Noisy Business Cycles*

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Abstract

This paper investigates a real-business-cycle economy that features dispersed information about the aggregate shocks to productivities, tastes, and monopoly power (the "fundamentals"). We first highlight why dispersed information is distinct from uncertainty about the fundamentals: it is only with dispersed information that agents can face uncertainty about the level of economic activity *beyond* the one they face about the fundamentals. We next show how this type of uncertainty can (i) contribute to significant noise in the business cycle even when agents are well informed about the fundamentals; (ii) increase inertia in the response of macroeconomic outcomes to aggregate productivity shocks; (iii) induce a negative short-run response of employment to aggregate productivity; (iv) formalize a certain type of demand shocks within an RBC economy; and (v) generate cyclical variation in observed Solow residuals and labor wedges. Turning to the normative properties, we show that none of the aforementioned properties are symptoms of inefficiency: if there are no mark-up shocks and information is fixed, the business cycle is constrained efficient. We conclude with discussing a number of potential extensions and policy implications.

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1 Introduction

There is a long tradition in macroeconomics, going back to Phelps (1970), Lucas (1972, 1975), Barro (1976), King (1982), and others, to use informational frictions to motivate why agents may be unaware of innovations to monetary policy, or of other shocks to the economy. This literature has recently been revived by Mankiw and Reis (2002), Sims (2003), and Woodford (2003a). The new generation has proposed alternative formalizations of informational frictions—infrequent updating in Mankiw-Reis, rational inattention in Sims—and has studied various positive implications.

Nevertheless, informational frictions seem to play no role other than justifying some uncertainty about the underlying economic fundamentals. In particular, it is unclear whether the *asymmetry* of information plays any distinct role. For example, what is crucial for the real effects of monetary shocks in Lucas (1972), Barro (1976), and Mankiw and Reis (2002) alike, is only that firms have imperfect information about the current monetary shocks, not that they have differential information. Furthermore, while most of this literature focuses on monetary shocks, we contend that the dispersion of information about the *real* shocks hitting the economy is more severe.[1] Finally, in many cases the *normative* implications of the dispersion of information remain unclear.[2]

In this paper, we contribute towards filling these gaps. We first highlight what, in our view, is the distinct nature, and modeling function, of dispersed information: as long as the equilibrium is unique, it is only with asymmetric information that agents can face uncertainty about the aggregate level of economic activity *beyond* the one they face about the economic fundamentals.[3]

This is crucial. Most macroeconomic models impose symmetric information. In so doing, and perhaps unintentionally, they reduce all the uncertainty that agents may face about current or future aggregate economic activity to the uncertainty they face about the few aggregate shocks to the model's fundamentals (such as technologies, tastes, or exogenous government policies). We contend, instead, that the uncertainty economic agents face in reality about current and future economic activity goes far beyond the one formalized in standard macroeconomic models. We thus propose that dispersed information is primarily a modeling device for formalizing a distinct type of uncertainty that agents may face when trying to forecast aggregate economic activity.

[1] Interestingly, in the Fed's Survey of Professional Forecasts (SPF), market analysts appear to have much more diverse expectations about real growth in consumption, output or corporate profits than about inflation.

[2] This is a gross and biased perspective on the literature; important qualifications will be made in due process.

[3] Note that we rule out sunspots. Also, throughout the paper we adopt the usual convention that "fundamentals" refer to the combination of technologies, preferences, endowments, market structures, and exogenous policies.

We then seek to illustrate how this distinct type of uncertainty can impact the business cycle. For this purpose, we find it best to abstract from nominal frictions and instead focus on a real-business-cycle economy. We finally study the normative properties of the equilibrium business cycle by comparing it to an appropriate constrained-efficiency benchmark.

Preview of model and results. Our model abstracts from capital accumulation, but allows for monopolistic power in the usual Dixit-Stiglitz fashion. It also allows for three types of aggregate shocks to the fundamentals of the economy: shocks to productivities, shocks to tastes, and shocks to monopoly power. This rich structure permit us to nest the RBC backbone of new-keynesian models, but is not crucial for any of our results: if he/she wishes so, the reader can restrict his attention to a competitive version of our model with only productivity shocks.

Our formalization of dispersed information builds on Lucas (1972) and Townsend (1983): firms and workers meet in informationally segmented locations, called "islands", and have to make their employment and production decisions while facing uncertainty about the underlying aggregate shocks. However, agents know their local fundamentals, which rules out the signal-extraction problems in Lucas (1972) and Townsend (1983) and instead permits us to concentrate on the role of trade linkages across the islands. In our model, different islands specialize in the production of different products, but households consume all products. Because of these trade linkages, the incentives of a firm or a worker in any given island depend on the local forecasts of the aggregate level of production in other islands. This captures concisely in our model the broader idea that the optimal behavior of any individual depends on her forecasts of the aggregate level of economic activity.

