Product Switching in a Model of Learning

Olga A. Timoshenko*
The George Washington University

November 15, 2012

Abstract

New exporters add and drop products with much greater frequency than old exporters. This paper rationalizes this behavior with a model of demand learning in which an exporter's profitability on the demand side is determined by a time-invariant firm-destination appeal index, and transient firm-destination-year preference shocks. New exporters must learn about their appeal indices in the presence of these shocks, and respond to fluctuations in demand by adding and dropping products more frequently than older exporters because they have less information about their attractiveness to consumers. Calibrated to match cross-section distribution of sales and scope, the model quantitatively accounts for the contribution of the extensive margins to aggregate Brazilian exports. The model predicts that in response to a decline in trade costs, existing exporters add new products and new exporters enter a destination. Counterfactual implies that the contribution of product adding to export growth resulting from trade liberalization is three times larger than the contribution of exporter entry.

Keywords: Learning, product switching, firm dynamics, exports, Brazil
JEL Classification: F12, F14, L11, L25

1 Introduction

Product-switching is widespread and economically important, and yet relatively little is known about how firms make product-switching decisions and how these decisions are affected by the firms' experiences. This paper rationalizes product-switching behavior with

*I am extremely grateful to Costas Arkolakis, Penny Goldberg, and Peter Schott for their substantial guidance and support. Lorenzo Caliendo, Rafael Dix-Carneiro, Illenin Kondo, and Logan Lewis provided helpful comments and suggestions, as did seminar participants at Yale University and the George Washington University. I thank Costas Arkolakis for providing me with the data for this research. Financial support from Yale University and the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged. This work was supported in part by the facilities and staff of the Yale University Faculty of Arts and Sciences High Performance Computing Center, and by the National Science Foundation under grant #CNS 08-21132 that partially funded acquisition of the facilities. First version of the paper: September 2011.

a model of demand learning, and demonstrates that this model provides a quantitatively appealing framework for policy analysis.

Product adding and dropping (product switching) is a substantial channel of microeconomic and macroeconomic adjustment in the economy, as was first argued in Bernard, Redding, and Schott (2010), and a large proportion of firms is engaged in product switching.[1] These firms adjust by adding and dropping products in response to changes in competition, regulations, trade costs and exchange rates. Such changes have been shown to translate into substantial productivity adjustments at firm level as well as at aggregate level, and to account extensively for the changes in traded volumes.[2]

Using an extensive data set on Brazilian exporters, I find that new exporters engage in substantially more product adding and dropping than old ones. The proportion of recently introduced products, and the share of exports arising from those new products, decline as the duration of export experience rises. An experienced exporter with a given value of export sales will derive a smaller share of its sales from new products and will add fewer products than a younger exporter with the same value of export sales. As I argue in this paper, the observed age dependence of product switching naturally arises from a model of demand learning.

In the model, a firm's profitability in a product in a given year depends on firm-product, firm-destination, and firm characteristics. On the supply side, a particular product's productivity is a function of firm "ability" and firm-product efficiency. On the demand side, a product's attractiveness in a particular export destination is determined by the firm-destination "appeal" as well as by transient firm-destination-year preference shocks. While productivity parameters are known to the firm upon entry, new firms must learn about the appeal of their brand in the presence of these shocks. A key insight of the model is that this learning leads to differential product-switching behavior among new and old firms: new firms respond to fluctuations in demand by adding and dropping products more frequently than older firms because they have less information about their attractiveness to consumers.

The model of demand learning is quantitatively consistent with the margins of export sales. Calibrated to match cross-sectional statistics on the export-sales and export-scope distributions of Brazilian exporters, the model predicts that exporter-turnover and product-switching margins are equally important in explaining aggregate Brazilian exports, as borne out in the data. Thus, a model of demand learning provides not only a solid rationalization for product-switching behavior, but also a quantitatively appealing framework for policy analysis, in particular for trade liberalizations.

The model yields striking quantitative implications regarding the role of product-switching and exporter-turnover margins in explaining the growth in exports resulting from a decline in trade costs. In response to a decline in trade costs, the model predicts that existing exporters will adjust by adding new products, and new exporters will react by entering a destination. In a counterfactual simulation of a bilateral trade liberalization, the calibrated model predicts that product adding contributes three times more than exporter entry does

[1]For example, see Bernard, Redding, and Schott (2010); Iacovone and Javorcik (2010). An exception to this literature is the work by Goldberg, Khandelwal, Pavcnik, and Topalova (2010b) where widespread product adding and infrequent product dropping among Indian firms were driven by heavy industrial regulations.

[2]Bernard, Redding, and Schott (2010, 2011); Goldberg, Khandelwal, Pavcnik, and Topalova (2010a,b); Gopinath and Neiman (2011)

to growth in export sales.

The choice to model learning on the demand side rather than the cost side, is motivated by the data on the evolution of a plant's physical productivity versus plant-level demand shock, analyzed by Foster, Haltiwanger, and Syverson (2008). On the one hand, the authors find that there is no substantial difference in physical productivity (measured as output per unit of input) across plants of different ages. In my model, a plant is equivalent to a firm. Thus, I assume that a firm's productivity level, referred to as the firm's "ability", stays constant throughout the firm's life-cycle. On the other hand, the authors find that the plant's demand shock is lower for younger plants than for the older ones. An increase in the demand component through the firm's life cycle suggests the possibility of learning occurring on the demand side.

Demand learning is not the only way to generate product switching among exporters, or among firms in general. Such behavior can be generated from a model with random productivity and demand shocks, as shown in the model of Bernard, Redding, and Schott (2010). In their model, product-specific demand shocks yield individual product adding and dropping, while productivity-specific shocks yield expansion or contraction of the range of products produced by a firm. A framework with random shocks, however, cannot deliver the age dependence of product switching conditional on size, which is the focus of this paper.[3]

This paper complements the line of research analyzing the role of a firm's age in its growth and expansion. The age dependence of the firm's growth rate conditional on size is a well documented fact (Evans, 1987), and this relationship can partially be accounted for by a decline in the frequency of product switching as firms age in the market, as suggested by the empirical finding of this paper. Arkolakis and Papageorgiou (2010) argue that a model with demand learning can predict the negative relation between the growth rate of a firm and its age conditional on size. I apply a similar concept of demand learning, but use it to analyze the age dependence of product-switching behavior.

While previous papers have focused on testing qualitative implications of an assumed learning mechanism (Albornoz, Pardo, Corcos, and Ornelas, 2011; Timoshenko, 2010), an important contribution of this paper lies in demonstrating that a model with demand learning performs well quantitatively. Some reduced-form quantitative evidence has been suggested in work by Ruhl and Willis (2008), in which the authors assume a deterministic learning process and demonstrate that such an assumption is helpful in predicting an increasing survival rate of exporters. However, in contrast to this earlier work, I incorporate an endogenous demand learning mechanism into a general equilibrium setup, and perform a quantitative evaluation of the learning model.

The paper will proceed as follows. In Section 2, I briefly present the data and document the patterns that motivate the theoretical analysis. In Section 3, I develop a model of demand learning and product choice, and study the model's implications for product-switching behavior. In Section 4, I perform a quantitative evaluation of the model. Section 5 presents the conclusions. All proofs are relegated to Appendix E.

[3] In the context of a model where a firm is associated with a single product, Arkolakis (2010) shows that age dependence of a firm's growth rate conditional on size cannot be generated from a model with random productivity evolution.

2 Empirical Trends

This section provides evidence on the importance of product-switching behavior in the context of Brazilian export data. The section also demonstrates how this behavior is affected by a firm's experience. First, I show that the contribution of product-switching margin is at least as large as the contribution of exporter-turnover margin in predicting the level and growth of Brazilian exports. Second, I show that, conditional on size, the share of added products and the share of export sales from added products decline with exporters' experience in a market.

2.1 Data

I use export sales data that come from customs declarations for merchandise exports by Brazilian exporters collected at SECEX (*Secretaria de Comercio Exterior*).[4] It is a four-dimensional panel data set for the period between 1990 and 2001, arranged by firm, product, destination and year. The data are reported in current USA dollars. Where appropriate, I convert exports into real terms (2001 USA dollars) using the annual USA consumer price index from the International Monetary Fund.

A product corresponds to a six-digit Harmonized Tariff System (HS) code.[5] I restrict the sample to include solely exports in manufacturing products, which results in a total of 4,637 products exported by Brazil between 1990 and 2001. Exports in manufacturing products comprise on average 91 percent of total Brazilian exports in a given year; on average 92 percent of exporters in a given year export at least one manufacturing product. Over the period 1990-2001, Brazilian manufacturing products have been exported to 240 destinations.

2.2 Extensive Margins of Brazilian Exports

The contribution of sales from new products by incumbent exporters is at least as large as the contribution of sales from new exporters in explaining the level of Brazilian exports, as can be seen from Table 1. In a given year, 4.4 percent of total Brazilian manufacturing exports arise from products introduced by incumbent exporters, that is, firms that exported in the previous and current year (third column of Table 1). The new-exporter margin amounts to a half of that quantity. Only 2.6 percent of total Brazilian manufacturing exports are accounted for by new entrants, that is, firms that began exporting in the current period.

A similar pattern emerges when we look at the level of exports to a particular destination (columns one and two of Table 1). For example, 4.5 percent of Brazilian manufacturing exports to the US, Brazil's top export destination, are generated by incumbent exporters introducing new products to the US. The new-exporter margin computed for exports to the US market is not as small as the margin computed for total Brazilian manufacturing exports, yet it remains below the new-product margin.

[4]For detailed description of the data set refer to Molinaz and Muendler (2009)

[5]The original data are reported at the eight-digit level, of which the first six digits correspond to the first six digits of the HS classification. To make the results comparable to other data sets, I aggregate the data to the six-digit HS level: six-digit HS codes are standardized across countries.

2.3 Margins of Brazilian Exports Growth

The role of product-switching margin versus exporter-turnover margin is even more pronounced in explaining the growth in Brazilian exports than it is in explaining the level of exports. Nearly 40 percent of the annual growth in Brazilian manufacturing exports arises from the product-switching margin, as can be seen from the third column of Table 2. The exporter-turnover margin, meanwhile, accounts for less than 15 percent of the annual growth in exports.

When we consider Brazilian exports to the individual destinations of Argentina and the US, the product-switching margin and the exporter-turnover margin are equally important in explaining the growth in export sales. Each margin accounts for 15 to 20 percent of the growth in annual export sales, as can be inferred from the first and second columns of Table 2.

2.4 Fraction of Sales from New Products

The sample mean of the intra-firm extensive margin is 20.0 percent, as shown in the third column of Table 3. An average surviving exporter derives 20.0 percent of its total export sales from newly added products. The magnitude of the margin is similar when computed for individual export destinations. For example, firms that export to the US derive 18.4 percent of exports to the US from the products they introduced into the US in a given year (second column of Table 3).

The magnitude of the intra-firm extensive margin varies among firms of different export ages. The export age is defined as the number of consecutive years a firm is observed exporting a positive amount of at least one good. While firms with two years of exporting derive 27.1 percent of sales from new goods, firms with 5 years of experience derive 14.3 percent of sales from new goods (third column in Table 4). A similar pattern emerges within individual destinations. Firms that have exported to the US for two years derive 24.6 percent of their US sales from products which are new to this destination, while firms that have exported for 5 years derive 12.7 percent of their sales from new products. Overall, the magnitude of the intra-firm extensive margin gradually declines with the firm's export age, suggesting that exporters are engaged in product switching less actively as they export for a longer period of time.

Results from the ordinary least squares (OLS) regression of the intra-firm extensive margin on export age and the logarithm of export sales confirm that the observed relation between the margin and the age is not a statistical artifact. One might explain the results in Table 4 by growth in sales of surviving firms. If a firm introduces on average the same number of products per year, sales from new products will account for less and less of total export sales as total firm's sales grow. In such a case the decline in the intra-firm extensive margin is not an indicator of less intense product switching. Thus, in order to claim that the intensity of product switching declines as export experience increases, it is important to control for the firm's total export sales. This is achieved in the above OLS regression by the inclusion of the logarithm of the firm's sales as a control variable. Controlling for the firm's total sales, the effect of export age on the intra-firm extensive margin remains negative and statistically significant (column (2) in Table 5). The result is preserved both at the aggregate (column (2)) and destination (column (4)) levels, confirming the conjecture that, conditional on sales, the longer a firm exports, the smaller is the fraction of export sales from new products.

2.5 Fraction of New Products

The fact that intensity of product switching declines with export experience can also be observed in the behavior of the share of new/added products, that is, the percentage of currently exported products a firm added between two consecutive periods. On average, close to one-third of products exported to the world by a Brazilian firm are newly added products (third column of Table 6).

The fraction of newly added products varies with the duration of a firm's export experience. In a firm that became an exporter a year ago, 36.1 percent of current products are newly added, while in a firm with five years of export experience, 26.3 percent of current product are new (third column in Table 7). The statistic seems to decline as export age rises for both young and old exporters, while it appears to be constant in the mid-range of export age. The statistic shows a much stronger declining pattern in age when only a subsample of those exporters that add a positive number of products in a given year is considered (sixth column in Table 7). A similar pattern exists within individual destinations.

Results from OLS regression of the fraction of new products on the export age and the logarithm of export sales (see Table 8) confirm that the observed relation between the fraction and the age is not a statistical artifact.

In the next section I will describe a model of product-switching behavior that is consistent with the observed patterns.

3 The Model

This section describes a model of learning with product switching, and discusses the model's implications for firms' quantity, scope, and market-participation decisions. The model is built on two key assumptions: Jovanovic's (1982) assumption of demand learning, and the assumption of product ordering in Arkolakis and Muendler (2010). The two assumptions are incorporated into Melitz's (2003) monopolistic competition framework with heterogeneous firms.

The world consists of $N+1$ countries. Time is discrete, and is denoted by t. Firms decide whether to enter or exit a particular market, how many products to sell in that market (scope decision), and in what quantities to sell those products (scale decision). A firm's products are faced with a demand shock in a given market, and the firm must make quantity and scope decisions before observing the shock. After the decisions are made, prices adjust to clear the markets and the firm infers its demand shock from the market clearing price. The firm makes quantity and scope decisions based on expectations about current demand shock, and these expectations are formed based on the mean and number of previously observed demand shocks. Products are imperfect substitutes.

