

# Wage Rigidity: A Solution to Several Asset Pricing Puzzles.\*

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## Abstract

In standard models wages are too volatile and returns too smooth. We make wages sticky through infrequent resetting, resulting in both (i) smoother wages and (ii) volatile returns. Furthermore, the model produces other puzzling features of financial data: (iii) high Sharpe Ratios, (iv) low and smooth interest rates, (v) time-varying equity volatility and premium, and (vi) a value premium. In standard models, highly pro-cyclical and volatile wages are a hedge. The residual - profit - becomes unrealistically smooth, as do returns. Smoother wages act like operating leverage, making profits more risky. Bad times and unproductive firms are especially risky because committed wage payments are high relative to output.

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# 1 Introduction

In standard production based models wage volatility is far too high relative to the data (e.g. Jermann (1998), Boldrin, Christiano, and Fisher (2001), Kaltenbrunner and Lochstoer (2010), Croce (2012)). These models also fail to match several important features of financial and accounting data: equity volatility is far too low<sup>1</sup>; expected returns and equity volatility are not time varying; the value premium is difficult to explain<sup>2</sup>; and profits are far too smooth while dividends are counter-cyclical and too volatile. We show that the failure to match wage volatility and the failure to match financial data are closely linked. We modify the standard model in a simple way: we reduce the frequency of wage resetting and increase complementarity of labor and capital (in line with empirical estimates). Our key finding is that this change not only makes average wages smoother but also greatly improves the model's ability to match financial and accounting data.

In the standard model wages are equal to the marginal product of labor, which is perfectly correlated with output and is fairly volatile. The standard model fails because wages act as a hedge for the shareholders of the firm. Profits are roughly equal to output minus wages, thus highly volatile and pro-cyclical wages make profits very smooth. Dividends are roughly equal to profits minus investment, because profits are smooth and investment is pro-cyclical, dividends are counter-cyclical. The firm appears too safe and its equity return is too smooth relative to the data. For example, in the standard model we use as the baseline, equity return volatility is below 3% compared to nearly 19% in the data.

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<sup>1</sup>Jermann (1998) and Boldrin, Christiano, and Fisher (2001) generate time-varying expected returns through internal habit, but the bulk of variations in the expected stock returns is due to the volatility of risk-free rate, and the risk-free rate is too volatile relative to the data.

<sup>2</sup>Zhang (2005) generates a sizable value premium in a partial equilibrium model through counter-cyclical price of risk and asymmetric adjustment costs. We find that value premium is negative in 'standard' model without wage rigidity even if we include asymmetric adjustment costs.

Infrequent wage resetting causes the average wage paid by firms to be equal to the weighted average of historical spot wages; this makes the average wage smoother than the marginal product of labor (See, e.g., Shimer (2005), Hall (2006), Gertler and Trigari (2009), and Rudanko (2009) for detailed discussions of wage rigidities in explaining unemployment dynamics). We view infrequent resetting as just one of several possible frictions that result in average wages being smoother than the marginal product of labor. When wages are smoother than the marginal product of labor, they are less of a hedge for the firm's shareholders. Profits, which are the residual after wages have been paid, are more volatile and dividends are pro-cyclical. This leads to a more volatile return on equity. When we combine infrequent wage resetting with labor adjustment costs the model produces equity return volatility of 16%, nearly as high as in the data.

Smooth wages act in the same way as operating (and even financial) leverage. Because equity is the residual, higher leverage implies riskier equity. However, this leverage is not constant through the business cycle. Because wages are smoother than output, leverage due to wages is higher in recessions than in expansions. Consistent with financial data, this leads to higher equity volatility and a higher expected equity premium during bad times. We show that aggregate wage growth can negatively forecast long horizon equity returns in the model and in the data. This happens because during bad (good) times, wages fall (rise) by less than output leading to an increase (decrease) in leverage and therefore equity risk. Our finding echoes Santos and Veronesi (2006) who show that labor income to consumption ratio is a good predictor of long horizon stock returns.

Similarly, the leverage due to wages is not constant in the cross-section. Low productivity firms have very high labor expenses relative to profit. Because low productivity firms are most at need to shed employees during recessions, low productivity firms are especially risky.

Value stocks are less productive than growth stocks and therefore earn a value premium. Our mechanism in generating value premium is different from Zhang (2005) in that we focus on the endogenous operating leverage effect induced by rigid wages which affects value firms more than growth firms, especially in economic downturns, while Zhang (2005) emphasizes the real frictions on firms' investment. Moreover, the growth rate shocks to aggregate productivity in our model are essentially the long-run risk shocks as in Bansal and Yaron (2004), whereas the pricing kernel in Zhang (2005) is effectively habit persistence as in Campbell and Cochrane (1999). To the best of our knowledge, we are the first to embed a realistic cross-section of firms in a general equilibrium model with long run risk style shocks<sup>3</sup>.

*Related literature* While the macroeconomic literature on wages and labor is quite large (for example Pissarides (1979)), there has been surprisingly little work done relating labor frictions to finance; notable examples are Uhlig (2007) and Merz and Yashiv (2007). Our model is most similar to Danthine and Donaldson (2002) who (to our knowledge) were the first to emphasize the operating leverage channel through which smoother wages can lead to higher equity volatility. Our model differs from Danthine and Donaldson (2002) in that 1) the staggered wage contract in our model endogenously implies smooth wages while Danthine and Donaldson (2002) assume an exogenous bargaining and risk sharing between households and the owner of capital; 2) our model is quantitatively closer to the data; 3) our model generates conditional variation in the time series and the cross section of expected returns while Danthine and Donaldson (2002) are silent on these hard to match moments.

Longstaff and Piazzesi (2004) also consider the importance of operating leverage for equity returns and volatility. While theirs is an endowment economy and they do not explicitly refer to their channel as labor income (they call it corporate fraction), the intuition is similar.

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<sup>3</sup>Gala (2011) studies the cross-section in a more standard general equilibrium model.

Dividends are modeled as a small but highly pro-cyclical and volatile component of aggregate consumption. Modeling the dividend this way allows the model to produce both high equity premia and high equity return volatility despite a relatively smooth consumption process.

Gourio (2007) notes that wages are smoother than output and explores the empirical implications of this for cross-sectional asset pricing. Because wages are smooth, profits should be volatile. He finds that profits are most volatile for low market-to-book (value) firms because they have a smaller gap between output and wage. These firms are therefore more risky. A factor model with the market and wage growth as the two factors does a reasonably good job at explaining the cross-section of asset returns.

Recent work by Kuehn, Petrosky-Nadeau, and Zhang (2011) and Li and Palomino (2012) is also closely related to our paper. Kuehn, Petrosky-Nadeau, and Zhang (2011) explore how search frictions in the style of Mortensen and Pissarides (1994) affect asset pricing in a general equilibrium setting with production. Like our model, they find that introducing frictions in the labor market can increase the model's equity volatility. Unlike our model, their channel works mostly through rare events (as in Barro (2006)) during which unemployment can spike up. On the other hand, Li and Palomino (2012) study nominal price and wage rigidity. They find that both types of rigidity increase expected equity returns, but wage rigidities have a larger impact. Shmalz (2012) explores the corporate finance implications of modeling labor as an implicit long term liability and shows that in response to unionization firms hold more cash and set lower financial leverage.

Our paper is also related to the literature on long run risk. While the wage and operating leverage channel can greatly improve equity volatility, it cannot alone bring the Sharpe Ratio of the standard model close enough to the data. Bansal and Yaron (2004) have shown that the combination of a high intertemporal elasticity of substitution and a very persistent

consumption growth rate can deliver a high Sharpe Ratio even with a low risk aversion. Croce (2012) and Kaltenbrunner and Lochstoer (2010) have shown that this can work in a production economy; our model is similar to both of these models but adds infrequent wage resetting. It is important to note that long run risk alone cannot produce time-varying excess returns or volatilities. Bansal and Yaron (2004) devote the second half of their paper to adding an exogenous state variable which controls the volatility of equity returns but is orthogonal to long run risk. Our model is able to produce time varying returns and volatilities endogenously.

Finally, our paper is related to the long literature on wage rigidities and unemployment dynamics. It has been shown that wage rigidities are crucial to explain U.S. labor market dynamics, e.g., Shimer (2005), Hall (2006), Gertler and Trigari (2009), Pissarides (2009), etc.<sup>4</sup> For example, Hall wrote, "The incorporation of wage stickiness makes employment realistically sensitive to driving forces." Our paper differs from these macro-papers in that we study asset pricing implications of staggered wage setting, while the models in labor economics fail to match the asset prices observed in the data; this is a problem endemic to most standard models, as observed by Mehra and Prescott (1985).

The rest of the paper is laid out as follows. In section 2 we write down the model. In section 3 we compare the results of a standard model to a model with infrequent wage resetting. We show that our preferred model can nearly match the equity volatility and equity premium observed in the data, as well as produce time-varying risk and a value premium.

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<sup>4</sup>In addition to wage rigidities, search and match frictions are also crucial to capture unemployment dynamics.

## 2 Model

In this section we describe our model. We begin with the household's problem. We then outline the firm's problem, the economy's key frictions are described there. Finally we define equilibrium.

### 2.1 Households

In the model financial markets are complete, therefore we consider one representative household who receives labor income, chooses between consumption and saving, and maximizes utility as in Epstein and Zin (1989).

$$U_t = \max \left( (1 - \beta)C_t^{1-\frac{1}{\psi}} + \beta E_t[U_{t+1}^{1-\theta}]^{\frac{1-\frac{1}{\psi}}{1-\theta}} \right)^{\frac{1}{1-\frac{1}{\psi}}} \quad (1)$$

$$W_{t+1} = (W_t + N_t * \bar{w}_t - C_t)R_{t+1} \quad (2)$$

where  $\bar{w}_t$  is the average wage in the economy,  $C_t$  is average consumption,  $W_t$  is the wealth held by the average household, and  $R_{t+1}$  is the return to a portfolio over all possible financial securities. For simplicity, we assume that aggregate labor supply is inelastic:  $N_t = 1$ <sup>5</sup> Risk aversion is given by  $\theta$  and the intertemporal elasticity of substitution by  $\psi$ .