We first establish that the general equilibrium of our economy can be represented as the Perfect Bayesian equilibrium of a certain fictitious game. Apart from providing us with a convenient solution method, this representation helps reveal the nature of the strategic interaction in our economy (i.e., how an agent's incentives depends on her expectations of other's actions) and helps isolate the different types of uncertainty faced by each individual agent.

In particular, we show how economic decisions in each island can be pinned down by the local economic fundamentals and the local expectations of aggregate economic activity. In this sense, workers and firms care about the aggregate shocks hitting the economy *only* to the extent that these shocks impact aggregate economic activity. This formalizes the broader idea that the key uncertainty agents face over the business cycle is not per se about the exogenous aggregate shocks but rather about the endogenous level of economic activity. But then note that, when agents share the same information, the equilibrium level of activity is pinned down by the economic fundamentals and the agents' common expectations of these fundamentals—agents cannot face any uncertainty about the aggregate level of economy beyond the one they face about the fundamentals. In contrast, when information is dispersed, and only then, agents can face additional uncertainty in forecasting the aggregate level of economic activity. This is simply because with asymmetric information an agent can be uncertain about the actions of other agents even if he himself happens to know perfectly both the true fundamentals and the other agents' expectations of the fundamentals.

We next isolate the key coefficient that regulates the equilibrium effects of this type of uncertainty. This coefficient identifies the degree of strategic complementarity in our economy: it summarizes how much economic activity in any island depends on the local forecasts of the aggregate activity. The origin of this strategic complementarity is neither any type of production externalities nor any other deviation from the standard RBC framework; it is merely the fact that agents (or islands) in our economy specialize in different products and trade with one another.

We then show that the strategic complementarity that originates in such trade linkages is irrelevant for the business cycle when information is commonly shared, but becomes crucial once information is dispersed. This may explain why the role of strategic complementarity has not been appreciated before within the standard RBC paradigm, even though it naturally emerges from specialization and trade. We proceed to show in more concrete terms how they matter for the positive properties of the business cycle when information is dispersed.

(i) Since agents are uncertain about the underlying aggregate shocks to productivity and other economic fundamentals, it is not surprising that there can be movements in aggregate employment and output that cannot be explained by the realized fundamentals: these fluctuations are just the product of common noise in the information of the agents. What is though surprising is that the contribution of this noise to the business cycle can be arbitrarily large even in situations that the agents are arbitrarily well informed about the underlying fundamentals. This possibility cannot obtain when information is commonly shared; but can obtain when information is sufficiently dispersed and the degree of strategic complementarity is sufficiently high.

(ii) The dispersion of information can also contribute to inertia in the response of macroeconomic outcomes to innovations in aggregate productivity shocks, or other shocks to fundamentals. Once again, this inertia can be high even when agents are arbitrarily well informed about the innovations to fundamentals, to the extent that these innovations are not common knowledge.

(iii) Some researchers have argued that aggregate employment responds negatively to aggregate productivity shocks in the data and have argued that this fact is inconsistent with RBC models but consistent with sticky-price models (e.g., Galí, 1999; Basu, Fernald and Kimball, 2006). Although whether this is a fact remains debatable (e.g., Christiano, Eichenbaum and Vigfusson, 2003), here we show how the dispersion of information can accommodate such a fact within the RBC paradigm: in our model, a negative short-run response of employment to innovations in aggregate productivity can obtain under parameterizations that guarantee that this response would have been positive had information been commonly shared.

(iv) Our noise-driven fluctuations help formalize a certain type of "demand shocks" within an RBC setting. Errors in forecasting economic activity can be interpreted as shocks in expectations of "aggregate demand". They help increase the relative volatility of employment while decreasing its correlation with output. Their contribution at high frequencies increase with the degree of strategic complementarity, but always vanishes at low frequencies. An identification strategy as in Blanchard and Quah (1989) or Galí (1999) would likely identify these shocks as "demand" shocks.

(v) Finally, our noise-driven fluctuations involve countercyclical variation in measured labor wedges and procyclical variation in Solow residuals. Once again, these cyclical variations are higher the stronger the strategic complementarity.

We hope that these results indicate the potential returns of introducing dispersed information in more quantitatively-oriented business-cycle models. We then conclude our contribution by studying the normative properties of the equilibrium business cycle. It is obvious that a planner could improve welfare if he could aggregate the information that is dispersed in the society, or otherwise collect and provide agents with more information. But can a planner improve upon the equilibrium allocations without changing the information structure?