3.1 Preferences and Demand

Each country j is populated with a measure L_j of infinitely-lived identical consumers. The preferences of a representative consumer in country j are given by the Dixit and Stiglitz (1977) constant elasticity of substitution (CES) utility function over consumption of the composite good C_{jt}

$$U_j = E\sum_{t=0}^{+\infty} \beta^t \ln\left(C_{jt}\right),$$

where β is the discount factor. Consumption of the composite good C_{jt} takes the CES form

$$C_{jt} = \left(\sum_{i=1}^{N+1} \int_{\Omega_{ijt}} \left(e^{a_{jt}(\omega)}\right)^{\frac{1}{\sigma}} c_{jt}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}},$$

where Ω_{ijt} is the mass of products (both imported and domestic) available for consumption in country j imported from country i in period t; $c_{jt}(\omega)$ is the consumption of a product $\omega \in \Omega_{ijt}$ in country j. σ is the elasticity of substitution between products. $a_{jt}(\omega)$ is a demand shock for product ω in country j.

Every product ω is offered in $G_{ijt}(\omega)$ differentiated varieties, thus $c_{jt}(\omega)$ is a product composite index which also takes the CES form

$$c_{jt}(\omega) = \left(\sum_{g=1}^{G_{ijt}(\omega)} c_{jgt}(\omega)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}.$$

g indexes varieties within product ω; $c_{jgt}(\omega)$ is the consumption of variety g of product ω in country j.

Aggregate price index P_{jt} associated with the consumption of the composite good C_{jt} is given by

$$P_{jt} = \left(\sum_{i=1}^{N} \int_{\Omega_{ijt}} e^{a_{jt}(\omega)} \sum_{g=1}^{G_{ijt}(\omega)} p_{jgt}(\omega)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}},$$

where $p_{jgt}(\omega)$ is the price of variety g of product ω in country j.

A consumer is endowed with one unit of labor that he inelastically supplies to the market. A consumer receives wage w_{jt} per unit of labor and owns a share of domestic firms. Given prices $p_{jgt}(\omega)$ and income, the level of $c_{jgt}(\omega)$ is chosen to minimize the cost of acquiring C_{jt} yielding demand equation

$$q_{jgt}(\omega) = e^{a_{jt}(\omega)} \frac{p_{jgt}(\omega)^{-\sigma}}{P_{jt}^{1-\sigma}} Y_{jt},$$

where Y_{jt} is the aggregate spending level defined as the sum of total labor income, $w_{jt}L_j$, and profits of domestic firms, Π_{jt}: $Y_{jt} = w_{jt}L_j + \Pi_{jt}$.

3.2 Production

At any point in time t there exists a continuum of firms. Upon entry, a firm is associated with a brand name, product ω (hereafter called brand), which becomes globally unique. Within its own brand, a firm can produce differentiated varieties of its product (hereafter called just products).

Firms differ in their ability level φ to produce products within their brand. All else being equal, a firm with a greater ability is able to produce each of its products at a lower cost than a less able firm.

There are two additional components that influence the profitability of a firm's products. First, a component that is product specific but the same across destinations and time, a product-specific productivity φ_g. Second, a component common across products within a firm but different across destinations and time, a destination-specific demand shock a_{jt}.

The destination demand shock is given by the sum of two components: time-invariant brand appeal index θ_j, and an inter-temporal preference shock ϵ_{jt}. The brand appeal index in a given destination is drawn from a normal distribution with mean $\bar{\theta}_j$ and variance σ_θ^2. The inter-temporal preference shock is drawn from a normal distribution with zero mean and variance σ_ϵ^2. The shocks are independently and identically distributed over time.

The brand appeal index is not known to the firm: before entering a market, a firm does not know how well perceived its product is going to be in that market. As the firm continues to supply the market, the appeal index is subject to inter-temporal preference shocks. Thus, a firm never observes its own appeal index, but must learn about it by observing demand shocks.

One of the simplifying assumptions I introduce in my analysis is the independence of the draws of the firm's appeal index across destinations. An alternative formulation that allows draws of the appeal index to be correlated across destinations is proposed in Albornoz, Pardo, Corcos, and Ornelas (2011) and also in Defevery, Heidz, and Larchx (2010) and Nguyen (2011). This modeling approach is essential for the analysis of exporters' sequential market-entry patterns, which is pursued in the aforementioned papers. Since the goal of this paper is to characterize product switching in a given destination conditional on entry, the assumption of correlated draws is not crucial to my analysis.

The level of current demand shock a_{jt} is not known to the firm at the time of making production decisions. Thus, the firm bases its decisions on its belief about the appeal index and, subsequently, about the demand shock.

Before entry into a market, a firm does not have any information regarding the appeal index and thus its prior belief about the appeal index is given by the distribution from which θ_j is initially drawn.

Upon observing n demand shocks that yield mean $\bar{a}_j$, the posterior distribution regarding the appeal index is given by the normal distribution with mean μ_{nj} and variance υ_n^2 (DeGroot, 2004) where

$$\begin{aligned} \mu_{nj} &= \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + n\sigma_\theta^2}\bar{\theta}_j + \frac{n\sigma_\theta^2}{\sigma_\epsilon^2 + n\sigma_\theta^2}\bar{a}_j, \\ \upsilon_n^2 &= \frac{\sigma_\epsilon^2\sigma_\theta^2}{\sigma_\epsilon^2 + n\sigma_\theta^2}. \end{aligned}$$

Firms from country i face fixed costs of supplying market j, $F_{ij}(G)$, which reflect advertising and marketing costs, costs associated with complying with regulatory standards, and market-research costs. These fixed costs increase with the number of products produced G (scope), yet there might be either economies or diseconomies of scope. Bernard, Redding, and Schott (2010) assume the fixed costs to be linear in scope, yet Arkolakis and Muendler (2010) estimate the incremental fixed costs to be increasing in scope. I am going to allow a fixed-cost structure that can potentially encompass all possible cases. Let $F_{ij}(G)$ be given by

$$F_{ij}(G) = f_{ij} \sum_{g=1}^{G} g^{\gamma}.$$

Since the incremental fixed costs are not constant, the decisions to export individual products are not independent across each other (as is true in the model of Bernard, Redding, and Schott (2010), for example). Thus, a firm will order products from the most to the least productive and will continue to add products until the variable profit from the marginal product is at least as large as the incremental fixed cost associated with it.

It is convenient to relabel products a firm can produce according to their efficiency in relation to each other. Let the subscript g on the product's productivity φ_g refer to the productivity rank of a product within the firm. $g = 1$ refers to a product a firm produces with the highest efficiency. As we move to lower ranks, the efficiency declines.

Further, the firm's ability φ influences φ_g to the extent that φ_g is increasing in φ, reflecting the fact that abler firms can produce all products with greater efficiency. Thus, productivities φ_g have to satisfy two properties: be increasing in φ, and be decreasing in g. Following Arkolakis and Muendler (2010), I am going to assume that φ_g takes functional form $\varphi_g = \frac{\varphi}{g^{\alpha}}$, where $\alpha > 0$.

All products are subject to the iceberg transportation cost τ_{ij}. τ_{ij} units of a good must be shipped from country i to j in order for one unit to arrive at destination j. For $i \neq j$ $\tau_{ij} > 1$, reflecting the fact that is it costly to ship products abroad. I further assume that in the home country firms do not face additional transportation costs beyond variable and fixed costs of production, $\tau_{ii} = 1$.

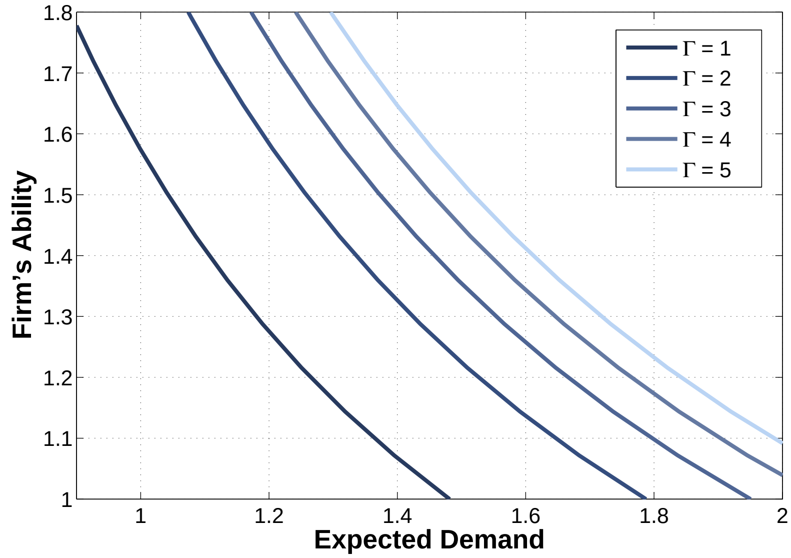
Conditional on entry into a market, firms choose the quantity of every good and the number of goods supplied to that market to maximize per-period expected profit given posterior belief about the appeal index. The resulting optimal quantity (scale) of product g supplied to market j from i is given by

$$q_{ijgt}(\varphi, \bar{a}_j, n) = \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma} \left(\frac{\varphi}{g^{\alpha}}\right)^{\sigma} b_j^{\sigma} \frac{P_{jt}^{\sigma-1} Y_{jt}}{(\tau_{ij} w_{it})^{\sigma}}, \tag{1}$$

where b_j^{σ} captures the effect of product appeal learning (demand learning) on optimal quantity. The complete characterization of the maximization problem is described in Appendix A.

b_j is defined as $E_{a_{jt}|\bar{a}_j,n}\left[e^{\frac{a_{jt}}{\sigma}}\right]$. It is the expected value of the exponent of current demand shock (normalized by the elasticity of substitution for convenience). The expectation is taken with respect to the current demand shock a_{jt} given the posterior distribution regarding the

Figure 1: **Scope Indifference Curves**



appeal index, which is completely described by $\bar{a}_j$ and n. I will refer to b_j^σ as the expected demand level.

The optimal scale decision yields expected variable profit from a product, which increases as the firm's ability and expected demand level rise, and decreases as the product's rank g rises:

$$\pi_{ijgt}(\varphi, \bar{a}_j, n) = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma}\left(\frac{\varphi}{g^\alpha}\right)^{\sigma-1} b_j^\sigma \frac{P_{jt}^{\sigma-1}Y_{jt}}{(\tau_{ij}w_{it})^{\sigma-1}}.$$

In determining the optimal number of products, a firm compares the expected variable profit for a given product to the incremental fixed costs and keeps adding products as long as the profit is greater than or equal to the costs.

Expected variable profit from a given product declines as the product's rank rises, while *ex ante* I do not impose any assumptions on the behavior of the incremental fixed costs. To ensure a well defined solution to the scope problem, the incremental fixed cost must decline no faster than the variable profit does, as the product's rank rises. This yields the following constraint on the parameters of the cost function: $\gamma > -\alpha(\sigma-1)$.

The optimal scope decision is then characterized by a set of scope indifference curves $\Gamma(\varphi^{\sigma-1}, b_j^\sigma) = g$ for $g \geq 1$ implicitly determined by equation (2) below and schematically depicted in Figure 1.

$$\varphi^{\sigma-1} b_j^\sigma = \frac{\sigma^\sigma}{(\sigma-1)^{\sigma-1}} f_{ij} g^{\gamma+\alpha(\sigma-1)} \frac{(\tau_{ij}w_{it})^{\sigma-1}}{P_{jt}^{\sigma-1}Y_{jt}}. \tag{2}$$

A firm with $\varphi^{\sigma-1}, b_j^\sigma$ such that $\Gamma(\varphi^{\sigma-1}, b_j^\sigma) = g$, makes zero net profit on the marginal product g, and thus sells g products. A firm with $\varphi^{\sigma-1}, b_j^\sigma$ such that $g < \Gamma(\varphi^{\sigma-1}, b_j^\sigma) < g+1$ makes positive net profit on the gth product, and considers adding another $g+1$st product.

Yet $g+1$st product cannot generate enough variable profit to cover the incremental fixed cost. Firms in this range thus produce g products.

Notice that a firm's ability φ stays constant throughout the firm's life cycle, while expected demand $b_j^\sigma(\bar{a}_j, n)$ adjusts as the firm observes new demand shocks.

The range of values $\varphi^{\sigma-1}$ and b_j^σ such that $\Gamma(\varphi^{\sigma-1}, b_j^\sigma) < 1$ is of special interest. Firms in the range of values below the scope indifference curve $\Gamma(\varphi^{\sigma-1}, b_j^\sigma) = 1$ make negative net profit on the first product they can potentially produce. In the static environment, such firms would immediately exit the market. In the dynamic learning environment considered in this paper, it would be optimal for some of these firms to incur one-period losses in exchange for observing current demand shock which has some information value. The following section describes the value of the firm, and the firm's optimal exit and entry decisions.

3.3 Firm Exit and Entry

Denote by $V_{ijt}(\varphi, \bar{a}_j, n)$ the continuation value of a firm in country i of being an exporter to market j.

An incumbent firm enters period t with the knowledge of its own ability φ, mean of the previously observed demand shocks $\bar{a}_j$, and the number of the observed shocks n. The firm must decide whether to participate in a given market in the current period, or exit. If the firm decides to participate in market j, it receives expected profit $\Pi_{ijt}(\varphi, \bar{a}_j, n)$ and expected discounted continuation value of being an exporter $\beta(1-\delta)E_{\bar{a'}_j|\bar{a}_j,n}V_{ijt}(\varphi, \bar{a}'_j, n+1)$, where δ is an exogenous probability of a shock that forces a firm to exit, and β is the rate at which a firm discounts future profits. If the firm decides to exit it receives the option value of non-exporting.

In the context of this model the option value of non-exporting is zero. A firm must export at least one product in a given period in order to observe current demand shock, and be able to update its own posterior belief about the appeal index. Once the firm exits market j, it will no longer receive any new information regarding its appeal index in market j. Thus, if it is optimal for a firm to exit in a given period, it will be optimal to stay out of the market for all subsequent periods, yielding zero value of exit.

