Optimal Taxation with Rent-Seeking[*]

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Abstract

Recent policy proposals have suggested taxing top incomes at very high rates on the grounds that some or all of the highest wage earners are engaged in socially unproductive or counterproductive activities, such as externality imposing speculation in the financial sector. To address this, we provide a model in which agents can choose between working in a traditional sector, where private and social products coincide, and a crowdable rent-seeking sector, where some or all of earned income reflects the capture of pre-existing output rather than increased production. We characterize Pareto optimal linear and non-linear income tax systems under the assumption that the social planner cannot or does not observe whether any given individual is a traditional worker or a rent-seeker. We find that optimal marginal taxes on the highest wage earners can remain modest even if all high earners are socially unproductive rent-seekers and the government has a strong intrinsic desire for progressive redistribution. Intuitively, taxing their effort at a lower rate stimulates their rent-seeking efforts, thereby keeping private returns for other potential rent-seekers low and discouraging further entry.

1 Introduction

The unwinding of the financial crisis over the past three years has exposed numerous examples of highly compensated individuals whose apparent contributions to social output proved illusory. Events like the recent housing bubble provide fertile ground for "rent-seeking" activity: pursuing personal enrichment by extracting a slice of the existing economic pie rather than by increasing the size of that pie. These highly salient examples of rent-seeking activities have inspired calls for a more steeply progressive tax code. For instance, Paul Krugman argued for higher taxes on "supersized incomes" in the context of discussing the profits from high speed trading, on the grounds that "it is hard to see how traders who place their orders one-thirtieth of a second faster than anyone else do anything to improve that social function."[1] Moreover, in various countries, the introduction of very high taxes (up to 90%) on bonus payments in the financial sector has been discussed on the grounds of similar rent-seeking arguments.

The argument behind such proposals is intuitively appealing. If much of the economic activity at high incomes is primarily socially unproductive rent-seeking or "skimming," then it would seem natural for a well designed income tax code to impose high marginal rates at high income levels.[2] This would discourage such behavior while simultaneously raising revenues which could be used, e.g., to lower taxes and encourage more productive effort at lower income levels.

The implications of such rent-seeking activities for optimal income taxation have not been studied formally, however. The idea that a particularly high level of rent-seeking behavior at high earnings levels should imply high tax rates at the top may seem intuitively correct, but is not well grounded in formal theory. The goal of this paper is therefore to provide a formal foundation for studying optimal taxation in economies with rent-seeking. Moreover, we aim at exploring the implications of rent-seeking for optimal taxes by comparing the taxes implied by traditional models with the those implied by models that explicitly incorporate rent-seeking.

By way of illustrating the model that we construct to address these issues, consider the following highly stylized economy. It consists of a continuum of individuals living on

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[1] New York Times Editorial, August 3, 2009.

[2] Bertrand and Mullainathan (2001) offer some formal evidence for rent-seeking activities among high earners. Their research indicates that the responsiveness of a CEO's pay to "lucky" increases in his or her company's profits is consistent with a crude "skimming" model of compensation. Philippon and Reshef (2006) argue that a substantial portion of financial sector compensation in their period of study represented transitory rents. Moreover, Kaplan and Rauh (2010) argue that wage-and-salary compensation almost certainly understates total compensation within the financial sector, as unrealized compensation is an increasingly important component of total compensation at the top end of the income distribution, particularly among financial sector "superstars," such as hedge fund managers and private equity investors.

large arable plain with a small creek flowing through it. As in Roy (1951), individuals are free to choose their occupation: they can either farm the plain or pan for gold in the creek. Unlike in Roy, however, the production technologies in the agricultural ("traditional") and the gold-panning ("rent-seeking") sectors are different. Farming is a standard, constant returns to scale activity: if an individual doubles her farming effort, her crop-output of doubles, and the economy's total crop output increases accordingly.

Gold-panning is different because the creek is small relative to the population and contains only a finite amount of gold. Because of this, there are decreasing returns to scale in gold-panning effort: increases in aggregate gold-panning increase the total gold output less than proportionally. Since any individual gold-panner collects a negligible proportion of the total gold output, each still has linear private returns to her efforts: if, e.g., she doubles her efforts, then she will "earn" twice as much gold. This linear private return strictly exceeds the social marginal returns from her gold-panning efforts, however, since a portion of the private return represents "skimming" of gold that other panners would otherwise have found. This wedge between the social and private returns to gold-panning is the crux of our model of rent-seeking.

Formally, if aggregate effort in the rent-seeking sector is E, then total output in the rent-seeking sector is $\mu(E)$, where $\mu(0) = 0$, $\mu'(E) \geq 0$ and $\mu''(E) \leq 0$ so that there are non-negative but decreasing returns to scale in rent-seeking. Since each unit of (equivalent) effort is equally productive, individual wages are proportional to the *average* return to rent-seeking effort, $\mu(E)/E$. When $\mu''(E) < 0$, the average return $\mu(E)/E$ exceeds the social marginal return to effort, $\mu'(E)$. Wages therefore exceed the social marginal product of effort.

Individuals differ in two dimensions: in their farming skill and in their gold-panning skill. θ measures their marginal return to effort—their wage—in the traditional farming sector. Rent-seeking wages are equal to $\varphi \mu(E)/E$, where φ measures the gold-panning skill of a given individual. The parameter φ thus measures her rent-seeking wages relative to other potential prospectors. The level of her rent-seeking wage depends on the aggregate efforts of all prospectors: as aggregate rent-seeking effort E rises, potential rent-seeking wages decrease along with $\mu(E)/E$.

Now consider the Pareto problem for optimal income taxation: design an income tax that maximizes some weighted average of the utilities of the individuals in the economy. In particular, suppose that the income tax does not condition on whether income is achieved in the rent-seeking or in the traditional sector. While such a restriction may seem ad hoc in this simple example, it can be easily motivated in a more realistic model where rent-seeking activities are not perfectly concentrated in particular (easily observ-

able) occupations. To a large degree, this also reflects the norm in existing tax codes, although calls for bonus taxes in the financial sector would represent a movement away from this norm.[3] This norm might reflect tradition, the lack of a reliable test for the type of income, or concerns about empowering a government to make the determination of just how productive individual workers or professions "really" are.

This is a more challenging problem than a standard Mirrlees (1971) optimal tax problem for two reasons. First, an additional complication arises from the wage distribution being endogenous. Fixing any given tax code, the decision of an individual about which sector to work in depends on the relative wages she can earn in the rent-seeking and traditional sectors. The former depends on how much effort other individuals are exerting in the rent-seeking sector. Solving for the outcomes induced by that tax code thus involves a fixed point problem: finding the level of aggregate rent-seeking effort E such that the wages induced by E lead to sectoral choices and effort such that aggregate rent-seeking effort is indeed E. The second challenge is that the distribution of skill-types is two-dimensional, so standard techniques typically do not apply (see Rochet and Chone, 1998). We address these challenges by observing that for any given aggregate rent-seeking effort E, the realized wage distribution is well defined, and, since taxes depend only on income, a standard single-crossing property holds. This allows us to treat the problem as a fixed point problem for E nested within a Mirrleesian optimal income tax problem.

We start by considering linear income tax schedules and compare the set of Pareto optimal marginal tax rates to the set of tax rates that would *appear* to be optimal for the same economy to a social planner who failed to take rent-seeking into account. For this purpose, we develop the notion of a "Self-Confirming Policy Equilibrium" (SCPE). Recall that, with rent-seeking, the wage distribution is endogenous to the tax code. A SCPE is a mutually-consistent tax policy/wage distribution pair such that a social planner who naively believes that the wage distribution is exogenous (as in a standard Mirrlees model) perceives the tax policy as optimal given the wage distribution induced by that policy. Our first result is that, with linear taxation, the set of Pareto optimal tax rates is shifted to the right compared to the SCPE set, formalizing the intuition that accounting for rent-seeking makes higher tax rates optimal *on average*.

We then turn to non-linear taxation, which allows us to address the effect of rent-seeking on the optimal *progressivity* of tax schedules. We first analyze the benchmark case where the economy *only* consists of a rent-seeking sector. Comparing the set of Pareto optimal and SCPE non-linear tax schedules, we find that the presence of rent-seeking *does*

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[3]We also recognize that the different treatment of capital and earned income and imperfections in accounting and monitoring can lead, in practice, to some sectoral difference in realized tax rates.

not affect optimal progressivity in this case: Given some Pareto weights, all marginal keep shares $1 - T'(y)$ in a Pareto optimum are scaled down compared to the SCPE by the factor $\beta(E) \equiv \mu'(E)E/\mu(E)$, which is the elasticity of rent-seeking output with respect to total rent-seeking effort. $\beta(E)$ measures the divergence between marginal and average product in the rent-seeking sector and thus captures the rent-seeking externality. Moreover, the top marginal tax rate is given by $1 - \beta(E)$, the Pigouvian corrective tax rate that makes agents fully internalize the rent-seeking externality.

We then demonstrate that these results are fundamentally changed when both sectors are present, and how misleading casual reasoning can be about the implications of rent-seeking for optimal taxation in a more general framework. In this case, marginal tax rates and hence progressivity of the tax schedule depend on the share of rent-seekers at a given wage. More surprisingly, the top marginal tax rate is *less* than the Pigouvian correction $1 - \beta(E)$ even if all top wage earners are rent-seekers and the governments strictly aims at redistributing towards low wage earners. We identify a *sectoral shift* effect as the key reason for this result: Taxing the top earners at a lower rate increases total rent-seeking effort E and therefore reduces private returns $\mu(E)/E$ in the rent-seeking sector. This prevents other agents from entering the socially less productive rent-seeking sector. Finally, we provide a quantitative example and show that this sectoral shift effect can be strong and induce top marginal tax rates that are *substantially* lower than the Pigouvian rate $1 - \beta(E)$ that a single sector model with rent-seekers only would have prescribed.

Related Literature. Our work builds on two major strands of the economics literature: the rent-seeking literature and the optimal income taxation literature. While rent-seeking is a conceptually important element of our model, our methods more closely track the optimal income taxation literature, notably Mirrlees (1971), Diamond and Mirrlees (1971a,b), and Diamond (1998). Until recently, the focus of the theoretical literature was on deriving results for a given assumed distribution of skills and social welfare function. Saez (2001) focused instead on inferring optimal taxes from observed income distributions. Moreover, Laroque (2005), Werning (2007) and Chone and Laroque (2010) study conditions under which an observer can test whether an existing set of taxes is or is not Pareto efficient. In the same spirit, we characterize the set of Pareto efficient tax policies rather than focusing on a particular social welfare function. In the context of rent-seeking, however, the wage distribution is endogenous to the tax code, so earlier tests—e.g., Werning (2007), who infers wage-cum-skill distributions from income distributions as a test of optimality—are potentially misleading. One might conclude that the tax code is indeed Pareto efficient given the inferred skill distribution under the (implicit and incorrect) assumption that the skill distribution is independent of the tax code. Our concept of a

self-confirming policy equilibrium, described above, is meant to capture this situation. It is closely related to the recent literature on self-confirming equilibria in learning models (e.g., Sargent, 2009, and Fudenberg and Levine, 2009).

Our paper also contributes to recent efforts to study optimal taxation under multi-dimensional private heterogeneity. In a recent study of the optimal income taxation of couples, Kleven, Kreiner and Saez (2009) have made progress along these lines (see also Scheuer (2011) for an application to entrepreneurial taxation). Their information structure is quite distinct from ours, however, as their second dimension of heterogeneity enters preferences additively rather than as a standard skill type.

We build on pioneering work in the rent-seeking literature including Tullock (1967), Krueger (1974), and Bhagwati (1980, 1982). Our model of rent-seeking is broad enough to include a wide range of activities, such as the patent races discussed in Dixit (1987), Loury (1979) and Dasgupta and Stiglitz (1980), socially useless but privately profitable financial speculation discussed by Arrow (1973) and Hirshleifer (1971), or externalities (see Sandmo (1975), who studies optimal commodity taxation in the presence of externalities). Relatedly, the structure we use to model compensation in the rent-seeking sector is borrowed from the search literature pioneered by Mortensen (1977). This is not coincidental: production in the rent-seeking sector in the example we discussed above is equivalent to "searching for gold." Hungerbuhler et al.'s (2008) recent work also introduces search in an optimal taxation problem, but their paper differs in that that "search" in their model is *for* employment rather than search *as* employment, as it is here.

Finally, our paper relates to the literature studying the equilibrium allocation of talent across different sectors when there are rents to be captured in some sectors (see e.g. Baumol, 1990, Murphy et al., 1991, and Acemoglu and Verdier, 1998). More recently, attention has focused on understanding the size of the financial sector when individuals face an occupational choice decision between becoming financiers or workers. In particular, Cahuc and Challe (2009) consider an OLG-model with asset bubbles in the financial market and show that inefficiently many workers may decide to become speculators, inducing increased inequality. However, none of these papers considers optimal tax policy to correct these equilibrium outcomes, which is the focus of our contribution. An important exception is the recent work by Philippon (2008, 2010), who considers an endogenous growth model with financiers, workers and entrepreneurs where financiers provide monitoring services to borrowing-constrained entrepreneurs. He analyzes the effect of linear, sector specific taxes on growth and shows that the second best optimum involves taxing the financial sector at the same rate as the nonfinancial sector. In contrast, we are interested in designing optimal tax policies driven by redistributive and corrective motives in the

presence of rent-seeking and do not consider growth effects of taxation nor sector-specific tax instruments.

Our paper proceeds as follows. Section 2 describes our modeling framework and the rent-seeking technology we study. Section 3 studies optimal linear taxation in this framework. It focuses on the divergence between the set of optimal and self-confirming linear tax rates in the presence of rent-seeking (Theorems 1 and 2). Sections 4 studies optimal non-linear taxation. Theorem 3 shows that rent-seeking does not, in itself, provide an argument for more progressive taxation: in an economy with a single rent-seeking sector, failing to take rent-seeking into account leads to tax rates that are suboptimally low but are nevertheless optimally progressive. Theorem 4 contains our most surprising result: it shows that under an intuitively plausible set of conditions in an economy with both rent-seeking and ordinary earnings, the top marginal tax rate is optimally less than the Pigouvian corrective tax rate even if the highest earners are all rent-seekers. Section 5 offers some numerical simulations that illustrate Theorem 4. Section 6 explores two extensions of our model: It shows that our results generalize to settings where the support of the skill distribution is unbounded above, and, under a natural set of conditions, to settings where rent-seekers also impose negative externalities on traditional workers. Section 7 concludes. Most proofs appear in the technical appendices.

2   Model and Approach

We consider an economy with two sectors: A traditional sector, where private and social marginal products coincide, and a rent-seeking sector, where the private marginal product exceeds the social marginal product. There is a unit-measure continuum of individual workers who can choose to work in either one of the two sectors. Each individual is endowed with a two-dimensional skill vector

$$(\theta, \varphi) \in \Theta \times \Phi, \quad \Theta = [\underline{\theta}, \overline{\theta}], \Phi = [\underline{\varphi}, \overline{\varphi}],$$

where θ captures an individual's skill in the traditional sector (which we also refer to as Θ-sector), and φ captures her skill in the rent-seeking sector (also referred to as Φ-sector).[4]

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[4]It is also possible to consider one dimensional skill distributions, in which an individual's Φ-sector skill φ can be viewed as a function of her Θ-sector skill θ. We focus on a fully two-dimensional skill distribution for several related reasons. First, it is more general: the one-dimension version of the model is a limiting case. Second, in contrast to the one-dimensional version, it generically permits both types of workers to co-exist at the same wage. Third, it allows for arbitrary skill-type correlations. Fourth, it eases a technical concern: a one-dimensional model with a smooth density function $f(\theta)$ will generically lead to discontinuities in the realized wage density at points in the wage distribution where individuals are indifferent

The distribution of individuals is described by a two-dimensional cdf $F : \Theta \times \Phi \to [0,1]$, with associated pdf $f(\theta, \varphi)$. Preferences are characterized by a continuously differentiable utility function $u(c,e)$ defined over consumption c and effort e with $u_c > 0$ and $u_e < 0$. In particular, we work with the specific form of quasilinear and isoelastic preferences, so that

$$u(c,e) = c - \frac{e^\gamma}{\gamma},$$

where $\gamma > 1$ and thus the wage elasticity of effort is constant and given by $\varepsilon \equiv 1/(\gamma - 1)$.

Each individual chooses the sector she works in so as to maximize her wage. We normalize the wage per unit of equivalent effort in sector Θ to 1, so $w = \theta$ for a Θ-sector worker with skill level θ. The wage per unit of equivalent effort in the Φ-sector is instead given by

$$w = \varphi \frac{\mu(E)}{E},$$

where E is the total equivalent effort in the Φ-sector, i.e.,

$$E = \int_{P(E)} \varphi e(\theta, \varphi) dF(\theta, \varphi), \quad \text{where} \ P(E) \equiv \left\{ (\theta, \varphi) \in \Theta \times \Phi \ \middle| \ \theta < \varphi \frac{\mu(E)}{E} \right\},$$

and $\mu(E)$ is the total output in the Φ-sector when aggregate sectoral effort is E. We assume μ to be twice continuously differentiable with $\mu(0) = 0$, $\mu'(E) > 0$ and $\mu''(E) < 0$. This captures the rent-seeking externality in a very general form. In particular, decreasing returns in the rent-seeking sector give rise to a divergence between the social marginal product of effort $\mu'(E)$ and the average product $\mu(E)/E > \mu'(E)$ that individuals face as their private wage. One extreme case (not considered here) would arise if $\mu(E) \equiv E$ for all E, so that the rent-seeking problem disappears and we find ourselves back in a standard Mirrlees economy since $\mu'(E) = \mu(E)/E = 1$. On the other hand, "pure" rent-seeking occurs when $\mu(E) \equiv \bar{\mu}$ for all E, so that there is a fixed rent to be captured in the rent-seeking sector and any effort there is in fact completely unproductive since $\mu'(E) = 0$.

We briefly provide two examples of standard rent-seeking settings that are captured by our general formulation.

Example 1 (Contests). *Consider N rent-seekers competing for a rent of value μ. Let the probability p_i that player $i \in \{1, ..., N\}$ wins the contest take the form proposed by Tullock (1980),*

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between working in the two sectors. We view this as both *a priori* and practically undesirable, the latter because it causes pervasive "bunching" of incomes.

i.e.

$$p_i(\varphi_i e_i, \varphi_{-i} e_{-i}) = \frac{\varphi_i e_i}{\sum_{j=1}^{N} \varphi_j e_j}.$$

Player i's expected payoff from the contest is therefore given by $\varphi_i e_i \mu / E$ with $E \equiv \sum_{j=1}^{N} \varphi_j e_j$. Hence, an individual rent-seeker faces a constant private expected return to effort $\varphi_i \mu / E$ whenever $\varphi_i e_i / E$ is small. In contrast, since some *individual obtains the rent for sure, the social return to effort is zero if $\mu = \overline{\mu}$ and $\varphi_i \mu'(E)$ if $\mu = \mu(E)$.*

Example 2 (Races). *Suppose individuals race to be the first to discover a rent with value $M(t)$ at time t. If the rent has not yet been discovered, the hazard rate for individual i is proportional to effective effort and thus given by $\lambda \varphi_i e_i$, where λ is a positive constant. In other words, if the rent has not yet been found, the probability that individual i will find it in the next small time increment dt is $\lambda \varphi_i e_i dt$. Hence, the probability that* some *rent-seeker will find it in dt is simply $\lambda E dt$. This implies that the time to discovery follows an exponential distribution with $p(t|E) = \lambda E \exp(-\lambda E t)$. Suppose the first individual to discover the rent captures the entire benefit $M(t)$. Then the expected payoff to an individual rent-seeker i can be computed as follows. To find the rent during a small increment of time dt around time t, two events have to happen: (i) the rent is discovered in dt, which happens with probability $p(t|E)dt$, and (ii) individual i is the one who finds the rent, which happens with probability $\varphi_i e_i / E$. Hence, the overall expected payoff from the race is*

$$\frac{\varphi_i e_i}{E} \int_0^\infty M(t) p(t|E) dt \equiv \varphi_i e_i \frac{\mu(E)}{E}$$

with $\mu(E) = \int_0^\infty M(t) p(t|E) dt$.

To characterize Pareto efficient and SCPE allocations, we define Pareto weights as follows. With $F_E(w) \equiv F(w, wE/\mu(E))$ denoting the cdf of the wage distribution induced by a given level of total effort E in the rent-seeking sector, we consider a set of welfare weights $\Psi(F_E(w))$, with $\Psi : [0, 1] \to [0, 1]$, $\Psi(0) = 0$, $\Psi(1) = 1$ and $\Psi(x)$ weakly increasing in x. The social planner maximizes $\int V(w) d\Psi(F_E(w))$ where $V(w)$ is the utility of all agents with wage w. The weighting function Ψ captures the possible redistributional motives in our framework and thus allows us to trace out the entire constrained Pareto frontier of the economy. Like the tax code, it treats any two individuals with the same wage as identical, so it expressly excludes caring about whether an individual is employed in the traditional or rent-seeking sector. Moreover, it depends on relative rather than absolute wages: if the entire distribution of wages shifts down, the welfare weights at any point in the distribution are unchanged.

For instance, if $\Psi(x) \equiv x$ for all $x \in [0, 1]$, then all individuals are weighted according to their population shares and there is no redistributive motive. We refer to

this benchmark case as "utilitarian" in the following. If $\Psi(x) \geq x$ for all $x \in [0,1]$, then $\Psi(F_E(w)) \geq F_E(w)$ for any $w \in [\underline{w}_E, \overline{w}_E]$, where $\underline{w}_E \equiv \max\{\underline{\theta}, \underline{\varphi}\mu(E)/E\}$ and $\overline{w}_E \equiv \max\{\overline{\theta}, \overline{\varphi}\mu(E)/E\}$ are the lowest and highest wages in the economy. Such Pareto weights thus describe the part of the Pareto frontier where the social planner at least weakly wants to redistribute from higher to lower wage individuals. We focus on this case in several of our results and call it a "regular" set of Pareto weights. We also sometimes refer to the resulting allocations as regular.