Building on a companion paper (Angeletos and La'O, 2008), we show that the answer to this question is negative as long as there are no monopolistic distortions. More generally, if subsidies correct the monopoly distortions or, at least, there is no variation in these distortions (no mark-up shocks), the equilibrium business cycle is efficient: the response of the economy to the underlying productivity and taste shocks, as well as to any noise, is just right. In this sense, the dispersion of information is not itself a source of inefficiency, and noise-driven fluctuations do not themselves justify government intervention, no matter their size. We then briefly comment how incorporating endogenous information aggregation could alter this result.

Layout. The remainder of the introduction discusses the related literature. Section 2 introduces the model. Section 3 characterizes the general equilibrium. Sections 4 and 5 explore the implications for business cycle. Section 6 discusses the properties of higher-order beliefs lying behind our results. Section 7 studies the normative properties of the business cycle. Section 8 concludes.

Related literature. The macroeconomics literature on informational frictions has a long history, a revived present, and—hopefully—a promising future. Recent contributions include Adam (2007), Amador and Weill (2007, 2008), Amato and Shin (2006), Angeletos and La'O (2008, 2009a), Angeletos and Pavan (2004, 2007, 2009), Bacchetta and Wincoop (2005), Collard and Dellas (2005), Hellwig (2002, 2005), Hellwig and Veldkamp (2008), Hellwig and Venkateswara (2008), Klenow and Willis (2007), Lorenzoni (2008, 2009), Luo (2008), Mackowiak and Wiederholt (2008, 2009), Mankiw and Reis (2002, 2006), Morris and Shin (2002, 2006), Moscarini (2004), Nimark (2008), Reis (2006, 2008), Rodina (2008), Sims (2003, 2006), Van Nieuwerburgh and Veldkamp (2006, 2008), Veldkamp (2006), Veldkamp and Woolfers (2007), and Woodford (2003a, 2008).

Most closely related to our paper are Morris and Shin (2002), Woodford (2003a), and Angeletos and Pavan (2007, 2009); these papers have all been important sources of inspiration, albeit in different ways. Morris and Shin (2002) were the first to highlight certain applied implications of asymmetric information and higher-order beliefs: they explored the sensitivity of higher-order beliefs to public information to explain why equilibrium outcomes in environments with strategic complementarity may feature a heightened sensitivity to noisy public news. Woodford (2003a) explored the inertia of higher-order beliefs to innovations in fundamentals to generate inertia in the response of prices to nominal shocks. Finally, Angeletos and Pavan (2007, 2009) provided a methodology for studying the positive and normative properties of a more general class of games with strategic complementarity and dispersed information.

Part of our contribution here is to show how the general equilibrium of a fully micro-founded real-business-cycle economy can be reduced to a game similar to those considered in the aforementioned papers and to identify what is the relevant degree of strategic complementarity. Once this "translation" is achieved, one can find close parallels between certain positive results of the present paper and certain positive results of these earlier papers. However, the details of this translation are important for appreciating the concrete positive implications of dispersed information for the business cycle. Furthermore, the precise micro-foundations are, clearly, indispensable if one wishes to understand the corresponding normative implications.

Our paper is also related to Beaudry and Portier (2004, 2006), Christiano et al. (2007), Jaimovich and Rebelo (2008), and Lorenzoni (2008). These papers consider certain types of expectations-driven, or noise-driven, fluctuations. However, the origin of these fluctuations is uncertainty about certain fundamentals, namely about future technological opportunities. These fluctuations obtain within representative-agent models, do not rest on asymmetric information, and are conceptually distinct from the ones we wish to emphasize here.

2 The model

There is a (unit-measure) continuum of households, or "families", each consisting of a consumer and a continuum of workers. There is a continuum of "islands", which define the boundaries of local labor markets as well as the "geography" of information: information is symmetric within an island, but asymmetric across islands. Each island is inhabited by a continuum of firms, which specialize in the production of differentiated commodities. Households are indexed by $h \in H = [0, 1]$; islands by $i \in I = [0, 1]$; firms and commodities by $(i, j) \in I \times J$; and periods by $t \in \{0, 1, 2, ...\}$.

Each period has two stages. In stage 1, each household sends a worker to each of the islands. Local labor markets then open, workers decide how much labor to supply, firms decide how much labor to demand, and local wages adjust so as to clear the local labor market. At this point, workers and firms in each island have perfect information regarding local productivity, but imperfect information regarding the productivities in other islands. After employment and production choices are sunk, workers return home and the economy transits to stage 2. At this point, all information that was previously dispersed becomes publicly known, and commodity markets open. Quantities are now pre-determined by the exogenous productivities and the endogenous employment choices made during stage 1, but prices adjust so as to clear product markets.

