

# A Nexus of Contracts Theory of Legal Entities

Kenneth Ayotte

Northwestern University School of Law\*

Henry Hansmann

Yale Law School

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## Abstract

In this paper, we develop a theory that explains why firms are so commonly organized as legal entities that are formally distinct from their owners. A legal entity permits an owner to create a firm as a bundle of contracts that can be transferred to someone else, but only if they are transferred together. This *bundled assignability* allows for a balancing of several potentially conflicting interests. First, the owner who assembles the contracts wants liquidity – that is, the ability to transfer the contracts and cash out. Second, the firm’s contractual counterparties want protection from opportunistic transfers that will reduce the value of the performance they’ve been promised. And third, the owner wants long-term commitments from the firm’s counterparties to protect the value of her investments in the bundle. Because transfers of equity interests in a legal entity will generally not be considered assignments of the entity’s contracts, entities reduce the contracting costs of creating bundled assignability. We find that owners will prefer bundled assignability when investments in the bundle are alienable from the owner; but when investments are specific to the owner, contracts that prohibit changes of control are optimal. We examine a sample of 287 supply and lease contracts. We find that bundled assignability is explicitly included in these contracts with great regularity, and that legal entities are the most common means of defining the bundles.

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# 1 Introduction: What Role For Legal Entities?

In modern economies, firms are commonly organized as legal entities that are separate from their stakeholders, and that can enter into contracts and hold property in their own name. The role of these entities has received little attention in the literature on the theory of the firm, which has focused on relationships among individuals and has largely omitted explicit analysis of entities (e.g. Coase (1937), Alchian and Demsetz (1972), Grossman and Hart (1986), Hart and Moore (1990)). Jensen and Meckling (1976) recognize the firm as a contracting *entity*, but offer no explanation for it:

“There is in a very real sense only a multitude of complex relationships (i.e. contracts) *between the legal fiction (the firm) and the owners of labor, material and capital inputs and the consumers of output.*”

(Emphasis added.) What, then, is the value of a legal entity as the center of the nexus of contracts? The question is made all the more salient by the ubiquity of firms in the modern economy that use a complicated web of legal entities to own assets that are ultimately under common control. The 100 largest U.S. public companies, for example, report an average of 243 subsidiaries, not including subsidiaries insufficiently significant to require public reporting. General Electric alone has approximately 1500 separately incorporated subsidiaries, most of which are wholly owned by General Electric. Why are these businesses organized as distinct legal entities, rather than as divisions of the parent company?

We offer answers to these questions that focus on the fact that a firm’s most valuable assets are often its contractual rights. Consider, for example, the movie rental company Netflix. The value of Netflix is based largely on its assemblage of contractual relationships. In particular, the DVDs that Netflix rents to its customers are acquired via contractual agreements with the major movie studios. These contracts require Netflix to make a small up-front payment to the studio for each DVD, and then contingency payments based on the number of times the movie is rented. Netflix provides streaming video to its subscribers by licensing content owned by movie studios using similar revenue sharing arrangements. All of the real estate it occupies is owned by other parties and used by Netflix pursuant to long-term lease contracts. Most of its revenues come through its pool of subscriber contracts.<sup>1</sup> In essence, Netflix is a bundle of contracts of which the incorporated legal entity, Netflix, is the nexus in the sense of being a common signatory to all of those contracts.<sup>2</sup>

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<sup>1</sup>See Netflix 10-K, 2008.

<sup>2</sup>Many firms own little to no physical assets at all, as our Netflix example illustrates. Broadway plays offer another conspicuous example. Each play that is produced is typically formed as a separate legal entity.

A noteworthy feature of these contractual agreements between Netflix and its counterparties (movie studios, landlords, and customers) is that they are bilateral – that is, they impose upon Netflix both rights and obligations, making the contracts simultaneously both assets and liabilities to the firm. We take as given that firms, for many potential reasons, find it advantageous to acquire inputs and provide outputs by contract, making their counterparties (i.e. their suppliers, employees, landlords, managers, customers, etc.) reliant on the quality of the firm’s future performance. A wide variety of contracts share this bilateral feature: common examples include leases, employment agreements, supply agreements, franchise agreements, and intellectual property licenses, to name a few.

It is this two-sided feature of contracts, and the resulting potential for two-sided opportunism, that gives rise to an important role for legal entities in the model we develop in this paper. In conducting business through an entity, the firm’s counterparties contract with an artificial person that maintains its identity when its owners change. This allows the owners of the firm to sell their interests freely when they have liquidity needs without requiring the consent of its counterparties to an assignment (i.e. a transfer) of the contract to a new owner. If this consent were not given, owners might not be able to realize the value of their specific investments in the firm, due to a holdup problem or a failure in bargaining. This might reduce the incentive of the owners to make non-contractible investments in the firm at the outset.

At the same time, because the legal entity is a common signatory for all the firm’s contracts, the owners can limit their own ability to act opportunistically. If allowed to assign contracts individually, the owners could threaten to assign contracts to less creditworthy firms with lower quality inputs. Less creditworthy firms have higher borrowing costs when they finance their assets at fair borrowing rates, so they see an assignment from a more creditworthy firm as an opportunity to obtain cheap financing. This, in turn, exposes counterparties to increased credit risk. Moreover, it might give owners an incentive to separate a bundle of contracts that are worth more together than apart. This opportunism problem can also reduce an owner’s incentive to make investments that increase the bundled value of the inputs.

In assembling a legal entity, and ensuring that the individual contracts in the bundle can not be transferred *by the entity*, an owner pledges to her counterparties that, while she may transfer her rights and obligations under the contract to a new owner, she can do so only if

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That entity has contracts with many individuals – including actors, musicians, stagehands, and a director – and also a rental contract with the theater where the work is performed. And of course it has contracts with ticket purchasers. But it rarely holds outright title to physical assets. The firm’s net value lies entirely in its assemblage of contracts.

the firm’s other contracts move along with it. The assembled value of the contracts provides, in effect, important assurance of prospective payment on the liability in question. In short, the entity in our theory provides a low-cost means of achieving *bundled assignability*. To provide empirical support for our theory, we examine assignment clauses in a sample of two classes of commercial contracts (supply agreements and leases) that have the bilateral feature of contracts in our model. We find that (1) bundled assignability is a regular feature of such contracts, and (2) legal entities are the most common means of creating and defining transferable bundles.

Our analysis uses the same economic forces as in the property rights theory of the firm (non-contractible specific investments in assets), but it also emphasizes financing considerations (the liquidity needs of owners, and the provision of financing by suppliers) as a crucial driving force behind legal entities, in contrast to the exclusive emphasis on assets in most of the theory of the firm literature. It offers insight not only into the economic and legal structure of firms, but also into the ways that restrictions on contract assignability are – and should be – affected by changes in the boundaries of the firm.

This work is not the only theory of legal entities that is based on interactions between assets and liabilities. One example is the theory of asset partitioning (Hansmann and Kraakman 2000a, 2000b; Hansmann, Kraakman, and Squire 2006). Counterparties to the contracts entered into with a given legal entity all have their contractual rights bonded by claims against a single common pool of assets, which consist of the other contractual rights and property rights held by the entity. Those claims, moreover, are made senior to the claims of the owners’ other personal or business creditors (by virtue of “entity shielding”). This literature argues that entity shielding can reduce the overall costs of asymmetric information by concentrating creditors’ claims on the bundles of assets that the creditors can most easily monitor. Our ongoing work in progress explores the connection between the asset partitioning and bundled assignability features that entities provide.

Another example is Iacobucci and Triantis (2007), which argues that the boundaries of legal entities can be driven by legal constraints requiring that certain decisions, such as capital structure, be made on an entity-wide basis. Separation of assets into different legal entities to achieve more tailoring of liabilities, however, may undermine the benefits of common control of assets. Closer to our work, another explanation focuses on “capital lock-in” (Blair, 2003). By limiting the rights of a firm’s owners to withdraw capital from the firm, corporate-type legal entities enhance the reliability of the firm’s assets as a bond for long-term investments by the firm’s employees, suppliers, creditors, and customers.

## 2 Legal Entities and Assignability of Contracts

A party's rights and obligations under a contract may or may not be transferable (or, as we will somewhat loosely say, assignable<sup>3</sup>) to a third party without the permission of the other party to the contract. For example, the rights of a promisee under a simple contract for payment of a definite sum of money are, as a default rule of contract law, generally presumed assignable. Contracts for labor services, in contrast, are generally presumed nonassignable by the employer. Whatever the default rule of law, the assignability of a contract can generally be altered by a specific provision in the contract itself. For example, although leaseholds are presumed assignable, it is extremely common for assignability to be curtailed by a clause in the lease prohibiting the tenant from assigning it without the consent of the landlord. Even when a promisor's obligations under a contract are assignable as a consequence of a default rule of law or a specific contractual provision, the promisor remains liable to the promisee after those obligations have been transferred to a third party, unless the promisee agrees (by means of a "novation" in the original contract or subsequently) that the original promisor will be excused from such continuing liability. When we say here that a contract is "assignable," we will take the further linguistic liberty of meaning that all of the assignor's rights and obligations under the contract can be assigned free of any residual liability for the assignor. Under the default rules of law, then, virtually all contracts are presumed non-assignable in this sense, and can be made assignable only by explicit contracting.