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<sup>5</sup>Although firms can adjust labor in our model, aggregate labor supply is fixed. In a frictionless model adding non-trivial labor supply does not require an additional state variable because labor supply can be solved for analytically through the household's first order condition. That is not the case in our model as we would need to keep track of lagged aggregate labor. Unfortunately adding an extra state variable is very difficult as the model's numerical solution is already very complex.

## 2.2 Firms

The interesting frictions in the model are on the firm's side. Firms (indexed by  $i$ ) choose investment and labor to maximize the present value of future dividend payments where the dividend payments are equal to the firm's output net of investment, wages, operating costs and adjustment costs. Output is produced from labor and capital. Firms hold beliefs about the discount factor  $M_{t+1}$ , which is determined in equilibrium.

### 2.2.1 The Wage Contract

In standard production models wages are reset each period and employees receive the marginal product of labor. We assume that any employee's wage will be reset this period with probability  $1 - \mu$ .<sup>6,7</sup> When  $\mu = 0$  our model is identical to models without rigidity: all wages are reset each period, each firm can freely choose the number of its employees, and each firm chooses  $N_t^i$  such that its marginal product of labor is equal to the wage. When  $\mu > 0$  we must differentiate between the spot wage ( $w_t$ ) which is paid to all employees resetting wages this year, the economy's average wage ( $\bar{w}_t$ ), and the firm's average wage ( $\bar{w}_t^i$ ). The firm's choice of employees may no longer make the marginal product equal to either average or spot wages. For example, in times of relatively high wages, firms are hesitant to hire many employees because this will saddle them with long term wage obligations. On the other hand, in times of relatively low wages, firms may hire extra employees since this will result in lower long term labor expenses.

When a firm hires a new employee in a year with spot wage  $w_t$ , with probability  $\mu$  it must

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<sup>6</sup>Note that this is independent of length of employment. This allows us to keep track of only the number of employees and the average wage as state variables, as opposed to keeping track of the number of employees and the wage of each tenure.

<sup>7</sup>Our way of modeling wage rigidity in spirit is similar to Gertler and Trigari (2009), but for tractability reasons, we do not model search and match frictions in the model.



pay this employee the same wage next year; on average this employee will keep the same wage for  $\frac{1}{1-\mu}$  years. All resetting employees come to the same labor market and the spot wage is selected to clear markets. The firm chooses its labor force  $N_t^i$  each period, however it is unable to fire employees under contract. In particular, firms can increase its labor force without limit but decrease up to a limit

$$N_t^i \geq \mu N_{t-1}^i. \quad (3)$$

These conditions lead to a natural formulation of the firm's average wage as the weighted average of the previous average wage and the spot wage:

$$\bar{w}_t^i N_t^i = w_t(N_t^i - \mu N_{t-1}^i) + \bar{w}_{t-1}^i \mu N_{t-1}^i \quad (4)$$

Here  $N_t^i - \mu N_{t-1}^i$  is the number of new employees the firm hires at the spot wage and  $\mu N_{t-1}^i$  is the number of tenured employees with average wage  $\bar{w}_{t-1}^i$ .

### 2.2.2 Technology

Firm  $i$ 's output is given by

$$Y_t^i = Z_t^i \left( \alpha (K_t^i)^\eta + (1 - \alpha) (Z_t N_t^i)^{\eta\rho} \right)^{\frac{1}{\eta}}. \quad (5)$$

Output is produced with CES technology from capital ( $K_t^i$ ) and labor ( $N_t^i$ ) where  $Z_t$  is labor augmenting aggregate productivity,  $Z_t^i$  is the firm's idiosyncratic productivity,  $\rho$  determines the degree of return to scale (constant return to scale if  $\rho = 1$ ),  $\frac{1}{1-\eta}$  is the elasticity of substitution between capital and labor (Cobb-Douglas production if  $\eta = 0$ ), and  $(1 - \alpha)\rho$  is

labor share in production.

The process for  $Z_t$  is non-stationary but its growth rate is stationary, this is in the spirit of the exogenous shock process in long run risk models of Bansal and Yaron (2004), Kaltenbrunner and Lochstoer (2010), and Croce (2012). The process for  $Z_t^i$  is stationary. Both are described in more detail in the calibration section.

### 2.2.3 Accounting

The equation for profit is

$$\Pi(K_t^i) = Y_t^i - \bar{w}_t^i N_t^i - \Psi_t^i \quad (6)$$

$\Pi(K_t^i)$  is profit, given by output less labor and operating costs<sup>8</sup>. Operating costs are defined as  $\Psi_t^i = f * K_t^i$ ; they depend on aggregate (but not firm specific) capital<sup>9</sup>. Labor costs are  $\bar{w}_t^i N_t^i$ .

The dividends paid by the firm is

$$D_t^i = \Pi(K_t^i) - I_t^i - \Phi(I_t^i, K_t^i) - \Xi(N_t^i, N_{t-1}^i), \quad (7)$$

which is profit less investment, capital adjustment costs and labor adjustment costs. Capital adjustment costs are given by  $\Phi(I_t^i, K_t^i) = v_t^i \left( \frac{I_t^i}{K_t^i} \right)^2 K_t^i$  where  $v_t^i = v^+$  if  $\frac{I_t^i}{K_t^i} > 0$  and  $v_t^i = v^-$  otherwise. Asymmetric costs have been shown to quantitatively help with the value premium by Zhang (2005). Labor adjustment costs are given by  $\Xi(N_t^i, N_{t-1}^i) = \xi(N_t^i - N_{t-1}^i)^2 w_t$ .

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<sup>8</sup>As there are no taxes or explicit interest expenses we do not differentiate between operating profit and net income and simply call it profit.

<sup>9</sup>Because productivity is non-stationary all model quantities are non-stationary and we cannot allow for a constant operating cost  $f$  as it would grow infinitely large or infinitely small relative to other quantities. All quantities in the model must be scaled by something that is co-integrated with the productivity level, such as aggregate capital. We have also experimented with using the spot wage ( $w_t$ ) or aggregate productivity ( $Z_t$ ) and the results appear insensitive to this.

### 2.2.4 The Firm's Problem

We will now formally write down firm i's problem. The firm maximizes the present discounted value of future dividends

$$V_t^i = \max_{I_t^i, N_t^i} E_t \left[ \sum_{j=0, \infty} M_{t+j} D_{t+j}^i \right], \quad (8)$$

subject to the standard capital accumulation equation

$$K_{t+1}^i = (1 - \delta) K_t^i + I_t^i, \quad (9)$$

as well as equations (3), (4), (5), (6), and (7).

We define the firm's return on capital as  $R_{t+1}^K = \frac{V_{t+1}}{V_t - D_t}$ . However, real world firms are financed by both debt and equity, with equity being the riskier, residual claim. To compare the model's equity return to empirical equity returns we lever the return on capital using the 2nd proposition of Modigliani and Miller (1958):  $R_{t+1}^E - R_t^f = (1 + \lambda)(R_{t+1}^K - R_t^f)$  where  $\lambda$  is the firm's debt to equity ratio<sup>10</sup>.

## 2.3 Equilibrium

We assume that there exists some underlying set of state variables  $S_t$  which is sufficient for this problem. Each firm's individual state variables are given by the vector  $S_t^i$ . Because the household is a representative agent, we are able to avoid explicitly solving the household's maximization problem and simply use the first order conditions to find  $M_{t+1}$  as an analytic

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<sup>10</sup>In doing so we assume firms keep leverage constant. We estimate the debt to equity ratio to be 0.59. Boldrin, Christiano, and Fisher (1999) include leverage in a production economy in the same way and provide additional discussion.

function of consumption or expectations of future consumption. For instance, with CRRA utility,  $M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\theta}$  while for Epstein-Zin utility  $M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{U_{t+1}}{E_t[U_{t+1}^{1-\theta}]^{\frac{1}{1-\theta}}} \right)^{\frac{1}{\psi}-\theta}$ .<sup>11</sup>

Equilibrium consists of:

- Beliefs about the transition function of the state variable and the shocks:  $S_{t+1} = f(S_t, Z_{t+1})$
- Beliefs about the realized stochastic discount factor as a function of the state variable and realized shocks:  $M(S_t, Z_{t+1})$
- Beliefs about the aggregate spot wage as a function of the state variable:  $w(S_t)$
- Firm policy functions (which depend on  $S_t$  and  $S_t^i$ ) for labor demand  $N_t^i$  and investment  $I_t^i$

It must also be the case that given the above policy functions all markets clear and the beliefs turn out to be rational:

- The firm's policy functions maximize the firm's problem given beliefs about the wages, the discount factor, and the state variable.
- The labor market clears:  $\sum N_t^i = 1$
- The goods market clears:  $C_t = \sum (\Pi_t^i + \Psi_t^i + \bar{w}_t^i N_t^i - I_t^i) = \sum D_t^i + \bar{w}_t^i N_t^i + \Phi_t^i + \Xi_t^i + \Psi_t^i$ .

Note that here we are assuming that all costs are paid by firms to individuals and are therefore consumed, the results look very similar if all costs are instead wasted.

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<sup>11</sup>Given a process for  $C_t$  we can recursively solve for all the necessary expectations to calculate  $M_{t+1}$ . The appendix provides more details.

- The beliefs about  $M_{t+1}$  are consistent with goods market clearing through the household's Euler Equation<sup>12</sup>.
- Beliefs about the transition of the state variables are correct. For instance if aggregate capital is part of the aggregate state vector, then it must be that  $K_{t+1} = (1 - \delta)K_t + \sum I_t^i$ .

