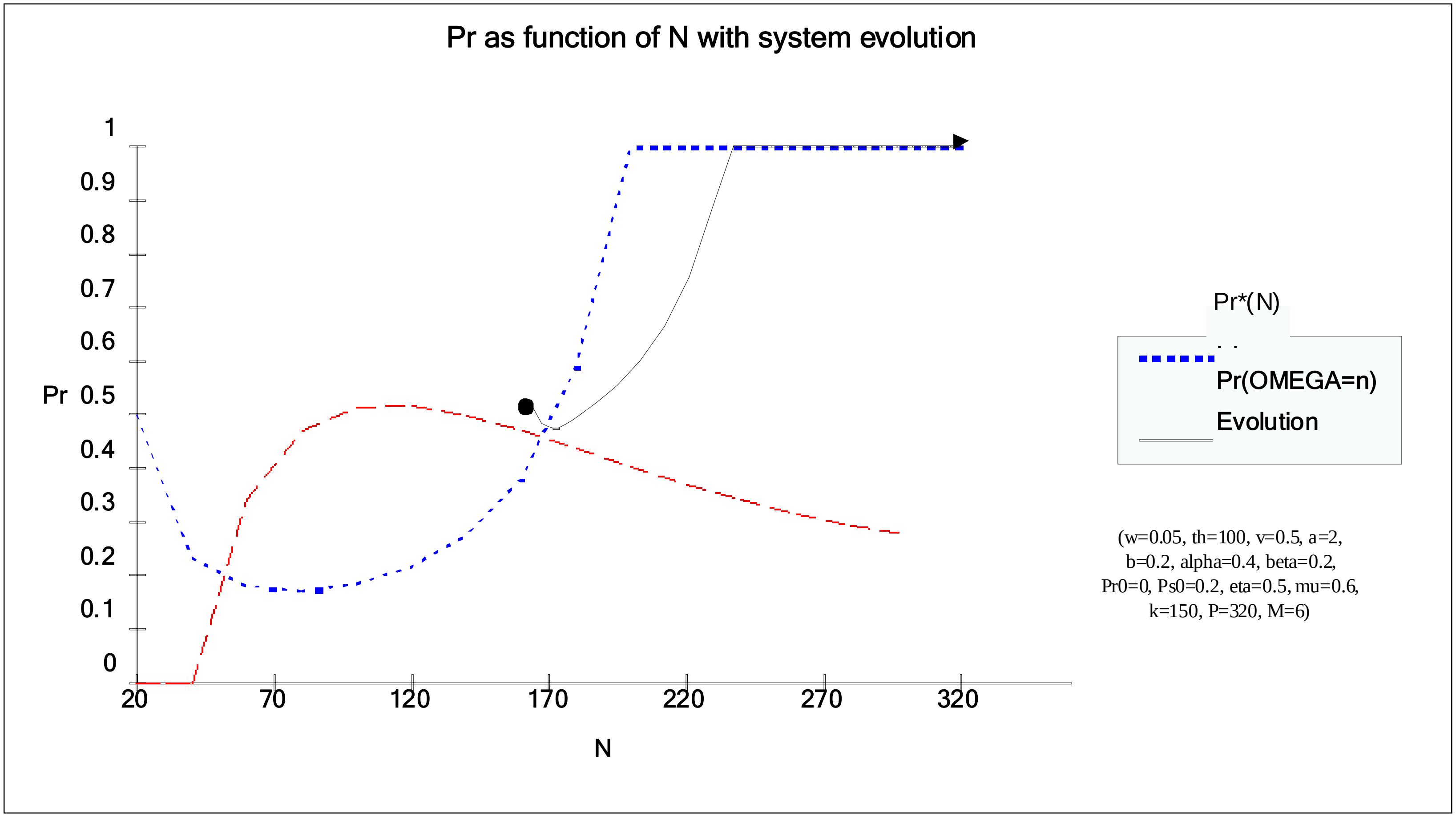

(Pr*,Ps*) parametric in N with Ps0 shifted up to 0.2
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Ps
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Pr
(w=0.1, th=100, v=1, a=2,
b=0.2, alpha=0.4, beta=0.2,
Pr0=0, Ps0=0.2, eta=0.5,
mu=0.6, k=150)