3   Optimal Linear Taxation

This section considers optimal linear taxes (t, T), where t is the marginal tax rate and T is the uniform lump-sum transfer. As discussed in the preceding section, the presence of rent-seeking makes the wage distribution endogenous to the tax code. A higher tax rate t induces lower effort at any wage, hence lower effort in the rent-seeking sector E. This lower effort E increases the private returns to rent-seeking $\mu(E)/E$, partially offsetting the effects of higher taxes. This endogeneity makes finding the T associated with any linear tax t non-trivial. We first solve for these lump-sum transfers. Then we formally define and solve for the set of Self-Confirming Policy Equilibrium (SCPE) tax rates—i.e., the set of tax rates which are an SCPE for *some* set of Pareto weights. This set turns out to be an interval under some mild regularity conditions.

   We then present results relating the set of SCPE linear tax rates to the set of Pareto optimal linear tax rates. Theorem 1 shows that the SCPE tax rates are "too low" in the following sense: the lowest SCPE tax rates are Pareto inefficient, and there are Pareto efficient tax rates strictly higher than any SCPE tax rate. If the economy only consists of a rent-seeking sector, we can characterize the SCPE and Pareto optimal tax rates explicitly and show that for a sufficiently strong rent-seeking externality (as parameterized by an elasticity $\beta(E) \equiv \mu'(E)E/\mu(E) \rightarrow 0$), *no* SCPE is Pareto optimal (Theorem 2 and its corollaries).

3.1   Feasible Linear Tax Allocations

Each individual takes E and hence her wage $w_{\theta,\varphi}(E) \equiv \max\left\{\theta, \varphi\frac{\mu(E)}{E}\right\}$ as given. For a given linear income tax (t, T), the individual solves

$$\max_{c,e} u(c,e) \ \text{ s.t. } \ c \leq (1-t)w_{\theta,\varphi}(E)e + T \tag{1}$$

with solution $c_{\theta,\varphi}(t, T; E)$, $e_{\theta,\varphi}(t, T; E)$ and indirect utility $V_{\theta,\varphi}(t, T; E) \equiv u(c_{\theta,\varphi}(t, T; E),$ $e_{\theta,\varphi}(t, T; E))$. Letting $P(E) \equiv \{(\theta, \varphi) | \theta < \varphi\mu(E)/E\}$, this leads us to the following definition:

Definition 1. *A **feasible linear tax allocation** is an allocation $\{c_{\theta,\varphi}(t, T; E), e_{\theta,\varphi}(t, T; E)\}$, a tax policy (t, T) and total equivalent effort in the Φ-sector E such that*
(i) $\{c_{\theta,\varphi}(t, T; E), e_{\theta,\varphi}(t, T; E)\}$ solves problem (1) given (t, T) and E,
(ii) the government budget balances:

$$T = t \left(\mu(E) + \int_{\Theta \times \Phi \backslash P(E)} \theta e_{\theta,\varphi}(t, T; E) dF(\theta, \varphi) \right), \tag{2}$$

and
(iii) total Φ-sector effort E is consistent with individual's choices:

$$E = \int_{P(E)} \varphi e_{\theta,\varphi}(t, T; E) dF(\theta, \varphi). \tag{3}$$

Note that finding feasible linear tax allocations involves solving a fixed point problem due to requirement (iii). In particular, a given linear tax policy (t, T) and total rent-seeking effort E determine after-tax wages $(1 - t)w_{\theta,\varphi}(E)$ and thus individual effort $e_{\theta,\varphi}(t, T; E)$. Then the *induced* total Φ-sector effort, given by the right-hand side of (3), has to be equal to the level of E that we started from for the allocation to be internally consistent.

We anticipate that the set of feasible linear tax allocations is a simple one-parameter family, parameterized by the level of taxes t. To establish this, however, it turns out to be more convenient to parameterize the set of feasible linear tax allocations via the level of effort in the Φ-sector, E, as the following lemma shows.

Lemma 1. *The set of feasible linear tax allocations is a one-parameter family indexed by E, with*

$$t(E) = 1 - \frac{E}{\mu(E)} \left(\frac{E}{k(E)} \right)^{\gamma - 1} \tag{4}$$

and

$$T(E) = \mu(E) + \left(\frac{E}{k(E)} \right)^{\gamma} \left[m(E) \left(\frac{E}{\mu(E)} \right)^{\frac{\gamma}{\gamma - 1}} \left(\left(\frac{k(E)}{E} \right)^{\gamma - 1} \frac{\mu(E)}{E} - 1 \right) - k(E) \right], \tag{5}$$

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where

$$k(E) \equiv \int_{P(E)} \varphi^{\frac{\gamma}{\gamma-1}} dF(\theta, \varphi) \quad and \quad m(E) \equiv \int_{\Theta \times \Phi \backslash P(E)} \theta^{\frac{\gamma}{\gamma-1}} dF(\theta, \varphi). \tag{6}$$

Proof. Notice first that fixing E also fixes $P(E)$ as well as $k(E)$ and $m(E)$ by equation (6). Using the functional form of preferences, we obtain

$$e_{\theta, \varphi}(t, T; E) = ((1-t)w_{\theta, \varphi}(E))^{\frac{1}{\gamma-1}} \tag{7}$$

and substituting in equation (3) yields

$$E = \int_{P(E)} \varphi \left((1-t)\frac{\mu(E)}{E} \varphi \right)^{\frac{1}{\gamma-1}} dF(\theta, \varphi) = \left((1-t)\frac{\mu(E)}{E} \right)^{\frac{1}{\gamma-1}} k(E) \tag{8}$$

by the definition of $k(E)$ in equation (6). Solving equation (8) for t yields equation (4). Finally, the unique lump-sum transfer $T(E)$ associated with the feasible linear tax allocation with Φ-sector effort E is defined by equation (2). Substituting equations (4), (6) and (7) yields (5). □

Equation (4) gives the unique tax rate $t(E)$ consistent with a feasible linear tax allocation with Φ-sector effort E. Note that $\mu(E)/E$ and $k(E)$ are decreasing in E (the latter because $E' > E \Rightarrow P(E') \subset P(E)$). Hence, $t(E)$ is decreasing in E, and the after tax wages $(1 - t(E))$ and $(1 - t(E))\frac{\mu(E)}{E}$ in the two sectors are *increasing* in E along the set of feasible linear tax allocations.

3.2 Self-Confirming Policy Equilibria and Pareto Optima

3.2.1 Definitions

A social planner who is aware of rent-seeking recognizes the endogeneity of the wage distribution with respect to tax policy and thus maximizes

$$\max_{t,T,E} \int_{\Theta \times \Phi} V_{\theta, \varphi}(t, T; E) d\Psi(F(\theta, \varphi)) \text{ s.t. } t \int_{\Theta \times \Phi} w_{\theta, \varphi}(E)e_{\theta, \varphi}(t, T; E)dF(\theta, \varphi) \geq T \tag{9}$$

for some given weighting function $\Psi(F)$,[5] which leads to the following definition:

Definition 2. *A **Pareto optimum with linear taxes** is a feasible linear tax allocation such that (t, T, E) solves program (9).*

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[5]Note that the budget constraints in (2) and (9) are equivalent for feasible linear tax allocations since

$$\int_{\Theta \times \Phi} w_{\theta, \varphi}(E)e_{\theta, \varphi}(t, T; E)dF(\theta, \varphi) = \mu(E) + \int_{\Theta \times \Phi \backslash P(E)} \theta e_{\theta, \varphi}(t, T; E)dF(\theta, \varphi).$$

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Hence, a sophisticated planner takes into account that changing tax policy will affect occupational choice and wages in the rent-seeking sector, and hence the overall wage distribution, which is equivalent to directly optimizing over E in addition to (t, T) within the set of feasible tax allocations.

In contrast, suppose a government or social planner is unaware of rent-seeking in the economy so that it takes the distribution of wages $w_{\theta,\varphi}(E)$ and thus E as given when optimizing over tax policy (t, T). Then it views its planning problem as the solution to the Pareto-program

$$\max_{t, T} \int_{\Theta \times \Phi} V_{\theta,\varphi}(t, T; E) d\Psi(F(\theta, \varphi)) \ \text{ s.t. } \ t \int_{\Theta \times \Phi} w_{\theta,\varphi}(E) e_{\theta,\varphi}(t, T; E) dF(\theta, \varphi) \geq T, \quad (10)$$

taking E as given and for some given set of Pareto-weights $\Psi(F)$. Based on this, we define a SCPE as follows:

Definition 3. *A **self-confirming policy equilibrium (SCPE) with linear taxes** is a feasible linear tax allocation such that (t, T) solves program (10), taking E as given.*

The idea behind this definition is that, for a given a tax policy (t, T) and preferences $u(c, e)$, the government is able to back out the wage distribution from the observed income distribution, as pointed out by Saez (2001). In the SCPE, the tax policy (t, T) is then indeed optimal given this wage distribution. In other words, the SCPE describes a fixed point where, when the government identifies the wage distribution from the equilibrium income distribution and tax policy and views it as fixed, the optimality of the equilibrium tax policy is confirmed. We are now ready to characterize the set of SCPE and compare it to the set of Pareto optima.

3.2.2 A Characterization of the Set of SCPE

Fixing a given E also fixes the resulting distribution $F_E(w)$ of wages $w_{\theta,\varphi}(E)$. In addition, define the lower and upper extremes in the support of the wage distribution (as a function of E) as

$$\underline{w}_E \equiv \inf_{\theta \in \Theta, \varphi \in \Phi} \{w_{\theta,\varphi}(E)\} = \max\left\{\underline{\theta}, \frac{\mu(E)}{E}\underline{\varphi}\right\}, \text{ and}$$

$$\overline{w}_E \equiv \sup_{\theta \in \Theta, \varphi \in \Phi} \{w_{\theta,\varphi}(E)\} = \max\left\{\overline{\theta}, \frac{\mu(E)}{E}\overline{\varphi}\right\}.$$

Then the following result provides a preliminary but useful characterization of the set of SCPE linear tax rates.

Lemma 2. *For a given E and the resulting wage distribution $F_E(w)$, the set of SCPE linear tax rates t is characterized by*

$$\frac{\underline{\xi}(E)}{\xi(E)} \leq \left[1 - \frac{t}{(\gamma - 1)(1 - t)}\right] \leq \frac{\overline{\xi}(E)}{\xi(E)} \tag{11}$$

with $\xi(E) \equiv \int_{\underline{w}_E}^{\overline{w}_E} w^{\frac{\gamma}{\gamma-1}} dF_E(w)$, $\underline{\xi}(E) \equiv \underline{w}_E^{\frac{\gamma}{\gamma-1}}$ and $\overline{\xi}(E) \equiv \overline{w}_E^{\frac{\gamma}{\gamma-1}}$.

Proof. Taking the distribution $F_E(w)$ of wages $w_{\theta,\varphi}(E) = \max\{\theta, \varphi \frac{\mu(E)}{E}\}$ as fixed, the planner believes that it faces the budget constraint:

$$T(t; E) = t \int_{\underline{w}_E}^{\overline{w}_E} w \left((1 - t)w\right)^{\frac{1}{\gamma-1}} dF_E(w) = t\nu(t; E),$$

where

$$\nu(t; E) \equiv (1 - t)^{\frac{1}{\gamma-1}} \int_{\underline{w}_E}^{\overline{w}_E} w^{\frac{\gamma}{\gamma-1}} dF_E(w)$$

is total output at E (using equation (7)). Since the planner (incorrectly) regards $F_E(w)$, $\overline{w}_E$ and $\underline{w}_E$ as being tax-independent, she believes that

$$\frac{\partial T(t; E)}{\partial t} = \nu(t; E) - t\frac{(1 - t)^{\frac{1}{\gamma-1}-1}}{(\gamma - 1)} \int_{\underline{w}_E}^{\overline{w}_E} w^{\frac{\gamma}{\gamma-1}} dF_E(w) = \nu(t; E)\left[1 - \frac{t}{(\gamma - 1)(1 - t)}\right]$$

Hence, given Pareto weights $\Psi(F_E(w))$, the planner attempts to solve:

$$\max_t T(t; E) + (1 - t)^{\frac{\gamma}{\gamma-1}} \frac{\gamma - 1}{\gamma} \int_{\underline{w}_E}^{\overline{w}_E} w^{\frac{\gamma}{\gamma-1}} d\Psi(F_E(w)).$$

The necessary condition for the planners problem,

$$(1 - t)^{\frac{1}{\gamma-1}} \int_{\underline{w}_E}^{\overline{w}_E} w^{\frac{\gamma}{\gamma-1}} d\Psi(F_E(w)) = \nu(t; E)\left[1 - \frac{t}{(\gamma - 1)(1 - t)}\right],$$

can therefore be satisfied for some $\Psi(F)$ if and only if:

$$(1 - t)^{\frac{1}{\gamma-1}} \underline{w}_E^{\frac{\gamma}{\gamma-1}} \leq \nu(t; E)\left[1 - \frac{t}{(\gamma - 1)(1 - t)}\right] \leq (1 - t)^{\frac{1}{\gamma-1}} \overline{w}_E^{\frac{\gamma}{\gamma-1}}.$$

Using the definition of $\nu(t; E)$ and rearranging yields equation (11). $\square$

The idea behind Lemma 2 is the following. For a fixed wage distribution, a naive social planner believes that the highest Pareto optimal linear tax rate is the one preferred by the lowest wage individuals with wage $\underline{w}_E$ and the lowest Pareto optimal tax rate is the one preferred by the highest wage earners with $\overline{w}_E$. All tax rates in between are also Pareto efficient since they would solve (10) for some Pareto weights function $\Psi(F)$. This gives rise to the bounds in equation (11). However, in fact the wage distribution

$F_E(w)$ depends on E and thus on t (through (4)). We can therefore denote the set of linear tax rates perceived as Pareto optimal given the observed wage distribution $F_{E(t)}(w)$ induced by some given t by $Y(t) \equiv [\underline{t}(t), \overline{t}(t)]$, which is the interval of tax rates satisfying inequality (11) in Lemma 2. Then a given tax rate t is a SCPE for some set of Pareto weights precisely when $t \in Y(t)$. We next explore properties of the set of SCPE linear tax rates that the correspondence $Y(t)$ gives rise to. We will then compare SCPE tax rates to the set of Pareto optimal ones.

Note first that, for any t, $\overline{t}(t) \in (0,1)$ and $\underline{t}(t) \in [-\infty, 0)$.[6] Furthermore, because $\lim_{t \to -\infty} 1 - \frac{t}{(\gamma-1)(1-t)} = \frac{\gamma}{\gamma-1}$, we have $\underline{t}(t) > -\infty$ if and only if $\frac{\overline{\zeta}(E)}{\underline{\zeta}(E)} < \frac{\gamma}{\gamma-1}$. We make the following distributional assumption to ensure that this is the case.

Assumption 1. *Let*

$$\lambda_\theta \equiv \int_{\Theta \times \Phi} \left(\frac{\theta}{\overline{\theta}} \right)^{\frac{\gamma}{\gamma-1}} dF(\theta, \varphi) \quad and \quad \lambda_\varphi \equiv \int_{\Theta \times \Phi} \left(\frac{\varphi}{\overline{\varphi}} \right)^{\frac{\gamma}{\gamma-1}} dF(\theta, \varphi).$$

Then we assume $\lambda_\theta, \lambda_\varphi > (\gamma - 1)/\gamma$.

Under this condition, we immediately obtain the following result:

Lemma 3. *Under Assumption 1, there exists a finite x such that $\underline{t}(t) > x > -\infty$ for all t.*

Proof. Observe that

$$\frac{\underline{\zeta}(E)}{\overline{\zeta}(E)} = \min \left\{ \int_{\underline{w}_E}^{\overline{w}_E} \left(\frac{w}{\overline{\theta}} \right)^{\frac{\gamma}{\gamma-1}} dF_E(w), \int_{\underline{w}_E}^{\overline{w}_E} \left(\frac{w}{\frac{\mu(E)}{E} \overline{\varphi}} \right)^{\frac{\gamma}{\gamma-1}} dF_E(w) \right\} \geq \min \{ \lambda_\theta, \lambda_\varphi \}.$$

Hence, under Assumption 1, $\frac{\overline{\zeta}(E)}{\underline{\zeta}(E)} < \frac{\gamma}{\gamma-1}$, and $\underline{t}(t(E)) > x > -\infty$, where $t(E)$ is the feasible linear tax allocation tax rate associated with E and

$$x = \frac{(\gamma - 1)(1 - \min\{\lambda_\theta, \lambda_\varphi\})}{1 + (\gamma - 1)(1 - \min\{\lambda_\theta, \lambda_\varphi\})}.$$

$\square$

The intuition behind Assumption 1 and Lemma 3 is as follows. As discussed above, the upper and lower bounds $\overline{t}(t)$ and $\underline{t}(t)$ are the linear tax rates preferred by the lowest and highest wage earners, respectively. They reflect a tradeoff between lowering the marginal tax rate t and the associated reduction in the lump-sum transfer T required by budget balance (2). The lowest wage earner always favors a positive tax rate $\overline{t}(t) \in (0,1)$

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[6]The zero bounds follow from the fact that with quasilinear and isoelastic preferences, the SCPE for utilitarian welfare weights with $\Psi(F) = F$ has $t = 0$.

and the associated positive lump-sum transfer $T > 0$. In contrast, the highest wage earners' preferred tax policy is always a wage subsidy $\underline{t}(t) < 0$ and a lump-sum tax (a negative lump-sum transfer $T < 0$). If the wage density falls off very quickly for higher wages, the highest wage earners' preferred wage subsidy is in fact infinite with $t \to -\infty$ because in this case financing such a wage subsidy does not require a large increase in the lump-sum tax. Assumption 1 rules out this case by requiring that the skill distribution has a sufficient mass of high-skilled types in both skill dimensions.

Assumption 1 thus ensures that the correspondence $Y(t)$ is finite-interval valued with $x < \underline{t}(t) \le \bar{t}(t) < 1$ for all t. Since $\underline{t}(t)$ and $\bar{t}(t)$ are both continuous, it is easy to show that there exists some lowest and highest fixed point of $Y(t)$, denoted $\underline{t}^{SC}$ and $\bar{t}^{SC}$. In fact, $\bar{t}(t)$ has a unique fixed point as the following lemma shows:

Lemma 4. *Consider the upper bound $\bar{t}(t)$ of the correspondence $Y(t)$. Then $d\bar{t}(t)/dt < 1$ at all points where $t = \bar{t}(t)$.*

Proof. See Appendix A.1. □

Lemma 4 implies that the upper bound of the correspondence $Y(t)$ can only "downward cross" the 45°-line, so that there must exist a unique crossing at $\bar{t}^{SC}$. Figure 1 illustrates the correspondence $Y(t)$ and the resulting set of SCPE tax rates for the case in which it is an interval given by $[\underline{t}^{SC}, \bar{t}^{SC}]$.[7] It lies between the (negative) tax rate at which the lower bound of the correspondence $Y(t)$ crosses the 45° line and the positive tax rate at which the top bound of the correspondence crosses it.

3.3 Comparing SCPE and Pareto Optimal Allocations

Intuitively, we expect that, since they fail to take into account the negative externality associated with effort in the rent-seeking sector, SCPE tax rates will be "too low" relative to Pareto optima. The following result establishes this formally. Specifically, it shows that the lowest SCPE tax rate (and anything below it) is Pareto inefficient, and that there are tax rates higher than any SCPE tax rate which are efficient.

Theorem 1. *(i) There exist Pareto optimal feasible linear tax allocations with $t > \bar{t}^{SC}$.*
(ii) Suppose that Assumption 1 is satisfied, so that $\underline{t}^{SC} > -\infty$ exists. Then any feasible linear tax allocation with $t \le \underline{t}^{SC}$ is Pareto inefficient.

Proof. See Appendix A.2. □

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[7]It is straightforward to provide conditions on fundamentals, namely λ_θ and λ_φ, so that $\underline{t}(t)$ also has a unique fixed point and the set of SCPE linear tax rates is indeed an interval.
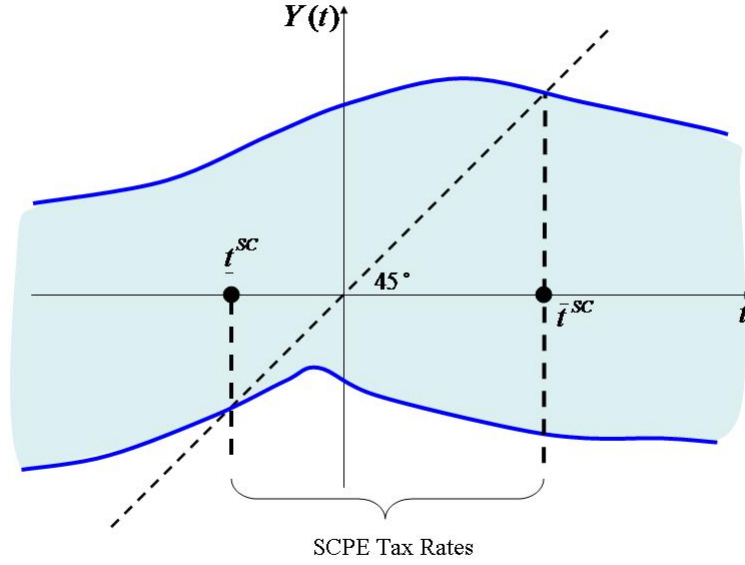
Figure 1: SCPE tax rates

Theorem 1 formalizes the intuition that the set of Pareto optimal linear tax rates is shifted to the right compared to the set of SCPE linear tax rates in terms of its bounds. Note that an interval structure of the set of SCPE tax rates is not required for this result. Theorem 1 only makes use of Assumption 1 (so that $\underline{t}(t) > x > -\infty$ for all t and hence $\underline{t}^{SC}$ exists) and Lemma 4 (so that $\bar{t}(t) \geq t$ for all $t \leq \bar{t}^{SC}$). Then the result can be shown to follow from the fact that a marginal tax increase has an additional positive welfare effect in the full Pareto program compared to the SCPE program since it reduces total rent-seeking effort E and thus shifts the wage distribution up.

