Firm Lifecycle Financing

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Abstract

This study presents a lifecycle model of firms based on endogenous changes in profitability that arise from successful product development. The model generates a lifecycle profile for investment and financing needs even in the absence of any financing frictions. Empirical tests using data on listed and unlisted U.K. firms support the main lifecycle predictions. In particular, the model generates the lifecycle profile of profitability changes observed in the data. Further, young firms realize more profitability jumps than older firms, and the effect of age on investment and financing decisions is stronger for young firms. This last result suggests that estimates from data sets with mostly mature public firms likely understate true firm lifecycle effects.

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1 Introduction

Firms are born, live, and die. The extant literature on firm lifecycles emphasizes the role of financing frictions, either in the form of costly external finance (see Cooley and Quadrini (2001) and Cabral and Mata (2003)) or in the form of agency costs (see DeAngelo, DeAngelo, and Stulz (2006) and DeAngelo, DeAngelo, and Stulz (2010)). However, Angelini and Generale (2008) document that costly external finance explains only a small portion of the effect of firm age on firm size. Further, agency conflicts between managers and owners are unlikely to explain the behavior of young private firms, as these firms are mostly managed by owner-entrepreneurs. In addition, models based on financing frictions cannot explain the fact that young firms realize profitability increases, on average.[1] Thus, the question of what drives the lifecycle behavior of firms remains partially unresolved.

This study presents a dynamic lifecycle model of firms based on endogenous profitability changes. The key feature is that firms spend resources on product development, which generates profitability increases. Even though the model contains no financing frictions, it captures the main lifecycle properties of financing and investment. Young firms spend resources on growth and market share; successful mature firms generate positive cash flows to investors; and firms exit as they either fail to generate quality improvements or as demand for their product wanes over time.[2]

The following key assumptions underpin the model. First, new entrants have low product quality, which limits the price an entrant can charge for its product. However, firms can expend resources on product development that, if successful, generate an outward shift in the demand curve for the firm's product.[3] Next, the success probability of product development depends on the ratio of product development expenses to the capital stock.[4] This lowers the incentive for large mature firms to invest in product development, implying that these firms generate fewer quality increases. Last, new entrants have higher output productivity than incumbents, ensuring that all firms eventually exit as they become less competitive over time.[5]

These features combine to generate an endogenous lifecycle profile of firms' profitability. Low product quality translates to lower profitability of entrants. However, the investment in product

[1] Author's calculation. Section 4.2 details how average profitability changes as a function of firm age.

[2] The economics of the model reflect the well-known product lifecycle theory in the marketing literature. See Kotler and Armstrong (2004), among others, for a discussion of the product lifecycle theory.

[3] This is a common assumption in the quality ladder literature. See Grossman and Helpman (1991a) and Grossman and Helpman (1991b).

[4] A similar alternative would be to assume that the success probability decreases with the quality level as in Aghion and Howitt (1992). Either assumption ensures that profitability - and therefore firm size - remains bounded.

[5] See Foster, Haltiwanger, and Syverson (2008) for evidence on the productivity advantage of entrants.

development by young firms implies that these firms generate positive profitability increases, as observed in the data. Profitability declines slowly as firms age due to the combination of the reduced incentives for quality improvements and the higher output productivity of new entrants. This lifecycle behavior of profitability generates a lifecycle for the firm's investment and financing decisions.

The model captures the salient features of a firm's lifecycle. In spite of their output productivity advantage, entrants are smaller than incumbents due to their quality disadvantage.[6] Low product quality combined with high product development expenses implies that young firms have negative cash flows, requiring them to obtain external finance. Young firms that successfully generate quality increases exhibit high sales growth and physical investment, while firms that fail to do so exit. High cash flows and low investment in physical capital and product development result in mature firms paying out a substantial portion of their earnings as dividends.

The analysis of a data set obtained by simulating the model verifies the key lifecycle features. Young firms exhibit more frequent quality increases and higher sales growth and investment rates. These firms also obtain external finance more frequently than mature firms. Mature firms pay high dividends, whereas young firms pay low dividends or none at all.[7] Furthermore, the effects of age on firms' policies are much stronger for young firms than mature firms. This reflects the fact that quality increases primarily affect the behavior of young firms. Instead, the lifecycle behavior of mature firms is mainly driven by their slow deterioration in competitiveness relative to new entrants.

This study tests these predictions using data on U.K. firms from the Bureau van Dijk Amadeus data set. This data set provides balance sheet and income statement data on public and private firms in the U.K. The underlying source for the data set comes from compulsory filings at the Companies House in the U.K. The main advantage of this data set is that it has much better coverage of small and young firms than data sets with mostly public firms such as Compustat.

The data reveal that firms with age less than five years realize profitability increases, on average. These increases are statistically and economically significant, with firms realizing a cumulative profit increase of about 5 percent from birth to age five. From then on, average profit changes remain mostly negative. These findings demonstrates that the lifecycle profile of profitability highlighted in the model matches that observed in the data.

[6] Models that focus only on productivity differences have difficulty generating this result, as firm size covaries positively with productivity.

[7] DeAngelo, DeAngelo, and Stulz (2006) and DeAngelo, DeAngelo, and Skinner (2009) argue that the lifecycle effect on dividend payments arises due to the agency costs associated with retaining free cash flow within the firm.

Using this data, a logit regression of profitability jumps on firm age and controls reveals that young firms generate profitability jumps more frequently than mature firms. This result remains robust to different definitions of profitability jumps. In addition, the effect of age on profitability jumps is stronger for younger firms, as in the simulated data set. This indicates that the effect of endogenous profitability changes is most potent for the youngest firms.

Regression analysis of the Amadeus data also reveals a clear lifecycle effect on investment, sales growth and external financing. Young firms have higher sales growth and investment rates, and they obtain external funds more frequently than mature firms. Further, repeating the analysis after separating the sample into young and mature firms reveals that age has a stronger effect on firms' policies for young firms than for mature firms, consistent with the model. This finding suggests that studies such as DeAngelo, DeAngelo, and Stulz (2006) and DeAngelo, DeAngelo, and Stulz (2010) may understate the effect of age on firms policies, as their data mostly consists of mature public corporations.

The above analysis suggests that endogenous profitability changes can generate the lifecycle effects observed in the data. However, the absence of any financing friction is a stark and unrealistic assumption. As such, an extension of the model incorporates costly external finance. Simulating the model with plausible calibrations for financing costs demonstrates that these financing costs have only a small effect on the age and quality distribution of firms, similar to the finding of Angelini and Generale (2008). This suggests that endogenous changes in profitability plays a bigger role in understanding lifecycle decisions of firms than external financing costs.

Other related studies on entrepreneurship and firm dynamics include Albuquerque and Hopenhayn (2004), who study firm dynamics in a model with optimal debt contracts. Bitler, Moskowitz, and Vissing-Jorgensen (2005) find that higher entrepreneurial ownership stakes lead to increased effort and performance. Cagetti and De Nardi (2006) demonstrate that a model of entrepreneurship in the presence of borrowing constraints can generate the observed distribution of wealth in the U.S. economy. The internal governance model of Acharya, Myers, and Rajan (2009) generates a lifecycle for dividend payments. However, their lifecycle begins from the IPO of the firm, whereas this study generates a firm lifecycle from the birth of the firm.

This study is organized as follows. Section 2 presents the model. Section 3 analyzes the data set obtained by simulating the model. Section 4 discusses the Bureau van Dijk data used in the study. Section 5 examines the main lifecycle features using this data. Section 6 extends the model to incorporate financing costs and Section 7 concludes.

2 Model

The model economy consists of a large number of heterogenous firms, each of which produces a differentiated product. The quantity produced by a firm varies with its productivity, capital stock and labor input. The price of its product depends on the product quality, with higher quality products earning a higher price.[8] Firms invest resources in physical investment and quality improvements in order to maximize the present value of dividends over their lifetimes.

2.1 Output, demand, and profits

Each firm uses capital, K, and labor, L, to produce a single product using a Cobb-Douglas production function. The average productivity of an existing firm of age a at time t is assumed to be fixed by its vintage $t-a$.[9] Firms with a later vintage have a higher average productivity level, reflecting the higher productivity of entrants documented by Foster, Haltiwanger, and Syverson (2008). The output of a firm of age a at time t is given by

$$Y_t(K, L, a) = (\mu_{t-a}\tilde{z})^{1-\alpha} K^{\alpha} L^{1-\alpha}, \tag{1}$$

where $\tilde{z}$ denotes transient shocks to productivity. This specification is standard in the investment literature and implies that the firm employs the optimal capital-to-labor ratio in the absence of labor adjustment costs.

The output of all firms are combined into a consumption aggregate, C, using a quality-weighted Dixit-Stiglitz aggregator with constant elasticity of substitution.

$$C = \left(\int_i q_i Y_i^{1-\nu} di \right)^{\frac{1}{1-\nu}},$$

where q_i denotes the quality level of good i. A product of higher quality generates higher utility

[8] One important distinction this study makes is to differentiate between output productivity and product quality. In the model, productivity measures the quantity of goods that can be produced by a unit of inputs and quality measures the relative importance of that good in the consumption aggregator. The profitability of a firm depends on both its productivity, which enables a firm to produce more, and its quality, which increases the demand for its good. The commonly used revenue-based measure of productivity in the literature measures the combination of output productivity and product quality. Other studies that make this distinction include Foster, Haltiwanger, and Syverson (2008) and Fasil (2009).