If a legal entity such as a corporation is a party to a contract, a transfer of ownership rights in the entity is not considered an assignment of the contract.<sup>4</sup> This rule is interpreted quite broadly. For example, the courts have held that the sale of all of the stock in a closely held corporation does not violate a contractual provision prohibiting the corporation from assigning the contract even when the stock is sold to a person to whom, previously, the counterparty to the contract had explicitly refused to permit the contract to be assigned.<sup>5</sup> Consequently, if

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<sup>3</sup>When we say that a contract is "assignable," we are using the term rather loosely from a legal point of view. In particular, by "assignable" we mean here that the transferee assumes all of the transferor's rights and obligations under the contract, while the transferor gives up all rights and is freed of all obligations. In legal terminology, this is to assume that all of the transferor's rights are assigned, and obligations are delegated, to the transferee, and in addition that the transfer is novated by the counterparty.

<sup>4</sup>*Baxter Healthcare Corp. v. O.R. Concepts, Inc.*, 7th Cir., 69 F.3d 785, 788 (1995) (change of ownership of stock does not constitute a variation of the selling corporation's contractual obligations and is not an assignment of the selling corporations' interest in an agreement); *Institut Pasteur v. Cambridge Biotech Corp.*, 1st Cir., 104 F.3d 489, 494, cert. denied, 117 S.Ct. 2551 (1997) (sale of stock in corporation doesn't constitute a violation of non-assignability provision in patent license); Note (1960).

<sup>5</sup>*Ser-Bye Corp. v. C.P. & G. Markets*, 78 Cal.App.2d 915, 179 P.2d 342 (1947); *Branmar Theater Co. v.*

the counterparty to a contract with a corporation wishes to limit the persons to whom control over the corporation can be sold, they must do this through specific language to that effect in the contract (a “change of control” clause); a non-assignment clause will not suffice.

These rules make it easy for contracting parties to provide that a given bundle of contracts will not be split up, while at the same time providing that control over that bundle of contracts as a whole can be freely assigned. To create bundled assignability via a legal entity, an owner could, first, set up a corporation (or some other entity), and hold the shares in the corporation. The corporation would then sign contracts with the firm’s counterparties that are individually non-assignable by the corporation. This simple contracting structure would create bundled assignability; the owner could assign the contracts as a bundle by selling the stock, but could not separate any individual contract from the bundle without permission from a counterparty.

To be sure, the parties could try to achieve the same ends using only contracts between flesh and blood individuals, without creating a separate legal person. The owner and her counterparties might, for example, make sure that each contract in the firm contains a clause that makes it individually non-assignable, but permits assignment as a bundle with an enumerated list of the businesses’ other contracts. As the business becomes more complex, however, and contracts come and go over time, this solution is unlikely to be feasible. To take one example, Boeing uses 700 different suppliers to create one of its airplanes. Attempting to identify and bundle each of the 700 supply contracts with the 699 other contracts would be messy, labor-intensive, and potentially fraught with error and ambiguities in identification. Moreover, as contracts change over time, each contract would need to anticipate these future contracts and identify them before they come into existence. In short, this is unlikely to be a practical solution in most realistic cases.

Contracts might also try to create bundled assignability using a general description of the bundle. The owner might, for example, sign contracts with her counterparties that prohibit individual assignment, but allow for assignment along with “all of the contracts of the movie rental business”. But the bundle of contracts that satisfy a general description can be ambiguous, and subject to manipulation by the owner and the counterparties. A counterparty might try to argue that the bundle is underinclusive, to exploit the potential to hold up the owner. These risks of ambiguity are particularly likely if the owner also owns other businesses that use assets and contracts in common.

In comparison, a bundle that references a legal entity creates substantially less ambiguity. The counterparty who is concerned about opportunism need only prove that there are con-

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Branmar, Inc., 264 A.2d 526 (Del.Ch.1970).

tracts written by the entity with which he contracted that are not being transferred in the sale. On the other side, the owner need only show that a contract not being assigned uses a different legal person as a signatory. This provides more assurance against opportunism on both sides of the transaction.

In short, a legal entity may not be necessary in all situations to create bundled assignability, but it is likely to be easier and more reliable than other alternatives. Creating bundled assignability with an entity requires only simple contracting terms to create, and is less subject to ex-post uncertainty when a transfer is to take place<sup>6</sup>.

The model that follows demonstrates the economic value of bundled assignability in a simple model with two contracts. As the model illustrates, when a person enters into a contract with an (artificial) legal entity rather than with an individual, the attributes of the entity that are important, and that are the reason for making the contract nonassignable, often reside not in the characteristics of the entity's owner(s) but in the other parties with whom the entity has contracts. In these situations, it is the bundle of contracts of which the entity is the nexus, rather than the owner(s) of the entity, that makes the entity unique as a (legal) person.

## 3 Model

### 3.1 Setup

The basic model takes place over four dates, 0,  $\frac{1}{2}$ , 1, and 2. At date 0, wealthless entrepreneur/managers (E) endowed with ideas for projects find suppliers with inputs required for production. At date  $\frac{1}{2}$ , entrepreneurs make investment decisions. At date 1, the state of the world and interim cash flows are realized, and relationships and means of production can be reorganized. At date 2, a final cash flow is realized and distributed according to the relevant contracts in place.

Suppose inputs come in two types (A and B), and two quality levels (high and low). A high quality input has a marginal cost  $r_h$  per period, while a low quality input costs  $r_l < r_h$  per period. To generate cash flow at at dates 1 or 2, a project must be started at date 0, and run continuously up to that date under the control of an entrepreneur, and with one unit of each type of input. Since an E has no cash, she must find a source of financing to acquire

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<sup>6</sup>In our companion piece, we discuss these issues in greater depth, including a discussion of other possibilities that would replicate bundled assignability without entities, and the advantages the legal entity would have over these possibilities. See Ayotte and Hansmann (2010).

these inputs. We assume that financing of each input must come, at least in part, from the individual supplier that provides it. We will refer to such contracts as *bilateral*, because each party is exposed to risk of non-performance by the other.<sup>7</sup> This assumption is crucial to the model and can be justified in several ways.<sup>8</sup>

We suppose that the date 1 cash flow to all projects is, for simplicity, riskless, and equal to the one-period opportunity cost of the assets used in production (so if a project uses one high quality input and one low, the date 1 cash flow is  $r_h + r_l$ ).<sup>9</sup> The value of output at date 2 is random, however, and can take the values 0 or  $X$ . The probability of the high cash flow at date 2 depends on the quality of the inputs, and the value of investments made by the entrepreneur to add value to the project.

We will assume two types of entrepreneurs in the economy, *good* and *average*. At date  $\frac{1}{2}$ , a good E can make an unobservable investment that adds value to her project. If a good E chooses to invest, she incurs a private cost  $c$ , drawn from a distribution  $G(c)$  over the support  $(0, \infty)$ . E's individual  $c$  is observable at date 0 to all parties but not contractible. We assume that average entrepreneurs have no ability to make investments. Both entrepreneur types have an outside option we normalize to zero.

To keep notation relatively simple and limit the number of cases to consider, we will assume that assets A and B always have symmetric effects on output; only the input qualities, whether investment has occurred, and the interaction between them will affect output. With this in mind, our notation will be represented as follows. If the entrepreneur invests (does not invest), the project will have probability of success  $\pi_j = q_j$  ( $\pi_j = p_j$ ). The subscript  $j \in \{h, m, l\}$  will denote total input quality. If a project uses two high (low) quality inputs from dates 1 to 2, it will have subscript  $h$  ( $l$ ). If one input is high quality and the other is low, we will use subscript  $m$ . We will make the following natural parameter value assumptions:

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<sup>7</sup>In bankruptcy law terminology, such contracts are called *executory contracts*, but we avoid using this term because of its more general definition in contract law.

<sup>8</sup>Possible reasons include: a required effort by the supplier; that the supplier may abscond with the money and be judgment proof; and that sellers know more about their goods than financiers do. Netflix, for example, expanded its library of DVD content dramatically after changing its business model from buying DVDs outright to negotiating revenue sharing arrangements with studios based on the quantity of rentals. (Shih, Kaufman and Spinola 2009)

<sup>9</sup>Equivalently, we could assume that E has some personal wealth to cover the first period cost of the supplies.



$$\begin{aligned}\pi_h &> \pi_m > \pi_l, \pi = \{p, q\} \\ q_j &> p_j, j \in \{h, m, l\}\end{aligned}$$

The first set of assumptions say that higher input quality leads to a higher probability of success, for a given investment decision; the second says that investment always increases the probability of success, for a given input quality pair.

To simplify the analysis further, suppose that without investment, the NPV of the firm is zero, independent of the quality of input type (assumption A1):

$$A1 : p_h X - 2r_h = p_m X - r_h - r_l = p_l X - 2r_l = 0$$

This also implies equalities that we will use later:

$$(p_h - p_m)X = (p_m - p_l)X = r_h - r_l$$

In other words, the high quality input is more valuable in production, but is also more expensive. Thus, the average entrepreneur is indifferent to the two input types if she pays a fair price to acquire them.

