

Accounting for Intermediates: Production Sharing and Trade in Value Added*

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Abstract

We combine input-output and bilateral trade data to compute the value added content of bilateral trade. The ratio of value added to gross exports (VAX ratio) is a measure of the intensity of production sharing. Across countries, export composition drives VAX ratios, with exporters of Manufactures having lower ratios. Across sectors, the VAX ratio for Manufactures is low relative Services, primarily because Services are used as an intermediate to produce manufacturing exports. Across bilateral partners, VAX ratios vary widely and contain information on both bilateral and triangular production chains. We document specifically that bilateral production linkages, not variation in the composition of exports, drives variation in bilateral VAX ratios. Finally, bilateral imbalances measured in value added differ from gross trade imbalances. Most prominently, the U.S.-China imbalance in 2004 is 30-40% smaller when measured in value added.

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Trade in intermediate inputs accounts for as much as two thirds of international trade. By linking production processes across borders, this input trade creates two distinct measurement challenges. First, conventional gross trade statistics tally the gross value of goods at each border crossing, rather than the net value added between border crossings. This well-known “double-counting” problem means that conventional data overstate the domestic (value added) content of exports. Second, multi-country production networks imply that intermediate goods can travel to their final destination by an indirect route. For example, if Japanese intermediates are assembled in China into final goods exported to the U.S., then Chinese bilateral gross exports embody third party (Japanese) content. Together, “double-counting” and multi-country production chains imply that there is a hidden structure of trade in value added underlying gross trade flows.

In this paper, we compute and analyze the value added content of trade. To do so, we construct a synthetic, global bilateral input-output table by combining input-output tables and bilateral trade data for many countries. This table describes how particular sectors in each destination country purchase intermediates from both home and individual foreign sources, as well as how each country sources final goods. Using this table, we split each country’s gross output according to the destination in which it is ultimately absorbed in final demand. We then use value added to output ratios from the source country to compute the value added associated with the implicit output transfer to each destination. This calculation requires a minimal set of assumptions and is consistent with the structure of standard trade models. The end result is a data set of “value added exports” that describes the destination where the value added produced in each source country is absorbed.

These data on the value added content of trade have many potential uses. Most directly, we compare them to gross bilateral trade flows to quantify the scope of production sharing. This approach to measuring production sharing yields comparable figures for many countries and sectors and respects the multilateral structure of production sharing. Further, because we use the national accounts definition of intermediates, our measures are easily translated into models.¹ This is important because the value added content of trade is a key theoretical object and calibration target in

¹This contrasts with alternative approaches, such as using data on trade in parts and components (e.g., Yeats (2001)) or trade between multinational parents and affiliates (e.g., Hanson, Mataloni, and Slaughter (2005)).

many trade and macroeconomic models. For example, value added exports can be used to calibrate “openness” and bilateral exposure to foreign shocks in international business cycle research.² For trade research, value added flows could be used to calibrate gravity-style trade models to allow for differences in trade patterns for final and intermediate goods. They could also be employed to calibrate many-country models of multi-stage production and vertical specialization, as in Yi (2003, 2010). And these applications only scratch the surface.

Our approach to measuring the value added content of trade draws upon a venerable literature on input-output accounting in models with multiple regions.³ It is also intimately related to recent efforts to measure the factor content of trade with traded intermediates. Treffer and Zhu (2005) develop a multi-country input-output framework to define a Vanek-consistent measure of the factor content of multilateral net exports.⁴ Our approach is distinguished from this work both by our focus on measuring value added (rather than factor quantities) and the emphasis we place on bilateral, gross trade (rather than multilateral, net trade). Belke and Wang (2006) and Daudin, Riffart, and Schweisguth (2008) also develop value added trade computations along the lines of those used in this paper, though our exposition and analysis differ considerably.⁵

Our work is also related to an active literature on measuring vertical specialization and the domestic content of exports.⁶ Aggregating across sectors and export destinations for each source country, the ratio of value added to gross exports can be interpreted as a metric of the domestic content of exports.⁷ Our domestic content metric generalizes the work by Hummels, Ishii, and Yi (2001). Hummels et al. compute the value added content of exports under the restrictive assump-

²See Bems, Johnson, and Yi (2010) for elaboration of this argument.

³Moses (1955) uses proportionality assumptions to split input use across sources, as we do, to build a multi-region model of the U.S. See also Isard (1951), Moses (1960), or Polenske (1995) for a review of the early literature.

⁴Reimer (2006) exposits an equivalent approach in a two country case. Johnson (2008) shows that these approaches are equivalent to computing the factor content of value added trade. The value added approach facilitates computation of the factor content of gross bilateral trade.

⁵Daudin, Riffart, and Schweisguth (2008) study the role of vertical specialization in generating regionalization in trade patterns, while Belke and Wang (2006) focus on measuring aggregate economic openness.

⁶See NRC (2006) for the U.S. See Dean, Fung, and Wang (2007), Chen, Cheng, Fung, and Lau (2008), and Koopman, Wang, and Wei (2008) for China. See Hummels, Ishii, and Yi (2001) and Miroudot and Ragoussis (2009) for changes in domestic content over time for mainly OECD countries. See Wang, Powers, and Wei (2009) on splitting up the value chain within Asia.

⁷Bilateral or sector level ratios of value added to exports do not have this domestic content interpretation.

tion that a country only exports final goods. By using input-output data for source and destination countries simultaneously, we are able to relax this assumption and allow each country to both import and export intermediate goods. While this generalization results in only minor adjustments in aggregate domestic content measurements in our data, we demonstrate that allowing two-way trade in intermediates is critically important for generating accurate bilateral value added flows.

Turning to our empirical results, we find that the ratio of value added to gross exports (VAX ratio) varies substantially across countries and sectors. Across sectors, we show that VAX ratios are substantially higher in Agriculture, Natural Resources, and Services than in Manufactures. This is mostly due to the fact that the manufacturing sector purchases inputs from non-manufacturing sectors, and therefore contains value added generated in those sectors. Across countries, the composition of trade drives aggregate VAX ratios, with countries that export Manufactures having lower aggregate VAX ratios. Aggregate VAX ratios do not covary strongly with income per capita, however, due to two offsetting effects. While richer countries tend to export Manufactures, which lowers their aggregate VAX ratios, they also export at higher VAX ratios within the manufacturing sector.⁸

Moving from aggregate to bilateral data, VAX ratios differ widely across partners for individual countries. For example, U.S. exports to Canada are about 40% smaller measured in value added terms than gross terms, whereas U.S. exports to France are essentially identical in gross and value added terms. These gaps arise for two main reasons. First, bilateral (“back-and-forth”) production sharing implies that value added trade is scaled down relative to gross trade. And these scaling factors differ greatly across bilateral partners. Second, multilateral (“triangular”) production sharing gives rise to indirect trade that occurs via countries that process intermediate goods. For some country pairs, bilateral VAX ratios are larger than one, as bilateral value added exports exceeds gross exports.

These adjustments imply that bilateral trade imbalances often differ in value added and gross terms. For example, the U.S.-China imbalance is approximately 30-40% smaller when measured

⁸VAX ratios within Manufactures are correlated with income because richer countries tend to export in sub-sectors with relatively high VAX ratios.

on a value added basis, while the U.S.-Japan imbalance is approximately 33% higher. These adjustments point to the importance of triangular production chains within Asia.

To illustrate the mechanisms at work in generating these results, we present two decompositions. In the first decomposition, we show that most of the variation in bilateral value added to export ratios arises due to production sharing, not variation in the composition of goods exported to different destinations. The second decomposition splits bilateral exports according to whether they are absorbed in the destination, embedded as intermediates in goods that are reflected back to the source country, or redirected to third countries embedded as intermediates in goods ultimately consumed there. Variation in the degree of absorption, reflection, and redirection across partners is an important driver of variation in bilateral value added to export ratios.

The rest of the paper is structured as follows. Section 1 presents the general accounting framework, defines our value added trade measures, and discusses the interpretation of value added to export ratios. Section 2 describes the data sources and assumptions we use to implement the accounting exercise. Section 3 presents our empirical results and Section 4 concludes.

1 The Value Added Content of Trade

In this section, we introduce the accounting framework and demonstrate how intermediate goods trade generates differences between gross and value added trade flows. We begin the section by presenting a general formulation of the framework with many goods and countries that we use in the calculations below. To aid intuition, we then exposit several results in stripped-down versions of this general framework with one composite sector and two or three countries. Results from these simple models carry over to the general model. We close by discussing insights from admitting multiple sectors.

1.1 The Value Added Content of Trade

Assume there are S sectors and N countries. Each country produces a single differentiated tradable good within each sector that is either used to satisfy final demand (equivalently, “consumed”) or used as an intermediate input in production. Output in each country is produced by combining local factor inputs with domestic and imported intermediate goods. Let the quantity of output produced in sector s of country i be denoted $q_i(s)$. Let the quantity of final goods from sector s in country i absorbed in destination j be $q_{ij}^c(s)$ and the quantity of intermediates from sector s in country i used to produce output in sector t in country j be $q_{ij}^m(s, t)$. Then markets clear in quantities: $q_i(s) = \sum_j q_{ij}^c(s) + \sum_j \sum_t q_{ij}^m(s, t)$.