Households. The utility of household h is given by

$$u_i = \sum_{t=0}^{\infty} \beta^t \left[U(C_{h,t}) - \int_I S_{i,t} V(n_{hi,t}) di, \right]$$

with

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \qquad \text{and} \qquad V(n) = \frac{n^{1+\epsilon}}{1+\epsilon}.$$

Here, $\gamma \geq 0$ parametrizes the income elasticity of labor supply,[4] $\epsilon \geq 0$ parameterizes the Frisch

[4]Note that risk aversion and intertemporal substitution play no role in our setting because all idiosyncratic risk is insurable and there is no capital (or any other channel for the economy to transfer resources across time). Rather, γ

elasticity of labor supply, $n_{hi,t}$ is the labor of the worker who gets located on island i during stage 1 of period t, $S_{h,t}$ is an island-specific shock to the disutility of labor, and $C_{h,t}$ is a composite of all the commodities that the household purchases and consumes during stage 2.

This composite, which also defines the numeraire used for wages and commodity prices, is given by the following nested CES structure:

$$C_{h,t} = \left[\int_I c_{hi,t}^{\frac{\rho-1}{\rho}} di \right]^{\frac{\rho}{\rho-1}}$$

where

$$c_{hi,t} = \left[\int_J c_{hij,t}^{\frac{\eta_{it}-1}{\eta_{it}}} dj \right]^{\frac{\eta_{it}}{\eta_{it}-1}}$$

and where $c_{hij,t}$ is the quantity household h consumes in period t of the commodity produced by firm j on island i. Here, η_{it} is a random variable that determines the period-t elasticity of demand faced by any individual firm within a given island i, while ρ is the elasticity of substitution across different islands. As will become clear later on, the reason for adopting this nested CES structure is two-fold. First, letting the within-island elasticity differ from the across-islands elasticity permits us to distinguish the degree of monopoly power (which will be determined by the former) from the degree of strategic complementarity (which will be determined by the latter). And second, letting the within-island elasticity to be random permits us to introduce mark-up shocks in the model.

Households own equal shares of all firms in the economy. The budget constraint of household h is thus given by the following:

$$\int_{I\times J} p_{ij,t} c_{hij,t} d(j,k) + B_{h,t+1} \leq \int_{J\times I} \pi_{ij,t} d(i,j) + \int_I w_{it} n_{hi,t} dk + R_t B_{h,t},$$

where $p_{ij,t}$ is the period-t price of the commodity produced by firm j on island i, $\pi_{ij,t}$ is the period-t profit of that firm, w_{it} is the period-t wage on island i, R_t is the period-t nominal gross rate of return on the riskless bond, and $B_{h,t}$ is the amount of bonds held in period t.

The objective of each household is simply to maximize expected utility subject to the budget and informational constraints faced by its members. Here, one should think of the worker-members of each family as solving a team problem: they share the same objective (family utility) but have different information sets when making their labor-supply choices. Formally, the household sends off during stage 1 its workers to different islands with bidding instructions on how to supply labor as a function of (i) the information that will be available to them at that stage and (ii) the wage that

controls the sensitivity of labor supply to income for given wage.

will prevail in their local labor market. In stage 2, the consumer-member collects all the income that the worker-member has collected and decides how much to consume in each of the commodities and how much to save (or borrow) in the riskless bond.

Asset markets. Asset markets operate in stage 2, along with commodity markets, when all information is commonly shared. This guarantees that asset prices do not convey any information. The sole role of the bond market in the model is then to price the risk-free rate. Moreover, because our economy admits a representative consumer, allowing households to trade risky assets in stage 2 would not affect any of the results.

Firms. The output of firm j on island i during period t is given by

$$q_{ij,t} = A_{i,t}(n_{ij,t})^{\theta}$$

where $A_{i,t}$ is the productivity in island i, $n_{ij,t}$ is the firm's employment, and $\theta \in (0,1)$ parameterizes the degree of diminishing returns in production. The firm's realized profit is given by

$$\pi_{ij,t} = p_{ij,t}q_{ij,t} - w_{i,t}n_{ij,t}$$

Finally, the objective of the firm is to maximize its expectation of the representative consumer's valuation of its profit, namely, its expectation of $U'(C_t)\pi_{ij,t}$.

Labor and product markets. Labor markets operate in stage 1, while product markets operate in stage 2. Because labor cannot move across islands, the clearing conditions for labor markets are as follows:

$$\int_J n_{ij,t}dj = \int_H n_{hi,t}dh \ \forall i$$

On the other hand, because commodities are traded beyond the geographical boundaries of islands, the clearing conditions for the product markets are as follows:

$$\int_H c_{hij,t}dh = q_{ij,t} \ \forall(i,j)$$

Fundamentals and information. Each island in our economy is subject to three types of shocks: shocks to the technology used by local firms (productivity shocks); shocks to the disutility of labor faced by local workers (taste shocks); and shocks to the elasticity of demand faced by local firms, translating to shocks in their monopoly power (mark-up shocks). We allow for both aggregate and idiosyncratic components to these shocks and, unless otherwise stated, for an arbitrary correlation between the shocks.