### 3.1.1 Entrepreneur liquidity shocks

An additional important driving force in our model is that a good E faces the possibility of a non-contractible liquidity shock  $L \in \{0, 1\}$ , which we model in the standard way. E incurs the liquidity shock at date 1 with probability  $0 < \lambda \leq 1$ . If the liquidity shock is not realized ( $L = 0$ ), E's utility is  $U(C_1, C_2) = C_1 + C_2$ , where  $C_1$  and  $C_2$  is E's consumption at dates 1 and 2, respectively. If the liquidity shock is realized ( $L = 1$ ), the entrepreneur derives no value from consumption at date 2 ( $U(C_1, C_2) = C_1$ ).

We will restrict consideration to assignment of contracts (for cash) to another entrepreneur as the exclusive means by which good E's can obtain liquidity. This assumption simply helps us to highlight the key trade-offs in the model in a simple way.<sup>10</sup> Entrepreneurs will not have any cash at date 1, but they will be able to borrow from a competitive pool of

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<sup>10</sup>E might try to borrow against the cash flow from the project to satisfy her liquidity needs without assigning the contracts. In a richer model, however, this would have limitations. A creditor might not be willing to lend against the full value of the cash flows if a (non-contractible) action were required between dates 1 and 2 to realize the cash flows, and only a party to a contract could take that action. If E pledged the entire cash flow to a creditor, she would have no incentive to require that a faulty input be replaced by the supplier, for example.

financiers, who provide liquidity by lending money to entrepreneur/assignees who acquire contracts. These financiers can observe the quality of the assets and the productivity of investments, so they lend against date 2 cash flows at fair rates at date 1. We assume financier claims on the entrepreneurs who acquire contracts will be junior in priority to the claims of suppliers.<sup>11</sup> We assume there are many potential average entrepreneurs as assignees, so a good entrepreneur/assignor has all the bargaining power with respect to an assignee.

### 3.1.2 Contracts

When a supplier contracts with an entrepreneur, we allow them to write bilateral contracts in one of the four classes below:

- *Short-term contracts (ST)*. If a short-term contract is written, E contracts for the input for one period, and agrees to pay  $r_k$  at date 1 to the supplier who supplies an input of type  $k \in \{h, l\}$ . She must sign a new contract at date 1 for access to an input between dates 1 and 2.
- *Two-period, non-assignable contracts (NA)*. In exchange for the right to use the input until date 2, E promises to repay  $r_k$  at date 1, and some amount  $F_{na}$  when the contract expires. E is not free to assign the contract at date 1 without the permission of the supplier.
- *Two-period, individually assignable contracts (IA)*. The contract has the same structure as above, but after paying  $r_k$  at date 1, E may assign the contract to another party. She may do this in one of two ways: she may assign a contract individually to another entrepreneur/assignee who holds a right to the other input type, or she may assign the two contracts together to an entrepreneur/assignee who holds no other rights to inputs.<sup>12</sup>

Assignment implies that the assignee assumes the full rights and obligations remaining under the contract. Specifically, the assignee has the right to use the input from dates

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<sup>11</sup>By virtue of asset partitioning, the suppliers to a legal entity would be senior in priority to the personal creditors of its owners. Thus, if the suppliers contracted with an entity created by the original entrepreneur, these suppliers would have a claim to the project's cash flows that is senior in priority to the claims of a financier who loaned money to the assignee personally to purchase the stock of the entity.

<sup>12</sup>This rules out the possibility that entrepreneurs might have other assets and liabilities that would affect the value of suppliers' claims. For example, an assignee might own other firms whose assets and liabilities might affect the value of a supplier's claim. This would introduce issues of asset partitioning (the priority of claims between two different businesses) which we leave for future work.

1 to 2, and must pay the supplier  $F_{ia}$  at date 2.

- *Two-period, bundled-assignable contracts (BA)*. The contract is similar to the two-period, individually assignable contract, except that E can transfer the contracts without the permission of either supplier only if she transfers both contracts together to the same assignee.

This contracting environment is admittedly restrictive. In particular, one restriction on our contracting space is that we do not allow E to commit to being *residually liable* for the suppliers' debts if the assignee fails to perform. In other words, if an assignee's project fails at date 2 and the suppliers' claims are not satisfied, they can not expect to recover any money from E. Allowing for this possibility in our model might limit individual assignment for the purposes of shifting credit risk to suppliers. To provide an effective guarantee to her suppliers, however, E would need to set aside proceeds from assigning contracts at date 1 to bond claims that may arise at date 2. Even if this commitment were contractible, it would likely limit E's ability to satisfy her liquidity needs at date 1<sup>13</sup>. Hence, we restrict consideration to possibilities that allow E to spend the full proceeds of assigned contracts freely at date 1.

To further simplify the exposition, we will make the assumption that bargaining between E and her suppliers is not possible at date 1. This implies that if a good E needs access to a particular input between dates 1 and 2, and/or the right to transfer the input to a new owner at date 1, she must contract for those rights at date 0. This assumption implies that inefficient assignments may occur if they are individually rational for E. It also implies that E will not be able to assign contracts to a new owner at date 1 unless the date 0 contract permits it.

The assumption of these strong bargaining frictions is not necessary to generate our main results. If frictionless, Coasean bargaining were possible at date 1 to eliminate any inefficient assignment or to permit any efficient assignment, the entrepreneur's noncontractible investments will be subject to holdup problems, as in Grossman and Hart (1986) and Hart and Moore (1990). Assuming that bargaining is not possible at date 1 is similar to assuming that the suppliers are able to capture all of the surplus from bargaining at date 1. Thus, the

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<sup>13</sup>In an ideal world, E might be able to assure her suppliers and satisfy her liquidity needs at date 1 under individually assignable contracts. For example, she might promise to hold any cash proceeds from assignment in escrow until date 2 for the benefit of her suppliers, and give these suppliers a senior claim on the money. She could then issue junior claims on the pot of money at date 1 to satisfy her liquidity needs. While theoretically plausible, this outcome strikes us as unrealistic. If the date 1 lender had incomplete information about the probability and/or amount of these supplier claims, for example, this solution would break down.

results in our simpler framework can be seen as a more stark version of the underinvestment that would be caused by holdup problems when bargaining is possible.

## 4 Investments in complementarities

To convey the basic intuition about the value of bundling, we will restrict consideration to investments that have value only if the two inputs used at date 0 are kept together until date 2. We will refer to these types of investments as *investments in complementarities*. This can be interpreted as an investment that specializes inputs to each other. For example, the entrepreneur who opens a restaurant might expend resources to train a manager to work with the wait staff, or to decorate the interior space to fit the unique style of the chef. We will analyze two investment types in this general class. The first, which we call *alienable* investments, are specific to the two inputs, but not to E; thus, E may sell her stake in the firm without affecting the investment's value as long as the inputs remain together. The second, which we call *inalienable*, additionally requires that E remain in control of the two original inputs until date 2.

### 4.1 Alienable investments in complementarities

The following inequality tells us when investment by E is efficient, for a given total asset quality  $j$ :

$$(q_j - p_j)X - c \geq 0$$

Let  $\bar{c}_j^{fb}$  denote the maximum  $c$  for which the inequality holds. This is given by

$$\bar{c}_j^{fb} = (q_j - p_j)X$$

Under alienable investments, the first-best efficient allocation is as follows: all entrepreneurs with asset quality  $j$  would choose to invest if and only if  $c \leq \bar{c}_j^{fb}$ . If an investment is made, the two inputs remain bundled together until date 2 to realize the value of the investment. If a good E invested at date  $\frac{1}{2}$  and  $L = 1$  at date 1, she assigns the contracts as a bundle to an average E and consumes any payment she receives. Thus, the first-best social welfare when  $c \leq \bar{c}_j^{fb}$  is simply  $(q_j - p_j)X - c$  and 0 otherwise.

There are several possible inefficiencies that may arise in our framework. First, a good E might not invest at date  $\frac{1}{2}$  when investment is efficient. Second, a good E might split up

the inputs inefficiently by assigning contracts to two different entrepreneurs at date 1, thus destroying the value of a complementarity investment made at date  $\frac{1}{2}$ . Third, a good E may be prevented from selling her interest in the firm when  $L = 1$ . This would be inefficient because E's equity stake is worthless to her when  $L = 1$ , but it might have value to an entrepreneur with no immediate liquidity needs.

In our framework, suppliers and assignees operate in competitive markets, so they will always break even in equilibrium. This implies that a good entrepreneur will fully internalize the costs of any inefficient actions in equilibrium, through a lower value of her equity stake in the firm. Thus, the contracts that are optimal for E will also be social welfare-maximizing.

#### 4.1.1 Average entrepreneurs

We begin our analysis by considering the problem of average entrepreneurs, who have no ability to make investments. These entrepreneurs will become the pool of potential assignees of contracts from good entrepreneurs.