The optimal market-participation decision for incumbent firms is thus a policy function associated with the following Bellman equation

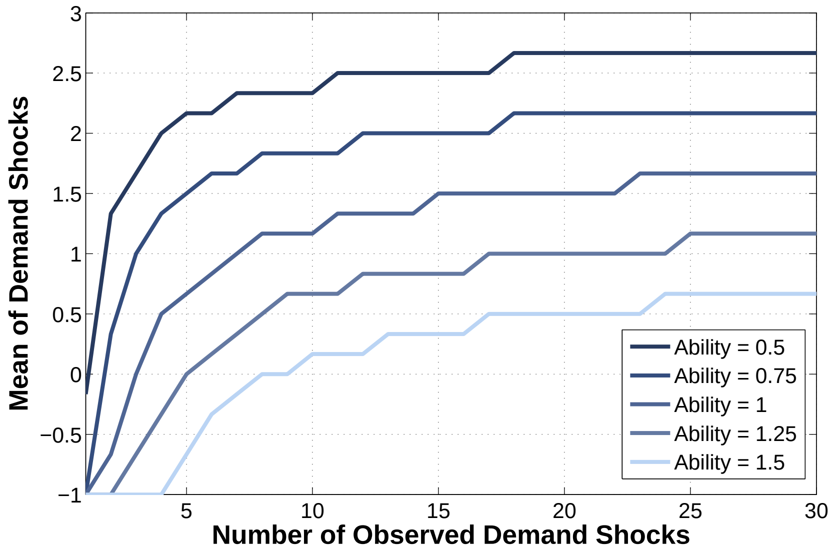
$$V_{ijt}(\varphi, \bar{a}_j, n) = \max\{\Pi_{ijt}(\varphi, \bar{a}_j, n) + \beta(1-\delta)E_{\bar{a}'_j|\bar{a}_j,n}V_{ijt+1}(\varphi, \bar{a}'_j, n+1), 0\}. \tag{3}$$

Proposition 1 *The optimal market-participation decision is given by a set of market-participation thresholds $\bar{a}^*_{ij}(\varphi, n)$ such that a firm from destination i continues to export to destination j when $\bar{a}_j \geq \bar{a}^*_{ij}(\varphi, n)$ and exits export market when $\bar{a}_j < \bar{a}^*_{ij}(\varphi, n)$. Policy function $\bar{a}^*_{ij}(\varphi, n)$ satisfies the following properties: (a) $\frac{\partial \bar{a}^*_{ij}(\varphi,n)}{\partial \varphi} < 0$, (b) $\bar{a}^*_{ij}(\varphi, n+1) > \bar{a}^*_{ij}(\varphi, n)$, and (c) $\lim_{n\to+\infty} \bar{a}^*_{ij}(\varphi, n) = \bar{a}^*_{ij}(\varphi)$.*

All proofs are relegated to Appendix E. The behavior of the market-participation thresholds, drawn in Figure 2, reflects the role of information and learning in the economy.

For a given ability level φ, the short-run threshold $\bar{a}^*_{ij}(\varphi, n)$, that is, when n is small, approaches the long-run value $\bar{a}^*_{ij}(\varphi, n)$, that is, when n approaches infinity, from below,

Figure 2: **Market Participation Thresholds**



thereby giving the firms time to learn about their true appeal index. I explain the exact meaning of this idea in the discussion below.

Consider the market-participation decision of an exporter in the limit, that is, when $n \to +\infty$ (in the long run). The behavior in the long run can be viewed as the behavior of firms should they have had perfect information about their own appeal index. As n approaches $+\infty$, the mean of the observed demand signals converges towards the true appeal index θ_j:

$$\lim_{n\to+\infty} \frac{\sum_{k=1}^{n} a_{j(t+k)}}{n} = \lim_{n\to+\infty} \frac{\sum_{k=1}^{n} \theta_j + \epsilon_{j(t+k)}}{n} = \theta_j + \lim_{n\to+\infty} \frac{\sum_{k=1}^{n} \epsilon_{j(t+k)}}{n} = \theta_j + E(\epsilon_{jt}) = \theta_j \quad (4)$$

Thus, in the long run, the mean of the demand signal can be viewed as the true appeal index of the firm, and the threshold level $\bar{a}^*_{ij}(\varphi)$ can be viewed as the exit threshold for the appeal index. Thus, had the firm had perfect information regarding the appeal index, it would have exited the export market immediately if $\theta_j < \bar{a}^*_{ij}(\varphi)$.

In the short run, the estimate of the appeal index, that is, the mean of the observed demand shocks, might be above or below the true value of the appeal index depending on the realization of random preference shocks. If the short-run value of the market-participation threshold stayed at the long-run level, a slight negative preference shock would force many firms out of the market immediately, while from the profitability perspective it would have been optimal for those firms to export. To accommodate the effect of occasional negative preference shocks on the estimate of the appeal index, the short-run value of the market exit threshold for a firm with productivity φ must be below its long-run equivalent. As a firm receives more and more demand signals, its estimate of its own appeal index becomes more accurate. Thus, a firm becomes less sensitive to occasional negative preference shocks. As a result the threshold level gradually increases to the long-run value.

Having described the continuation value of being an exporter and optimal market participation by incumbents, I now turn to a description of the entry of new firms.

Following Chaney (2008), I assume that in every country j there exists an exogenous mass of prospective entrants J. Upon entry every firm draws its ability level φ from a Pareto distribution with the scale parameter φ_{min} and the shape parameter ξ.[6] After observing its ability level a firm in market i decides in which market j to participate by comparing the expected value of entry $V_{ij}^{E}(\varphi)$ to the cost of entry. Since I do not assume the presence of sunk market-entry costs, the entry cost is zero. Furthermore, since there is no sunk entry cost, decisions to enter destinations operate independently of each other.

A firm enters a particular market whenever the expected value of entry is greater than zero. The expected value of entry into a market is given by the sum of the initial expected profit in that market and the expected continuation value of being an exporter

$$V_{ij}^{E}(\varphi) = \Pi_{ijt}(\varphi, 0, 0) + \beta(1-\delta)E_{\bar{a}_j'|0,0}V_{ijt}(\varphi, \bar{a}_j', 1).$$

Since prior to entry a firm has not observed any demand shocks, the value of the mean demand shocks is normalized to zero, and the number of the observed demand shocks is zero. The expectation of a firm's initial profits is computed based on the firm's prior belief about the demand parameter given by the distribution from which the appeal index is drawn. Notice that since $\Pi_{ijt}(\varphi, 0, 0)$ is increasing in φ, so is the expected value of entry. Thus productivity entry threshold φ_{ij}^{*} is determined by equating the expected value of entry to zero:

$$V_{ij}^{E}(\varphi_{ij}^{*}) = 0.$$

A firm with an ability level above φ_{ij}^{*} will enter destination j and begin serving that market. A firm with an ability level below φ_{ij}^{*} for all destinations j will exit the economy forever.

3.4 Determination of Equilibrium

I am going to solve for the stationary general equilibrium in the case of symmetric countries. Thus, I assume that all countries are of equal size $L_j = L$; they have the same number of entrants $J_j = J$; they are separated by equal trade cost $\tau_{ij} = \tau, > 1$ for $i \neq j$ and $\tau_{ii} = 1$. Firms in every country face the same fixed cost structure $f_{ij} = f$. The fundamental distribution of the appeal index is also the same across countries, $\bar{\theta}_j = \bar{\theta}$. Wage w is normalized to 1.

For a given set of parameters of the model, a symmetric stationary equilibrium is given by ability entry thresholds into the domestic and export markets, φ_d^{*} and φ_x^{*}, mean demand-shock exit thresholds from the domestic and export markets, $\bar{a}_d^{*}(\varphi, n)$ and $\bar{a}_x^{*}(\varphi, n)$, scope indifference curves in the domestic and export markets, $\Gamma_d(\varphi, b^{\sigma})$ and $\Gamma_x(\varphi, b^{\sigma})$, aggregate price level and expenditure level P and Y, masses of domestic and exporting firms, M_d and M_x, and a measure function in the domestic and export markets, $m_d(\varphi, \bar{a}, n, a)$ and $m_x(\varphi, \bar{a}, n, a)$. In what follows I describe how all equilibrium values are determined.

[6] For simplicity I assume that parameters of Pareto distribution are the same across countries. An alternative approach is taken by Eaton, Kortum, and Kramarz (2011), where differences in scale parameters across countries reflect differences in technological advancement among them. While such an assumption is more realistic, it is not crucial for the analysis pursued in this paper.

For a given value of P and Y all equilibrium thresholds and scope indifference curves are given by the solution to the firm's maximization problem. Given the equilibrium thresholds and firms' beliefs about the distribution of demand shocks, Proposition 2 below describes the equilibrium measure function of firms, $m_i(\varphi, \bar{a}, n, a)$ for $i \in \{d, x\}$.

Proposition 2 *The equilibrium measure function of firms $m_i(\varphi, \bar{a}, n, a)$, $i \in \{d, x\}$, is given by*

$$m_i(\varphi, \bar{a}, n, a) = \frac{1}{\sqrt{\upsilon_n^2 + \sigma_\epsilon^2}} \phi \left(\frac{a - \mu_n}{\sqrt{\upsilon_n^2 + \sigma_\epsilon^2}} \right) m_i(\varphi, \bar{a}, n),$$

where $m_i(\varphi, \bar{a}, n)$ is such that

$$m_i(\varphi, 0, 0) = J \left(\frac{\varphi_{min}}{\varphi_i^*} \right)^\xi \frac{\xi (\varphi_i^*)^\xi}{\varphi^{\xi+1}},$$

and

$$m_i(\varphi, \bar{a}, n+1) = \int_{\bar{a}_i^*(\varphi, n)}^{+\infty} m_i(\varphi, \bar{a}', n) \frac{1}{\sqrt{\upsilon_n^2 + \sigma_\epsilon^2}} \phi \left(\frac{\bar{a}(n+1) - \bar{a}' n - \mu_n}{\sqrt{\upsilon_n^2 + \sigma_\epsilon^2}} \right) d\bar{a}'.$$

Using the equilibrium measure function, the equilibrium mass of firms can be written as (see Appendix B)

$$M_i(\varphi_i^*) = J \left(\frac{\varphi_{min}}{\varphi_i^*} \right)^\xi \int_{\varphi_i^*}^{+\infty} \frac{\xi (\varphi_i^*)^\xi}{\varphi^{\xi+1}} H_i(\varphi) d\varphi, \tag{5}$$

where

$$H_i(\varphi) = \sum_{n=0}^{+\infty} \int_{\bar{a}_i^*(\varphi, n)}^{+\infty} \ldots \int_{\bar{a}_i^*(\varphi, 1)}^{+\infty} p(\bar{a}|\bar{a}_{n-1}) p(\bar{a}_{n-1}|\bar{a}_{n-2}) p(\bar{a}_1) d\bar{a}_1 \ldots d\bar{a}_{n-1} d\bar{a}$$

with $p(\bar{a}_k|\bar{a}_{k-1})$ being transitional densities of the form $p(\bar{a}_k|\bar{a}_{k-1}) = \frac{1}{\sqrt{\upsilon_{k-1}^2 + \sigma_\epsilon^2}} \phi \left(\frac{\bar{a}_k k - \bar{a}_{k-1}(k-1) - \mu_{k-1}}{\sqrt{\upsilon_{k-1}^2 + \sigma_\epsilon^2}} \right)$. Since normal densities take positive values, $H_i(\varphi)$ is positive for all φ.

From equation (5) observe that an increase in φ_i^* increases the lower bound of integration. Since the integrand is a positive function, the value of the integral declines. Thus, $M_i(\varphi_i^*)$ declines in φ_i^*.

Define by M the total mass of varieties available in the economy. M is given by

$$M(\varphi_d^*, \varphi_x^*) = M_d(\varphi_d^*) + N M_x(\varphi_x^*). \tag{6}$$

In solving for equilibrium values of P, Y, M_d, and M_x I will use results in Proposition 3 below.

Proposition 3 *There exist $u_d^* = \varphi_d^* P Y^{\frac{1}{\sigma-1}}$ and $u_x^* = \frac{\varphi_x^* P Y^{\frac{1}{\sigma-1}}}{\tau}$ such that*

(a) $u_d^* = u_x^* = u^*$;

(b) u^* is completely determined by the exogenous parameters of the model;

(c) Equilibrium mean profit level of firms is equal across destinations and is completely determined by u^*;

(d) Equilibrium mean revenue level of firms is equal across destinations and is completely determined by u^*.

Proposition 3 states that equilibrium values of P, Y, φ_d^*, and φ_x^* can be normalized in such a way as to yield a single variable u^*. This variable is completely determined by the exogenous parameters of the model, and is useful in characterizing the mean profit and revenue level of firms in a destination as a function of exogenous parameters only.

Notice that part (a) of Proposition 3 implies $\varphi_x^* = \tau\varphi_d^*$. Thus, the equilibrium mass of varieties in equation (6) can be written as a function of a single threshold φ_d^*:

$$M(\varphi_d^*) = M_d(\varphi_d^*) + NM_x(\tau\varphi_d^*). \tag{7}$$

Using results from parts (c) and (d) of Proposition 3, and the rational expectations of firms about the aggregate price level, the goods market clearing condition can be written as (see Appendix C)

$$L + M\tilde{\pi} = M\tilde{r}, \tag{8}$$

where $\tilde{\pi}$ is the mean profit level across firms in a given destination, $\tilde{\pi} = E_{m_d}(\pi_d(\varphi, \bar{a}, n, a)) = E_{m_x}(\pi_x(\varphi, \bar{a}, n, a))$. $\tilde{r}$ is the mean revenue level across firms in a given destination, $\tilde{r} = E_{m_d}(r_d(\varphi, \bar{a}, n, a)) = E_{m_x}(r_x(\varphi, \bar{a}, n, a))$.

Equation (8) determines equilibrium mass of varieties as a function of exogenous parameters only

$$M = \frac{L}{\tilde{r} - \tilde{\pi}}.$$

The equilibrium threshold φ_d^* is then a solution to $M = M_d(\varphi_d^*) + NM_x(\tau\varphi_d^*)$.

Given φ_d^*, $\varphi_x^* = \tau\varphi_d^*$, the mass of domestic and exporting firms is determined by equation (5), $Y = L + M\tilde{\pi}$; $P = \frac{u^*}{\varphi_d^* Y^{\frac{1}{\sigma-1}}}$. This completes the characterization of equilibrium.

3.5 Properties of the Solution

Here, I will demonstrate the properties of the optimal solution for the stationary symmetric equilibrium described in the previous section.

In demonstrating how learning affects the optimal scale and scope decisions of firms, it is instructive to compute how scale will respond to an additional demand-shock signal.

Proposition 4 *In the steady state there exists a threshold level of mean demand shock $\hat{a}$ given by $\bar{\theta} + \frac{\sigma_\theta^2}{2\sigma}$, such that scale adjustment in response to an additional demand shock, defined as $\frac{q_g(\varphi,\bar{a},n+1)}{q_g(\varphi,\bar{a},n)}$, is greater than 1 if $\bar{a} > \hat{a}$, $\frac{q_g(\varphi,\bar{a},n+1)}{q_g(\varphi,\bar{a},n)} < 1$ if $\bar{a} < \hat{a}$, and $\frac{q_g(\varphi,\bar{a},n+1)}{q_g(\varphi,\bar{a},n)} = 1$ if $\bar{a} = \hat{a}$.*

Proposition 4 states that firms with a sufficiently high realized value of the mean demand shock are prompted to expand the scale of products in response to an additional demand shock, while firms with a sufficiently low realized value of the mean demand shocks are prompted to contract the scale of products in response to an additional shock. The intuition for the obtained result is as follows.