If we evaluate these quantity flows at a common price, say $p_i(s)$, then we can rewrite the market clearing condition in value terms as:

$$y_i(s) = \sum_j c_{ij}(s) + \sum_j \sum_t m_{ij}(s, t), \quad (1)$$

where $y_i(s) \equiv p_i(s)q_i(s)$, $c_{ij}(s) \equiv p_i(s)q_{ij}^c(s)$, and $m_{ij}(s, t) \equiv p_i(s)q_{ij}^m(s, t)$ are the value of production, final demand, and intermediate goods shipments. Gross bilateral exports, denoted $x_{ij}(s)$, include goods destined for both final and intermediate use abroad: $x_{ij}(s) = c_{ij}(s) + \sum_t m_{ij}(s, t)$. Then (1) equivalently says that output is divided between domestic final use, domestic intermediate use, and gross exports.

To express market clearing conditions for many countries and sectors in a compact form, we define a series of matrices and vectors. Collect the total value of production in each sector in the $S \times 1$ vector y_i and allocate this output to final and intermediate use. Denote country i 's final demand for its own goods by $S \times 1$ vector c_{ii} and shipments of final goods from i to country j by the $S \times 1$ vector c_{ij} . Further, denote use of intermediate inputs from i by country j by $A_{ij}y_j$, where A_{ij} is an $S \times S$ input-output matrix with elements $A_{ij}(s, t) = m_{ij}(s, t)/y_j(t)$. Gross exports from i to j ($i \neq j$) are then $x_{ij} = c_{ij} + A_{ij}y_j$.

With this notation in hand, we collect information on intermediate goods sourcing and final

goods flows in vector/matrix form:

$$A \equiv \begin{pmatrix} A_{11} & A_{12} & \dots & A_{1N} \\ A_{21} & A_{22} & \dots & A_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & \dots & A_{NN} \end{pmatrix}, \quad y \equiv \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}, \quad c_j \equiv \begin{pmatrix} c_{1j} \\ c_{2j} \\ \vdots \\ c_{Nj} \end{pmatrix}.$$

Then, we write the $S \times N$ goods market clearing conditions as:

$$y = Ay + \sum_j c_j. \quad (2)$$

This is the classic representation of an input-output system, where total output is split between intermediate and final use. Whereas a typical input-output system focuses on sectoral linkages within a single economy, this system is expanded to trace intermediate goods linkages across countries and sectors. We therefore refer to A as the global bilateral input-output matrix.

Using this system, we can write output as:

$$y = \sum_j (I - A)^{-1} c_j. \quad (3)$$

To interpret this expression, $(I - A)^{-1}$ is the ‘‘Leontief inverse’’ of the input-output matrix. The Leontief inverse can be expressed as a geometric series: $(I - A)^{-1} = \sum_{k=0}^{\infty} A^k$. Multiplying by the final demand vector, the zero-order term c_j is the direct output absorbed as final goods, the first-order term $[I + A]c_j$ is the direct output absorbed plus the intermediates used to produce that output, the second-order term $[I + A + A^2]c_j$ includes the additional intermediates used to produce the first round of intermediates (Ac_j), and the sequence continues as such. Therefore, $(I - A)^{-1}c_j$ is the vector of output used both directly and indirectly to produce final goods absorbed in country j .

Equation (3) thus decomposes output from each source country i into the amount of output from the source used to produce final goods absorbed in country j . To make this explicit, we

define:

$$\begin{pmatrix} y_{1j} \\ y_{2j} \\ \vdots \\ y_{Nj} \end{pmatrix} \equiv (I - A)^{-1} c_j, \quad (4)$$

where y_{ij} is the $S \times 1$ vector of output from i used to produce final goods absorbed in j . To calculate the value added associated with these implicit output transfers, define the ratio of value added to output for each sector within country i , as $r_i(t) = 1 - \sum_j \sum_s A_{ji}(s, t)$.

With this notation in hand, we can now define value added exports and the value added to export ratio, ‘‘VAX ratio’’, as a measure of the value added content of trade.

Definition 1 (Value Added Exports). The total value added produced in sector s in source country i and absorbed in destination country j is $va_{ij}(s) = r_i(s)y_{ij}(s)$. Total value added produced in i and absorbed in j is then $va_{ij} = \sum_s va_{ij}(s)$.

Definition 2 (VAX Ratio). The sector-level bilateral value added to export ratio is given by $va_{ij}(s)/x_{ij}(s)$. The aggregate bilateral value added to export ratio is $va_{ij}/\iota x_{ij}$, where ι is a $1 \times S$ vector of ones.

1.2 Discussion

We turn to special cases to interpret value added trade flows and the value added content of trade. We use a two country model to develop intuition for the value added content of trade calculations and link our analysis to previous work on the domestic content of exports (equivalently, vertical specialization) by Hummels, Ishii, and Yi (2001). We then use a stylized three country model to demonstrate how our framework tracks value added through the multi-country production chain, even if that value added travels to its final destination via third countries. We also discuss the interpretation of VAX ratios in multi-sector models.

1.2.1 Two Countries, One Sector Per Country

Suppose that there are now only two countries, and each country produces a single differentiated aggregate good. Then the analog to the output decomposition (3) is:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \left[I - \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \right]^{-1} \begin{pmatrix} c_{11} \\ c_{21} \end{pmatrix} + \left[I - \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \right]^{-1} \begin{pmatrix} c_{12} \\ c_{22} \end{pmatrix}. \quad (5)$$

This system describes how the gross output of each country is embodied in final consumption in each of the two countries. To unpack this result, we solve for the breakdown of country 1's production:

$$y_1 = y_{11} + y_{12} \quad (6)$$

with $y_{11} = M_1 \left(c_{11} + \frac{\alpha_{12}}{1 - \alpha_{22}} c_{21} \right)$ and $y_{12} = M_1 \left(\frac{\alpha_{12}}{1 - \alpha_{22}} c_{22} + c_{12} \right)$,

where $M_1 \equiv \left(1 - \alpha_{11} - \frac{\alpha_{12}\alpha_{21}}{1 - \alpha_{22}} \right)^{-1} \geq 1$ is an intermediate goods multiplier that describes the total amount of gross output from country 1 required to produce one unit of country 1's net output.⁹

The first term (y_{11}) is the total amount of country 1's output that is required to produce final goods absorbed in country 1. This term includes both output dedicated to satisfy country 1's demand for its own final goods ($M_1 c_{11}$), as well as output needed to satisfy country 1's demand for country 2 final goods ($M_1 \frac{\alpha_{12}}{1 - \alpha_{22}} c_{21}$).¹⁰ The second term (y_{12}) has a similar interpretation in terms of country 2's demand.¹¹ Because (6) geographically decomposes country 1's output, we can translate this into a decomposition of value added: $va_1 = va_{11} + va_{12}$, where $va_{ij} = [1 - \alpha_{11} - \alpha_{21}]y_{ij}$ is value added generated by country i that is absorbed in country j .

⁹This multiplier is greater than one because output is "used up" in the production process. Without exported intermediates ($\alpha_{12} = 0$), this multiplier would be $(1 - \alpha_{11})^{-1}$. The additional term reflects the fact that intermediate goods sourced from country 2 contain output produced by country 1.

¹⁰To export final goods c_{21} requires producing $(1 - \alpha_{22})^{-1}c_{21}$ units of country 2 output, which itself requires $\alpha_{12}(1 - \alpha_{22})^{-1}c_{21}$ units of country 1's output as intermediates. To produce this country 1 output requires M_1 times $\alpha_{12}(1 - \alpha_{22})^{-1}c_{21}$ units of country 1's output overall, because some output is used up in the production process.

¹¹To highlight how the output decomposition depends on cross-border intermediate linkages, note that if $\alpha_{12} = 0$ the output decomposition would be: $y_{11} = (1 - \alpha_{11})^{-1}c_{11}$ and $y_{12} = (1 - \alpha_{11})^{-1}c_{12}$. In this counter-factual case, output of country 1 is only used to produce final goods originating in country 1.

There are four output concepts underlying flows from country 1 to country 2: (1) final goods c_{12} , (2) gross exports x_{12} , (3) implicit output transfers y_{12} , and (4) value added exports va_{12} . We pause here to clarify the relationship between them. To begin, note that $x_{12} = c_{12} + \alpha_{12}y_2$, so $c_{12} \leq x_{12}$ when there are exported intermediates. Further, using the output decomposition for country 2 ($y_2 = y_{22} + y_{21}$), we decompose gross exports as: $x_{12} = \alpha_{12}y_{21} + (c_{12} + \alpha_{12}y_{22})$. Multiplying both sides of the expression by $(1 - \alpha_{11})^{-1}$ then translates exports into the gross output required to produce them.¹² It is straightforward to show that $y_{12} = (1 - \alpha_{11})^{-1}(c_{12} + \alpha_{12}y_{22})$. Therefore, $y_{12} = (1 - \alpha_{11})^{-1}x_{12} - (1 - \alpha_{11})^{-1}\alpha_{12}y_{21}$. So the implicit output transferred from country 1 to country 2 is equal to the gross output required to produce exports minus the gross output that is reflected back embedded in country 2 goods that are absorbed by country 1.¹³ Finally, we note that $va_{12} \leq y_{12}$, because the value added to output ratio is bounded above by one.