## 3 Results

### 3.1 Calibration

We solve the model at an annual frequency using a variation of the Krusell and Smith (1998) algorithm, we discuss the solution method in the appendix. The model requires us to choose the preference parameters:  $\beta$  (time discount factor),  $\theta$  (risk aversion),  $\psi$  (intertemporal elasticity of substitution); the technology parameters:  $\alpha$  and  $\rho$  (jointly determine labor share in output and degree of return to scale),  $\frac{1}{1-\eta}$  (elasticity of substitution between labor and capital),  $\delta$  (depreciation),  $f$  (operating cost); the adjustment cost parameters:  $\nu^+$  (upward capital adjustment cost),  $\nu^-$  (downward capital adjustment cost),  $\xi$  (labor adjustment cost). Finally we must choose our key parameter  $\mu$  which determines the frequency of wage resetting. In Table 1 we present parameter choices for five models of interest: (i) a standard model with Cobb-Douglas technology where all wages are reset each year ( $\mu = 0$ ); (ii) a model with Cobb-Douglas technology where wages are reset once every four years on average ( $\mu = 0.75$ ); (iii) a model with a calibrated elasticity of substitution between labor and capital but where all wages are reset each year; (iv) a model with a calibrated elasticity of substitution between

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<sup>12</sup>For example, with CRRA  $M_{t+1} = \beta \left( \frac{\sum D_{t+1,i} + \bar{w}_t N_{t+1,i} + \Phi_{t+1,i} + \Xi_{t+1,i} + \Psi_{t+1,i}}{\sum D_{t,i} + \bar{w}_t N_{t,i} + \Phi_{t,i} + \Xi_{t,i} + \Psi_{t,i}} \right)^{-\theta}$

labor and capital where wages are reset once every four years on average; (v) and finally a model with a calibrated elasticity of substitution between labor and capital where wages are reset once every four years on average and where firms face labor adjustment costs ( $\xi = 0.5$ ). Additionally we must choose a process for aggregate productivity shocks and idiosyncratic productivity shocks. Below we justify our choices of these parameters.

**Preferences** We set  $\beta = 0.995$  to match the level of the risk free rate. We set  $\theta = 8$  to get a reasonably high Sharpe Ratio while keeping risk aversion within the range recommended by Mehra and Prescott (1985). The intertemporal elasticity of substitution  $\psi$  also helps with the Sharpe Ratio, it is set to 2; Bansal and Yaron (2004) show that values above 1 are required for the long run risk channel to match asset pricing moments.

**Technology** The technology parameters are fairly standard and we use numbers consistent with prior literature. We jointly choose  $\alpha = 0.25$  and  $\rho = 0.853$  so that labor share and profit share are consistent with empirical estimates<sup>13</sup>. Similarly we set  $\eta = -1$  to match empirical estimates of the elasticity of substitution between labor and capital<sup>14</sup>, however we also experiment with Cobb-Douglas technology ( $\eta = 0$ ). We set  $\delta = 0.1$  to match annual depreciation.

**Operating Cost**  $\Psi_t = f * K_t$  is a fixed cost from the perspective of the firm, however it depends on the aggregate state of the economy, in particular aggregate capital. We choose  $f$  to match the average market-to-book ratio in the economy<sup>15</sup>, which we estimate to be

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<sup>13</sup>Labor share is  $(1 - \alpha)\rho = 0.64$ . Profit share is  $(1 - \alpha)(1 - \rho) = 0.11$  implying returns to scale of 0.89. Gomes (2001) uses 0.95 citing estimates of just under 1 by Burnside (1996). Burnside, Eichenbaum, and Rebelo (1995) estimate it to be between 0.8 and 0.9. Khan and Thomas (2008) use .896, justifying it by matching the capital to output ratio. Bachmann, Caballero, and Engel (2011) use 0.82, justifying it by matching the revenue elasticity of capital.

<sup>14</sup>The elasticity of substitution between labor and capital is  $\frac{1}{1-\eta} = 0.5$ . In a survey article Chirinko (2008) argues this elasticity is between 0.4 and 0.6.

<sup>15</sup>This is the market-to-book ratio for the entire firm value (the enterprise value). From Compustat we calculate the market-to-book ratio for equity to be 1.64 and the book debt to market equity ratio to be 0.59.

Table 1: Calibration

All model parameters are listed in this table. Note that most parameters are shared by all models and only five parameters ( $\eta$ ,  $v^+$ ,  $f$ ,  $\mu$ , and  $\xi$ ) vary across models.

Parameter	Description	I	II	III	IV	V
		Cobb-Douglas	Cobb-Douglas	Calibrated CES	Calibrated CES	Calibrated CES
		Frequent Resetting	Infrequent Resetting	Frequent Resetting	Infrequent Resetting	Infrequent Resetting
						Labor Adj Cost
Preferences						
$\beta$	Time Preference	0.995	0.995	0.995	0.995	0.995
$\theta$	Risk Aversion	8	8	8	8	8
$\psi$	IES	2	2	2	2	2
Production						
$(1 - \alpha)\rho$	Labor Share	0.64	0.64	0.64	0.64	0.64
$\alpha + \rho - \alpha\rho$	Returns to Scale	0.89	0.89	0.89	0.89	0.89
$\frac{1}{1-\eta}$	Labor Capital Elasticity	1.00	1.00	0.50	0.50	0.50
$\delta$	Depreciation	0.10	0.10	0.10	0.10	0.10
$v^+$	Upward Adj. Cost	0	0	0.80	1.10	1.50
$\frac{v^+}{v^-}$	Asymmetry in Adj.	3	3	3	3	3
$f$	Operating Cost	0.039	0.036	0.061	0.065	0.057
$\mu$	Probability no Resetting	0	0.75	0	0.75	0.75
$\xi$	Labor Adj. Cost	0	0	0	0	0.50

1.34. While we think it is realistic for this cost to increase when aggregate capital is higher (during expansions), the results are not sensitive to this assumption. The results look very similar when  $\Psi_t$  is simply growing at the same rate as the economy<sup>16</sup>.

**Capital Adjustment Costs** Within each model, we choose capital adjustment costs ( $\nu^+$  and  $\nu^-$ ) to match the volatility of aggregate investment. Models with a higher  $\mu$  (smoother wages) require a higher adjustment cost for investment volatility to match the data, therefore the level of adjustment cost is different across models.

Zhang (2005) argues that downward adjustment costs are higher than upward adjustment costs and shows this can be helpful in explaining the value premium puzzle; because empirical evidence is scarce, he sets their ratio  $\frac{\nu^-}{\nu^+} = 10$ . We set  $\frac{\nu^-}{\nu^+} = 3$ . Higher adjustment costs always help increase equity volatility and the value premium but they decrease aggregate investment volatility. This restriction on matching aggregate investment limits how much work capital adjustment costs can do in helping to match financial moments. However, for a given level of aggregate investment volatility, we find that asymmetric adjustment costs lead to somewhat higher equity volatility and value premium than symmetric costs.

**Labor Adjustment Costs** In the model with  $\mu = 0.75$  and no labor adjustment cost ( $\xi = 0$ ) the cross-sectional variation in employment is far too high. Therefore, we choose the labor adjustment cost ( $\xi = 0.5$ ) to roughly match this cross-sectional variation. One issue with matching cross-sectional moments is determining whether the scale of a production unit in our model is analogous to a plant, a firm, or something in between; this is closely related to the calibration of idiosyncratic shocks, which is discussed below. While both the variation in employment growth and the variation in investment rate are likely to be scale dependent,

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Sweeney, Warga, and Winters (1997) find that outside of the Volcker period aggregate market to book values for debt are very close to one. These numbers imply a market to book of 1.34 for enterprise value.

<sup>16</sup>Recall that the model is non-stationary, therefore  $\Psi_t$  cannot be a constant and must be scaled by something that is cointegrated with the size of the economy.



their ratio is less likely to be so. Thus, to avoid issues of scale, we will target the ratio of volatility of employment growth to the volatility of investment rate, this quantity is roughly 4 in the data.

The total cost of adjustment (labor and capital) in our model with highest costs is close to 2% of output. We view this as reasonable, this also falls towards the low end of estimates provided in Hamermesh and Pfann (1996).

**Productivity Shocks** In order for the long run risk channel to produce high Sharpe ratios, aggregate productivity must be non-stationary with a stationary growth rate. We specify the growth rate of aggregate productivity ( $\Delta Z_t$ ) to be a symmetric 3-state Markov process with autocorrelation 0.5, unconditional mean of 0.02, and unconditional standard deviation of 0.037. We choose these numbers to roughly match the autocorrelation, growth rate, and standard deviation of output. Aggregate productivity is then  $Z_{t+1} = Z_t + \Delta Z_{t+1}$  which is consistent with long run risk<sup>17</sup>.

The volatility of idiosyncratic productivity shocks  $Z_t^i$  depends on the model's scale, that is which real world production unit (firm, plant) is analogous to the model's production unit. There is no consensus on the right scale to use, for example the annual autocorrelation and unconditional standard deviation are 0.69 and 0.40 in Zhang (2005), 0.62 and 0.19 in Gomes (2001), 0.86 and 0.04 in Khan and Thomas (2008), while Pastor and Veronesi (2003) estimate that the volatility of firm-level profitability rose from 10% per year in the early 1960s to 45% in the late 1990s. We have experimented with various idiosyncratic shocks and find that our aggregate results are virtually unaffected by the size of these shocks. In our model  $Z_t^i$  is a 2-state Markov process with autocorrelation and unconditional standard deviation of 0.6 and 0.4 respectively.

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<sup>17</sup>Our process for TFP growth is analogous to a discretized AR(1) process, Bansal and Yaron (2004) use a slightly more complicated formulation where consumption growth is ARMA(1,1).

**Frequency of wage resetting** In standard models wages are reset once per period and employees receive the marginal product of labor as compensation. This corresponds to the  $\mu = 0$  case. However, wages are far too volatile in these models relative to the data. We choose the frequency of resetting to roughly match the volatility of wages in the data. This results in  $\mu = 0.75$  or an average resetting frequency of four years.