In general, the aggregate fundamentals of the economy in any given period t are identified by the entire joint distribution of the shocks $(A_{it}, S_{it}, \eta_{it})$ in the cross-section of islands.[5] Let Ψ_t denote the aforementioned distribution. The standard practice in macroeconomics would be to assume that Ψ_t is commonly known in the beginning of period t; Ψ_t would then identify the exogenous aggregate state for period t and the equilibrium values of all aggregate variables in that period would be functions of Ψ_t alone. In contrast, we wish to consider situations where information about Ψ_t is dispersed during most of period t. We thus assume that agents in different islands observe only noisy private (local) signals about Ψ_t in stage 1, when they have to make their decentralized employment and production choices. On the other hand, we assume that Ψ_t becomes common known in stage 2, when agents meet in the centralized commodity and financial markets.

For much of our analysis we do not need to make any special assumptions about the signals that may be available to each island. For example, we can impose a Gaussian structure as in Morris and Shin (2002). Alternatively, we could allow some islands to be perfectly informed and others to be imperfectly informed, mimicking the idea in Mankiw and Reis (2002) that only a fraction of the agents update their information sets in any given point of time. To some extent, we could even interpret the noise in these signals as the product of rational inattention a la Sims (2003). More generally, we do not expect the details of the origins of noise to be crucial for our results.[6]

We thus start by allowing a rather arbitrary information structure. We nevertheless need a precise notation and formalization.[7] First, we let ω_t denote the "type" of an island during period t. This variable encodes all the information available to an island about the local shocks hitting that island as well as about the cross-sectional distribution of shocks and information in the economy. Next, we let Ω_t denote the distribution of ω_t in the cross-section of islands. This variable identifies the aggregate state of the economy during period t; note that the aggregate state now includes not only the cross-sectional distribution Ψ_t of the shocks but also the cross-sectional distributions of the information (signals). Finally, we let $\mathcal{S}_\omega$ denote the set of possible types for each island, $\mathcal{S}_\Omega$ the set of probability distributions over $\mathcal{S}_\omega$, and $\mathcal{P}(\cdot|\cdot)$ a probability measure over $\mathcal{S}_\Omega^2$.[8]

[5]For the special case that the shocks are jointly log-normal with time-invariant second moments, this distribution is conveniently parameterized by the mean values of the shocks.

[6]The only complication is that any "micro-foundation" of the informational frictions renders the information structure endogenous to other parameters of the economy. However, since we lack enough guidance on the particular form of this endogeneity, abstracting from it appears to be an excellent benchmark for the purposes of this paper.

[7]The formalization we use here builds on the one in Angeletos and Pavan (2009).

[8]To avoid getting distracted by purely technical issues regarding measurability and the like, our proofs treat $\mathcal{S}_\omega$

We can then formalize the information structure as follows. In the beginning of period t, and conditional on Ω_{t-1}, Nature draws a distribution $\Omega_t \in \mathcal{S}_\Omega$ using the measure $\mathcal{P}(\Omega_t|\Omega_{t-1})$.[9] Nature then uses Ω_t to make independent draws of $\omega_t \in \mathcal{S}_\omega$, one for each island. In the beginning of period t, before they make their current-period employment and production choices, agents in any given island get to see only their own ω_t; in general, this informs them perfectly about their local shocks, but only imperfectly about the underlying aggregate state Ω_t. In the end of the period, however, Ω_t becomes commonly known (ensuring that Ψ_t also becomes commonly known).

The key informational friction in our model is that agents face uncertainty about the underlying aggregate state Ω_t. Whether they face uncertainty about their own local shocks is immaterial for the type of effects we analyze in this paper. Merely for convenience, then, we assume that the agents of an island learn their own local shocks in stage 1. We can thus express the shocks as functions of ω_t: we denote with $A(\omega_t)$ the local productivity shock, with $S(\omega_t)$ the local taste shock, and with $\eta(\omega_t)$ the local mark-up shock.

3 Equilibrium

In this section we characterize the equilibrium of the economy. We first provide a convenient game-theoretic representation of the general equilibrium. We then use this representation in order to identify the distinct type of uncertainty introduced by dispersed information and the degree of strategic complementarity that underlies the general equilibrium of our economy.