It is straightforward to see that average entrepreneurs can do no better than writing short-term contracts with suppliers. Since there is no investment decision, and assets are not specific, there is no risk that access to valuable inputs will be lost at date 1. Moreover, since financial markets are frictionless, there is no benefit to long-term financing at date 0. The entrepreneur will always have access to inputs at a competitive price in the spot market at date 1. We summarize this intuition in the following lemma:

**Lemma 1** *For average entrepreneurs, a (non-unique) set of optimal contracts with suppliers is a series of short-term contracts: a supplier of an asset of quality  $k \in \{h, l\}$  from dates 0 to 1 is promised a repayment of  $r_k$  at date 1. At date 1, an average entrepreneur will acquire 2 new inputs to continue the project from dates 1 to 2.*

The proof is straightforward and thus omitted. Given that long-term contracts can do no better than short-term contracts, we assume that average E's will write short-term contracts at date 0. As a result, they will stand ready to be assignees of contracts, whether individually or bundled, from good entrepreneurs who wish to assign at date 1.<sup>14</sup>

#### 4.1.2 Good entrepreneurs: the contracting problem

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<sup>14</sup>In equilibrium, of course, competition among assignees to purchase contracts drives the profits from this strategy to zero.

We now turn to the analysis for good entrepreneurs, who have the possibility of investing in the bundle. Given E's limited wealth and need for outside financing, underinvestment will occur in general: the highest  $c$  type such that investment can be implemented, for any contracting arrangement, will always be lower than the first-best. This occurs because E will not internalize the benefit of her investment on the supplier/creditors, who have a claim on the final cash flow and benefit from a higher probability of success. Given this underinvestment problem, a contracting class that elicits investment for more  $c$ -types, all else equal, will be more efficient. Thus, for each contracting class, we will begin by solving for the highest  $c$ -type for which investment can be implemented in equilibrium.<sup>15</sup>

We will then conduct a more complete comparison of the four contracting classes. We now analyze the four contracting options for the entrepreneur in turn, beginning with the two contract forms that do not allow free transferability of the inputs by E at date 1.

**Short-term contracts (ST)** In our simple environment, it is clear that short-term contracts will never elicit any costly investment by a good entrepreneur. Under short-term contracts, E must recontract for access to the inputs at date 1. Given our assumption that bargaining over new terms is not possible at date 1, E can never realize the value of any specific investment in the inputs. This means that investment requires long-term contracting. Formally, the highest  $c$ -type that invests is  $\bar{c}_{st} = 0$ .

**Non-assignable contracts (NA)** A second alternative is to guarantee that the entrepreneur has control rights over the input for two periods, by signing a long-term contract. This would give the entrepreneur some payoff from her investment, but only in states of the world in which she does not suffer the liquidity shock.

Formally, we will work backward, starting from E's date 1 problem. Given our assumption that bargaining to permit assignment at date 1 is not possible, E's payoff will be 0 when  $L = 1$ , since her contracts do not allow transfer. When  $L = 0$ , she will realize the full value of her equity in the firm by holding the contracts until date 2. Her equity stake has a date 1 expected value equal to:

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<sup>15</sup>Our game can give rise to multiple equilibria, depending on the suppliers' date 0 beliefs about E's actions at dates  $\frac{1}{2}$  and 1. For example, suppliers might believe that E will not invest at date  $\frac{1}{2}$ , and will set higher contract prices given this belief. Given the higher contract price, E may find it optimal to shirk. We will focus on equilibria where suppliers have optimistic beliefs: they will choose the lowest  $F$  such that their participation constraints will be satisfied, given that E will take utility-maximizing actions at dates  $\frac{1}{2}$  and 1.

$$\pi_j(X - 2F_{na}),$$

where  $\pi_j = q_j$  if E invested at date  $\frac{1}{2}$ , and  $\pi_j = p_j$  if she did not.

Looking backward to date  $\frac{1}{2}$ , E will choose to invest if and only if her incentive compatibility constraint is satisfied:

$$(1 - \lambda)(q_j - p_j)(X - 2F_{na}) \geq c \quad (1)$$

Note that the left hand side of the constraint, the increase in E's expected payoff from investing, is reduced by the factor  $(1 - \lambda)$ : E realizes value from the investment only if  $L = 0$ .

We can now return to date 0, to solve for the promised date 2 payment  $F_{na}$  that satisfies the suppliers' participation constraint, given their beliefs about E's actions at date  $\frac{1}{2}$  and 1. If a supplier anticipates investment, a supplier of an input of quality  $k$  to a firm with total input quality  $j$  will face the following participation constraint:

$$q_j F_{na} \geq r_k$$

In equilibrium, the suppliers' participation constraints will be binding, so  $F_{na} = \frac{r_k}{q_j}$ . The suppliers' beliefs will be consistent with equilibrium play as long as E prefers to invest, given  $F_{na}$ : that is, (1) must be satisfied for  $F_{na} = \frac{r_k}{q_j}$ . Thus, the highest  $c$  type for which investment can be implemented under non-assignable contracts,  $\bar{c}_{na}$ , is the investment cost for which the suppliers' participation constraints and E's incentive compatibility constraint are binding:

$$\bar{c}_{na} = (1 - \lambda)(q_j - p_j)(X - 2\frac{r_k}{q_j}) \quad (2)$$

To recap, under non-assignable contracts, E is not able to transfer her interest in the firm by assigning the contracts. As a result, she receives no return from her investment when she has liquidity needs ( $L = 1$ ). This reduces her incentive to make the investment at date  $\frac{1}{2}$ .

**Individually assignable contracts (IA)** Now suppose that, as a means of eliminating the bargaining and underinvestment problem caused by E's liquidity needs, the entrepreneur contracts for the ability to freely assign the two bilateral contracts at date 1. If she contracts for the ability to assign her contracts individually, she will have the option at date 1 to assign the contracts individually (to two different assignees) or to assign them as a bundle (to the same assignee). Since the liquidity shock is assumed to be non-contractible, E might choose to assign contracts rather than keep them if it increases her payoff, even if she has no liquidity need at date 1. (If E is indifferent between assigning and keeping the contracts at date 1, we will assume she assigns.)

**E's date 1 problem** E's incentives to assign contracts at date 1 will depend on the total input quality. First, consider an entrepreneur who starts with two low quality inputs, and agrees to repay each supplier some amount  $F_{ia}$  at date 2. If E chooses to assign her contracts as a bundle at date 1, she will receive a sale price equal to the expected value of the project:

$$\pi_l(X - 2F_{ia}),$$

where  $\pi_l = q_l$  if she invested and  $\pi_l = p_l$  if she did not. If E attempts to assign a single contract individually, an assignee would be willing to pay E some transfer price  $t_l$ . Given that E has all the bargaining power with respect to assignees, this payment will be equal to the increase in the assignee's expected payoff from using the assigned contract instead of acquiring a contract in the marketplace at a fair rate at date 1. If the assignee takes a contract with inputs quality  $l$  and has total asset quality  $j$  between dates 1 and 2, this value is given by

$$t_l = p_j\left(\frac{r_l}{p_j} - F_{ia}\right) = r_l - p_j F_{ia}$$

Intuitively, this is the expected value to the assignee of the subsidized financing that comes from using the assigned contract instead of a market-rate contract. It is clear from the expression that an assignee is willing to pay more for an assigned contract with a fixed  $F_a$  ( $r_l - p_l F_a > r_l - p_m F_a$ ) if her other input is low quality (thus making  $p_j = p_l$ ) than if it is high quality ( $p_j = p_m$ ), because this makes the financing subsidy larger. Hence, E will always choose an assignee whose other input is low quality. Because investments are in complementarities, a productive investment has no effect on the assignee's project when she takes on one but not both of the inputs, so it has no effect on  $t_l$ . Thus, E's expected payoff from assigning both of her contracts individually is  $2t_l$ .

This leads to the following lemma:

**Lemma 2** *If a good E starts a firm with low quality inputs, then there is no risk of opportunistic individual assignment at date 1: when E does not invest, she is indifferent between bundled and individual assignment. When she invests, she strictly prefers bundled assignment to individual assignment at date 1.*

**Proof.** See appendix. ■

The lemma demonstrates that differences in input quality are necessary to generate an opportunistic assignment problem. If the firm's initial creditworthiness is no better than its



potential assignees, then E will never prefer to assign a contract inefficiently. When inputs are low quality, then, IA contracts will be equivalent to BA contracts.

With this in hand, we now consider the opposite extreme, when the entrepreneur starts with two high-quality inputs.<sup>16</sup> If the entrepreneur chooses to assign her contracts as a bundle at date 1, an average entrepreneur is willing to pay the project's entire expected value:

$$\pi_h(X - 2F_{ia})$$

Next, suppose the entrepreneur chooses to assign her contracts individually. Each entrepreneur/ assignee is willing to pay up to the increase in his expected payoff,  $t_h$ :

$$t_h = p_m \left( \frac{r_h}{p_m} - F_{ia} \right) = r_h - p_m F_{ia}$$

So the entrepreneur's total payoff from individual assignment is  $2t_h = 2(r_h - p_m F_{ia})$ . The entrepreneur prefers individual assignment if and only if the difference between her payoff from individual and bundled assignment positive. Using A1 and rearranging, E will prefer individual assignment to bundled assignment if and only if

$$(p_h - p_m) \frac{2F_{ia}}{X} - (\pi_h - p_h) \frac{X - 2F_{ia}}{X} > 0 \quad (3)$$

for any  $F_{ia} \leq \frac{r_h}{p_l}$ . The expressions  $\frac{X-2F_{ia}}{X}$  and  $\frac{2F_{ia}}{X}$  are the equity-to-assets and debt-to-assets ratios, respectively. They reflect the initial capital structure of the project, and thus the percentage share of the final cash flow allocated to E and the suppliers, respectively. Note that the first term in the expression is always strictly positive, and reflects the expected value redistributed from the suppliers by individual assignment at date 1, normalized by the maximum project payoff. Both suppliers' claims lose value from assignment: instead of being paired with another high quality input, each supplier after assignment will be paired with another input of lower quality and thus exposed to greater credit risk.