We also believe that this number is realistic, for example assistant professors, investment bankers, and corporate lawyers all wait approximately this long to be promoted. Rich and Tracy (2004) estimate that a majority of labor contracts last between two and five years with a mean of three years and cite several major renewals (United Auto Workers, United Steel Workers) which are at the top of the range. Even if explicit contracts are written for a shorter period than our calibration (or not written at all), we believe that four years is a reasonable estimate of how long the real wage of many employees stays unchanged. For example, if the costs of replacing employees (for employers) and the costs of finding a new job (for employees) are high, the status quo will remain, keeping wages the same without an explicit contract. Another example are workers who receive small raises every year (keeping their real wage constant or growing slowly), these workers do not experience major changes to their income until they are promoted, or let go, or move to another job. Hall (1982) estimates an average job duration of eight years for American workers, Abraham and Farber (1987) estimate similar numbers just for non-unionized workers (presumably unionized workers have even longer durations).

### 3.2 The Standard Model

In this section we will discuss the model where production is Cobb-Douglas and wages are reset once per year ( $\mu = 0$ ). This is a standard real business cycle model with the addition of

long run risk. This model is most similar to Croce (2012) and Kaltenbrunner and Lochstoer (2010). Like other RBC models (for example see Prescott (1986)), this model does a good job at matching most standard macroeconomic moments, this can be seen in Panel B of Table 2. One important exception to this model’s success in reproducing macro-economic moments is wage behavior, which is in the fourth row. In this model the wage is equal to the marginal product of labor. The resulting volatility of the wage is twice as high as in the data and it is perfectly correlated with output, compared to a correlation of only 0.22 in the data. Because labor income comprises such a large fraction of output, this flaw is very significant quantitatively and is responsible for many of this model’s failures at matching financial data; these are discussed below. Our goal is to fix this flaw.

Unlike the first generation of RBC models, which did a very poor job matching financial moments (the well known Equity Premium Puzzle), this model can produce a high Sharpe Ratio through the long run risk channel. First proposed by Bansal and Yaron (2004) in an endowment economy and later incorporated into a production economy by Croce (2012) and Kaltenbrunner and Lochstoer (2010), the long run risk channel makes the economy appear risky to households because (i) a high intertemporal elasticity of substitution makes households care not only about instantaneous shocks to consumption growth but also shocks to expectations of future consumption growth, (ii) shocks to the growth rate (as opposed to the level in standard models) of productivity are persistent causing expectations of future consumption growth to vary over time. As can be seen in Panel C of Table 4, the model’s Sharpe Ratio is 0.38 despite a risk aversion of only 8; this is similar to the 0.36 in the data. For comparison, most models without long run risk produce Sharpe Ratios below 0.1 unless risk aversion is significantly higher.

However this model still has several important flaws. Most evident is the volatility of

equity returns which is nearly 19% in the data but below 3% in the model (second row in Panel C of Table 4). Because the volatility of equity is so low, the equity premium (which is the Sharpe Ratio multiplied by the volatility of equity) is also quite low.

Another criticism of long run risk models, as well as standard real business cycle models, is that they cannot endogenously produce variations in risk premia or equity volatility across the business cycle. As can be seen in Panel D of Table 4, in the data (first row) the expected equity premium and equity volatility during bad times are more than twice as high as during good times. The standard model (second row) produces virtually no variation in either of these quantities. To fix this, Bansal and Yaron (2004) introduce stochastic volatility into their model through an additional state variable. As will be seen below, a smoother wage endogenously produces variation in the volatility of equity and the equity premium.

Additionally, the standard model does very poorly on several important accounting moments, as can be seen in Panels A and B of Table 4. The model has profit volatility that is too low and dividend volatility that is far too high. For example the volatility of dividend growth is 4.3% in the data but 193% in the model! Furthermore, dividends in the model are counter-cyclical while they are highly pro-cyclical in the data. A smoother wage will result in improved model performance on these accounting moments.

Finally, the standard model performs poorly when we consider cross-sectional asset pricing. The well known value premium puzzle is that low market-to-book (value) stocks have higher average returns than high market-to-book (growth) stocks. However, the opposite is true in the standard model: growth stocks have higher average returns. These results are in Panel C of Table 4. Here too, smoother wages will result in an improvement relative to the standard model.

### 3.3 Elasticity of substitution between capital and labor

Traditionally production based models have used Cobb-Douglas production functions ( $\eta = 0$ ). This implies that the elasticity of substitution between labor and capital ( $\frac{1}{1-\eta}$ ) is one. Empirical estimates of this elasticity are below one ( $\eta < 0$ ). A survey article by Chirinko (2008) discusses various efforts to estimate this elasticity and argues it is between 0.4 and 0.6; we follow this by setting it to 0.5 in our best model. A lower elasticity strengthens complementarity between labor and capital while a higher elasticity makes them substitutes<sup>18</sup>.

Consider a model where wages are frequently reset ( $\mu = 0$ ), in this case the wage is still equal to the marginal product of labor, regardless of the elasticity of substitution between labor and capital. The marginal product of labor is:

$$\frac{\partial Y_t}{\partial N_t} = \rho(1 - \alpha)Y_t^{1-\eta}Z_t^{\rho\eta}N_t^{\rho\eta-1}$$

To get intuition for how  $\eta$  affects the wage we perform a simple experiment. Holding constant  $N_t$  and  $K_t$  while letting  $Z_t$  vary, we compute the volatility of output and of the marginal product of labor. We present a ratio of the two volatilities and their correlation in Table 3. As the complementarity between capital and labor gets stronger, the marginal product of labor (and equivalently the wage) become less volatile relative to output. Nevertheless, the marginal product of capital and output are nearly perfectly correlated even for extreme values of  $\eta$ .

Note that empirical estimates suggest that capital and labor are indeed compliments with  $\eta = -1$ . Thus, even without other frictions, a properly calibrated  $\eta$  can improve the volatility of wages in the model. This is evident in Panel D of Table 2, where we present

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<sup>18</sup>For output defined as in our model,  $\lim_{\eta \rightarrow -\infty} Y_t = \min(K_t, Z_t N_t)$  making labor and capital perfect compliments and  $\lim_{\eta \rightarrow \infty} Y_t = \max(K_t, Z_t N_t)$  making them perfect substitutes.

a model with  $\eta = -1$  but frequent resetting ( $\mu = 0$ ). Here the volatility of wages falls to 1.59%, compared to 1.71% in the standard model.

This also helps to improve the model’s asset pricing and accounting performance presented in Table 4. Comparing the second ( $\eta = 0$ ) and fourth ( $\eta = -1$ ) rows we see that the model’s profits are now more volatile (Panel A), dividends are less volatile and less counter-cyclical (Panel B), stock returns are more volatile (Panel C) and are conditionally higher in bad times (Panel D). In particular, the volatility of equity returns rises to 5.78% compared to 2.93% in the standard model.

All of the above changes bring the model closer to the data. The reason for these improvements is that we have broken the tight link between wages and output present in the standard model. In the next section we introduce infrequent wage setting, it will further break this link and will further improve the model’s performance. Because the intuition for the two channels is very similar, we hold off discussion until the next section.

### 3.4 Infrequent resetting of wages

In this section we will discuss the models in which wages are reset less frequently than once per year ( $\mu = 0.75$ ). We will show that this fixes or greatly reduces all of the problems with the standard model listed above.

Because wages are set infrequently, the average wage is no longer equal to the marginal product of labor but rather a weighted average of past spot wages, this results in average wages being much smoother than the marginal product of labor. We believe that almost any model in which average wages are smoother than the marginal product of labor will have results qualitatively similar to those discussed below. One recent example is Kuehn, Petrosky-Nadeau, and Zhang (2011) where search frictions lead to sticky wages. We view

infrequent wage setting as one of several mechanisms responsible for the relatively smooth wages in the data.

### 3.4.1 Unconditional Asset Pricing Moments

When  $\mu > 0$  there are two relevant wages. The average employee who did not reset receives the average wage from the previous period  $\bar{w}_{t-1}$ . All resetting employees receive the spot wage  $w_t$ , which clears the labor market. The average wage is a weighted average between last year's wage and this year's spot wage. Consistent with the empirical findings of Pissarides (2009), in our model the spot wage is more volatile than the average wage. Despite this, the average wage is still smoother than the marginal product of labor. The fourth row in Table 2 shows that all models with infrequent resetting (Panels C,E,F) are able to match the volatility of wages in the data. Recall that reducing the elasticity between labor and capital also enabled the model to decrease wage volatility, however it could not reduce the correlation between wages and output. On the other hand, infrequent resetting decreases both volatility and correlation. In our best model (Panel F), the correlation between wages and output is 0.81, still short of 0.22 in the data, but an improvement on perfect correlation in standard models (Panels B,D)<sup>19</sup>.

Profits and dividends are presented in Panels A and B of Table 4. Profits are approximately equal to output minus wages. In a standard model wages are highly volatile and highly pro-cyclical (the marginal product of labor is perfectly correlated with output). This results in profits being too smooth. Dividends are approximately equal to profits minus investment. Because profits are relatively small in magnitude while investment is pro-cyclical,

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<sup>19</sup>Because our model overestimates the correlation, it likely underestimates the importance of the wage-leverage channel for asset pricing. A simple fix is to include an additional shock to the average wage, which would further disassociate it from output. However, this will result in longer computation times and make the model less transparent.

dividends in a standard model are counter-cyclical and highly volatile, exactly the opposite of what we observe in the data. Because profits are smooth and dividends are counter-cyclical, the firm's equity is also very smooth in standard models. In other words, pro-cyclical wages act as a hedge for the firm's shareholders, making equity seem very safe.

When wages become smoother and less correlated with the marginal product of labor, profits become more volatile and more pro-cyclical relative to the standard model. Volatile and pro-cyclical profits lead to pro-cyclical dividends. We can see that complementarity between labor and capital alone (fourth row), and infrequent wage setting alone (third row) each increase the volatility of profits (Panel B), decrease the volatility of dividends (Panel C), and make dividends more pro-cyclical (Panel C). In models combining the two (fifth and sixth rows) profit volatility is near the data and dividends exhibit the right amount of pro-cyclicality. Dividends are also less volatile than the standard model, although still more volatile than the data<sup>20</sup>.

