Patent Disclosure in Standard Setting*

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Abstract

In this paper we analyze the timing of patent disclosure by a patent holder during the process of industry standard setting. In a non-cooperative model of communication with asymmetric information we endogenize patent holdup to study the effect of patent strength, the productivity of industry standard setting, and a standard setting organization's IPR disclosure rules. We find that late disclosure is more likely in more productive standard setting organizations and in less competitive industries. The enforcement of antitrust laws against deceptive conduct in standard setting organizations results in earlier disclosure.

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1 Introduction

Industry or product standards are developed and implemented to facilitate the interoperability of products and increase their value to customers.[1] They also have a social function by improving the rate of diffusion of new technologies[2] and eliminating mis-coordination among producers.[3] Recent empirical research investigates the effects of both collaboration and competition among firms participating in standard setting organizations (SSO) on the success and outcome of this process. Leiponen (2008) and Leiponen and Bar (2008), for instance, show that (social and political) connections are important determinants of the ability to contribute to a standard setting process.[4] On the other hand, conflicting and vested interests—arising from problems of asymmetric information or tensions due to fierce product market competition—can have a significant impact on the process.

This effect is likely to be amplified if the standard incorporates intellectual property (IP) (Weiss and Sirbu, 1990; Farrell and Klemperer, 2007:2026). Feldman, Graham, and Simcoe (2009), for example, document that patents disclosed in SSOs are highly litigated and that the litigation rates are correlated with the business structure of the disclosing firms. Baron and Pohlmann (2010) use a large set of essential-patent declarations to analyze the effect of patent pools on patent disclosure. Such disclosure of IP—especially when delayed—may be used as a strategic variable as it can provide the owner of IP with a bargaining leverage over prospective users of IP—often referred

[1]See, e.g., the discussions of standards and network effects in Scotchmer (2004) or Shapiro and Varian (1998), or the collected works of Stan Liebowitz and Stephen Margolis (Lewin, 2002).

[2]Rysman and Simcoe (2008) show that patents disclosed in SSOs receive up to twice as many citations as other patents in the same sector and conclude that such institutions play a crucial role in leading to a bandwagon process among adopters (especially in the ICT industry).

[3]See the discussion in Farrell and Klemperer (2007:2026f) and the literature cited therein

[4]For an earlier case study on the development of the packet switching standard X.25 in computer communication see Sirbu and Zwimpfer (1985).

to as patent holdup or ambush.[5] In this paper we endogenize the magnitude of patent holdup and study how competition and existence of a valid patent affect the strategic use of disclosure of IP.

Lerner and Tirole (2006) analyze how technology sponsors choose the SSO that maximizes the chances of getting their technologies adopted by final users, and Chiao, Lerner, and Tirole (2007) study (theoretically and empirically) the relationship between IPR disclosure rules[6]—a patent holder's obligation to reveal its intellectual property before a final choice is made—and the level of licensing prices. Yet, little has been written on to what extent the scope for "opportunistic" patent disclosure undermines the work of an SSO, a forum for reaching consensus under competitive and strategic tensions. The work by Simcoe (2008) and Farrell and Simcoe (2009) are first contributions. They highlight the impact of strategic interests on the delay of standard adoption.

In our analysis, we focus on the opportunistic patent disclosure to study how competition and the threat of patent holdup affects the timing of patent disclosure, and eventually the quality of a standard and the timing of its adoption. We present a dynamic model with asymmetric information based on Stein (2008),[7] in which two product-market competitors are engaged in the process of standard setting. They take turns in suggesting new standard components that are outcome of stochastic innovation process. We make two main assumptions: First, ideas for components are complementary insofar as a firm can find a new standard component only if the other firm has suggested a component in the previous round (e.g., Hellmann and Perotti, 2010; Stein, 2008). Second, the model's information structure is asymmetric as the initial standard

[5]See Farrell, Hayes, Shapiro, and Sullivan (2007); Farrell and Shapiro (2008); Ganglmair, Froeb, and Werden (2010); Lemley and Shapiro (2007); Shapiro (2010); Tarantino (2010), among others.

[6]See Annex 2 in http://ec.europa.eu/competition/consultations/2010_horizontals/microsoft_en.pdf.

[7]The model in Hellmann and Perotti (2010) shares many features with Stein (2008). We work with the latter because it can easily be extended to model standard setting with intellectual property.

component is a patent-protected technology and the patent holder must decide when to disclose the patent. Chiao, Lerner, and Tirole (2007:911) report that "due to the [...] complexity of patent portfolios, rivals frequently could not determine 'the needle in the haystack': that is, which patents were relevant to a given standardization effort." We assume that only the patent holder has knowledge of the patent. The identification of a patent that is relevant to the development of a specific standard imposes significant search costs on the firms participating in an SSO, especially when firms with very large patent portfolios are involved in the discussion.[8] Therefore, unless declared by its holder, the existence of the relevant intellectual property rights is hardly anticipated by firms. The rival, however, might be aware of the possibility of a patent. For the baseline results we consider "naïve" rivals who expect patents with probability zero (see also Kobayashi and Wright, 2009; Shapiro, 2010) ("unawareness") and then extend our analysis to the case where rivals expect an essential patent to exist with strictly positive probability ("awareness").

We design our model to capture two key factors that drive a patent holder's decision to disclose. By disclosing early, the patent holder gains from higher productivity of the standardization process, but loses part of her bargaining leverage from patent holdup. The first factor refers to the *benefits* of disclosure of intellectual property. As the patent may contain valuable technical information that provides a deeper understanding of the functioning of a certain technology, disclosure can be fruitfully exploited during the standard setting process. Chiao, Lerner, and Tirole (2007:911) find that, according to SSO members, by "highlighting the relevant patents or applications, [...] firms felt they were disclosing to competitors valuable information about [...] their future

[8]Search costs may turn out to be burdensome even for the patent holders. During a public hearing conducted by the Department of Justice and the Federal Trade Commission in 2007, expert panelists reported that "[c]omplying with different disclosure policies in different SSOs can be costly to IP holders, especially for those with large patent portfolios," and that "if an SSO's disclosure policy is too burdensome, IP holders won't come to the table because of the high cost." (U.S. Dep't of Justice & Fed. Trade Comm'n, 2007:43)

technological strategies." Our model captures this by an increased effectiveness of the standard setting process, meaning an increased probability with which a firm finds a new component or technology for the standard and the other firm agrees to its inclusion.

The second factor refers to the *costs* of patent disclosure. The owner of intellectual property of an essential part of the standard can require from other firms producing within the scope of the standard the payment of license fees. The amount of these fees will depend on the strength of the patent (Farrell and Shapiro, 2008) and the extent to which other firms have relied on the standard to be adopted and started manufacturing final products based on the present state of the standard.[9] The patented standard component is therefore locked in by virtue of producers having invested in standard-specific design. We assume that the later the patent holder discloses the patent, the more the patented technology is locked in, and the less likely it is replaced with an alternative.

The existing literature on the ensuing problem of patent holdup—sometimes also referred to as "patent ambush"—in standard setting[10,11] has assumed the *magnitude* of holdup to be exogenous. Patents enable innovators to earn monopoly rents on their

[9]DeLacey, Herman, Kiron, and Lerner (2006) document the long development of the xDSL and IEEE 802.11 standards. More specifically, when discussing the process of standard 802.11n definition (which improved the 802.11g version), DeLacey, Herman, Kiron, and Lerner (2006:13ff) present the case of Belkin, which had been shipping "pre-N" products for over a year before the final specification of the standard was certified.

[10]The patent holdup problem is a greatly debated issue in the law and economics literature, and with dissonant positions. To give two remarkable examples, Lemley and Shapiro (2007) stress the adverse impact of holdup on licensing decisions in industries with complex products, whereas Geradin (2009) claims that the real impact of patent holdup on the correct functioning of standard setting organizations is over-rated. We take a neutral stance and assume that a holdup problem may arise, although its incidence on the standard setting process is endogenous and depends on the timing of patent disclosure.

[11]Remarkably, many of the cases regarding SSOs deal with disclosure issues: In the FTC matters against Dell Computer Corp. (FTC order *Dell Computer Corp., FTC Docket NO. C-3658, 121 F.T.C. 616 (1996)*) and Rambus Inc., *FTC v. Rambus Inc., 522 F.3d 456 (D.C. Cir. 2008)*, the European Commission against Rambus (*"Antitrust: Commission confirms sending a Statement of Objections to Rambus", MEMO/07/330, http://europa.eu/rapid/pressReleasesAction.do?reference=MEMO/07/330*), or *Broadcom Corp. v. Qualcomm Inc., 501 F.3d 297 (3d Cir. 2007)*, accusers contended that patentees failed to comply to the disclosure rule of the SSO where the standardization process took place.

innovations. To our knowledge our model is the first to endogenize patent holdup in standard setting. We assume patent strength to be given, however, view the bargaining leverage as being contingent on whether or not the patented technology is included in the standard. With early disclosure a suitable alternative for the patented-protected component can be found without discarding the entire standard as the standard setting process is still at an early stage. Delaying disclosure locks in the patent-protected component as finding a suitable alternative becomes less likely. In combination with producers being locked in by having invested in standard-specific design, the innovator obtains a higher bargaining leverage over producers the later it discloses the patent.

For the baseline model we first consider an SSO with an IP policy, i.e., disclosure rule, that requires the patent holder to disclose the patent to the SSO. Failure to do so results in a waiver of IP rights—we refer to this case as *ex-ante disclosure rule*. This means that if by the end of the standard setting process the patent is not disclosed, the patent holder loses its bargaining leverage over manufacturers that sell standard-compliance products.[12] We find two sets of results:

First, a valid patent is a necessary condition for the patent holder to delay disclosure, i.e., not disclose at the beginning of the standard setting process. A valid patent is not sufficient, though. The *productivity* of the standardization process, i.e., the success probability of innovation, meaning the probability with which firms find further components to add to the standard, is another key factor. If in the absence of disclosure

[12]For example, see the European Commission's press release on the Rambus case (*"Antitrust: Commission accepts commitments from Rambus lowering memory chip royalty rates", IP/09/1897, http://europa.eu/rapid/pressReleasesAction.do?reference=IP/09/1897*) and the United States Court of Appeals for the Federal Circuit decision on *Qualcomm Inc. v. Broadcom Corp., Docket Number 07-1545. Nos. 2007-1545, 2008-1162. http://caselaw.findlaw.com/us-federal-circuit/1150919.html ("[W]e agree with the district court that, '[a] duty to speak can arise from a group relationship in which the working policy of disclosure of related intellectual property rights ('IPR') is treated by the group as a whole as imposing an obligation to disclose information [. . .].' [. . .] In these circumstances, we conclude that it was within the district court's authority [. . .] to determine that Qualcomm's misconduct falls within the doctrine of waiver. [. . .] remand with instructions to enter an unenforceability remedy limited in scope to any [standard]-compliant products.").*

the standard setting process is relatively productive, the patent holder is willing to forego the gains from a rise in productivity and obtain a higher bargaining leverage instead. A small effect on productivity of the process implies a delay of disclosure.

For the second set of results, we disentangle the effect of the degree of product market competition on the functioning of the standard setting process and the timing of disclosure. We show that in a highly competitive industry collaborative standard setting cannot be sustained. Intuitively, strong competitive pressures impair the agents' incentives to cooperate on the development of a standard. For an intermediate level of competition the procedure of standard development becomes viable again and disclosure is not strategically delayed. Moreover, lower levels of competition render disclosure more and more profitable. The intuition is that if competitive pressures are fierce, the gains from holdup cannot be large. Tough competition implies that firms profits are modest, and so are the rents that can be extracted from competitors via licensing. Conversely, as competition softens, larger product market profits give a strong incentive to delay disclosure so to recoup higher licensing fees.

When we relax the assumption that patent holders waive their IP rights when not disclosing the patent during the standard setting process—we refer to this case as *ex-post disclosure rule*—we find that patent disclosure is delayed even more. The underlying story is simple. The costs of not disclosing the patent, particularly, the threat of losing one's bargaining leverage when missing the window of opportunity, are lower with an SSO's IPR policy that does not sanction ex-post disclosure.