To directly compare value added exports to gross exports, we compute the VAX ratio:

$$\begin{aligned} \frac{va_{12}}{x_{12}} &= \frac{(1 - \alpha_{11} - \alpha_{21})y_{12}}{x_{12}} \\ &= \frac{1 - \alpha_{11} - \alpha_{21}}{1 - \alpha_{11}} \left(\frac{x_{12} - \alpha_{12}y_{21}}{x_{12}} \right), \end{aligned} \quad (7)$$

where the second line follows from the discussion in the previous paragraph. The difference $x_{12} - \alpha_{12}y_{21}$ is exports less reflected intermediates, or equivalently the portion of exports genuinely consumed abroad. The VAX ratio will always be less than one, so value added exports are scaled down relative to gross exports.

The VAX ratio for a country can be thought of as a metric of the “domestic content of exports.” Indeed, it is closely related to previous approaches to measuring domestic content in the literature. To see this, note that the VAX ratio has two components. The first component, $\frac{1 - \alpha_{11} - \alpha_{21}}{1 - \alpha_{11}}$, is equivalent to a metric of domestic content developed in Hummels, Ishii, and Yi (2001).¹⁴ This

¹²This follows from manipulation of the market clearing condition for country 1: $y_1 = (1 - \alpha_{11})^{-1}(c_{11} + x_{12})$.

¹³Note that if $\alpha_{12} = 0$, then $y_{12} = (1 - \alpha_{11})^{-1}x_{12}$, so the gross output required to produce exports equals the actual amount of output transferred from country 1 to country 2.

¹⁴Hummels et al. focus their discussion on measuring vertical specialization or the “import content of exports,” which is given by $\alpha_{21}(1 - \alpha_{11})^{-1}$. Domestic content is then one minus the import content of exports. Though we discuss these concepts here in a scalar case, they generalize in a straightforward way to models with many sectors.

metric captures the value added associated with the gross output needed to produce exports as a fraction of total exports. The Hummels-Ishii-Yi metric is equal to the VAX ratio only when country 2 does not use imported intermediates ($\alpha_{12} = 0$), and therefore country 1 exports final goods alone. In contrast, with two-way trade in intermediates the Hummels-Ishii-Yi metric overstates the amount of domestic value added that is generated per unit of exports. The second component of the VAX ratio adjusts for two-way trade in intermediates by allowing some exports to be dedicated to producing goods that are ultimately consumed at home. That is, it allows for a portion of exports to be reflected back to the source rather than absorbed abroad.

1.2.2 Three Countries, One Sector Per Country

While the two country framework illustrates the basic discrepancy between value added and gross trade flows, additional insights emerge as one introduces a third country to the mix. We focus on a special, algebraically straightforward case that illustrates how the accounting framework tracks the final destination at which value added by a given country is consumed even if this value circulates through a multi-country production chain en route to its final destination. We construct the special case to approximate a stylized account of production chains between the U.S. and Asia.¹⁵

Let country 1 be the U.S., country 2 be China, and country 3 be Japan. Further, assume that China imports intermediates from the U.S. and Japan and exports only final consumption goods only to the U.S. For simplicity, we assume that the U.S. and Japan do not export any final goods and only export intermediates to China. This configuration of production can be represented as:

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & 0 \\ 0 & \alpha_{22} & 0 \\ 0 & \alpha_{32} & \alpha_{33} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} + \begin{pmatrix} c_{11} \\ c_{22} + c_{21} \\ c_{33} \end{pmatrix}. \quad (8)$$

¹⁵This example was inspired by Linden, Kraemer, and Dedrick (2007), who trace the iPod production chain. The iPod combines U.S. intellectual property from Apple with a Japanese display and disk drive, which is manufactured in China. These components are assembled in China and the iPod is shipped to the U.S.

This then can be solved to yield the following three-equation system:

$$\begin{aligned}
y_1 &= \underbrace{\frac{1}{1 - \alpha_{11}} c_{11}}_{y_{11}} + \underbrace{\frac{\alpha_{12}}{(1 - \alpha_{11})(1 - \alpha_{22})} c_{21}}_{y_{12}} + \underbrace{\frac{\alpha_{12}}{(1 - \alpha_{11})(1 - \alpha_{22})} c_{22}}_{y_{12}} \\
y_2 &= \underbrace{\frac{1}{1 - \alpha_{22}} c_{21}}_{y_{21}} + \underbrace{\frac{1}{1 - \alpha_{22}} c_{22}}_{y_{22}} \\
y_3 &= \underbrace{\frac{\alpha_{32}}{(1 - \alpha_{33})(1 - \alpha_{22})} c_{21}}_{y_{31}} + \underbrace{\frac{\alpha_{32}}{(1 - \alpha_{33})(1 - \alpha_{22})} c_{22}}_{y_{32}} + \underbrace{\frac{1}{1 - \alpha_{33}} c_{33}}_{y_{33}}.
\end{aligned} \tag{9}$$

This system provides the implicit output transfers needed to calculate value added flows.

Two points are interesting to note. First, as in the two-country case above, U.S. demand for U.S. output has both a direct component $\frac{1}{(1 - \alpha_{11})} c_{11}$, and an indirect component $\frac{\alpha_{12}}{(1 - \alpha_{11})(1 - \alpha_{22})} c_{21}$ that accounts for the fact that U.S. imports of final goods from China include embedded U.S. content. Thus, a larger share of U.S. output is ultimately absorbed at home than bilateral trade statistics would indicate. Correspondingly, Chinese bilateral exports overstate the true Chinese content shipped to the U.S. due to bilateral U.S.-China production sharing.

The second point is that, although Japan does not export directly to the U.S., the U.S. does import Japanese content embedded in Chinese exports to the U.S. This effect is the result of multi-country production chains, and was absent in the two country case analyzed above. In the equation for Japan (country 3), this effect appears as $\frac{\alpha_{32}}{(1 - \alpha_{33})(1 - \alpha_{22})} c_{21}$.

Because Chinese exports to the U.S. contain both U.S. and Japanese content, the bilateral VAX ratio of China-U.S. trade is:

$$\frac{va_{21}}{x_{21}} = 1 - \left(\frac{va_{31} + \alpha_{12} y_{21}}{x_{21}} \right) < 1. \tag{10}$$

This illustrates that the bilateral VAX ratio removes both the Japanese value added (va_{31}) and U.S. intermediate goods ($\alpha_{12} y_{21}$) from Chinese exports to the U.S.¹⁶ Turning to Japan, it has positive

¹⁶U.S. imports from China contain U.S. content because the U.S. exports intermediates to China and imports final goods from China. Thus, U.S. intermediates are reflected back to the US and constitute a portion of the value added

value added exports to the U.S. and zero direct bilateral exports. Therefore, the bilateral VAX ratio for Japan-U.S. trade is undefined, or practically infinite for small bilateral exports. This extreme ratio illustrates another general lesson. Though the aggregate VAX ratio is bounded by one for each country, bilateral VAX ratios may be greater than one when an exporter sends intermediates abroad to be processed and delivered to a third country. Thus, bilateral VAX ratios pick up the influence of both bilateral and multilateral production sharing relationships.

When bilateral VAX ratios vary across partners, bilateral value added balances do not equal bilateral trade imbalances. To illustrate this, we define $tb_{12} \equiv x_{12} - x_{21}$ and $vab_{12} \equiv va_{12} - va_{21}$ to be bilateral U.S.-China trade and value added balances. In this special case, where the configuration of production is given by (8), these balances are related as follows:

$$tb_{12} + \alpha_{32}y_{21} = vab_{12}. \quad (11)$$

That is, $tb_{12} < vab_{12}$. So assuming the U.S. runs a trade deficit with China in this example, then it will run a smaller deficit with China in value added terms due to the fact that Chinese bilateral trade contains Japanese content ($\alpha_{32}y_{21}$). As a corollary, the U.S.'s bilateral balance with Japan will be distorted in the opposite direction.

To generalize this result, we can write any given bilateral value added balance as:

$$\begin{aligned} vab_{ij} &= \frac{va_{ij}}{x_{ij}}x_{ij} - \frac{va_{ji}}{x_{ji}}x_{ji} \\ &= \frac{1}{2}(x_{ij} + x_{ji}) \left[\frac{va_{ij}}{x_{ij}} - \frac{va_{ji}}{x_{ji}} \right] + \frac{1}{2} \left(\frac{va_{ij}}{x_{ij}} + \frac{va_{ji}}{x_{ji}} \right) [x_{ij} - x_{ji}]. \end{aligned} \quad (12)$$

The first term adjusts the value added balance due to differences in VAX ratios between exports and imports. When the VAX ratio for exports is high relative to imports, the value added balance is naturally pushed in a positive direction. Note here that this is true even if gross trade is balanced. The second term adjusts the value added balance based on the average level of VAX ratios. Starting from an initial imbalance, the value added balance is scaled up or down relative to the trade balance,

that the U.S. purchases from itself.

depending on whether VAX ratios are greater than or less than one (on average). So differences in VAX ratios between partners within a bilateral relationship and the absolute level of the VAX ratios between partners both influence the size of the adjustment in converting gross imbalances to value added terms.