Upon entry, a firm's expectations about its own appeal index are given by the mean $\bar{\theta}$ of the distribution, from which the parameter is drawn. After some demand shocks have been observed, a firm learns whether its true appeal index is higher or lower than the baseline value $\bar{\theta}$. Suppose the realized mean of the demand shock $\bar{a}$ is higher than the value originally believed ($\bar{a} > \hat{a}$, to be exact, as stated in Proposition 4). Conditional on ability φ and $\bar{a}$, an additional demand shock reassures the firm that its appeal index is higher than originally thought (through the channel of reducing the variance of the posterior belief υ_n^2, which declines in n). As a result, the firm expands the scale of products produced. However, if the realized mean of the demand shock $\bar{a}$ is lower than the originally-believed value, the firm adjusts to the new information by contracting the scale of every product.

The results of Proposition 4 are helpful in understanding the mechanism by which demand learning induces firms to add and drop products. As the scale of a given product expands or contracts in response to an additional demand shock, so does the variable profit generated by that product. Since the product's incremental fixed cost remains unchanged, changes in the product's profitability translate into changes in the range of products sold by a firm.

Consider a firm with $\varphi^{\sigma-1}$ and b^σ such that $g < \Gamma(\varphi^{\sigma-1}, b^\sigma) < g+1$. As discussed before, such a firm will produce g products and will generate a positive net profit from gth product. The firm does not yet sell $g+1$st product since the variable profit is not sufficient to cover the incremental fixed costs. As the firm receives an additional demand signal, the expected demand adjusts appropriately. On the one hand, for the firms with $\bar{a} > \bar{\theta} + \frac{\sigma_\theta^2}{2\sigma}$, expected demand increases in n conditional on $\bar{a}$. Thus, an additional demand signal will increase the profitability of the marginal $g+1$st product without affecting its marginal variable cost. When the increase in b^σ is sufficiently large, the firm will add an additional product. On the other hand, for firms with $\bar{a} < \bar{\theta} + \frac{\sigma_\theta^2}{2\sigma}$, expected demand declines in n conditional on $\bar{a}$. Thus, an additional demand signal will reduce the profitability of the marginal product g without affecting its marginal variable cost. When the decline in b^σ is sufficiently large, the firm will drop its marginal product g.

Proposition 4 describes how the process of learning affects a firm's decisions to adjust the scale of products produced: as firms learn, they either expand or contract the scale of their products. Proposition 5 below states that, conditional on size, scale-adjustment decision is age dependent, where in the context of this paper age corresponds to the number of observed demand shocks in a given market.

Proposition 5 *In the steady state there exists a threshold level of expected demand $\hat{b}^\sigma$ given by $\bar{\theta} + \frac{\sigma_\theta^2+\sigma_\epsilon^2}{2\sigma}$, such that conditional on b^σ, $\frac{q_g(\varphi,\bar{a}(b^\sigma),n+1)}{q_g(\varphi,\bar{a}(b^\sigma),n)}$ declines in n for $b^\sigma > \hat{b}^\sigma$, increases in n for $b^\sigma < \hat{b}^\sigma$, and stays constant for $b^\sigma = \hat{b}^\sigma$.*

In analyzing the age dependence of the scale-adjustment decision we need to control for the initial scale of the firm. Note from equation (1) that the optimal scale is completely determined by the firm's ability φ and expected demand b^σ. Firms with the same expected demand, however, vary in their mean demand-shock level and age (the number of observed demand signals). Proposition 5 states that scale adjustment varies with age, conditional on the same initial expected demand value. Younger firms with high expected demand expand at a higher rate than older firms with the same initial expected demand. Younger firms with low expected demand contract at a lower rate than older firms with the same initial expected demand.

The expected demand threshold $\hat{b}^{\sigma} = \bar{\theta} + \frac{\sigma_\theta^2 + \sigma_\epsilon^2}{2\sigma}$ defined in Proposition 5 has an intimate relation to the mean demand-shock threshold $\hat{a} = \bar{\theta} + \frac{\sigma_\theta^2}{2\sigma}$ defined in Proposition 4: when $\bar{a} = \hat{a}$, $b^{\sigma} = \hat{b}^{\sigma}$. Thus, firms with expected demand above the threshold demand are precisely those firms which have a realized mean demand shock above the baseline value. Such firms can be thought of as the firms which have underestimated their true appeal index. As soon as they discover that they are more appealing to consumers than originally believed, they rapidly expand. An older firm with the same expected demand is more convinced of its appeal index, and thus does not adjust the scale by as much as a younger firm which has just discovered its high appeal index and wants to expand.

Rather the opposite story emerges in the firms that overestimated their demand level: firms with the realized expected demand $b^{\sigma} < \hat{b}^{\sigma}$ or the realized mean of demand shocks $\bar{a} < \hat{a}$. As they discover their mean demand shock to be below the baseline value, they respond by contracting. Older firms, however, contract by more since they are more certain, after receiving a greater number of negative demand shocks, that they are of low appeal to consumers. Younger firms are not as strongly convinced that their products are less liked than they originally believed them to be. Thus, as young firms discover with greater certainty that they are of the low-appeal type, they gradually adjust product scale downward.

The age dependence of the scale decision translates into the age dependence of product-adding and -dropping decisions. Conditional on the firm's ability and expected demand, younger firms adjust scale by a greater amount than older firms. As a result, younger firms will see a larger increase in all their products' profitability than will older firms. Thus, younger firms will be prompted to add more products than older firms. Since the initial scope of the firms was the same, younger firms will exhibit a greater share of added products within the range of exported products.

The model yields predictions regarding the response of a firm's intensive and extensive margins to a change in trade costs. Proposition 6 below establishes the effect of a change in trade costs on individual product scale and sales.

Proposition 6 *In a symmetric stationary equilibrium* $\frac{dq_{gx}}{d\tau} < 0$ *and* $\frac{dr_{gx}}{d\tau} < 0$, $\frac{dr_{gd}}{d\tau} > 0$, $\frac{dq_{gd}}{d\tau} > 0$ *if* $\frac{N}{\tau^{\xi} + N} > \frac{1}{\sigma - 1}$ *and* $\frac{dq_{gd}}{d\tau} < 0$ *otherwise.*

Proposition 6 describes how the within-firm intensive margin adjusts in response to changes in trade costs. First, in response to a decline in trade costs, firms will adjust the scale at which they produce their products: the quantity of each exported good unambiguously increases, while the response of the quantity of each domestically supplied good depends on the number of trading partners, the original trade costs, and the elasticity of substitution. Second, the changes to the scale of products sold unambiguously translate into changes in per-product revenue. Sales from exported goods increase, while sales from domestically supplied goods unambiguously decline in response to a decline in trade costs.

Consider the case of an exporter facing a decline in trade cost. Since sales (and, as a result, variable profit) of every potential product increase without affecting the incremental fixed market-entry cost, it might become optimal for a firm to expand the range of produced products. Similarly, a firm selling to the domestic market will experience a decline in variable profit and might find it optimal to reduce the number of products supplied.

There are two potential channels that affect the scope of supplied products (extensive margin) due to a change in trade costs. First, as discussed above, an increase or decrease in a product's profitability might induce the marginal product to surpass a scope threshold, resulting in product adding or dropping. Second, the scope thresholds are themselves affected by a change in trade costs. For an export market, the scope indifference curves move toward the origin in response to a decline in trade costs, inducing firms to add products. For a domestic market, the scope indifference curves move away from the origin in response to a decline in trade costs, inducing firms to drop products (see Appendix D).

The relative importance of the intensive versus extensive margins of a firm's adjustment in predicting changes in trade flows are evaluated numerically in the following section. I calibrate the symmetric two-country stationary equilibrium to match Brazilian world exports. I use the calibrated model to evaluate its ability to predict the margins of trade flows, and perform a counterfactual simulation of a decline in trade costs to study the relative importance of the margins as predicted by the model.

4 Quantitative Analysis

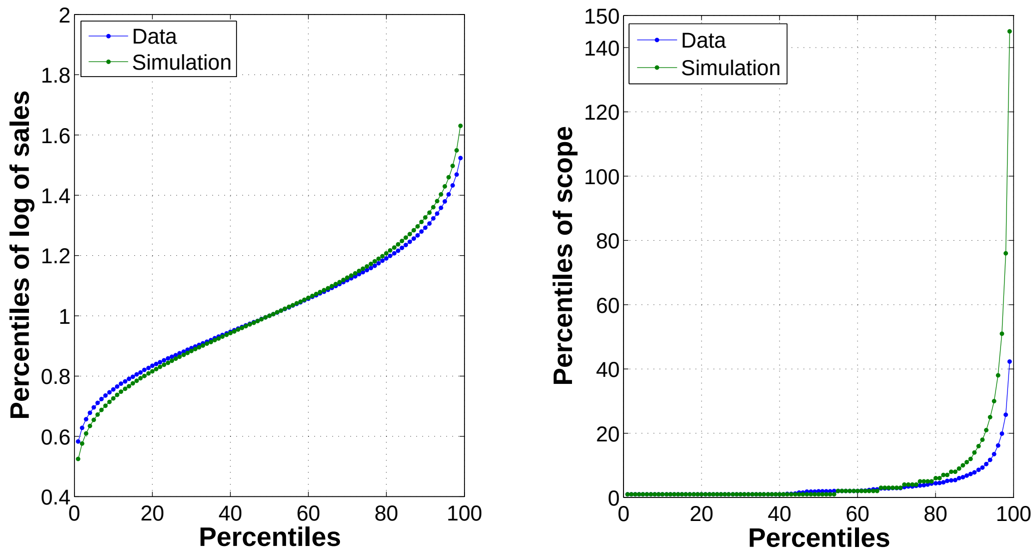
I perform a quantitative analysis of the model to answer the following two questions. First, can a model with demand learning, parameterized to match cross-section distributions of sales and scope, account for the extensive and intensive margins of aggregate export flows? Second, what is the relative importance of extensive versus intensive margins in generating export-sales growth due to trade liberalization?

I use the indirect inference method of Gourieroux and Monfort (1996) to calibrate the parameters of the model that govern the cost structure and the distribution of shocks: f, α, γ, σ_θ^2, and σ_ϵ^2. The method involves solving the maximization problem for each parameter guess. Firm-level data is simulated based on the optimal solution to the firm's problem. From the simulated data a number of moments is computed and compared to the identical moments from the data. The sum of the squares of the percentage deviation of the simulated moments from the data moments is used as the criterion function. Parameters are then chosen to minimize the criterion.

The following five moments are chosen from the data and help identify the five parameters mentioned above: the mean and the standard deviation of the logarithm of the firm's total export sales, the sales-weighted growth rate of firm-product exports, the fraction of multi-product exporters, and the fraction of product-switching exporters. The moments' values are reported in the first column of Table 9.

To solve the firm's maximization problem, I calibrate three parameters of the model: β, σ, and δ. The firm's discount factor is given by β and is set to 0.9606, which is consistent with the period length of a year assumed in the simulations. The elasticity of substitution across varieties is given by σ and is set to 7.49. This value is reported in Broda and Weinstein (2006) as the mean elasticity of substitution across five-digit SITC product categories. The exogenous death rate of firms is given by δ and is set to 0.03. Three percent corresponds to the annual exit rate among Brazilian exporters in the top 5 percent of export sales distribution. Further, the following parameters are normalized to the corresponding values: $\bar{\theta} = 0$, $\xi = 7.4$, $\varphi_{min} = 1$.

Figure 3: **Simulated and Actual Distributions**



4.1 Calibration Results

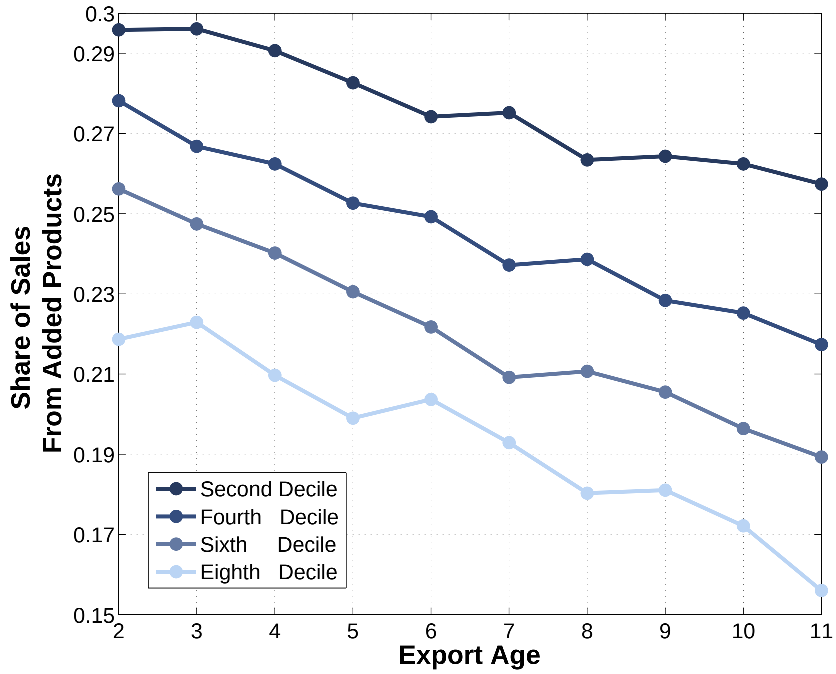
The calibration yields parameter values reported in Table 10. The simulated data moments are reported in the second column of Table 9. The model slightly underpredicts the fraction of product-switching exporters, due to a trade-off between the firm's size and the frequency of product switching. A lower value of $\alpha(\sigma-1)+\gamma$ moves scope indifference curves closer together, as can be inferred from the indifference curve equation (2). Thus, for a given dynamic in demand beliefs, a firm would be more likely to add and drop products for lower values of the coefficient $\alpha(\sigma-1)+\gamma$. At the same time, a lower value of $\alpha(\sigma-1)+\gamma$ moves indifference curves closer to the origin. Thus, for a given ability level and demand belief, a firm will export a greater number of products that would yield higher sales.

Parameterized as described above, the model matches well the cross-section distributions of scope and the logarithm of sales, as depicted in Figure 3. The thicker tail in the scope distribution is a result of the size and product-switching trade-off described above. The model slightly overpredicts the proportion of large firms in the economy, but matches the frequency of product-switching behavior better.