Our results contribute not only to the discussion of strategic patent disclosure and holdup in standard setting, but has implications for the general literature on knowledge sharing and diffusion (Anton and Yao, 2002, 2004; Haeussler, Jiang, Thursby, and Thursby, 2009; Hellmann and Perotti, 2010; Stein, 2008; von Hippel, 1987). von Hippel (1987), for instance, in an early contribution studies the problem of technical know-how trading among technicians of competing firms and shows, by means of case studies, that cooperative communications between competitors can take place, however, such conversations are not sustainable when very harsh competition is at work.[13] We deliver the analogous result that tough competition impedes firms' discussions and prevents collaborative standard setting. With a focus on the complementarity of information[14] Haeussler, Jiang, Thursby, and Thursby (2009) build a model of knowledge diffusion among academic scientists. Their model shares with ours the feature that complementary information is needed to solve a problem and that such information is exchanged among competing agents. They assume that each agent can quit the info sharing game with its own solution to the problem, whereas we rule this out; a successful standard setting process requires collaboration of all parties involved.

The structure of the paper is as follows: In Section 2 we introduce our extension of the model by Stein (2008). In Section 3 we define the first best outcome and show that in cooperative equilibrium a standard setting process cannot be sustained if competition is too fierce. In Section 4 we analyze the non-cooparative equilibria of our baseline model with unawareness and consider the extension of ex-post disclosure in Section 5. In Section 6 we consider the case of awareness. We discuss computational results for the disclosure timing and the length of the standardization process in Section 7. We conclude in Section 8. The formal proofs of the results are relegated to the Appendix.

2 Basic Model

We draw on the basic setup in Stein (2008) and add disclosure of IP to the model. There are two firms, A (she) and B (he), that engage in a process of industry standard

[13] von Hippel (1987) makes the example of the aerospace industry, where firms competing for an important government contract report not to trade information with rivals.

[14] See also Hellmann and Perotti (2010) or Stein (2008).

definition by means of exchanging ideas and technologies. They take turns with A moving at stages $t = 1, 3, 5, \ldots$ and B moving at stages $t = 2, 4, 6, \ldots$. At each stage $t \geq 2$, the firm to move has an opportunity to develop a new non-protected technology χ_t (i.e., candidate component of the industry standard).

2.1 Information Structure

At the initial stage $t = 1$, firm A has access to a patent-protected technology χ_1. At stage $t = 1$ and any future odd stage, she has three options: She can (1) *stop* the process (stay quiet and reveal neither the technology nor the patent), (2) *disclose* (reveal both the technology and the fact that it is patent-protected), or (3) *continue* the process (communicate the technology to B but keep the fact that it is patent-protected to herself).[15] These actions imply the following for the structure of the standard setting process.

1. If A *stops* at any odd t, firm B cannot develop χ_{t+1} and the game ends. This assumption embeds a strong form of complementarity into the production function for components of the standard. A useful new technology and component of the standard can be produced by a firm only if there is access to a prior technology.

2. If A *discloses* at any odd t, meaning revealing the existence of the patent on technology χ_1 and communicating the new technology χ_t, firm B will with probability q develop a new technology and standard component χ_{t+1} at $t + 1$. If B fails, with probability $1 - q$, the game stops. If successful, firm B can either *continue* by truthfully revealing technology χ_{t+1}, after which it is A's turn in $t + 2$; or *stop*. At $t + 2$, firm A will have disclosed and is left with the decision to either *stop* or *continue*.

[15] Note that A can choose not to disclose the patent at $t = 1$ but reconsider her decision at $t = 3, 5, \ldots$

3. If A *continues* at any odd t and has disclosed at an earlier stage, the game continues as described above. If A has not yet disclosed the patent and at t decides to continue and therefore keep its existence to herself, firm B develops a new technology and standard component χ_{t+1} with probability $p < q$.

The structure of the game is depicted in Figure 1.[16] The process continues until one firm fails to produce a new component or decides to *stop*. For the baseline results we assume that B is not aware of the possibility that the initial component is patent-protected (Kobayashi and Wright, 2009; Shapiro, 2010). This implies that, as long as A has not disclosed her patent, B will at any even t anticipate that both parties have at $t+1, t+2, \ldots$ a probability p of finding a new component for the standard. We relax this assumption in Section 6.

[FIGURE 1 ABOUT HERE]

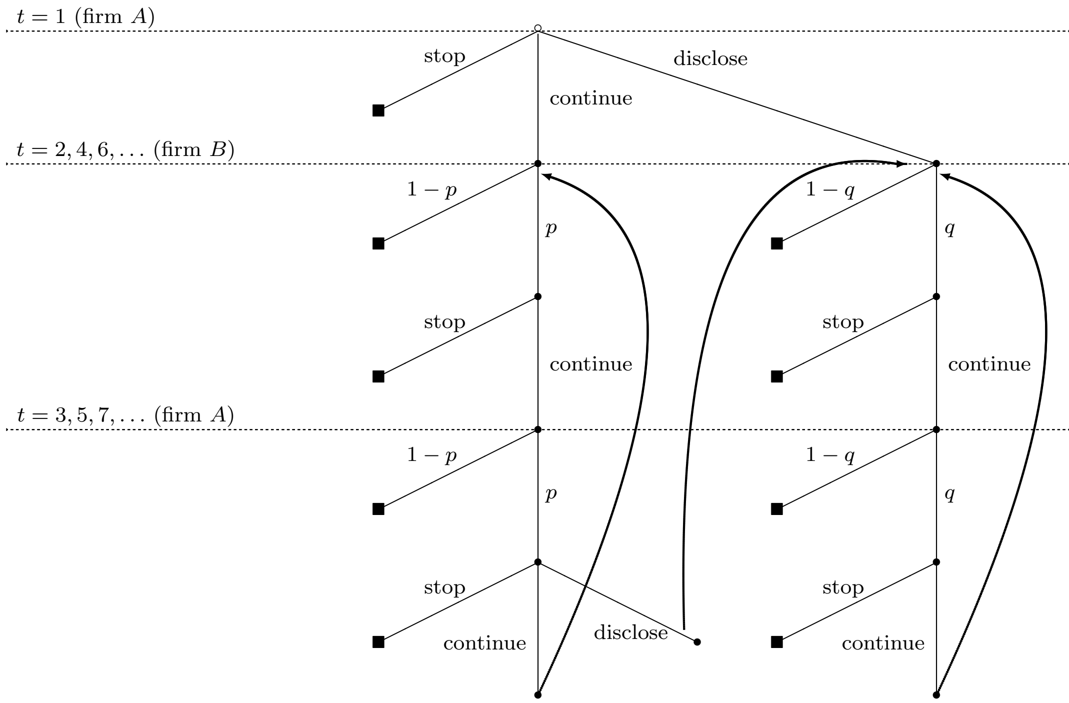
2.2 Payoffs

The longer firms communicate and therefore the more components they add to a standard—let that number of communication rounds and standard components be denoted by n_S—the better the standard eventually becomes. We follow Stein (2008) and design the parties' payoff functions trying to provide the specific competitive setting in the product market as well as introducing to the model the main forces that characterize the functioning of SSO. Market profits realize only after the standardization process is brought to an end and the standard adopted. The timing of conversation and competition is depicted in Figure 2.

[FIGURE 2 ABOUT HERE]

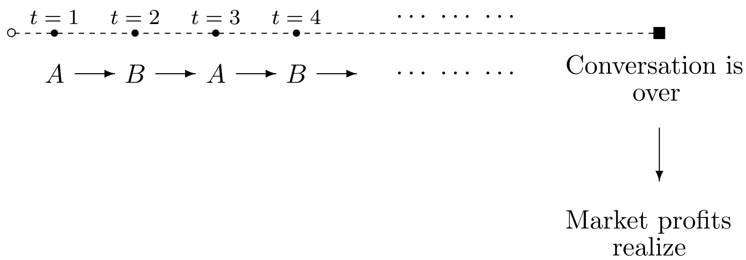
[16] The left arm of this game tree depicts the game structure in Stein (2008), the right arm represents our extension of his model, accounting for intellectual property in the communication of a standard setting process.

Figure 1: Conversation Game with Patent Disclosure



Standard Setting Organization's Rules: The SSO of the baseline model has two main rules: The *disclosure rule* and the *licensing rule*. The disclosure rule prescribes that A must declare relevant IPRs before the conversation on the composition of the standard is over and the standard technology is adopted on the product market. In line with the legal evidence of courts imposing waivers on firms that have tried to enforce undisclosed IP, the rule on the licensing regime prescribes that A can set a license fee only if she discloses the patent before the end of the standardization process. We refer to this as *ex-ante disclosure rule*. In a later extension of the model—*ex-post disclosure rule*—we will relax this assumption and assume that A can levy a fee even if she declares her IPRs after the end of the conversation with B.

Figure 2: Timing of Standard Setting and Competition



Product market competition: The higher the number of components, n_S, and the higher the quality of the standard, the lower the costs the parties incur in production. More specifically, having access to an n_S-standard, the parties can manufacture the product at cost $(1 - h(n_S))$. Here, $h(n_S)$ is an increasing function, with $h(0) = 0$ and $\lim_{n_S \to \infty} h(n_S) = 1$, that captures the total cost savings associated with n_S components. Also, a party that develops a new technology but decides not to communicate it manufactures the product at cost $(1 - h(n_S + 1)) < (1 - h(n_S))$ and has therefore a cost advantage over its rival.

We assume that firms A and B each face a market of unit mass and that all customers into the market have a reservation value of one. Moreover, there is a fractional overlap of size θ in A's and B's customer bases, with $0 < \theta < 1$. In other words, A and B have a monopoly on a fraction $(1 - \theta)$ of their customers, but compete for the remaining fraction θ. The products are otherwise undifferentiated and competition is à la Bertrand.

The effect of shading the existence of a relevant patent on firms' payoffs is driven by two factors.

Productivity: When A discloses to B the existence of the patent, the probability that either party in subsequent periods finds new components for the standard is higher than when the patent is hidden, $q > p$. The standardization process becomes more productive, creating a shared interest in communicating the patent as soon as possible.

Holdup: The holdup problem of manufacturers who employ patent-protected technologies is characterized as follows: If patent-holder firm A does not sell a license for the initial patent-protected technology, χ_1, then manufacturer B infringes if selling his products. This threat gives firm A a bargaining leverage that maps into the license fee the parties will negotiate once the standard has been adopted and production commences.

Let $\sigma \in (0, 1)$ be the fraction of B's profits firm A can extract by means of license fees. It depends on two factors: (1) Let $\tau \geq 1$ the timing of disclosure. As more and more components χ_t are added to the standard, the initial technology χ_1, upon which the standard is built, becomes more essential and the degree of lock-in increases. This implies that the later firm A discloses the patent, the more difficult it becomes to find an adequate substitute for the patented technology. Also, the longer the standard setting process takes the greater the degree to which manufacturers are locked in, having relied on a non-proprietary standard. (2) Let $\alpha > 0$ denote the strength of the patent. Suppose no adequate substitute can be found for the patented technology, then firm A's bargaining leverage will eventually depend on how weak or strong the patent is (Farrell and Shapiro, 2008). We assume that $\sigma = \sigma(\alpha, \tau)$ is continuous and increasing in α and τ with $\sigma(0, \tau) = 0$, $\sigma(\alpha, 1) = 0$ and $\lim_{\tau \to \infty} \sigma(\alpha, \tau) = \alpha$.

A measure for the quality of the standard is its number of components, n_S. If the standardization process stops because either firm fails to find a new component, then both firms have access to the same information and $n_A = n_B = n_S$, where n_i is the number of components firm i is aware of. Alternatively, if party i finds a new component but decides to stop the standardization process without revealing it, then

$n_i = n_S + 1 > n_S = n_{-i}$. That firm then has an advantage over its competitor because it can manufacture the product at a lower price. Put together, the assumptions above yield payoffs of

$$U_A = (1-\theta)\, h(n_A) + \theta \max\{0, h(n_A) - h(n_B)\} + \sigma(\alpha, \tau) U_B \tag{1}$$

for firm A and

$$U_B = [(1-\theta)\, h(n_B) + \theta \max\{0, h(n_B) - h(n_A)\}]\,(1 - \sigma(\alpha, \tau)) \tag{2}$$

for firm B. The first part of equation (1) reflects the fact that for a fraction $(1-\theta)$ of her customers, A is a monopolist and charges the full reservation value of one; with costs of $(1 - h(n_A))$. Her profits per customer are thus $h(n_A)$. On the remaining fraction θ of her customers, where A's and B's consumer bases overlap, Bertrand competition implies that A makes a profit only if her costs are strictly below those of B; analogously for B in equation (2). The third term in (1) reflects the fact that by enforcing her IPR, firm A can extract a share $\sigma(\alpha, \tau)$ of B's profits.

3 First Best and Cooperative Equilibrium

In a first-best world, firm A discloses her patent at $t = 1$ and both A and B communicate their respective ideas for standard components until they fail to find further ideas. The intuition for this is straightforward. As more components increase the quality of the standard and lower the costs of production, communication is socially desirable. Disclosure of the patent increases the productivity of this process.