1.2.3 Many Countries, Many Sectors

The interpretation of aggregate value added exports and VAX ratios developed in the one-sector examples in previous sections carries over to the many country, multi-sector framework in Section 1.1. One important distinction between the one-sector and multi-sector frameworks is that VAX ratio at the sector level cannot be interpreted as the domestic content of exports. At the sector level, the VAX ratio is the ratio of exported value added produced in a sector as a share gross exports from that sector. Rather than summarizing the domestic content of exports, the sectoral VAX ratio reflects a combination of influences.

First, the sectoral VAX ratio incorporates information on how individual sectors engage in trade. Consider a situation in which producers in one sector sell intermediates to purchasers in another sector, who in turn produce goods for export.¹⁷ In this case, the intermediate goods suppliers engage in trade indirectly. Hence, we observe no direct exports from the intermediate goods supplier, but do observe value added exports because value added from that sector is embedded in the purchaser's goods. Thus, value added exports from a particular sector may be physically embodied in goods exported from that sector or embodied in exports of other sectors. High ratios of value added exports to gross trade (possibly above one) at the sector level are evidence of indirect participation in trade. Low ratios instead indicate that a given sector's gross exports embody value added produced outside that sector.

Second, the sectoral VAX ratio is influenced by how individual sectors fit into cross-border production chains. Consider now a situation in which producers in a sector export their output directly, without further processing in other sectors. To simplify the analysis, let us further assume

¹⁷For example, the "raw milk" sector in our data has near zero exports, but raw milk is sold to the "dairy products" sector, which does export.

that this sector does not purchase intermediates from other domestic sectors. Then the sectoral VAX ratio depends on the sector's connection to foreign production chains. Specifically, the VAX ratio will be determined by whether exported output is absorbed abroad or used to produce foreign goods that are ultimately absorbed at home. If exports are largely absorbed abroad, one would see a relatively high VAX ratio.

Third, the sectoral VAX ratio depends on the general value added to gross output ratio of a sector. This is evident, as value added exports of a sector are equal to the output of that sector absorbed abroad times the value added to output ratio of that sector. Since manufacturing sectors generally have low value added to output ratios relative to other sectors, this will show up in sectoral VAX ratios as well.

Though these influences are difficult to separate empirically, we discuss evidence below that sheds light on the relative importance of these channels.

2 Data

Our data source is the GTAP 7.1 Data Base assembled by the Global Trade Analysis Project at Purdue University. This data is compiled based on three main sources: (1) World Bank and IMF macroeconomic and Balance of Payments statistics; (2) United Nations Commodity Trade Statistics (Comtrade) Database; and (3) input-output tables based on national statistical sources. To reconcile data from these different sources, GTAP researchers adjust the input-output tables to be consistent with international data sources.¹⁸ The GTAP data includes bilateral trade statistics and input-output tables for 94 countries plus 19 composite regions covering 57 sectors in 2004.¹⁹

¹⁸See the GTAP website at <http://www.gtap.agecon.purdue.edu/> for documentation of the source data. Since raw input-output tables are based on national statistical sources, they inherit all the shortcomings of those sources. For example, import tables are often constructed using a "proportionality" assumption whereby the imported input table is assumed to be proportional to the overall aggregate input-output table.

¹⁹GTAP assigns composite regions "representative" input-output tables, constructed from input-output tables of similar countries. Composite regions do not play an important role in our results, accounting for 5% of world trade and 3% of world value added. To measure bilateral services trade, GTAP uses OECD data where available and imputes bilateral services trade elsewhere. Because services account for less than 18% of exports for the median country, our results are likely to be insensitive to moderate mismeasurement of services trade.

Regarding sector definitions, there are 18 Agriculture and Natural Resources sectors, 24 Manufactures sectors, and 15 Services sectors.

In the data, we have information on 6 objects for each country:

1. y_i is a 57×1 vector of total gross production.
2. c_{Di} is a 57×1 vector of domestic final demand.
3. c_{Ii} is a 57×1 vector of domestic final import demand.
4. A_{ii} is a 57×57 domestic input-output matrix, with elements $A_{ii}(s, t)$.
5. A_{Ii} is a 57×57 import input-output matrix, with elements $A_{Ii}(s, t) = \sum_{j \neq i} A_{ji}(s, t)$.
6. $\{x_{ij}\}$ is a collection of 57×1 bilateral export vectors for exports from i to j .

The definition of “final demand” is based on the national accounts, including consumption, investment, and government purchases. We value each country’s output at a single set of prices, regardless of where that output is shipped or how it is used. This ensures that the value of production revenue equals expenditure.²⁰ Following input-output conventions, we use “basic prices,” defined as price received by a producer (minus tax payable or plus subsidy receivable by the producer).²¹

To set up the accounting framework, we use bilateral trade data to split the imported intermediates matrix A_{Ii} and the imported final goods vector c_{Ii} into bilateral input-output matrices A_{ji} and final demand vectors c_{ji} . Within each sector, we assume that imports from each source country are split between final and intermediate in proportion to the overall split of imports between final and intermediate use in the destination. Further, conditional on being allocated to intermediate use, we assume that imported intermediates from each source are split across purchasing sectors in

²⁰Put differently, while quantity choices may reflect price differences across destinations or uses that arise due to transport costs, tariffs, and markups, we value the resulting quantity flows at a single set of prices.

²¹In our framework, the level of value added differs from the one used in national accounts. We calculate value added as output at basic prices minus intermediates at basic prices, whereas the national accounts calculate value added as output at basic prices minus intermediates at purchaser’s prices.

proportion to overall imported intermediate use in the destination. Formally, for goods from sector s used by sector t , we define bilateral input-output matrices and consumption import vectors:

$$A_{ji}(s, t) = A_{Ii}(s, t) \left(\frac{x_{ji}(s)}{\sum_j x_{ji}(s)} \right) \quad \text{and} \quad c_{ji}(s) = c_{Ii}(s) \left(\frac{x_{ji}(s)}{\sum_j x_{ji}(s)} \right).$$

This assumption implies that all variation in total bilateral intermediate and final goods flows arises due to variation in the composition of imports across partners. For example, we would find that US imports from Canada are intermediate goods intensive because most imports from Canada are goods that are on average used as intermediates (e.g., auto parts).

In the main calculation, we assume that production techniques and input requirements are the same for exports and domestically absorbed final goods. This assumption is problematic for countries that have large export processing sectors. These processing sectors (almost by definition) produce distinct goods for foreign markets with different input requirements and lower value added to output ratios than the rest of the economy. Ignoring this fact tends to overstate the value added content of exports.

As an alternative calculation, we relax this assumption for China and Mexico, two prominent countries with large export processing sectors (roughly two thirds of exported Manufactures originates in these sectors) and key trading partners with the U.S.²² We present supplementary calculations below that adjust the value added content of exports using an adaptation of a procedure from Koopman, Wang, and Wei (2008). The basic idea is to measure the share of exports and imports that flow through the export processing sector, and then impute separate input-output coefficients for the processing sector so as to be consistent with these flows. Details of the procedure are presented in Appendix A. We then compute the value added content of trade using a new input-output system that includes these amended tables.²³

²²For Mexico, we classify exports originating from maquiladoras as processing exports. For China, we use estimates from Koopman, Wang, and Wei (2008) constructed from Chinese trade statistics, obtained from Zhi Wang.

²³We perform this calculation at a higher level of aggregation than our baseline calculation, with three composite sectors. We believe the results are not very sensitive to aggregation, as aggregate value added flows are nearly identical in the original, unadjusted data whether computed using 57 sectors or 3 composite sectors.

3 Empirical Results

3.1 Multilateral Value Added Exports

Table 1 reports aggregate VAX ratios for each country, grouped by region.²⁴ Across countries, value added exports represent about 73% of gross exports. The magnitude of the adjustment varies both across and within regions. At the regional level, VAX ratios are lowest for Europe (broadly defined) and East Asia, and higher in the Americas, South Asia and Oceania, and the Middle East and Africa. Looking within regions, the new E.U. members (e.g., Estonia, Hungary, Slovakia, and the Czech Republic) stand out as having low VAX ratios in Central-Eastern Europe, while Japan stands out with a high VAX ratio relative to East Asia.

For China and Mexico, we report two separate calculations of the VAX ratio in the table, one computed without adjusting for processing trade and a second adjusted for processing trade.²⁵ VAX ratios for both China and Mexico fall substantially when we adjust for export processing trade, from 0.70 to 0.59 for China and from 0.67 to 0.52 for Mexico. This brings the ratios for China and Mexico in line with other emerging markets such as South Korea or Hungary, and is evidence of the low value added to export ratios within each country's processing sector.²⁶

Moving down a level of disaggregation, we report VAX ratios for three composite sectors by country in Table 1 as well. The three sectors are: Agriculture and Natural Resources, Manufacturing, and Services. VAX ratios are typically greater than or equal to one in the Agriculture and Natural Resources and Services sectors, and markedly less than one in Manufacturing. This cross-sector variation is primarily due differences in the manner in which each sector engages in trade, rather than differences across sectors in the degree of participation in cross-border production shar-

²⁴We omit ratios for composite regions from the table.