The relationship between wages, profits, and dividends is also evident in figure 1 where we plot impulse responses to a positive productivity shock for a frictionless model and for our best model. In panel A, wages respond much slower to a productivity shock in our best model, needing around ten years to fully catch up to the frictionless model. On the other hand, the short term jump in profits (Panel B) in our best model is three times that of the frictionless model. Finally, dividends (Panel C) actually fall in response to a positive productivity shock in the frictionless model because profit is relatively small and smooth while investment is pro-cyclical. In our best model dividends respond positively to a positive productivity shock

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<sup>20</sup>There are many reasons why firms choose to smooth dividends in the real world that are outside of our model. Furthermore, the model dividends are not directly comparable to real world dividends because model dividends include all cash flows to equity, such as share repurchases and equity issuance. A dividend adjustment cost would result in smoother dividends, however this would both complicate the numerical solution and reduce transparency.



and the magnitude is much smaller than in the frictionless model.

With smoother wages no longer being as strong of a hedge, equity volatility is closer to the data as well. Panel C of Table 4 shows that the model with infrequent resetting but no labor adjustment costs has an equity volatility of 13.13% compared to 2.89% in the standard model. Adding labor adjustment costs makes equity even riskier because adjusting down in a time of high wages is a less attractive option. Our best model, combining infrequent resetting and labor adjustment costs, has an equity volatility of 15.58% which is close to the 18.85% in the data. A common shortcoming of production models that were able to produce high equity volatilities has been that they also implied volatile risk free rates (for example [Jermann \(1998\)](#) and [Boldrin, Christiano, and Fisher \(2001\)](#)). Note that the risk free rate is sufficiently smooth in all of our models, this is due to our choice of the intertemporal elasticity of substitution. Furthermore, due to the high volatility of equity, our model is not only able to match the high Sharpe Ratio (like [Croce \(2012\)](#) and [Kaltenbrunner and Lochstoer \(2010\)](#)) but also (nearly match) the actual equity premium. In our best model, combining infrequent resetting with labor adjustment costs results in an equity premium of 5.92% compared to 6.76% in the data. On the other hand the standard model, despite having a high Sharpe Ratio, has an equity premium of only 1.08%.

### **3.4.2 Conditional Asset Pricing Moments**

It is well known that financial moments exhibit conditional variation. The volatility of equity returns tends to be autocorrelated, it is also higher in recessions than expansions. For example, in our sample volatility was 19.34% following periods of low GDP growth and 16.61% following periods of high GDP growth. While volatility is related to the quantity of risk, the price of risk appears to be higher during recessions as well. The combination of high

volatility and high Sharpe Ratios causes expected excess equity returns to be much higher during recessions than during expansions. A large literature has documented that expected returns are predictable, with business cycle related variables such as the term spread, the default spread, the dividend yield, and the consumption wealth ratio all having predictive power.

In our model wages act as operating leverage, making equity (the residual) riskier. However because wages are smoother than output, the amount of operating leverage is time varying. In particular wages are relatively high during bad times, making bad times especially risky. Panel D of Table 4 compares equity volatility and equity premia during bad times (bottom 25% of GDP growth) and good times (top 25% of GDP growth). Although the model cannot produce time-varying Sharpe Ratios, the model does produce higher volatility and higher equity risk during bad times, as in the data. The expected 5 year equity returns (annualized) are 6.13% and 4.82% in recessions versus expansions in our best model, compared to 9% and 3.81% in the data.

While GDP growth is a simple predictor, our model suggests that a more natural predictor should be related to wages and operating leverage. In particular, wage growth should be negatively related to expected excess returns. Periods of high wage growth (expansions) are times of relatively low operating leverage because output grows by more than wages; these are times when expected excess returns are low. Conversely, periods of low wage growth (recessions) are times of relatively high operating leverage because output falls by more than wages; these are times when expected excess returns are high.

We find that past wage growth is a very good predictor of expected returns in our model as well as in the data. Table 5 reports results from long horizon regressions where the equity return in excess of the risk free rate over the following  $T$  years is regressed on wage growth

realized today. The pattern in our model is similar to the data, although the coefficients are smaller in magnitude. In both model and data the magnitude of coefficients,  $R^2$ , and t-statistics all rise with horizon. Comparing a one year to a ten year horizon, the coefficient on wage growth rises from -0.72 to -5.60 in our model and -2.44 to -18.65 in the data; the  $R^2$  rises from 0.03 to 0.21 in our model and 0.02 to 0.30 in the data; the t-statistic rises from 0.63 to 2.58 in our model and 1.60 to 3.04 in the data. In a companion paper we show that traditional variables known to forecast long-horizon returns (dividend yield, term spread, default spread, and the consumption wealth ratio) do not subsume wage growth when considered simultaneously.

### 3.4.3 Cross-sectional Asset Pricing Moments

The value premium puzzle is another empirical anomaly that is difficult for models to explain. Stocks with low market-to-book ratios (value stocks) have higher average returns than stocks with high market-to-book ratios (growth stocks). We compute each firm's book-to-market ratio and sort firms into quintile portfolios similar to Fama and French (1992), we present several statistics for each portfolio in Panels A (data) and B (best model) of Table 6<sup>21</sup>.

Just as operating leverage due to wages varies through time to produce conditional variation in aggregate returns, operating leverage due to wages also varies cross-sectionally. To summarize operating leverage due to wages we compute the profit to labor expenses ratio for each quintile, this is in the third column of Table 6. Value (high book-to-market) firms have much lower profit to labor expenses than growth (low book-to-market) firms, that is value firms are burdened with high labor expenses. The profit to labor expenses ratio is 0.45

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<sup>21</sup>The spread in book-to-market in our model is far smaller than the data in part because our firm level productivity shock contains only a high and a low state and is therefore too simple. We have experimented with more realistic distributions and the spread in book-to-market becomes larger.

for value and 0.70 for growth in the data, compared to 0.12 for value and 0.63 for growth in our best model. The reason value stocks are riskier in this environment is that during bad times wages are relatively high and all firms want to reduce labor. This is especially true for low productivity firms, who also tend to be value firms. However reducing labor is costly, therefore low productivity firms suffer disproportionately during recessions.

We define the value premium as the difference in average returns between the top quintile and bottom quintile of stocks sorted on book-to-market. As can be seen in Panel C of Table 4, the standard model produces a negative value premium, that is value stocks have lower average returns than growth stocks. Neither higher complementarity between labor and capital or infrequent resetting do much better. On the other hand combining the two with labor adjustment costs produces a positive value premium of 1.70%. While this is still short of the 4.46% in the data, we view this as a significant improvement over the standard model<sup>22</sup>.

Notably Zhang (2005), Carlson, Fisher, and Giammarino (2004), and Cooper (2006) also generate a sizable value premium. However, our model differs from these papers in important ways. First, the above are partial equilibrium models in which equilibrium prices are exogenously specified, while ours is a full-fledged general equilibrium model (to our best knowledge, the first to embed a realistic cross-section of firms in a long-run risk world). In general equilibrium, prices dampen the firms' investment demand making it harder to generate large cross-sectional risk dispersion. In fact, specification III (calibrated CES without wage rigidity) is a standard investment-based model with capital adjustment costs in general equilibrium, but the value premium in this model is tiny. Second, our model hinges on firms' wage expense being rigid, for which we provide direct evidence from Compustat firms, while

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<sup>22</sup>One additional reason for the value premium is financial leverage: value firms have higher leverage than growth firms as can be seen in the fourth column of Table 6. Since we assume constant leverage for all firms, this is missing from the model.

the above models employ fixed production costs which are hard to identify.

## 4 Conclusion

In standard models wages are far too volatile and pro-cyclical relative to the data. Wages therefore act as a hedge for the firm's owners, making profits too smooth and dividends counter-cyclical. As a result, the equity volatility in the data is nearly ten times that of standard models.

We smooth the average wage by introducing infrequent wage resetting and higher complementarity between labor and capital into the standard model. As a result profits and dividends both look much closer to the data, as does the volatility of equity returns. The same channel allows us to bring the model closer to explaining several other unresolved puzzles in financial data. The model is able to produce equity volatility and equity returns that are counter-cyclical, as in the data. Furthermore, value stocks have higher average returns than growth stocks in our model, as in the data.

## A Data

Stock returns are from the Center for Research in Security Prices (CRSP) and accounting information is from the CRSP/Compustat Merged Annual Industrial Files. Our accounting sample is from 1975 to 2008, all other data is from 1954 to 2008. We exclude from the sample any firm-year observation with missing data or for which total assets or the gross capital stock are either zero or negative. In addition, as standard, we omit firms whose primary SIC classification is between 4900 and 4999 (regulated firms) or between 6000 and 6999 (financial firms). The gross domestic fixed investment price deflator is from NIPA table 1.1.9. GDP is real gross domestic product from NIPA table 1.1.6; real consumption is nondurable consumption from NIPA table 2.3.5, scaled by implicit price for nondurable expenditures from NIPA table 2.3.4. Investment is investment in private non-residential fixed assets from NIPA table 4.7. Capital is private non-residential fixed assets from NIPA table 4.1. Both investment and capital are scaled by investment price deflator to get real terms. Wage is compensation of employees from NIPA table 6.2 divided by hours worked by full-time and part-time employees from NIPA table 6.9. Annual dividend is aggregated over monthly dividend from Robert Shiller's webpage: <http://www.econ.yale.edu/shiller/data.htm>. CRSP value-weighted market returns and risk free rates are from Ken French webpage: [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). Firm profit is net sales (SALE) minus the sum cost of good sold (COGS) and selling, general and administrative expense (XSGA). We aggregate all firms' profit scaled by investment price deflator to get total real profit.

## B Numerical Solution

### Making the Model Stationary

Note that the model is not stationary. In order to solve it numerically, we must rewrite it in terms of stationary quantities. We will show that a normalizing all non-stationary variables by  $Z_t^\rho$  implies a stationary competitive equilibrium. We will do this in two steps. First we will show that if the firm believes that the stochastic discount factor is stationary and that aggregate quantities (in particular the spot wage) normalized by  $Z_t^\rho$  are stationary than the firm's policy functions normalized by  $Z_t^\rho$  will also be stationary. Second we show that these policy functions imply that these aggregate quantities are indeed stationary when normalized by  $Z_t^\rho$ .