3.4 Example: A One Sector Rent-Seeking Economy

For illustrative purposes, we briefly consider the special case where the economy only consists of the rent-seeking sector. This case would emerge if all the skill density was concentrated in the φ-dimension, giving rise to a continuous cdf $F(\varphi)$ on $\Phi = [\underline{\varphi}, \overline{\varphi}]$. The following result provides a simple comparison between the SCPE and Pareto optimal linear tax allocations for a fixed set of Pareto weights $\Psi(F)$.

Theorem 2. *Consider a one sector rent-seeking economy and suppose Assumption 1 holds.[8] Then*

---

[8]More precisely, Assumption 1 becomes $\lambda_\varphi \equiv \int_{\underline{\varphi}}^{\overline{\varphi}} \left(\frac{\varphi}{\overline{\varphi}}\right)^{\frac{\gamma}{\gamma-1}} dF(\varphi) > (\gamma - 1)/\gamma$ in this special case.

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for any set of Pareto-weights $\Psi(F)$, there is a unique SCPE tax rate t^{SC} and a unique Pareto optimal tax rate t^{PO} such that

$$1 - t^{SC} = \left(1 + (\gamma - 1) \left(1 - \frac{\int_\Phi \varphi^{\frac{\gamma}{\gamma-1}} d\Psi(F(\varphi))}{\int_\Phi \varphi^{\frac{\gamma}{\gamma-1}} dF(\varphi)} \right) \right)^{-1} \tag{12}$$

and

$$1 - t^{PO} = \beta(E^{PO})(1 - t^{SC}), \tag{13}$$

where E^{PO} is the level of rent-seeking equivalent effort at the Pareto optimum given $\Psi(F)$ and

$$\beta(E) \equiv \frac{\mu'(E) E}{\mu(E)} < 1$$

denotes the elasticity of aggregate output in the rent-seeking sector with respect to E.

Proof. See Appendix A.3. □

If the economy only consists of a rent-seeking sector, the formulas for the optimal t^{SC} and t^{PO} for any given $\Psi(F)$ take a very intuitive form. In particular, note that t^{SC} can be expressed only in terms of fundamentals and is completely independent of the rent-seeking technology $\mu(E)$ and is decreasing in the term $\int_\Phi \varphi^{\frac{\gamma}{\gamma-1}} d\Psi(F(\varphi)) \Big/ \int_\Phi \varphi^{\frac{\gamma}{\gamma-1}} dF(\varphi)$, which measures the redistributive motives implied by $\Psi(F)$. Notably, if $\Psi(F)$ is regular (i.e. $\Psi(F) \geq F$ for all $F \in [0,1]$), then $t^{SC} \geq 0$ and t^{SC} increases as $\Psi(F)$ shifts more weight to lower skilled individuals. t^{SC} is also increasing (in absolute value) in γ, which is inversely related to the wage elasticity of effort $\varepsilon = 1/(\gamma - 1)$.

As equation (13) makes clear, t^{PO} shares these comparative statics with respect to redistributive motives with t^{SC}, but in addition is such that the keep share $1 - t^{PO}$ is scaled down compared to the SCPE by the elasticity of the rent-seeking technology $\beta(E)$. This elasticity captures the divergence between the marginal product $\mu'(E)$ and the private returns $\mu(E)/E$ and hence the rent-seeking externality. Since $\beta(E) < 1$, $t^{PO} > t^{SC}$ for any given set of Pareto weights $\Psi(F)$. A special case arises for utilitarian welfare, so that the redistributive motives disappear.

Corollary 1. *Suppose $\Psi(F)$ is utilitarian with $\Psi(F) = F$ for all $F \in [0,1]$. Then $t^{SC} = 0$ and $t^{PO} = 1 - \beta(E^{PO}) > 0$.*

The utilitarian case isolates the pure corrective motive for taxation in our framework. While the SCPE tax rate is zero in this case, the Pareto optimum is associated with a

strictly positive, Pigouvian tax that makes agents internalize and is increasing in the rent-seeking externality. Notably, in the extreme case of $\mu(E) = \overline{\mu}$ so that $\beta(E) = 0$ (a "pure" rent-seeking economy), $t^{PO} = 1$ and all effort is completely crowded out. On the other hand, if $\beta(E) = 1$ because $\mu(E) = E$, the rent-seeking problem would disappear and $t^{PO} = t^{SC} = 0$.

Theorem 2 immediately implies the following result for the entire sets of SCPE and Pareto optimal linear tax rates in a one sector rent-seeking economy:

Corollary 2. *In a one sector rent-seeking economy, the set of SCPE with linear taxes is independent of the structure of the production function $\mu(E)$ and given by the interval*

$$
\left[1 - \frac{1}{\gamma\left(1 - \frac{\overline{k}}{k}\right) + \frac{\overline{k}}{k}}, 1 - \frac{1}{\gamma\left(1 - \frac{\underline{k}}{k}\right) + \frac{\underline{k}}{k}} \right],
$$

where $k = \int_{\Phi} \varphi^{\frac{\gamma}{\gamma-1}} dF(\varphi)$, $\underline{k} = \underline{\varphi}^{\frac{\gamma}{\gamma-1}}$ and $\overline{k} = \overline{\varphi}^{\frac{\gamma}{\gamma-1}}$. The set of Pareto-optimal linear tax rates is

$$
\left[1 - \frac{\beta(E^{PO})}{\left(\gamma\left(1 - \frac{\overline{k}}{k}\right) + \frac{\overline{k}}{k}\right)}, 1 - \frac{\beta(E^{PO})}{\left(\gamma\left(1 - \frac{\underline{k}}{k}\right) + \frac{\underline{k}}{k}\right)} \right].
$$

The sets of both SCPE and Pareto optimal linear tax rates are always both intervals in a one-sector economy and the interval of Pareto optimal tax rates is shifted to the right compared to the SCPE interval. For instance, consider the rent-seeking technology $\mu(E) = E^{\beta}$ with $\beta \in (0, 1)$, so that the elasticity $\beta(E) = \beta$ is constant. Then for sufficiently low β, every Pareto efficient tax rate is higher than every SCPE tax rate, so that the entire set of SCPE tax rates is Pareto inefficient.

This single sector linear tax model bears a close relationship to Sandmo's (1975) model of optimal *commodity* taxation in the presence of atmospheric externalities. The major difference is that linear income taxation allows for uniform lump sum taxes and transfers, which are ruled out by the commodity taxes analyzed by Sandmo (1975). Only if we further restricted our linear income tax to a proportional tax would our framework in this special case become formally equivalent to a version of Sandmo's model.[9]

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[9]Specifically, it would become equivalent to a one good, continuous type-space version of Sandmo's (1975) model with individual utilities given by

$$
u(c, e, C) = c - \Lambda(C)e^{\gamma},
$$

where C is aggregate consumption, and $\Lambda(C)$, which captures the consumption externality, is related in a simple way to the rent-seeking technology $\mu(E)$ in our model. Of course, this version of the model is trivial: the "optimal" tax is entirely pinned down by the social budget constraint.

4 Optimal Non-Linear Taxation

The analysis above indicates that—as one would have expected—taking rent-seeking into account prescribes higher levels of (linear) taxation. In this section, we extend our analysis to non-linear taxation. This constitutes a methodological contribution as we show how our notions or SCPE and Pareto optimality can be operationalized with non-linear taxation in our rent-seeking framework. In addition, by relaxing the assumption that all agents in the economy face the same marginal tax rates, it also allows us to address a number of relevant policy questions using our model. For instance, how does rent-seeking affect marginal tax rates at different income levels? In other words, what are the implications of rent-seeking for the optimal progressivity of the tax schedule?

We first show that, with non-linear taxation, marginal tax rates depend on the share of rent-seekers at a given wage level. However, in the special case of a one sector model where all agents are rent-seekers, rent-seeking does *not* affect the optimal degree of progressivity in the tax system. This is even though taxing high incomes at a higher rate allows for additional redistribution towards lower wage earners through two channels in such an economy: it generates additional tax revenue that can be transferred to lower incomes, but it *also* increases everyone's wage by discouraging effort and thus reducing total E and increasing the private returns to effort $\mu(E)/E$. We demonstrate that this second channel does nonetheless not lead to a more progressive tax schedule, and any impetus that rent-seeking arguments can provide for an enhanced (or decreased) progressivity of the tax schedule must therefore result from the sectoral composition of workers at different skill levels.

Second, we study a particularly salient aspect of progressivity by exploring the optimal top marginal tax rate in a two-sector model (with both traditional effort and rent-seeking). Our main insight here is that, even if the highest wage earners are all rent-seekers and the social planner has a preference for redistribution towards lower wage earners, the optimal top marginal tax rate is *less* than the Pigouvian rate $1 - \beta(E)$ that would let agents fully internalize the rent-seeking externality, as derived in the previous section. The key reason is a sectoral shift effect that we discuss in detail below: Taxing the top earners (or any rent-seekers) at a lower rate increases total rent-seeking effort E and therefore reduces private returns in the rent-seeking sector $\mu(E)/E$. This prevents other agents from entering the socially less productive rent-seeking sector. In our quantitative analysis in the following section 5, we show that this sectoral shift effect can be strong and induce top marginal tax rates that are *substantially* lower than the Pigouvian rate $1 - \beta(E)$ that a single sector model with rent-seekers only would have prescribed.

Third, we consider the efficiency of SCPE non-linear tax schedules. In the linear taxation framework discussed in the preceding section, there existed SCPE tax rates that were Pareto efficient and others that were Pareto inefficient (unless with a single sector the rent-seeking externality was so strong that the entire set of SCPE was inefficient). In contrast, we show here that under general conditions no regular SCPE is Pareto-optimal with non-linear taxes. Thus, the more flexible non-linear tax instrument allows for Pareto improvements from taking rent-seeking into account under a much wider range of circumstances.

4.1 A Decomposition and Definitions

We start by defining SCPE and Pareto optima when the tax schedule can be non-linear and thus allocations are only constrained by resource and incentive constraints. For this purpose, it turns out to be useful to decompose the problem of finding SCPE or Pareto optimal allocations into two steps: The first (referred to as "inner" problem) involves finding the optimal resource feasible and incentive compatible allocation for a fixed level of rent-seeking effort E and thus a fixed wage distribution with

$$F_E(w) \equiv F\left(w, w\frac{E}{\mu(E)}\right),\tag{14}$$

$$f_E^\theta(w) \equiv \int_{\underline{\varphi}}^{w\frac{E}{\mu(E)}} f(w, \varphi)d\varphi, \quad f_E^\varphi(w) \equiv \frac{E}{\mu(E)}\int_{\underline{\theta}}^{w} f\left(\theta, w\frac{E}{\mu(E)}\right)d\theta,\tag{15}$$

and $f_E(w) \equiv f_E^\theta(w) + f_E^\varphi(w)$ and $[\underline{w}_E, \overline{w}_E]$ with

$$\underline{w}_E \equiv \max\left\{\underline{\theta}, \underline{\varphi}\frac{\mu(E)}{E}\right\} \quad \text{and} \quad \overline{w}_E \equiv \max\left\{\overline{\theta}, \overline{\varphi}\frac{\mu(E)}{E}\right\}.$$

Note that E also determines the occupational choice of all individuals and therefore the sectoral composition of the economy, so that we call $f_E^\theta(w)$ the density of wages in the traditional sector and $f_E^\varphi(w)$ in the rent-seeking sector conditional on E. $f_E(w)$ is thus the aggregate wage density for a given E, adding the densities of individuals with a given wage in both sectors.

The second step then involves finding the optimal (or, in the case of an SCPE, consistent) level of E. We refer to this step as the "outer" problem.

4.1.1 Pareto Optima with Non-linear Taxes

Since the wage distribution is fixed for given E, the inner problem for the Pareto optimum is an almost standard Mirrlees problem with the only complication that we have to take into account the sectoral composition of the economy. More precisely, the induced level of equivalent effort in the rent-seeking sector has to be consistent with the level of E that we started from. For some given Pareto weights $\Psi(F)$, we therefore define the inner problem as follows:

$$W(E) \equiv \max_{V(w),e(w)} \int_{\underline{w}_E}^{\overline{w}_E} V(w)d\Psi(F_E(w)) \tag{16}$$

s.t.

$$V'(w) - \frac{e(w)^\gamma}{w} = 0 \quad \forall w \in [\underline{w}_E, \overline{w}_E] \tag{17}$$

$$\mu(E) - \int_{\underline{w}_E}^{\overline{w}_E} we(w)f_E^\varphi(w)dw = 0 \tag{18}$$

$$\int_{\underline{w}_E}^{\overline{w}_E} we(w)f_E(w)dw - \int_{\underline{w}_E}^{\overline{w}_E} \left(V(w) + \frac{e(w)^\gamma}{\gamma}\right)f_E(w)dw \geq 0. \tag{19}$$

We employ the standard Mirrleesian approach of optimizing directly over allocations, i.e. over effort $e(w)$ and consumption $c(w)$ profiles. It is convenient to write allocations in terms of utilities $V(w) \equiv c(w) - e(w)^\gamma/\gamma$ and efforts $e(w)$ and then to infer consumption.

The social planner then maximizes some weighted average of the individuals' utilities $V(w)$ subject to a set of constraints. (19) is a standard resource constraint and constraint (18) guarantees that total effort in the rent-seeking sector indeed sums up to E (or, equivalently, the sum of all incomes in the rent-seeking sector equals $\mu(E)$). Finally, the allocation $V(w), e(w)$ needs to be incentive compatible, i.e.

$$V(w) \geq V(w') + \frac{e(w')}{\gamma}\left(1 - \left(\frac{w'}{w}\right)^\gamma\right) \quad \forall w, w' \in [\underline{w}_E, \overline{w}_E]. \tag{20}$$

It is a well-known result that the global incentive constraints (20) are equivalent to the local incentive constraints (17) and the monotonicity constraint that income $y(w) \equiv we(w)$ must be non-decreasing in w.[10] We follow the standard approach of dropping the monotonicity constraint and checking ex-post that it is satisfied. If the solution to problem (16) to (19) does not satisfy it, optimal bunching would need to be considered.

Once a solution $V(w), e(w)$ to the inner problem has been found, the resulting welfare

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[10]See, for instance, Fudenberg and Tirole (1991), Theorems 7.2 and 7.3.

is given by $W(E)$, so that the outer problem for the Pareto problem simply becomes

$$\max_{E} W(E). \tag{21}$$

This leads us to the following definition:

Definition 4. *A **Pareto optimum** with non-linear taxes is a level of total equivalent rent-seeking effort E and an allocation $V(w), e(w)$ such that (i) E solves the outer problem (21) and (ii) $V(w), e(w)$ solves the inner problem (16) to (19) given E.*

Note that marginal tax rates $T'(y(w))$ can be backed out from an allocation $V(w), e(w)$ by using the workers' first order condition

$$1 - T'(y(w)) = \frac{e(w)^{\gamma-1}}{w}. \tag{22}$$

4.1.2 Self-Confirming Policy Equilibria with Non-linear Taxes

Suppose the social planner does not take into account rent-seeking in the economy. Then for a fixed level of E and hence a fixed wage distribution, she views the optimal tax problem as a standard Mirrlees problem, so that the inner problem becomes

$$\tilde{W}(E) \equiv \max_{V(w), e(w)} \int_{\underline{w}_E}^{\overline{w}_E} V(w) d\Psi(F_E(w))$$

s.t.

$$V'(w) - \frac{e(w)^{\gamma}}{w} = 0 \ \ \forall w$$

$$\int_{\underline{w}_E}^{\overline{w}_E} we(w) f_E(w) dw - \int_{\underline{w}_E}^{\overline{w}_E} \left(V(w) + \frac{e(w)^{\gamma}}{\gamma} \right) f_E(w) dw \geq 0.$$

Hence, the inner problem for a SCPE is a strictly relaxed version of the inner problem for the Pareto optimum, dropping constraint (18). This is because the naive social planner is not aware of the fact that the wage distribution is endogenous or that total rent-seeking effort has to hit E. However, for the equilibrium to be self-confirming, when computing the total rent-seeking effort implied by the solution $e(w)$, namely

$$\tilde{E}(E) = \frac{E}{\mu(E)} \int_{\underline{w}_E}^{\overline{w}_E} we(w) f_E^{\varphi}(w) dw, \tag{23}$$

then we have to be at a fixed point such that $E = \tilde{E}(E)$. In other words, in a SCPE, the planner takes the wage distribution as fixed and designs an optimal tax schedule given

this distribution. Then the wage distribution induced by this tax schedule has to be equal to the original wage distribution, so that the planner finds herself confirmed in the view that the wage distribution is fixed (even though it actually is not once we were to move away from the fixed point). We thus have the following definition:

Definition 5. *A **Self-Confirming Policy Equilibrium (SCPE)** with non-linear taxes is a level of total equivalent rent-seeking effort E and an allocation $V(w), e(w)$ such that (i) E is a fixed point of $\tilde{E}(E)$ defined in (23) and (ii) $V(w), e(w)$ solves the inner problem (16) s.t. (17) and (19) given E.*

Hence, while the inner problem for a SCPE is a relaxed version of the inner problem for a Pareto optimum, the outer problem is in fact a fixed point problem rather than an optimization.[11]

4.2   Marginal Tax Rate Formulas from the Inner Problems

In this subsection, we demonstrate that our approach allows us to derive transparent formulas for optimal marginal tax rates both for Pareto optima and SCPE conditional on E and thus a wage distribution. In fact, given some Pareto weights $\Psi(F)$, we have the following result based on solving the inner problems (16) s.t. (17) and (19) (and (18) in the case of a Pareto optimum):

Proposition 1. *Fix E and let ξ denote the multiplier on constraint (18) in the Pareto problem (16) to (19). Then*

$$1 - T'(y(w)) = \left(1 - \xi \frac{f_E^\varphi(w)}{f_E(w)}\right)\left(1 + \gamma \frac{\Psi(F_E(w)) - F_E(w)}{w f_E(w)}\right)^{-1} \tag{24}$$

for all $w \in [\underline{w}_E, \overline{w}_E]$ at a Pareto optimum. Instead, in a SCPE

$$1 - T'(y(w)) = \left(1 + \gamma \frac{\Psi(F_E(w)) - F_E(w)}{w f_E(w)}\right)^{-1}. \tag{25}$$

Proof. See Appendix B.1. □

Let us start with interpreting the formula for marginal tax rates in a SCPE as given by (25). $T'(y(w)) \geq 0$ at all income levels if and only if $\Psi(F)$ is regular, and it is increasing

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[11]Note that an equivalent way of describing a SCPE in our framework is to define it is a level of E and an allocation $V(w), e(w)$ such that, given E, the allocation $V(w), e(w)$ solves problem (16) to (19) *including* constraint (18), but (18) is not binding at the solution. The latter condition makes sure that we are at a fixed point of $\tilde{E}(E)$.

in $\Psi(F_E(w)) - F_E(w)$, i.e. in the degree to which $\Psi(F)$ shifts weight to lower wage individuals compared to $F_E(w)$. This captures the redistributive effect of an increase in the marginal tax rate at w. Moreover, $T'(y(w))$ is decreasing in the wage elasticity of effort $\varepsilon = 1/(\gamma - 1)$ and the wage density $f_E(w)$, which are both related to the distortionary effects at w (see also Diamond (1998)).

Interestingly, the formula for marginal tax rates at a Pareto optimum shares this structure, but adds to it a corrective factor that transparently captures the Pigouvian motive for taxation in our framework. Notably, it is such that all marginal keep shares $1 - T'(y(w))$ are scaled down by $1 - \xi f_E^\varphi(w)/f_E(w)$, where ξ is the Lagrangian on constraint (18) and $f_E^\varphi(w)/f_E(w)$ is the share of rent-seekers at wage level w. This is intuitive as it is saying that the optimal correction, which makes agents internalize the rent-seeking externality, is proportional to the fraction of rent-seekers at w and the shadow cost of the rent-seeking constraint (18). In particular, it disappears if $f_E^\varphi(w) = 0$ at w, or if constraint (18) does not bind, which would be the case at a SCPE.

However, the comparison between marginal tax rates at SCPE and Pareto optima, even for the same weighting function $\Psi(F)$, is not straightforward since ξ and E are in fact endogenous, with the former depending on E, which is in turn determined from the respective outer problems. Since Pareto optima and SCPE will in general involve different levels of E, they will also differ in their wage distributions $F_E(w)$ and thus $f_E(w)$ and $f_E^\varphi(w)$. We will therefore next explore the determination of E (and thus ξ) by considering the outer problems in more detail. The following immediate implications of Proposition 1 will be prove useful for this.

Corollary 3. *The top marginal tax rate $T'(y(\overline{w}_E))$ is zero in any SCPE. In any Pareto optimum, it is given by*

$$T'(y(\overline{w}_E)) = \xi \frac{f_E^\varphi(\overline{w}_E)}{f_E(\overline{w}_E)}.$$

Notably, $T'(y(\overline{w}_E)) = \xi$ if all top earners are rent-seekers.