3.1 Definition

Because of the symmetry of preferences across households, and the symmetry of technologies and information within each island, we can talk of a typical worker and firm for each island; that is, it is without any loss of generality to impose symmetry in the choices of workers and firms within each island. Finally, because each family sends workers to every island and receives profits from every firm in the economy, each family's income is fully diversified during stage 2. This guarantees that our model admits a representative consumer and that no trading takes place in the financial market. To simplify the exposition, we thus set $B_t = 0$ and abstract from the financial market.

and $\mathcal{S}_\Omega$ as if they were finite sets. However, none of our results hinges on this restriction.

[9] Note that we have imposed that the aggregate state Ω_t follows a Markov process; apart from complicating the notation, nothing changes if we let the aforementioned probability measure depend on all past aggregate states.

Because of the absence of capital and the Markov assumption on the process for the aggregate state, Ω_{t-1} summarizes all the payoff-relevant public information as of the beginning of period t. Recall then that the additional information that becomes available to an island in stage 1 of that period is only ω_t. As a result, for any given island, the labor supply of the local workers, the labor demand and the level of production of the local firms, and the wage that clears the local labor market, all can depend on the past aggregate state Ω_{t-1} and the island's current local state ω_t, but not the current aggregate state Ω_t. On the other hand, the prices that clear the commodity markets in stage 2, and all aggregate outcomes, do depend on the current aggregate state Ω_t. We thus define an equilibrium as follows.

Definition 1. *An equilibrium consists of an employment strategy $n : \mathcal{S}_\omega \times \mathcal{S}_\Omega \to \mathbb{R}_+$ a production strategy $q : \mathcal{S}_\omega \times \mathcal{S}_\Omega \to \mathbb{R}_+$, a wage function $w : \mathcal{S}_\omega \times \mathcal{S}_\Omega \to \mathbb{R}_+$, an aggregate output function $Q : \mathcal{S}_\Omega^2 \to \mathbb{R}_+$, an aggregate employment function $N : \mathcal{S}_\Omega^2 \to \mathbb{R}_+$, a price function $p : \mathcal{S}_\omega \times \mathcal{S}_\Omega^2 \to \mathbb{R}_+$, and a consumption strategy $c : \mathbb{R}_+^3 \to \mathbb{R}_+$, such that the following are true:*

(i) The price function p is normalized so that

$$P(\Omega_t, \Omega_{t-1}) \equiv \left[\int p(\omega, \Omega_t, \Omega_{t-1})^{1-\rho} d\Omega_t(\omega)\right]^{\frac{1}{1-\rho}} = 1$$

for all (Ω_t, Ω_{t-1}).

(ii) The quantity $c(p, p', Q)$ is the representative consumer's optimal demand for any commodity whose price is p when the price of all other commodities from the same island is p' and the aggregate output (income) is Q.

(iii) When the current aggregate state is Ω_t and the past aggregate state is Ω_{t-1}, the price that clears the market for the product of the typical firm from island ω_t is $p(\omega_t, \Omega_t, \Omega_{t-1})$; the employment and output levels of that firm are, respectively, $n(\omega_t, \Omega_{t-1})$ and $q(\omega_t, \Omega_{t-1})$, with $q(\omega_t, \Omega_{t-1}) = A(\omega_t) n(\omega_t, \Omega_{t-1})^\theta$; and the aggregate output and employment indices are, respectively,

$$\begin{aligned} Q(\Omega_t, \Omega_{t-1}) &= \left\{\int q(\omega, \Omega_{t-1})^{\frac{\rho-1}{\rho}} d\Omega_t(\omega)\right\}^{\frac{\rho}{\rho-1}} \\ N(\Omega_t, \Omega_{t-1}) &= \int n(\omega, \Omega_{t-1}) d\Omega_t(\omega) \end{aligned}$$

(iv) The quantities $n(\omega_t, \Omega_{t-1})$ and $q(\omega_t, \Omega_{t-1})$ are optimal from the perspective of the typical firm in island ω_t, taking into account that firms in other islands are behaving according to the same strategies, that the local wage is given by $w(\omega_t, \Omega_{t-1})$, that prices will be determined in stage 2 so as

to clear all product markets, that the representative consumer will behave according to consumption strategy c, and that aggregate income will be given by $Q(\Omega_t, \Omega_{t-1})$.

(v) The local wage $w(\omega_t, \Omega_{t-1})$ is such that the quantity $n(\omega_t, \Omega_{t-1})$ is also the optimal labor supply of the typical worker in an island of type ω_t.

Note that condition (i) simply means that the numeraire for our economy is the CES composite defined when we introduced preferences. The rest of the conditions then represent a hybrid of a Walrasian equilibrium for the complete-information exchange economy that obtains in stage 2, once production choices are fixed, and a subgame-perfect equilibrium for the incomplete-information game played among different islands in stage 1.