The firm's problem is:

$$\begin{aligned}
V(Z_t^i, K_t^i, N_{t-1}^i, \bar{W}_{t-1}^i; Z_t, K_t, S_t, \bar{W}_{t-1}) &= \max_{I_t^i, N_t^i} Z_t^i (\alpha(K_t^i)^\eta + (1 - \alpha)(Z_t N_t^i)^\rho)^\frac{1}{\eta} \\
&\quad - \left( \bar{W}_{t-1}^i N_{t-1}^i \mu + W_t (N_t^i - N_{t-1}^i \mu) \right) - W_t f \\
&\quad - v_t \left( \frac{I_t^i}{K_t^i} - \delta \right)^2 K_t^i - \xi (N_t^i - N_{t-1}^i)^2 W_t \\
&\quad + E_t[M_{t+1} V(Z_{t+1}^i, K_{t+1}^i, N_t^i, \bar{W}_t^i; Z_{t+1}, K_{t+1}, S_{t+1}, \bar{W}_t)]
\end{aligned} \tag{10}$$

Where  $Z_t^i$  is the idiosyncratic productivity,  $K_t^i$  is the firm's individual capital,  $N_{t-1}^i$  is the firm's employment last period,  $\bar{W}_{t-1}^i$  is the firm's average wage last period,  $Z_t$  is aggregate productivity,  $\bar{W}_{t-1}$  is the aggregate average wage from last period, and  $W_t$  is the spot wage this period. Following Krusell and Smith (1998) the state space potentially contains all information about the joint distribution of capital and productivity.  $K_t$  and  $S_t$  summarize this distribution. We explicitly write its first moment  $K_t$  as an aggregate state variable and

let  $S_t$  be a vector of any other relevant moments normalized by the mean (ie the normalized second moment is  $E[(K_t^i - K_t)^2]/K_t^2$ ).

On the right of this equation the first line contains output, the second line labor expenses and operating costs, the third line adjustment costs of capital and labor, and the fourth the firm's continuation value.

Households have beliefs about the evolution of the aggregate quantities  $M_{t+1}$ ,  $K_t$ , and  $S_t$  and about the spot wage as a function of the aggregate state. Aggregate wage evolves as  $\bar{W}_t = \mu \bar{W}_{t-1} + (1 - \mu)W_t$ . The individual state variables evolve as:

$$\begin{aligned} K_{t+1}^i &= (1 - \delta)K_t^i + I_t^i \\ \bar{W}_t^i &= \frac{\bar{W}_{t-1}^i N_{t-1}^i \mu + (N_t^i - N_{t-1}^i \mu)W_t}{N_t^i} \end{aligned} \quad (11)$$

Let us define  $k_t^i = \frac{K_t^i}{Z_t^\rho}$ ,  $k_t = \frac{K_t}{Z_t^\rho}$ ,  $i_t^i = \frac{I_t^i}{Z_t^\rho}$ ,  $w_t = \frac{W_t}{Z_t^\rho}$ ,  $\bar{w}_t^i = \frac{\bar{W}_t^i}{Z_{t+1}^\rho}$ , and  $\bar{w}_t = \frac{\bar{W}_t}{Z_{t+1}^\rho}$  (not that the timing of  $\bar{w}_t^i$  and  $\bar{w}_t$  differs from the others). We will now show by induction that the value function is linear in  $Z_t^\rho$ . Suppose this is true at  $t+1$ :

$$V(Z_{t+1}^i, K_{t+1}^i, N_t^i, \bar{W}_t^i; Z_{t+1}, K_{t+1}, S_{t+1}, \bar{W}_t) = Z_{t+1}^\rho V(Z_{t+1}^i, k_{t+1}^i, N_t^i, \bar{w}_t^i; 1, k_{t+1}, S_{t+1}, \bar{w}_t)$$

Then we can rewrite the firm's problem as:

$$\begin{aligned} V(Z_t^i, k_t^i, N_{t-1}^i, \bar{w}_{t-1}^i; 1, k_t, S_t, \bar{w}_{t-1}) &= \max_{i_t^i, N_t^i} Z_t^i (\alpha(k_t^i)^\eta + (1 - \alpha)(N_t^i)^{\rho\eta})^{\frac{1}{\eta}} \\ &\quad - (\bar{w}_{t-1}^i N_{t-1}^i \mu + w_t(N_t^i - N_{t-1}^i \mu)) - w_t f \\ &\quad - v_t \left( \frac{i_t^i}{k_t^i} - \delta \right)^2 k_t^i - \xi (N_t^i - N_{t-1}^i)^2 w_t \\ &\quad + E_t \left[ \left( \frac{Z_{t+1}}{Z_t} \right)^\rho M_{t+1} V(Z_{t+1}^i, k_{t+1}^i, N_t^i, \bar{w}_t^i; 1, k_{t+1}, S_{t+1}, \bar{w}_t) \right] \end{aligned} \quad (12)$$



where the aggregate wage evolves as  $\bar{w}_t = (\mu\bar{w}_{t-1} + (1-\mu)w_t) \left(\frac{Z_{t+1}}{Z_t}\right)^{-\rho}$  and the individual state variables evolve as:

$$\begin{aligned} k_{t+1}^i &= ((1-\delta)k_t^i + i_t^i) \left(\frac{Z_{t+1}}{Z_t}\right)^{-\rho} \\ \bar{w}_t^i &= \left(\frac{\bar{w}_{t-1}^i N_{t-1}^i \mu + (N_t^i - N_{t-1}^i \mu) w_t}{N_t^i}\right) \left(\frac{Z_{t+1}}{Z_t}\right)^{-\rho} \end{aligned} \quad (13)$$

As long as  $\left(\frac{Z_{t+1}}{Z_t}\right)^\rho$ ,  $M_{t+1}$ ,  $k_{t+1}$ , and  $w_{t+1}$  are stationary this is a well defined stationary problem where the firm's optimal policy ( $i_t^i$  and  $N_t^i$ ) will also be stationary. But this implies that  $k_{t+1}^i$  and  $k_{t+1} = \sum k_{t+1}^i$  are stationary as well, confirming the firm's beliefs.

It is similarly straight forward to show that the stochastic discount factor is stationary. First of all note that  $M_{t+1}$  is related to the growth rate of consumption, so it should be stationary. More formally:

$$\begin{aligned} U_t &= \left(C_t^{1-\frac{1}{\psi}} + \beta E_t[U_{t+1}^{1-\theta}]^{\frac{1-\frac{1}{\psi}}{1-\theta}}\right)^{\frac{1}{1-\frac{1}{\psi}}} \\ M_{t+1} &= \beta \left(\frac{U_{t+1}}{E_t[U_{t+1}^{1-\theta}]^{\frac{1}{1-\theta}}}\right)^{\frac{1}{\psi}-\theta} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \end{aligned} \quad (14)$$

Define  $c_t = \frac{C_t}{Z_t^\rho}$  and  $u_t = \frac{U_t}{Z_t^\rho}$  and note that the firm's optimal policy implies that  $c_t$  is stationary. Now we can rewrite the above equations as:

$$\begin{aligned} u_t &= \left(c_t^{1-\frac{1}{\psi}} + \beta E_t\left[\left(\frac{Z_{t+1}}{Z_t}\right)^\rho u_{t+1}^{1-\theta}\right]^{\frac{1-\frac{1}{\psi}}{1-\theta}}\right)^{\frac{1}{1-\frac{1}{\psi}}} \\ M_{t+1} &= \beta \left(\frac{\left(\frac{Z_{t+1}}{Z_t}\right)^\rho u_{t+1}}{E_t\left[\left(\frac{Z_{t+1}}{Z_t}\right)^\rho u_{t+1}^{1-\theta}\right]^{\frac{1}{1-\theta}}}\right)^{\frac{1}{\psi}-\theta} \left(\frac{c_{t+1}}{c_t}\right)^{-\frac{1}{\psi}} \left(\frac{Z_{t+1}}{Z_t}\right)^{-\frac{\rho}{\psi}} \end{aligned} \quad (15)$$

which are stationary as long as  $c_t$  is stationary.

Next, we must show that the spot wage is stationary. The firm's first order condition for

labor implies:

$$w_t = Z_t^i \left( \alpha (k_t^i)^\eta + (1 - \alpha) (N_t^i)^{\rho\eta} \right)^{\frac{1-\eta}{\eta}} (1 - \alpha) \rho (N_t^i)^{\rho\eta-1} + E_t \left[ \left( \frac{Z_{t+1}}{Z_t} \right)^\rho M_{t+1} \frac{\partial V_{t+1}}{\partial N_t^i} \right] \quad (16)$$

For every firm, the right hand side is well defined and stationary, therefore the wage is too. To jointly find the wage and each firm's choice of  $N_t^i$  one must solve a system of  $N$  equations.  $N-1$  equations where the right hand side of the first order condition for firm 1 is set equal to firm  $i$  ( $i=2, N$ ), and the labor market clearing equation  $\sum N_t^i = 1$ .

There remains one last complication, is  $S_t$  stationary? This is related to a more general problem of the validity and accuracy of the Krusell and Smith (1998) algorithm. We cannot give an explicit answer as it is not clear what exactly  $S_t$  must contain. Krusell and Smith (1998) argue that  $S_t$  should contain higher order moments of the distribution since they fully describe the distribution. Since we define  $S_t$  to be normalized by its first moment, it is likely that these normalized higher moments are stationary. We have also checked the behavior of several simulated higher order moments and they appear stationary. In practice our numerical algorithm (described in the next section) only considers the first moment so  $S_t$  is an empty set, which is stationary by definition.

## Numerical Algorithm

We will now describe the numerical algorithm used to solve the stationary problem above. We will first describe the algorithm used to solve a model with CRRA utility and then the extension necessary to solve the recursive utility version. The algorithm is a variation of the algorithm in Krusell and Smith (1998).