The second term, also normalized by  $X$ , reflects the entrepreneur's share of the added value from the investment when the contracts are kept together. Note that if E did not invest, then  $\pi_h = p_h$ , so the second term is zero, and E will always prefer to assign. The following lemma notes some of the comparative statics that drive the entrepreneur's incentive to assign contracts individually rather than as a bundle:

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<sup>16</sup>We omit formal analysis of the case where E uses one high quality and one low quality input, but the analysis of this case is very similar to the case of 2 high quality inputs; as long as one of the inputs is high quality, the opportunistic assignment problem will occur with respect to the low quality input.

**Lemma 3** *Suppose the entrepreneur starts a project with two high-quality inputs. If E does not invest, then she always prefers individual assignment to bundled assignment at date 1. If E invests, she may or may not prefer individual assignment to bundled assignment at date 1. Conditional on investment, E is more likely to prefer individual assignment when:*

- a) *The value of the investment is lower ( $q_h - p_h$  is lower);*
- b) *The gains from opportunistic assignment are higher ( $p_h - p_m$  is higher)*
- c) *E's equity stake in the project is lower ( $\frac{X-2F_{ia}}{X}$  is lower, or equivalently,  $\frac{2F_{ia}}{X}$  is higher)*

**Proof.** Evident by inspection of (3). ■

Parts (a) of the lemma is intuitive: given that the value lost by individual assignment is the value of the complementarity investment, individual assignment is more likely to be optimal for E when the investment adds less value. Part (b) is also intuitive, since E balances the lost investment gains with the payoff from opportunistic assignment. The larger the increase in credit risk shifted to the suppliers, the larger the subsidy the assignee receives from the contract. Because E has the bargaining power with the assignees, E will capture this subsidy. Thus, opportunistic assignment ultimately transfers wealth from the suppliers to E.

Part (c) of the lemma suggests that the identity of the firm's claimholders is an important determinant of E's incentive to assign. Though the total benefit of the complementarity investment is  $(q_h - p_h)X$ , E only receives a fraction  $\frac{X-2F_{ia}}{X}$  of this value, through her equity stake in the project. The rest of the benefit of the investment accrues to the suppliers. As a result, higher leverage increases E's incentive for opportunistic assignment. Conversely, the larger the project leverage ( $\frac{2F_{ia}}{X}$ ), the larger the transfer payments  $2t_h$  would be as a fraction of the project value. This effect also biases E in favor of individual assignment.

**E's date  $\frac{1}{2}$  problem** Having solved for E's date 1 assignment decision, we can now look backward to her investment decision at date  $\frac{1}{2}$ . When (3) holds following investment, E prefers to assign individually at date 1. Knowing that the investment has no value when contracts are assigned individually, it is clear that E will never invest at date  $\frac{1}{2}$ .

If (3) does not hold after investment, E will keep the bundle together at date 1. Thus, E may have the incentive to invest at date  $\frac{1}{2}$  if  $c$  is sufficiently low. Her incentive compatibility constraint for investment is

$$q_h(X - 2F_{ia}) - 2t_h \geq c \tag{4}$$

Substituting for  $t_h$ , this can be rewritten as

$$(q_h - p_h)(X - 2F_{ia}) - 2(p_h - p_m)F_{ia} \geq c \quad (5)$$

**The date 0 problem** We can now return to date 0, to solve for  $F_{ia}$ . If the suppliers anticipate that E will invest at date  $\frac{1}{2}$  and assign as a bundle at date 1, the suppliers' participation constraints will be satisfied by setting  $F_{ia} = \frac{r_h}{q_h}$ . This will constitute equilibrium play as long as (12) is satisfied when  $F_{ia} = \frac{r_h}{q_h}$ .<sup>17</sup> Thus, when E uses 2 high quality inputs, the highest  $c$ -type for which investment can be implemented under IA contracts,  $\bar{c}_{ia}$ , is given by:

$$\bar{c}_{ia} = (q_h - p_h)(X - 2\frac{r_h}{q_h}) - 2(p_h - p_m)\frac{r_h}{q_h} \quad (6)$$

With this efficiency benchmark in hand, we can now turn to bundled assignable contracts and compare cutoff types, to see which class of contracts give the strongest incentives for investment by E.

**Bundled-assignable contracts (BA)** When contracts are bundled assignable, the entrepreneur can freely transfer them without the permission of her suppliers. Thus, the underinvestment problem caused by E's liquidity shock is eliminated, and the entrepreneur can satisfy her liquidity needs by assigning the contracts as a bundle. At the same time, the bundling can limit some of the opportunism problems inherent in assignable contracts and preserve E's incentive to invest in complementarities.

Under BA contracts, E's date 1 problem is straightforward. If she chooses to assign, she must assign the contracts as a bundle. When  $L = 1$ , she strictly prefers to sell, and when  $L = 0$  she is indifferent; hence, we can assume, for expositional convenience, that she will always assign the bundle. She will receive a price equal to the project's expected value to an entrepreneur/assignee:

$$\pi_j(X - 2F_{ba}),$$

where  $\pi_l = q_l$  if she invested and  $\pi_l = p_l$  if she did not. Next, we return to the date  $\frac{1}{2}$  investment decision. The entrepreneur's incentive compatibility constraint for investment under BA is the following:

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<sup>17</sup>Equilibrium also requires that E prefers not to assign individually at date 1, but if E prefers to invest, then this condition will hold *a fortiori*.

$$(q_j - p_j)(X - 2F_{ba}) \geq c. \quad (7)$$

Finally, we can return to the date 0 contracting problem between E and the suppliers. As above, the suppliers' participation constraints will be satisfied at  $F_{ba} = \frac{r_k}{q_j}$ , as long as E's incentive compatibility constraint (7) is satisfied when  $F_{ba} = \frac{r_k}{q_j}$ . Thus, the highest  $\bar{c}_{ba}$  for which investment will occur is

$$\bar{c}_{ba} = (q_j - p_j)\left(X - 2\frac{r_k}{q_j}\right). \quad (8)$$

### 4.1.3 Comparing the contracting classes

Having solved for the investment cutoff values for the 4 contracting classes, we can analyze the efficiency of these forms more fully in the following proposition:

**Proposition 4** *When investments are alienable investments in complementarities between the two inputs, BA contracts weakly dominate all other contracting classes:*

- a) *For any c-type and input qualities, E's expected utility under the optimal set of BA contracts is at least as high as her expected utility under any ST, NA, or IA contracts.*
- b) *If E uses at least one high quality input, there exists a positive measure of c-types such that  $\max\{\bar{c}_{st}, \bar{c}_{na}, \bar{c}_{ia}\} < c \leq \bar{c}_{ba}$ . For these parameter values, E will invest under BA but will not invest under any other contracting class, and her expected utility is strictly higher under BA than under any ST, NA or IA contracts.*
- c) *In any equilibrium such that E invests under BA, her expected utility is equal to social welfare under the first-best allocation.*

**Proof.** See the appendix. ■

The proposition says that BA is the best of all worlds when investments are alienable investments in complementarities. BA gives maximal incentives to invest at date  $\frac{1}{2}$  and involves no inefficiencies at date 1. Intuitively, BA is superior to NA because E is able to realize some of the value of her investment in states where she is liquidity-constrained. The ability to assign the bundle of contracts is more efficient at date 1, because it allows a liquidity-constrained entrepreneur to transfer ownership of the firm to an entrepreneur who values the project's future cash flows more than she does. BA also gives E better incentives to invest at date  $\frac{1}{2}$ , because she knows she can realize value from her investment even in states of the world when she needs liquidity.

If E uses low quality inputs, BA and IA are equivalent, because E never has incentive to assign individually under IA. If E uses a high quality input, however, BA is superior to IA, due to the opportunistic assignment problem under IA. BA forces the entrepreneur to commit to a particular group of inputs in constructing a firm, and ties her payoff to the bundled value of those inputs. Individual assignability, when initial input quality is high, gives the entrepreneur the option to cash out profitably by splitting up the inputs and diverting value from her suppliers. If the investment has low value relative to the gains from opportunistic assignment, E might prefer to break up a bundle that is complementary in value at date 1. Knowing this, E will never invest to increase the value of the bundle at date  $\frac{1}{2}$ .

Even if E has the incentive to keep a bundle together after investing, the ability to assign individually can still reduce E's incentive to invest, by increasing her payoff in states of the world where she did not invest. If E shirks on investing and the assets have low complementarity as a result, individual assignability allows E to liquidate the firm for a more attractive price through opportunistic assignment. The reduced penalty for E in shirking reduces her incentive to invest in complementarities.