[9] Allowing the productivity of all existing firms to grow at a constant rate only affects the rate at which firm values are detrended. It will have no effect on the firm's optimal policies. For simplicity, I assume no productivity growth for incumbents.

per unit of output. Normalizing the aggregate price level to 1, the price charged for good i is given by

$$P(Y_t) = q_i Y_t^{-\nu} C_t^{\nu}, \tag{2}$$

where the mark-up ν depends on the elasticity of substitution between two goods. This specification implies that a product of higher quality commands a higher price for the same quantity, reflecting its higher weight in the consumption aggregate, C.

Let w_t denote the wage level. The gross profit of a firm is given by

$$\Pi_t(K, a, \tilde{z}; q_i) = \max_L P(Y_t)Y_t - w_t L, \tag{3}$$

where the price, $P(Y_t)$, and output, Y_t, are given by equations (2) and (1), respectively. Some algebra yields the following expression for profits:

$$\Pi_t(K, \tilde{z}, a; q_i) = (1-\theta)\left(\frac{\theta}{w_t}\right)^{\frac{\theta}{1-\theta}} \left[q_i C_t^{\nu} (\mu_{z,t-a}\tilde{z})^{1-\nu} K^{\alpha(1-\nu)}\right]^{\frac{1}{1-\theta}}, \tag{4}$$

where $\theta = (1-\alpha)(1-\nu)$. The mark-up in the pricing equation (2) leads to decreasing returns to scale in the profit function, implying a bounded optimal firm size. In comparison, studies that follow Lucas (1978) generate an optimal firm size based on the assumption of a non-reproducible factor such as managerial talent.

The above profit function trends with the consumption aggregator, C_t, and the wage level, w_t. In order to solve for the value function and simulate the economy, one needs to detrend the profit function as in Eberly, Rebelo, and Vincent (2008). The appropriate detrending variable, X_t, is given by:

$$X_t = \left(\frac{\mu_t}{w_t}\right)^{\frac{\theta}{\nu}} C_t. \tag{5}$$

Detrending by X_t accounts for the productivity growth of new entrants, and the trend growth of the consumption aggregator and the wage level. Using lower case letters to denote the detrended variables, let $\pi = \frac{\Pi_t}{X_t}$ and $k = \frac{K_t}{X_t}$. The detrended profit function is given by

$$\pi(k, \tilde{z}, a; q_i) = (1-\theta)(\theta)^{\frac{\theta}{1-\theta}} \left[q_i (1+g_\mu)^{-a} \tilde{z}^{1-\nu} k^{\alpha(1-\nu)}\right]^{\frac{1}{1-\theta}}, \tag{6}$$

where g_μ denotes the productivity growth rate of entrants. The subsequent analysis employs this detrended profit function.

2.2 Endogenous product quality

The above discussion examines the effect of firm age on profitability while treating product quality, q_i, as fixed. However, firms spend significant resources on product development. These include not only research and development expenditures, but also expenditures such as advertising that can potentially increase the demand for a good.

In the model, firms have product quality levels q_i indexed by i. The quality levels are linear in the index, i, which ranges from 1 to an upper limit $\bar{n}$.[10] New firms enter the economy with a product quality index of 1. Each period, a firm spends resources on product development, denoted by s. If the firm's product development was successful it realizes an increase in product quality, with probability of success increasing in s.[11] Thus, a firm with a series of good draws sees a rapid increase in product quality while a firm with a series of bad draws languishes at a low quality level. Formally, the evolution of product quality is given by the following equation:

$$\begin{aligned} q_i' &= q_{i+1} \text{ with probability } p(s) \text{ and,} \\ &= q_i \text{ with probability } 1 - p(s), \end{aligned} \tag{7}$$

where q_i' denotes the next period quality level of a firm with current quality level q_i.

The success probability of generating a quality increase is parameterized as follows:

$$p(s) = 1 - \exp\left(-b\frac{s}{k}\right), \tag{8}$$

where b is a parameter that influences the success rate. The above exponential function provides a parsimonious approach to modeling success rates. It implies that the marginal probability of success decreases as s increases. Warusawitharana (2010) employs a similar specification.

The scaling of the success probability with the capital stock, k, helps limit firm growth. This

[10] The assumption of an upper limit on product quality is for technical reasons. In effect, the framework for the evolution of product quality implies that the model will have an endogenous upper limit beyond which firms will no longer invest in product development.

[11] This setup reflects the basic structure of the quality ladder literature. Klette and Kortum (2004) and Lentz and Mortensen (2008) presents a different approach where each firm can increase the number of products they sell. Opp (2010) examines asset prices in an endogenous growth model with quality ladders.

scaling implies that, due to the decreasing returns to scale in the profit function (6), firms will reduce investment in product development as their quality level (and therefore capital stock) increases. Thus, successful firms will find it increasingly more costly to generate further successes. Without this, or some other, limit on quality increases, most firms will converge to the highest quality index, $\bar{n}$.

2.3 Discussion of lifecycle mechanics

Although notationally complex, the economics underlying the model are quite simple. The evolution of profitability with firm age drives the lifecycle in the model. Profitability depends on changes in the firm's output productivity and product quality. Equation (6) presents the effect of firm age on productivity, while equations (7) and (8) specify how product quality evolves over time.

The higher output productivity of new firms implies that the competitiveness of existing firms slowly deteriorates over time. However, this effect is offset by stochastic increases in product quality, which translates into higher demand for the product. Young firms enter the economy with quality index 1. These firms can realize a large increase in firm value through a quality increase, leading them to expend significant resources in product development. Firms increase physical investment following a quality increase as the increased demand for the good increases the optimal capital stock. At the beginning, the increases in product quality will overcome the effect of relative productivity declines, resulting in increasing profitability with age. As quality increases and firms mature, they reduce spending on product development, and the effect of firm age on productivity dominates the effect of quality increases. These changes in profitability with firm age generate the key lifecycle features of the model.

2.4 Other firm policies

2.4.1 Investment

Each period, firms can invest in new capital. Denote new investment by i. The detrended next period capital of a firm is given by

$$k'(1+g_x) = k(1-\delta) + i,$$

where δ denotes the depreciation rate.[12] In addition, firms face a quadratic adjustment cost of investment, given by $\lambda\frac{i^2}{2k}$. This adjustment cost is commonly used in the investment literature (see Hayashi (1982)) and helps limit the volatility of investment.

2.4.2 Entry and exit

A fixed cost of operating each period, c, implies that firms will exit if their value falls below a certain threshold. Firms exit each period after they realize their idiosyncratic shock $\tilde{z}$. Exiting firms sell their capital at a discounted price f. Firms decide to exit optimally if their continuation value is lower than the value they would obtain by selling their capital.

New firms enter the economy with quality index i. They have an initial capital stock of K_0 and begin operations immediately. In the steady state, the rate of entry is assumed to equal the rate of exit.

2.4.3 Taxation

The model incorporates a simplified tax structure. Firms pay linear taxes on operating profits net of fixed costs, depreciation, and product development expenses. The last term reflects the tax deductibility of research and development and advertising expenses. Denote the tax rate by τ. The taxes payable by a firm is given by

$$\tau\left(\pi(k,\tilde{z},a;q_i) - c - \delta k - s\right).$$

2.5 Firm value

Firm value and optimal policies can be derived from a sequence of Bellman equations indexed by the quality index, i. The value function for quality level q_i is given by:

$$\begin{aligned} v(k,\tilde{z},a;q_i) &= \max_{i,s,k'} \; d + \beta(1+g_x)E_z[jv_c(k',\tilde{z}',a+1;q_{i+1}) + (1-j)v_c(k',\tilde{z}',a+1;q_i)], \\ s.t. \quad (1+g_x)k' &= k(1-\delta) + i, \\ j &\sim \mathcal{B}\left(p(s)\right), \\ \text{with} \quad d &= (1-\tau)\left(\pi(k,\tilde{z},a;q_n) - c - s\right) + \tau\delta k - i - \lambda\frac{i^2}{2k}, \end{aligned}$$

[12] The $(1+g_x)$ term arises from the division by X_t to detrend the capital accumulation equation.

where j is a binary variable that equals 1 if the firm obtains a quality increase. The success probability, $p(s)$, is given in equation (8). The $(1 + g_x)$ terms appear in the above expression to take into account the effect of growth as in Eberly, Rebelo, and Vincent (2008). The possibility of exit implies that the continuation value equals

$$v_c(k, \tilde{z}, a; q_i) = \max\{v(k, \tilde{z}, a; q_i), fk\}.$$

In economic terms, the above Bellman equation states that the value of a firm in a given quality stage equals the dividend payment plus the expected future value of the firm. Negative dividend values correspond to equity issuance. The expected future value of the firm takes into account that, with probability $p(s)$, the firm may realize an increase in the quality index. The discount rate for the future remains unchanged over the lifecycle, reflecting the finding of Moskowitz and Vissing-Jorgensen (2002) that private firms generate similar returns as public firms. Each firm chooses its physical investment and product development expense to maximize value.

The following proposition establishes the existence and uniqueness of the sequence of value functions indexed by the quality index.

Proposition 1 *There exists a sequence of unique value functions $v(k, \tilde{z}, a; q_i)$.*

Proof. See Appendix A. ■

The optimal policies of a firm can be derived from the above sequence of Bellman equations.

2.6 Optimal product development expenses

Firms trade off the cost of product development expenses for the higher probability of an increase in the quality index. The firm value increase with the quality index reflects both the ability of the firm to sell current output at a higher price, and the ability of the firm to invest and increase production in the future to take advantage of the shift in the demand curve.