The aggregate state space is potentially infinite because it contains the full distribution

of capital across firms. We follow Krusell and Smith (1998) and summarize it by the average aggregate capital  $k_t$  and the state of aggregate productivity  $\Delta Z_t$ ; because past wages matter, we augment the aggregate state space with the previous period's average wage  $\bar{w}_{t-1}$ . Each of these is put on a grid, with the grid sizes of 20 for capital and 9 for past wage. Productivity is a 3-state Markov process. We also discretize the firm's individual state space with grid sizes of 25 for individual capital ( $k_t^i$ ), 11 for last period's labor ( $N_{t-1}^i$ ), and 5 for last period's average wage ( $\bar{w}_{t-1}^i$ ). Individual productivity is a 2-state Markov process. We chose these grid sizes after careful experimentation to determine which grid sizes had the most effect on Euler equation errors and predictive  $R^2$ .

For each point in the aggregate state space  $(k_t, \bar{w}_{t-1}, \Delta Z_t)$  we start out with an initial belief about consumption, spot wages, and investment ( $c_t$ ,  $w_t$ , and  $i_t$ ); note that this non-parametric approach is different from Krusell and Smith (1998)<sup>23</sup>. From these we can solve for aggregate capital next period  $k_{t+1} = ((1 - \delta)k_t + i_t) \left( \frac{Z_{t+1}}{Z_t} \right)^{-\rho}$  for each realization of the shock. Combining  $k_{t+1}$  with beliefs about consumption as a function of capital we can also solve for the stochastic discount factor next period:  $M_{t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\theta} \left( \frac{Z_{t+1}}{Z_t} \right)^{-\theta\rho}$ . This is enough information to solve the stationary problem described in the previous section. We solve the problem by value function iteration with the output being policies and market values of each firm for each point in the state space.

The next step is to use the policy functions to simulate the economy. We simulate the economy for 5500 periods (we throw away the initial 500 periods). In addition to the long

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<sup>23</sup>The standard Krusell and Smith (1998) algorithm instead assumes a functional form for the transition, such as  $\log(\bar{k}_{t+1}) = A(Z_t) + B(Z_t)\log(\bar{k}_t)$  and forms beliefs only about the coefficients  $A(Z_t)$  and  $B(Z_t)$  however we find that this approach does not converge in many cases due to incorrect beliefs about off-equilibrium situations and that our approach works better. Without heterogeneity and infrequent resetting we would not need beliefs about  $w_t$  because it would just be the marginal product of aggregate capital. Similarly, we would not need beliefs about  $c_t$  as we could solve for it from  $y_t = c_t + i_t$  where  $y_t$  is aggregate output, however aggregate output is no longer a simple analytic function of aggregate capital.

simulation, we start off the model in each point of the aggregate state space. We must do this because unlike Krusell and Smith (1998), the beliefs in our algorithm are non-parametric and during the model's typical behavior it does not visit every possible point in the state space. From the simulation we form simulation implied beliefs about  $c_t$ ,  $w_t$ , and  $i_t$  at each point in the aggregate state space by averaging over all periods in which the economy was sufficiently close to that point in the state space. Our updated beliefs are a weighted average of the old beliefs and the new simulation implied beliefs<sup>24</sup>. With these updated beliefs we again solve the firm's dynamic program; we continue doing this until convergence.

In order to solve this model with recursive preferences an additional step is required. Knowing  $c_t$  and  $k_{t+1}$  as functions of the aggregate state is not alone enough to know  $M_{t+1}$  because in addition to consumption growth, it depends on the household's value function next period:  $M_{t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\frac{1}{\psi}} \left( \frac{U_{t+1}}{E_t[U_{t+1}^{1-\theta}]^{\frac{1}{1-\theta}}} \right)^{\frac{1}{\psi}-\theta}$ . However this problem is not difficult to overcome. After each simulation step we use beliefs about  $c_t$  and  $k_{t+1}$  to recursively solve for the household's value function at each point in the state space. This is again done through value function iteration, however as there are no choice variables this recursion is very quick.

We perform the standard checks proposed by Krusell and Smith (1998) to make sure we have found the equilibrium. Although our beliefs are non-parametric, we can still compute an  $R^2$  analogous to a regression; all of the  $R^2$  are above 0.999. We have also checked that an additional state variable (either the cross-sectional standard deviation of capital or lagged capital) does not alter the results.

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<sup>24</sup>The weight on the old belief is often required to be very high in order for the algorithm to converge. This is because while rational equilibria exist, they are only weakly stable in the sense described by Marcet and Sargent (1989). However, we find that this is only a problem when capital adjustment costs are very close to zero.

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Figure 1: Impulse responses

This figure plots the impulse responses of the aggregate wage, aggregate profit, and aggregate dividend to a positive productivity shock. The solid line represents the standard model (Cobb-Douglas production and no wage rigidity) while the dashed line represents our best model (calibrated CES production, wage rigidity, and labor adjustment costs). The x-axis is time after the shock, the y-axis is model quantity at any time relative to time zero.

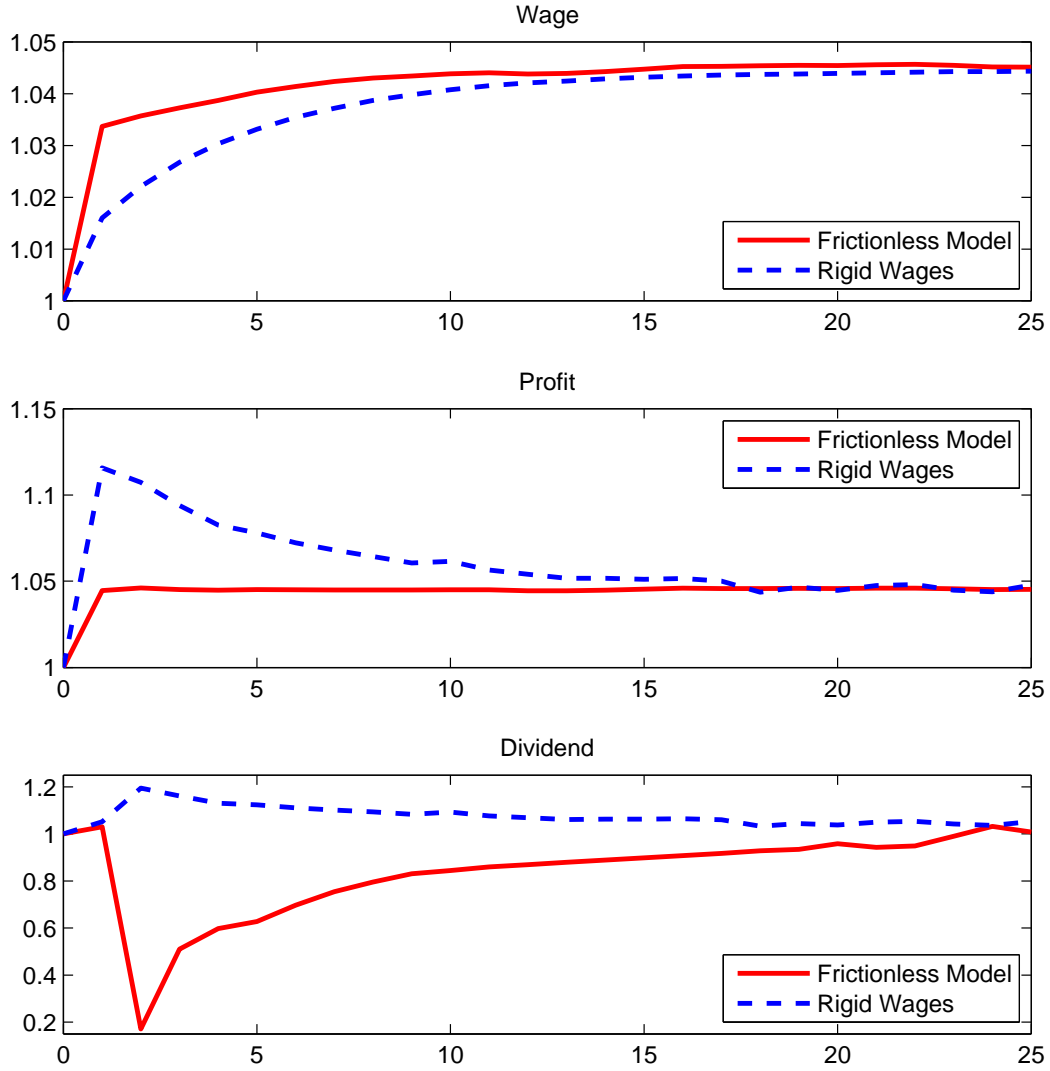


Table 2: Macroeconomic moments

This table compares macroeconomic moments from the data (1954-2008) to several versions of our model. All reported correlations are with HP filtered GDP ( $y$ ) except for growth rates of variables, in these cases correlations are reported with the growth rate of GDP. In the data  $w$  is compensation per hour. The models in Panels B and C have Cobb-Douglas technology ( $\frac{1}{1-\eta} = 1$ ) and in Panels D, E, and F have a calibrated elasticity of substitution between labor and capital ( $\frac{1}{1-\eta} = 0.5$ ). Panels B and D are frictionless models with no wage rigidity ( $\mu = 0$ ) or labor adjustment costs ( $\xi = 0$ ). Panels C and E have wage rigidity ( $\mu = 0.75$ ) but no labor adjustment costs ( $\xi = 0$ ). Panel F presents the model with wage rigidity ( $\mu = 0.75$ ) and labor adjustment costs ( $\xi = 0.5$ ). Note that the model in Panel B is analogous to the standard model while the one in Panel F is our preferred model.