Parameterized to match cross-section moments of sales and scope distributions, the model accurately predicts the role of margins in explaining trade flows, and the role of export age in explaining product-switching behavior. As borne out in the data, the model predicts a greater importance of new-products margin over new-exporters margin in accounting for total Brazilian exports as described in Table 11. The first column replicates the first column of Table 1, while the second column reports the results from the simulated data. Although the model slightly underpredicts the magnitudes of both of the extensive margins, the share of sales arising from new products introduced by incumbent exporters is three times lager than the share of sales arising from the new exporters.

The model predicts age dependence of product-switching behavior as depicted in Figure 4. Each curve in the Figure plots the relationship between the fraction of sales from added

Figure 4: **Age Dependence of Products Switching: Simulations**



products and the firm's export age for firms belonging to a given export-sales decile. For example, a firm with export sales in the sixth decile will derive 26 percent of sales from new products in the second year of being an exporter, while the same firm will derive only 19 percent of sales from new products when it has been exporting for 11 consecutive years.

4.2 Counterfactual Analysis

In this section I use the parameterized model to perform a counterfactual analysis of trade-liberalization reform that yields 4.8 percent growth in export sales. Counterfactual simulations imply a substantial adjustment on the product-switching margin. 31 percent of the growth in total exports is attributed to the firms' expansion of the range of exported products, while only 10 percent can be attributed to exporter-turnover margin. (See Table 12).

The relative importance of the product-switching margin in relation to the exporter-turnover margin predicted by the simulations is comparable to the magnitudes found in the literature. For example, in their study of Indian trade liberalization in the 1990's, Goldberg, Khandelwal, Pavcnik, and Topalova (2010a) find that the product-switching margin accounts for nearly 25 percent of total Indian manufacturing output growth in the 1990's.

5 Conclusion

This paper identifies new patterns in the product-switching behavior of exporters and develops a model of demand learning to explain them, both qualitatively and quantitatively. Specifically, I incorporate a model of demand learning into a general equilibrium framework and perform a quantitative assessment of the learning model in terms of its ability to predict the data.

Using customs-record data on the universe of Brazilian exporters, I begin by documenting the greater importance of product-switching over exporter-turnover margin in export sales. I contribute to the literature by documenting this pattern in the context of export data as opposed to domestic or import data. I then document a new pattern in the product-switching behavior of exporters: conditional on size, younger exporters engage in product switching more frequently than exporters with more experience. The frequency of product switching is defined in terms of two statistics: the share of sales from added products, and the share of added products in the current product mix. Conditional on size, both variables decline as the age of exporting increases.

The demand-learning model provides a natural explanation for the age dependence of product-switching behavior. Products of individual firms are characterized by their appeal index, which is not known to the firms *ex ante* and is subject to inter-temporal preference shocks. Upon observing the sum of their appeal index and preference shocks (the demand shocks), firms learn their true appeal index. Variations in inter-temporal preference shocks translate into variations in the observed mean of the demand shocks, which further translate into product adding and dropping. The convergence of the mean of the observed demand shocks to the true appeal index generates age dependence of product switching.

Finally, the paper shows that the learning model, calibrated to match cross-sectional statistics for the sales and the scope distributions of exporters, quantitatively predicts the product-switching and exporter-turnover margins in export trade flows. The paper also rationalizes the age dependence of product switching conditional on size. These findings contribute to a fast-growing literature that focuses on demand learning to study various aspects of firm growth and survival.

References

Albornoz, Facundo, Hector F. Calvo Pardo, Gregory Corcos, and Emanuel Ornelas. 2011. "Sequential Exporting." Working paper.

Arkolakis, Costas. 2010. "A Unified Theory of Firm Selection and Growth." Working Paper.

Arkolakis, Costas and Mark-Andreas Muendler. 2010. "The Extensive Margin of Exporting Products: A Firm-Level Analysis." Working Paper.

Arkolakis, Costas and Theodore Papageorgiou. 2010. "Firm Selection, Growth and Learning." Working Paper.

Bernard, Andrew B., Stephen J. Redding, and Peter K. Schott. 2010. "Multi-Product Firms and Product Switching." *American Economic Review* 100 (1):70–97.

Bernard, Andrew J., Stephen Redding, and Peter Schott. 2011. "Multi-Product Firms and Trade Liberalization." *Quarterly Journal of Economics* 126 (3):1271–1318.

Broda, Cristian and David Weinstein. 2006. "Globalization and the Gains from Variety." *Quarterly Journal of Economics* 121 (2):541–585.

Chaney, Thomas. 2008. "Distorted Gravity: The Intensive and the Extensive Margins of International Trade." *American Economic Review* 98 (4):1707–1721.

Defevery, Fabrice, Benedikt Heidz, and Mario Larchx. 2010. "Spatial Exporter Dynamics." Working Paper.

DeGroot, Morris. 2004. *Optimal Statistical Decisions*. Hoboken, N.J., wiley classics library ed. ed.

Dixit, Avinash K. and Joseph E. Stiglitz. 1977. "Monopolistic Competition and Optimum Product Diversity." *American Economic Review* 67 (3):297–308.

Eaton, Jonathan, Samuel Kortum, and Francis Kramarz. 2011. "An Anatomy of International Trade: Evidence from French Firms." *Econometrica* 79 (5):1453–1498.

Evans, David S. 1987. "Tests of Alternative Theories of Firm Growth." *Journal of Political Economy* 95 (4):657–674.

Foster, Lucia, John Haltiwanger, and Chad Syverson. 2008. "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?" *American Economic Review* 98 (1):394–425.

Goldberg, Pinelopi, Amit Khandelwal, Nina Pavcnik, and Petia Topalova. 2010a. "Imported Intermediate Inputs and Domestic Product Growth: Evidence from India." *Quarterly Journal of Economics* 125 (4):1727–1767.

———. 2010b. "Multiproduct Firms and Product Turnover in the Developing World: Evidence from India." *Review of Economics and Statistics* 92 (4):1042–1049.

Gopinath, Gita and Brent Neiman. 2011. "Trade Adjustment and Productivity in Large Crises." NBER Working Paper 16958.

Gourieroux, Christian and Alain Monfort. 1996. *Simulation-Based Econometric Models*. Oxford University Press.

Iacovone, Leonardo and Beata S. Javorcik. 2010. "Multi-Products Exporters: Product Churning, Uncertainty and Export Discoveries." *The Economic Journal* 120 (544):481–499.

Jovanovic, Boyan. 1982. "Selection and the Evolution of Industry." *Econometrica* 50 (3):649–670.

Melitz, Marc J. 2003. "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica* 71 (6):1695–1725.

Molinaz, Danielken and Marc-Andreas Muendler. 2009. "Preparing to Export." Working Paper.

Nguyen, Daniel X. 2011. "Demand Uncertainty: Exporting Delays and Exporting Failures." Working paper.

Ruhl, Kim J. and Jonathan J. Willis. 2008. "New Exporter Dynamics." Working paper.

Timoshenko, Olga. 2010. "State Dependence in Export Market Participation: Does Exporting Age Matter?" Working Paper.

Appendices

A Firm's Maximization Problem

The profit-maximization problem of a firm in destination j based in country i with ability φ, mean of observed demand shocks $\bar{a}_j$, and number of observed shocks n, takes the following form

$$\Pi_{ijt}(\varphi, \bar{a}_j, n) \equiv \max_{q_{ijgt}, G_{ijt} \geq 1} E_{a_{jt}|\bar{a}_j, n} \left[\sum_{g=1}^{G_{ijt}} \left(q_{ijgt}^{\frac{\sigma-1}{\sigma}} e^{\frac{a_{jt}}{\sigma}} P_{jt}^{\frac{\sigma-1}{\sigma}} Y_{jt}^{\frac{1}{\sigma}} - \frac{\tau w_{it} g^{\alpha}}{\varphi} q_{ijgt} \right) - f_{ij} \sum_{g=1}^{G_{ijt}} g^{\gamma} \right].$$

Note that the expectation operator in the objective function refers to the random variable $e^{\frac{a_{jt}}{\sigma}}$. Since in relation to $e^{\frac{a_{jt}}{\sigma}}$ other variables are treated as non-random, and as such the objective is linear in $e^{\frac{a_{jt}}{\sigma}}$, we can apply the expectation operator directly to the variable. Denote by $b_j = E_{a_{jt}|\bar{a}_j, n}\left[e^{\frac{a_{jt}}{\sigma}}\right]$.

The maximization can be split into a two-step procedure. First, given the number of products G_{ijt}, solve for the optimal quantity of a given product q_{ijgt}. Second, given the optimal quantity, solve for the optimal number of products.

Taking the first-order conditions of the objective function with respect to q_{ijgt}, keeping G_{ijt} fixed, yields optimal quantity decision

$$q_{ijgt}(\varphi, \bar{a}_j, n) = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \left(\frac{\varphi}{g^{\alpha}}\right)^{\sigma} b_j^{\sigma} \frac{P_{jt}^{\sigma-1} Y_{jt}}{(\tau_{ij} w_{it})^{\sigma}}.$$

Substitute the optimal quantity decision back into the objective function to obtain

$$\Pi_{ijt}(\varphi, \bar{a}_j, n) = \sum_{g=1}^{G_{ijt}} \frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}} \left(\frac{\varphi}{g^{\alpha}}\right)^{\sigma-1} b_j^{\sigma} \frac{P_{jt}^{\sigma-1} Y_{jt}}{(\tau_{ij} w_{it})^{\sigma-1}} - f_{ij} \sum_{g=1}^{G_{ijt}} g^{\gamma}.$$

Compare the variable profit from a given product to the incremental fixed cost. For a firm to profitably produce a product, it must be that the variable profit is greater than or equal to the marginal fixed cost of supplying that product to a market

$$\frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}} \left(\frac{\varphi}{g^{\alpha}}\right)^{\sigma-1} b_j^{\sigma} \frac{P_{jt}^{\sigma-1} Y_{jt}}{(\tau_{ij} w_{it})^{\sigma-1}} > f_{ij} g^{\gamma}. \tag{9}$$

Rearrange equation (9) in the following way:

$$\varphi^{\sigma-1} b_j^{\sigma} > \frac{\sigma^{\sigma}}{(\sigma-1)^{\sigma-1}} f_{ij} g^{\gamma+\alpha(\sigma-1)} \frac{(\tau_{ij} w_{it})^{\sigma-1}}{P_{jt}^{\sigma-1} Y_{jt}}. \tag{10}$$

At the point of making a scope decision, the left-hand side of inequality (10) is constant and positive. The right-hand side uniformly increases from zero to $+\infty$ if $\gamma + \alpha(\sigma-1) > 0$, or declines from $+\infty$ to zero if $\gamma + \alpha(\sigma-1) < 0$. In the latter case the maximization problem does not have a well defined and economically meaningful solution for all possible ability levels. For example, for a firm with high enough φ, such that for $g = 1$ inequality (10) is satisfied, it would be optimal to produce an infinite number of products. Thus, for the maximization problem to have a well defined solution I assume $\gamma + \alpha(\sigma - 1) > 0$. In looking at inequality (9) the condition implies that the incremental fixed cost must decline more slowly than the variable profit, or must increase in rank g.

With $\gamma + \alpha(\sigma - 1) > 0$, the right-hand side of inequality (10) increases from zero to $+\infty$, and a firm will keep adding products as long as inequality (10) is satisfied.

Note that when $\varphi^{\sigma-1}b_j^\sigma$ are such that inequality (10) holds with equality for some g, a firm will make zero net profit on the last (marginal) product g. For a firm with $\varphi^{\sigma-1}b_j^\sigma$ slightly below this level, the inequality will not hold and the production of g will not be profitable. Thus, the firm will produce $g - 1$ products. However, for a firm with $\varphi^{\sigma-1}b_j^\sigma$ slightly above this level, the inequality will be satisfied – implying that production of the gth product will bring a positive net profit. The firm will produce g products until $\varphi^{\sigma-1}b_j^\sigma$ increases sufficiently to make zero net profit on the next $g + 1$st product. Thus,

$$\varphi^{\sigma-1}b_j^\sigma = \frac{\sigma^\sigma}{(\sigma-1)^{\sigma-1}} f_{ij} g^{\gamma+\alpha(\sigma-1)} \frac{(\tau_{ij}w_{it})^{\sigma-1}}{P_{jt}^{\sigma-1}Y_{jt}}$$

implicitly defines scope indifference curves in the $(\varphi^{\sigma-1}, b_j^\sigma)$ space for each scope level g. Denote these indifference curves by $\Gamma(\varphi^{\sigma-1}, b_j^\sigma) = g$. A firm with $\varphi^{\sigma-1}, b_j^\sigma$ such that $g \leq \Gamma(\varphi^{\sigma-1}, b_j^\sigma) < g + 1$ will produce exactly g products.

B Equilibrium Mass of Firms

Using the measure function $m_i(\varphi, \bar{a}, n)$, the equilibrium mass of firms is defined as

$$M_i = \sum_{n=0}^{+\infty} \int_{\varphi_i^*}^{+\infty} \int_{\bar{a}_i^*(\varphi,n)}^{+\infty} m_i(\varphi, \bar{a}, n) d\varphi d\bar{a}. \tag{11}$$

By recursive substitution using equations (18) and (19), $m_i(\varphi, \bar{a}, n)$ can be expressed as a function of demand exit thresholds and posterior distributions in the following way

$$m_i(\varphi, \bar{a}, n) = J\left(\frac{\varphi_{min}}{\varphi_i^*}\right)^\xi \frac{\xi(\varphi_i^*)^\xi}{\varphi^{\xi+1}} \int_{\bar{a}_i^*(\varphi,n-1)}^{+\infty} ... \int_{\bar{a}_i^*(\varphi,1)}^{+\infty} p(\bar{a}|\bar{a}_{n-1})p(\bar{a}_{n-1}|\bar{a}_{n-2})p(\bar{a}_1)d\bar{a}_1 d...\bar{a}_{n-1}, \tag{12}$$

where $p(\bar{a}_k|\bar{a}_{k-1}) = \frac{1}{\sqrt{\upsilon_{k-1}^2+\sigma_\epsilon^2}}\phi\left(\frac{\bar{a}_k k - \bar{a}_{k-1}(k-1) - \mu_{k-1}}{\sqrt{\upsilon_{k-1}^2+\sigma_\epsilon^2}}\right)$ is the transitional density from state $(\bar{a}_{k-1}, k-1)$ to state $(\bar{a}_k, k)$, and $p(\bar{a}_1) = \frac{1}{\sqrt{\sigma_\theta^2+\sigma_\epsilon^2}}\phi\left(\frac{\bar{a}_1 - \bar{\theta}_i}{\sqrt{\sigma_\theta^2+\sigma_\epsilon^2}}\right)$.