The following proposition establishes the optimality condition for product development expenses:

Proposition 2 *The first order condition for product development implies that*

$$\begin{aligned} \textit{marginal cost of funds} &= \textit{marginal benefit of product development expenses} \\ \Rightarrow \qquad 1-\tau &= \beta(1+g_x)E\left[\frac{v(k',\tilde{z}',a+1;q_{i+1})-v(k',\tilde{z}',a+1;q_i)}{k}\right]b(1-p(s)). \end{aligned}$$

Proof. See Appendix A. ■

Bolton, Chen, and Wang (2009) obtain a similar expression for investment in their model. The marginal benefit from product development expenses increases with the expected increase in firm value from a quality increase per unit of current capital. This expression decreases as quality increases, due to the decreasing returns to scale nature of the profit function. This leads to lower product development expenses. The $(1-p(s))$ term implies that an increase in the success probability $p(s)$ lowers the marginal benefit, thereby ensuring an interior solution for product development expenses. The marginal cost of funds in the above expression equals $1-\tau$, reflecting the tax deductibility of product development expenses.

3 Model simulation and analysis

The model links age to firms' policies through the evolution of profitability over the lifecycle. The stochastic nature of the lifecycle and the complexity of the model makes it difficult to generate explicit propositions relating age to the firm policies. As such, this section investigates the lifecycle effects in the model using data obtained from simulating a calibrated version of the model.

3.1 Calibration

The calibration of the model parameters employs values either estimated or commonly used in the literature where possible. The remaining parameters are set to generate plausible values for moments of interest.

The following parameters are set to values commonly used in the literature. The capital share of output, α, equals 0.33. The price markup parameter ν equals 0.25. These imply that $\theta = 0.5$. The depreciation rate is set at an annual rate of 10 percent. The discount rate $\beta = 0.952$, corresponding to a annual real return to capital (equity) of 5 percent.

The corporate tax rate is set at 20 percent. This follows the marginal tax rate applicable for

small companies (defined as those with profits less than 300,000 pounds) in the U.K., which ranged from 19 to 21 percent over the sample period.[13]

The adjustment cost parameter λ equals 3. This value is slightly higher than the estimates obtained by Cooper and Haltiwanger (2006). However, their model includes a range of other adjustment costs as well, while the model in this study has only a quadratic term. The resale value of capital is set at 0.8, in between the estimate of 0.6 obtained by Hennessy and Whited (2005) and the estimate of 0.95 obtained by Cooper and Haltiwanger (2006).

The success rate parameter for quality improvements, s, is set to 3.5. Warusawitharana (2010) estimates this parameter using research and development expenses by public firms and obtains estimates between 2 and 5. This parameter translates to mean product development expense of about 34 percent of capital for firms of age 1.

The remaining parameters are calibrated to generate plausible moments. The autocorrelation and standard deviation of productivity shocks are set to 0.6 and 0.05, respectively. The quality level at index 1, $q_1 = 0.14$. This translates to an average value of Tobin's Q of about 1.3. An increase in the quality index increases the quality level by 0.0025. The fixed costs generate an exit rate of about 4.2%.

3.2 Model solution and simulation

The sequence of value functions given in equation (9) can be solved by backward induction. First, solve the value function at quality index $\bar{n}$, and then solve backwards for all quality indices from $\bar{n} - 1$ to 1. The solution employs the optimal product development policy given in Proposition 2.

The simulated data set is obtained by simulating the model economy with 2000 firms over a period of 80 years. Observations in the first 30 years are discarded as a burn-in sample. The simulation uses the optimal policies derived from solving the sequence of value functions. Each period, exiting firms are replaced by new entrants. This simulated sample provides a steady state cross-section of firms that can be employed to investigate firm policies in the model. Appendix B provides further detail on the numerical solution of the model and the construction of the simulated sample.

[13]The marginal tax rate increases to 30 percent for companies with profits over 1.5 million pounds.

3.3 Lifecycle profile of profitability changes

Figure 1 presents average profitability changes as a function of age from the simulated data set. The underlying data excludes the first and last years of a firm's life. The figure demonstrates a strong lifecycle effect on profitability changes for firms. Young firms obtain sharp profitability increases, reflecting quality increases from successful product development. As firms age, they realize fewer quality increases. Beyond five years of age, the competitive pressures from the productivity advantage of new entrants dominates the effect of quality increases. This leads to negative average profitability changes. Overall, the figure demonstrates that the model captures a strong lifecycle effect on profitability changes of firms.

3.4 Regression analysis of simulated data

This section reports the results of various regressions of firms' policies on age and other controls using the simulated data set. These regressions shed light on the impact of firm age on policies of interest in the model and provide a comparison with the subsequent results obtained using data from U.K. firms. The regression specification exactly matches that used with the actual data. Whited (2006), Bloom, Bond, and van Reenen (2007), and Warusawitharana (2008), among others, use the same approach of comparing regression results from a simulated model with the results from actual data.

Table 1 presents the results of regressions of various financial policies on firm age and controls. Panel A presents linear panel regressions for sales growth, investment, and dividends. Panel B presents logit regressions on whether the firm obtained a quality increase and on whether it obtained external financing. The availability of actual data constrain the controls used in the regressions.[14] All the regressors are statistically significant at the 5 percent level, reflecting the importance of the regressors in the model.

The results demonstrate that younger firms generate quality increases at a much higher rate than older firms, reflecting the higher product development expenses of young firms. The higher rate of quality increases translates a higher sales growth rate. Further, firms respond to the quality increases by increasing investment, as seen by the higher investment rates of younger firms. Increased product development expenses and high investment rates imply that young firms require external financing more often. In contrast, mature successful firms become self-financing and pay

[14] For instance, the Bureau van Dijk data on firms does not include firm value as most of the firms are privately held. This makes it impossible to construct Tobin's Q with the actual data.

dividends as they generate higher cash flows and have lower investment rates and product development expenses.[15] Taken together, these findings demonstrate that the model captures a clear lifecycle effect on firms' policies.

The analysis demonstrates a linear effect of age on the firm's policies. However, the impact of age may differ over the firm's lifecycle. Young firms will obtain quality increases at a higher rate than mature firms who, for the most part, will have already achieved a high quality level. Thus, changes in age translate to bigger changes in product quality and profitability for young firms. This implies that firm age will have a greater effect on the policies of young firms than mature firms. This hypothesis can be tested by replicating the lifecycle regressions after separating the simulated sample into young and mature firms.

Table 2 present the results of replicating the analysis in Table 1 on firms with age less than or equal to its median value. Table 3 presents the corresponding results for firms with age greater than the median value. In both tables, Panel A presents linear panel regressions for sales growth, investment, and dividends; Panel B presents logit regressions on whether the firm obtained a quality increase and on whether it obtained external financing.

The results indicate a strong effect of age on firms' policies for firms with age below its median value. In this subsample, a firm that is one year younger will have higher product development expenses, resulting in more frequent quality increases. These quality increases lead to higher sales growth and investment rates. The lower cash flows and high expenditures imply that younger firms obtain external finance more frequently and that they pay less dividends. Further, the effect of age is economically larger for this sample for all the regressions except the investment regression.

In contrast, the quality ladder mechanism has no effect on the mature firms. The undefined coefficients on the logit regression on quality increases arises due to the absence of any quality increases in this subsample.[16] Instead, the lifecycle effect for mature firms is driven by the increasing output productivity of new entrants, which weakens the competitive position of firms as they age. This results in mature firms reducing investment, and therefore sales growth, as they age. However, these firms continue to generate positive cash flows, enabling them to increase dividends with age.

[15] DeAngelo, DeAngelo, and Stulz (2006) obtain a similar result with Compustat data using retained earnings as a proxy for the firm's lifecycle stage.

[16] This stark results highlights a limitation of the model. Many large public corporations generate demand increases through the introduction of new or improved products. Extending the model to incorporate differences in the ability of firms to generate quality increases would enable the model to incorporate quality increases by large firms. However, young firms would continue to generate quality increases at a higher rate in the extended model.

4 Data

This study tests the predictions of the model using data from the Amadeus database maintained by Bureau van Dijk. This data set provides balance sheet and income statements for public and private firms in many European countries. One limitation of the data set is that it does not include information on dividends or product development expenses such as research and development and advertising.

The analysis uses data on firms from the U.K. to avoid any cross country differences. Focusing only on U.K. firms helps mitigate any cohort effects that may arise from the introduction of the euro in continental Europe. Further, the accounting regulations in the U.K. require firms to file annual accounts at the Companies House. These legally required filings provide the source data for the Amadeus data set.[17]

4.1 Sample

The sample period extends from 1997 to 2007, as the data set contains few observations in the years prior to 1997. Firms with missing values for total assets, year of incorporation, or revenue are excluded from the sample. Firms with less than 5 employees are also excluded to eliminate self-employed individuals that have chosen to incorporate as a firm. The sample excludes any firms with maximum total assets less than 500,000 pounds over the sample period. The sample also excludes observations of firms that are either inactive or in bankruptcy proceedings, observations with accounting periods other than one year, and observations of financial firms, as identified by 2-digit SIC codes. All observations are rescaled to take into account different units of observations for different firms in the data set.[18]

The study uses the following variable definitions. Firm age is measured from the year of incorporation. Firm size equals the log of total assets. Sales growth is defined as the growth rate of operating revenues. Profitability equals operating profits before interest and depreciation divided by lagged total assets. Physical investment is constructed as fixed assets minus lagged fixed assets plus depreciation divided by lagged fixed assets. All variables except firm age and size are Winsorized at the 95 percent level to reduce the impact of outliers.