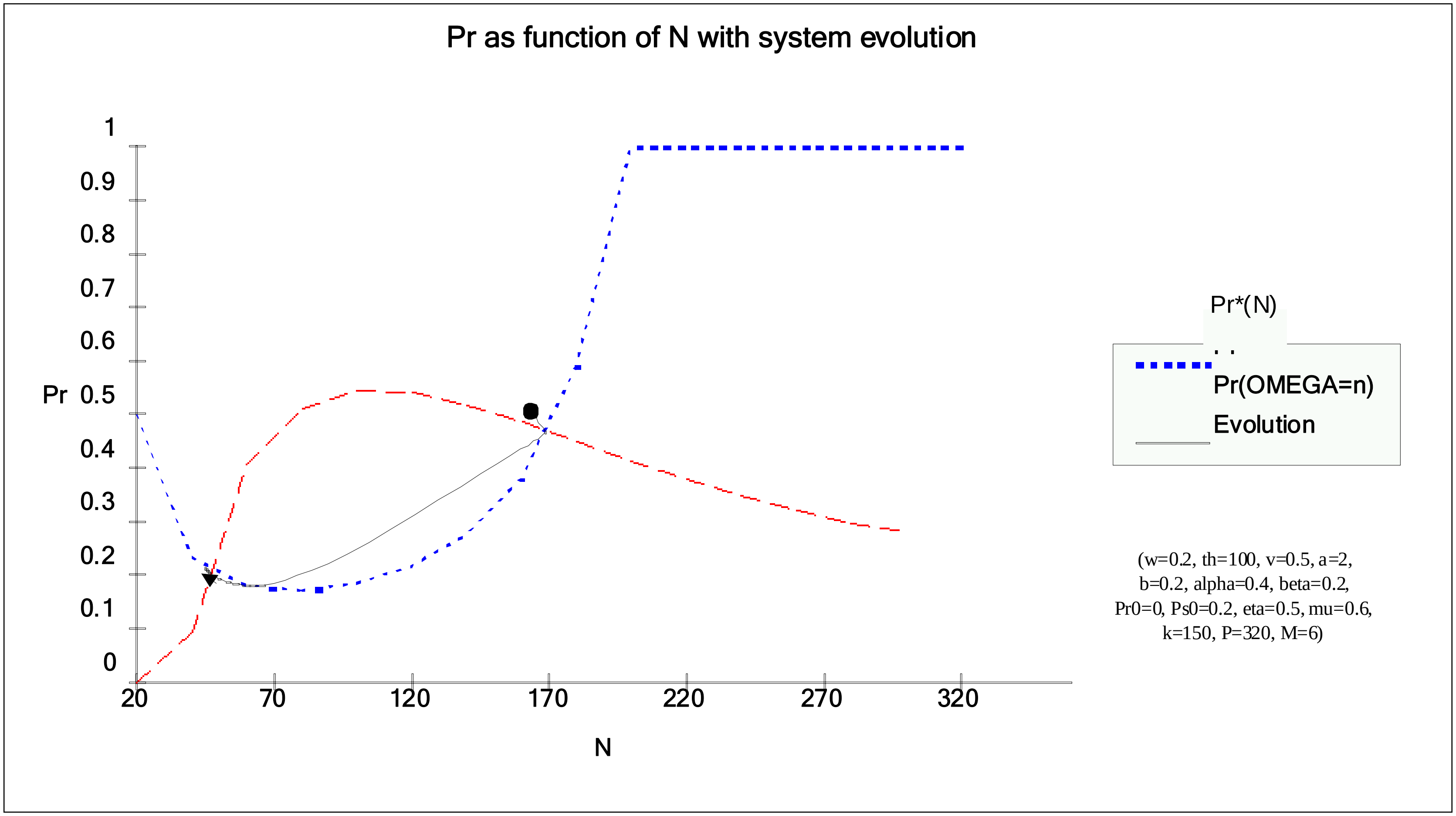

(dN/dt=0) surface over N, Pr
(N/P)*
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
N
20
60
100
140
180
220
260
300
Pr
0
0.5
1
0.9-1
0.8-0.9
0.7-0.8
0.6-0.7
0.5-0.6
0.4-0.5
0.3-0.4
0.2-0.3
0.1-0.2
0-0.1
(w=0.05, th=100, v=1,
a=2, b=0.2, alpha=0.4,
beta=0.2, Pr0=0,
Ps0=0.1, eta=0.5,
mu=0.6, k=150, P=320,
M=6)


Pr as function of N with system evolution
Pr
N
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20
70
120
170
220
270
320
(N'=0)-locus
Pr*(N)
Pr(OMEGA=n)
Evolution
(w=0.05, th=100, v=1, a=2, b=0.2, alpha=0.4, beta=0.2, Pr0=0, Ps0=0.1, eta=0.5, mu=0.6, k=150, P=320, M=6)


Pr as function of N with system evolution
Pr
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20
70
120
170
220
270
320
N
Pr*(N)
Pr(OMEGA=n)
Evolution
(w=0.05, th=100, v=1, a=2,
b=0.2, alpha=0.4, beta=0.2,
Pr0=0, Ps0=0.28, eta=0.5,
mu=0.6, k=150, P=320, M=6)


Pr as function of N with system evolution
1
0.9
0.8
0.7
0.6
Pr 0.5
0.4
0.3
0.2
0.1
0
20
70
120
170
220
270
320
N
Pr*(N)
Pr(OMEGA=n)
Evolution
(w=0.05, th=100, v=1.4, a=2,
b=0.2, alpha=0.4, beta=0.2,
Pr0=0, Ps0=0.28, eta=0.5,
mu=0.6, k=150, P=320, M=6)


Pr as function of N with system evolution
1
0.9
0.8
0.7
0.6
Pr 0.5
0.4
0.3
0.2
0.1
0
20
70
120
170
220
270
320
N
Pr*(N)
Pr(OMEGA=n)
Evolution
(w=0.05, th=100, v=0.5, a=2,
b=0.2, alpha=0.4, beta=0.2,
Pr0=0, Ps0=0.2, eta=0.5, mu=0.6,
k=150, P=320, M=6)



Pr as function of N with system evolution
Pr
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20
70
120
170
220
270
320
N
Pr*(N)
Pr(OMEGA=n)
Evolution
(w=0.2, th=100, v=0.5, a=2,
b=0.2, alpha=0.4, beta=0.2,
Pr0=0, Ps0=0.2, eta=0.5, mu=0.6,
k=150, P=320, M=6)