²⁵In the calculation adjusted for processing trade in China and Mexico, VAX ratios in all countries change relative to the unadjusted benchmark calculation. The absolute size of the changes in aggregate VAX ratios is very small, with a median of 0.016 and 90% of changes less than 0.053. Therefore, we report only one set of ratios for all countries other than China and Mexico.

²⁶For the processing sector, we estimate that China's VAX ratios is 0.13, while Mexico's VAX ratio is 0.08. These ratios measure the value added produced within the processing sector as a share of processing exports. These ratios represent a lower bound on the domestic content of processing exports, since the processing sector purchases intermediates from other domestic sectors.

ing. Further, differences in value added to output ratios across sectors are also an important source of variation.

To sort through these influences, we refer back to Section 1.2.3. Recall sectoral VAX ratios would tend to be low when exports are used to produce foreign goods that are ultimately absorbed at home. If we were to assume that all output was absorbed abroad, then the sectoral analog to the aggregate Hummels-Ishii-Yi calculation would be approximately equal to the sectoral VAX ratio. To be precise, the output used to produce exports would be $(I - A_{ii})^{-1} \left(\sum_{j \neq i} x_{ij} \right)$ and element-wise multiplication by sectoral value added to output ratios would yield a sectoral VAX ratio. In our data, this counter-factual calculation yields ratios that are very close to the actual VAX ratios. As such, differences across sectors in the degree of foreign absorption of exports does not appear to drive the VAX ratios. Further, we note that differences in value added to export ratios also cannot explain the full variation in VAX ratios across sectors. In the data, the value added to output ratio in Manufactures is roughly 0.25 lower than in Agriculture and Natural Resources and Services sectors. This goes part of the way toward explaining differences in VAX ratios across sectors, but falls substantially short.

What remains is the mode via which sectors engage in trade. Recall that we observe gross exports from a given sector only if output from that sector crosses an international border with no further processing. With this in mind, it is obvious that VAX ratios are greater than one when a sector exports value added embodied in another sector's goods. In the data, it appears that Manufactures, which are directly traded, embody substantial value added from the other sectors. One implication of this fact is that the composition of aggregate value added flows differs from that of gross trade. Figure 1 summarizes this fact by plotting the share of Manufactures and Services in both types of trade for the 10 largest exporters. The role of Manufactures in value added trade is diminished, while Services is increased by a roughly equivalent amount.²⁷ The upshot is that Services are far more exposed to international commerce than one would think based on gross trade statistics.

²⁷Agriculture and Natural Resources constitutes a roughly equal share of value added and gross trade.

To organize the cross-country variation in the data, we construct a “between-within” decomposition of the aggregate VAX ratio. The decomposition is constructed relative to a reference country as follows:

$$\begin{aligned}
 VAX_i - \overline{VAX} = & \underbrace{\sum_s [VAX_i(s) - \overline{VAX}(s)] \left(\frac{\omega_i(s) + \bar{\omega}(s)}{2} \right)}_{\text{Within Term}} \\
 & + \underbrace{\sum_s [\omega_i(s) - \bar{\omega}(s)] \left(\frac{VAX_i(s) + \overline{VAX}(s)}{2} \right)}_{\text{Between Term}}, \quad (13)
 \end{aligned}$$

where s denotes sector, i denotes country, and $\omega(s)$ and $VAX(s)$ are the export share and VAX ratio in sector s . Bars denote reference country variables, which are constructed based on global composites.²⁸ In this decomposition, the Within Term varies primarily due to differences in VAX ratios within sectors across countries, while the Between Term is influenced mainly by differences in the sector composition of trade. To isolate compositional shifts between Manufactures and non-Manufactures, we calculate the decomposition using two composite sectors, pooling Services plus Agriculture and Natural Resources into a single composite non-manufacturing sector.

Cross-country variation in aggregate VAX ratios is to a large extent driven by variation in the composition of exports. In fact, a simple variance decomposition indicates that the Between Term “accounts for” roughly 100% of the variance in the aggregate VAX ratio if we split the covariance of the Between and Within Terms equally.²⁹ The Between Term is dominant because of the large differences in VAX ratios across sectors. Countries that export predominantly Manufactures, the sector with the lowest VAX ratio, tend to have low aggregate VAX ratios as well.

Despite this strong composition effect, aggregate VAX ratios are only weakly related to the overall level of economic development. Panel A in Table 2 reports that a one log point increase in income per capita is associated with a fall in domestic content of 0.8 percentage points, though

²⁸Reference country VAX ratios for each sector are the ratios of value added exports to gross exports for the world as a whole. Export shares are the share of each sector in total world exports.

²⁹Specifically, the variance breaks down as follows: $\text{var}(\text{Agg. VAX}) = 0.01$, $\text{var}(\text{Within}) = 0.03$, $\text{var}(\text{Between}) = 0.04$, and $\text{cov}(\text{Within}, \text{Between}) = -0.03$.

this correlation is not quite significantly different from zero at conventional significance levels.³⁰ This weak aggregate correlation is a manifestation of two offsetting effects. First, richer countries tend to have exports concentrated in Manufactures, which has a relatively low VAX ratio. Second, richer countries tend to export with higher VAX ratios than poorer countries within composite sectors, particularly within Manufactures.

To illustrate these offsetting effects, we project the Between Term and the Within Term separately on exporter income to quantify the relative contribution of each to the overall correlation. In Panel A of Table 2, we see that there is a strong negative correlation of the Between Term with exporter income. That is, countries systematically shift toward manufacturing (which has lower value added to output on average) as they grow richer and this depresses the aggregate VAX ratios. The effect of this on overall VAX ratios is obscured because the Within Term is significantly positively correlated with exporter income. This positive correlation is mostly due to the fact that rich countries have higher VAX ratios within Manufactures. Panel B of Table 2 reports the correlation of VAX ratios for Manufactures with income per capita and splits this into Between and Within Terms as above.³¹ The positive correlation between Manufactures VAX ratios and income is itself driven by a positive composition (“between”) effect, wherein richer countries tend to specialize in manufacturing sectors with high VAX ratios.

3.2 Bilateral Value Added Exports and Balances

For a particular exporter, bilateral VAX ratios differ widely across destinations. For concreteness, we graphically present bilateral value added to trade ratios for the two largest exporters, the U.S. and Germany, in Figure 2. In the figure, value added to import ratios are VAX ratios for each country exporting to the U.S./Germany, while value added to export ratios are recorded for U.S./German

³⁰The p-value for a two-sided test that the correlation does not equal zero is 14%. In this regression, we omit outliers Belgium, Luxembourg, and Singapore. If these three countries are included, the correlation roughly doubles in size and becomes highly significant.

³¹VAX ratios for the non-Manufactures composite are positively correlated with income per capita, but the correlation is not significant. Therefore, we do not report these results separately.

exports to each country.³²

Looking at the U.S., there is wide variation in VAX ratios. For some partners, value added exports are quite close to gross exports. For example, the difference between gross and value added exports to the U.K. amounts to only 3% of gross exports. For others, gross trade either overstates or understates the bilateral exchange of value added. Value added exports to Canada are \$77 billion (40%) smaller than gross exports, and value added exports to Mexico are \$40-\$50 billion (36-44%) smaller. Value added trade falls by a similar proportional amount, between 30-50%, relative to gross trade for countries like Ireland, Korea, and Taiwan, which are well-cited examples of production sharing partners. At the other end of the spectrum, several countries have VAX ratios toward the U.S. above one. For example, countries on Europe's Eastern periphery (see Russia) have bilateral VAX ratios above one mainly because they supply intermediates to Western European countries that then end up being consumed in the U.S. Further, commodity producers (see Australia) also often have ratios above one.

The U.S. data are representative of general patterns in the data.³³ Looking at Germany, discrepancies between value added and gross trade also vary in meaningful ways across partners. Value added trade is scaled down quite substantially for the vast majority of its large European partners, in contrast to the U.S. This surely is an indication of the integrated structure of production within the European Union and its neighbors. Consistent with anecdotal evidence, this is most pronounced for the Czech Republic and Hungary. Geography appears to play a substantial role, as trade with partners of similar income levels such as the U.S. and Japan is relatively less distorted.

One consequence of these trade adjustments is that bilateral trade balances differ when measured in gross versus value added terms. Figure 3 displays three measures of bilateral balances for the U.S.: the bilateral trade balance, the bilateral value added balance, and the bilateral value added balance adjusted for processing exports in China and Mexico. In interpreting this figure,

³²We display data for the 15 largest trade partners for each country plus additional countries selected for illustration purposes, including adjusted and unadjusted bilateral VAX ratios for China and Mexico. In line with the aggregate results, adjusting for processing trade lowers bilateral VAX ratios vis-a-vis these countries but has only modest effects on ratios for other countries.

³³The median bilateral VAX ratio in the data is 0.91, and the 10th – 90th percentile range is 0.59 to 2.07. Approximately 40% of the bilateral VAX ratios are greater than one.

it is important to keep in mind that multilateral trade balances equal the multilateral value added balance for each country. Therefore, a decline in the bilateral value added balance relative to the gross trade balance for one country necessarily implies an increase for some other country.