## 4.2 Inalienable investments in complementarities

To this point, we have assumed that investment by E was specific to the original inputs, but not specific to E. In this environment, bundled assignability is the best of all worlds, allowing E to be fully liquid by giving her ability to assign the contracts, but protecting suppliers from opportunism by promising to keep the inputs together. In this section, we consider an investment that creates some tension between E's need for liquidity and the suppliers' demand for protection against opportunism. We will assume in this section that investment is specific, not only to the suppliers, but also to E. For example, E might invest in her own ability to manage the relationships with her suppliers. To realize the value of such an investment, she must maintain an equity stake in the firm and have control over the original inputs until date 2. Changing the nature of the investment will have important effects on E's choice of contracts.

**ST, NA, and IA contracts** Note that the solution to E's problem under ST contracts and NA contracts will not change when the investment becomes inalienable; neither of these contracting classes gives E the ability to transfer contracts to a new owner in any event, so alienability by E has no effect on the optimal contracting problem. While the solution to E's optimal contracting problem will change under IA, free assignability under inalienability

creates the same opportunistic assignment problems as IA under alienability. As a result, IA will be dominated by BA for the same reasons as above. Thus, we omit analysis of individual assignability in the main text for brevity; the interested reader may consult the proof of the proposition that follows in the appendix. We now consider BA contracts more formally.

**BA contracts** We start by considering E's date 1 problem. First, note that if  $L = 1$ , she will always transfer her interest in the firm, whether or not she has invested at date  $\frac{1}{2}$ ; though the investment's value is destroyed, this is the only way for E to realize any value from her equity stake in the firm. A buyer will pay a price equal to the project's expected value, which is always

$$p_j(X - 2F_{ba})$$

whether or not investment occurred at date  $\frac{1}{2}$ .

If  $L = 0$ , she will be indifferent between assigning the bundle and keeping the bundle if she did not invest. If she did invest, she strictly prefers to keep the contracts, since they are worth more under her control than under a new entrepreneur. Her expected payoff if she keeps the 2 contracts is

$$\pi_j(X - 2F_{ba})$$

where  $\pi_j = q_j$  if she invested and  $\pi_j = p_j$  if she did not.

This implies that E's incentive compatibility constraint at date  $\frac{1}{2}$  is

$$(1 - \lambda)(q_j - p_j)(X - 2F_{na}) \geq c$$

The IC constraint is reduced by the factor  $(1 - \lambda)$ , because E realizes the benefit of her investment only when  $L = 0$ .

Returning back to date 0, the suppliers' participation constraints, given a belief that E will invest at date  $\frac{1}{2}$ , is given by

$$(\lambda p_j + (1 - \lambda)q_j)F'_{ba} = r_k$$

As before, the suppliers' participation constraints will be satisfied at  $F'_{ba} = \frac{r_k}{\lambda p_j + (1 - \lambda)q_j}$ , as long as E's incentive compatibility constraint is satisfied when  $F'_{ba} = \frac{r_k}{\lambda p_j + (1 - \lambda)q_j}$ . Thus, the highest  $c$  for which investment will occur is

$$\bar{c}'_{ba} = (1 - \lambda)(q_j - p_j)\left(X - 2\frac{r_k}{\lambda p_j + (1 - \lambda)q_j}\right) \quad (9)$$

It is instructive to compare this expression to the investment cutoff for non-assignable contracts, which we restate here for convenience:

$$\bar{c}_{na} = (1 - \lambda)(q_j - p_j)\left(X - 2\frac{r_k}{q_j}\right)$$

We compare bundled assignability and non-assignability in the following lemma:

**Lemma 5** *Suppose investments by E are inalienable investments in complementarities. Then  $\bar{c}_{na} > \bar{c}'_{ba}$ : more c-types invest under non-assignable contracts than under bundled assignable contracts.*

**Proof.** Evident by inspection of  $\bar{c}_{na}$  and  $\bar{c}'_{ba}$ , since  $q_j > p_j$ . ■

Note that both investment cutoff expressions include the multiplicative term  $(1 - \lambda)$ , reflecting that investment produces value for E only when  $L = 0$ . The only difference between the two expressions is the suppliers' date 2 required repayment, which is higher under BA. Under BA, if E chooses to invest at date  $\frac{1}{2}$ , and she has a liquidity shock date 1, she will assign the contracts as a bundle to cash out her interest in the firm. Because the investment is specific to E, bundled assignment destroys the value of the investment and exposes suppliers to greater risk of default; they compensate for this with a higher required repayment. Though E can not satisfy her liquidity needs at date 1 under NA, this contracting mode provides suppliers with more protection by ensuring that the value of the investment is preserved. The lower cost of supplies, in turn, gives E greater incentive to invest.

With this intuition in hand, we can compare the 4 contracting classes under inalienable investment in the following proposition:

**Proposition 6** *When investments are inalienable investments in complementarities, NA contracts weakly dominate all other contracting classes:*

a) *For any c-type and input qualities, E's expected utility under the optimal NA contracts is at least as high as her expected utility under any ST, IA or BA contracts.*

b) *For any input qualities, there exist a positive measure of c-types such that  $\max\{\bar{c}_{st}, \bar{c}_{ia}, \bar{c}_{ba}\} < c \leq \bar{c}_{na}$ . For these parameter values, E will invest under NA but will not invest under any other contracting class, and E's expected utility is strictly higher under NA than her expected utility under any ST, IA or BA contracts.*

When investments are in E's human capital and thus inalienable, non-assignable contracts are preferred to all other contracting classes. Non-assignability is valuable because it protects suppliers from transfers of control that would increase the firm's default risk. When

supplier/creditors are more protected, they are willing to invest in the firm at lower cost (lower  $F$ ). The lower cost of supplies, in turn, create stronger incentives for E to invest, because her payoff from the investment is larger in the event that  $L = 0$ .

For  $c$ -types that are low enough to elicit investment under BA, NA, or IA the entrepreneur is indifferent between the contracting classes. Her expected utility is always

$$(1 - \lambda)(q_j - p_j)X - c$$

This expression confirms that E is not able to realize any value from her investment in equilibrium, directly or indirectly, when she suffers a liquidity shock. If she attempts to assign the firm's contracts to a new owner, the value of the investment is lost. If she remains in control of the firm, she must wait to receive the proceeds of the investment; by assumption, future cash flows are worthless to her when she needs liquidity.

The analysis of inalienable investments suggests that when owners' investments are more important to the ongoing value of the firm, we would expect to see more contracts that restrict assignability, even as a bundle. For example, contracting parties might prefer to include *change of control* clauses in their contracts that prohibit owners from selling their equity in the firm to a new owner, so as to tie owners to the bundle along with the other inputs.

## 5 Law and Assignability

In this model, the reason why counterparties wish to restrict the firm's ability to assign their contracts essentially lies in credit risk. And it is the collection of other contracts to which the firm is a party that keeps its credit risk low. This is only one of various reasons why a firm's counterparties may be concerned about the other contracts in the bundle held by the firm.<sup>18</sup> It is perhaps the most important reason, however, and it seems to help explain the law's presumptions as to whether various corporate transactions constitute assignments of the firm's contracts.

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<sup>18</sup>Another reason might be that counterparties to a firm's contracts derive personal – and perhaps nonpecuniary – value from their association with each other. A professor of economics, for example, might insist on only a relatively modest salary for a position on the Harvard faculty, since the major benefit of that position will be the personal rewards of associating (and being associated) with other members of that faculty. If Harvard were free to assign her contract to an aspiring state university with a mediocre faculty, Harvard might find that a profitable action – though the loss to the professor might exceed the price that Harvard could extract from the state university, rendering the assignment inefficient.



## 5.1 Assignments in Bankruptcy

Under bankruptcy law, contracts that are otherwise nonassignable by their explicit terms are held to be individually assignable in bankruptcy. Our model helps illustrate why the law might favor assignability in bankruptcy states.<sup>19</sup> In the model, the owner of the firm has the strongest incentive for opportunistic assignment when her contractual rights are to higher quality assets, and when her capital structure is less leveraged. In other words, the benefits of bundling contracts are largest for firms that are both economically and financially healthy.<sup>20</sup>

While we have not modeled the case explicitly for the sake of brevity, it is intuitive that E might prefer individual assignability to all other contracting classes if her investments are sufficiently likely to have value *outside* its current bundle. For example, a growing firm might occupy a leased corporate headquarters, and invest in the internal layout of the floors of the building to facilitate collaboration among its employees. Once the firm grows and requires a larger space, it might want to capture the investment in the office layout by assigning the lease to a smaller firm. Intuitively, IA is the only contracting class in our model that would allow E to capture value from such an investment, since none of the other classes permit assignment away from the date 0 bundle.