A first question is whether the first-best outcome can be implemented in a cooperative equilibrium. The parties' joint payoffs are

$$U^C = \begin{cases} U_A + U_B &= 2(1-\theta)\,h(n_A) & \text{if } n_A = n_B \\ U_i + U_j &= h(n_i) + (1-2\theta)\,h(n_j) & \text{if } n_i > n_j \end{cases} \tag{3}$$

In the latter case firm $i = A, B$ has not continued and revealed an idea. For $\theta = \frac{1}{2}$, the two expressions for U^C are equivalent. For any $\theta > \frac{1}{2}$, however, the joint payoffs from (cooperatively) not continuing (so that $n_i > n_j$) are higher than from continuing. We show in the following proposition that disclosure and communication of ideas is not part of a cooperative equilibrium if θ is sufficiently high. In other words, in a highly competitive industry, collaborative standard setting cannot be sustained.

PROPOSITION 1 (Cooperative Equilibrium)**.** *If competition is too high (for sufficiently high values of θ) there is no communication in the cooperative equilibrium.*

The formal proof of this result is relegated to the appendix. For a parametric example, suppose $h(n) = 1 - \beta^n$ with $0 < \beta < 1$. The joint payoffs from stopping the process are strictly larger than from continuing if

$$2(1-\theta)\left[1 - \frac{\beta^t(1-q)}{1-\beta q}\right] < 2(1-\theta) - (1+\beta-2\theta)\,\beta^{t-1}. \tag{4}$$

This greatly simplifies to

$$\frac{1+\beta q}{2} < \theta. \tag{5}$$

For the remainder of this paper we restrict attention to sufficiently low degrees of competition, $\theta < \frac{1+\beta q}{2}$. If communication for all t cannot be implemented in a cooperative equilibrium, it will not be implementable in a non-cooperative equilibrium, which is what we analyze in the next section.

4 The Case of Ex-Ante Disclosure Rule

The analysis of non-cooperative equilibria demonstrates how patent disclosure and the scope for holdup affect the firms' incentives to communicate in a standard setting process. We proceed as follows: We first shed light on their incentives to continue communication after the patent has been disclosed[17] and then derive conditions for firm A to disclose her patent.

4.1 Post-Disclosure Communication

Suppose firm A disclosed the patent at stage τ so that success probability (of finding a new component) is q. We first consider the case for B and then turn to firm A.

If at $t \geq \tau + 1$, B *continues* and the game moves along the equilibrium path, i.e., always continue, until either A or B fail to find a new component, firm B's expected payoffs are given by

$$\mathrm{E}_t U_B(continue@t|\tau) = (1 - \sigma(\alpha, \tau))\,(1 - \theta)\, H(t|q)$$

where

$$H(t|q) = \sum_{i=0}^{\infty} q^i\,(1 - q)\, h(t + i). \tag{6}$$

This expression is increasing in probability q.[18] The intuition behind (6) is as follows:

[17] This is analogous to the steps in Stein (2008) but for probability $q > p$ and sharing rule $\sigma(\alpha, \tau)$ of B's profits.

[18] More generally, $H(t|x)$ is increasing in x. The derivative of $H(t|x)$ with respect to x is equal to:

$$\sum_{i=0}^{\infty} x^i \left(\frac{i(1-x)}{x} - 1 \right) h(t+i),$$

which, after some manipulation, can be rewritten as

$$\sum_{i=0}^{\infty} i x^{i-1} (h(t+i+1) - h(t+i)) > 0.$$

With probability $(1-q)$, there will be no further ideas after time t, so the standard has t components with a total cost-cutting value of $h(t)$ for both parties; with probability $q(1-q)$, there will be exactly one further idea after t, so the standard has $t+1$ components with a total cost-cutting value of $h(t+1)$; with probability $q^2(1-q)$ there are exactly two further components, and so forth. By contrast, suppose that firm B considers deviating from the equilibrium strategy, i.e., *stop* at stage t. His payoffs in this case are equal to

$$U_B(stop@t|\tau) = (1-\sigma(\alpha,\tau))\left[h(t) - \theta h(t-1)\right].$$

This expression reflects the fact that if B stops, he keeps idea χ_t to himself and has therefore a production cost advantage over A. This allows him to not only earn a profit of $(1-\theta)\,h(t)$ in the monopoly market, but also a profit of $\theta\left[h(t)-h(t-1)\right]$ in the competitive market.[19] Because of A's patent holdup, firm B keeps only a fraction $(1-\sigma(\alpha,\tau))$ of his profits.

For firm B to *continue* the conversation, $\mathrm{E}_t U_B(continue@t|\tau) \geq U_B(stop@t|\tau)$ must hold for all values of $t > \tau$. This condition is satisfied if and only if

$$\frac{H(t|q) - h(t-1)}{h(t) - h(t-1)} \geq \frac{1}{1-\theta}. \tag{7}$$

We derive firm's A condition to *continue* the communication analogously. If at $t \geq \tau + 2$, A *continues* and the game moves along the equilibrium path until either A or B fail to find new components, firm A's expected payoffs are given by

$$\mathrm{E}_t U_A(continue@t|\tau) = (1-\theta)\,(1+\sigma(\alpha,\tau))\,H(t|q). \tag{8}$$

[19] In the Bertrand game, B underbids A by offering a price $1-h(t-1)$. His production costs are $1-h(t)$

The expression is the same as for firm B, except that instead of "paying" a fraction $\sigma(\alpha,\tau)$, firm A receives a fraction $\sigma(\alpha,\tau)$ of B's profits. Now, suppose that firm A considers deviating from her equilibrium strategy, i.e., *stop* at stage t. In this case, her payoffs are

$$U_A(stop@t|\tau) = h(t) - \theta h(t-1) + (1-\theta)\,\sigma(\alpha,\tau)h(t-1). \tag{9}$$

It reflects her monopoly and competition profits as well as her share from B's monopoly market profits.[20] For firm A to always *continue* the process, $\mathrm{E}_t U_A(continue@t|\tau) \geq U_A(stop@t|\tau)$ must hold for all values of $t > \tau$. This is satisfied if and only if

$$(1+\sigma(\alpha,\tau))\,\frac{H(t|q) - h(t-1)}{h(t) - h(t-1)} \geq \frac{1}{1-\theta}. \tag{10}$$

We can conclude from conditions (7) and (10) that, after disclosure, if $\sigma > 0$, firm A's incentives to *continue* the standardization process are stronger than firm B's. Her condition to continue is therefore never binding.[21]

Continuing the conversation allows A to gain twice from increased productivity, directly and indirectly. A longer communication leads to a better standard and lower production costs for both firms. This has a direct positive impact on A's profits. The indirect effect arises from the fact that A extracts part of B's profits by means of license fees. Accordingly, we find that after disclosure A is more eager to continue the communication than B.

The analysis of the firms' post-disclosure incentives shows that whether or not

[20] Note that in case of A stopping and not communicating the last component χ_t so that the standard consists of only $t-1$ components, B's competition profits are equal to zero.

[21] Equation (7) is binding. For the parametric example in Section 3 (which uses a functional form for $h(\cdot)$ employed by Stein (2008)), *continue* is the non-cooperative equilibrium strategy for both firms if $\beta q \geq \theta$. The condition for sustaining a non-cooperative equilibrium is more restrictive than for a cooperative equilibrium, $\theta < \frac{1+\beta q}{2}$.

continuing the standard setting process can be sustained in equilibrium does not depend on the threat of patent holdup as (7) is independent of σ.

4.2 Patent Disclosure

We now turn to firm A's decision to disclose the patent. In the cooperative equilibrium, she reveals the information about the patent right away to (jointly) benefit from increased productivity of the standard setting process. For the main results of this paper, we ask the following: Does firm A ever have an incentive to delay disclosure? And if so, what are the conditions for such delayed disclosure?

We have assumed that firm B is not aware of the possibility of a patent. He has incomplete knowledge of firm A's action set as he does not anticipate firm A's choice to *disclose*. By Stein (2008), firm B's pre-disclosure condition to continue is

$$\frac{H(t|p) - h(t-1)}{h(t) - h(t-1)} \geq \frac{1}{1-\theta} \tag{11}$$

with $H(t|p)$ given as in equation (6) for probability p instead of q. Because $q > p$, for a given t, $H(t|q) > H(t|p)$, so that, for $q \geq p$ and $\alpha \geq 0$, condition (11) implies condition (7), and condition (7) implies condition (10).

Condition (11) gives rise to two cases that we need to consider: The first, which we call "unconstrained disclosure" and analyze below, is the case when (11) holds. This means if firm A decides to *continue* communication but not *disclose* the patent, then pre-disclosure communication is sustainable as an equilibrium, as firm B will *continue* the process. The second case, which we call "constrained disclosure," is when condition (11) is violated. This means that firm B does not have an incentive to continue prior to disclosure. Firm A's decision is thus constrained by the anticipation of B stopping the process. As we will see later, A can salvage the standard setting process by disclosing early on.

4.2.1 Unconstrained Disclosure

At every odd stage t, firm A has to decide[22] whether to *disclose* right away, so that $\tau = t$, and realize expected payoffs

$$\mathrm{E}_t U_A(disclose@t) = (1-\theta)\,(1+\sigma(\alpha,t))\,H(t|q), \tag{12}$$

or postpone disclosure, meaning *continue* at t and *disclose* at $t+2$ with expected payoffs

$$\begin{aligned}\mathrm{E}_t U_A(disclose@t+2) &= (1-\theta)\,[(1-p)\,h(t) + p\,(1-p)\,h(t+1)] + \\ &\quad (1-\theta)\,p^2\,(1+\sigma(\alpha,t+2))\,H(t+2|q). \end{aligned} \tag{13}$$

Firm A faces the following trade-off: On the one hand, the continuation value after disclosure increases the later disclosure takes place, indeed $\sigma(\alpha,t) < \sigma(\alpha,t+2)$ and $H(t|q) < H(t+2|q)$. On the other hand, by postponing disclosure one round, A loses the gains associated with disclosure at t and $t+1$, characterized by the possibility to expropriate a fraction $\sigma(\alpha,t)$ of B's profits at an increased probability $q > p$. The expected value at t of the gains from disclosure at $t+2$ are discounted by p^2, which is the probability the standardization process reaches stage $t+2$.

Provided a regularity condition, discussed below, in Proposition 2 we provide a simple necessary and sufficient condition for firm A to delay disclosure to a later period,

[22]Note that *stop*ping is dominated. We assume that (11) holds. Moreover, (11) implies (7) implies (10), where the latter implies that after disclosure firm A's *continue* dominates firm A's *stop*. Because $\sigma = 0$, prior to disclosure firm A's payoffs from stopping are lower than after disclosure, so that stopping is less attractive and condition (10) sufficient for pre-disclosure stopping to be dominated.

so that $\tau > 1$. We also show that firm A will eventually want to disclose the patent—τ is finite—meaning that unless the process is terminated due to either firm's failure to find a new component, the patent will always be disclosed.

PROPOSITION 2 (Unconstrained Disclosure).
Let $\mathrm{E}_t U_A(disclose@t)$ *and* $\mathrm{E}_t U_A(disclose@t+2)$ *intersect at most once. If*

$$\mathrm{E}_1 U_A(disclose@3) \geq \mathrm{E}_1 U_A(disclose@1) \tag{14}$$

then firm A delays disclosure. There exists a finite disclosure date $\tau > 1$*. If (14) does not hold, the patent is disclosed at the outset of the standardization process and* $\tau = 1$*.*

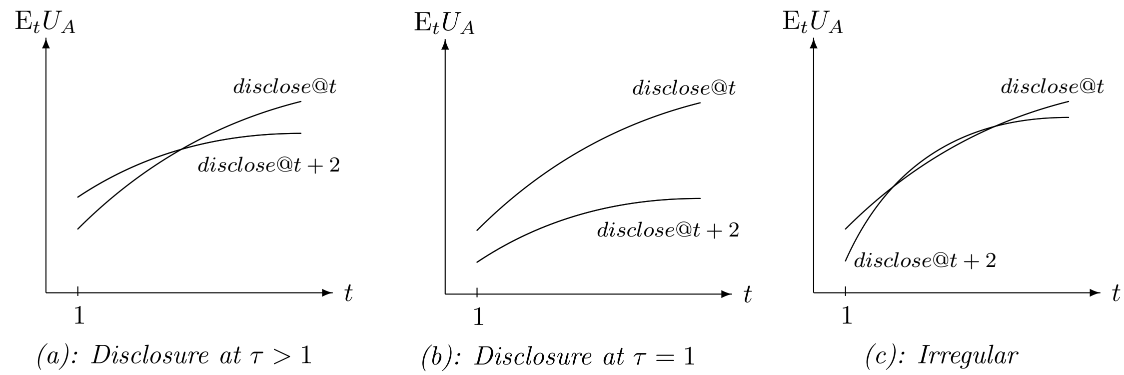
The simple condition in (14) states that if at $t = 1$ firm A's expected payoffs from postponing disclosure one round to $t = 3$ are at least as high as from disclosing right away, then disclosure will be delayed at least one round. We reformulate firm A's disclosure problem as an optimal stopping problem and, using results provided by Stokey and Lucas (1989), we show that a stopping rule exists. Such a case is depicted in panel *(a)* of Figure 3. We plot the graphs for expected payoffs at t from disclosure at t and disclosure at $t+2$ over time.