Comparing these alternate measures, there are large shifts in bilateral balances in Asia. Most prominently, the U.S. deficit with China falls by roughly 30-40% (\$35-50 billion), while the deficit with Japan rises by around 33% (\$17-18 billion). The end result is that the value added balances (adjusted for processing trade) are nearly equal for Japan and China. Looking elsewhere within Emerging Asia, U.S. deficits with Taiwan and South Korea also rise and U.S. surpluses with Australia and Singapore fall. Together, adjustments in these five countries (Australia, Japan, Singapore, South Korea, and Taiwan) nearly exactly add up to the fall in the U.S.-China deficit, which points to triangular production sharing within Asia with these countries feeding intermediates to China that are then embodied in Chinese exports to the U.S.

To understand these adjustments, we focus on the U.S.-China and U.S.-Japan balances with reference to the decomposition of the value added balance in Equation (12). First, looking at China, the VAX ratio for U.S. exports to China exceeds the VAX ratio for imports by about 8% in the unadjusted calculation and 4% in the adjusted calculation. This tends to raise the value added balance relative to the trade balance, though only modestly (by \$10 billion without adjustment and \$5 billion with adjustment).³⁴ Second, the value added content of both bilateral U.S. exports and imports to/from China are well below one. The simple average VAX ratio across exports and imports is 0.80 without adjustment and 0.66 with adjustment. If VAX ratios for both exports and imports were equal to this average level, this would imply value added deficits 20% or 34% smaller than the gross deficits. This second “level effect” accounts for most of the adjustment from gross to value added balances for China (between \$25-\$44 billion of the total change). In contrast, for Japan, this level effect is virtually nil, as the simple average VAX ratio is near one (literally, 0.98 without adjustment and 1.00 with adjustment). The U.S. deficit with Japan rises in value added terms mainly because the ratio of value added imports to gross imports is high relative to the ratio

³⁴If gross trade were (counterfactually) balanced between the U.S. and China, the value added balance would show a surplus due to this force alone.

of value added exports to gross exports (the VAX ratio for imports is 0.16 higher than for exports in both calculations).

3.3 Inspecting the Mechanism: Bilateral Decompositions

To demonstrate that production sharing drives variation in bilateral VAX ratios, we construct two decompositions in the data. The first decomposition splits variation in bilateral VAX ratios into components arising from differences in the composition of exports across destinations and differences in bilateral production sharing relations. The second decomposition looks directly at how output circulates within cross-border production chains by (approximately) splitting bilateral exports into components absorbed and consumed in the destination, reflected back and ultimately consumed in the source, and redirected and ultimately consumed in a third destination.

To construct the first decomposition, we express the bilateral VAX ratio as:

$$\begin{aligned} \frac{va_{ij}}{\iota x_{ij}} &= \frac{\iota (I - A_{ii} - A_{Ii}) y_{ij}}{\iota x_{ij}} \\ &= \underbrace{\frac{\iota (I - A_{ii} - A_{Ii}) (I - A_{ii})^{-1} x_{ij}}{\iota x_{ij}}}_{\text{Bilateral HIY (BHIY)}} + \underbrace{\frac{\iota (I - A_{ii} - A_{Ii}) (y_{ij} - (I - A_{ii})^{-1} x_{ij})}{\iota x_{ij}}}_{\text{Production Sharing Adjustment (PSA)}} \end{aligned} \quad (14)$$

The first term is equivalent to the Hummels-Ishii-Yi measure of the domestic content of exports calculated using bilateral exports. For a given source country, it varies only due to variation in the composition of the export basket across destinations.

The second term is a production sharing adjustment. This adjustment depends on the difference between the amount of country i output consumed in j , y_{ij} , and the gross output from i required to produce bilateral exports to j , $(I - A_{ii})^{-1} x_{ij}$. When $y_{ij} < (I - A_{ii})^{-1} x_{ij}$, the VAX ratio is smaller than the bilateral HIY benchmark. This situation arises when country i 's intermediate goods shipped to country j are either reflected back to itself embedded in foreign produced final goods or intermediate goods used to produce domestic final goods, or redirected to third destinations embedded in country j 's goods. When $y_{ij} > (I - A_{ii})^{-1} x_{ij}$, the VAX ratio is larger than the

HIY benchmark. This situation arises when country i ships intermediates to some third country that then (directly or indirectly) embeds those goods in final goods absorbed in country j .

To quantify the role of each term in explaining bilateral VAX Ratios, we decompose the variance of the bilateral VAX Ratio for each exporter across destinations, $\text{var}_i \left(\frac{va_{ij}}{\iota x_{ij}} \right)$, into variation due to the BHIY Term versus the PSA Term. Table 3 reports the share of the total variance accounted for by the BHIY and PSA terms for representative exporters.³⁵ The production sharing adjustment (PSA Term) evidently dominates the decomposition. This implies that variation in production sharing relations across partners, not export composition across destinations, drives the bilateral VAX ratio. Put differently, bilateral VAX ratios are determined not by what an exporter sends to any given destination, but rather how those goods are used abroad. In concrete terms, even though the U.S. sends automobile parts to both Canada and Germany, the U.S. VAX ratio with Canada is lower than with Germany because Canada is part of a cross-border production chain with the U.S.

To look at production chains more directly, we construct a second decomposition that splits bilateral exports according to whether they are absorbed, reflected, or redirected by the destination to which they are sent. We construct the decomposition using the division of bilateral exports into final and intermediate goods along with the output decomposition for the foreign destination:

$$\begin{aligned} \iota x_{ij} &= \iota (c_{ij} + A_{ij}y_j) \\ &= \underbrace{\iota (c_{ij} + A_{ij}y_{jj})}_{\text{Absorption}} + \underbrace{\iota A_{ij}y_{ji}}_{\text{Reflection}} + \underbrace{\sum_{k \neq j,i} \iota A_{ij}y_{jk}}_{\text{Redirection}} \end{aligned} \quad (15)$$

The first term captures the portion of bilateral exports absorbed and consumed in destination j , including both final goods from country i and intermediates from i embodied in country j 's consumption of its own goods. The second term captures the reflection of country i 's intermediates back to itself embodied in country j goods. The third term is the summation of country i 's inter-

³⁵In the table, we split the covariance equally between the BHIY and PSA Terms. Because the covariance is small, our conclusions are not sensitive to how we split the covariance.

mediates embodied in j 's goods that are consumed in all other destinations, i.e., redirected to third destinations.³⁶

We report the results of this decomposition for informative bilateral pairs in Table 4. Looking at the upper left portion of the table, we see that Japan's exports to China are primarily either absorbed in China or redirected to the U.S. Comparing Japan's trade with China to that with the U.S., we see that Japanese exports to the U.S. are nearly exclusively absorbed by the U.S., indicating minimal bilateral U.S.-Japan production sharing. In contrast, looking at the upper right panel, we see that large portions of U.S. exports to Canada and Mexico are reflected back to the U.S. for final consumption. Looking at the lower left panel, we see that sharing a common border with two different countries does not necessarily imply tight bilateral production sharing relationships. German exports to France are primarily absorbed there, while nearly half of exports to the Czech Republic are reflected or redirected. Finally, in the lower right corner, we see that Korea is engaged in triangular trade with the U.S. and other destinations via China. In contrast, a larger share of Korean exports to Japan are eventually consumed there. These results are consistent with our priors regarding the role of China as a production sharing hub in Asia.³⁷

4 Concluding Remarks

Intermediate goods trade is a large and growing feature of the international economy. Quantification of cross-border production linkages is therefore central to answering a range of important empirical questions in international trade and international macroeconomics. This requires going beyond specific examples or country/regional studies to develop a complete, global portrait of production sharing patterns. This paper provides such a portrait using input-output and trade data to

³⁶This decomposition is only approximate, because the output split used in constructing the decomposition is influenced by the entire structure of cross-border linkages. Nonetheless, this decomposition is informative as it returns shares that are consistent with the zero order and first round effects of the Leontief matrix inversion (i.e., $[I + A]$) describing how final goods absorbed in each destination are produced. We prefer the decomposition in the text to this alternative "first-order approximation" of the production structure because it adds up to bilateral exports.

³⁷These decompositions are computed without adjusting for processing trade in China. Adjusting for processing trade tends to amplify reflection and redirection effects. Thus, our table understates the amount of redirection within Asia and reflection in U.S.-Mexico trade.

compute bilateral trade in value added. We document significant differences between value added and gross trade flows, differences that reflect heterogeneity in production sharing relationships. We look forward to applying this data in future work to deepen our understanding of the consequences of production sharing.