Substitute equation (12) in equation (11) to obtain

$$\begin{aligned} M_i &= \sum_{n=0}^{+\infty} \int_{\varphi_i^*}^{+\infty} \int_{\bar{a}_i^*(\varphi,n)}^{+\infty} J\left(\frac{\varphi_{min}}{\varphi_i^*}\right)^\xi \frac{\xi(\varphi_i^*)^\xi}{\varphi^{\xi+1}} \int_{\bar{a}_i^*(\varphi,n-1)}^{+\infty} ... \int_{\bar{a}_i^*(\varphi,1)}^{+\infty} p(\bar{a}|\bar{a}_{n-1})p(\bar{a}_{n-1}|\bar{a}_{n-2})p(\bar{a}_1)d\bar{a}_1...d\bar{a}_{n-1}d\bar{a}d\varphi \\ &= J\left(\frac{\varphi_{min}}{\varphi_i^*}\right)^\xi \int_{\varphi_i^*}^{+\infty} \frac{\xi(\varphi_i^*)^\xi}{\varphi^{\xi+1}} \sum_{n=0}^{+\infty} \int_{\bar{a}_i^*(\varphi,n)}^{+\infty} ... \int_{\bar{a}_i^*(\varphi,1)}^{+\infty} p(\bar{a}|\bar{a}_{n-1})p(\bar{a}_{n-1}|\bar{a}_{n-2})p(\bar{a}_1)d\bar{a}_1...d\bar{a}_{n-1}d\bar{a}d\varphi \end{aligned}$$

Denote

$$H_i(\varphi) = \sum_{n=0}^{+\infty} \int_{\bar{a}_i^*(\varphi,n)}^{+\infty} ... \int_{\bar{a}_i^*(\varphi,1)}^{+\infty} p(\bar{a}|\bar{a}_{n-1})p(\bar{a}_{n-1}|\bar{a}_{n-2})p(\bar{a}_1)d\bar{a}_1...d\bar{a}_{n-1}d\bar{a},$$

yielding

$$M_i(\varphi_i^*) = J\left(\frac{\varphi_{min}}{\varphi_i^*}\right)^{\xi} \int_{\varphi_i^*}^{+\infty} \frac{\xi(\varphi_i^*)^{\xi}}{\varphi^{\xi+1}} H_i(\varphi)d\varphi.$$

C Goods Market Clearing Condition

The aggregate expenditure level Y is defined as the sum of labor payments and dividends from national firms

$$Y = L + \Pi_d + N\Pi_x, \tag{13}$$

where Π_d denotes profit from firms selling to the domestic market, and Π_x denotes profit from exports to a given destination. Total profits from a destination can further be expressed as the product of the mass of firms selling to that destination and the average profit level of those firms, as shown below.

$$\begin{aligned}
\Pi_i &= \sum_{n=0}^{+\infty} \int_{\varphi^*}^{+\infty} \int_{\bar{a}_i^*(\varphi,n)}^{+\infty} \int_{-\infty}^{+\infty} \pi_i(\varphi,\bar{a},n,a)m_i(\varphi,\bar{a},n,a)dad\bar{a}d\varphi = \\
&= M_i \sum_{n=0}^{+\infty} \int_{\varphi^*}^{+\infty} \int_{\bar{a}_i^*(\varphi,n)}^{+\infty} \int_{-\infty}^{+\infty} \pi_i(\varphi,\bar{a},n,a)\varsigma_i(\varphi,\bar{a},n,a)dad\bar{a}d\varphi = \\
&= M_i\tilde{\pi}_i,
\end{aligned}$$

where M_i is the mass of firms selling to destination i, and $\varsigma_i(\varphi,\bar{a},n,a)$ is the density associated with measure $m_i(\varphi,\bar{a},n,a)$ and defined as $\frac{m_i(\varphi,\bar{a},n,a)}{\sum_{n=0}^{+\infty}\int_{\varphi^*}^{+\infty}\int_{\bar{a}_i^*(\varphi,n)}^{+\infty}\int_{-\infty}^{+\infty} m_i(\varphi,\bar{a},n,a)dad\bar{a}d\varphi} = \frac{m_i(\varphi,\bar{a},n,a)}{M_i}$. $\tilde{\pi}_i$ is the mean profit level. As stated in part (c) of Proposition 3, $\tilde{\pi}_i$s are equal across destinations. Thus, equation (13) can be written as

$$Y = L + (M_d + NM_x)\tilde{\pi}, \tag{14}$$

yielding the left-hand side of the equation (8) in the text.

I will show that Y also equals the total revenue of firms using goods market clearing conditions and rational expectations of the firms.

Rational expectations of the firms imply that the price index that firms take as given P, results from individual prices that are set in equilibrium. Consider the definition of the price index

$$\begin{aligned}
P^{1-\sigma} &= \sum_{n=0}^{+\infty} \int_{\varphi_d^*}^{+\infty} \int_{\bar{a}_d^*(\varphi,n)}^{+\infty} \int_{-\infty}^{+\infty} e^a \sum_{g=1}^{G(\varphi,\bar{a},n,a)} p_{dg}^{1-\sigma} m_d(\varphi,\bar{a},n,a)dad\bar{a}d\varphi + \\
&+N \sum_{n=0}^{+\infty} \int_{\varphi_x^*}^{+\infty} \int_{\bar{a}_x^*(\varphi,n)}^{+\infty} \int_{-\infty}^{+\infty} e^a \sum_{g=1}^{G(\varphi,\bar{a},n,a)} p_{xg}^{1-\sigma} m_x(\varphi,\bar{a},n,a)dad\bar{a}d\varphi.
\end{aligned}$$

Market clearing implies optimal price level given by

$$p_{ig} = \frac{\sigma}{\sigma-1}\tau b^{-1}\frac{g^{\alpha}}{\varphi}e^{\frac{a}{\sigma}}.$$

Substitute p_{ig} into the price index to obtain

$$\begin{aligned}
P^{1-\sigma} &= \sum_{n=0}^{+\infty}\int_{\varphi_d^*}^{+\infty}\int_{\bar{a}_d^*(\varphi,n)}^{+\infty}\int_{-\infty}^{+\infty}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\varphi^{\sigma-1}b^{\sigma-1}e^{\frac{a}{\sigma}}\sum_{g=1}^{G(\varphi,\bar{a},n)}g^{\alpha(1-\sigma)}M_d\varsigma_d(\varphi,\bar{a},n,a)dad\bar{a}d\varphi + \\
&\quad +N\sum_{n=0}^{+\infty}\int_{\varphi_d^*}^{+\infty}\int_{\bar{a}_d^*(\varphi,n)}^{+\infty}\int_{-\infty}^{+\infty}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left(\frac{\varphi}{\tau}\right)^{\sigma-1}b^{\sigma-1}e^{\frac{a}{\sigma}}\sum_{g=1}^{G(\varphi,\bar{a},n)}g^{\alpha(1-\sigma)}M_x\varsigma_x(\varphi,\bar{a},n,a)dad\bar{a}d\varphi = \\
&= \sum_{n=0}^{+\infty}\int_{\varphi_d^*}^{+\infty}\int_{\bar{a}_d^*(\varphi,n)}^{+\infty}\int_{-\infty}^{+\infty}\frac{r_d(\varphi,\bar{a},n,a)}{P^{\sigma-1}Y}M_d\varsigma_d(\varphi,\bar{a},n,a)dad\bar{a}d\varphi + \\
&\quad +N\sum_{n=0}^{+\infty}\int_{\varphi_d^*}^{+\infty}\int_{\bar{a}_d^*(\varphi,n)}^{+\infty}\int_{-\infty}^{+\infty}\frac{r_x(\varphi,\bar{a},n,a)}{P^{\sigma-1}Y}M_x\varsigma_x(\varphi,\bar{a},n,a)dad\bar{a}d\varphi = \\
&= \frac{M_d\tilde{r}_d + NM_x\tilde{r}_x}{P^{\sigma-1}Y}.
\end{aligned}$$

Together with the result of part (d) of Proposition 3, the last equality yields

$$Y = (M_d + NM_x)\tilde{r}. \tag{15}$$

Combining equation (14) and (26), and using $M = M_d + NM_x$, we obtain

$$L + M\tilde{\pi} = M\tilde{r}.$$

D Scope Indifference Curves and Trade Costs

Consider equation (2) describing the scope indifference curves. The curves are parameterized by the endogenous variable $B = \frac{\tau^{\sigma-1}}{P^{\sigma}Y}$. The higher the value of B, the higher should be the value of $\varphi^{\sigma-1}b^{\sigma}$ to yield the same scope level g. Thus in response to an increase in B, the level curves will move away from the origin. As a result, a firm in the state (φ, b) will find itself lying on a lower scope indifference curve and will be forced to drop marginal products. In a similar way, a decline in the value of B will move level curves $\Gamma(\varphi^{\sigma-1}b^{\sigma}) = g$ toward the origin, inducing firms to add marginal products.

I will show below that $\frac{dB_x}{d\tau} > 0$ and $\frac{dB_d}{d\tau} < 0$.

$$B_x = \frac{\tau^{\sigma-1}}{P^{\sigma}Y} = \left(\frac{\varphi_x^*}{u^*}\right)^{\sigma-1}.$$

Since, as shown in Appendix 6, $\frac{d\varphi_x^*}{d\tau} > 0 \Rightarrow \frac{dB_x}{d\tau} > 0$. In response to a decline in trade costs, B_x declines, shifting scope indifference curves toward the origin, and inducing firms to add products.

$$B_d = \frac{1}{P^{\sigma}Y} = \left(\frac{\varphi_d^*}{u^*}\right)^{\sigma-1}.$$

Since, as shown in Appendix 6, $\frac{d\varphi_d^*}{d\tau} < 0 \Rightarrow \frac{dB_x}{d\tau} < 0$. In response to a decline in trade costs, B_d increases, shifting scope indifference curves away from the origin and inducing firms to drop products.

E Proofs of Propositions

Proof of Proposition 1. Consider Bellman equation (3) written in the steady state:

$$V_{ij}(\varphi, \bar{a}_j, n) = \max\{\Pi_{ij}(\varphi, \bar{a}_j, n) + \beta(1-\delta)E_{\bar{a}'_j|\bar{a}_j,n}V_{ij}(\varphi, \bar{a}'_j, n+1), 0\}.$$

In the region of the state space where it is optimal for the firm to continue exporting, value function V_{ij} is given by $\tilde{V}_{ij}$ that satisfies

$$\tilde{V}_{ij}(\varphi, \bar{a}_j, n) = \Pi_{ij}(\varphi, \bar{a}_j, n) + \beta(1-\delta)E_{\bar{a}'_j|\bar{a}_j,n}\tilde{V}_{ij}(\varphi, \bar{a}'_j, n+1).$$

In the explicit form, $\tilde{V}_{ij}$ is given by

$$\tilde{V}_{ij}(\varphi, \bar{a}_j, n) = \sum_{k=0}^{+\infty}(\beta(1-\delta))^k E_{\bar{a}^k_j|\bar{a}_j,n}\Pi_{ij}(\varphi, \bar{a}^k_j, k+n), \tag{16}$$

where $\bar{a}^0_j \equiv \bar{a}_j$. Conditional on the sate $(\bar{a}_j, n)$, $\bar{a}^k_j$ is normally distributed with mean $\frac{k\mu_{nj}(\bar{a}_j)}{n+k} + \frac{\bar{a}_j n}{n+k}$ and variance $\frac{\upsilon_n^2 k^2}{(t+k)^2} + \frac{k\sigma_\epsilon^2}{(t+k)^2}$. As k goes to infinity this distribution converges to a well defined one given by $N\left(\mu_{nj}(\bar{a}_j), \upsilon_n^2\right)$. Next, note that as the number of signals observed approaches infinity, the expected demand b^σ remains bounded: $\lim_{n\to+\infty} b^\sigma(\bar{a}, n) = \exp\left(\bar{a} + \frac{\sigma_\epsilon^2}{2\sigma}\right)$. Thus, for a given $\bar{a}_j$ in equation (16), $E_{\bar{a}^k_j|\bar{a}_j,n}\Pi_{ij}(\varphi, \bar{a}^k_j, k+n)$ is well defined for all k and remains bounded as k approaches infinity. Thus, $\tilde{V}_{ij}(\varphi, \bar{a}_j, n)$ is a well defined function, and the market-participation thresholds $\bar{a}^*_{ij}(\varphi, n)$ are implicitly determined by

$$\tilde{V}_{ij}(\varphi, \bar{a}_j, n) = 0. \tag{17}$$

Since the profit function $\Pi_{ij}(\varphi, \bar{a}_j, n)$ is monotonically increasing in $\bar{a}_j$, by equation (16) so is $\tilde{V}_{ij}(\varphi, \bar{a}_j, n)$. Thus, for given values of φ and n, equation (17) determines a unique threshold $\bar{a}^*_{ij}(\varphi, n)$.

Since the profit function $\Pi_{ij}(\varphi, \bar{a}_j, n)$ is monotonically increasing in φ, by equation (16) so is $\tilde{V}_{ij}(\varphi, \bar{a}_j, n)$. Since, for a given n, $\tilde{V}_{ij}(\varphi, \bar{a}_j, n)$ increases in both φ and $\bar{a}_j$, it must be true that $\bar{a}^*_{ij}(\varphi, n)$ declines in φ for a given n: $\frac{\partial \bar{a}^*_{ij}(\varphi,n)}{\partial\varphi} < 0$.

Since the variances of $\bar{a}^k_j$s in equation (16) decline as n increases, and since the profit function is convex in $\bar{a}^k_j$, $\tilde{V}_{ij}(\varphi, \bar{a}_j, n)$ declines as n increases. Thus, for a given φ, $\bar{a}^*_{ij}(\varphi, n)$ must increase with n to maintain the equality in (17): $\bar{a}^*_{ij}(\varphi, n+1) > \bar{a}^*_{ij}(\varphi, n)$.