[FIGURE 3 ABOUT HERE]

Condition (**??**) above is a sufficient condition for delayed disclosure. Consider panels *(b)* and *(c)* of Figure 3 when the condition is violated. Because in the limit the expected payoffs from delaying are strictly smaller than from disclosing, the two graphs for expected payoffs (disclosure at t and disclosure at $t+2$) never intersect or intersect twice.[23] In the former case, the simple condition is not only sufficient for

[23]The two graphs could be tangents or intersect more than twice. For the argument, cases are of no relevance.

Figure 3: Expected Payoffs From Delaying Disclosure: Three Cases



delayed disclosure but also necessary, because if it does not hold firm A will disclose right away as her expected payoffs will always be strictly higher than the expected payoffs from delaying. In the latter case, we cannot rule out that, although disclosing right away is the dominant action at $t = 1$, firm A waits until a point when delaying is dominant (as the graph for disclosure at $t + 2$ lies above the graph for disclosure at t). We rule out this case by assumption of a regularity condition.

In the following two propositions we refine the existence result of an optimal stopping rule. We provide the formal proofs in the appendix. In Proposition 3 we show that the presence of valid intellectual property and ensuing threat of patent holdup is a necessary condition for the delay of disclosure. This seems tautological. Without intellectual property there is no intellectual property to disclose. The crucial point is that intellectual property is valid in the sense that (i) it can be enforced, meaning that it is not invalidated by means of an SSO's IPR rules or antitrust agencies' intervention, and (ii) it is "strong" enough. Without the prospect of a bargaining leverage arising from IPR, firm A has no incentive to jeopardize the productivity of the standard setting process by not revealing.

PROPOSITION 3. *Let $q > p > 0$. Enforced intellectual property and ensuing patent* *holdup is a necessary condition for the delay of patent disclosure, i.e.,* $\alpha > 0$ *so that* $\sigma(\alpha, t) > 0$ *for all* $t > 1$.

The presence of valid intellectual property is a necessary condition for delayed disclosure. However, it is not sufficient. We show in Proposition 4 that, given $\alpha > 0$, there is a lower bound $\bar{p} < q$ for the pre-disclosure success probability p (a measure for the pre-disclosure productivity of the standard setting process) such that condition (14) holds and disclosure is delayed for all $p \geq \bar{p}$; and (14) is violated for all $p < \bar{p}$ so that $\tau = 1$.

PROPOSITION 4. *Let* $\alpha > 0$ *and* $q > 0$. *If the pre-disclosure success probability* p *is not too low, i.e., for* $\bar{p} \leq p < q$ *with* $\bar{p} > 0$, *condition (14) holds and disclosure of the patent is delayed.*

If the baseline probability to continue the conversation (p) is relatively high, then the cost of delaying disclosure is small: A exploits the sufficiently high productivity of the communication to postpone patent's revelation until the marginal gains from holdup are exhausted.

For the remainder of this section we assume that the pre-disclosure success probability is sufficiently high so that the patent is delayed, $p \geq \bar{p}$ and $\tau > 1$. By Proposition 2, the disclosure stage τ is such that the expected payoffs from disclosure in t in equation (12) are at least as high as the expected payoffs from disclosure in $t + 2$ in equation (13) for all $t \geq \tau$ and strictly smaller for all $t < \tau$. In Proposition 5 we provide comparative statics for firm A's propensity to delay disclosure.

PROPOSITION 5. *The patent holder is more inclined to delay disclosure of her patent the higher the pre-disclosure success probability* p *is. The strength of the patent,* α, *has an ambiguous effect on the propensity to disclose the patent. If the effect of patent strength on the patent holder's bargaining leverage is sufficiently increasing with* *delayed disclosure, so that*

$$\frac{\sigma_\alpha(\alpha,\tilde{t})H(\tilde{t}|q)}{\sigma_\alpha(\alpha,\tilde{t}+2)H(\tilde{t}+2|q)} < p^2, \tag{15}$$

then patent strength α has a delaying effect on patent disclosure.

In Proposition 3 we showed that the existence of enforceable IPRs is a necessary condition for delay of disclosure, so that $\tau > 1$ and the patent holdup problem arises; in Proposition 4 we provided a sufficient condition for delayed disclosure. Whether or not these two factors of the standard setting process—patent strength α and the pre-disclosure productivity of the process, p—have a positive effect on the patent holder's propensity to delay disclosure, is discussed in Proposition 5.

As in Proposition 4, in Proposition 5 the effect of an increase in p on the propensity to delay disclosure is clear. However, the impact of patent strength is ambiguous and will eventually depend on the bargaining technology determining the shape of σ. A stronger patent increases both the gains from disclosing today and the ones from later disclosure, which are discounted by p^2. If the latter are sufficiently large, so to offset the cost of time, then patent strength delays disclosure. We come back to this point, when we discuss a parameterized version of the model.

4.2.2 Constrained Disclosure

We now analyze the case when prior to disclosure a communication equilibrium cannot be sustained because (11) is not satisfied for all t. This means that if firm A were to *continue* at some t, firm B would *stop* at $t+1$.

PROPOSITION 6 (Constrained Disclosure)**.** *Let condition (11) be violated for some $t \geq 1$.*

1. *If condition (7) is violated for some $t > \tau \geq 1$, then firm A will* stop *at $t = 1$.*

2. *If condition (7) holds for all $t > \tau \geq 1$, then firm A will* disclose *at $t = 1$, and the process continues until one of the parties fails to come up with a new standard component.*

There are three main implications to take away from Proposition 6. *First*, for a high degree of competition, so that (7) is violated and post-disclosure communication cannot be sustained, the standardization process is never initiated. Consider the parameterization introduced above. For all $\frac{1+\beta q}{2} \geq \theta > \beta q$, the parties jointly benefit from standardization but, in a noncooperative game, cannot sustain the process.

Second, for degrees of competition that allow for the process to be initiated, $\beta q \geq \theta > \beta p$, we observe immediate disclosure. This means firm A forsakes her rent-seeking possibilities. The intuition for the last result is straightforward. For high degrees of competition, firm B's monopoly profits are relatively low. Because firm A can extract rents only from B's monopoly profits—the parties' profits from the market on which they compete are tiny or zero—if competition is fierce the gains from holdup are small, and more than outweighed by the gains from disclosing right away to increase the efficiency of the standardization process.

Third, not surprisingly, a very inefficient pre-disclosure process will provide little incentive for firm A to delay disclosure. As we can conclude from Proposition 6, a sufficiently high success probability, $p \geq \frac{\theta}{\beta}$, so that condition (11) holds, is necessary for firm A to delay disclosure and engage in rent-seeking or holdup activities.

5 The Case of Ex-Post Disclosure Rule

In this extension of the baseline results we relax the assumption that firm A, when not disclosing the patent prior to the end of the standard setting process, waives her IPR so that $\sigma = 0$. This means, if the patent has not been disclosed when firm either A or

B at stage t *stops* communication, A can disclose ex post and $\sigma = \sigma(\alpha, t)$. Likewise, when the parties *continue* the conversation at t but either fails to find a component for the standard, $\sigma = \sigma(\alpha, t)$. Moreover, we focus on the case of unconstrained disclosure, so that (11) holds.

The parties' post-disclosure incentives are unaffected. Pre-disclosure payoffs for firm A, however, will change. They are

$$\widehat{U}_A(stop@t) = U_A(stop@t|\tau) \tag{16}$$

in equation (9) if *stop*. Firm A's pre-disclosure payoffs from *stop* are the same as the post-disclosure payoffs in the baseline mode; indeed, A can now enforce her patent even after the process ends. For the same reason, A's payoffs are

$$\mathrm{E}_t\widehat{U}_A(continue@t) = (1-\theta)\,\widehat{H}(t|p), \tag{17}$$

if *continue* with

$$\widehat{H}(t|p) = \sum_{i=0}^{\infty} p^i\,(1-p)\,h(t+1)\,(1+\sigma(\alpha, t+i))\,. \tag{18}$$

We show in Lemma 1 that if firm B's pre-disclosure communication condition in equation (11) holds, meaning that as long as the patent is not disclosed, firm B will not stop communication, then for firm A to *stop* is dominated by to *continue*. With condition (11), neither B nor A have an incentive to stop the standardization process. The proof is relegated to the appendix.

LEMMA 1. *Condition (11) implies* stop *by firm A to be strictly dominated.*

With this result in mind, we can concentrate on firm A's decision to either *continue* or *disclose*. As in the previous section, firm A, at every odd stage t has to decide whether to disclose right away and realize expected payoffs

$$\mathrm{E}_t\widehat{U}_A(disclose@t) = \mathrm{E}_tU_A(disclose@t) \tag{19}$$

in equation (12), or postpone disclosure by one round, meaning continue at t and disclose at $t+2$ with expected payoffs of

$$\begin{aligned}\mathrm{E}_t\widehat{U}_A(disclose@t+2) \quad &= \quad (1-\theta)\sum_{i=0}^{1} p^i\,(1-p)\,(1+\sigma(\alpha,t+i))\,h(t+i) + \\ &\quad\quad (1-\theta)\,p^2\,(1+\sigma(\alpha,t+2))\,H(t+2|q).\end{aligned} \tag{20}$$

Comparing the expected payoffs from delaying disclosure for the cases of ex-ante disclosure in the previous section (equation (13)) and this section's ex-post disclosure (equation (20)), we see that if firm A does not lose her bargaining leverage by missing the window of opportunity to disclose, the costs of delaying disclosure by one round are lower. In the ex-ante disclosure case, these costs result from (i) a lower success probability, $p < q$, and (ii) losing bargaining leverage, $\sigma = 0$, if either party fails to find a new component in $t+1$ or $t+2$. In equation (20) firm A does not lose her bargaining leverage, thus only the first cost factor applies.

Allowing for ex-post disclosure without depriving firm A of her bargaining leverage increases firm A's benefits from delaying disclosure and affects the results in Propositions 4 and 5. Proposition 7 summarizes—the proof is relegated to the appendix.

PROPOSITION 7. *Let $\alpha > 0$ and $q > 0$. In the case of ex-post disclosure, if the pre-disclosure success probability p is not too low, i.e., for $\hat{p} \leq p < q$ with $\hat{p} > 0$, disclosure of the patent is delayed, and the disclosure stage is $\hat{\tau} > 1$. Moreover, not sanctioning non-disclosure of intellectual property by enforcing it results in stronger* *incentives to delay disclosure. Also, if* $p \in [\hat{p}, \bar{p})$*, the patent is disclosed at stage* $t = 1$ *under ex-ante disclosure yet delayed under ex-post disclosure.*

The results in Proposition 7 together with the discussion above show that allowing for ex-post disclosure results in firm A's weaker incentives to disclose early in the standardization process. This has two main policy implications: The first is for standardization consortia and regards the choice of the disclosure rule. The SSOs that wish to limit the scope for opportunistic patent disclosure should specify that the declaration of relevant IPRs must happen before the end of the standardization process. However, this prescription needs to be enforced, and here comes the second implication, for antitrust agencies: Punishing the deceptive conduct of a patent holder that fails to comply with an early-disclosure rule is a necessary condition to limit patent ambush.