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Table 1: VAX Ratios by Country and Sector

Country	Code	Aggregate	Composite Sector			Country	Code	Aggregate	Composite Sector		
			Ag.& Nat.R.	Manuf.	Services				Ag.& Nat.R.	Manuf.	Services
Central & Eastern Europe						North & South America					
Albania	alb	0.79	2.10	0.44	0.97	Argentina	arg	0.84	1.27	0.40	2.26
Armenia	arm	0.67	1.21	0.46	1.12	Bolivia	bol	0.85	1.08	0.24	1.79
Azerbaijan	aze	0.86	1.14	0.18	1.08	Brazil	bra	0.86	0.95	0.51	3.27
Belarus	blr	0.69	5.69	0.35	4.25	Canada	can	0.70	1.00	0.44	1.97
Bulgaria	bgr	0.63	0.85	0.38	1.17	Chile	chl	0.80	0.92	0.46	2.31
Croatia	hrv	0.71	1.04	0.52	0.92	Colombia	col	0.86	0.92	0.51	2.16
Czech Republic	cze	0.59	1.52	0.43	1.51	Costa Rica	cri	0.69	0.68	0.37	2.23
Estonia	est	0.53	1.07	0.34	0.94	Ecuador	ecu	0.90	0.90	0.37	3.30
Georgia	geo	0.77	1.23	0.38	1.44	Guatemala	gtm	0.79	0.82	0.43	1.83
Hungary	hun	0.54	0.96	0.38	1.39	Mexico	mex	0.67	0.69	0.65	0.93
Kazakhstan	kaz	0.78	0.53	0.50	3.26	Mexico (adjusted)	mex_adj	0.52	0.88	0.41	1.27
Kyrgyzstan	kgz	0.70	0.78	0.49	1.01	Nicaragua	nic	0.74	1.12	0.38	2.04
Latvia	lva	0.64	0.84	0.51	0.96	Panama	pan	0.84	1.06	0.36	0.91
Lithuania	ltu	0.63	0.95	0.46	1.23	Paraguay	pry	0.84	0.91	0.28	1.07
Poland	pol	0.70	1.34	0.52	1.57	Peru	per	0.93	0.99	0.72	1.78
Romania	rou	0.70	2.58	0.48	1.95	United States	usa	0.77	0.86	0.49	1.58
Russian Federation	rus	0.87	0.99	0.41	2.49	Uruguay	ury	0.71	1.31	0.42	1.30
Slovakia	svk	0.55	1.29	0.39	1.77	Venezuela	ven	0.89	1.06	0.29	5.54
Slovenia	svn	0.64	2.26	0.44	1.59	South Asia & Oceania					
Ukraine	ukr	0.67	0.92	0.27	2.67	Australia	aus	0.86	0.87	0.50	1.64
East Asia						Bangladesh	bgd	0.75	5.06	0.43	2.66
Cambodia	khm	0.62	3.86	0.40	1.26	India	ind	0.81	1.80	0.46	1.68
China	chn	0.70	4.11	0.46	2.75	New Zealand	nzl	0.82	1.56	0.43	1.60
China (adjusted)	chn_adj	0.59	3.90	0.40	1.97	Pakistan	pak	0.82	4.70	0.39	2.18
Hong Kong	hkg	0.73	49.74	0.38	0.84	Sri Lanka	lka	0.66	1.10	0.42	1.31
Indonesia	idn	0.79	1.47	0.45	2.39	Western Europe					
Japan	jpn	0.85	2.70	0.53	3.93	Austria	aut	0.67	2.09	0.49	1.01
Korea	kor	0.63	2.53	0.46	2.62	Belgium	bel	0.48	0.54	0.32	1.29
Lao	lao	0.74	1.97	0.33	0.91	Cyprus	cyp	0.77	1.18	0.64	0.79
Malaysia	mys	0.59	1.53	0.41	1.87	Denmark	dnk	0.73	1.27	0.53	1.01
Philippines	phl	0.58	1.55	0.44	2.15	Finland	fin	0.72	3.83	0.50	1.52
Singapore	sgp	0.37	0.40	0.25	0.80	France	fra	0.73	1.17	0.47	1.79
Taiwan	twn	0.58	1.36	0.39	3.18	Germany	deu	0.74	1.56	0.47	2.52
Thailand	tha	0.60	3.64	0.38	1.52	Greece	grc	0.77	1.44	0.56	0.82
Vietnam	vnm	0.58	1.04	0.35	1.26	Ireland	irl	0.66	2.05	0.46	1.11
Middle East & Africa						Italy	ita	0.77	2.18	0.53	1.77
Botswana	bwa	0.88	0.91	0.57	1.17	Luxembourg	lux	0.40	0.83	0.43	0.39
Egypt	egy	0.81	2.69	0.43	0.79	Malta	mlt	0.63	0.71	0.62	0.64
Ethiopia	eth	0.76	1.03	0.18	0.80	Netherlands	nld	0.69	0.96	0.43	1.29
Iran	irn	0.95	1.09	0.26	1.74	Norway	nor	0.87	0.91	0.47	1.41
Madagascar	mdg	0.75	0.91	0.50	1.02	Portugal	prt	0.68	2.25	0.46	1.17
Malawi	mwi	0.72	0.56	0.49	3.70	Spain	esp	0.75	1.19	0.46	1.32
Mauritius	mus	0.72	0.87	0.59	0.86	Sweden	swe	0.72	1.94	0.43	1.84
Morocco	mar	0.78	1.26	0.50	1.12	Switzerland	che	0.67	0.74	0.44	1.43
Mozambique	moz	0.76	1.25	0.35	1.49	United Kingdom	gbr	0.79	1.05	0.51	1.24
Nigeria	nga	0.94	0.95	0.59	0.92	Medians by Region					
Senegal	sen	0.73	1.04	0.48	1.02	Central & Eastern Europe		0.68	1.10	0.43	1.42
South Africa	zaf	0.80	0.62	0.45	2.96	East Asia		0.62	1.97	0.40	1.87
Tanzania	tza	0.81	1.07	0.26	1.19	Middle East & Africa		0.77	1.03	0.45	1.21
Tunisia	tun	0.69	1.43	0.38	1.45	North & South America		0.84	0.95	0.42	1.97
Turkey	tur	0.76	1.25	0.51	1.46	South Asia & Oceania		0.81	1.68	0.43	1.66
Uganda	uga	0.83	0.89	0.35	1.24	Western Europe		0.72	1.19	0.47	1.29
Zambia	zmb	0.78	1.02	0.25	9.29	Overall		0.73	1.09	0.44	1.46
Zimbabwe	zwe	0.69	0.58	0.44	2.69						

Source: Authors' calculations based on GTAP Database Version 7.1. Data is for 2004.

Table 2: Aggregate and Manufacturing VAX Decompositions

Panel A: Aggregate VAX Decomposition			
	$VAX_i - \overline{VAX}$	Within Term	Between Term
Log Income Per Capita	-0.008 (0.005)	0.028** (0.011)	-0.036*** (0.013)
R^2	0.02	0.07	0.08
N	90	90	90
Panel B: Manufacturing VAX Decomposition			
	$VAX_i - \overline{VAX}$	Within Term	Between Term
Log Income Per Capita	0.018*** (0.006)	-0.007 (0.009)	0.025*** (0.008)
R^2	0.11	0.01	0.12
N	89	89	89

Robust standard errors are in parentheses. Significance levels: * $p < .1$, ** $p < .05$, *** $p < .01$. Constants included in all regressions. Income per capita equals exporter value added per capita, where value added is calculated using our data and population is from the GTAP 7.1 database. Belgium, Luxembourg, and Singapore excluded in Panel A and Botswana, Hong Kong, Paraguay, and Peru excluded in Panel B as outliers.

Table 3: Bilateral VAX Ratio: Bilateral HIY vs. Production Sharing Adjustment
Variance Decomposition

Exporter	BHIY Term	PSA Term
U.S.	5%	95%
Germany	5%	95%
Japan	1%	99%
China	9%	91%
Argentina	1%	99%
France	8%	92%
Hungary	5%	95%
India	7%	93%
Portugal	9%	91%
Median Country	3%	97%

See the text for details regarding the decomposition. The Median Country is the median statistic for all 93 countries in the data.

Table 4: Decomposing Trade: Absorption, Reflection, and Redirection

Japan exports to:				U.S. exports to:			
China		U.S.		Mexico		Canada	
China	64.5%	U.S.	92.7%	Mexico	72.3%	Canada	68.9%
U.S.	11.1%	Canada	1.4%	U.S.	22.1%	U.S.	24.1%
Japan	4.3%	Mexico	0.7%	Canada	0.9%	U.K.	0.7%
Germany	2.5%	Japan	0.6%	Germany	0.4%	Japan	0.7%

Germany exports to:				Korea exports to:			
France		Czech Rep.		China		Japan	
France	74.8%	Czech Republic	57.7%	China	61.3%	Japan	83.1%
Germany	3.6%	Germany	11.7%	U.S.	12.1%	U.S.	4.7%
U.K.	2.8%	U.K.	3.0%	Japan	4.7%	China	2.3%
U.S.	2.6%	U.S.	2.6%	Germany	2.7%	Germany	1.0%

See the text for details regarding the decomposition. The entries in the table describe the approximate share of bilateral exports to each destination that are ultimately consumed in that destination. Shares do not sum to one because we include only the top four destinations for each bilateral pair. Data is for 2004.