Since $\lim_{n\to+\infty} b^\sigma(\bar{a}, n) = \exp\left(\bar{a} + \frac{\sigma_\epsilon^2}{2\sigma}\right)$, there exists a limit to $\Pi_{ij}(\varphi, \bar{a}_j, n)$ as n approaches infinity, and thus there exists a limit to $\tilde{V}_{ij}(\varphi, \bar{a}_j, n)$ as n approaches infinity yielding the existence of the limit for $\bar{a}^*_{ij}(\varphi, n)$: $\lim_{n\to\infty} \bar{a}^*_{ij}(\varphi, n) = \bar{a}^*_{ij}(\varphi)$. ■

Proof of Proposition 2. To describe the measure function $m_i(\varphi, \bar{a}, n, a)$ for $i \in \{d, x\}$, note that it can be written as

$$m_i(\varphi, \bar{a}, n, a) = \varsigma(a|\varphi, \bar{a}, n) m_i(\varphi, \bar{a}, n),$$

where $\varsigma(a|\varphi, \bar{a}, n)$ is the probability of current demand shock to be a for a firm of type $(\varphi, \bar{a}, n)$. The distribution of demand shock for a firm of type $(\varphi, \bar{a}, n)$ is given by the firm's posterior distribution: $N(\mu_n, \upsilon_n^2 + \sigma_\epsilon^2)$. Thus, $\varsigma(a|\varphi, \bar{a}, n)$ is given by $\frac{1}{\sqrt{\upsilon_n^2+\sigma_\epsilon^2}} \phi\left(\frac{a-\mu_n}{\sqrt{\upsilon_n^2+\sigma_\epsilon^2}}\right)$.

$m_i(\varphi, \bar{a}, n)$ is the measure of firms of type $(\varphi, \bar{a}, n)$. All firms enter with $\bar{a} = 0$ and $n = 0$, the mass of entrants is J, out of which only firms with $\varphi > \varphi_j^*$ drawn from a Pareto distribution enter. Thus, the mass of entrants is $J\left(\frac{\varphi_{min}}{\varphi_i^*}\right)^\xi$. Conditional on entry, the ability level follows Pareto distribution with support $[\varphi_j^*, +\infty)$. Thus, the initial condition is given by

$$m_i(\varphi, 0, 0) = J\left(\frac{\varphi_{min}}{\varphi_i^*}\right)^\xi \frac{\xi(\varphi_i^*)^\xi}{\varphi^{\xi+1}} \tag{18}$$

for $\varphi > \varphi_i^*$ and $m_i(\varphi, 0, 0) = 0$ otherwise.

Next, I describe the transition rule to state $(\varphi, \bar{a}, n+1)$. A firm in state $(\varphi, \bar{a}', n)$ can transfer to state $(\varphi, \bar{a}, n+1)$ with the appropriate demand shock a given by $a = \bar{a}(n+1) - \bar{a}'n$. Conditional on $(\varphi, \bar{a}', n)$, the demand shock is distributed $N(\mu_n, \upsilon_n^2 + \sigma_\epsilon^2)$. Thus, by the law of large numbers, the measure of firms in state $(\varphi, \bar{a}', n)$ that transitions to state $(\varphi, \bar{a}, n+1)$ is given by $m_i(\varphi, \bar{a}', n)\frac{1}{\sqrt{\upsilon_n^2+\sigma_\epsilon^2}}\phi\left(\frac{\bar{a}(n+1)-\bar{a}'n-\mu_n}{\sqrt{\upsilon_n^2+\sigma_\epsilon^2}}\right)$, implying the transition rule given by

$$m_i(\varphi, \bar{a}, n+1) = \int_{\bar{a}_i^*(\varphi,n)}^{+\infty} m_i(\varphi, \bar{a}', n) \frac{1}{\sqrt{\upsilon_n^2 + \sigma_\epsilon^2}} \phi\left(\frac{\bar{a}(n+1) - \bar{a}'n - \mu_n}{\sqrt{\upsilon_n^2 + \sigma_\epsilon^2}}\right) d\bar{a}'. \tag{19}$$

■

Proof of Proposition 3: Parts (a) and (b). Consider per-period firm's profit described in Appendix A

$$\Pi_{ijt}(\varphi, \bar{a}_j, n) = \sum_{g=1}^{G_{ijt}(\varphi^{\sigma-1} b_j^\sigma)} \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} \left(\frac{\varphi}{g^\alpha}\right)^{\sigma-1} b_j^\sigma \frac{P_{jt}^{\sigma-1} Y_{jt}}{(\tau_{ij} w_{it})^{\sigma-1}} - f_{ij} \sum_{g=1}^{G_{ijt}(\varphi^{\sigma-1} b_j^\sigma)} g^\gamma.$$

In the symmetric stationary equilibrium, the profit can be written as

$$\Pi(\varphi, \bar{a}, n) = \sum_{g=1}^{G(\varphi^{\sigma-1} b^\sigma)} \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} \left(\frac{\varphi}{g^\alpha}\right)^{\sigma-1} b^\sigma \frac{P^{\sigma-1} Y}{(\tau)^{\sigma-1}} - f \sum_{g=1}^{G(\varphi^{\sigma-1} b^\sigma)} g^\gamma,$$

where $\tau > 1$ for an export market, and $\tau = 1$ for domestic market.

A firm's dynamic behavior is determined by the evolution of beliefs about the demand parameter described by $\bar{a}$, and n. Ability φ and aggregate variables P and Y, stay constant through the firm's life cycle. Thus, in characterizing the firm's dynamic behavior, consider normalization $u = \frac{\varphi P Y^{\frac{1}{\sigma-1}}}{\tau}$. Using normalization u, a firm's profit can be written as

$$\Pi(u, \bar{a}, n) = \sum_{g=1}^{G(u,b^\sigma)} \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} g^{\alpha(1-\sigma)} b^\sigma u^{\sigma-1} - f \sum_{g=1}^{G(u,b^\sigma)} g^\gamma.$$

The convenience of this representation is that given u, there are no more endogenous equilibrium objects determining a firm's profit. Also notice that conditional on u, the profits from domestic and foreign markets appear identical: the effect of transportation costs τ is taken into account in defining u.

Using normalization u, a firm's market participation problem can be written as

$$V(u, \bar{a}, n) = \max \left\{ \sum_{g=1}^{G(u,b^\sigma)} \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} g^{\alpha(1-\sigma)} b^\sigma u^{\sigma-1} - f \sum_{g=1}^{G(u,b^\sigma)} g^\gamma + \beta(1-\delta) E_{\bar{a}'|\bar{a},n} V(u, \bar{a}', n+1), 0 \right\}.$$

Observe that conditional on u, the problems for domestic and export markets are identical.

The solution to this problem is characterized by a set of market exit thresholds $\bar{a}^*(u, n)$, which, as expected, do not depend on a specific destination. Conditional on u the thresholds are completely determined by the exogenous parameters of the model.

The uniqueness of the market exit thresholds implies $\bar{a}^*(u, n) = \bar{a}_i^*(\varphi, n)$. A firm in state (u, n) exits the market when $\bar{a} < \bar{a}^*(u, n) = \bar{a}^*\left(\frac{\varphi P Y^{\frac{1}{\sigma-1}}}{\tau}, n\right)$. Thus, a firm with ability $\varphi = \frac{\tau u}{P Y^{\frac{1}{\sigma-1}}}$ exits the market when $\bar{a} < \bar{a}^*\left(\frac{\varphi P Y^{\frac{1}{\sigma-1}}}{\tau}, n\right)$. At the same time, the exit threshold is given by $\bar{a}_i^*(\varphi, n)$. Uniqueness of the exit threshold implies $\bar{a}^*\left(\frac{\varphi P Y^{\frac{1}{\sigma-1}}}{\tau}, n\right) = \bar{a}_i^*(\varphi, n)$, or $\bar{a}^*(u, n) = \bar{a}_i^*(\varphi, n)$.

A firm's value of entry into either the domestic or an export market can be written as

$$V^E(u) = \Pi(u, 0, 0) + \beta(1-\delta) E_{\bar{a}'|0,0} V(u, \bar{a}', 1). \tag{20}$$

The entry threshold u^* is determined by $V^E(u^*) = 0$. Note that conditional on u, equation (20) does not involve any endogenous variables. Thus, u^* is completely determined by the exogenous parameters of the model. Furthermore, it is equal across destinations.

A firm will enter a given destination whenever $u \geq u^*$, or equivalently $\varphi \geq \frac{\tau u^*}{P Y^{\frac{1}{\sigma-1}}}$. Since the entry thresholds φ_d^* and φ_x^* are unique, it must be true that $\varphi_d^* = \frac{u^*}{P Y^{\frac{1}{\sigma-1}}}$ and $\varphi_x^* = \frac{\tau u^*}{P Y^{\frac{1}{\sigma-1}}}$.
■

Proof of Proposition 3: Parts (c) and (d). Denote by $r_i(\varphi, \bar{a}, n, a)$ revenue of firms in state $(\varphi, \bar{a}, n, a)$ in market i. The mean revenue level of firms in destination i is defined as

$$\tilde{r}_i = \sum_{n=0}^{+\infty} \int_{\varphi_i^*}^{+\infty} \int_{\bar{a}_i^*(\varphi,n)}^{+\infty} \int_{-\infty}^{+\infty} r_i(\varphi, \bar{a}, n, a) \frac{m_i(\varphi, \bar{a}, n, a)}{M_i} da\, d\bar{a}\, d\varphi,$$

where $\frac{m_i(\varphi,\bar{a},n,a)}{M_i}$ defines the density associated with measure $m_i(\varphi,\bar{a},n,a)$. Substitute equilibrium revenue, substitute the measure function defined in Proposition 2, integrate out a to obtain

$$\tilde{r}_i = \sum_{n=0}^{+\infty} \int_{\varphi_i^*}^{+\infty} \int_{\bar{a}_i^*(\varphi,n)}^{+\infty} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \frac{\varphi^{\sigma-1} P^{\sigma-1} Y}{\tau^{\sigma-1}} b^{\sigma} \sum_{g=1}^{G} g^{\alpha(1-\sigma)} \frac{m_i(\varphi,\bar{a},n)}{M_i} d\bar{a} d\varphi.$$

Using recursive definition of $m_i(\varphi,\bar{a},n)$ described in Appendix B, mean revenue level can be further written as

$$\tilde{r}_i = J\left(\frac{\varphi_{min}}{\varphi_i^*}\right)^{\xi} \int_{\varphi_i^*}^{+\infty} \frac{\xi(\varphi_i^*)^{\xi}}{\varphi^{\xi+1}} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \frac{\varphi^{\sigma-1} P^{\sigma-1} Y}{\tau^{\sigma-1}} \sum_{n=0}^{+\infty} \int_{\bar{a}_i^*(\varphi,n)}^{+\infty} b^{\sigma} \sum_{g=1}^{G} g^{\alpha(1-\sigma)} \frac{h_i(\varphi,\bar{a},n)}{M_i} d\bar{a} d\varphi,$$

where $h_i(\varphi,\bar{a},n) = \int_{\bar{a}_i^*(\varphi,n-1)}^{+\infty} \dots \int_{\bar{a}_i^*(\varphi,1)}^{+\infty} p(\bar{a}|\bar{a}_{n-1}) p(\bar{a}_{n-1}|\bar{a}_{n-2}) p(\bar{a}_1) d\bar{a}_1 d\dots\bar{a}_{n-1}$ (individual components are defined in Appendix B).

Consider the change of variables $u = \frac{\varphi P Y^{\frac{1}{\sigma-1}}}{\tau}$. $h_i(\varphi,\bar{a},n)$ depends on φ through thresholds $\bar{a}_i^*(\varphi,n)$ which are equal to $\bar{a}^*(u,n)$. Thus $h_i(\varphi,\bar{a},n) = h(u,\bar{a},n)$.

Further,

$$\frac{d\varphi}{\varphi^{\xi+1}} = \frac{\tau du \left(PY^{\frac{1}{\sigma-1}}\right)^{\xi+1}}{PY^{\frac{1}{\sigma-1}} (\tau u)^{\xi+1}}.$$

First, apply the transformation to the mass of firms M_i defined in Appendix B:

$$M_i(\varphi_i^*) = J\left(\frac{\varphi_{min}}{\varphi_i^*}\right)^{\xi} \int_{\varphi_i^*}^{+\infty} \frac{\xi(\varphi_i^*)^{\xi}}{\varphi^{\xi+1}} H_i(\varphi) d\varphi = \tag{21}$$

$$= J\left(\varphi_{min}\right)^{\xi} \xi \left(\frac{PY^{\frac{1}{\sigma-1}}}{\tau}\right)^{\xi} \int_{u^*}^{+\infty} \frac{1}{u^{\xi+1}} H(u) du. \tag{22}$$

Next, apply the transformations to the mean revenue equation which now can be written as

$$\tilde{r}_i = \frac{\int_{u^*}^{+\infty} \frac{1}{u^{\xi+1}} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} u^{\sigma-1} \sum_{n=0}^{+\infty} \int_{\bar{a}^*(u,n)}^{+\infty} b^{\sigma} \sum_{g=1}^{G} g^{\alpha(1-\sigma)} h(u,\bar{a},n) d\bar{a} du}{\int_{u^*}^{+\infty} \frac{1}{u^{\xi+1}} H(u) du}. \tag{23}$$

Equation (23) completely determines mean revenue in destination i as a function of the exogenous parameters only. Note that the right-hand side of equation (23) is independent of i, implying that the mean revenue is the same across destinations. Similar calculations apply to the mean profit level. ■

Proof of Proposition 4. Recall that $q_{ijg}(\varphi,\bar{a}_j,n)$ is given by

$q_{ijg}(\varphi, \bar{a}_j, n) = \left(\frac{\sigma-1}{\sigma}\right)^\sigma \left(\frac{\varphi}{g^\alpha}\right)^\sigma b_j^\sigma \frac{P_j^{\sigma-1} Y_j}{(\tau_{ij} w_i)^\sigma}$, where $b_j = E_{a_{jt}|\bar{a}_j, n}\left[e^{\frac{a_{jt}}{\sigma}}\right]$. Using posterior distribution to compute the expectation b_j^σ, $b_j^\sigma = \exp\left[\mu_{jn} + \frac{1}{2}\left(\frac{\upsilon_n^2 + \sigma_\epsilon^2}{\sigma}\right)\right]$. Thus,

$$\begin{aligned} \frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} &= \frac{\left(\frac{\sigma-1}{\sigma}\right)^\sigma \left(\frac{\varphi}{g^\alpha}\right)^\sigma \exp\left[\mu_{j,n+1} + \frac{1}{2}\left(\frac{\upsilon_{n+1}^2 + \sigma_\epsilon^2}{\sigma}\right)\right] \frac{P_j^{\sigma-1} Y_j}{(\tau_{ij} w_i)^\sigma}}{\left(\frac{\sigma-1}{\sigma}\right)^\sigma \left(\frac{\varphi}{g^\alpha}\right)^\sigma \exp\left[\mu_{jn} + \frac{1}{2}\left(\frac{\upsilon_n^2 + \sigma_\epsilon^2}{\sigma}\right)\right] \frac{P_j^{\sigma-1} Y_j}{(\tau_{ij} w_i)^\sigma}} = \\ &= \exp\left(\mu_{j,n+1} - \mu_{jn} + \frac{\upsilon_{n+1}^2 - \upsilon_n^2}{2\sigma}\right). \end{aligned}$$

Substitute for μ_{jn}, $\mu_{j,n+1}$, υ_n^2, υ_{n+1}^2, rearrange terms to obtain

$$\frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} = \exp\left(\frac{\lambda(2\sigma(\bar{a}_j - \bar{\theta}_j) - \sigma_\theta^2)}{2\sigma(1 + (n+1)\lambda)(1 + n\lambda)}\right), \tag{24}$$

where $\lambda = \frac{\sigma_\theta^2}{\sigma_\epsilon^2}$. The relation of $\frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)}$ to 1 depends on the sign of $\frac{\lambda(2\sigma(\bar{a}_j - \bar{\theta}_j) - \sigma_\theta^2)}{2\sigma(1+(n+1)\lambda)(1+n\lambda)}$. Notice that the denominator is always positive. The numerator is linearly increasing in $\bar{a}_j$ and equals zero when $\bar{a}_j = \bar{\theta}_j + \frac{\sigma_\theta^2}{2\sigma}$. Thus, the threshold level $\hat{a}_j = \bar{\theta}_j + \frac{\sigma_\theta^2}{2\sigma}$.