6 Patent Disclosure with Awareness

We now turn to the case of B's awareness. We assume that, until firm A discloses the patent, firm B expects the initial technology to be patent-protected with probability $\pi > 0$. Firm A's payoffs for *stop*, *continue*, and *disclose* are not affected by this. Moreover, $\pi > 0$ does not affect firm B's post-disclosure communication incentives, meaning that condition (7) still applies. Likewise, his pre-disclosure payoffs from *stop* at t, $\widetilde{U}_B(stop@t)$, do not change,

$$\widetilde{U}_B(stop@t) = h(t) - \theta h(t-1). \tag{21}$$

His expected payoffs from *continue* at t, $\mathrm{E}_t\widetilde{U}_B(continue@t)$, are equal to

$$\mathrm{E}_t\widetilde{U}_B(continue@t) = (1-\theta)\left[H(t|p) + \pi\left[\widetilde{H}(t|p,q) - H(t|p)\right]\right] \tag{22}$$

with $H(t|p)$ defined in equation (6) for $q = p$ and

$$\widetilde{H}(t|p,q) \;=\; \sum_{i=0}^{\tau-t-1} p^i\,(1-p)\,h(t+i) + p^{\tau-t}\,(1-\sigma(\alpha,\tau))\,H(\tau|q). \qquad (23)$$

If at t, firm B anticipates disclosure at τ, then the communication process continues at most $\tau - t - 1$ periods. Disclosure stage τ is reached with probability $p^{\tau-t}$. Once firm A has disclosed the patent, firm B keeps a share $1-\sigma(\alpha,\tau)$ of his profits stemming from post-disclosure communication, $(1-\theta)\,H(\tau|q)$.

At t, firm B will *continue* if and only if $\mathrm{E}_t\widetilde{U}_B(continue@t) \geq \widetilde{U}_B(stop@t)$,

$$\frac{H(t|p) + \pi\left[\widetilde{H}(t|p,q) - H(t|p)\right] - h(t-1)}{h(t) - h(t-1)} \geq \frac{1}{1-\theta}. \qquad (24)$$

This pre-disclosure communication condition (24) in the case with awareness is analogous to the pre-disclosure communication condition (11) in the case without awareness. Note that for $\pi = 0$, these two conditions are identical. Further note that under awareness, $\pi > 0$, firm B's pre-disclosure communication condition is less restrictive if $\widetilde{H}(t|p,q) > H(t|p)$ and more restrictive otherwise. Because $\widetilde{H}(t|p,q)$ is increasing in q and $H(t|p)$ is independent of q, (24) is less likely to be more restrictive than (11) for higher q. Awareness relaxes the B's pre-disclosure communication constraint if q is sufficiently large. The reverse is true for the pre-disclosure success probability. The difference $\widetilde{H}(t|p,q) - H(t|p)$ is decreasing in p, and and (24) is more likely to be more restrictive than (11) with higher p. Likewise for α. Because σ is increasing in α, firm B keeps a smaller fraction of the post-disclosure payoffs, and (24) is more likely to be more restrictive than (11) with higher α.

For the effect of the disclosure date τ on firm B's per-disclosure decision, we take the first derivative of $\widetilde{H}(t|p,q)$ with respect to τ and obtain

$$\begin{aligned}\frac{\partial \widetilde{H}(t|p,q)}{\partial \tau} &= \frac{\tau}{p^2}(1-p)\,h(\tau-t-1)+\frac{1}{p}\frac{\partial h(\tau-t-1)}{\partial \tau}+\\ &\quad (1-\sigma(\alpha,\tau))\sum_{i=0}^{\infty}q^i(1-q)\left(\frac{\partial h(\tau+i)}{\partial \tau}+h(\tau+i)\right)-\\ &\quad \frac{\partial \sigma(\alpha,\tau)}{\partial \tau}\sum_{i=0}^{\infty}q^i(1-q)\,h(\tau+i). \qquad (25)\end{aligned}$$

The sign of the derivative is ambiguous. Because $H(t|p)$ is independent of τ, condition (24) can thus be more or less restrictive with higher τ than (11).

For firm A, the disclosure decision with awareness is affected by firm B's awareness of patents, $\pi > 0$, insofar as the disclosure decision becomes a constrained maximization problem in the following sense. First, let the optimal disclosure date given unawareness ($\pi = 0$) as determined in Proposition 2 (for unconstrained disclosure with condition (11) satisfied) be denoted by τ^*. If for $\tau = \tau^*$ condition (24) holds for all $t \leq \tau^*$, then as B anticipates disclosure at $t = \tau^*$, he will *continue* for all $t \leq \tau^*$, and A will optimally disclose at $t = \tau^*$. In this case, B's communication condition in (24) is slack. Awareness does not constrain firm A's optimization over τ, and she chooses $\tau = \tau^*$ as in the case of unawareness.

Alternatively, suppose that (24) does not hold for $t = \tau^*$ and all $t \leq \tau^*$. This means that if B anticipates A to disclose at $\tau = \tau^*$, he will not continue the standardization process but *stop* instead. In equilibrium, disclosure cannot be at $\tau = \tau^*$. In this case, condition (24) is binding and B's awareness constrains A's optimization over τ. In Proposition 8 provides the results for equilibrium disclosure with awareness, $\tilde{\tau}$.

PROPOSITION 8 (Disclosure With Awareness)**.** *Suppose condition (7) is satisfied for all t. Let $\pi > 0$ and $\hat{\tau}$ the highest value for $\tau' \leq \tau^*$ such that (24) is satisfied at*

$\tau' \geq 1$. *Then the optimal disclosure stage is equal to* $\tilde{\tau} = \hat{\tau} \leq \tau^*$, *if* $\hat{\tau}$ *exists, and* $\tilde{\tau} = 1$ *otherwise.*

We show in Proposition 8 that if B's awareness constrains firm A's disclosure decision, then in equilibrium there can be no disclosure after τ^*. This is because A cannot commit to disclose at any t later than $t = \tau^*$. The intuition for this is straightforward. Suppose that firm B's pre-disclosure communication condition in (24) is satisfied for some $\tau'' > \tau^*$. Firm B will continue communication for all $t \leq \tau''$ if firm A can credibly commit to delay disclosure until τ'', but because firm A's payoffs are independent of firm B's awareness, once firm A reaches $t = \tau^*$ she will disclose and *not* delay until τ''. Therefore, any disclosure period after τ^* is not subgame-perfect.

If the post-disclosure communication condition (7) is violated so that after disclosure the standard setting process will not be continued in equilibrium, then firm A will *stop* immediately.

PROPOSITION 9. *If condition (7) is violated for some* $t > \tau$, *then firm* A *will* stop *at* $t = 1$.

Propositions 8 and 9 give the disclosure timing under awareness. We summarize the implications of awareness in the following corollary.

COROLLARY 1 (Effect of Awareness on Disclosure)**.** *If the firm not holding a patent is aware of the possibility that the other firm holds a patent, then in equilibrium this awareness* ($\pi > 0$) *will not delay patent disclosure more than in the case of unawareness* ($\pi = 0$). *If with any effect, awareness induces the patent holder to disclose the patent earlier.*

7 Computational Results

In this final section we provide a number of computational results. For the parameterized version of the model we assume that $h(t) = 1 - \beta^t$ and

$$\sigma(\alpha, t) = \left(1 - \gamma^{t-1}\right) \alpha. \tag{26}$$

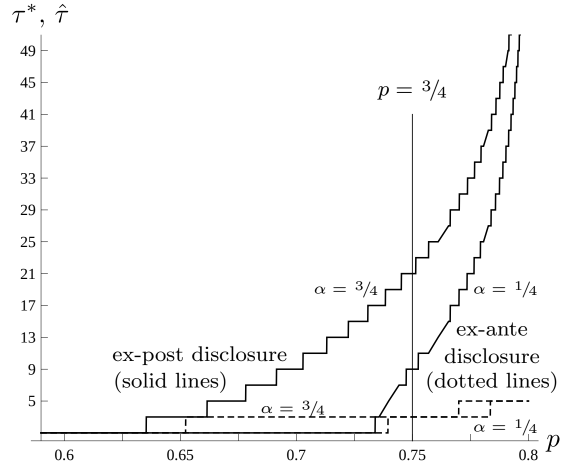
We first calibrate the model under the assumption of patent unawareness and use the parameter values in Table 1. Note that the condition in equation (11) is satisfied as long as $p \geq \frac{\theta}{\beta} = \frac{5}{16}$ (Stein, 2008). Recall that τ^* denotes disclosure with an ex-ante disclosure rule in the case of unawareness (Propositions 2 and 6), $\hat{\tau}$ denotes disclosure with an ex-post disclosure rule in the case of unawareness (Proposition 7), and $\tilde{\tau}$ denotes dislosure in the case of awareness (Propositions 8 and 9).

Table 1: Calibration – Model with Patent Unawareness

Calibration						Cutoffs		Disclose		Duration	
α	β	γ	p	q	θ	$\bar{p}$	$\hat{p}$	τ^*	$\hat{\tau}$	$\bar{T}$	$\hat{T}$
3/4	4/5	3/4	3/4	4/5	1/4	0.652	0.635	3	21	4.6	4
1/4	4/5	3/4	3/4	4/5	1/4	0.739	0.734	3	9	4.6	4.1
3/4	4/5	3/4	3/5	4/5	1/4	0.652	0.635	1	1	5	5

Table 1 provides results for the baseline model with an ex-ante disclosure rule and unawarenes: (1) the critical values $\bar{p}$ and $\hat{p}$ (Propositions 4 and 7) for the lower bound of the pre-disclosure success probability that yields a delay of disclosure, (2) equilibrium disclosure, τ^* and $\hat{\tau}$, and (3) the expected duration of the standardization process. Given $p = 3/4$ and $\alpha = 3/4$ (respectively, $1/4$), firm A will disclose in $t = \tau^* = 3$ in the baseline scenario with the imposed IP waiver, and in $t = \hat{\tau} = 21$ (respectively, 9) in the extension with ex-post disclosure. Instead, if $p = 3/5$ and thus below the critical values (Proposition 4) then disclosure takes place at $t = \tau^* = \hat{\tau} = 1$ in both

Figure 4: Equilibrium Disclosure with Unawareness



the disclosure rules considered. Later disclosure as result of an ex-post disclosure rule implies a shorter expected duration of the standardization process ($\overline{T}$ with ex-ante disclosure rule and $\widehat{T}$ with ex-post disclosure rule)[24]. As long as the patent is not disclosed, the success probability of finding a new component is p and below the post-disclosure success probability. The process is thus more likely to end due to failure of finding a new component, resulting in a shorter expected duration.

In Figure 4 we plot as function of pre-disclosure success probability p the time of ex-ante disclosure τ^* with the IP waiver (dotted lines) and ex-post disclosure $\hat{\tau}$ (solid lines). We see a weak effect of p on the disclosure timing in the baseline case (see Proposition 5). For the extension with ex-post disclosure this effect is much more pronounced. We can also see that with higher patent strenght (α) firms disclose later, given p.

[FIGURE 4 ABOUT HERE]

[24]The expected duration of the standardization process is computed by means of the following formula

$$1 + \sum_{i=0}^{\tau-2} p^i(1-p)i + p^{\tau-1}\sum_{i=0}^{\infty} q^i(1-q)(\tau-1+i).$$

It ranges between 4 (if disclosure is delayed to infinity) and 5 (if immediate disclosure).

Propositions 1 and 6 show that the impact of product market competition on the standardization process is monotonic: a large value of θ jeopardizes agents' incentives to communicate, a low value θ makes the conversation sustainable and spurs disclosure. The parameter values in Table 1 are such that (11) and (7) hold true and A's disclosure decision is unconstrained. In Table 2, we look at the value of τ^* in the baseline case with ex-ante disclosure as θ increases. If $\theta = {}^{16}\!/_{25} > 1/4$ then (11) is violated but (7) holds true: A can salvage the process by disclosing right away, because B would then have incentive to communicate back his idea in the next period. If $\theta = {}^{4}\!/_{5} > {}^{16}\!/_{25}$ neither (11) nor (7) are satisfied, so A stops the process in $t = 1$.

Table 2: Calibration – Constrained Disclosure

Calibration						Disclose
α	β	γ	p	q	θ	τ^*
$^3/_4$	$^4/_5$	$^3/_4$	$^3/_4$	$^4/_5$	$^{16}/_{25}$	1
$^3/_4$	$^4/_5$	$^3/_4$	$^3/_4$	$^4/_5$	$^4/_5$	$\nexists$ (stop at $t = 1$)

Finally, we analyze the impact of patent awareness on the timing of disclosure. Table 3 provides an example in which B knows with probability $\pi = {}^{1}\!/_{2}$ that A has a patent on χ_1 with $\theta = {}^{11}\!/_{20}$. Under unawareness, disclosure takes place at $\tau^* = 3$ and (11) holds true for all values of t. With awareness, condition (24) is not satisfied at $\tau^* = 3$, while it holds for $\tau = 1$ and $\tau \geq 9$. By Proposition 8 we know that in such a case $t = \tilde{\tau} = 1$, so the expected duration of the standardization process is longer under awareness.