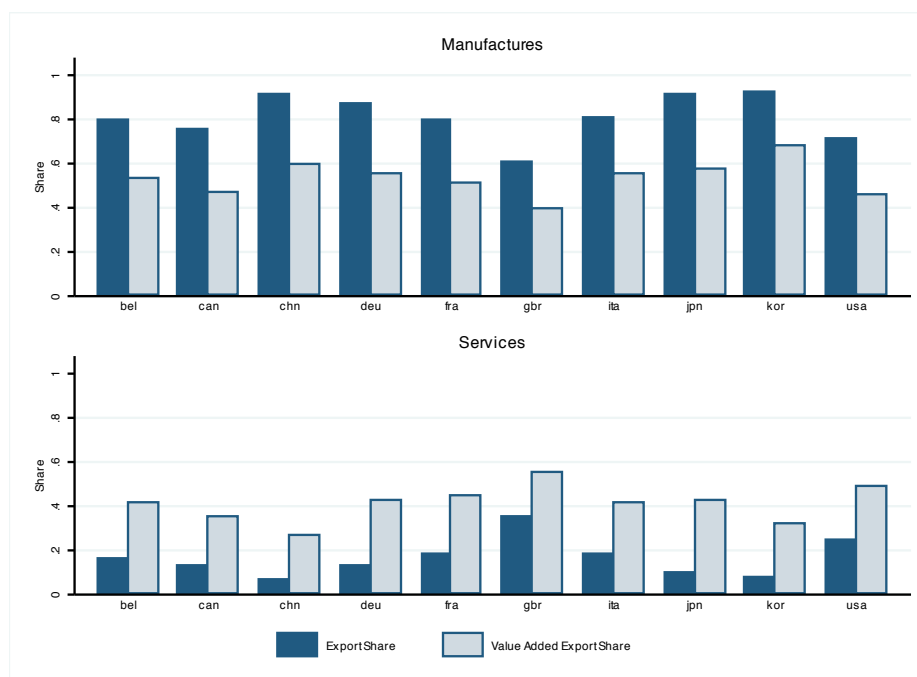


Figure 1: Composite Sector Shares of Gross Exports and Value Added Exports, by Country (2004)

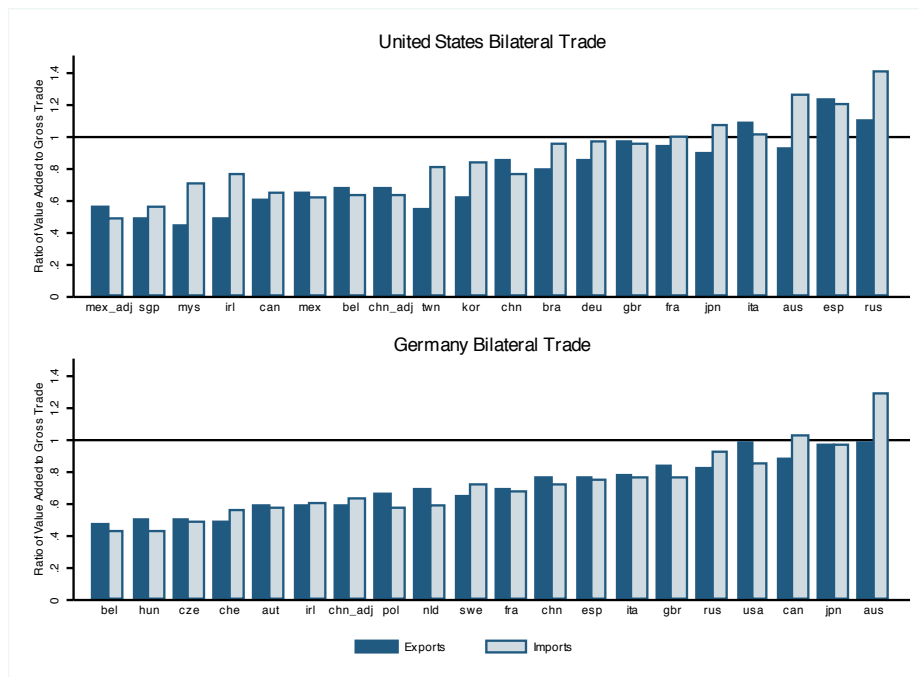


Figure 2: Value Added to Gross Trade Ratios for the United States and Germany, by Partner (2004)

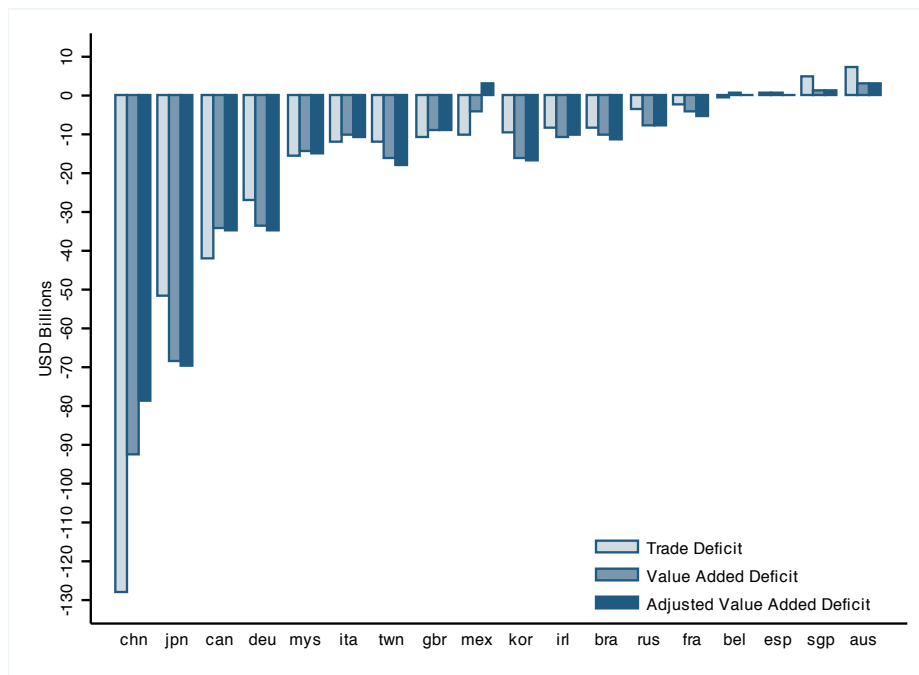


Figure 3: Bilateral Trade and Value Added Balances for the United States, by Partner (2004)

Appendix A

The basic idea behind the adjustment for processing trade is to split the aggregate economy into separate processing and non-processing units, each with its own input-output structure. Both sectors use domestic and imported intermediates, but they differ in terms of intermediate input intensity and the source (domestic versus imported) of intermediates. Further, all output in the export processing sector is exported.

From the input-output data, we observe the domestic intermediate use matrix m_{ii} and import use matrix as m_{Ii} for the economy as a whole. From trade data, we observe total exports originating from and imported intermediates used by the processing sector, denoted x_i^P and \bar{m}_{Ii}^P respectively. Output in the non-processing sector, denoted y_i^N , is calculated by subtracting x_i^P from total output in the input-output accounts. We seek separate intermediate use matrices for the two sectors $\{m_{ii}^N, m_{ii}^P, m_{Ii}^N, m_{Ii}^P\}$ and value added by sector $\{va_i^N, va_i^P\}$ that satisfy:

$$m_{ii} = m_{ii}^N + m_{ii}^P \quad (\text{A1})$$

$$m_{Ii} = m_{ii}^N + m_{Ii}^P \quad (\text{A2})$$

$$y_i^N = va_i^N + \iota [m_{ii}^N + m_{Ii}^N] \quad (\text{A3})$$

$$x_i^P = va_i^P + \iota [m_{ii}^P + m_{Ii}^P] \quad (\text{A4})$$

$$\bar{m}_{Ii}^P = m_{Ii}^P \iota', \quad (\text{A5})$$

where ι is a conformable row vector of ones and ι' is its transpose.³⁸

If there are N sectors, then there are $4(N \times N) + 2N$ unknowns and only $2(N \times N) + 3N$ constraints so we cannot solve directly for the unknown coefficients. We therefore follow Koopman, Wang, and Wei (2008) and use a constrained minimization routine to impute the unknown coefficients, where the objective function minimizes squared deviations between imputed values and target values. Target values are set by splitting intermediate use and value added across processing

³⁸These constraints differ from those used by Koopman, Wang, and Wei (2008) in that we use the domestic and import intermediate use matrices separately, whereas they pool this information into a single overall use matrix.

and non-processing sectors according to their shares in total output.

With the resulting split tables, we use bilateral trade data as in the main text to construct bilateral sourcing matrices and the global input-output table.³⁹ In performing the calculation, we use processing trade shares from Koopman, Wang, and Wei (2008) for China. For Mexico, we obtain trade data for the maquiladora sector from the Bank of Mexico.⁴⁰ Due to concerns about the quality of disaggregate data and the accuracy of the imputation procedure for individual sectors, we aggregate the data to 3 composite sectors prior to imputing coefficients. Because bilateral value added trade results are essentially identical in the main data when computed with 57 sectors or 3 composite sectors, we believe aggregation does not result in diminished accuracy.

³⁹In the resulting system, China and Mexico effectively have $2N$ sectors, where each of the N sectors is separated into processing and non-processing sub-sectors.

⁴⁰Data is available at: <http://www.banxico.org.mx/polmoneinflacion/estadisticas/balanzaPagos/balanzaPagos.html>.