Let us expand on what we mean by this. When firms in an island decide how much labor to employ and how much to produce during stage 1, they face uncertainty about the prices at which they will sell their product during stage 2 and hence they face uncertainty about the marginal return to labor. Similarly, when workers in an island decide how much labor to supply, they face uncertainty about the real income their household will have in stage 2 and hence face uncertainty about the marginal value of the wealth that they can generate by working more. But then note that firms and workers in each island can anticipate that the prices that clear the commodity markets in stage 2 and the realized level of real income are, in equilibrium, determined by the level of employment and production in other islands. This suggests that we can solve for the general equilibrium of the economy by reducing it to a certain game, where the incentives of firms and workers in an island depend on their expectations of the choices of firms and workers in other islands. We implement this solution strategy in the following.

Remark. To simplify notation, we often use q_{it} as a short-cut for $q(\omega_t, \Omega_{t-1})$, Q_t as a short-cut for $Q(\Omega_t, \Omega_{t-1})$, $\mathbb{E}_{it}$ as a short-cut for $\mathbb{E}[\cdot|\omega_t, \Omega_{t-1}]$, and so on; also, we drop the indices h and j, because we know that allocations are identical across households, or across firms within an island.

3.2 Characterization

Towards solving for the equilibrium, consider first how the economy behaves in stage 2. The optimal demand of the representative consumer for a commodity from island i whose price is p_{it} when the price of other commodities in the same island is p'_{it} is given by the following:

$$c_{it} = \left(\frac{p_{it}}{p'_{it}}\right)^{-\eta_{it}} \left(\frac{p'_{it}}{P_t}\right)^{-\rho} C_t,$$

where $P_t = 1$ by our choice of numeraire.[10] In equilibrium, $C_t = Q_t$. It follows that the equilibrium consumption strategy is given by $c(p, p', Q) = p^{-\eta}(p')^{\eta-\rho} Q$. Equivalently, the inverse demand function faced by a firm during period t is

$$p_{it} = (p'_{it})^{1-\frac{\rho}{\eta_{it}}} q_{it}^{-\frac{1}{\eta_{it}}} Q_t^{\frac{1}{\eta_t}} \tag{1}$$

Consider now stage 1. Given that the marginal value of nominal income for the representative household is $U'(C_t)$ and that $C_t = Q_t$ in equilibrium, the objective of the firm is simply

$$\mathbb{E}_{it}\left[U'(Q_t)(p_{it}q_{it} - w_{it}n_{it})\right].$$

Using (1), we conclude the typical firm on island ω_t maximizes the following objective:

$$\mathbb{E}_{it}\left[U'(Q_t)\left((p'_{it})^{1-\frac{\rho}{\eta_{it}}} Q_t^{\frac{1}{\eta_{it}}} q_{it}^{1-\frac{1}{\eta_{it}}} - w_{it}n_{it}\right)\right], \tag{2}$$

where $q_{it} = A_{it}n_{it}^{\theta}$. As long as $1 > (1 - \frac{1}{\eta_t})\theta > 0$ (which we assume to be always the case), the above objective is a strictly concave function of n_t, which guarantees that the solution to the firm's problem is unique and that the corresponding first-order condition is both necessary and sufficient. This condition is simply given by equating the expected marginal cost and revenue of labor, evaluated under local expectation of the equilibrium pricing kernel:

$$\mathbb{E}_{it}\left[U'(Q_{it})\right] w_{it} = \left(\frac{\eta_{it}-1}{\eta_{it}}\right)\mathbb{E}_{it}\left[U'(Q_t)(p'_{it})^{1-\frac{\rho}{\eta_{it}}}\left(\frac{Q_t}{q_{it}}\right)^{\frac{1}{\eta_{it}}}\right]\left(\theta A_{it}n_{it}^{\theta-1}\right). \tag{3}$$

Next, note that, since all firms within an island set the same price in equilibrium, it must be that $p'_{it} = p_{it}$. Along with (1), this gives

$$p'_{it} = p_{it} = \left(\frac{q_{it}}{Q_t}\right)^{-\frac{1}{\rho}}. \tag{4}$$

This simply states that the equilibrium price of the typical commodity of an island relative to the numeraire is equal to the MRS between that commodity and the numeraire. Finally, note that the optimal labor supply of the typical worker on island i is given by equating the local wage with the MRS between the numeraire and leisure:

$$w_{it} = \frac{S_{it}n_{it}^{\epsilon}}{\mathbb{E}_{it}\left[U'(Q_t)\right]} \tag{5}$$

[10] To understand this condition, note that $c'_{it} = \left(\frac{p'_{it}}{P_t}\right)^{-\rho} C_t$ is the demand for the busket of commodities produced by a particular island; the demand for the commodity of a particular firm in that islands is then $c_{it} = \left(\frac{p_{it}}{p'_{it}}\right)^{-\eta_i} c'_{it}$.