Table 3: Calibration - Model with Patent Awareness

Calibration							Disclose		Duration	
α	β	γ	p	q	π	θ	τ^*	$\tilde{\tau}$	$\overline{T}$	$\widetilde{T}$
$^3/_4$	$^4/_5$	$^3/_4$	$^3/_4$	$^4/_5$	$^1/_2$	$^{11}/_{20}$	3	1	4.6	5

8 Concluding Remarks

We have presented a model of communication with asymmetric information, based on the work by Stein (2008), with which we endogenize the magnitude of patent holdup to study the effect on the timing of patent disclosure of patent strength, the productivity of industry standard setting, and a standard setting organization's IPR disclosure rules. We find that late disclosure is more likely in more productive standard setting organizations and in less competitive industries. The intuition for the former result is that delaying patent disclosure increases the patent holder's bargaining leverage, which in turn results in higher license fees the more valuable the standard is. The latter result arises from the observation that rent extraction via opportunistic licensing is the more profitable the higher are the firms' market profits. Moreover, enforcing a standard setting organization's IPR disclosure rules (i.e., ex-ante disclosure rule), results in earlier disclosure. Recent litigation and the ongoing debate on the role of antitrust in standard setting[25] underscore the relevance of our results for the evaluation of legal and organizational policy.

[25] See chapter 2 in U.S. Dep't of Justice & Fed. Trade Comm'n (2007).

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A Appendix

Proof of Proposition 1

Proof. We assume a cooperative equilibrium with disclosure at $t = 1$ exists, implying that communication of ideas for components at all stages. We show that for sufficiently high θ the joint payoffs from continuing communication are smaller than from not continuing, i.e.,

$$\mathrm{E}U^C(continue@t) < U^C(stop@t) \tag{A.1}$$

for some t. As the expected joint payoffs are higher when the probability of success is $q > p$, it is straightforward to assume that A has disclosed that patent at $t = 1$. It suffices to show that there are values of θ such that the condition in (A.1) holds for some t. The joint payoffs from continuing are

$$\mathrm{E}U^C(continue@t) = 2\,(1-\theta)\sum_{i=0}^{\infty} q^i\,(1-q)\,h(t+i),$$

the joint payoffs from stopping are

$$U^C(stop@t) = h(t) + (1-2\theta)\,h(t-1).$$

By $h(t) > h(t-1)$, $U^C(stop@t) > 0$ for all θ; $\mathrm{E}U^C(continue@t) = 0$ for $\theta = 1$ and strictly positive otherwise. The critical value $\theta^C(q, h(\cdot))$ for which $\mathrm{E}U^C(continue@t) = U^C(stop@t)$ is strictly smaller than unity so that there are some $\theta > \theta^C(q, h(\cdot))$ for which (A.1) holds. Note, also, that this critical value is strictly larger than 0.5. Suppose for a moment that

$$\mathrm{E}\tilde{U}^C(continue@t) = 2\,(1-\theta)\sum_{i=0}^{\infty} q^i\,(1-q)\,h(t) = 2\,(1-\theta)\,h(t).$$

$\mathrm{E}\tilde{U}^C(continue@t) = U^C(stop@t)$ for $\theta = 0.5$, and the condition in equation (A.1) holds for $\theta > 0.5$. Because $h(t) < h(t+i)$ for all $i > 0$, $\mathrm{E}U^C(continue@t) > \mathrm{E}\tilde{U}^C(continue@t)$ and get $\theta^C(q, h(\cdot)) > 0.5$. Q.E.D.

Proof of Proposition 2

Proof. For the sake of this proof, we assume that $t \in (0, 1) \subset \mathbb{R}_+$, so that t draws on real numbers bigger than unity. This simplifies the analysis without loss of generality. Moreover, for notational simplicity, let $\mathrm{E}_t(@t) := \mathrm{E}_t U_A(disclose@t)$ and $\mathrm{E}_t(@t{+}2) := \mathrm{E}_t U_A(disclose@t{+}2)$. Consider the following properties of the expected payoff functions $\mathrm{E}_t(@t)$ in equation (12) and $\mathrm{E}_t(@t+2)$ in equation (13).

P1. $\mathrm{E}_t(@t)$ and $\mathrm{E}_t(@t+2)$ are strictly increasing in t because $\sigma(\alpha, t)$ (for $\alpha > 0$), $h(t)$, $h(t+1)$, and $H(t|q)$ are strictly increasing in t.

P2. Because $\lim_{t\to\infty} h(t+k) = 1$ for all $k \geq 0$ and $\lim_{t\to\infty} \sigma(\alpha, t) = \alpha$, we get

$$\lim_{t\to\infty} \mathrm{E}_t(@t) = (1-\theta)(1+\alpha), \tag{A.2}$$

$$\lim_{t\to\infty} \mathrm{E}_t(@t+2) = (1-\theta)\left(1+p^2\alpha\right), \tag{A.3}$$

P3. The value of $\mathrm{E}_t(@t)$ lies in a bounded space,

$$\mathrm{E}_t(@t) \in [\mathrm{E}_1(@1), \mathrm{E}_\infty(@\infty))$$

with $\mathrm{E}_1(@1) = (1-\theta)\, H(1|q) > 0$.

LEMMA A.1. *In the limit, the expected payoffs from delaying disclosure one round are* strictly *smaller than the payoffs from disclosing right away,* $\lim_{t\to\infty} \mathrm{E}_t(@t) > \lim_{t\to\infty} \mathrm{E}_t(@t+2)$.

Proof. By **P3** and $p < q \leq 1$. Q.E.D.

LEMMA A.2. *If* $\mathrm{E}_1(@1) < \mathrm{E}_1(@3)$, *then there exists a finite value* $\tilde{t} > 1$ *such that* $\mathrm{E}_t(@t+2) \leq \mathrm{E}_t(@t)$ *for all* $t \geq \tilde{t}$ *and* $\mathrm{E}_t(@t+2) > \mathrm{E}_t(@t)$ *for all* $t < \tilde{t}$.

Proof. By **P3** and the intermediate value theorem. Q.E.D.

LEMMA A.3. *The assumption that* $\mathrm{E}_t(@t)$ *and* $\mathrm{E}_t(@t+2)$ *intersect at most once implies that if condition (14) does not hold and* $\mathrm{E}_1(@1) > \mathrm{E}_1(@3)$ *then* $\mathrm{E}_t(@t) \geq \mathrm{E}_t(@t+2)$ *for all* $t \geq 1$.

Proof. By Lemma A.1. This situation is depicted in panel *(b)* of Figure 3. Q.E.D.

The proof for claim 2 of the proposition follows straight from Lemma A.3, which states that in t an expected-profit maximizing firm A prefers disclosing in t to waiting one round and disclosing in $t+2$. This result holds for all t, hence, firm A in $t+2$ prefers disclosing in $t+2$ to waiting one round and disclosing in $t+4$. Anticipating her stage-$t+2$ decision in t, the firm in t prefers disclosing in t to waiting two rounds and disclosing in $t+4$; and so forth. By this argument, firm A will disclose the patent in $t = 1$.

P4. Expected payoffs $\mathrm{E}_t(@t+2)$ can be rewritten as an increasing function of $\mathrm{E}_t(@t)$:

$$\mathrm{E}_t(@t+2) := p^2 \rho(\mathrm{E}_t(@t)) \tag{A.4}$$

with

$$\rho(\mathrm{E}_t(@t) := \mathrm{E}_t(@t) + (1-\theta)\left(\phi + \frac{\sum_{k=0}^{1} p^k\,(1-p)\, h(t+k)}{p^2}\right) \tag{A.5}$$

where

$$\phi = (1+\sigma(\alpha, t+2))\, H(t+2|q) - (1+\sigma(\alpha,t))\, H(t|q)$$

and

$$\mathrm{E}_{t+2}(@t+2) = \mathrm{E}_t(@t) + (1-\theta)\,\phi = (1-\theta)\,(1+\sigma(\alpha, t+2))\, H(t+2|q).$$

With property **P4** we can formulate firm A's present value maximization problem in a recursive fashion. For further notational simplicity, let $D := \mathrm{E}_t(@t)$ and $\rho(D) := \rho(\mathrm{E}_t(@t)) = \mathrm{E}_t(@t+2)/p^2$. Consequently, the problem of firm A can be rewritten as

$$\mathcal{P}: \quad V(D) = \max\left\{D, p^2 V[\rho(D)]\right\}. \tag{A.6}$$

Moreover, let $\underline{D} = \mathrm{E}_1(@1)$, $\tilde{D} = \mathrm{E}_t(@\tilde{t})$, and $\rho(\tilde{D}) = \mathrm{E}_t(@\tilde{t}+2)/p^2$ with $\tilde{t}$ defined in Lemma A.2. In $\mathcal{P}$, D is the state variable and the objective is to determine the timing of disclosure. Three of the necessary conditions, guaranteeing that a fixed point that solves $\mathcal{P}$ exists and is unique (Stokey and Lucas, 1989), hold true:

NC1. D takes values in a bounded set [by **P3**]

NC2. $\rho(D)$ is increasing in D [by **P4**]

NC3. $\exists \tilde{D} : \rho(\tilde{D}) = \tilde{D}/p^2$ [by Lemma A.2]

These three conditions are necessary to establish the existence of a functional fixed point to the stopping problem we are analyzing. Yet, the additional condition that has to be discussed regards the initial condition, that is, the condition on the value of the payoffs associated with disclosure right away instead of waiting until $t = 3$. Two cases must be distinguished, depending on whether the initial condition prescribes immediate disclosure or not.

Case (i) If $\rho(\underline{D}) < \underline{D}/p^2$, then, by Lemma A.3 the initial condition prescribes that disclosure should take place right away.

In the following we study case (ii), in which at $t = 1$ the agent finds it profitable to delay disclosure. The objective of the analysis that follows is to show that a function (or simple rule) that prescribes to disclose at some $\tau \geq \tilde{t} > 1$ exists and is unique. Note that because the disclosure stage τ is restricted to odd integers, but $\tilde{t}$ can be any real number larger than unity, τ is defined as

$$\tau = \begin{cases} \lceil \tilde{t} \rceil & \text{if } \lceil \tilde{t} \rceil \text{ is an odd integer} \\ \lceil \tilde{t} \rceil + 1 & \text{if otherwise} \end{cases} \tag{A.7}$$

For **Case (ii)** we assume $\rho(\underline{D}) \geq \underline{D}$ (i.e., equation (14) holds) and $\mathrm{E}_t(@t)$ and $\mathrm{E}_t(@t+2)$ intersect at most once. Referring to **NC3**, this is the case if

$$\forall D > \tilde{D} : p^2\rho(D) < D \quad \text{and} \quad \forall D < \tilde{D} : p^2\rho(D) > D.$$

The contraction mapping theorem can be applied and a simple stopping rule exists. To show this, we first prove that Blackwell's monotonicity and discounting conditions are satisfied (Blackwell, 1965). An operator T is a contraction mapping if the following two conditions hold:[26]

[26] In the following we use $f(\cdot)$ and $g(\cdot)$ to denote the candidate solution to our functional fixed point problem.

Monotonicity: $\forall x,\ f(x) \le g(x)$ then $Tf(x) \le Tg(x)$ for all x.

Monotonicity is satisfied because, if $\forall x\ f(x) \le g(x)$, then $p^2 f(\rho(D)) \le p^2 g(\rho(D))$ and $\max\left\{D, p^2[f(\rho(D))]\right\} \le \max\left\{D, p^2[g(\rho(D))]\right\}$. Monotonicity implies that if $f(x) \le g(x)$ then the objective function for which $\max\left\{D, p^2[g(\rho(D))]\right\}$ is the maximized value is uniformly higher than the function for which $\max\left\{D, p^2[f(\rho(D))]\right\}$ is the maximized value.

Discounting: For a scalar a define $(f+a)(x) = f(x)+a$. $\exists\beta \in (0,1)$, $T(f+a)(x) \le Tf(x) + \beta a$, for all $f, a \ge 0$ and x in the state space.

Discounting is satisfied because the following holds:

$$\begin{aligned}\max\left\{D, p^2[f(\rho(D)) + a]\right\} &\le \max\left\{D + p^2 a, p^2[f(\rho(D)) + a]\right\} \\ &= \max\{D, p^2[f(\rho(D))]\} + p^2 a.\end{aligned}$$

Consequently, the functional problem $\mathcal{P}$ has a unique fixed point $V(\cdot)$. In other words, we can identify a unique function that solves the maximization problem in $\mathcal{P}$ for each value of the state variable; such a function provides a rule that prescribes the optimal decision (disclose/delay) depending on the value of the state variable D. More specifically, the functional fixed point $V(\cdot)$ is increasing, meaning that if $f(D') \le f(D'')\ \forall D' \le D''$, then $p^2 f(D') \le p^2 f(D'')$ and $\max\left\{D, p^2 f(\rho(D'))\right\} \le \max\left\{D, p^2 f(\rho(D''))\right\}$.