[17] The source data for the U.K. data is the same as that for the FAME database, which is also maintained by Bureau van Dijk. Brav (2009) provides a detailed description of how Bureau van Dijk constructs the FAME database.

[18] Amadeus reports most observations of firms in UK in units of pounds. But, some firms have values reported in thousands of pounds and a few firms have values reported in millions of pounds.

One key variable in the subsequent analysis is a profitability jump dummy that attempts to capture whether firms realize large profitability increases. The profitability jump dummy equals 1 if the difference between the current period profitability level and the average profitability over the past three years is greater than 0.10. Computing the difference using the average over the past three years helps mitigate misclassification errors arising from a one-period decrease in profitability that reverses in the next period. Section 5.3 examines the robustness of the results to different definitions of the profitability jump dummy.

One limitation of the data is that it does not provide direct measures of whether the firm obtains external finance. As such, this study constructs two measures of financing using balance sheet data. The equity issue dummy variable equals one if the firm's contributed capital was greater than last period's contributed capital plus 2 percent. The external financing dummy variable equals one if the sum of the firms contributed capital, debt, and bank loans was greater than the corresponding last period value plus 2 percent. Although these measures are not ideal, they provide a good approximation for whether the firm obtained external finance.

4.2 Profitability changes over the lifecycle

The model emphasizes the role of profitability changes over the lifecycle as the main driver of lifecycle behavior by firms. As such, it is clearly important to examine whether the observed profitability changes over the lifecycle are consistent with those implied by the model. Figure 2 plots the mean change in firms' profitability from age t to $t+1$ as a function of age, t.[19] The dashed (blue) lines represent the 95 percent confidence interval around the estimated sample means. Figure 1 presents the corresponding figure from the simulated data set.

The figure demonstrates that firms with age less than or equal to 5 years generate profitability increases, on average. These profitability increases are statistically and economically significant, with a cumulative profitability increase of more than 5 percent over this period. Average profitability changes become negative, for the most part, after firms become more than 10 years of age. This lifecycle profile of profitability changes matches that generated by the model - young firms generate profitability increases from improvements in product quality while mature firms face a slow decline in profitability. Section 5 uses regression analysis to examine whether the data supports the lifecycle effects predicted by the model.

[19] The lines begins with age 2. Although firms of age 1 generate profitability increases, on average, the very small number of observations with age 1 results a in wide confidence interval around the estimate.

4.3 Summary statistics

Table 4 presents the summary statistics for the data used in the regression analysis. Panel A presents the summary statistics for the entire sample. Panels B and C present the summary statistics for the young and mature firm subsamples, respectively. Young firms comprise those with age less than or equal to its median value, while mature firms comprise those with age greater than its median value.

The summary statistics reveal that firms realize profitability increases of more than .10 about once every 8 years. However, young firms realize profitability jumps at a substantially higher rate than mature firms. This suggests that the average profitability increases of young firms documented in Figure 2 arises from the higher rate of profitability jumps realized by young firms.

The other summary statistics also demonstrate a marked effect of age on firms' policies. On average, young firms have higher sales growth, profitability, and investment rates than mature firms. Young firms also obtain external financing at a higher frequency than mature firms. This difference is particularly notable for equity issues. The differences in the mean values across the young and mature firm subsamples are statistically significant at the 5 percent level. These findings are broadly consistent with the lifecycle model detailed in Section 2.

5 Results

This section presents the findings from estimating the lifecycle regressions on data from Amadeus. Comparison of these regression results with those obtained using simulated data in Section 3.3 informs whether the lifecycle features of the model reflect the actual data. Unfortunately, the data set does not include information on product development expenses such as research and development or advertising. As such, it is not possible to test this prediction directly. However, an analysis of the effect of firm age on policies such as investment and equity issuance sheds light on whether endogenous changes in profitability can generate the lifecycle effects of interest.

5.1 Full sample analysis

Table 5 presents the results of the lifecycle regressions. Panel A presents the panel regressions with sales growth and investment as dependent variables. Panel B presents the logit regressions with the probability jump dummy and two measures of external financing as dependent variables. The

regressions include year dummies and industry dummies constructed at the 2-digit SIC level. The reported standard errors are heteroskedasticity robust and adjust for clustering at the firm level. These results mimic the analysis of the simulated sample reported in Table 1 with the exception of the dividend regression.[20]

Young firms realize profitability jumps - defined as a profitability increase of more than 0.10 from its lagged 3-year moving average - at a higher rate than older firms. In economic terms, the results translate to about a 0.10 percent increase in the probability of obtaining a profitability jump for a firm that is one year younger. This compares with an overall probability of obtaining profitability jumps of 12.3 percent. This finding supports the mechanism highlighted in the model, where higher product development expenses generate more frequent quality increases for young firms.

The analysis also reveals that the lifecycle stage of the firm strongly affects sales growth and investment. Young firms exhibit higher sales growth and investment rates, even after accounting for their smaller size. These differences are statistically and economically significant. A firm that is one year younger will have, on average, a 0.31 percent and 0.22 percent higher sales growth and investment rates, respectively. These results suggest that the profitability jumps obtained by younger firms lead to higher sales growth and investment rates, as in the model.

The logit regressions reveal that younger firms obtain external financing more frequently.[21] This effect is particularly striking for the equity issue dummy. A firm that is one year younger obtains equity financing at a 0.09 percent higher rate, compared to an overall rate of equity financing of 5.7 percent. Defining external financing using book values of debt and equity generates a statistically significant - albeit economically weaker - effect of firm age.

Comparison of the estimates for the control variables provides another gauge of the model. The sales growth and profitability coefficients obtained using the actual data have the same sign and significance as those obtained with the simulated data. This indicates that the ability of the model to capture the lifecycle effect does not come at the expense of the effects of profitability and sales growth on firms' policies.

[20]The Amadeus data does not include dividend payments.

[21]DeAngelo, DeAngelo, and Skinner (2009) find a significant effect of firm lifecycles on secondary equity issuance. Their sample consists of public firms, whereas the sample in this study consists mostly of private firms.

5.2 Split sample analysis

The model predicts that lifecycle effects will be much stronger for young firms than mature firms, as verified by the analysis of the simulated data. This section examines this prediction using the Amadeus data.

Table 6 reports the results of replicating the lifecycle regressions reported in the previous section on firms with age less than or equal to its median value in the data set. The regressions include year and industry dummies at the 2-digit SIC code level, and the standard errors adjust for heteroskedasticity and cluster at the firm level. Table 2 presents the corresponding results with the simulated data.

The results indicate a much stronger effect of age for young firms. Within this subsample, a firm that is one year younger has about a 0.35 percent higher probability of obtaining profitability jumps. Both in absolute terms and relative to the subsample average, this is much higher than the corresponding age effect on profitability jumps for the entire sample. This matches the age effect observed in the model, where quality increases from product development main affects young firms.

The analysis also reveals a much stronger age effect on sales growth and investment. In economic terms, a firm that is one year younger will have a 2.6 percent higher sales growth rate and a investment rate that is about 0.6 percent higher in this subsample. The logit regressions reported in Panel B also demonstrate a stronger effect of firm age on equity issues for young firms. However, age becomes insignificant when external financing is measured using the book values of debt plus equity in this subsample.

Table 7 presents the results of the lifecycle regressions on firms with age greater than its median value. The regressions include year and industry dummies at the 2-digit SIC code level, and the standard errors adjust for heteroskedasticity and cluster at the firm level. Table 3 presents the corresponding results with the simulated data.

The results reveal that the effect of age on firms is much weaker for this subsample. The age coefficients for all the regressions except for external financing are significantly less negative than those obtained with young firms in Table 6. These findings demonstrate that the effect of age weakens significantly when one focuses on more mature firms. This reflects the pattern observed in the model, where changes in product quality has little effect on mature firms. Instead, lifecycle effects arise in mature firms due to the slow decline in profitability arising from the output productivity advantage of new entrants.

Furthermore, the mature firm subsample results have uniformly lower adjusted and pseudo R-

squared values than the young firm subsample. This indicates that the predictors in the regression have lower overall explanatory power, consistent with a smaller role for the firm's lifecycle stage. As before, the profitability and sales growth coefficients for the mature firm subsample have similar qualitative properties in the actual and simulated data sets.

5.3 Robustness

The results obtained from the analysis of the simulated data set remain robust to changes in the calibrated parameter values. Although the estimated coefficients change, the main findings remain. Young firms obtain more frequent quality increases, which result in higher sales growth and investment rates. High product development expenses and physical investment lead young firms to obtain external finance more frequently. Further, the effect of firm age on these policies is much more pronounced for younger firms.

One key finding from the Amadeus data set is that younger firms realize profitability jumps more frequently. Table 8 examines the robustness of this result to changing the definition of profitability jumps to require a profitability increases of 0.05, 0.15, and 0.20, respectively. Changing the cutoff value for the profitability jump dummy has little effect on the result. As before, younger firms realize more profitability jumps, with the effect of age being stronger for the young firm subsample. Further, in unreported results, defining profitability jumps using the increase in profitability from the last period value leads to similar results.

The empirical results obtained using the Amadeus data set remain robust to the following changes. Lowering the Winsorization cutoff to 1 percent results in somewhat lower adjusted R^2 values and lower point estimates for the control variables. Using a higher value for the firm size cutoff in the sample leads to similar results. Changing the definition of the equity issue and external financing dummies to require either 0.1 or 5 percent growth in book debt and book equity, respectively, generates similar findings.[22] Changing the cutoff values used in identifying young and mature firms to the 40th and 60th percentiles also leads to similar results.