A rule making contracts assignable in bankruptcy, even though those contracts would otherwise be nonassignable, is in effect a means of putting a clause in the contract that says it is assignable if and only if the outcome for the firm is poor. Conditional assignability of this sort involves a smaller threat of opportunistic assignment, and hence is less of a threat to the interests of the firm's counterparty. Moreover, the decision to assign contracts in bankruptcy is overseen by a judge charged with permitting assignments only if there is "adequate assurance of future performance," which limits opportunistic assignment.<sup>21</sup> Conditional assignability allows owners to capture the value of investments that are worth more when a bundle is split up. Investments of this kind are more likely to arise in economically and financially distressed firms, since the going-concern value of a firm in bankruptcy is often negative.

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<sup>19</sup>We should be careful to note here that our model does not explain why the bankruptcy code makes this contingent assignability in bankruptcy states a mandatory rule, as opposed to a default rule that parties could contract around.

<sup>20</sup>In our informal companion piece, we consider the possibility that the investment in an input (or bundle of inputs) could be more valuable if the input(s) are separated from their current bundle. This provides a different justification for permitting assignment in bankruptcy, since it is more likely in bankruptcy states that going-concern value is negative, and contracts are more valuable when split up. Over-riding an anti-assignment clause in these states can prevent holdup, and hence enhance investment incentives, when an investment is worth more outside the bundle.

<sup>21</sup>Bankruptcy Code 365(f)(2)(B)

The exception helps prove the rule. Some types of contracts that are nonassignable outside of bankruptcy are also generally held to be nonassignable in bankruptcy. Licenses for intellectual property are an example. This is understandable. The incentive for opportunistic assignment of such licenses is different from those considered in this model. The threat is that the original licensee will assign the license to another firm that will make broader or different use of it than the original licensee would have, thus effectively taking from the licensor more than was intended to be granted by the contract with the original licensee. Since this threat is no less severe in bankruptcy, there is less reason to alter the rules of assignability in bankruptcy with respect to these types of contracts.

## 5.2 Asset Sales

A merger of two firms effectively involves giving a common nexus to two bundles of contracts that were formerly held separately. Hence it doesn't provide an opportunity for the kind of opportunistic separation of complementary contracts explored by our model. This illustrates why a merger or consolidation is generally presumed not to offend non-assignability provisions in a firm's contracts.

But the courts' approach to a sale of a firm's assets is more flexible. While an attempt to transfer contracts to a new entity in a sale of corporate assets is generally considered an assignment of the contracts involved, the courts will sometimes hold otherwise when the sale involves substantially all of the firm's assets, on the grounds that the effect on the firm's contractual counterparties is essentially equivalent to that of a merger. The law might have chosen a different rule here, holding that a sale of assets always involves an assignment of the contracts involved, on the grounds that the firm can choose the formal procedure for merger if it wants to avoid triggering anti-assignment clauses, and that insistence on treating these transactional forms differently makes it easier for contracting parties to specify when assignment is or is not acceptable (for example, by explicitly providing that certain types of mergers are or are not to be considered permissible assignments of the contract). This would parallel the approach taken to sales of stock. The reason for taking a less formal approach to asset sales, apparently, is that tax considerations (and perhaps corporate governance considerations, such as shareholder voting rights, that don't directly affect the interests of contractual counterparties) may make it very expensive for the firm to structure a transaction as a merger rather than as a sale of assets. Consequently, it's understandable that courts are sometimes prepared to hold that a corporate asset sale can proceed without triggering the holdup rights and bargaining failures that might occur as a result of anti-assignment clauses.

## 6 Empirical Evidence

Our theory explains why parties might prefer to structure their contracts to prevent individual assignability yet permit bundled assignability. We have also argued that legal entities are useful in creating and identifying the bundle of assets and contracts that can be freely transferred. In this section, we present preliminary evidence from the assignment clauses of commercial contracts. For the purposes of this draft, we restrict consideration to two basic questions. First, do contracting parties actually contract for bundled assignability in practice? Second, do they use entities as a means of achieving bundled assignability?

We examine 287 supply and lease contracts from public firms, gathered from the SEC Edgar database, between 2007 and 2009. We restrict consideration to supply contracts and leases. These contract types are likely to fit the underlying assumptions of our theory, since they are typically bilateral executory contracts, with inputs that have potential to become specific and complementary to the firm. For this draft, we also restrict attention to the “debtor” side of the contract (the tenant or buyer).

Table 1 presents the results on individual anti-assignment clauses. The data show that contracting parties routinely contract out of individual assignability: the “debtor” side of the contract is prevented from individually assigning its rights and obligations under the contract in 95.5% of the contracts in our sample. The percentage of leases that are non-assignable by the tenant is higher than the percentage of supply contracts that are non-assignable by the buyer (99.2% vs. 91.7%, respectively). This may be true because leases, having a larger pool of potential users, are more subject to an opportunistic transfer problem than a supply contract.

Table 2 reports summary statistics on bundled assignability. We define a contract as “bundled assignable” if the contract restricts individual assignability, yet permits free assignment (under some conditions) if the contract is assigned along with some other asset(s) or contract(s) in the firm. We create two definitions of bundled assignability. In the first, the contract is coded as explicitly bundled assignable if the contract is individually non-assignable and explicitly permits assignment with a designated bundle. In the second, the contract is coded as implicitly bundled assignable if the contract is individually non-assignable, but does not explicitly restrict assignability in the event of a merger, acquisition, or change of control. As we have discussed, this generally creates bundled assignability under the default rules of law, since mergers and acquisitions are not held to be violations of individual anti-assignment clauses.

We find that bundled assignability is also very common in our sample: 85.7% of the

contracts in our sample create some form of bundled assignability for the debtor party to the contract. About 63% of contracts create this bundled assignability explicitly, by identifying bundles with which the contract can be assigned.

In Table 3, we report summary statistics for those contracts that explicitly permit bundled assignability, to see whether contracting parties use entities, or some other means, to define the bundles with which the contract may be assigned. If the contract permits assignment in the event of a merger, acquisition, or sale of “all or substantially all assets” of the contracting entity, then an entity is being used to define the bundle. Alternatively, if the contract allows for assignment along with a specifically identified asset or contract, or if the contract provides a general description of a bundle (“the business/segment to which this agreement relates”) we consider the bundle definition to be non-entity-based. Some contracts use multiple definitions of the bundle with which the contract can travel; we record all definitions used by the parties in a given contract.

Of the contracts that create bundled assignability explicitly, 93.9% of these include an entity-based definition of a bundle; 63.5% use entity-based bundles exclusively. Nevertheless, we do find evidence that bundles are sometimes defined in a way that would not require entities: 36.5% of the bundled-assignable contracts in our sample include at least one definition of a bundle that does not use an entity, and 6.1% of these contracts use only non-entity based definitions of bundles .

These simple summary statistics provide preliminary evidence that contracting parties are aware of the economic forces underlying our theory. Contracting parties contract for bundled assignability with great regularity in practice. When parties create bundled assignability, they usually, though not exclusively, use entities to define the bundle with which the contract can travel. This suggests that legal entities are a valuable, though not unique, device used to balance the owner’s need for liquidity against counterparties’ demand for protection against opportunism.

## 7 Conclusion

A legal entity permits an entrepreneur to create a firm as a bundle of contracts that can be transferred to someone else, but only if they are transferred together. This bundled assignability allows for a balancing of several potentially conflicting interests. First, the entrepreneur who assembles the contracts wants liquidity – that is, the ability to transfer the contracts and cash out her interest in them. Second, the counterparties to the firm’s contracts – the firm’s

employees, suppliers, creditors, and customers – want protection from opportunistic transfers that will reduce the value of the performance they’ve been promised. And third, the entrepreneur wants long-term commitments from the firm’s counterparties to protect the value of her investments in integrating them to the firm. By providing that transfers of equity interests in the entity will generally not be considered assignments of the firm’s contracts, organizational law provides a flexible tool that permits easy modulation of the trade-off among these interests. An appreciation of this role of legal entities not only refines our theories of the firm, but provides guidance in shaping legal doctrine concerning the effects of various types of control transactions on a firm’s contractual rights and obligations.

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## Appendix A: Tables

**Table 1: Explicit individual non-assignability, debtor party.** Table 1 reports the percentage of contracts that explicitly impose restrictions on assignment of the contract on an individual basis. The sample includes 287 lease and supply agreements from the SEC Edgar database between 2007 and 2009, filed as a "Material Contract" (Exhibit 10). The debtor party is the buyer in a supply contract, and the tenant in a lease contract.

Contract type	Number of contracts	Number of individually non-assignable contracts	Percentage of individually non-assignable contracts
Supply	145	133	91.7%
Lease	142	141	99.3%
Total	287	274	95.5%

**Table 2: Bundled assignability, debtor party.** Table 2 reports the percentage of contracts that allow for bundled assignability by the debtor party to the contract. A contract is coded as explicitly bundled-assignable if the contract is both (a) explicitly individually non-assignable (using the same criteria as in Table 1), and (b) explicitly permits assignment of the contract (possibly under specified conditions) if assigned to a party acquiring all or some subset of the assets or contracts of the debtor party. A contract is coded as implicitly bundled-assignable if it is both (a) individually non-assignable and (b) does not explicitly restrict assignment of the contract in the event of a merger, acquisition, or change in control of the debtor party. The sample is described in Table 1.