To conclude the proof, we determine the optimal simple disclosure rule by following the next two steps, where, as above, $f(\cdot)$ denotes a candidate fixed point solution.

1. Assume $\forall D > \tilde{D} : f(D) \le D$. Then

$$\max\{D, p^2 f(\rho(D))\} \le \max\{D, p^2 f(D/p^2)\} = p^2 \max\{D/p^2, f(D/p^2)\} = D,$$

   meaning that once we start with a function f that satisfies the assumption all the future iterations stick to it and the same happens to the fixed point. This implies that $\forall D > \tilde{D} : V(D) = D$. Hence, A should disclose for all $D > \tilde{D}$.

2. Assume $\forall D < \tilde{D} : f(D) > D$. Then

$$\begin{aligned}\max\{D, p^2 f(\rho(D))\} \ge \max\{D, p^2 f(D/p^2)\} &= p^2 \max\{D/p^2, f(D/p^2)\} \\ &= p^2 f(D/p^2) > p^2 D/p^2 = D.\end{aligned}$$

   Also in this case, once we start with a function f that satisfies the assumption the future iterations and the fixed point stick to it. This implies that $\forall D < \tilde{D} : V(D) > D$, that is, A should not disclose for all $D < \tilde{D}$.

Therefore, the optimal rule prescribes disclosure if and only if

$$\forall D \ge \tilde{D} : \rho(\tilde{D}) = \tilde{D}/p^2.$$

Strictly speaking, such a rule suggests to disclose at the lowest $t \ge \tilde{t}$, where $\tilde{t}$ is defined in Lemma A.2 and t an odd integer. Q.E.D.

Proof of Proposition 3

Proof. By Proposition 2, and the regularity condition therein, the necessary and sufficient condition for firm A to delay patent disclosure at $t = 1$, so that $\tau > 1$, is

$$\mathrm{E}_1 U_A(disclose@1) \leq \mathrm{E}_1 U_A(disclose@3).$$

After some manipulation, we can rewrite this as

$$\sum_{k=0}^{1} \left[q^k (1-q) - p^k (1-p)\right] h(1+k) \leq \left[(1+\sigma(\alpha,3))\, p^2 - q^2\right] \sum_{k=0}^{\infty} q^k (1-q)\, h(3+k).$$

For the proof of the proposition, we show that, given $q > p$, the necessary and sufficient condition for delayed disclosure is not satisfied. This means, for $\alpha = 0$ so that $\sigma(\alpha, 3) = 0$, we show that

$$\left[q^2 - p^2\right] \sum_{k=0}^{\infty} q^k (1-q)\, h(3+k) + \sum_{k=0}^{1} \left[q^k (1-q) - p^k (1-p)\right] h(1+k) > 0.$$

This expression can be rearranged to read

$$\sum_{k=0}^{\infty} q^k (1-q)\, h(1+k) - \sum_{k=0}^{1} p^k (1-p)\, h(1+k) - p^2 \sum_{k=0}^{\infty} q^k (1-q)\, h(3+k) > 0$$

and, by the definition of $H(t|q)$ in equation (6) for $t = 1$ and $t = 3$,

$$H(1|q) - \sum_{k=0}^{1} p^k (1-p)\, h(1+k) - p^2 H(3|q) > 0. \tag{A.8}$$

To show that this last inequality holds for all $q > p$, first note that

$$H(1|q) = \sum_{k=0}^{1} q^k (1-q)\, h(1+k) + q^2 H(3|q).$$

If

$$\sum_{k=0}^{1} q^k (1-q)\, h(1+k) + q^2 H(3|q) > \sum_{k=0}^{1} p^k (1-p)\, h(1+k) + p^2 H(3|q) \tag{A.9}$$

then (A.8) holds with strict equality and condition (14) in Proposition 2 is violated for $\alpha = 0$. We can rewrite (A.9) as

$$h(1) + q\left[h(2) - h(1)\right] + q^2\left[H(3|q) - h(2)\right] > h(1) + p\left[h(2) - h(1)\right] + p^2\left[H(3|q) - h(2)\right].$$

It holds if

$$H(3|q) - h(2) > 0. \tag{A.10}$$

Because $H(3|q) = (1-q)\, h(3) + \sum_{k=1}^{\infty} q^k\,(1-q)\, h(3+k)$, (A.10) holds true if and only if

$$h(3) - h(2) + \sum_{k=1}^{\infty} q^k\,(1-q)\, h(3+k) - qh(3).$$

We can further expand the summation to get

$$h(3) - h(2) + q\,[h(4) - h(3)] + \sum_{k=2}^{\infty} q^k\,(1-q)\, h(3+k) - q^2 h(4)$$

and

$$q^0\,[h(3) - h(2)] + q^1\,[h(4) - h(3)] + q^2\,[h(5) - h(4)] + \sum_{k=3}^{\infty} q^k\,(1-q)\, h(3+k) - q^3 h(5).$$

As we continue the expansion, the last term, $q^i h(2+i)$ is equal to zero in the limit, since $i \to \infty$. All other terms, $q^i\,[h(3+i) - h(2+i)]$ are strictly positive so that (A.10) holds true. Q.E.D.

Proof of Proposition 4

Proof. We prove the claim by applying the intermediate value theorem. First, note that $\mathrm{E}_t U_A(disclose@1) - \mathrm{E}_t U_A(disclose@3)$, or

$$(1-\theta)\left\{\sum_{k=0}^{1}\left[q^k\,(1-q) - p^k\,(1-p)\right] h(1+k) - \left[(1+\sigma(\alpha,3))\,p^2 - q^2\right] H(3|q)\right\}, \tag{A.11}$$

is strictly positive for $q > 0$ and $p = 0$. The expression in (A.11) can be rewritten[27] as

$$(1-\theta)[H(1|q) - h(1)] > 0.$$

This inequality holds by equation (7) for $t = 1$ and because $\theta > 0$ and $h(0) = 0$. Note that equation (7) holds by equation (11), which is the underlying assumption of this section's analysis.

If, instead, $p = q$, then (A.11) is reduced to

$$-q^2\sigma(\alpha,3)H(3,q)(1-\theta) < 0.$$

For a given q, (A.11) is continuous in p and strictly decreasing in p with the first derivative

[27]Note, that $\lim_{p\to 0} p^0 = 1$.

with respect to p,

$$\begin{aligned} -(1-\theta)\left\{\sum_{k=0}^{1}\left[kp^{k-1}(1-p)-p^k\right]h(1+k)+2p\,(1+\sigma(\alpha,3))\,H(3|q)\right\} &= \\ -(1-\theta)\left\{[-h(1)+(1-2p)\,h(2)]+2p\,(1+\sigma(\alpha,3))\,H(3|q)\right\} &= \\ -(1-\theta)\left\{\left[h(2)-h(1)+2p\sum_{k=0}^{\infty}q^k\,[h(3+k)-h(2+k)]\right]+2p\sigma(\alpha,3)H(3|q)\right\} &< 0. \end{aligned}$$

To summarize, $\mathrm{E}_t U_A(disclose@1) - \mathrm{E}_t U_A(disclose@3)$ is strictly positive (firm A discloses at $t=1$) for $p=0$ and strictly negative (firm A delays disclosure) for $p=q$; moreover, it is continuous and strictly decreasing in p. Hence, by the intermediate value theorem, there exists a value $\bar{p} := \bar{p}(q,\sigma(\cdot),h(\cdot))$ with $\bar{p}\in(0,q)$ for the pre-disclosure probability p such that $\mathrm{E}_t U_A(disclose@1) > \mathrm{E}_t U_A(disclose@3)$ and disclosure at $t=1$ for all $p<\bar{p}$; and $\mathrm{E}_t U_A(disclose@1) \le \mathrm{E}_t U_A(disclose@3)$ and disclosure at a later stage for all $p \ge \bar{p}$. Q.E.D.

Proof of Proposition 5

Proof. By Lemma A.2, $\tilde{t}$ is such that

$$F := \mathrm{E}_{\tilde{t}} U_A(disclose@\tilde{t}) - \mathrm{E}_{\tilde{t}} U_A(disclose@\tilde{t}+2) = 0.$$

We can define τ as

$$\tau = \begin{cases} \lceil \tilde{t} \rceil & \text{if } \lceil \tilde{t} \rceil \text{ is an odd integer} \\ \lceil \tilde{t} \rceil + 1 & \text{if otherwise} \end{cases} \tag{A.12}$$

By this definition, an increase in $\tilde{t}$ is a measure for firm A's *propensity* to delay disclosure.

By the implicit function theorem,

$$\frac{d\tilde{t}}{dp} = -\left.\frac{\partial F}{\partial p}\right/\frac{\partial F}{\partial \tilde{t}}$$

and

$$\frac{d\tilde{t}}{d\alpha} = -\left.\frac{\partial F}{\partial \alpha}\right/\frac{\partial F}{\partial \tilde{t}}\ .$$

By Lemma A.2, $F>0$ for $t>\tilde{t}$ and $F<0$ for $t<\tilde{t}$. Hence, F is increasing in t at $\tilde{t}$; $\frac{\partial F}{\partial \tilde{t}} > 0$. Moreover,

$$\frac{\partial F}{\partial p} = -(1-\theta)\sum_{i=0}^{\infty} q^i\left[h(\tilde{t}+2+i)-h(\tilde{t}+1+i)\right] < 0.$$

Hence,

$$\frac{d\tilde{t}}{dp} = -\left.\frac{\partial F}{\partial p}\right/\frac{\partial F}{\partial \tilde{t}} > 0$$

and $\tilde{t}$, as a measure for the propensity to delay disclosure, is increasing in the pre-disclosure success probability p.

For the effect of α on $\tilde{t}$, we find that

$$\frac{\partial F}{\partial \alpha} = (1-\theta)\left[\frac{\partial \sigma(\alpha,\tilde{t})}{\partial \alpha} H(\tilde{t}|q) - p^2 \frac{\partial \sigma(\alpha,\tilde{t}+2)}{\partial \alpha} H(\tilde{t}+2|q)\right].$$

This expression is negative, and $d\tilde{t}/d\alpha > 0$, if and only if condition (15) holds true. Q.E.D.

Proof of Proposition 6

Proof. We begin the proof by showing that if condition (11) does not hold for *all* t, then firm B will *stop* at $t = 2$. Put differently, if (11) holds for all $t < \hat{t}$, but is violated for $t \geq \hat{t}$, communication does not continue until $t = \hat{t} - 1$. This is by a simple backward-induction argument. Let $\hat{t}$ be even so that firm B is the one to stop (the following argument applies also to an odd $\hat{t}$). At $\hat{t} - 1$, firm A will either *stop* or *continue*. If she stops, her payoffs are $U_A(stop@\hat{t}-1) = (1-\theta)\, h(\hat{t}-1) + \theta[h(\hat{t}-1) - h(\hat{t}-2)]$. If she continues, with probability p she expects firm B to have another idea but *stop* the process. Her respective payoffs are $(1-\theta)\, h(\hat{t}-1)$. With probability $1-p$, she expects firm B to fail; her respective payoffs are $(1-\theta)\, h(\hat{t}-1)$. Then, her payoffs from *continue* are equal to $\mathrm{E}_{\hat{t}-1} U_A(continue@\hat{t}-1) = (1-\theta)\, h(\hat{t}-1)$. Because $h(\hat{t}-1) > h(\hat{t}-2)$, $U_A(stop@\hat{t}-1) > \mathrm{E}_{\hat{t}-1} U_A(continue@\hat{t}-1)$.

Anticipating that firm B stops at $\hat{t}$ induces firm A to stop at $\hat{t} - 1$. At $\hat{t} - 2$, firm B decides whether to *continue* or *stop*. By the very same argument, anticipating that firm A stops at $\hat{t}-1$ induces firm B to stop at $\hat{t}-2$. The process unravels, and firm B stops at $t = 2$ if condition (11) is violated for some t.

Because $\mathrm{E}_1 U_A(continue@1) < U_A(stop@1)$ (given that condition (11) does not hold), firm A will not continue at $t = 1$ absent disclosure. But she may decide to disclose the patent at $t = 1$. Indeed, disclosure can only happen at $t = 1$; because if A continues without disclosing at $t = 1$ the game ends at $t = 2$. Moreover, given that disclosure takes place at $t = 1$, $\sigma(\alpha, 1) = 0$.

We can now provide the proof of the proposition: If after disclosure a communication equilibrium cannot be sustained (that is, if (7) does not hold for all t), then firm A will not disclose. Instead, if (7) holds, then firm A discloses at $t = 1$.