The estimated age effects become insignificant when the regression specification is changed to include firm fixed effects. This arises due to the fact that the sample spans only 10 years, and eliminating firm fixed effects takes out much of the interesting variation in firm age, which ranges from 1 to 151 years in the sample. Effectively, a fixed effects specification absorbs the difference in

[22] The coefficient on age for equity issue for the mature firm subsample becomes negative at the 10 percent level when the equity issue dummy is defined using the 5 percent growth rate.

behavior between a firm with an average age of 5 years and a firm with an average age of 30 years into the fixed effect, thereby ignoring this variation.

6 Effect of costly external finance

The prior analysis demonstrates that endogenous changes in profitability can generate firm lifecycle behavior in a setting with no financing frictions. But, the absence of financing frictions is an unrealistic assumption. A natural response, therefore, would be to argue that financing frictions dominate the effects of profitability changes. However, the finding of Angelini and Generale (2008) that costly external finance explains only a small part of the dependency between firm age and size suggests otherwise. This section tackles the above response by evaluating simulated data from an extension of the model that incorporates costly external finance.

6.1 External financing costs

For simplicity, the model incorporates external financing costs in a reduced form manner. Although such an approach cannot explain why these financing costs arise, it is sufficient to examine the effects of financing constraints on the firm's policies.

6.1.1 Equity issuance costs

Firms face a proportional cost of issuing equity in the model, with proportionality constant ϕ. This cost can be thought of as reflecting an illiquidity discount associated with investing in private firms. It could also reflect information costs associated with such an investment. This cost does not necessarily capture the investment banking and other fees associated with a public equity issuance as the model is mainly focused on small private firms.

6.1.2 Borrowing costs

Firms do not face any costs of obtaining debt in the model. However, the interest rate on their debt increases with the leverage of the firm, denoted by l. This captures in a reduced form the idea that firms with higher leverage are more likely to default.[23] The interest rate function is parameterized

[23]The structural model with endogenous default in Hennessy and Whited (2007) generates a positive relationship between leverage and interest rates.

as follows:

$$r(l) = r_0(1 + \psi l). \tag{9}$$

The interest rate at zero leverage, r_0, is set such that the marginal cost of borrowing net of tax benefits at zero leverage equals the marginal cost of internal funds. This assumption reflects the high interest costs faced by private firms and implies that firms do not borrow solely for tax purposes in the model.

6.1.3 Financing hierarchy

These assumptions generate a financing hierarchy (pecking order) in the model as in Myers and Majluf (1984). Internal funds provide the cheapest form of finance, followed by debt. As the marginal cost of debt increases with leverage, firms will eventually turn to external equity. The marginal cost of funds increases smoothly as firms move from internal funds to external equity. A similar hierarchy would arise if one assumed a constant interest rate and an arbitrary borrowing limit. This financing hierarchy ensures that external finance - either debt or equity - is costlier than internal funds.

6.2 Calibration and analysis

6.2.1 Calibration

The cost of external equity, ϕ, equals 0.1. Gomes (2001) calibrates this value to 0.07 to reflect investment banking fees for public equity issues. In contrast, Cooley and Quadrini (2001) calibrate external financing costs to a much higher value of 0.3. However, they find that their results are insensitive to whether they calibrate this cost to 0.1 or 0.3.

The interest rate is calibrated to increase with leverage at a rate of 10 basis points per 1% increase in leverage. Meisenzahl (2009) finds that a 1% increase in leverage results in about a 1.1 basis points increase in interest rates using data from the Survey of Small Business Finances.

Due to the financing hierarchy, firms are more sensitive to the costs of debt than the costs of external equity. As such, calibrating the cost of debt to a relatively high value ensures that the analysis errs in favor of finding a strong effect for costly external finance. In comparison, increasing the cost of external equity has a relatively small effect on the degree of financing costs faced by the firm.

6.2.2 Results

This study examines the effect of financing costs by comparing the distribution for product quality and firm age with and without external financing costs.

Figure 3 presents the distribution of product quality from the simulated data set with and without costly external finance. The presence of external financing costs lowers the average quality level in the economy. The simulated economy with financing costs has more firms with quality indices 2 and 3, and less firms with quality indices greater than 3.[24] This reflects the effects of increased costs of funds on optimal product development expenses detailed in Proposition 2.

Figure 4 plots the corresponding age distributions for the simulated economies with and without external financing costs. Financing costs shifts the age distribution to the left with a greater mass of firms with age less than 18 and a smaller mass with age more than 18. This arises partly due to the effect of financing costs on the quality distribution. Lower average quality indices lowers firm value and increases exit. In addition, financing costs also lowers firm value - thereby increasing exit - for firms of the same quality index by increasing their cost of funds when they need to obtain external finance.

However, the overall effect of external finance costs on the quality and age distributions is quite small. The financing costs lower average quality level from 2.96 to 2.89 and lowers the average firm age from 12.1 to 11.9 years. In unreported results, replicating the analysis in Tables 1 to 3 on the simulated data set with financing costs generates similar findings. These findings demonstrate that while financing costs do affect firm's lifecycle behavior in the model, this effect does not subsume that of endogenous profitability changes.

7 Conclusion

This study documents a lifecycle profile for profitability changes of firms. On average, firms realize profitability increases in their early years, followed by a slow decline in profitability with age. This lifecycle profile of profitability arises endogenously in a model where investment in product development generates outward shifts in the demand curve faced by firms. The model generates the lifecycle behavior of firms observed in the data. In particular, young firms realize profitability

[24] The financing costs also leads to fewer firms with quality index 1. This arises due to costly external finance lowering physical investment for entrants. In subsequent periods, firms that fail to innovate will invest more in product development per unit of capital than comparable firms in the economy without financing costs. This leads to a smaller mass of firms with quality index 1.

jumps more frequently, have higher investment and sales growth, and obtain external financing more frequently. Further, the effect of age on firms' decisions is more pronounced for younger firms.

More generally, the model highlights the importance of endogenous profitability changes on firm dynamics. The existing literature on financial policies of firms typically considers cash flows as entirely exogenously determined (for instance, see Miao (2005), Gamba and Triantis (2008) and Bolton, Chen, and Wang (2009)). Further research into understanding the effect of endogenous profitability changes on firm dynamics may prove fruitful.

Appendix

A Proofs

Proposition 1 *There exists a sequence of unique value functions $v(k, \tilde{z}, a; q_i)$.*

Proof.

The proof employs backward induction over the quality indices i from $\bar{n}$ to 1. The value function for a firm in the i^{th} quality stage is given by the following Bellman equation:

$$
\begin{aligned}
v(k, \tilde{z}, a; q_i) &= \max_{i,s,k'} \; d + \beta(1+g_x)(E_z[j v_c(k', \tilde{z}', a+1; q_{i+1}) + (1-j) v_c(k', \tilde{z}', a+1; q_i)]), \\
s.t. \quad (1+g_x)k' &= k(1-\delta) + i, \\
j &\sim \mathcal{B}\left(p(s)\right), \\
\text{with} \quad d &= (1-\tau)\left(\pi(k, \tilde{z}, a; q_n) - c - s\right) + \tau\delta k - i - \lambda\frac{i^2}{2k}, \\
v_c(k, \tilde{z}, a; q_i) &= \max\{v(k, \tilde{z}, a; q_i), fk\}.
\end{aligned}
$$

At the highest quality index, $i = \bar{n}$, optimal product development expenses $s = 0$, by construction, as the firm cannot obtain a further quality increase. Therefore, the value function for quality index $\bar{n}$ satisfies the conditions of Theorem 9.6 in Stokey and Lucas (1989), implying the existence and uniqueness of the value function $v(k, \tilde{z}, a; q_{\bar{n}})$.

Assume that a unique value function exists for quality index $i+1$. The Bellman equation given in equation (A.1) follows a contraction mapping. The results in Bertsekas (2000, Chapter 7) imply the existence and uniqueness of the value function for quality index i. This completes the proof. ■

Proposition 2 *The first order condition for product development implies that*

$$
\begin{aligned}
\textit{marginal cost of funds} &= \textit{marginal benefit of product development expenses} \\
\Rightarrow \qquad 1-\tau &= \beta(1+g_x)E\left[\frac{v(k', l', \tilde{z}', a+1; q_{i+1}) - v(k', l', \tilde{z}', a+1; q_i)}{k}\right] b(1-p(s)).
\end{aligned}
$$

Proof.

Taking first order conditions from the above Bellman equation, one obtains that,

marginal cost of funds = marginal benefit of product development expenses.

In the absence of financing costs, the marginal cost of funds (L.H.S of the above equation) is trivially given by

$$\text{L.H.S} = 1 - \tau$$

The marginal benefit of product development expenses (R.H.S. of the above equation) is given by:

$$\begin{aligned}\text{R.H.S.} &= \frac{\partial(\beta(1+g_x)(E_z[jv_c(k',l',\tilde{z}',a+1;q_{i+1})+(1-j)v_c(k',l',\tilde{z}',a+1;q_i)]))}{\partial s} \\ &= \beta(1+g_x)(E_z[v_c(k',l',\tilde{z}',a+1;q_{i+1})-v_c(k',l',\tilde{z}',a+1;q_i)])\frac{\partial E[j]}{\partial s}.\end{aligned}$$

Some algebra yields that

$$\frac{\partial E[j]}{\partial s} = \frac{\partial p(s)}{\partial s} = \frac{b}{k}(1-p(s)).$$

Substituting this into the previous expression completes the proof. ■

B Model solution and simulation

The optimal policies of the firm are obtained using value function iteration to solve for the above sequence of Bellman equations indexed by quality level. The value function iteration employs the optimal product development expense given in Proposition 2. The solution for investment is carried out numerically over a grid of values for capital.