Contract type	Number of contracts	Percentage of explicitly bundled-assignable contracts	Percentage of explicitly or implicitly bundled-assignable contracts
Supply	145	63.4%	89.0%
Lease	142	62.7%	82.4%
Total	287	63.1%	85.7%

**Table 3: Entity and non-entity bundle definitions, debtor party.** Table 3 reports percentages of explicitly bundled-assignable contracts that use entity-based and non-entity-based definitions of bundles with which the contract may be assigned. The sample includes only those contracts that are coded as explicitly bundled assignable, as reported in Table 2. A bundle is defined as an entity bundle if assignment is permitted in the event of a merger, acquisition, or a sale of all or substantially all of the assets of the debtor party to the contract. A bundle is defined as a non-entity bundle if assignment is permitted with (a) specific asset(s) and/or contract(s), or (b) a general definition of a bundle that does not specifically reference the debtor entity, such as "business" or "segment".

Contract type	Number of contracts	Entity bundles only	Entity and non-entity bundles	Non-entity bundles only
Supply	92	37.0%	52.1%	10.9%
Lease	89	91.0%	7.9%	1.1%
Total	181	63.5%	30.4%	6.1%

Appendix: Proofs

Lemma 2

The entrepreneur's date 1 expected payoff after individual assignment is  $2t_l$ , irrespective of whether the investment was made. The lowest possible date 1 expected payoff from bundled assignment occurs when the investment is not made, which is given by  $p_l(X - 2F_{ia})$ . Applying A1, this can be rewritten as  $2(r_l - p_l F_{ia}) \geq 2(r_l - p_j F_{ia}) = 2t_l$ . Thus, the expected payoff from bundled assignment is always at least as large as the payoff from individual assignment.

Proposition 4

**Proof.** First, consider any  $c$ -types that do not invest. Without investment, projects are zero-NPV by assumption A1. This implies that, under any solution satisfying the suppliers' participation constraints, E's date 1 equity value can not exceed zero. Thus, for these types, all contracting classes are equal and give E an expected utility of zero.

Next, we compare the investment cost cutoff values under BA to the other contracting classes. Comparing the cutoff types (8) and (6) when inputs are high quality ( $k = h$  and  $j = h$ ), we can see that  $\bar{c}_{ba} > \bar{c}_{ia}$  if and only if

$$2(p_h - p_m) \frac{r_h}{q_h} > 0$$

which is always true, since  $p_h > p_m$ . Similarly, inspection of (8) and (2) shows that  $\bar{c}_{ba} > \bar{c}_{na}$  for any  $\lambda > 0$ . Finally,  $\bar{c}_{ba} > 0$ , since  $q_h > p_h$ , and applying A1,  $X - 2\frac{r_h}{q_h} > X - 2\frac{r_h}{p_h} = 0$ . Thus, there exist a positive measure of  $c$ -types such that  $\max\{\bar{c}_{ia}, \bar{c}_{na}\} < c \leq \bar{c}_{ba}$ . (Trivially, BA elicits investment for strictly more  $c$ -types than ST contracts, since no  $c > 0$  will invest under ST contracts.)

We have shown that, when inputs are high quality, there is a strictly positive measure of  $c$ -types for which E will invest under BA but will not invest under any other contracting class. For  $c$ -types in this range ( $\min\{\bar{c}_{na}, \bar{c}_{ia}\} < c \leq \bar{c}_{ba}$ ), E's expected utility is zero under the other contracting classes but is strictly positive in equilibrium under BA. To see this, note that if E chooses to shirk rather than invest, her expected utility under BA is

$$p_j(X - 2\frac{r_k}{q_j})$$

which is strictly positive by A1 and  $q_j > p_j$  for all  $j$ . By revealed preference, then, E's expected utility when she invests is also strictly positive for any  $c \leq \bar{c}_{ba}$ . In this range of  $c$ -types, E strictly prefers BA to all other contracting classes.



Next, consider any  $c$ -type such that E invests in equilibrium under BA and some other contracting class. Under BA, when investments are alienable, there are no ex-post inefficiencies at date 1: when  $L = 1$ , E will realize the full value of the firm by assigning the bundle and spending the proceeds. By the definition of BA, all valuable bundles will be kept together, so no investment value is lost. If there are no ex-post inefficient actions taken after date 0, E's expected utility as of date 0 in equilibrium under BA is  $(q_j - p_j)X - c$ . This is the first-best social welfare, so E's expected utility can be no higher under any other contracting class in any equilibrium. Hence, we have shown that BA weakly dominates all other contracting classes. ■

Proposition 6

First, it is obvious that ST contracts will be dominated by all other contract forms, so we will focus on the remaining three. Before proving the proposition, we first solve for the cutoff type under IA contracts and show that IA contracts are weakly dominated by BA contracts. Then we show that BA contracts are weakly dominated by NA contracts.

First, note that when E uses two low quality inputs, IA and BA are equivalent. Suppose, then, that E uses two high quality inputs.

We begin with E's date 1 problem. As above, E will prefer individual assignment to keeping the bundle together if and only if (3) holds, restated here:

$$(p_h - p_m)\frac{2F_{ia}}{X} - (\pi_h - p_h)\frac{X - 2F_{ia}}{X} > 0 \quad (10)$$

When investments are inalienable, however,  $\pi_h = q_h$  only in the event that E invested *and*  $L = 0$ . If both events occur, E will strictly prefer to hold the contracts rather than assign them as a bundle, because the contracts are worth more under her control. If E invested but  $L = 1$ , she will always assign in some form at date 1. But doing this eliminates the value of the investment, so  $\pi_h = p_h$ . Given that this is the case, (3) suggests that E always prefers individual assignment to bundled assignment when investment is inalienable.

If (3) holds for  $\pi_h = q_h$ , E will always assign individually in all states at date 1; thus, she will never invest at date  $\frac{1}{2}$ . If (3) does not hold for  $\pi_h = q_h$ , E may have the incentive to invest. Her incentive compatibility constraint for investment is

$$(1 - \lambda)(q_j(X - 2F'_{ia}) - 2t_h) \geq c \quad (11)$$

Substituting for  $t_h$ , this can be rewritten as

$$(1 - \lambda)((q_h - p_h)(X - 2F'_{ia}) - 2(p_h - p_m)F'_{ia}) \geq c \quad (12)$$

We can now solve for  $F'_{ia}$ . Using the same logic as above, if the suppliers anticipate investment,  $F'_{ia} = \frac{r_h}{(1-\lambda)q_h + \lambda p_m}$ . Note that  $F'_{ia} > F'_{ba}$ , because individual assignment to an average E will occur when  $L = 1$ . Substituting, E's IC constraint for investment under IA contracts is the following:

$$\bar{c}'_{ia} = (1 - \lambda)((q_h - p_h)(X - 2\frac{r_h}{(1-\lambda)q_h + \lambda p_m}) - 2(p_h - p_m)\frac{r_h}{(1-\lambda)q_h + \lambda p_m})$$

Because  $F'_{ia} > F'_{ba}$ , and since  $2(p_h - p_m)\frac{r_h}{(1-\lambda)q_h + \lambda p_m} > 0$ , it can be verified by inspection that  $\bar{c}'_{ia} < \bar{c}'_{ba}$ . Thus, for  $c$  such that  $\bar{c}'_{ia} < c < \bar{c}'_{ba}$ , E's expected utility is higher under BA than IA, using the same arguments as in the proposition above.

Next, consider any  $c$  type that invests under both BA and IA. Under BA, E's equilibrium expected utility when E invests is given by

$$\lambda p_j(X - 2F'_{ba}) + (1 - \lambda)q_j(X - 2F'_{ba}) - c$$

where  $F'_{ba} = \frac{r_k}{\lambda p_j + (1-\lambda)q_j}$ . Under IA, when inputs are high quality, E's equilibrium expected utility when E invests is given by

$$\lambda(2r_h - 2p_m F'_{ia}) + (1 - \lambda)q_h(X - 2F'_{ia}) - c$$

where  $F'_{ia} = \frac{r_h}{\lambda p_m + (1-\lambda)q_h}$ .

It can be shown that both of these expressions can be reduced to the same value:

$$(1 - \lambda)(q_h - p_h)X - c$$

Summarizing, BA is superior to IA when both inputs are high quality and  $\bar{c}'_{ia} < c < \bar{c}'_{ba}$ . In all other cases, BA and IA are equivalent. Hence, BA weakly dominates IA, so we can focus on the choice between BA and NA.

BA vs. NA contracts

From Lemma 5, we know that  $\bar{c}'_{ba} < \bar{c}_{na}$ , so there are some types that would invest under NA that do not invest under BA. Using similar arguments as above, we know that E strictly prefers NA in this range. We also know that for any  $c$  that does not invest under NA or BA, E's expected utility is zero. Finally, consider  $c$  types that invest under both NA and BA.

Under NA, E's equilibrium expected utility is given by

$$(1 - \lambda)q_j(X - 2F_{na}) - c$$

where  $F_{na} = \frac{r_k}{q_j}$ . But this expression can also be reduced to

$$(1 - \lambda)(q_h - p_h)X - c$$

so NA is equivalent to BA and IA conditional on investment. Thus, NA weakly dominates BA for any input qualities.