Thus, while SCPE share the typical "no distortion at the top" property with a standard optimal taxation problem, a Pareto optimum will impose a top marginal tax rate that still reflects the corrective motive for taxation in our framework, which crucially depends on the value of ξ and the share of rent-seekers at the top. The same is true for all wage levels in the case of utilitarian welfare:

Corollary 4. *Suppose Ψ is utilitarian, i.e. $\Psi(F) = F$ for all $F \in [0, 1]$. Then*

$$T'(y(w)) = \xi \frac{f_E^\varphi(w)}{f_E(w)} \quad \forall w \in [\underline{w}_E, \overline{w}_E]$$

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in any Pareto optimum and $T'(y(w)) = 0$ for all $w \in [\underline{w}_E, \overline{w}_E]$ in any SCPE.

Given our quasilinear preferences, all redistributive motives disappear for utilitarian welfare, so that the corresponding SCPE involves no taxation whatsoever (the laissez-faire equilibrium). The utilitarian Pareto optimum in contrast involves a marginal tax rate that again exclusively reflects the corrective motive for taxation, just like the top marginal tax rate in the case of general Pareto weights.

4.3 Optimal Size of the Rent Seeking Sector from the Outer Problem

We now turn to the outer problem to determine the equilibrium level of the rent-seeking sector E and thus ξ, which has turned out to be a key input in the marginal tax rate formula (24).

4.3.1 A General Formula

We start with the following decomposition of the welfare effect of a marginal increase in E from the outer problem for a Pareto optimum (21):

Proposition 2. *For any given Pareto optimum (i.e. any given set of Pareto weights $\Psi(F)$), the welfare effect of a marginal change in total equivalent rent-seeking effort E can be decomposed as follows:*

$$W'(E) = \xi \mu'(E) + \xi S + Z, \tag{26}$$

where

$$S \equiv \int_{\underline{w}_E}^{\overline{w}_E} we(w) \frac{df_E^\theta(w)}{dE} dw \tag{27}$$

is the sectoral shift effect and

$$Z \equiv -\frac{1 - \beta(E)}{E} \left((1 - \xi)\mu(E) - D_1 - \xi D_2 \right) \tag{28}$$

is the wage shift effect with

$$D_1 \equiv \int_{\underline{w}_E}^{\overline{w}_E} e(w)^\gamma \left(\Psi(F_E(w)) - F_E(w) \right) \frac{d}{dw} \left(\frac{f_E^\varphi(w)}{f_E(w)} \right) dw \tag{29}$$

and

$$D_2 \equiv \int_{\underline{w}_E}^{\overline{w}_E} w^2 e'(w) \frac{f_E^\varphi(w) f_E^\theta(w)}{f_E(w)} dw. \tag{30}$$
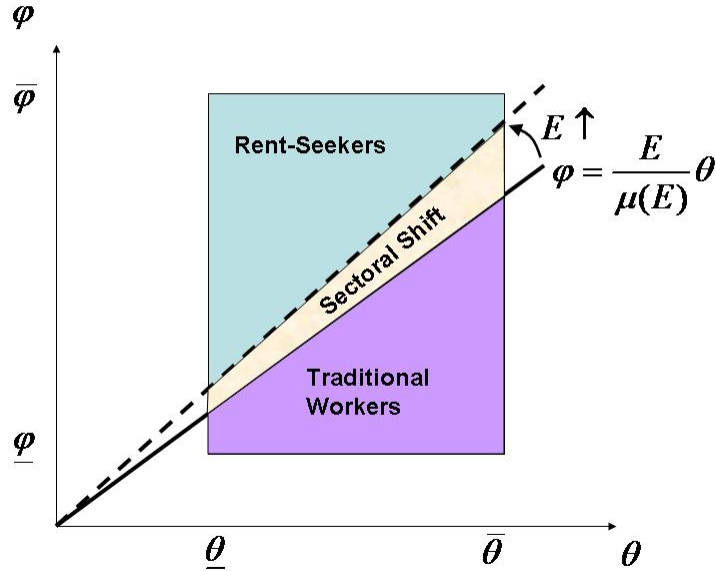
Proof. See Appendix B.2. □

Figure 2: The sectoral shift effect

Proposition (2) shows that a change in E leads to a change in welfare $W(E)$ that can be divided into three effects. First, there is a direct effect on constraint (18), captured by the first term in (26). Second, there is a sectoral shift effect S given by equation (27). In particular, since a marginal increase in E reduces the private returns $\mu(E)/E$ and thus wages in the rent-seeking sector, individuals who were indifferent between being a worker or a rent-seeker before the change will leave the rent-seeking sector and move to the traditional sector. Then S measures the total income that is shifted to the traditional sector through their move, holding wages and the effort schedule fixed. This effect is key to our analysis in the following and illustrated in Figure 2. Note that, by (15),

$$\frac{df_E^\theta(w)}{dE} = \frac{1}{\mu(E)} \left(1 - \beta(E)\right) w f \left(w, w \frac{E}{\mu(E)}\right) > 0$$

so that $S > 0$ whenever $f(\theta, \varphi)$ has full support on $[\underline{\theta}, \overline{\theta}] \times [\underline{\varphi}, \overline{\varphi}]$. Intuitively, since private returns $\mu(E)/E$ exceed the social marginal product $\mu'(E)$ in the rent-seeking sector, the sectoral shift effect is always welfare improving and S therefore positive. Observe that it would disappear in a one sector economy where all agents are rent-seekers and hence $f_E^\theta(w) = 0$ for all w and E.

Finally, the third effect in (26) is the wage shift effect Z. It results from the fact that, as

observed above, increasing E reduces the wages in the rent-seeking sector as $\mu(E)/E$ falls. This leads to a downward shift in the rent-seeking and overall wage distributions $f_E^\varphi(w)$ and $f_E(w)$, even when keeping the occupational choice of agents fixed. This effect is the most involved, which is why we present an intuitive derivation of its decomposition in equations (28), (29) and (30) in the following subsection. Appendix B.2 provides a different proof based on the Lagrangian for the inner problem.

4.3.2   Understanding the Wage Shift Effect

Directly computing the wage shift effect by brute force, as in Appendix B.2, is cumbersome. We therefore present a variational argument to derive the decomposition in Proposition 2. To that end, first recall that the wage of a rent-seeker is $\varphi\mu(E)/E$. Hence,

$$\frac{dw}{dE} = -\varphi\frac{\mu(E)}{E^2}\left(1 - \frac{\mu'(E)E}{\mu(E)}\right) = -\frac{1-\beta(E)}{E}w, \tag{31}$$

where $\beta(E) = \mu'(E)E/\mu(E)$ is the output elasticity of the rent-seeking technology and hence $1 - \beta(E)$ is the Pigouvian corrective tax that would let agents fully internalize the rent-seeking externality. A small shift ΔE in E thus changes a given rent-seeker's wage from w to $w - ((1-\beta(E))/E)w\Delta E$. The wage shift effect is the welfare consequence of such a shift.

By the envelope theorem, we can compute the welfare effect of this wage shift by holding the optimal schedules $e(w)$ and $V(w)$ constant. The wage shift thus involves moving rent-seekers to effort

$$e\left(w - \frac{1-\beta(E)}{E}w\Delta E\right) \approx e(w) - e'(w)\frac{1-\beta(E)}{E}w\Delta E$$

and to utility

$$V\left(w - \frac{1-\beta(E)}{E}w\Delta E\right) \approx V(w) - V'(w)\frac{1-\beta(E)}{E}w\Delta E.$$

It is easier to compute the effects of this shift by breaking it into two sequential sub-shifts, which we define pointwise at each wage w. The first sub-shift holds the wage w constant, and changes the schedules $e(w)$ and $V(w)$ for *all workers* in both the rent-seeking and traditional sectors. The second sub-shift *re-allocates* effort and utility between wage w rent-seekers and wage w traditional workers while at the same time changing the wage of the rent-seekers only. These sub-shifts are constructed so that the first sub-shift absorbs

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the total change in e and V induced by the change in E at each wage, while the second sub-shift involves a *pure* re-allocation of e and V across sectors. Formally, we define:

Sub-Shift 1: At each w, let $e(w)$ and $V(w)$ change to $\tilde{e}(w)$ and $\tilde{V}(w)$ for all agents, with

$$\tilde{e}(w) \equiv e(w) - \frac{f_E^\varphi(w)}{f_E(w)}e'(w)\frac{1 - \beta(E)}{E}w\Delta E,$$

and

$$\tilde{V}(w) \equiv V(w) - \frac{f_E^\varphi(w)}{f_E(w)}V'(w)\frac{1 - \beta(E)}{E}w\Delta E.$$

Sub-Shift 2: At each w, let the wage change from w to $w - \frac{1-\beta(E)}{E}w\Delta E$ for the rent-seekers only (and revert to the original effort schedule $e(w)$). Their effort therefore changes from $\tilde{e}(w)$ to

$$e\left(w - \frac{1 - \beta(E)}{E}w\Delta E\right) \approx e(w) - e'(w)\frac{1 - \beta(E)}{E}w\Delta E.$$

The effort of wage w traditional workers reverts from $\tilde{e}(w)$ back to $e(w)$ in this sub-shift. Similarly, utility of the rent-seekers changes from $\tilde{V}(w)$ to

$$V\left(w - \frac{1 - \beta(E)}{E}w\Delta E\right) \approx V(w) - V'(w)\frac{1 - \beta(E)}{E}w\Delta E,$$

and the utility of traditional workers changes from $\tilde{V}(w)$ back to $V(w)$.

By construction, the total change in effort $e(w)$ and utility $V(w)$ by (original) wage w workers in sub-shift 2 is exactly zero. To see this, note that each rent-seeker's effort changes by

$$\left(e(w) - e'(w)\frac{1 - \beta(E)}{E}w\Delta E\right) - \left(e(w) - e'(w)\frac{1 - \beta(E)}{E}w\Delta E\frac{f_E^\varphi(w)}{f_E(w)}\right)$$

$$= -e'(w)\frac{1 - \beta(E)}{E}\frac{f_E^\theta(w)}{f_E(w)}w\Delta E,$$

and each traditional worker's effort changes by

$$e(w) - \tilde{e}(w) = e'(w)\frac{1 - \beta(E)}{E}\frac{f_E^\varphi(e)}{f_E(w)}w\Delta E.$$

These are equal in absolute value and have opposite signs when weighted by the masses $f_E^\varphi(w)$ and $f_E^\theta(w)$ of rent-seekers and traditional workers at wage w, respectively. An analogous argument shows that our decomposition into two sub-shifts makes sure that

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the total change in utility $V(w)$ among (original) wage w workers in sub-shift 2 is also zero.

This is a useful decomposition precisely because the welfare consequences of sub-shift 1 are zero by the envelope theorem. Sub-shift 2 is where all of the welfare effects occur, and this sub-shift involves only a pointwise *re-allocation* of $e(w)$ and $V(w)$ across individuals within the two sectors. We can therefore compute the welfare consequences of the wage shift effect (i.e. sub-shift 2) as follows.[12]

1. Because total utility $V(w)$ and effort $e(w)$ across both sectors are held constant at each w, there are no welfare effects from changing $V(w)$ in (16) or (19) or $e(w)$ in (19), where the changes are weighted by the total population density $f_E(w)$.

2. The Pareto weights effect that captures the change in $\Psi(F_E(w))$ in (16), which results from the wage shift within the rent-seeking sector, is exactly zero at each wage w. To wit: the change in the Pareto weight on the wage w rent-seekers is

$$-\psi(F_E(w)) f_E(w) f_E^\theta(w) \frac{1-\beta(E)}{E} w,$$

where $f_E^\theta(w)((1-\beta(E))/E)w$ measures the mass of traditional workers between w and $w - ((1-\beta(E))/E)w$, i.e. those for whom the rent-seekers used to have a higher wage and now have a lower wage. The change in Pareto weight on the wage w traditional workers is, similarly,

$$\psi(F_E(w)) f_E(w) f_E^\varphi(w) \frac{1-\beta(E)}{E} w.$$

Weighting these terms by the sectoral densities $f_E^\varphi(w)$ and $f_E^\theta(w)$ shows that they are equal in absolute value and and have opposite signs.

3. The direct wage effect from the change in rent-seeking wages in (18) and (19) is

$$-(1-\xi)we(w)f_E^\varphi(w)\frac{1-\beta(E)}{E}. \tag{32}$$

Integrating this across all wages yields

$$-(1-\xi)\frac{\mu(E)}{E}(1-\beta(E)). \tag{33}$$

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[12]We drop the ΔE-terms here, so that the effects are interpreted in "per unit change in E" terms.

4. The effect of the change in $e(w)$ on (18) is

$$\xi w^2 e'(w) \frac{f_E^\varphi(w) f_E^\theta(w)}{f_E(w)} \frac{1 - \beta(E)}{E} \tag{34}$$

and again integrating over all wages gives

$$\xi \frac{1 - \beta(E)}{E} \int_{\underline{w}_E}^{\overline{w}_E} w^2 e'(w) \frac{f_E^\varphi(w) f_E^\theta(w)}{f_E(w)} dw \equiv \xi \frac{1 - \beta(E)}{E} D_2 \tag{35}$$

D_2 thus captures the effect of the effort re-allocation in the rent-seeking sector. Note that it would disappear in a one sector rent-seeking economy with $f_E^\theta(w) = 0$ for all w.

5. Finally, consider the effect on the incentive constraints (17). To compute these, notice that the incentive constraints are, by construction, satisfied by the original and the final allocations. The incentive effects in sub-shift 2 are therefore equal and opposite to the incentive effects in sub-shift 1, which are easy to compute:

$$\int_{\underline{w}_E}^{\overline{w}_E} \frac{f_E^\varphi(w)}{f_E(w)} \frac{1 - \beta(E)}{E} \left(w V'(w) \eta'(w) + \gamma e(w)^{\gamma - 1} e'(w) \eta(w) \right) dw$$

$$= -\frac{1 - \beta(E)}{E} \int_{\underline{w}_E}^{\overline{w}_E} \left(-\frac{f_E^\varphi(w)}{f_E(w)} \frac{d}{dw} \left[e(w)^\gamma \eta(w) \right] \right) dw, \tag{36}$$

where $\eta(w)$ is the multiplier on the incentive constraint (17) at w, and $\eta(w) = \Psi(F_E(w)) - F_E(w)$ from the necessary conditions for $V(w)$. The incentive effects of sub-shift 2 are equal and opposite to this, i.e. given by $((1 - \beta(E))/E) D_1$, where, after integrating by parts (and using $\eta(\underline{w}_E) = \eta(\overline{w}_E) = 0$),

$$D_1 \equiv \int_{\underline{w}_E}^{\overline{w}_E} \frac{d}{dw} \left(\frac{f_E^\varphi(w)}{f_E(w)} \right) e(w)^\gamma \left(\Psi(F_E(w)) - F_E(w) \right) dw \tag{37}$$

Hence, D_1 captures the incentive effects of the wage shift in the rent-seeking sector, and it disappears whenever the share of rent-seekers is constant across wages, as would be the case in a one sector economy.

Putting these effects together, we see that the total welfare effect of the wage shift in the rent-seeking sector is

$$Z = -\frac{1 - \beta(E)}{E} \left((1 - \xi) \mu(E) - D_1 - \xi D_2 \right) \tag{38}$$

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as claimed in Proposition 2.

4.3.3 Example: A One Sector Rent-Seeking Economy

Before considering the general implications of Proposition 2 for ξ and thus marginal tax rates in any Pareto optimum, let us again turn to the special benchmark case where all agents are rent-seekers. In particular, suppose all the skill density is concentrated in the φ-dimension with pdf $f(\varphi)$ and cdf $F(\varphi)$ so that $f_E^\theta(w) = 0$ for all w and E. Then obviously $S = D_1 = D_2 = 0$, so that setting $W'(E) = 0$ at the Pareto optimum and (26) implies

$$\xi = 1 - \beta(E).$$

This leads to the following comparison between Pareto optimum and SCPE for given Pareto weights $\Psi(F)$:

Theorem 3. *Consider a one sector rent-seeking economy. Then*

$$1 - T'(y(\varphi)) = \beta(E) \left(1 + \gamma \frac{\Psi(F(\varphi)) - F(\varphi)}{\varphi f(\varphi)} \right)^{-1}$$

in a Pareto optimum and

$$1 - T'(y(\varphi)) = \left(1 + \gamma \frac{\Psi(F(\varphi)) - F(\varphi)}{\varphi f(\varphi)} \right)^{-1}$$

in a SCPE given $\Psi(F)$, for all $\varphi \in \Phi$.

Proof. The result immediately follows from (i) equations (24) and (25) with $f_E^\varphi(w) = f_E(w)$ in Proposition 1, (ii) the fact that $w = \varphi\mu(E)/E$, $F_E(w) = F_E(\varphi\mu(E)/E) = F(\varphi)$ and thus $f_E(w) = f(\varphi)E/\mu(E)$, and (iii) $\xi = 1 - \beta(E)$ by equation (26) in Proposition 2 (setting $W'(E) = 0$ for a Pareto optimum) since $S = D_1 = D_2 = 0$ in a one sector rent-seeking economy. $\square$

Theorem 3 shows that the marginal tax rate formula for a SCPE in a one sector rent-seeking economy shares the same structure as the general formula in Proposition 1, but is now given explicitly in terms of fundamentals, namely the skill distribution $f(\varphi)$, the redistributional motives captured by $\Psi(F)$ and the elasticity of effort $\varepsilon = 1/(\gamma - 1)$. In a Pareto optimum, the marginal keep share at each skill level is scaled down compared to the SCPE by the Pigouvian corrective factor $\beta(E)$, similar to what we observed in Theorem 2 for the case of linear taxation.

Observing that this correction is *uniform* across individuals immediately leads to the following corollary:

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Corollary 5. *For any given set of Pareto-weights $\Psi(F)$ and any given skill type φ, the marginal tax rate is higher in the Pareto optimum compared to the SCPE. The progressivity of the tax schedule, as measured by the ratio of marginal keep shares*

$$\frac{1 - T'(y(\varphi))}{1 - T'(y(\varphi'))} \quad \text{for any } \varphi, \varphi' \in \Phi,$$

is the same in the Pareto-optimum and SCPE.

Corollary 5 shows that rent-seeking does not, in and of itself, provide a motive for increased progressivity in marginal tax rates, at least not given our preference assumptions. Note, however, that the comparison of progressivities is based only on the relationship between marginal rates at different income levels. Two systems with a 20% and 40% flat tax rate, respectively, which are used to finance lump-sum transfers are thus treated as equally progressive. Moreover, it is important to note that incomes $y(\varphi)$ will be different in a Pareto optimum and a SCPE, even for the same skill type φ and Pareto weights. Hence, the result is specific to *individuals*, not income levels. This is not particularly problematic, however, in our framework. It is natural to consider measures of progressivity that are scale independent (so that, e.g., changing the units of income does not affect the measure). And it is straightforward to show that the pre-tax income in the SCPE is simply a proportional reduction of the pre-tax income in the Pareto optimum under the hypotheses of Corollary 5. The same result would apply to incomes for any scale-independent progressivity measure.

The result in Corollary 5 is particularly surprising in view of the fact that, in an economy with rent-seeking, taxing higher wage earners at higher rates allows for additional redistribution through two channels: The first is standard and results from the additional tax revenue that can be transferred to lower incomes. In addition, however, redistribution can now also occur by affecting wages *directly*. A higher marginal tax rate on high wage earners discourages their effort and thus reduces E. This in turn increases $\mu(E)/E$ and thus everyone's wage, including the wages of the bottom earners. Nevertheless, as the result shows, this additional effect does not affect the optimal progressivity of the tax schedule.

Theorem 3 implies that the top marginal tax rate is $T'(y(\overline{\varphi})) = 1 - \beta(E) > 0$ in any Pareto optimum and zero in any SCPE. Thus, in a one sector economy, the top rate is exactly equal to the Pigouvian corrective tax. The same result is true for the entire tax schedule with utilitarian welfare:

Corollary 6. *Consider a one sector rent-seeking economy and suppose $\Psi(F)$ is utilitarian. Then*

$T'(y(\varphi)) = 0$ *in any SCPE and* $T'(y(\varphi)) = 1 - \beta(E) > 0$ *in any Pareto optimum for all* $\varphi \in [\underline{\varphi}, \overline{\varphi}]$.

Hence, with utilitarian welfare, the optimal tax schedule in fact involves a flat marginal tax rate and collapses back to the linear taxation case discussed in Corollary 1.

4.4   Top Marginal Tax Rates

Let us return to the general case of a two sector economy, so that the sectoral shift effect S and the re-allocation and incentive components of the wage shift effect, namely D_1 and D_2, do not disappear. Then we first have the following result:

Proposition 3. *$\xi > 0$ in any regular Pareto optimum.*

Proof. See Appendix B.3. □

The proof of Proposition 3 involves showing that the wage shift effect Z from Proposition 2 is negative whenever the Pareto weights imply a weak redistributive motive from high to low wage agents, i.e. when $\Psi(F)$ is regular. This is intuitive since Z measures the welfare effect of a wage reduction for a part of the population, namely all rent-seekers. Since $\mu'(E)$ and S are positive, $\xi > 0$ then follows directly from setting $W'(E) = 0$ in (26). The rent-seeking problem thus leads to a strictly positive top marginal tax rate $T'(y(\overline{w}_E)) = \xi f_E^{\varphi}(w)/f_E(w)$ at any regular Pareto optimum in which the share of rent-seekers is non-zero at the top.

Furthermore, using (29) and (30) in (26) and setting $W'(E) = 0$ yields

$$\xi = (1 - \beta(E)) \frac{\mu(E) - D_1}{\mu(E) + SE + (1 - \beta(E)) D_2}. \tag{39}$$

The following Theorem summarizes the implications of these insights for the top marginal tax rate:

Theorem 4. *Consider any regular Pareto optimum with the following properties:*
(i) effort $e(w)$ is weakly increasing in w and
(ii) the share of rent seekers $f_E^{\varphi}(w)/f_E(w)$ is weakly increasing in w.
Then

$$0 \leq T'(y(\overline{w}_E)) = \xi \frac{f_E^{\varphi}(\overline{w}_E)}{f_E(\overline{w}_E)} < 1 - \beta(E)$$

even if all top earners are rent-seekers (and the first inequality is strict whenever $f_E^{\varphi}(\overline{w}_E) > 0$).