The simulation sample is constructed using the value function solutions over 8 different product quality values. Adding more quality indices will have little impact as very few of the firms in the simulated sample reach the 8th quality index level. The numerical solution is carried using the following grid sizes: a profitability shock grid with 5 values, a capital grid with 50 values, and an age grid of 1 to 35. The maximum age level does not bind in the simulation. The construction of the simulated sample follows fairly trivially from the numerical solution.

Firms exit endogenously in the simulation when their exit value exceeds the continuation value. Each firm that exits is replaced with a new firm of age 1 with capital stock level at the lowest grid value. New entrants are assigned a random profitability shock level. These new entrants (firms

with age 1) are discarded from the simulated data set to ensure that the assumptions regarding entry have no effect on the results.

Incorporating the financing friction substantially complicates the numerical solution. Not only does this add another dimension (leverage) over which firms optimize, the leverage at which a firm of given size, age, and quality index obtains external equity has to be solved within the model. Further, the optimal leverage choices do not necessarily lie on the leverage grid, requiring linear interpolation in both the value function solution and the model simulation. These complications result in the numerical solution for the model with financing frictions taking about 12 hours to solve.

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Table 1: Estimates from simulated data - All firms

The table reports the results obtained from estimating the lifecycle equations on all observations from the simulated data set. Table 5 presents the corresponding results obtained from the Amadeus data set. Panel A presents the panel regressions on sales growth, investment, and dividends. Panel B presents logit regressions on quality jumps, and external financing. Section 3.2 details the construction of the simulated data set. The simulation sample excludes new entrants. The standard errors are heteroskedasticity robust. * and ** denote significance at the 5 and 1 percent confidence levels, respectively. Sales growth, profitability, investment, and dividends are reported in percentage terms in Panel A.

Panel A: Panel regressions

	Sales growth	Investment	Dividends
Age	-0.970**	-1.470**	1.110**
	(0.000)	(0.010)	(0.000)
Size	-3.13**	-7.69**	1.01**
	(0.07)	(0.05)	(0.06)
Profitability		0.17**	0.28**
		(0.00)	(0.00)
Sales growth		0.07**	-0.01**
		(0.00)	(0.00)
Observations	97719	97719	97719
Adjusted R-squared	0.35	0.91	0.72

Panel B: Logit regressions

	Quality increase	External financing
Age	-0.154**	-0.549**
	(0.003)	(0.008)
Size	-2.85**	-6.56**
	(0.04)	(0.09)
Profitability		-7.78**
		(0.25)
Sales growth		0.54**
		(0.16)
Observations	97719	97719
Pseudo R-squared	0.33	0.69

Table 2: Estimates from simulated data - Young firms

The table reports the results obtained from estimating the lifecycle equations on observations with firm age less than or equal to its median value in the simulated data set. Table 6 presents the corresponding results obtained from the Amadeus data set. Panel A presents the panel regressions on sales growth, investment, and dividends. Panel B presents logit regressions on quality jumps, and external financing. Section 3.2 details the construction of the simulated data set. The simulation sample excludes new entrants. The standard errors are heteroskedasticity robust. * and ** denote significance at the 5 and 1 percent confidence levels, respectively. Sales growth, profitability, investment, and dividends are reported in percentage terms in Panel A.

Panel A: Panel regressions

	Sales growth	Investment	Dividends
Age	-1.320**	-1.390**	2.540**
	(0.030)	(0.010)	(0.010)
Size	-3.34**	-15.12**	-9.77**
	(0.27)	(0.05)	(0.11)
Profitability		0.09**	0.29**
		(0.00)	(0.00)
Sales growth		0.00**	-0.02**
		(0.00)	(0.00)
Observations	52277	52277	52277
Adjusted R-squared	0.25	0.98	0.70

Panel B: Logit regressions

	Quality increase	External financing
Age	-0.369**	-1.210**
	(0.025)	(0.052)
Size	-1.65**	-3.94**
	(0.16)	(0.61)
Profitability		-13.32**
		(0.35)
Sales growth		0.20
		(0.20)
Observations	52277	52277
Pseudo R-squared	0.31	0.70

Table 3: Estimates from simulated data - Mature firms

The table reports the results obtained from estimating the lifecycle equations on observations with firm age greater than its median value in the simulated data set. Table 7 presents the corresponding results obtained from the Amadeus data set. Panel A presents the panel regressions on sales growth, investment, and dividends. Panel B presents logit regressions on quality jumps, and external financing. In the simulated sample, no mature firms obtain quality jumps. Section 3.2 details the construction of the simulated data set. The simulation sample excludes new entrants. The standard errors are heteroskedasticity robust. * and ** denote significance at the 5 and 1 percent confidence levels, respectively. Sales growth, profitability, investment, and dividends are reported in percentage terms in Panel A.

Panel A: Panel regressions

	Sales growth	Investment	Dividends
Age	-0.240**	-0.370**	0.470**
	(0.010)	(0.010)	(0.010)
Size	10.86**	13.95**	-13.75**
	(0.16)	(0.14)	(0.15)
Profitability		0.27**	0.49**
		(0.00)	(0.00)
Sales growth		0.01**	-0.00*
		(0.00)	(0.00)
Observations	45442	45442	45442
Adjusted R-squared	0.15	0.89	0.61

Panel B: Logit regressions

	Quality increase	External financing
Age	-	-0.015
	(-)	(0.014)
Size	-	-0.53**
	(-)	(0.23)
Profitability	-	-1.97*
	(-)	(0.80)
Sales growth	-	0.37
	(-)	(0.53)
Observations	-	45442
Pseudo R-squared	-	0.01

Table 4: Summary statistics

The table presents summary statistics for variables of interest from the Amadeus data set. Panels A, B, and C report values for all firms, firms with age less than or equal to its median, and firms with age greater than its median, respectively. Section 4 details the construction of the sample. Firm age is measured from the date of incorporation. Sales growth equals the annual growth rate of revenue. Investment equals the growth rate of fixed assets adjusted for depreciation. Profitability jump equals 1 if the firms profitability was more than 10 percent higher than the average value over the past three years. Equity issuance equals 1 if the firms book equity is greater than the corresponding value for the previous year plus 2 percent. External financing equals 1 if the sum of book debt and book equity is greater than the corresponding sum for the previous year plus 2 percent. All continuous variables (except firm age) are Winsorized at the 95th percentile to reduce the impact of outliers.

Panel A: All firms

	Age	Sales growth	Profitability	Investment	Prof. jump	Equity issue	External financing
Mean	23.2	0.139	0.109	0.363	0.123	0.057	0.470
Median	16.2	0.065	0.093	0.142	-	-	-
St. dev.	21.2	0.334	0.162	0.567	0.329	0.232	0.499
Obs. (1000s)	275	275	275	221	123	241	180

Panel B: Young firms

	Age	Sales growth	Profitability	Investment	Prof. jump	Equity issue	External financing
Mean	8.7	0.201	0.120	0.445	0.156	0.078	0.484
Median	8.2	0.096	0.103	0.169	-	-	-
St. dev.	4.0	0.389	0.184	0.647	0.363	0.268	0.500
Obs. (1000s)	137	137	137	106	51	114	84

Panel C: Mature firms

	Age	Sales growth	Profitability	Investment	Prof. jump	Equity issue	External financing
Mean	37.4	0.078	0.098	0.288	0.102	0.038	0.458
Median	30.0	0.045	0.084	0.126	-	-	-
St. dev.	21.9	0.256	0.135	0.470	0.302	0.192	0.499
Obs. (1000s)	139	139	139	116	76	127	96

Table 5: Estimates from U.K. data - All firms

The table reports the results obtained from estimating the lifecycle equations on the Amadeus data set. Panel A presents the panel regressions on sales growth and investment. Panel B presents logit regressions on profitability jumps, equity issuance, and external financing. Section 4 details the construction of the sample and the variable definitions. All regressions include year and 2-digit SIC code industry dummies. Standard errors are heteroskedasticity robust and clustered at the firm level. * and ** denote significance at the 5 and 1 percent confidence levels, respectively. Sales growth, profitability, and investment are reported in percentage terms in Panel A.

Panel A: Panel regressions

	Sales growth	Investment
Age	-0.310**	-0.220**
	(0.000)	(0.010)
Size	0.44**	-2.27**
	(0.06)	(0.11)
Profitability		0.27**
		(0.01)
Sales growth		0.28**
		(0.01)
Observations	275399	221185
Adjusted R-squared	0.04	0.10

Panel B: Logit regressions

	Profitability jump	Equity issue	External financing
Age	-0.009**	-0.012**	-0.002**
	(0.001)	(0.001)	(0.000)
Size	-0.21**	0.18**	0.07**
	(0.01)	(0.01)	(0.00)
Profitability		-2.42**	-1.37**
		(0.08)	(0.04)
Sales growth		1.10**	0.81**
		(0.02)	(0.02)
Observations	127284	241300	180828
Pseudo R-squared	0.05	0.08	0.02

Table 6: Estimates from U.K. data - Young firms

The table reports the results obtained from estimating the lifecycle equations on the Amadeus data set for observations with age less than or equal to its median value, 16. Panel A presents the panel regressions on sales growth and investment. Panel B presents logit regressions on profitability jumps, equity issuance, and external financing. Section 4 details the construction of the sample and the variable definitions. All regressions include year and 2-digit SIC code industry dummies. Standard errors are heteroskedasticity robust and clustered at the firm level. * and ** denote significance at the 5 and 1 percent confidence levels, respectively. Sales growth, profitability, and investment are reported in percentage terms in Panel A.