1. First, note that if (7) does not hold for all t, then firm B will stop at $t = 2$. This is by the argument provided above for pre-disclosure communication. firm A's payoffs if she stops are $U_A(stop@1) = (1-\theta)\, h(1) + \theta[h(1) - h(0)] = h(1)$. Her payoffs for disclosure are $\mathrm{E}_1 U_A(disclose@1) = (1-\theta)\, h(1)$. For all $\theta > 0$, $U_A(stop@1) > \mathrm{E}_1 U_A(disclose@1)$ and firm A stops.

2. Second, condition (7) implies condition (10); after disclosure a communication equilibrium can be sustained. firm A's payoffs for disclosure are

$$\mathrm{E}_1 U_A(disclose@1) = \mathrm{E}_1 U_A(continue@1|1) = (1-\theta)\, H(1|q).$$

She will disclose if $(1-\theta)\, H(1|q) \geq h(1) = U_A(stop@1)$ or, for $h(0)=0$,

$$\frac{H(1|q)}{h(1)} = \frac{H(1|q)-h(0)}{h(1)-h(0)} \geq \frac{1}{1-\theta}.$$

If (7) holds for all t, then it holds for $t=1$, and the above condition holds. This concludes the proof. Q.E.D.

Proof of Lemma 1

Proof. *Stop* is dominated by *continue* if $\mathrm{E}_t\widehat{U}_A(continue@t) \geq \widehat{U}_A(stop@t)$ if and only if

$$\frac{\widehat{H}(t|p)-(1+\sigma(\alpha,t))\, h(t-1)}{h(t)-h(t-1)} \geq \frac{1}{1-\theta}. \tag{A.13}$$

We assume condition (11) holds; and (11) implies (A.13) if and only if

$$\widehat{H}(t|p)-(1+\sigma(\alpha,t))\, h(t-1) \geq H(t|p)-h(t-1)$$

or

$$\sum_{i=0}^{\infty} p^i\,(1-p)\, h(t+i)\sigma(\alpha,t+i) \geq \sigma(\alpha,t)h(t-1). \tag{A.14}$$

Because

$$\sum_{i=0}^{\infty} p^i\,(1-p)\, h(t+i) = h(t) + \sum_{i=0}^{\infty} p^i\,(1-p)\,[h(t+i)-h(t)] > h(t-1)$$

and $\sigma(\alpha,t+i)$ an increasing weight on the LHS, (A.14) holds with strict equality. Q.E.D.

Proof of Proposition 7

Proof. We want to show that there exists a cutoff value $\hat{p}$ for pre-sucess probability p such that (i) greater values of p trigger disclosure delay at $t=1$ and (ii) such a cutoff value is lower than in the case with ex-ante disclosure ($\bar{p}$).

We apply the intermediate value theorem. First, note that

$$\mathrm{E}_1\widehat{U}_A(disclose@1) - \mathrm{E}_1\widehat{U}_A(disclose@3),$$

or

$$(1-\theta)\left[H(1|q) - \sum_{k=0}^{1} p^k\,(1-p)\, h(1+k)(1+\sigma(\alpha,1+k)) - p^2(1+\sigma(\alpha,3)H(3|q)\right], \tag{A.15}$$

is strictly positive for $q>0$ and $p=0$. The expression in (A.15) can be rewritten[28] as

$$(1-\theta)[H(1|q)-h(1)] > 0.$$

[28]Note, that $\lim_{p\to 0} p^0 = 1$.

This inequality holds by equation (7) for $t = 1$ and because $\theta > 0$ and $h(0) = 0$. Note that equation (7) holds by equation (11), which is the underlying assumption of this section's analysis.

If, instead, $p = q$, then (A.15) is reduced to

$$- (1-\theta)\left[q^2\sigma(\alpha,3)H(3,q) + \sum_{k=0}^{1} q^k (1-q)\, h(1+k)(1+\sigma(\alpha,1+k))\right] \;<\; - (1-\theta)[q^2\sigma(\alpha,3)H(3,q) \;<\; 0.$$

For a given q, (A.15) is continuous in p and strictly decreasing in p with the first derivative with respect to p equal to

$$- (1-\theta)\left\{\sum_{k=0}^{1}\left[kp^{k-1}(1-p) - p^k\right](1+\sigma(\alpha,1+k))h(1+k) + 2p\,(1+\sigma(\alpha,3))\,H(3|q)\right\} =$$
$$- (1-\theta)\left\{[-h(1) + (1-2p)\,h(2)(1+\sigma(\alpha,2))] + 2p\,(1+\sigma(\alpha,3))\,H(3|q)\right\} =$$
$$- (1-\theta)\left\{(1+\sigma(\alpha,2))h(2) - h(1) + 2p\sum_{k=0}^{\infty} q^k\,[h(3+k)(1+\sigma(\alpha,3+k)) - h(2+k)(1+\sigma(\alpha,2+k))]\right\} \;<\; 0.$$

To summarize, $\mathrm{E}_1\widehat{U}_A(disclose@1) - \mathrm{E}_1\widehat{U}_A(disclose@3)$ is strictly positive (firm A discloses at $t = 1$) for $p = 0$ and strictly negative (firm A delays disclosure) for $p = q$; moreover, it is continuous and strictly decreasing in p. Hence, by the intermediate value theorem, there exists a value $\hat{p} := \hat{p}(q, \sigma(\cdot), h(\cdot))$ with $\hat{p} \in (0, q)$ for the pre-disclosure probability p such that $\mathrm{E}_1\widehat{U}_A(disclose@1) > \mathrm{E}_1\widehat{U}_A(disclose@3)$ and disclosure at $\hat{\tau} = 1$ for all $p < \hat{p}$; and $\mathrm{E}_1\widehat{U}_A(disclose@1) \leq \mathrm{E}_1\widehat{U}_A(disclose@3)$ and disclosure at a later stage for all $p \geq \hat{p}$.

Finally, since for $p = 0$, the expressions in (A.11) and (A.15) take the same value and for $p = q$ (A.15) is lower than (A.11), $\hat{p}$ is smaller than $\bar{p}$ (the cutoff value defined in the proof of Proposition 4). This implies that for $p \in [\hat{p}, \bar{p})$ firm A delays disclosure under ex-post disclosure while it discloseses her patent under ex-ante disclosure at stage $t = 1$. Q.E.D.

Proof of Proposition 8

Proof. For the proof of the proposition, consider Figure A.1 below. It depicts

$$\phi(\tau) = \frac{H(t|p) + \pi\left[\widetilde{H}(t|p,q) - H(t|p)\right] - h(t-1)}{h(t) - h(t-1)} - \frac{1}{1-\theta},$$

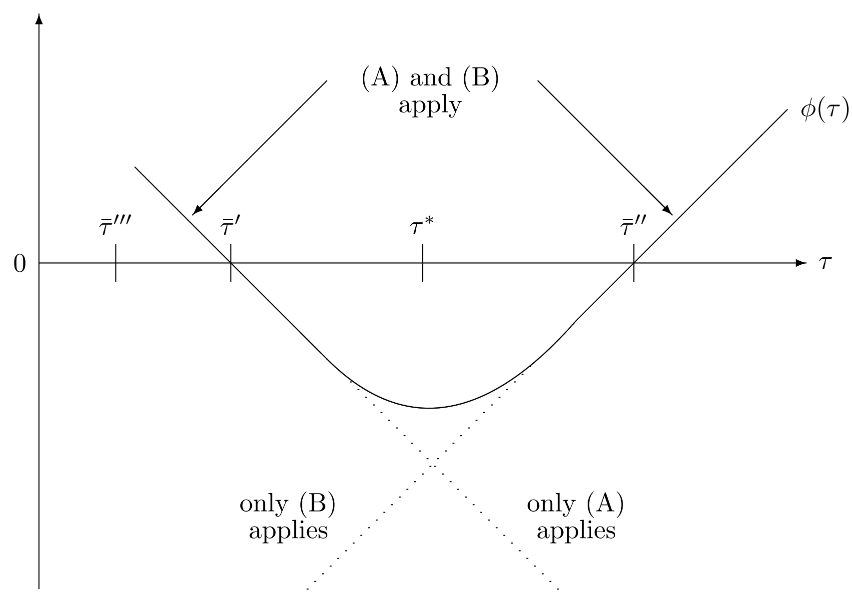
the difference between the LHS and the RHS in condition (24). Condition (24) holds if $\phi \geq 0$ and is violated otherwise. Suppose $\phi < 0$, then there are two non mutually exclusive cases:

(A). There exist some $\bar{\tau}' < \tau^*$ such that (24) holds at $\bar{\tau}'$ for all $t \leq \bar{\tau}'$.

(B). There exist some $\bar{\tau}'' > \tau^*$ such that (24) holds at $\bar{\tau}''$ for all $t \leq \bar{\tau}''$.

In Figure A.1, the downward sloping part of the solid curve (with the dotted extension) represents case (A): Given $\phi(\tau) < 0$ at τ^*, there exist values $\bar{\tau}' < \tau^*$ such that $\phi(\tau) \geq 0$. The upward sloping part of the solid curve (with the dotted extension) represents case (B): Given $\phi(\tau) < 0$ at τ^*, there exist values $\bar{\tau}'' > \tau^*$ such that $\phi(\tau) \geq 0$. Moreover, if neither (A) nor (B) is true, then there is no $\tau \geq 1$ such that (24) is satisfied.

Figure A.1: Cases for Condition (24) in Proposition 8



We show the proof of the proposition with the following four steps:

1. If $\phi(\tau^*) \geq 0$, then $\tau = \tau^*$, as shown in the text.

The following two steps relate to cases (B) and (A), respectively:

2. Patent disclosure is not later than τ^*, $\tau \not> \tau^*$. First, note that if there is no such $\bar{\tau}''$ such that $\phi(\bar{\tau}'') \geq 0$, then firm B will not continue until $\bar{\tau}''$. Second, suppose case (B), at least one $\bar{\tau}'' > \tau^*$ exists, and (24) holds at $\bar{\tau}''$ but not at τ^*. For communication to continue until $\bar{\tau}''$, disclosing at $\bar{\tau}''$ must be an optimal strategy for firm A. But, by Proposition 2, firm A discloses at τ^*. Because firm A cannot commit to disclosing at $\bar{\tau}''$, firm B will anticipate disclosure at τ^*, and thus not continue communication. Hence, $\tau \not> \tau^*$.

3. Suppose case (A), at least one $\bar{\tau}' < \tau^*$ exists, and (24) holds at $\bar{\tau}'$ but not at τ^*. Moreover, suppose there is a $\bar{\tau}''' < \bar{\tau}'$ such that (24) holds. By Proposition 2, firm A prefers disclosure at $\bar{\tau}'$ over $\bar{\tau}'''$, or more generally, prefers to delay disclosure for all $t \leq \tau^*$. If there is a range of $\tau' < \tau^*$ such that (24) holds, firm A will disclose at the highest of these values, denoted by $\bar{\tau}$,

$$\bar{\tau} \equiv \max\left\{\tau' < \tau^* : \text{condition (24) holds at } \tau'\right\}.$$

Finally, it is not an optimal strategy for A to disclose at some $\tau > \hat{\tau}$ because firm B anticipating disclosure at $\tau > \bar{\tau}$ will not continue communication.

The last step covers to the situation where (24) is violated and (A) does not apply.

4. If no $\bar{\tau}' \geq 1$ exists, so that $\bar{\tau}$ does not exist, then firm A will disclose at $t = 1$. This case is analogous to the second claim in Proposition 6. Any strategy with later disclosure will induce B to discontinue communication at $t = 2$. firm A discloses at $t = 1$ to salvage the standardization process because post-disclosure communication continues as condition (7) is assumed to hold. Note that the existence of a $\tau'' > \tau^*$ (case (B)) is irrelevant, because as shown above, $\tau \not> \tau^*$.

To summarize: If (24) holds then $\bar{\tau} = \tau^*$ and firm A discloses at $\tilde{\tau} = \bar{\tau} = \tau^*$. If (24) does not hold, then, if it exists (case (A) applies), $\bar{\tau} < \tau^*$ and $\tilde{\tau} = \bar{\tau}$. If $\bar{\tau}$ does not exist (case (A) does not apply), then $\tilde{\tau} = 1$. Q.E.D.

Proof of Proposition 9

Proof. Note that $H(t|q) > \widetilde{H}(t|p, q)$ so that (24) implies (7). This means, if (7) does not hold, then (24) is violated. The proof is analogous to the proof of the first claim in Proposition 6. Q.E.D.