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Theorem 4 provides a surprising result for our general framework: Even if the top earners in the economy are *all* rent-seekers, the optimal top marginal tax rate is *less* than the full Pigouvian correction $1 - \beta(E)$. This contrasts with the result in Theorem 3 for a one sector economy. Indeed, we showed there that the marginal keep share at the top of the income distribution was $1 - \beta(E)$, so the after-tax hourly wage of the highest earners was simply $\overline{\varphi}\mu'(E)$ – i.e. exactly equal to the marginal social product of effort. In other words, with a single rent-seeking sector, the optimal top rate was "non-distortionary": it was positive and exactly equal to the Pigouvian correction for the rent-seeking externality.

One might be tempted to expect a similar result to apply to the more general two sector model when only rent-seekers are the top earners. In fact, since the top earners are all rent-seekers, rent-seeking imposes a negative externality, and the government has a desire to redistribute from high-earners to low earners, this seems like a clear case for high marginal tax rates on high earners, as discussed in the introduction. As Theorem 4 demonstrates, however, this intuition is not complete. The key reason is the additional sectoral shift effect not present in a one sector economy: By lowering the marginal tax rate on the top earning rent-seekers, total equivalent effort E increases and thus wages in the rent-seeking sector fall. As a consequence, some agents now find it profitable to leave the rent-seeking sector and become traditional workers. Since the traditional sector is socially more productive, this shift is always welfare enhancing ($S > 0$).

As discussed above, the increase in total rent-seeking equivalent effort E has additional effects in a two-sector economy, which result from the fact that agents in both sectors must be treated the same conditional on the wage w, namely the effort re-allocation and incentive effects D_1 and D_2.[13] The assumptions in Theorem 4 make sure that these effects go in the same direction as the sectoral shift effect, so that both D_1 and D_2 are also positive. Note, however, that these are only sufficient assumptions, so that $\xi < 1 - \beta(E)$ is possible even when they are violated for some wage levels. In the quantitative analysis in section 5, we will verify these assumptions and demonstrate that the top marginal tax rate can be *substantially* lower than what the full Pigouvian correction would have suggested due to the sectoral shift effect present in our framework.

When $\Psi(F)$ is utilitarian, the incentive effect D_1 vanishes so that the assumptions in Theorem 4 can be relaxed as follows:

Corollary 7. *Consider a utilitarian Pareto optimum with the property that effort $e(w)$ is weakly*

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[13]The proof of Proposition 2 in Appendix B.2 makes the no-discrimination constraints underlying these effects explicit.

increasing in w. Then

$$0 \leq T'(y(w)) = \xi \frac{f_E^\varphi(w)}{f_E(w)} < (1 - \beta(E)) \frac{f_E^\varphi(w)}{f_E(w)} \quad \forall w.$$

In this case, the marginal tax rate is less than the Pigouvian corrective tax multiplied by the share of rent-seekers at w no matter how the rent-seekers are located within the wage distribution.

4.5 Inefficiency of SCPE with Non-linear Taxes

In our analysis of linear taxation with rent-seeking, we demonstrated that the set of Pareto optimal linear tax rates was shifted upwards compared to the SCPE set, but there could exist some overlap so that some SCPE were in fact also Pareto optimal. The following final result shows how this is changed under non-linear taxation:

Theorem 5. *Any regular SCPE with a non-zero share of rent-seekers at the bottom or top wage is Pareto inefficient.*

Proof. See Appendix B.4. □

With non-linear taxation, regular SCPE are Pareto dominated in a broad set of circumstances. The proof is based on two observations. First, all regular SCPE have a non-decreasing tax schedule, since $T'(\cdot) \geq 0$ by (25). Second, all Pareto optima with a non-decreasing tax schedules have a strictly positive marginal tax rate at the highest (lowest) income when there are rent-seekers at the highest (lowest) wage. Since SCPEs always involve zero marginal tax rates at the extremes, no regular SCPE can be Pareto optimal.

5 A Numerical Example

This section first parameterizes a stylized two-sector economy with a rent-seeking sector that is socially completely unproductive at the margin. We then compute a Pareto optimal and a SCPE tax system for this economy for a government with a particular set of welfare weights. Despite the fact that the highest earners are all rent-seekers in this economy, we find that marginal taxes in the Pareto optimum remain modest and display approximately the same degree of progressivity at high incomes as in the corresponding SCPE. We finally discuss the quantitative results.

5.1 A Parametrization

In order to compute optimal and SCPE tax systems, we specify a two-dimensional skill distribution, Pareto weights, the elasticity of labor supply ε and the output function $\mu(E)$. We take the labor supply elasticity to be $\varepsilon = .7$ (so that $\gamma \approx 2.4$) and set $\mu(E) = \bar{\mu} = 10$. Hence, we consider the extreme case of a fixed rent to be captured in the rent-seeking sector, so that all rent-seeking effort there is entirely unproductive at the margin. Note that this would imply a Pigouvian corrective tax rate of $1 - \beta(E) = 100\%$.

We use a skill distribution on support $\Theta \times \Phi = [5, 20] \times [4, 20]$ which is independent across the two dimensions in our baseline example, so that $F(\theta, \varphi) = F_\theta(\theta) F_\varphi(\varphi)$. We further assume that F_θ and F_φ are Pareto distributions with Pareto parameters $\alpha_\theta = 4$ and $\alpha_\varphi = 3.4$, respectively. This is consistent with empirical evidence on the income distribution of the highest earners (see e.g. Saez, 2001).[14] We truncate both distributions at the top of the support and renormalize accordingly. Furthermore, to prevent bunching at $w = \bar{\theta} = 20$, we re-scale F_θ so that $f_\theta(\bar{\theta}) = 0$, and renormalize accordingly.

Finally, we assume Pareto weights of the form $\Psi(F) = 1 - (1 - F)^\rho$. The parameter ρ thus characterizes the magnitude of the government's desire for redistribution: $\rho = 1$ for a utilitarian social planner, and $\rho \to \infty$ for a Rawlsian one. We take $\rho = 1.3$, so that the government has a motive for redistribution from high to low wage workers and $\Psi(F)$ is regular.

5.2 Simulation Results

Figure 3 shows the marginal tax rate $T'(y(w))$, the tax schedule $T(y(w))$, the average tax rate $T(y(w))/y(w)$ and the share of rent-seekers $f_E^\varphi(w)/f_E(w)$ as a function of the wage w both for the Pareto optimum and the SCPE resulting from our parametrization above. It indicates that optimal tax rates are higher than the SCPE tax rates. This leads individuals at a given wage to exert less effort relative to the SCPE. The total rent-seeking effort E in the Pareto optimum is consequently lower than in the SCPE (by approximately 3%), and the pre-tax wages of the rent-seekers are higher (also by about 3%). This explains why the support of the wage distribution is extended further at the top in the Pareto optimum compared to the SCPE. However, the total rent-seeking output $\bar{\mu}$ as a share of total income is barely changed at the Pareto optimum compared to the SCPE (35.1% as opposed to 32.4% at the SCPE). The same observation holds for the share of rent-seekers:

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[14]In particular, Saez (2001) argues that the top tail of the income distribution is approximated by a Pareto distribution with parameter $k = 2$. So long as the top earners are all rent-seekers, it is straightforward to show that this implies an identified skill distribution for φ that is Pareto with parameter $\alpha_\varphi = k(1+\varepsilon) = 3.4$.
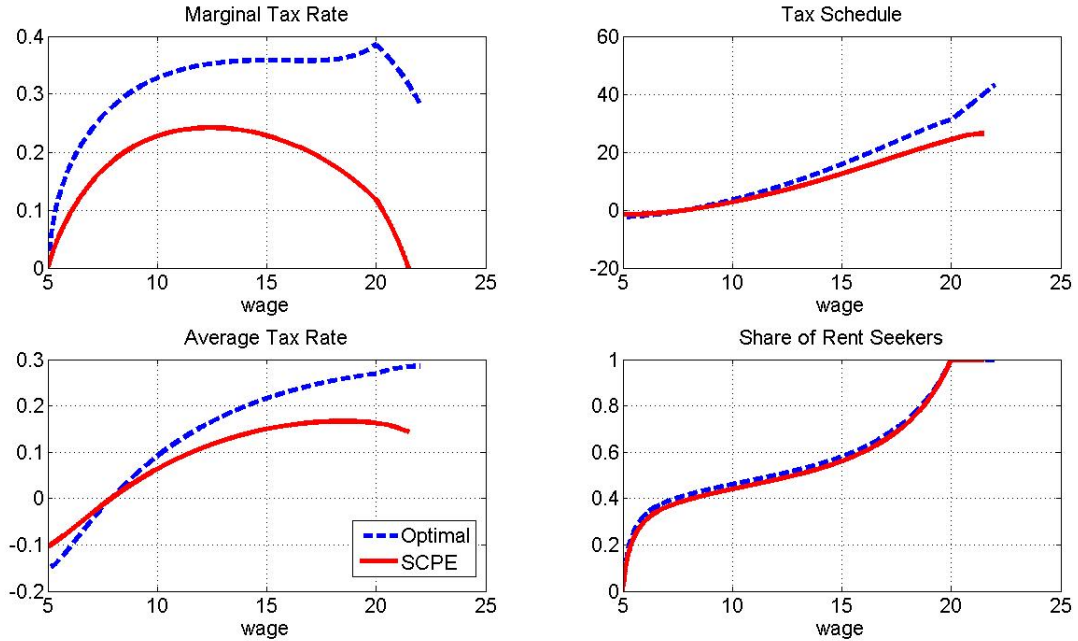
Figure 3: Marginal and average tax rates as a function of the wage

it is even slightly higher in aggregate at the Pareto optimum (40%) than the SCPE (37.2%).

Since $\bar{\theta} = 20$, all agents earning a wage higher than 20 are exclusively rent-seekers. Thus, in both the SCPE and the Pareto optimum, the top earners are in the socially completely unproductive rent-seeking sector. Given that the government has a strict desire to redistribute to low earners, this seems like a "slam-dunk" case for high – and highly progressive – marginal tax rates on high earners. In fact, the full Pigouvian corrective tax rate would be 100% in this example. Yet, Figure 3 indicates decidedly modest top marginal tax rates in the Pareto optimum: they are less than 40%, and even decreasing to less than 30% for the very top earners. As discussed in detail above, the *sectoral shift* effect provides the key intuition. Raising taxes on the highest earners reduces their effort. Since they are all rent-seeking, a reduction in their effort raises $\mu(E)/E$ and the private returns to rent-seeking effort. This makes rent-seeking more appealing to traditional sector workers, some of whom shift into the rent-seeking sector. Since the social marginal returns to rent-seeking are lower than the returns to traditional work, this shift is strictly undesirable. In line with Theorem 4, the presence of the sectoral shift effect therefore leads to top marginal tax rates that are strictly *less* than the Pigouvian correction and, as Figure 3 indicates, *substantially* less.

Figure 4 presents the same results as a function of income $y(w)$ as opposed to the wage w. Even though the Pareto optimum induces higher wages as seen in Figure 3, the higher
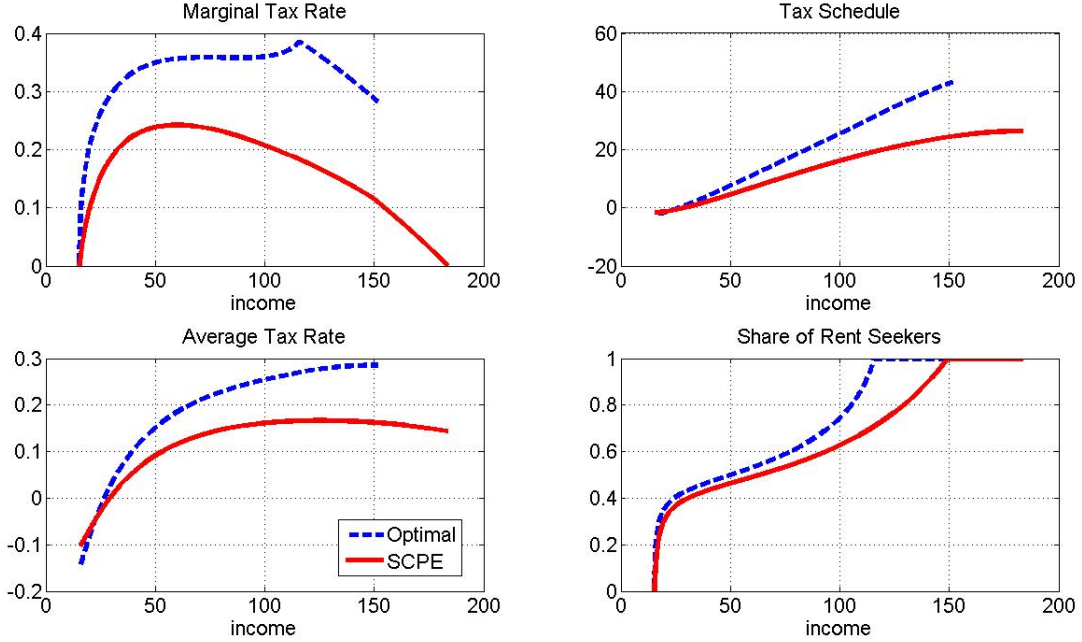
37

Figure 4: Marginal and average tax rates as a function of income

marginal tax rates discourage effort and therefore the support of the income distribution does not extend as far at the top as in the SCPE. Otherwise, similar qualitative results obtain.

The left panel in Figure 5 demonstrates that the assumptions in Theorem 4 are satisfied in our numerical example: individual effort $e(w)$ is increasing in the wage w, and the share of rent-seekers is increasing as seen above, so that the additional re-allocation and incentive effects go in the same direction as the sectoral shift effect in pushing the top marginal tax rate below the full Pigouvian rate. A fortiori, income $y(w) = we(w)$ is therefore strictly increasing in the wage, so that the monotonicity constraint is satisfied and bunching does not need to be considered. The right panel in Figure 5 compares optimal progressivity of the tax schedules in the Pareto optimum and SCPE, as measured by the rate of change of the average tax rate as a function of income. It demonstrates that the optimal progressivity of marginal tax rates at high incomes is barely different in the two systems, despite the fact that the Pareto optimum fully accounts for the fact that the top earners are socially completely unproductive whereas the SCPE does not.

Figure 6 plots welfare as a function of the two-dimensional type space (θ, φ). Clearly, welfare is strictly increasing in θ and independent of φ for the traditional workers, and vice versa for the rent-seekers. The resulting kink occurs along the line of indifferent workers with $\theta = \varphi\mu(E)/E$. As one can see, the Pareto optimum for a given Ψ does
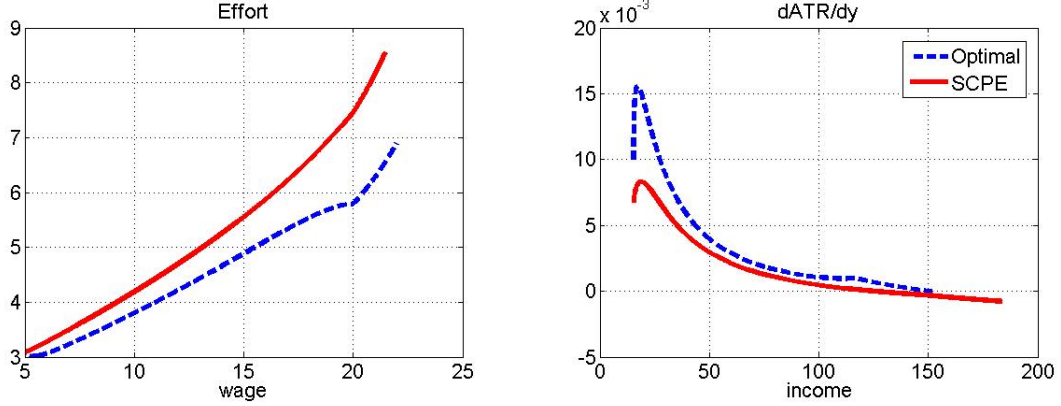
Figure 5: Effort as a function of the wage and progressivity as a function of income

not represent a Pareto improvement over the SCPE *for the same* $\Psi(\cdot)$ (although such an improvement exists by Theorem 5). In contrast, it is such that high earners are made substantially worse off, but low earners are made better off relative to the SCPE. As the lower right panel shows, however, most of the skill density is concentrated among the low skilled in both dimensions in our parametrization, so that the Pareto optimum induces higher welfare than the SCPE as measured by the criterion Ψ.

Finally, figure 7 shows the results from a different parametrization in which individual skills are positively correlated across the two dimensions rather than independent. In particular, (θ, φ) follow a joint bivariate Pareto distribution with common Pareto parameter $\alpha = 4$ and a correlation between θ and φ of $1/\alpha = .25$.[15] All other parameters remain as before. The figure indicates that the optimal and SCPE tax policies are not very different in this specification compared to the independent skills case. Marginal tax rates are slightly lower than before but follow a similar pattern.

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[15]The bivariate Pareto distribution has a joint density function

$$f(\theta, \varphi) = \alpha(\alpha + 1) \left(\underline{\theta}\,\underline{\varphi} \right)^{\alpha+1} \left(\underline{\varphi}\theta + \underline{\theta}\varphi - \underline{\theta}\,\underline{\varphi} \right)^{-(\alpha+2)}.$$

We truncate it at the top and use the same support as in the independent skills case. The joint density function is such that both marginal distributions are Pareto with parameter α.
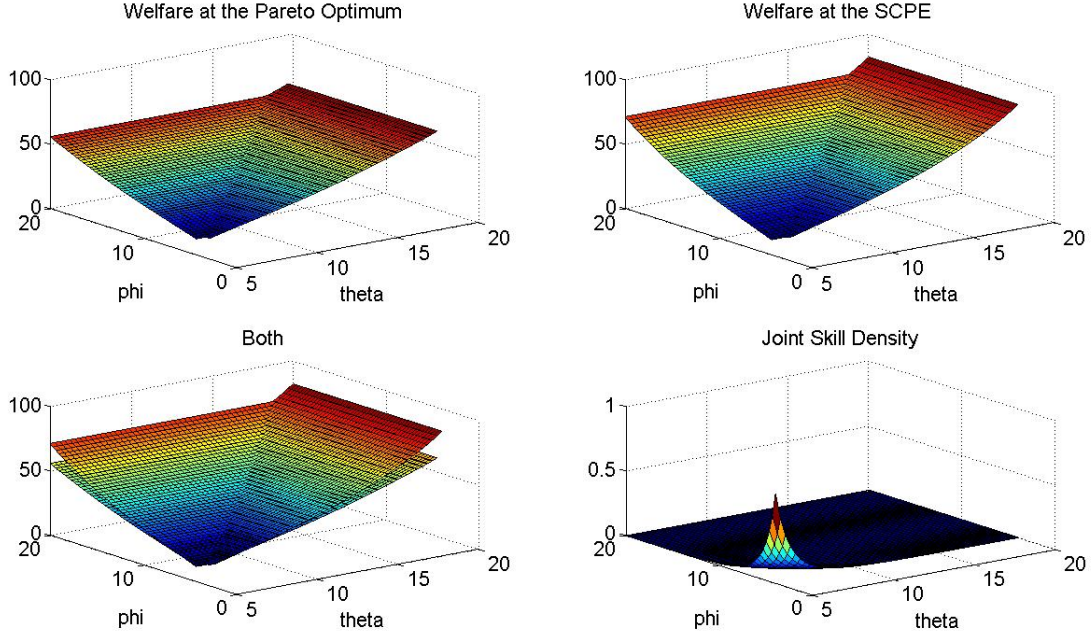
Figure 6: Welfare as a function of (θ, φ)

6 Extensions

6.1 Unbounded Support

In this section, we first demonstrate that our results readily extend to the case of a skill (and thus wage) distribution with unbounded support. In particular, suppose that

$$\lim_{w \to \infty} \frac{f_E^{\varphi}(w)}{f_E(w)} = 1$$

so that there are only rent-seekers at the very top of the income distribution. Moreover, assume that the top tail of the wage distribution converges to a Pareto distribution with parameter α, so that

$$\lim_{w \to \infty} \frac{1 - F_E(w)}{w f_E(w)} = \frac{1}{\alpha}.$$

Then we can use equations (24) and (25) in Proposition 1 to derive asymptotic marginal tax rates for $w \to \infty$ under one minor additional technical assumption. Specifically, for the Lagrangian underlying the the proof of Proposition 1 to be well-defined, we require the limiting condition
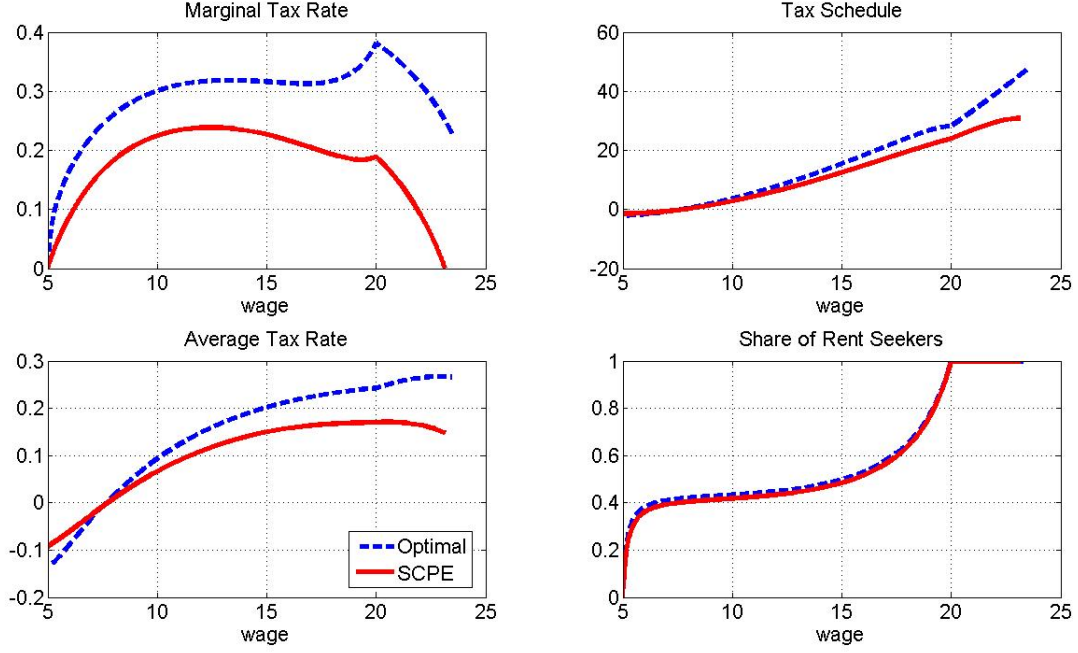
$$\lim_{w \to \infty} V(w)(1 - F_E(w)) = 0, \tag{40}$$

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Figure 7: Positively correlated skills (θ, φ)

i.e. $V(w)$ must not grow at a rate higher than α (since α is the rate at which $1 - F_E(w)$ declines if the top tail of the wage distribution is approximated by a Pareto distribution). It is easy to see that, if this limiting condition were violated, $\int_{\underline{w}_E}^{\infty} V(w)dF_E(w) = \infty$ so that a utilitarian planner could achieve infinite welfare, which we find easy to rule out.