Panel A: Panel regressions

	Sales growth	Investment
Age	-2.650**	-0.640**
	(0.030)	(0.060)
Size	-0.03	-3.70**
	(0.08)	(0.15)
Profitability		0.27**
		(0.02)
Sales growth		0.30**
		(0.01)
Observations	136505	105535
Adjusted R-squared	0.08	0.11

Panel B: Logit regressions

	Profitability jump	Equity issue	External financing
Age	-0.027**	-0.070**	-0.003
	(0.005)	(0.004)	(0.002)
Size	-0.19**	0.16**	0.05**
	(0.01)	(0.01)	(0.00)
Profitability		-2.22**	-1.55**
		(0.08)	(0.05)
Sales growth		0.88**	0.82**
		(0.03)	(0.02)
Observations	50729	113963	84353
Pseudo R-squared	0.04	0.09	0.03

Table 7: Estimates from U.K. data - Mature firms

The table reports the results obtained from estimating the lifecycle equations on the Amadeus data set for observations with age greater than its median value, 16. Panel A presents the panel regressions on sales growth, and investment. Panel B presents logit regressions on profitability jumps, equity issuance, and external financing. Section 4 details the construction of the sample and the variable definitions. All regressions include year and 2-digit SIC code industry dummies. Standard errors are heteroskedasticity robust and clustered at the firm level. * and ** denote significance at the 5 and 1 percent confidence levels, respectively. Sales growth, profitability, and investment are reported in percentage terms in Panel A.

Panel A: Panel regressions

	Sales growth	Investment
Age	-0.060**	-0.140**
	(0.000)	(0.010)
Size	0.93**	-0.36**
	(0.05)	(0.12)
Profitability		0.27**
		(0.02)
Sales growth		0.22**
		(0.01)
Observations	138894	115650
Adjusted R-squared	0.01	0.05

Panel B: Logit regressions

	Profitability jump	Equity issue	External financing
Age	-0.006**	-0.000	-0.003**
	(0.001)	(0.001)	(0.000)
Size	-0.21**	0.23**	0.10**
	(0.01)	(0.01)	(0.00)
Profitability		-2.59**	-1.12**
		(0.16)	(0.06)
Sales growth		0.98**	0.77**
		(0.05)	(0.03)
Observations	76525	127270	96475
Pseudo R-squared	0.04	0.06	0.01

Table 8: Profitability jumps

The table reports the results obtained from estimating the logit regression for profitability jumps, defined using different cutoff values, on the Amadeus data set. Panels A, B, and C present the results for all firms, firms with age less than or equal to its median value, and firms with age above its median value, respectively. The profitability jump variable equals 1 in each column if profitability increased by more than 0.05, 0.15, and 0.20 from the average value over the past three years. Section 4 details the construction of the sample and the variable definitions. All regressions include year and 2-digit SIC code industry dummies. Standard errors are heteroskedasticity robust and clustered at the firm level. * and ** denote significance at the 5 and 1 percent confidence levels, respectively.

Panel A: All firms

	Profitability jump		
	> 0.05	> 0.15	> 0.20
Age	-0.005**	-0.012**	-0.016**
	(0.000)	(0.001)	(0.001)
Size	-0.14**	-0.25**	-0.29**
	(0.01)	(0.01)	(0.01)
Observations	127284	127284	127284
Pseudo R-squared	0.03	0.06	0.07

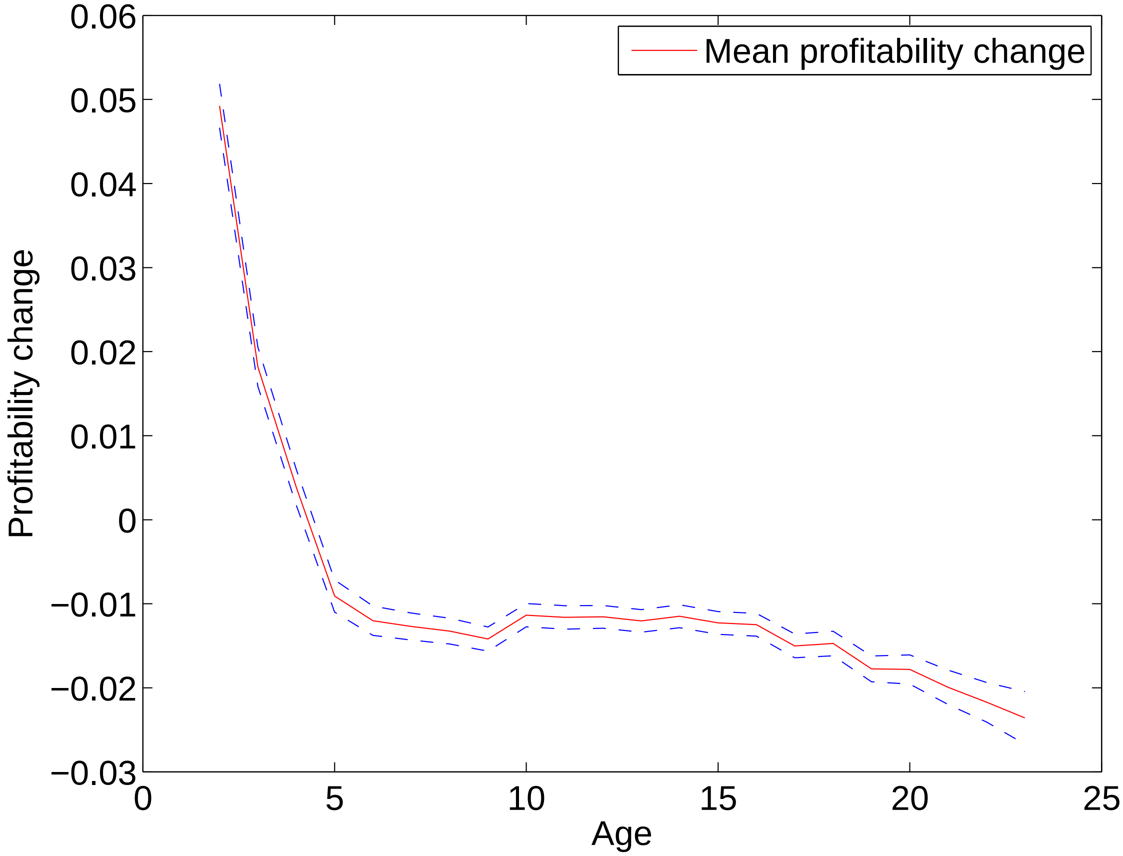
Panel B: Young firms

	Profitability jump		
	> 0.05	> 0.15	> 0.20
Age	-0.018**	-0.038**	-0.043**
	(0.004)	(0.006)	(0.007)
Size	-0.13**	-0.24**	-0.27**
	(0.01)	(0.01)	(0.01)
Observations	50741	50667	50667
Pseudo R-squared	0.02	0.05	0.06

Panel C: Mature firms

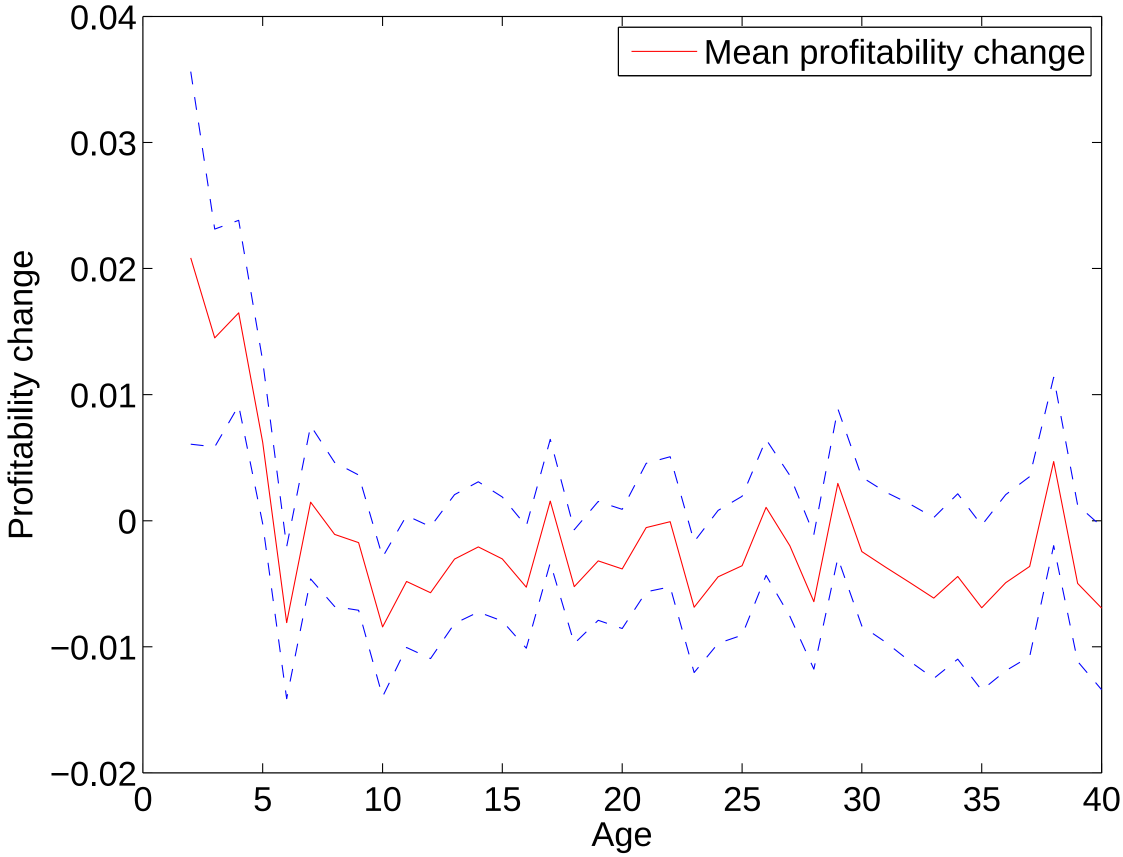
	Profitability jump		
	> 0.05	> 0.15	> 0.20
Age	-0.003**	-0.007**	-0.009**
	(0.001)	(0.001)	(0.001)
Size	-0.15**	-0.26**	-0.31**
	(0.01)	(0.01)	(0.02)
Observations	76525	76504	76402
Pseudo R-squared	0.03	0.05	0.06