Panel A: Data			
	$\sigma(x)$	$\rho(x, y)$	AC(x)
y	1.58	1.00	0.32
c	1.13	0.84	0.35
i	5.13	0.80	0.42
w	0.79	0.22	0.42
$\Delta c$	1.50	0.42	0.22
$\Delta i$	6.40	0.62	0.27
i-k	0.82	0.50	0.78

Panel B: Cobb-Douglas, no wage rigidity, no labor adjustment cost			
	$\sigma(x)$	$\rho(x, y)$	AC(x)
y	1.71	1.00	0.54
c	1.12	0.81	0.31
i	4.81	0.89	0.44
w	1.71	1.00	0.54
$\Delta c$	2.09	0.84	0.46
$\Delta i$	6.57	0.81	0.29
i-k	1.62	0.07	0.92

Panel C: Cobb-Douglas, wage rigidity, no labor adjustment cost			
	$\sigma(x)$	$\rho(x, y)$	AC(x)
y	1.90	1.00	0.55
c	1.50	0.95	0.64
i	5.25	0.88	0.33
w	0.82	0.85	0.67
$\Delta c$	2.32	0.94	0.75
$\Delta i$	7.29	0.84	0.14
i-k	1.60	0.05	0.89

Panel D: Calibrated CES, no wage rigidity, no labor adjustment cost			
	$\sigma(x)$	$\rho(x, y)$	AC(x)
y	1.89	1.00	0.51
c	1.00	0.91	0.48
i	4.31	0.95	0.48
w	1.59	1.00	0.51
$\Delta c$	1.87	0.91	0.65
$\Delta i$	5.80	0.90	0.39
i-k	1.54	0.09	0.93

Panel E: Calibrated CES, wage rigidity, no labor adjustment cost			
	$\sigma(x)$	$\rho(x, y)$	AC(x)
y	1.91	1.00	0.55
c	1.50	0.95	0.64
i	5.25	0.88	0.33
w	0.82	0.85	0.67
$\Delta c$	2.32	0.94	0.75
$\Delta i$	7.29	0.84	0.13
i-k	1.60	0.11	0.89

Panel F: Calibrated CES, wage rigidity, labor adjustment cost			
	$\sigma(x)$	$\rho(x, y)$	AC(x)
y	1.89	1.00	0.54
c	1.54	0.96	0.63
i	5.24	0.86	0.29
w	0.81	0.81	0.71
$\Delta c$	2.35	0.96	0.73
$\Delta i$	7.33	0.84	0.10
i-k	1.54	0.09	0.88

Table 3: Labor capital elasticity and the wage

Here we set  $N = 1$ ,  $K = 1.5$ ,  $\alpha = 0.25$ ,  $\rho = 0.8533$ ,  $Z_t \sim N(1, 0.1)$ . We set  $K$  to roughly equal to average capital in our best model. We set the volatility of  $Z_t$  higher than in the model only to highlight the effect of  $\eta$  on volatility of wages, qualitatively everything remains the same for lower volatility of  $Z_t$ . All other parameters are identical to our calibration. We compute  $Y_t = (\alpha K^\rho + (1 - \alpha) * (Z_t N)^{\rho\eta})^{\frac{1}{\eta}}$  and  $\frac{\partial Y_t}{\partial N} = \rho(1 - \alpha)Y^{1-\eta}Z^{\rho\eta}N^{\rho\eta-1}$  and report the ratios of their volatilities, and their correlation. Note that  $\eta = 0$  corresponds to Cobb-Douglas production.

$\eta$	$\frac{\sigma(\frac{\partial Y}{\partial N})}{\sigma(Y)}$	$\frac{\sigma(\log(\frac{\partial Y}{\partial N}))}{\sigma(\log(Y))}$	$\sigma(\frac{\partial Y}{\partial N}, Y)$
-5.0	0.64	0.79	0.995
-1.0	0.55	0.78	0.999
-0.5	0.58	0.87	1.000
0.0	0.64	1.00	1.000
0.5	0.73	1.21	1.000
1.0	0.85	1.51	1.000
5.0	3.21	14.06	0.998

Table 4: Accounting and financial moments

This table compares the data (1954-2008) to several versions of our model. The standard frictionless model is in row two, a model with just wage rigidity is in row three, a model with just a calibrated CES is in row four, a model with both wage rigidity and a calibrated CES is in row five, finally a model with wage rigidity, a calibrated CES, and labor adjustment costs is in row six. All reported correlations are with HP filtered GDP ( $y$ ) except for growth rates of variables, in these cases correlations are reported with the growth rate of GDP. In Panel C, the value premium is defined as the difference in average returns between firms in the top quintile and bottom quintile of a book-to-market sorting. In panel D, a recession is defined as bottom 25% of GDP growth, while an expansion is top 25%. Conditional returns, standard deviations, and Sharpe Ratios are calculated for five years after a recession or expansion.

Panel A: Profits

$\frac{1}{1-\eta}$	$\mu$	$\xi$	$\sigma(\Delta\pi)$	$\rho(\Delta\pi, \Delta y)$	$AC(\Delta\pi)$	$\sigma(\pi)$	$\rho(\pi, y)$	$AC(\pi)$
Data			8.85	0.35	0.18	6.74	0.62	0.30
1.0	0	0	3.34	0.99	0.51	2.34	1.00	0.54
1.0	0.75	0	6.12	0.89	0.40	4.64	0.97	0.49
0.5	0	0	5.55	0.99	0.41	4.11	0.99	0.51
0.5	0.75	0	9.59	0.87	0.41	7.42	0.95	0.51
0.5	0.75	0.5	9.19	0.87	0.39	6.98	0.94	0.48

Panel B: Dividends

$\frac{1}{1-\eta}$	$\mu$	$\xi$	$\sigma(\Delta D)$	$\rho(\Delta D, \Delta y)$	$AC(\Delta D)$	$\sigma(D)$	$\rho(D, y)$	$AC(D)$
Data			4.28	0.29	0.61	3.31	0.53	0.59
1.0	0	0	192.94	-0.18	-0.28	140.82	-0.45	0.06
1.0	0.75	0	16.42	0.44	-0.28	11.49	0.57	0.01
0.5	0	0	14.40	-0.09	-0.17	10.18	-0.08	0.11
0.5	0.75	0	85.49	0.34	-0.03	65.35	0.39	0.12
0.5	0.75	0.5	76.78	0.33	0.02	57.64	0.39	0.21

Panel C: Unconditional financial moments

$\frac{1}{1-\eta}$	$\mu$	$\xi$	$E[R_f]$	$\sigma(R_f)$	$E[R_e - R_f]$	$\sigma(R_e - R_f)$	SR	$E[R_V - R_G]$
Data			1.59	1.92	6.76	18.85	0.36	4.46
1.0	0	0	1.17	0.62	1.11	2.89	0.38	-0.04
1.0	0.75	0	1.18	0.62	2.35	6.18	0.38	-0.03
0.5	0	0	1.50	0.94	2.23	5.78	0.38	0.09
0.5	0.75	0	1.46	0.95	5.00	13.13	0.38	-0.15
0.5	0.75	0.5	1.12	0.96	5.92	15.58	0.38	1.70

Panel D: Conditional financial moments

$\frac{1}{1-\eta}$	$\mu$	$\xi$	$E[R_e - R_f R]$	$E[R_e - R_f E]$	$\sigma(R_e - R_f R)$	$\sigma(R_e - R_f E)$
Data			9.00	3.81	19.34	16.61
1.0	0	0	1.08	1.15	2.76	2.93
1.0	0.75	0	2.19	2.06 <sup>45</sup>	5.78	5.43
0.5	0	0	2.40	2.20	6.31	5.79
0.5	0.75	0	5.03	4.22	13.24	11.12
0.5	0.75	0.5	6.13	4.82	16.14	12.69

Table 5: Long horizon predictability

This table presents results from the regression  $R_{t+1,t+T} - R_{t,t+T-1}^f = a + b * \Delta Wage$  for the data and the model. The data is for the period 1955-2008. To be comparable to the data, in the model we report the average results from 34 regressions, each over an independent 53 year period. The model is our "best" model, with wage rigidity ( $\mu = 0.75$ ), a calibrated CES ( $\frac{1}{1-\eta} = 0.5$ ), and labor adjustment costs ( $\xi = 0.5$ ).

T	Data			Model		
	$b$	$t(b)$	$R^2$	$b$	$t(b)$	$R^2$
1	-2.44	1.60	0.02	-0.72	0.63	0.03
2	-6.05	2.32	0.11	-1.39	0.93	0.05
3	-7.48	2.22	0.14	-2.08	1.22	0.08
4	-8.59	2.09	0.15	-2.63	1.45	0.10
5	-10.61	2.66	0.18	-3.12	1.65	0.13
6	-11.51	2.67	0.19	-3.60	1.81	0.14
7	-11.94	2.68	0.18	-4.13	1.99	0.15
8	-14.23	2.73	0.22	-4.54	2.15	0.17
9	-16.06	2.62	0.23	-5.08	2.37	0.19
10	-18.65	3.04	0.30	-5.60	2.58	0.21

Table 6: The value premium

This table presents average book-to-market of equity, profit to labor expenses ratio, financial leverage, and mean excess return for five portfolios sorted on book-to-market. The top panel contains data and the bottom panel results from our "best" model, with  $\mu = 0.9$  and  $\xi = 0.75$ . To compute book-to-market of equity in the model we assume that the book and market value of debt are the same and set book-to-market of equity to be the inverse of  $\frac{V-D}{1-D}$  where  $V$  is the market value of the full firm,  $K$  is the firm's capital, and  $D = 0.37$  is the constant debt to value ratio. Profits are defined as Revenue-COGS-SGA from Compustat, labor expenses are employees from Compustat multiplied by the average wage for the firm's industry from NIPA.

Panel A: Data

	B/M	$\frac{Profit}{LaborExpense}$	$\frac{Debt}{Debt+Equity}$	$E[R^i - R^f]$
Growth	0.20	0.70	0.19	4.72
2	0.47	0.62	0.33	5.49
3	0.70	0.61	0.42	6.34
4	0.97	0.54	0.49	8.49
Value	2.85	0.45	0.78	9.19
V-G	2.65	-0.25	0.59	4.46

Panel B: Model

	B/M	$\frac{Profit}{LaborExpense}$	$\frac{Debt}{Debt+Equity}$	$E[R^i - R^f]$
Growth	0.43	0.63	0.37	5.55
2	0.49	0.58	0.37	5.85
3	0.63	0.41	0.37	6.27
4	1.06	0.13	0.37	6.88
Value	1.19	0.12	0.37	7.25
V-G	0.76	-0.51	0.00	1.70