Under this same limiting assumption, Propositions 2 and 3 also go through, and $0 < \xi < 1 - \beta(E)$ under the same conditions as in the bounded support case. The following Proposition summarizes the asymptotic marginal tax rate results based on these insights:

Proposition 4. *Suppose the share of rent-seekers converges to one and the wage distribution converges to a Pareto distribution with parameter α for $w \to \infty$. Assume also that (40) is satisfied. Then the asymptotic marginal tax rate in the Pareto optimum is*

$$\lim_{w \to \infty} T'_{PO}(y(w)) = \frac{\xi\alpha + \gamma(1 - \Psi'(1))}{\alpha + \gamma(1 - \Psi'(1))},$$

and in the SCPE

$$\lim_{w \to \infty} T'_{SCPE}(y(w)) = \frac{\gamma(1 - \Psi'(1))}{\alpha + \gamma(1 - \Psi'(1))}.$$

The full the Pigouvian asymptotic marginal tax rate is (from the one-sector model, e.g., Theorem

$$\lim_{w \to \infty} T'_{Pigou}(y(w)) = \frac{(1 - \beta(E))\alpha + \gamma(1 - \Psi'(1))}{\alpha + \gamma(1 - \Psi'(1))}.$$

Putting these together shows that

$$\lim_{w \to \infty} T'_{SCPE}(y(w)) < \lim_{w \to \infty} T'_{PO}(y(w)) < \lim_{w \to \infty} T'_{Pigou}(y(w))$$

whenever Ψ is regular (which implies $\Psi'(1) \leq 1$) and the conditions in Theorem 4 are satisfied. Notice that the asymptotic tax rate formulas further simplify in the special case emphasized in Saez (2001) where no welfare weight is put on the top earners so that $\Psi'(F) = 0$ for $F \to 1$.

6.2   Cross-Sectoral Externalities

Suppose that, in addition to imposing an externality on other rent-seekers, rent-seeking also imposes a negative externality on traditional workers. In this subsection, we address this case by taking aggregate output in the traditional sector to be $E_\theta \Xi(E)$, where E_θ is aggregate traditional sector effort, E (as before) is aggregate rent-seeking effort, and $\Xi'(E) < 0$, so that the cross-sectoral externality is negative. Rent-seeking output is still $\mu(E)$.[16]

It is straightforward to generalize the "wage shift" and "sectoral shift" effects discussed in section 4.3 to this case. To that end, note that an "inner" problem for this economy is essentially the same as before (i.e., equations (16)-(19)).[17] Also, for a Θ-sector worker in this economy,

$$\frac{dw}{dE} = -\frac{\beta_\Xi(E)}{E}w, \quad \text{where} \quad \beta_\Xi(E) \equiv -\frac{E}{\Xi(E)}\frac{d\Xi(E)}{dE} > 0.$$

(As before, $dw/dE = -\frac{1 - \beta(E)}{E}w$ for Φ-sector workers.)

6.2.1   The Generalized Sectoral Shift Effect

The sectoral shift effect is the welfare effect of the between-sector shift of workers induced by a change in E, holding constant their effort, utilities, and wages. This effect is

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[16]It is worth noting that this framework is, in fact, *the most general* version of a two sector model in which only one sector causes (negative) externalities.

[17]The only difference is that now $f_E^\theta(w) \equiv \frac{1}{\Xi(E)} \int_{\underline{\varphi}}^{w\frac{E}{\mu(E)}} f(w/\Xi(E), \varphi)d\varphi$.
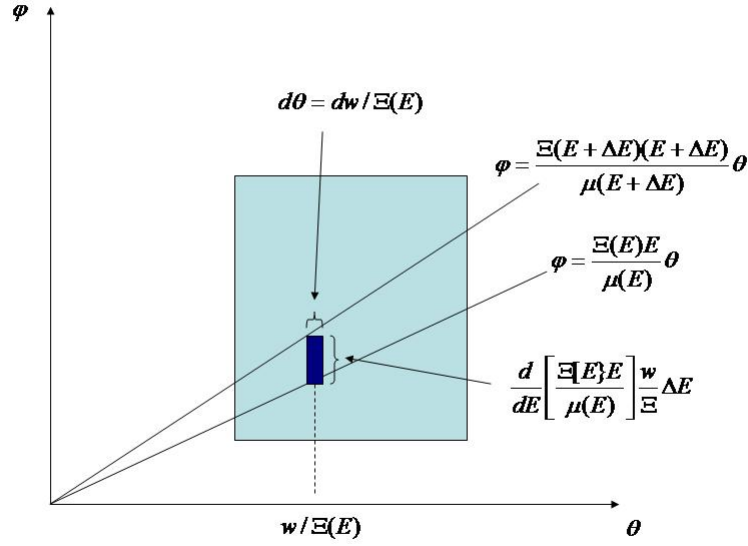
Figure 8: Computing the sectoral shift effect with cross sector externalities

illustrated in Figure 8 for a small change in E. It works out to be:

$$S_{gen} = \int_{\underline{w}_E}^{\overline{w}_E} we(w) \left(\frac{1}{\Xi(E)} \right) \left(\frac{d}{dE} \left[\frac{\Xi(E)E}{\mu(E)} \right] \right) f \left(\frac{w}{\Xi(E)}, w \frac{E}{\mu(E)} \right) \frac{dw}{\Xi(E)}$$

$$= \frac{\Delta\beta(E)}{\mu(E)\Xi(E)} \int_{\underline{w}_E}^{\overline{w}_E} w^2 e(w) f \left(\frac{w}{\Xi(E)}, w \frac{E}{\mu(E)} \right) dw,$$

where

$$\Delta\beta(E) \equiv (1 - \beta(E)) - \beta_{\Xi}(E)$$

measures the difference between the "within sector" externality and the "across sector" externalities imposed by rent-seeking effort. The sign of S_{gen} is equal to the sign of $\Delta\beta(E)$.

6.2.2 The Generalized Wage Shift Effect

There are two components to the wage shift effect here: the one computed in section 4.3.2 which results from the change in Φ-sector wages, and a new one which results from the change in Θ-sector wages. The latter can be computed in essentially the same way the former was computed: by decomposing the effect into a zero-welfare sub-shift 1 which

adjusts aggregate e and V for individuals in both sectors with wage w,[18] and a welfare changing sub-shift 2 which re-allocates e and V across sectors at each w while changing w in the Θ-sector.

As in section 4.3.2, only the analogs of steps 3 to 5 yield non-zero welfare effects of, respectively, $-\frac{\beta_\Xi(E)}{E}Y_\theta$, $-\xi\frac{\beta_\Xi(E)}{E}D_2$, and $-\frac{\beta_\Xi(E)}{E}D_1$, where Y_θ is the Θ-sector output. Putting these together gives the second component of the wage shift effect:

$$Z_\theta = -\frac{\beta_\Xi(E)}{E}\left(Y_\theta + D_1 + \xi D_2\right) \tag{41}$$

It is worth comparing and contrasting the three terms in (41) with the three terms in (38). First, the terms in (41) are multiplied by $\beta_\Xi(E)$ instead of $1 - \beta(E)$. This reflects the different magnitudes of the wage changes in the two sectors. Second, the D_1 and D_2 terms have *opposite* signs in (41) and (38). In the case of D_2, this is because the Φ-sector effort changes in sub-shift 2 are in the opposite directions. To wit: in both cases, sub-shift 1 involves $e(w)$ shifting down in both sectors, and sub-shift 2 then involves $e(w)$ shifting down further in one sector and reverting back up to its original level in the other. The "reverting" sector is Φ here instead of Θ in 4.3.2. In the case of D_1, the sign change simply comes from the fact that sub-shift 1 involves weights $\frac{f_E^\theta(w)}{f_E^\varphi(w)}$ instead of $\frac{f_E^\varphi(w)}{f_E^\varphi(w)}$, and $\frac{d}{dE}\left(\frac{f_E^\theta(w)}{f_E(w)}\right) \equiv -\frac{d}{dE}\left(\frac{f_E^\varphi(w)}{f_E(w)}\right)$. The first terms in (41) and (38) have the same signs: a decrease in wages in either sector has a negative welfare effect.

Adding (41) and (38) gives the total wage shift effect in the generalized model:

$$Z_{gen} = -\left(\frac{1-\beta(E)}{E}\mu(E) + \frac{\beta_\Xi(E)}{E}Y_\theta\right) + \frac{1-\beta(E)}{E}\xi\mu(E) + \frac{\Delta\beta(E)}{E}(D_1 + \xi D_2). \tag{42}$$

6.2.3 Generalized Results

The preceding results yield the following welfare change induced by a small increase in E:

$$W'(E) = \xi\mu'(E) + \xi S_{gen} + Z_{gen}$$

$$= \xi S_{gen} + \xi\left(\frac{\mu(E)}{E} + \frac{\Delta\beta(E)}{E}D_2\right) - \frac{\mu(E)}{E}\left(1 - \beta(E) + \frac{Y^\theta\beta_\Xi(E)}{\mu(E)}\right) + \frac{\Delta\beta(E)}{E}D_1.$$

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[18]This requires $e(w) \to e(w) - \frac{f_E^\theta(w)}{f_E(w)}e'(w)\frac{\beta_\Xi(E)}{E}w\Delta E$. As compared with the shifts in section 4.3.2, only the weights $\frac{f_E^\theta(w)}{f_E(w)}$ and $\frac{f_E^\varphi(w)}{f_E(w)}$ differ.

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The term $\frac{\mu(E)}{E}\left((1-\beta(E))+\frac{Y_\theta}{\mu(E)}\beta_\Xi(E)\right)$ is easily recognized as the tax rate t_{Pig} which corrects the Pigouvian externality (i.e., the wedge between the private marginal product $\mu(E)/E$ and the social marginal product $\mu'(E)+\Xi'(E)E_\theta$ of Φ-sector effort). Therefore the first order condition for the outer problem yields:

$$W'(E)=0 \quad \Rightarrow \quad \xi = t_{Pig}\left(\frac{\mu(E)-\frac{\Delta\beta(E)}{t_{Pig}}D_1}{\mu(E)+ES+\Delta\beta(E)D_2}\right). \tag{43}$$

Since the sign of S is the same as the sign of $\Delta\beta(E)$, this[19] directly implies that Theorem 4 generalizes whenever $\Delta\beta(E) > 0$—i.e., whenever the "within sector" externality is greater than the across-sector externality—as in the following Proposition.

Proposition 5. *Suppose that $\Delta\beta(E) > 0$, and consider any regular Pareto optimum with the following properties:*
(i) effort $e(w)$ is weakly increasing in w and
(ii) the share of rent seekers $f_E^\varphi(w)/f_E(w)$ is weakly increasing in w.
Then
$$0 \le T'(y(\overline{w}_E)) < t_{Pig}$$
even if all top earners are rent-seekers.

Equation (43) can also be used to study the $\Delta\beta(E) \le 0$ case. If $\Delta\beta(E) = 0$, then the top marginal tax rate is exactly equal to the Pigouvian tax t_{Pig}—whether the additional conditions of Proposition 5 hold or not. If $\Delta\beta(E) < 0$, then if those conditions *do* hold, Expression (43) implies that the top marginal tax rate will *exceed* the Pigouvian corrective tax as long as the multiplier ξ remains positive. This is intuitive. If the across-sector externality dominates, then the sectoral shift effect reverses sign, providing an additional reason to tax the top earning rent-seekers: lowering the effort of these workers not only corrects an externality, raising the absolute returns to effort for every other worker in the economy, but it *also* raises the *relative* returns to effort in the more productive sector and encourages a beneficial re-allocation of the effort among other workers.

7 Conclusion

Our results indicate that, although the presence of rent-seeking behavior leads to higher taxes than would otherwise be optimal, it does not necessarily imply that taxes should

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[19]Along with the generalization of Proposition 3 which follows using the same methods.

be more steeply progressive. This is true even when rent seeking is an activity pursued primarily, or even exclusively, by the highest earners. One implication is that income taxation alone is at best an imperfect tool for addressing rent seeking externalities, even when that rent-seeking is known to be concentrated in an easily identified portion of the income distribution.

Our model and analysis illustrate how the techniques of optimal income taxation can be applied to economies with rent-seeking. The techniques we develop are likely to be fruitful, however, in a broader class of related environments. These would include, for example, environments with positive externalities and environments in which individuals can exert both traditional and rent-seeking effort.

We address rent-seeking because we view it as a qualitatively important phenomenon which occurs in a broad range of settings. Beyond the traditional notion of rent-seeking within or through governments and legal systems, we view it as potentially important in: finance, wherein individuals compete to exploit a potentially limited set of arbitrage opportunities; in publishing, where, for example, a textbook author can earn large infra-marginal rents by producing a text which is only marginally better than an existing one; in pharmaceuticals, where, for example, patent races can provide large rewards for a new drug which is developed only incrementally sooner than it otherwise would have been; and in many other areas. Our paper does not attempt to address the quantitative importance of rent-seeking, but we view this as an important direction for future research.

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A   Proofs for Section 3

A.1   Proof of Lemma 4

We prove the lemma by proving the following two claims:

1. If $\underline{\varphi}\frac{\mu(E(t))}{E(t)} \geq \underline{\theta}$, then $\frac{d\bar{t}(t)}{dt} < 0$.

2. If $\underline{\varphi}\frac{\mu(E(t))}{E(t)} < \underline{\theta}$, then $\frac{d\bar{t}(t)}{dt} < 1$ at points where $t = \bar{t}(t)$.

Proof. Note that $\bar{t}(t)$ solves

$$\frac{w_{E(t)}^{\frac{\gamma}{\gamma-1}}}{\int_{\underline{w}_{E(t)}}^{\overline{w}_{E(t)}} w^{\frac{\gamma}{\gamma-1}} dF_{E(t)}(w)} = \left[1 - \frac{\bar{t}}{(\gamma-1)(1-\bar{t})}\right]. \tag{44}$$

The right-hand-side of (44) is decreasing in $\bar{t}$. When $\underline{\varphi}\frac{\mu(E(t))}{E(t)} \geq \underline{\theta}$, $\underline{w}_{E(t)} = \underline{\varphi}\frac{\mu(E(t))}{E(t)}$, so the left-hand side of (44) is increasing in t and, hence, $\bar{t}(t)$ is decreasing in t. This establishes the first claim.

Towards establishing the second claim, invert equation (44) to yield:

$$\frac{\int_{\underline{w}_{E(t)}}^{\overline{w}_{E(t)}} w^{\frac{\gamma}{\gamma-1}} dF_{E(t)}(w)}{\underline{w}_{E(t)}^{\frac{\gamma}{\gamma-1}}} = \left[\frac{1-\bar{t}}{1 - \frac{\gamma}{(\gamma-1)}\bar{t}}\right]. \tag{45}$$

Note that

$$\int_{\underline{w}_{E(t)}}^{\overline{w}_{E(t)}} w^{\frac{\gamma}{\gamma-1}} dF_{E(t)}(w) = \left\{\left(\frac{\mu(E)}{E}\right)^{\frac{\gamma}{\gamma-1}} \int_{\varphi=\underline{\varphi}}^{\overline{\varphi}} \int_{\theta=\underline{\theta}}^{\varphi\frac{\mu(E)}{E}} \varphi^{\frac{\gamma}{\gamma-1}} f(\theta,\varphi) d\theta d\varphi + \int_{\varphi=\underline{\varphi}}^{\overline{\varphi}} \int_{\varphi\frac{\mu(E)}{E}}^{\theta=\overline{\theta}} \theta^{\frac{\gamma}{\gamma-1}} f(\theta,\varphi) d\theta d\varphi\right\},$$

so that

$$\frac{d}{dt}\int_{\underline{w}_{E(t)}}^{\overline{w}_{E(t)}} w^{\frac{\gamma}{\gamma-1}} dF_{E(t)}(w)$$

$$= \left(\frac{1}{t'(E)}\right)\frac{\gamma}{\gamma-1}\left(\frac{\mu'(E(t))}{\mu(E(t))} - \frac{1}{E}\right)\left(\frac{\mu(E(t))}{E(t)}\right)^{\frac{\gamma}{\gamma-1}} \int_{\varphi=\underline{\varphi}}^{\overline{\varphi}} \int_{\theta=\underline{\theta}}^{\varphi\frac{\mu(E(t))}{E(t)}} \varphi^{\frac{\gamma}{\gamma-1}} f(\theta,\varphi) d\theta d\varphi.$$

Since $\int_{\varphi=\underline{\varphi}}^{\overline{\varphi}} \int_{\theta=\underline{\theta}}^{\varphi \frac{\mu(E(t))}{E(t)}} \theta^{\frac{\gamma}{\gamma-1}} f(\theta, \varphi) d\theta d\varphi \geq 0$:

$$0 < \frac{\frac{d}{dt} \int_{\underline{w}_{E(t)}}^{\overline{w}_{E(t)}} w^{\frac{\gamma}{\gamma-1}} dF_{E(t)}(w)}{\int_{\underline{w}_{E(t)}}^{\overline{w}_{E(t)}} w^{\frac{\gamma}{\gamma-1}} dF_{E(t)}(w)} \leq \left(-\frac{1}{t'(E)}\right) \frac{\gamma}{\gamma-1} \left(\frac{1}{E} - \frac{\mu'(E(t))}{\mu(E(t))}\right). \tag{46}$$

Next, compute

$$\left(\frac{\frac{d}{d\bar{t}}\left(\frac{1-\bar{t}}{1-\frac{\gamma}{(\gamma-1)}\bar{t}}\right)}{\left(\frac{1-\bar{t}}{1-\frac{\gamma}{(\gamma-1)}\bar{t}}\right)}\right)^{-1} = (1-\bar{t})(\gamma-1)\left(1 - \frac{\gamma}{\gamma-1}\bar{t}\right). \tag{47}$$

To show that $\frac{d\bar{t}(t)}{dt} < 1$, it suffices to show that the rate of change of the left hand side of (45) is smaller than the rate of change of the right, or, from (46) and (47), that:

$$\left(-\frac{1}{t'(E)}\right) \frac{\gamma}{\gamma-1} \left(\frac{1}{E} - \frac{\mu'(E(\bar{t}))}{\mu(E(\bar{t}))}\right) (1-\bar{t})(\gamma-1)\left(1 - \frac{\gamma}{\gamma-1}\bar{t}\right) < 1, \tag{48}$$

i.e., that the the rate of change of the (log of the) left-hand-side of is smaller than the rate of change of the (log of the) right-hand-side.