Figure 1: Simulated age profile of profitability changes



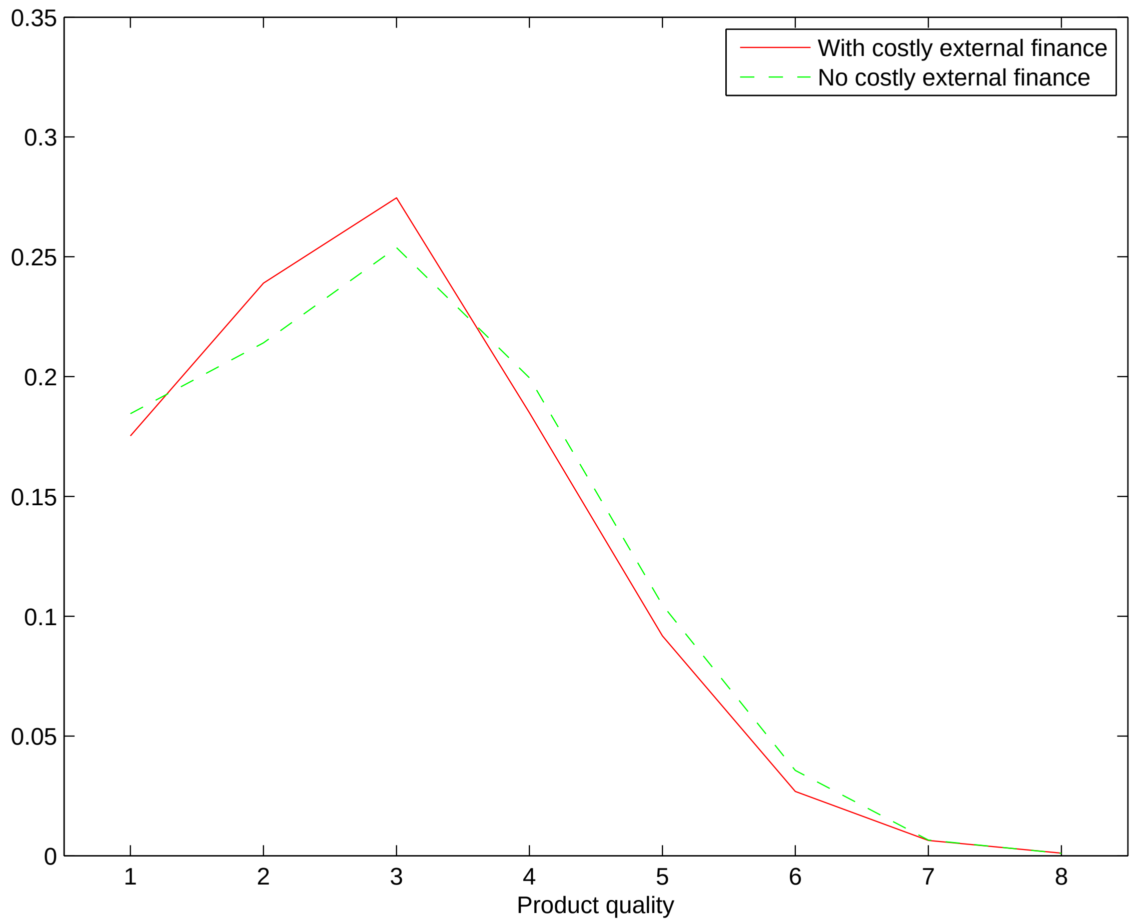
Using the simulated data set, the figure plots the mean change in profitability from age t to $t+1$ as a function of age, t. The solid (red) line plots the mean change while the dashed (blue) lines plot the 95 percent confidence intervals for the mean profitability change. Sections 3.1 and 3.2 detail the construction of the simulated data set. The sample includes firms with ages 2 to 23, above which the number of firms in the simulated data set declines below 500.

Figure 2: Age profile of profitability changes



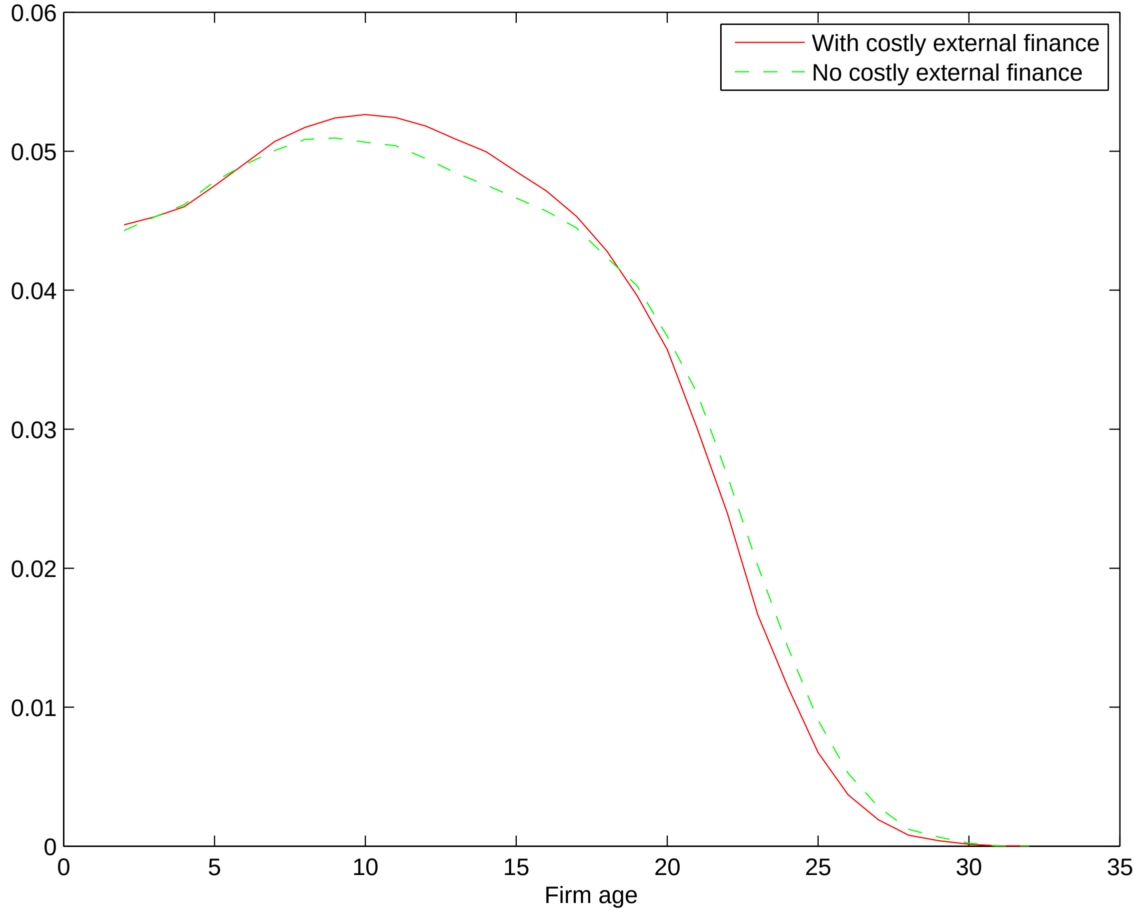
The figure plots the mean change in profitability from age t to $t+1$ as a function of age, t. The solid (red) line plots the mean change while the dashed (blue) lines plot the 95 percent confidence intervals for the mean profitability change. Section 4 details the construction of the sample using the Amadeus data set and the variable definitions. The sample includes firms with ages 2 to 40. Firms with age above 40 continue to exhibit mostly negative mean profitability changes; the confidence interval this estimate steadily increases with age due to the decrease in the number of observations.

Figure 3: Quality distribution of firms



The figure plots the quality distribution of firms in the simulated economy. The solid (red) line plots the quality distribution under the full model and the dashed (green) line plots the quality distribution assuming no costs of external finance. Section 3.2 details the construction of the simulated data set.

Figure 4: Age distribution of firms



The figure plots the age distribution of firms in the simulated economy. The solid (red) line plots the age distribution under the full model and the dashed (green) line plots the age distribution assuming no costs of external finance. Section 3.2 details the construction of the simulated data set.