When $\bar{a}_j > \hat{a}_j$, $\frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} > 0 \Rightarrow \exp\left(\frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)}\right) > 1 \Rightarrow \frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} > 1$.

When $\bar{a}_j < \hat{a}_j$, $\frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} < 0 \Rightarrow \exp\left(\frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)}\right) < 1 \Rightarrow \frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} < 1$.

When $\bar{a}_j = \hat{a}_j$, $\frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} = 0 \Rightarrow \exp\left(\frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)}\right) = 1 \Rightarrow \frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} = 1$. ■

Proof of Proposition 5. Recall that b_j^σ is expressed in terms of $\bar{a}_j$ and n as

$$b_j^\sigma = \exp\left[\mu_{jn} + \frac{1}{2}\left(\frac{\upsilon_n^2 + \sigma_\epsilon^2}{\sigma}\right)\right].$$

Manipulating the equation yields

$$\bar{a}_j - \bar{\theta}_j = \frac{\ln b_j^\sigma (1 + \lambda n)}{\lambda n} - \frac{\bar{\theta}_j (1 + \lambda n)}{\lambda n} - \frac{\sigma_\theta^2}{2\sigma\lambda n} - \frac{\sigma_\epsilon^2 (1 + \lambda n)}{2\sigma\lambda n}. \tag{25}$$

Substitute equation (25) into equation (24) to obtain

$$\frac{q_{ijg}(\varphi, \bar{a}_j, n+1)}{q_{ijg}(\varphi, \bar{a}_j, n)} = \exp\left(\frac{\ln b_j^\sigma - \bar{\theta}_j - \frac{\sigma_\theta^2 + \sigma_\epsilon^2}{2\sigma}}{(1 + (1+n)\lambda)}\right).$$

For $b_j^\sigma = \bar{\theta}_j + \frac{\sigma_\theta^2 + \sigma_\epsilon^2}{2\sigma}$ (which corresponds to $\bar{a}_j = \bar{\theta}_j + \frac{\sigma_\theta^2}{2\sigma}$), the exponent equals 1 and thus the scale does not change with n. For $b_j^\sigma > \hat{b}_j^\sigma$, the numerator in the exponent is positive, thus the ratio declines in n. For $b_j^\sigma < \hat{b}_j^\sigma$, the numerator in the exponent is negative, thus the ratio increases in n. ■

Proof of Proposition 6. In proving the result I am going to use the following three equations

$$u^* = \varphi_d^* P Y^{\frac{1}{\sigma-1}} \tag{26}$$

$$u^* = \varphi_x^* P Y^{\frac{1}{\sigma-1}} \frac{1}{\tau} \tag{27}$$

$$K(u^*) = \left(PY^{\frac{1}{\sigma-1}}\right)^{\xi} + N\frac{\left(PY^{\frac{1}{\sigma-1}}\right)^{\xi}}{\tau^{\xi}}. \tag{28}$$

Equation (28) is obtained by manipulating $M = M_d(\varphi_d^*) + N M_x(\varphi_x^*)$, where expression of $M_i(\varphi_i^*)$ in terms of u^* is used (equation (22)). $K(u^*) = \frac{M}{J(\varphi_{min})^{\xi}\xi\int_{u^*}^{+\infty}\frac{1}{u^{\xi+1}}H(\varphi)du}$.

An important thing to note is that u^* is completely determined by the exogenous parameters of the model (excluding τ) by Proposition 3, and thus does not respond to variation in τ. Denote by $X = PY^{\frac{1}{\sigma-1}}$.

The optimal product sales are given by

$$r_{gi}(\varphi, \bar{a}, n, a) = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left(\frac{\varphi}{g^{\alpha}}\right)^{\sigma-1} b^{\sigma-1}\frac{P^{\sigma-1}Y}{\tau^{\sigma-1}}e^{\frac{a}{\sigma}}.$$

Using equations (26) or (27), they can be written as

$$r_{gi}(\varphi, \bar{a}, n, a) = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left(\frac{\varphi}{g^{\alpha}}\right)^{\sigma-1} b^{\sigma-1}\left(\frac{u^*}{\varphi_i^*}\right)^{\sigma-1} e^{\frac{a}{\sigma}}.$$

Thus, the sign of the derivative of $r_{gi}(\varphi, \bar{a}, n, a)$ with respect to τ is the same as the sign of the derivative of φ_i^* with respect to τ, which I compute below.

Total differentiation of equation (28) yields

$$\frac{dX}{X}\frac{\tau}{d\tau} = \frac{N}{\tau^{\xi}+N}.$$

Thus, $0 < \frac{dX}{X}\frac{\tau}{d\tau} < 1$. Total differentiation of equation (26) yields

$$\frac{d\varphi_d^*}{\varphi_d^*}\frac{\tau}{d\tau} = -\frac{dX}{X}\frac{\tau}{d\tau}.$$

Thus, $\frac{d\varphi_d^*}{d\tau} < 0$ implying $\frac{dr_{gd}}{d\tau} > 0$.

Total differentiation of equation (27) yields

$$\frac{d\varphi_x^*}{\varphi_x^*}\frac{\tau}{d\tau} = 1 - \frac{dX}{X}\frac{\tau}{d\tau}.$$

Since $0 < \frac{dX}{X}\frac{\tau}{d\tau} < 1$, $\frac{d\varphi_x^*}{d\tau} > 0$ implying $\frac{dr_{gx}}{d\tau} < 0$.

Consider characterizing the response of a product's sales which are given by

$$q_{gi}(\varphi, \bar{a}, n) = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{\varphi}{g^{\alpha}}\right)^{\sigma} b^{\sigma}\frac{P^{\sigma-1}Y}{\tau^{\sigma}},$$

and using equations (26) or (27), q_{gi} can be written as

$$q_{gi}(\varphi, \bar{a}, n) = \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma} \left(\frac{\varphi}{g^{\alpha}}\right)^{\sigma} b^{\sigma} \left(\frac{u^*}{\varphi_i^*}\right)^{\sigma-1} \frac{1}{\tau}. \tag{29}$$

Differentiate equation (29) with respect to τ to obtain

$$\frac{dq_{gi}}{d\tau} = \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma} \left(\frac{\varphi}{g^{\alpha}}\right)^{\sigma} b^{\sigma} \frac{u^{*(\sigma-1)}}{\varphi_i^{*(\sigma-1)}\tau^2} \left[-(\sigma - 1)\frac{d\varphi_i^*}{\varphi_i^*}\frac{\tau}{d\tau} - 1\right].$$

Since $\frac{d\varphi_x^*}{d\tau} > 0$, $\frac{dq_{gx}}{d\tau} < 0$.

For the domestic market $-(\sigma - 1)\frac{d\varphi_i^*}{\varphi_i^*}\frac{\tau}{d\tau} - 1 = (\sigma - 1)\frac{N}{\tau^{\xi}+N} - 1$. Thus, $\frac{d\varphi_d^*}{d\tau} > 0$ when $\frac{N}{\tau^{\xi}+N} > \frac{1}{\sigma-1}$, and negative otherwise. ■

Table 1: Sources of Brazilian Manufacturing Exports to Argentina, the US, and the World in an Average Year (percent of export sales)

Contribution of	Argentina	US	World
New Exporters	6.4	4.2	2.6
New Products by Incumbent Exporters	7.5	4.5	4.4
Incumbent Products by Incumbent Exporters	86.2	91.5	93

Note: The table reports the share of Brazilian exports in manufacturing products in a given year that arises either from new exporters, or from the sales of products previously not exported by continuing exporters, or from the sales of previously exported products by continuing exporters. A new product is defined at the exporter level. Mean across annual observations.

Table 2: Decomposition of Brazilian Manufacturing Export-Sales Growth to Argentina, the US, and the World in an Average Year (percent)

Margin	Argentina	US	World
Exporter-Turnover	5.25	0.61	0.42
Product-Switching	5.19	0.58	1.12
Intensive	13.30	2.68	1.38
Total	23.73	3.87	2.87

Note: The table reports the decomposition of the growth rate of real exports in manufacturing products by margins of trade. Mean across annual observations.

Table 3: Mean of the Fraction of Export Sales from New Products among Firms Exporting to Argentina, the US, and the World

	Argentina	US	World
Intra-Firm Extensive Margin	17.9	18.4	20.0

Note: The table reports the percentage of an exporter's sales derived from products introduced between two consecutive periods (intra-firm extensive margin). Sample of all surviving exporters is considered.

Table 4: Mean of the Fraction of Export Sales from New Products among Firms Exporting to Argentina, the US, and the World: Decomposition by Export Age

Export Age	Argentina	US	World
2	24.2	24.6	27.1
3	17.5	17.6	19.8
4	15.2	14.4	16.0
5	13.6	12.7	14.3
6	13.7	10.3	14.1
7	13.7	11.8	12.9
8	12.6	10.6	12.7
9	10.2	9.7	11.8
10	9.0	9.7	9.8
11	8.2	6.6	7.6

Note: The table reports the intra-firm extensive margin computed by export-age categories. Sample of all surviving exporters with defined export age is considered.

Table 5: OLS Regression. The Dependent Variable is the Fraction of Sales From New Products

	(1)	(2)	(3)	(4)
Export Age	-0.025***	-0.018***	-0.016***	-0.013***
	(0.001)	(0.001)	(0.0003)	(0.0003)
Log Exports		-0.026***		-0.018***
		(0.001)		(0.0003)
Constant	0.294***	0.561***	0.217***	0.402***
	(0.003)	(0.007)	(0.001)	(0.004)
R^2	0.02	0.05	0.01	0.02
Number of Obs.	56,638	56,638	225,569	225,569
Level of Obs.	Firm-Year	Firm-Year	Firm-Dest.-Year	Firm-Dest.-Year

Note: The table reports the results of OLS regression. The dependent variable is the fraction of an exporter's sales derived from products introduced between two consecutive periods. A sample of surviving exporters is considered.
*** Statistically significant at the 1% level.

Table 6: Mean of the Proportion of Newly Added Products among Firms Exporting to Argentina, the US, and the World

	Argentina	US	World
Percent of new products	28.3	23.1	29.4

Note: The table reports the percentage of currently exported products an exporter added between two consecutive periods. Sample of all surviving firms is considered.

Table 7: Mean of the Fraction of Newly Added Products among Firms Exporting to Argentina, the US, and the World: Decomposition by Export Age

	All Incumbent Exporters			Incumbent Exporters that Added Products		
Export Age	Argentina	US	World	Argentina	US	World
2	33.3	31.0	36.1	66.0	69.0	67.5
3	27.8	25.8	30.6	57.4	62.4	59.1
4	25.8	22.8	27.5	53.0	58.5	54.6
5	25.0	21.6	26.3	49.6	54.3	51.6
6	27.0	20.0	27.5	50.9	53.1	51.5
7	27.5	22.4	26.9	49.1	53.0	49.8
8	25.5	19.5	27.2	46.1	50.4	47.9
9	23.5	20.7	26.9	42.6	48.6	47.9
10	23.0	21.6	25.4	40.9	50.2	46.5
11	22.1	21.0	24.6	40.3	45.4	43.2

Note: The table reports the percentage of currently exported products an exporter added between two consecutive periods computed by export age categories. A sample of surviving exporters with defined export age is considered.

Table 8: OLS Regression. The Dependent Variable is the Fraction of Added Products in an Exporter's Product Mix

	(1)	(2)	(3)	(4)
Export Age	-0.016***	-0.015***	-0.009***	-0.008***
	(0.001)	(0.001)	(0.0003)	(0.0004)
Log Exports		-0.002***		-0.004***
		(0.001)		(0.0003)
Constant	0.370***	0.388***	0.270***	0.314***
	(0.003)	(0.008)	(0.001)	(0.004)
R^2	0.01	0.01	0.003	0.004
Number of Obs.	56,638	56,638	225,569	225,569
Level of Obs.	Firm-Year	Firm-Year	Firm-Dest.-Year	Firm-Dest.-Year

Note: The table reports the results of OLS regression. The dependent variable is the fraction of added products in an exporter's product mix. A sample of surviving exporters is considered.

*** Statistically significant at the 1% level.

Table 9: Data and Simulation Moments

Moment	Data	Simulation
1. Mean of log-sales	11.57	11.57
2. Standard deviation of log-sales	2.63	2.69
3. Growth rate of firm-product sales (percentage points)	1.40	1.31
4. Share of multi-product exporters	0.59	0.46
5. Share of product-switching exporters	0.50	0.34
Criterion		0.16

Note: Statistics are computed at the level of total firm exports (aggregated across destinations). For the first three moments, the mean is taken across firm-year observations. For the last two moments the mean is taken across annual observations.

Table 10: Parameter Values

Parameter	α	γ	σ_θ	σ_ϵ	f
Value	0.15	0.02	2.50	14.33	27,422.10

Table 11: Decomposition of Brazilian exports (percent of export sales)

Margin	Data	Simulation
New Exporters	2.6	0.5
New Products by Incumbent Exporters	4.4	1.7
Incumbent Products by Incumbent Exporters	93	97.8

Table 12: Margins of Export-Sales Growth: Simulation

Growth of exports	Exporter-Turnover Margin	Product-Switching Margin	Intensive Margin
4.8%	0.5%	1.5%	2.8%