To that end, use equation (4) to observe that

$$(1-t)\frac{\mu(E)}{E} = \left(\frac{E}{k(E)}\right)^{\gamma-1}.$$

Since $k(E)$ is decreasing in E,

$$-t'(E)\frac{\mu(E)}{E} + (1-t)\left(\frac{\mu'(E)}{E} - \frac{\mu(E)}{E^2}\right) > (\gamma-1)\left(\frac{E}{k(E)}\right)^{\gamma-1}\frac{1}{E} = \frac{(\gamma-1)}{E}(1-t)\frac{\mu(E)}{E},$$

whereby

$$-t'(E) > (1-t)\left(\frac{\gamma}{E} - \frac{\mu'(E)}{\mu(E)}\right) > (1-t)\gamma\left(\frac{1}{E} - \frac{\mu'(E)}{\mu(E)}\right),$$

and

$$\left(-\frac{1}{t'(E)}\right)\gamma\left(\frac{1}{E} - \frac{\mu'(E)}{\mu(E)}\right)(1-t) < 1.$$

It follows immediately from the fact that $\bar{t} > 0$ that:

$$\left(-\frac{1}{t'(E)}\right)\frac{\gamma}{\gamma-1}\left(\frac{1}{E} - \frac{\mu'(E(\bar{t}))}{\mu(E(\bar{t}))}\right)(1-\bar{t})(\gamma-1)\left(1 - \frac{\gamma}{\gamma-1}\bar{t}\right) < \left(1 - \frac{\gamma}{\gamma-1}\bar{t}\right) < 1,$$

establishing the second claim. □

A.2 Proof of Theorem 1

Consider an individual with a given skill type (θ, φ). Then the preferred SCPE tax rate for this individual maximizes her indirect utility

$$U_{\theta,\varphi}(t) \equiv V_{\theta,\varphi}(t, T(t); E(t)) = t(1-t)^{\frac{1}{\gamma-1}} \int_{\underline{w}_{E(t)}}^{\overline{w}_{E(t)}} w^{\frac{\gamma}{\gamma-1}} dF_{E(t)}(w) + \frac{\gamma-1}{\gamma}(1-t)^{\frac{\gamma}{\gamma-1}} \left(w_{\theta,\varphi}(E(t))\right)^{\frac{\gamma}{\gamma-1}} \quad (49)$$

where we used equations (4) and (5) to find $E(t)$ and $T(t) \equiv T(E(t))$. From the point of view of a social planner who does not recognize the endogeneity of the wage distribution $F_E(w)$ (and $\underline{w}_E$, $\overline{w}_E$), the "naive" derivative with respect to t is:

$$
\begin{aligned}
\overline{U}'_{\theta,\varphi}(t) &= \frac{dt(1-t)^{\frac{1}{\gamma-1}}}{dt} \int_{\underline{w}_E}^{\overline{w}_E} w^{\frac{\gamma}{\gamma-1}} dF_E(w) + \frac{\gamma-1}{\gamma}\left(\frac{d(1-t)^{\frac{\gamma}{\gamma-1}}}{dt}\right) \left(w_{\theta,\varphi}(E(t))\right)^{\frac{\gamma}{\gamma-1}} \\
&= (1-t)^{\frac{1}{\gamma-1}} \left[\left(1 - \frac{1}{\gamma-1}\frac{t}{1-t}\right) \int_{\underline{w}_E}^{\overline{w}_E} w^{\frac{\gamma}{\gamma-1}} dF_E(w) - w_{\theta,\varphi}^{\frac{\gamma}{\gamma-1}}\right],
\end{aligned} \quad (50)
$$

which is decreasing in $w_{\theta,\varphi}$.[20] The upper bar indicates that the wage distribution is kept fixed here, rather than seen as dependent on t.

The following auxiliary result will be useful.

Lemma 5. *Under Assumption 1, $\overline{U}_{\theta,\varphi}(t)$ is single-peaked in t for any (θ, φ) and fixed wage distribution $F_E(w)$.*

Proof. Whenever $t < 1$, the sign of $\overline{U}'_{\theta,\varphi}(t)$ is determined by the sign of the square-bracketed term in (50). This term is continuous and strictly decreasing in t, is negative as $t \to 1$, and, under Assumption 1, is positive as $t \to -\infty$. $\overline{U}'_{\theta,\varphi}(t)$ is therefore single peaked at the unique $\tilde{t}$ for which this term is zero. $\qquad\square$

The *actual* increase in the indirect utility of an individual with skill (θ, φ), taking the endogeneity of wages into account, is:

$$
\begin{aligned}
U'_{\theta,\varphi}(t) &= \frac{dt(1-t)^{\frac{1}{\gamma-1}}}{dt} \int_{\underline{w}_{E(t)}}^{\overline{w}_{E(t)}} w^{\frac{\gamma}{\gamma-1}} dF_{E(t)}(w) + \frac{\gamma-1}{\gamma}\frac{d(1-t)^{\frac{\gamma}{\gamma-1}} w_{\theta,\varphi}(E(t))^{\frac{\gamma}{\gamma-1}}}{dt} \\
&\quad + t(1-t)^{\frac{1}{\gamma-1}} \frac{d\int_{\underline{w}_{E(t)}}^{\overline{w}_{E(t)}} w^{\frac{\gamma}{\gamma-1}} dF_{E(t)}(w)}{dt} + \frac{\gamma-1}{\gamma}(1-t)^{\frac{\gamma}{\gamma-1}}\frac{dw_{\theta,\varphi}(E(t))^{\frac{\gamma}{\gamma-1}}}{dt} \\
&= \overline{U}'_{\theta,\varphi}(t) + t(1-t)^{\frac{1}{\gamma-1}} \frac{d\int_{\underline{w}_{E(t)}}^{\overline{w}_{E(t)}} w^{\frac{\gamma}{\gamma-1}} dF_{E(t)}(w)}{dt} + \frac{\gamma-1}{\gamma}(1-t)^{\frac{\gamma}{\gamma-1}}\frac{dw_{\theta,\varphi}(E(t))^{\frac{\gamma}{\gamma-1}}}{dt} \\
&> \overline{U}'_{\theta,\varphi}(t). \quad\quad (51)
\end{aligned}
$$

The inequality follows from the fact that that $\int_{\underline{w}_{E(t)}}^{\overline{w}_{E(t)}} w^{\frac{\gamma}{\gamma-1}} dF_{E(t)}(w)$ is strictly increasing in t and $w_{\theta,\varphi}(E(t))$ is nondecreasing in t for all (θ, φ).

Fixing any $t \leq \underline{t}^{SC}$, the correspondence $Y(t)$ lies strictly above t by Lemma 4 (i) and Assumption 1. By Lemma 5, then $\overline{U}'_{\theta,\varphi}(t) \geq 0$. (The perceived peak of any individual's utility function is Pareto optimal, so t

---

[20]More precisely, whenever (θ, φ) and (θ', φ') such that $w_{\theta,\varphi}(E(t)) > w_{\theta',\varphi'}(E(t))$, then $\overline{U}'_{\theta,\varphi}(t) < \overline{U}'_{\theta',\varphi'}(t)$.

is to the the left of the peak.) We conclude that $U'_{\theta,\varphi}(t) > \bar{U}'_{\theta,\varphi}(t) \geq 0$ for all (θ, φ) in $\Theta \times \Phi$, and therefore that a marginal tax increase at any $t \leq \underline{t}^{SC}$ is Pareto improving, which proves part (ii) of the Lemma.

By Lemma 4 (i), $\bar{t}(t) \geq t$ for any $t \leq \bar{t}^{SC}$. So, by Lemma 5 and equation (51), $0 \leq \bar{U}'_{\underline{\theta},\underline{\varphi}}(t) < U'_{\underline{\theta},\underline{\varphi}}$. Hence, there exists a tax rate $t > \bar{t}^{SC}$ which the $(\underline{\theta}, \underline{\varphi})$ type strictly prefers to all $t \leq \bar{t}^{SC}$, and the tax rate $\bar{t}^{PO}$ which maximizes the well-being of the lowest skill type (and thus is Pareto optimal) has $\bar{t}^{PO} > \bar{t}^{SC}$. (Note that $\bar{t}^{SC}$ is well defined, since $[\bar{t}^{SC}, 1]$ is compact and $U_{\underline{\theta},\underline{\varphi}}(t)$ is continuous in t .) This proves part (i) of the Lemma.

A.3   Proof of Theorem 2

For the SCPE, the planner takes the wage distribution at some E as given. Fix *any* feasible E and let $w_\varphi = \varphi \mu(E)/E$. Then the SCPE problem is

$$\max_t \int_\Phi \overline{U}_\varphi(t) d\Psi(F(\varphi)),$$

where

$$\overline{U}_\varphi(t) = t(1-t)^{\frac{1}{\gamma-1}} \int_\Phi w_\varphi^{\frac{\gamma}{\gamma-1}} dF(\varphi) + \frac{\gamma-1}{\gamma}(1-t)^{\frac{\gamma}{\gamma-1}} w_\varphi^{\frac{\gamma}{\gamma-1}},$$

as above. The solution

$$\frac{t^{SC}}{1-t^{SC}} = (\gamma-1)\left(1 - \frac{\int_\Phi \varphi^{\frac{\gamma}{\gamma-1}} d\Psi(F(\varphi))}{\int_\Phi \varphi^{\frac{\gamma}{\gamma-1}} dF(\varphi)}\right) \tag{52}$$

exists, since

$$\left(1 - \frac{\int_\Phi \varphi^{\frac{\gamma}{\gamma-1}} d\Psi(F(\varphi))}{\int_\Phi \varphi^{\frac{\gamma}{\gamma-1}} dF(\varphi)}\right) \geq \left(1 - \frac{\int_\Phi \overline{\varphi}^{\frac{\gamma}{\gamma-1}} dF(\varphi)}{\int_\Phi \varphi^{\frac{\gamma}{\gamma-1}} dF(\varphi)}\right) > -1,$$

by Assumption 1. Since t^{SC} is independent of E, the unique SCPE is the feasible linear tax allocation with $t = t^{SC}$ (and $E = E(t^{SC})$, $T = T(t^{SC})$).

For the full Pareto program, it is more convenient to write the problem as

$$\max_{t,T,E} T + \frac{\gamma-1}{\gamma}(1-t)^{\frac{\gamma}{\gamma-1}} \left(\frac{\mu(E)}{E}\right)^{\frac{\gamma}{\gamma-1}} \int_\Phi \varphi^{\frac{\gamma}{\gamma-1}} d\Psi(F(\varphi))$$

subject to:

$$T = t(1-t)^{\frac{1}{\gamma-1}} \left(\frac{\mu(E)}{E}\right)^{\frac{\gamma}{\gamma-1}} \int_\Phi \varphi^{\frac{\gamma}{\gamma-1}} dF(\varphi)$$

and

$$\mu(E) = (1-t)^{\frac{1}{\gamma-1}} \left(\frac{\mu(E)}{E}\right)^{\frac{\gamma}{\gamma-1}} \int_\Phi \varphi^{\frac{\gamma}{\gamma-1}} dF(\varphi).$$

Attaching multipliers κ and η, the first order condition for T implies $\kappa = 1$, and the first order condition for t can be rearranged to

$$t^{PO} = \frac{(\gamma-1)\left(1 - \frac{\int_\Phi \varphi^{\frac{\gamma}{\gamma-1}} d\Psi(F(\varphi))}{\int_\Phi \varphi^{\frac{\gamma}{\gamma-1}} dF(\varphi)}\right) + \eta}{(\gamma-1)\left(1 - \frac{\int_\Phi \varphi^{\frac{\gamma}{\gamma-1}} d\Psi(F(\varphi))}{\int_\Phi \varphi^{\frac{\gamma}{\gamma-1}} dF(\varphi)}\right) + 1}.$$

Moreover, combining the first order conditions for t and E yields $\eta = 1 - \mu'(E^{PO})E^{PO}/\mu(E^{PO}) > 0$, where E^{PO} is the level of E at the Pareto optimum given Ψ. Solving equation (52) for $1 - t^{SC}$ and comparing $1 - t^{PO}$ to $1 - t^{SC}$ yields the result.

B Proofs for Section 4

B.1 Proof of Proposition 1

For the following, it will be convenient to rewrite the inner problem for the Pareto optimum as follows:

$$W(E) \equiv \max_{V_\theta(w), e_\theta(w), V_\varphi(w), e_\varphi(w)} \int_{\underline{w}_E}^{\overline{w}_E} V_\theta(w)\psi(F_E(w))f_E^\theta(w)dw + \int_{\underline{w}_E}^{\overline{w}_E} V_\varphi(w)\psi(F_E(w))f_E^\varphi(w)dw \tag{53}$$

s.t.

$$V_\theta'(w) - \frac{e_\theta(w)^\gamma}{w} = 0, \quad V_\varphi'(w) - \frac{e_\varphi(w)^\gamma}{w} = 0 \quad \forall w \tag{54}$$

$$\mu(E) - \int_{\underline{w}_E}^{\overline{w}_E} we_\varphi(w)f_E^\varphi(w)dw = 0 \tag{55}$$

$$\int_{\underline{w}_E}^{\overline{w}_E} \left(we_\theta(w) - (V_\theta(w) + e_\theta(w)^\gamma/\gamma)\right) f_E^\theta(w)dw$$
$$+ \int_{\underline{w}_E}^{\overline{w}_E} \left(we_\varphi(w) - (V_\varphi(w) + e_\varphi(w)^\gamma/\gamma)\right) f_E^\varphi(w)dw \quad \geq \quad 0 \tag{56}$$

$$e_\theta(w) = e_\varphi(w), \quad V_\theta(w) = V_\varphi(w) \quad \forall w. \tag{57}$$

Due to the additional no-discrimination constraints (57), it is obvious that problems (16)-(19) and (53)-(57) are equivalent. We attach multipliers $\eta_\theta(w)$ and $\eta_\varphi(w)$ to the two incentive constraints (54), ξ to (55) and $\delta_e(w), \delta_V(w)$ to the no discrimination constraints (57).

The Lagrangian corresponding to (53)-(57) is (after integrating by parts (54))

$$\begin{aligned}
\mathcal{L} &= \int_{\underline{w}_E}^{\overline{w}_E} V_\theta(w)\left(\psi(F_E(w)) - 1\right)f_E^\theta(w)dw + \int_{\underline{w}_E}^{\overline{w}_E} V_\varphi(w)\left(\psi(F_E(w)) - 1\right)f_E^\varphi(w)dw \\
&\quad - \int_{\underline{w}_E}^{\overline{w}_E} V_\theta(w)\eta_\theta'(w)dw - \int_{\underline{w}_E}^{\overline{w}_E} \frac{e_\theta(w)^\gamma}{w}\eta_\theta(w)dw \\
&\quad - \int_{\underline{w}_E}^{\overline{w}_E} V_\varphi(w)\eta_\varphi'(w)dw - \int_{\underline{w}_E}^{\overline{w}_E} \frac{e_\varphi(w)^\gamma}{w}\eta_\varphi(w)dw \\
&\quad + (1 - \xi)\int_{\underline{w}_E}^{\overline{w}_E} we_\varphi(w)f_E^\varphi(w)dw + \int_{\underline{w}_E}^{\overline{w}_E} we_\theta(w)f_E^\theta(w)dw + \xi\mu(E) \\
&\quad - \int_{\underline{w}_E}^{\overline{w}_E} e_\theta(w)^\gamma/\gamma f_E^\theta(w)dw - \int_{\underline{w}_E}^{\overline{w}_E} e_\varphi(w)^\gamma/\gamma f_E^\varphi(w)dw \\
&\quad + \int_{\underline{w}_E}^{\overline{w}_E} \delta_e(w)(e_\varphi(w) - e_\theta(w))dw + \int_{\underline{w}_E}^{\overline{w}_E} \delta_V(w)(V_\varphi(w) - V_\theta(w))dw. \tag{58}
\end{aligned}$$

The first order conditions for $V_\theta(w)$ and $V_\varphi(w)$ are

$$(\psi(F_E(w)) - 1)f_E^\theta(w) = \eta'_\theta(w) + \delta_V(w) \tag{59}$$

$$(\psi(F_E(w)) - 1)f_E^\varphi(w) = \eta'_\varphi(w) - \delta_V(w). \tag{60}$$

Adding them yields

$$\eta'_\theta(w) + \eta'_\varphi(w) = (\psi(F_E(w)) - 1)f_E(w) \tag{61}$$

and hence

$$\eta_\theta(w) + \eta_\varphi(w) = \Psi(F_E(w)) - F_E(w). \tag{62}$$

The first order conditions for $e_\theta(w)$ and $e_\varphi(w)$ are

$$\left(w - e_\theta(w)^{\gamma-1}\right)f_E^\theta(w) = \gamma\eta_\theta(w)\frac{e_\theta(w)^{\gamma-1}}{w} + \delta_e(w) \tag{63}$$

$$\left((1-\xi)w - e_\varphi(w)^{\gamma-1}\right)f_E^\varphi(w) = \gamma\eta_\varphi(w)\frac{e_\varphi(w)^{\gamma-1}}{w} - \delta_e(w) \tag{64}$$

and adding and using (57) yields

$$\left(w - e(w)^{\gamma-1}\right)f_E(w) - \xi w f_E^\varphi(w) = \gamma(\eta_\theta(w) + \eta_\varphi(w))\frac{e(w)^{\gamma-1}}{w}. \tag{65}$$

Rearranging and substituting (62) gives

$$\frac{e(w)^{\gamma-1}}{w} = \frac{f_E(w) - \xi f_E^\varphi(w)}{f_E(w) + \gamma(\Psi(F_E(w)) - F_E(w))/w}.$$

Note that, from the worker's problem

$$\max_e we - T(we) - \frac{e^\gamma}{\gamma},$$

we obtain

$$1 - T'(we(w)) = \frac{e(w)^{\gamma-1}}{w} \tag{66}$$

and hence the result in (24). (25) immediately follows from the fact that the inner problem for the SCPE is the same as for the Pareto optimum, only dropping constraint (18).

B.2   Proof of Proposition 2

We prove the result using the following two Lemmas:

Lemma 6. *For any given set of Pareto weights, the welfare effect of a marginal change in aggregate rent seeking effort E can be decomposed as follows:*

$$W'(E) = \xi\mu'(E) + \xi S + Z, \tag{67}$$

where

$$S \equiv \int_{\underline{w}_E}^{\overline{w}_E} we(w)\frac{df_E^\theta(w)}{dE}dw \tag{68}$$

54

and

$$Z = -\frac{1-\beta(E)}{E}\left((1-\xi)\mu(E) - D\right) \tag{69}$$

with

$$D \equiv \int_{\underline{w}_E}^{\overline{w}_E} w V_\varphi'(w)\delta_V(w)dw + \int_{\underline{w}_E}^{\overline{w}_E} w e_\varphi'(w)\delta_e(w)dw. \tag{70}$$

Proof. Using (58),

$$
\begin{aligned}
W'(E) &= \int_{\underline{w}_E}^{\overline{w}_E} V_\theta(w)\psi'(F_E(w))\frac{dF_E(w)}{dE}f_E^\theta(w)dw + \int_{\underline{w}_E}^{\overline{w}_E} V_\theta(w)\left(\psi(F_E(w)) - 1\right)\frac{df_E^\theta(w)}{dE}dw \\
&\quad + \int_{\underline{w}_E}^{\overline{w}_E} V_\varphi(w)\psi'(F_E(w))\frac{dF_E(w)}{dE}f_E^\varphi(w)dw + \int_{\underline{w}_E}^{\overline{w}_E} V_\varphi(w)\left(\psi(F_E(w)) - 1\right)\frac{df_E^\varphi(w)}{dE}dw \\
&\quad + (1-\xi)\int_{\underline{w}_E}^{\overline{w}_E} w e_\varphi(w)\frac{df_E^\varphi(w)}{dE}dw + \int_{\underline{w}_E}^{\overline{w}_E} w e_\theta(w)\frac{df_E^\theta(w)}{dE}dw + \xi\mu'(E) \\
&\quad - \int_{\underline{w}_E}^{\overline{w}_E} e_\theta(w)^\gamma/\gamma\frac{df_E^\theta(w)}{dE}dw - \int_{\underline{w}_E}^{\overline{w}_E} e_\varphi(w)^\gamma/\gamma\frac{df_E^\varphi(w)}{dE}dw + B
\end{aligned}
$$

with

$$
\begin{aligned}
B &\equiv \frac{d\overline{w}_E}{dE}\left[V_\varphi(\overline{w}_E)\left(\psi(F_E(\overline{w}_E)) - 1\right) + (1-\xi)\overline{w}_E e_\varphi(\overline{w}_E) - e_\varphi(\overline{w}_E)^\gamma/\gamma\right]f_E^\varphi(\overline{w}_E) \\
&\quad - \frac{d\underline{w}_E}{dE}\left[V_\varphi(\underline{w}_E)\left(\psi(F_E(\underline{w}_E)) - 1\right) + (1-\xi)\underline{w}_E e_\varphi(\underline{w}_E) - e_\varphi(\underline{w}_E)^\gamma/\gamma\right]f_E^\varphi(\underline{w}_E) \\
&= \frac{1}{E}\left(\mu'(E) - \frac{\mu(E)}{E}\right)\left[\left[V_\varphi(\overline{w}_E)\left(\psi(F_E(\overline{w}_E)) - 1\right) + (1-\xi)\overline{w}_E e_\varphi(\overline{w}_E) - e_\varphi(\overline{w}_E)^\gamma/\gamma\right]f_E^\varphi(\overline{w}_E)\overline{\varphi}\right. \\
&\quad \left. - \left[V_\varphi(\underline{w}_E)\left(\psi(F_E(\underline{w}_E)) - 1\right) + (1-\xi)\underline{w}_E e_\varphi(\underline{w}_E) - e_\varphi(\underline{w}_E)^\gamma/\gamma\right]f_E^\varphi(\underline{w}_E)\underline{\varphi}\right] \tag{71}
\end{aligned}
$$

since variations in $\overline{w}_E$ and $\underline{w}_E$ only affect the rent seekers. Let us rearrange this as follows:

$$
\begin{aligned}
W'(E) &= \int_{\underline{w}_E}^{\overline{w}_E} V_\theta(w)\psi'(F_E(w))\frac{dF_E(w)}{dE}f_E^\theta(w)dw + \int_{\underline{w}_E}^{\overline{w}_E} V_\varphi(w)\psi'(F_E(w))\frac{dF_E(w)}{dE}f_E^\varphi(w)dw \\
&\quad + \int_{\underline{w}_E}^{\overline{w}_E}\left(V_\varphi(w)\left(\psi(F_E(w)) - 1\right) + (1-\xi)w e_\varphi(w) - \frac{e_\varphi(w)^\gamma}{\gamma}\right)\frac{df_E(w)}{dE}dw \\
&\quad - \int_{\underline{w}_E}^{\overline{w}_E}\left(V_\varphi(w)\left(\psi(F_E(w)) - 1\right) + (1-\xi)w e_\varphi(w) - \frac{e_\varphi(w)^\gamma}{\gamma}\right)\frac{df_E^\theta(w)}{dE}dw \\
&\quad + \int_{\underline{w}_E}^{\overline{w}_E}\left(V_\theta(w)\left(\psi(F_E(w)) - 1\right) + w e_\theta(w) - \frac{e_\theta(w)^\gamma}{\gamma}\right)\frac{df_E^\theta(w)}{dE}dw + \xi\mu'(E) + B \\
&= \int_{\underline{w}_E}^{\overline{w}_E} V_\varphi(w)\psi'(F_E(w))\frac{dF_E(w)}{dE}f_E(w)dw \\
&\quad + \int_{\underline{w}_E}^{\overline{w}_E}\left(V_\varphi(w)\left(\psi(F_E(w)) - 1\right) + (1-\xi)w e_\varphi(w) - \frac{e_\varphi(w)^\gamma}{\gamma}\right)\frac{df_E(w)}{dE}dw \\
&\quad + \xi\int_{\underline{w}_E}^{\overline{w}_E} w e_\varphi(w)\frac{df_E^\theta(w)}{dE}dw + \xi\mu'(E) + B, \tag{72}
\end{aligned}
$$

where we used the no discrimination constraints (57). We can now integrate the second line by parts and

55

get

$$\tilde{B} - \int_{\underline{w}_E}^{\overline{w}_E} \left(V_{\varphi}'(w)(\psi(F_E(w)) - 1) + V_{\varphi}(w)\psi'(F_E(w))f_E(w) \right) \frac{dF_E(w)}{dE} dw$$

$$- (1 - \xi) \int_{\underline{w}_E}^{\overline{w}_E} e_{\varphi}(w) \frac{dF_E(w)}{dE} dw - \int_{\underline{w}_E}^{\overline{w}_E} e_{\varphi}'(w) \left(w - e_{\varphi}(w)^{\gamma - 1} \right) \frac{dF_E(w)}{dE} dw$$

with

$$\tilde{B} \equiv \left[V_{\varphi}(\overline{w}_E) \left(\psi(F_E(\overline{w}_E)) - 1 \right) + (1 - \xi)\overline{w}_E e_{\varphi}(\overline{w}_E) - e_{\varphi}(\overline{w}_E)^{\gamma}/\gamma \right] \frac{dF_E(\overline{w}_E)}{dE}$$

$$- \left[V_{\varphi}(\underline{w}_E) \left(\psi(F_E(\underline{w}_E)) - 1 \right) + (1 - \xi)\underline{w}_E e_{\varphi}(\underline{w}_E) - e_{\varphi}(\underline{w}_E)^{\gamma}/\gamma \right] \frac{dF_E(\underline{w}_E)}{dE}. \tag{73}$$

Substituting in (72) and cancelling terms yields

$$
\begin{aligned}
W'(E) &= \xi\mu'(E) - (1 - \xi) \int_{\underline{w}_E}^{\overline{w}_E} e_{\varphi}(w) \frac{dF_E(w)}{dE} dw + \xi \int_{\underline{w}_E}^{\overline{w}_E} we_{\theta}(w) \frac{df_E^{\theta}(w)}{dE} dw \\
&\quad - \int_{\underline{w}_E}^{\overline{w}_E} V_{\varphi}'(w)(\psi(F_E(w)) - 1) \frac{dF_E(w)}{dE} dw \\
&\quad - \int_{\underline{w}_E}^{\overline{w}_E} e_{\varphi}'(w) \left((1 - \xi)w - e_{\varphi}(w)^{\gamma - 1} \right) \frac{dF_E(w)}{dE} dw + B + \tilde{B}. \tag{74}
\end{aligned}
$$

Moreover, note that, from (14) and (15),

$$
\begin{aligned}
\frac{dF_E(w)}{dE} &= \frac{d}{dE} \left(\frac{E}{\mu(E)} \right) w \frac{\partial F\left(w, wE/\mu(E)\right)}{\partial \varphi} = \frac{d}{dE} \left(\frac{E}{\mu(E)} \right) w \int_{\underline{\theta}}^{w} f\left(\theta, w \frac{E}{\mu(E)} \right) d\theta \\
&= \frac{\mu(E)}{E} \frac{d}{dE} \left(\frac{E}{\mu(E)} \right) w f_E^{\varphi}(w) = \frac{1}{E} \left(1 - \frac{\mu'(E)E}{\mu(E)} \right) w f_E^{\varphi}(w),
\end{aligned}
$$

so that $B = -\tilde{B}$ and (74) becomes

$$
\begin{aligned}
W'(E) &= \xi\mu'(E) + \xi \int_{\underline{w}_E}^{\overline{w}_E} we_{\theta}(w) \frac{df_E^{\theta}(w)}{dE} dw - \frac{1}{E} \left(1 - \frac{\mu'(E)E}{\mu(E)} \right) \left[(1 - \xi) \int_{\underline{w}_E}^{\overline{w}_E} e_{\varphi}(w) w f_E^{\varphi}(w) dw \right. \\
&\quad + \int_{\underline{w}_E}^{\overline{w}_E} V_{\varphi}'(w)(\psi(F_E(w)) - 1) w f_E^{\varphi}(w) dw \\
&\quad \left. + \int_{\underline{w}_E}^{\overline{w}_E} e_{\varphi}'(w) \left((1 - \xi)w - e_{\varphi}(w)^{\gamma - 1} \right) w f_E^{\varphi}(w) dw \right]. \tag{75}
\end{aligned}
$$

Let us next use the first order conditions for $V_{\varphi}(w)$ and $e_{\varphi}(w)$, (60) and (64), to substitute in (75) to obtain

$$
\begin{aligned}
W'(E) &= \xi\mu'(E) + \xi \int_{\underline{w}_E}^{\overline{w}_E} we_{\theta}(w) \frac{df_E^{\theta}(w)}{dE} dw - \frac{1}{E} \left(1 - \frac{\mu'(E)E}{\mu(E)} \right) \left[(1 - \xi) \int_{\underline{w}_E}^{\overline{w}_E} e_{\varphi}(w) w f_E^{\varphi}(w) dw \right. \\
&\quad + \int_{\underline{w}_E}^{\overline{w}_E} V_{\varphi}'(w)w \left(\eta_{\varphi}'(w) - \delta_V(w) \right) dw \\
&\quad \left. + \int_{\underline{w}_E}^{\overline{w}_E} we_{\varphi}'(w) \left(\gamma\eta_{\varphi}(w) \frac{e_{\varphi}(w)^{\gamma - 1}}{w} - \delta_e(w) \right) dw \right]. \tag{76}
\end{aligned}
$$

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Finally, integrating by parts in the third line gives

$$\int_{\underline{w}_E}^{\overline{w}_E} \gamma e_\varphi'(w) e_\varphi(w)^{\gamma-1} \eta_\varphi(w) dw = -\int_{\underline{w}_E}^{\overline{w}_E} e_\varphi(w)^\gamma \eta_\varphi'(w) dw$$

and substituting in (76) and using the incentive constraints (17) leaves us with

$$
\begin{aligned}
W'(E) &= \xi \mu'(E) + \xi \int_{\underline{w}_E}^{\overline{w}_E} we(w) \frac{df_E^\theta(w)}{dE} dw - \frac{1}{E} \left(1 - \frac{\mu'(E)E}{\mu(E)}\right) \left[(1-\xi) \int_{\underline{w}_E}^{\overline{w}_E} e(w) w f_E^\varphi(w) dw \right.\\
&\quad \left. - \int_{\underline{w}_E}^{\overline{w}_E} wV'(w) \delta_V(w) dw - \int_{\underline{w}_E}^{\overline{w}_E} we'(w) \delta_e(w) dw \right]\\
&= \xi \left(\frac{\mu(E)}{E} + S\right) - \frac{\mu(E)}{E} \left(1 - \frac{\mu'(E)E}{\mu(E)}\right) \left(1 - \frac{D}{\mu(E)}\right),
\end{aligned}
\tag{77}
$$

which completes the proof of the first lemma. □

This first Lemma demonstrates that term D in the wage shift effect Z captures the presence of the no-discrimination constraints (70). The following, second lemma derives a further decomposition of this no-discrimination effect:

Lemma 7. *The no-discrimination effect D in (69) can be further decomposed as follows:*

$$D = D_1 + \xi D_2 \tag{78}$$

with

$$D_1 \equiv \int_{\underline{w}_E}^{\overline{w}_E} e(w)^\gamma \left(\Psi(F_E(w)) - F_E(w)\right) \frac{d}{dw} \left(\frac{f_E^\varphi(w)}{f_E(w)}\right) dw \tag{79}$$

and

$$D_2 \equiv \int_{\underline{w}_E}^{\overline{w}_E} w^2 e'(w) \frac{f_E^\varphi(w) f_E^\theta(w)}{f_E(w)} dw. \tag{80}$$

Proof. Subtract (59) from (60) to get

$$\delta_V(w) = (\psi(F_E(w)) - 1) \frac{\Delta(w)}{2} - \frac{\eta_\theta'(w) - \eta_\varphi'(w)}{2}$$

with $\Delta(w) \equiv f_E^\theta(w) - f_E^\varphi(w)$. Similarly,

$$\delta_e(w) = \left(w - e(w)^{\gamma-1}\right) \frac{\Delta(w)}{2} + \xi \frac{w f_E^\varphi(w)}{2} - \gamma \frac{\eta_\theta(w) - \eta_\varphi(w)}{2} \frac{e(w)^{\gamma-1}}{w}.$$

Substituting in the definition of D and using $wV'(w) = e(w)^\gamma$ from (17) yields

$$
\begin{aligned}
D &= \int_{\underline{w}_E}^{\overline{w}_E} e(w)^\gamma (\psi(F_E(w)) - 1) \frac{\Delta(w)}{2} dw + \int_{\underline{w}_E}^{\overline{w}_E} we'(w) \left(w - e(w)^{\gamma-1}\right) \frac{\Delta(w)}{2} dw + \xi \int_{\underline{w}_E}^{\overline{w}_E} w^2 e'(w) \frac{f_E^\varphi(w)}{2} dw\\
&\quad - \int_{\underline{w}_E}^{\overline{w}_E} e(w)^\gamma \frac{\eta_\theta'(w) - \eta_\varphi'(w)}{2} - \int_{\underline{w}_E}^{\overline{w}_E} \gamma e'(w) e(w)^{\gamma-1} \frac{\eta_\theta(w) - \eta_\varphi(w)}{2}.
\end{aligned}
\tag{81}
$$

The second line vanishes after integrating the last integral by parts. Next, integrating the first integral in

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(81) by parts yields

$$\int_{\underline{w}_E}^{\overline{w}_E} e(w)^\gamma \frac{\Delta(w)}{2f_E(w)} \left(\psi(F_E(w) - 1)\right) f_E(w) dw$$

$$= -\int_{\underline{w}_E}^{\overline{w}_E} \left(\gamma e'(w) e(w)^{\gamma-1} \frac{\Delta(w)}{2f_E(w)} + e(w)^\gamma \frac{d}{dw}\left(\frac{\Delta(w)}{2f_E(w)}\right)\right) \left(\Psi(F_E(w)) - F_E(w)\right) dw$$

and substituting in (81) gives

$$D = -\int_{\underline{w}_E}^{\overline{w}_E} e(w)^\gamma \frac{d}{dw}\left(\frac{\Delta(w)}{2f_E(w)}\right) \left(\Psi(F_E(w)) - F_E(w)\right) dw$$

$$+ \int_{\underline{w}_E}^{\overline{w}_E} we'(w) \left(w - e(w)^{\gamma-1} - \gamma e(w)^{\gamma-1} \frac{\Psi(F_E(w)) - F_E(w)}{wf_E(w)}\right) \frac{\Delta(w)}{2} dw$$

$$+ \xi \int_{\underline{w}_E}^{\overline{w}_E} w^2 e'(w) \frac{f_E^\varphi(w)}{2} dw.$$

Note that, by (62) and (65),

$$w - e(w)^{\gamma-1} - \gamma e(w)^{\gamma-1} \frac{\Psi(F_E(w)) - F_E(w)}{wf_E(w)} = \xi w \frac{f_E^\varphi(w)}{f_E(w)}.$$

Hence,

$$D = -\int_{\underline{w}_E}^{\overline{w}_E} e(w)^\gamma \frac{d}{dw}\left(\frac{\Delta(w)}{2f_E(w)}\right) \left(\Psi(F_E(w)) - F_E(w)\right) dw$$

$$+ \xi \int_{\underline{w}_E}^{\overline{w}_E} w^2 e'(w) \left(\frac{f_E^\varphi(w)}{f_E(w)} \frac{\Delta(w)}{2} + \frac{f_E^\varphi(w)}{2}\right) dw$$

$$= -\int_{\underline{w}_E}^{\overline{w}_E} e(w)^\gamma \frac{d}{dw}\left(\frac{f_E(w) - 2f_E^\varphi(w)}{2f_E(w)}\right) \left(\Psi(F_E(w)) - F_E(w)\right) dw$$

$$+ \xi \int_{\underline{w}_E}^{\overline{w}_E} w^2 e'(w) \frac{f_E^\varphi(w)\left(\Delta(w) + f_E(w)\right)}{2f_E(w)} dw$$

$$= \int_{\underline{w}_E}^{\overline{w}_E} e(w)^\gamma \frac{d}{dw}\left(\frac{f_E^\varphi(w)}{f_E(w)}\right) \left(\Psi(F_E(w)) - F_E(w)\right) dw + \xi \int_{\underline{w}_E}^{\overline{w}_E} w^2 e'(w) \frac{f_E^\varphi(w)f_E^\theta(w)}{f_E(w)} dw$$

$$= D_1 + \xi D_2,$$

which completes the proof of the second lemma and thus of Proposition 2. $\qquad \square$

B.3 Proof of Proposition 3

From Proposition 2,

$$Z = -\frac{1 - \beta(E)}{E} \left((1 - \xi)\mu(E) - D_1 - \xi D_2\right). \tag{82}$$

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Let us decompose

$$\mu(E)(1-\xi) = \mu(E) - \xi \int_{\underline{w}_E}^{\overline{w}_E} we(w)\left(1 - \frac{f_E^\theta(w)}{f_E(w)}\right)f_E^\varphi(w)dw - \xi \int_{\underline{w}_E}^{\overline{w}_E} we(w)\frac{f_E^\theta(w)f_E^\varphi(w)}{f_E(w)}dw$$

$$= \int_{\underline{w}_E}^{\overline{w}_E} we(w)\left(f_E^\varphi(w) - \xi\frac{(f_E^\varphi(w))^2}{f_E(w)}\right)dw - \xi \int_{\underline{w}_E}^{\overline{w}_E} we(w)\frac{f_E^\theta(w)f_E^\varphi(w)}{f_E(w)}dw$$

$$= \int_{\underline{w}_E}^{\overline{w}_E} \frac{f_E^\varphi(w)}{f_E(w)}we(w)\left(f_E(w) - \xi f_E^\varphi(w)\right)dw - \xi \int_{\underline{w}_E}^{\overline{w}_E} we(w)\frac{f_E^\theta(w)f_E^\varphi(w)}{f_E(w)}dw$$

$$= \int_{\underline{w}_E}^{\overline{w}_E} \frac{f_E^\varphi(w)}{f_E(w)}\gamma\frac{e(w)^\gamma}{w}\eta(w)dw + \int_{\underline{w}_E}^{\overline{w}_E} e^\gamma(w)f_E^\varphi(w)dw - \xi \int_{\underline{w}_E}^{\overline{w}_E} we(w)\frac{f_E^\theta(w)f_E^\varphi(w)}{f_E(w)}dw$$

$$\equiv G_1 + G_2 - \xi G_3$$

where the last step follows from the first order condition for $e(w)$, equation (65), which can be rearranged to

$$\frac{e^\gamma(w)}{w}[wf_E(w) + \gamma\eta(w)] = we(w)\left(f_E(w) - \xi f_E^\varphi(w)\right) \tag{83}$$

with $\eta(w) \equiv \eta_\theta(w) + \eta_\varphi(w) = \Psi(F_E(w)) - F_E(w)$. Hence, the wage shift effect Z can be rewritten as

$$Z = -\frac{1-\beta(E)}{E}(G_1 + G_2 - D_1 - \xi(D_2 + G_3)). \tag{84}$$

Moreover, recall the first order condition for $V(w)$, equation (61),

$$\psi(F_E(w))f_E(w) - f_E(w) = \eta'(w) \quad \Rightarrow \quad \psi(F_E(w))f_E^\varphi(w) = f_E^\varphi(w) + \eta'(w)\frac{f_E^\varphi(w)}{f_E(w)}$$

and compute

$$
\begin{aligned}
G_2 - D_1 + G_1 &= G_2 + \int_{\underline{w}_E}^{\overline{w}_E} \frac{d}{dw}[e(w)^\gamma\eta(w)]\frac{f_E^\varphi(w)}{f_E(w)}dw + G_1 \\
&= G_2 + \int_{\underline{w}_E}^{\overline{w}_E} e(w)^\gamma\eta'(w)\frac{f_E^\varphi(w)}{f_E(w)}dw + \int_{\underline{w}_E}^{\overline{w}_E} \gamma\eta(w)\frac{e(w)^{\gamma-1}}{w}(e'(w)w + e(w))\frac{f_E^\varphi(w)}{f_E(w)}dw \\
&= \int_{\underline{w}_E}^{\overline{w}_E} e(w)^\gamma\psi(F_E(w))f_E^\varphi(w)dw + \int_{\underline{w}_E}^{\overline{w}_E} \eta(w)\gamma(1 - T'(y(w)))y'(w)\frac{f_E^\varphi(w)}{f_E(w)}dw \\
&= \int_{\underline{w}_E}^{\overline{w}_E} wV'(w)\psi(F_E(w))f_E^\varphi(w)dw + \int_{\underline{w}_E}^{\overline{w}_E} \eta(w)\gamma c'(w)\frac{f_E^\varphi(w)}{f_E(w)}dw \equiv G_4 + G_5,
\end{aligned}
$$

where the first line follows from integrating by parts D_1, the third line uses the first order condition for $V(w)$ and the workers' first order condition (66), and the last line uses the incentive constraints (17) and $c(w) \equiv y(w) - T(y(w))$. This leads us to

$$Z = -\frac{1-\beta(E)}{E}(G_4 + G_5 - \xi(D_2 + G_3)). \tag{85}$$

Solving $W'(E) = 0$ for ξ yields

$$\xi = (1 - \beta(E))\frac{G_4 + G_5}{\mu'(E)E + SE + (1 - \beta(E))(D_2 + G_3)}. \tag{86}$$

Note that $G_4 > 0$ since $V'(w) > 0$ from (17). Also, $G_5 \geq 0$ whenever $\Psi(F)$ is regular so that $\eta(w) \geq 0$ because $c'(w) = (1 - T'(y(w)))y'(w) \geq 0$ since $T'(y(w)) \leq 1$ and $y'(w) \geq 0$ for all w. Finally,

$$
\begin{aligned}
D_2 + G_3 &= \int_{\underline{w}_E}^{\overline{w}_E} (we'(w) + e(w))w \frac{f_E^\theta(w)f_E^\varphi(w)}{f_E(w)} dw \\
&= \int_{\underline{w}_E}^{\overline{w}_E} y'(w)w \frac{f_E^\theta(w)f_E^\varphi(w)}{f_E(w)} dw \geq 0
\end{aligned}
$$

since $y'(w) \geq 0$. Hence, $\xi > 0$ for any regular Pareto optimum.

B.4 Proof of Theorem 5

Note first that any regular SCPE has a non-decreasing tax schedule $T(y(w))$ since, from (25),

$$
1 - T'(y(w)) = \left(1 + \gamma \frac{\Psi(F_E(w)) - F_E(w)}{w f_E(w)}\right)^{-1} \leq 1
$$

if $\Psi(F) \geq F$, so that $T'(y(w)) \geq 0$ for all w. Then the following auxiliary lemma is useful to prove the theorem:

Lemma 8. *Any Pareto optimum with a non-decreasing tax schedule $T(y)$ has $\xi > 0$.*

Proof. Consider any Pareto optimum with non-negative marginal tax rates. Then $W'(E) = 0$ for some set of Pareto weights. Suppose, by way of contradiction, that $\xi \leq 0$ for the associated inner problem. By the first order condition for $e(w)$ (65), this implies:

$$
\frac{(1 - T'(y(w)))\gamma\eta(w)}{f_E(w)} \geq wT'(y(w)).
$$

Hence,

$$
\begin{aligned}
G_5 &= \int_{\underline{w}_E}^{\overline{w}_E} \frac{\gamma\eta(w)(1 - T'(w))}{f_E(w)} y'(w) f_E^\varphi(w) dw \\
&\geq \int_{\underline{w}_E}^{\overline{w}_E} wT'(y(w))y'(w)f_E^\varphi(w) dw = \int_{\underline{w}_E}^{\overline{w}_E} \frac{dT(y(w))}{dw} w f_E^\varphi(w) dw \geq 0.
\end{aligned}
$$

Since $G_4 > 0$, this implies $G_4 + G_5 > 0$, hence from (85)

$$
W'(E) = \xi\mu'(E) + \xi S - \frac{1 - \beta(E)}{E}[G_4 + G_4 - \xi(D_2 + G_3)] < 0,
$$

since $D_2 + G_3 \geq 0$ and we assumed $\xi \leq 0$. This contradicts $W'(E) = 0$. □

Consider any SCPE. It has $T'(y(\underline{w}_E)) = T'(y(\overline{w}_E)) = 0$ by equation (25). By assumption, it has a non-decreasing tax-schedule, so, by the above lemma $\xi > 0$ for the inner part of the Pareto problem for any $\Psi(\cdot)$. Since $f_E^\varphi(w) > 0$ at $\underline{w}_E$ or $\overline{w}_E$ or both, equation (24) implies $T'(y(\underline{w}_E)) > 0$ or $T'(y(\overline{w}_E)) > 0$ or both. Hence, there are no weights $\Psi(\cdot)$ for which the SCPE solves the Pareto problem.