Conditions (4) and (5) give the equilibrium prices and wages as functions of the equilibrium allocation. Using these conditions into condition (3), we conclude that the equilibrium allocation is pinned down by the following condition:

$$S_{it} n_{it}^{\epsilon} = \left(\frac{\eta_{it} - 1}{\eta_{it}} \right) \mathbb{E}_{it} \left[U' \left(Q_t \right) \left(\frac{q_{it}}{Q_t} \right)^{-\frac{1}{\rho}} \right] \left(\theta A_{it} n_{it}^{\theta - 1} \right). \tag{6}$$

This condition has a simple interpretation: it equates the private cost and benefit of effort in each island. To see this, note that the left-hand side is simply the marginal disutility of an extra unit of labor in island i; as for the right-hand side, $\frac{\eta_{it}-1}{\eta_{it}}$ is the reciprocal of the local monopolistic mark-up, $U' \left(Q_t \right) \left(\frac{q_{it}}{Q_t} \right)^{-\frac{1}{\rho}}$ is the marginal utility of an extra unit of the typical local commodity, and $\theta A_{it} n_{it}^{\theta-1}$ is the corresponding marginal product of labor.

Note that condition (6) expresses the equilibrium levels of local employment n_{it} and local output q_{it} in relation to the local shocks and the local expectations of aggregate output Q_t. Using the production function, $q_{it} = A_{it} n_{it}^{\theta}$, to eliminate n_{it} in this condition, and reverting to the more precise notation of Definition 1 (i.e., replacing q_{it} with $q(\omega_t, \Omega_{t-1})$, Q_t with $Q(\Omega_t, \Omega_{t-1})$, A_{it} with $A(\omega_t)$, and so on), we reach the following result.

Proposition 1. *Let*

$$f(\omega) \equiv \log \left\{ \theta^{\frac{\theta}{1-\theta+\epsilon+\gamma\theta}} \left(\frac{\eta(\omega) - 1}{\eta(\omega)} \right)^{\frac{\theta}{1-\theta+\epsilon+\gamma\theta}} S(\omega)^{-\frac{\theta}{1-\theta+\epsilon+\gamma\theta}} A(\omega)^{\frac{1+\epsilon}{1-\theta+\epsilon+\gamma\theta}} \right\}$$

be a composite of all the local shocks hitting an island of type ω and define the coefficient

$$\alpha \equiv \frac{\frac{1}{\rho} - \gamma}{\frac{1}{\rho} + \frac{1-\theta+\epsilon}{\theta}} < 1$$

(i) The equilibrium levels of local and aggregate output are the solution to the following fixed-point problem:

$$\log q\left(\omega_t, \Omega_{t-1}\right) = (1-\alpha) f(\omega_t) + \alpha \log \left\{ \mathbb{E} \left[Q(\Omega_t, \Omega_{t-1})^{\frac{1}{\rho}-\gamma} \middle| \omega_t, \Omega_{t-1} \right]^{\frac{1}{\frac{1}{\rho}-\gamma}} \right\} \quad \forall (\omega_t, \Omega_{t-1}) \tag{7}$$

and

$$Q(\Omega_t, \Omega_{t-1}) = \left[\int q(\omega, \Omega_{t-1})^{\frac{\rho-1}{\rho}} d\Omega_t(\omega) \right]^{\frac{\rho}{\rho-1}} \quad \forall (\Omega_t, \Omega_{t-1}). \tag{8}$$

(ii) The equilibrium levels of local and aggregate employment are given by

$$n(\omega_t, \Omega_{t-1}) = \left(\frac{q(\omega, \Omega_{t-1})}{A(\omega_t)} \right)^{\frac{1}{\theta-1}} \quad \textit{and} \quad N(\Omega_t, \Omega_{t-1}) = \int n(\omega_t, \Omega_{t-1}) d\Omega(\omega);$$

the equilibrium wage rate by

$$w(\omega_t, \Omega_{t-1}) = \theta \frac{q(\omega_t, \Omega_{t-1})}{n(\omega_t, \Omega_{t-1})};$$

and the equilibrium prices by

$$p(\omega_t, \Omega_t, \Omega_{t-1}) = \left(\frac{q(\omega_t, \Omega_{t-1})}{Q(\Omega_t, \Omega_{t-1})} \right)^{-\frac{1}{\rho}}.$$

This result proves that the general equilibrium of our economy reduces to a simple fixed-point relation between local and aggregate output. In so doing, it offers a game-theoretic representation of our economy: the aforementioned fixed point coincides with the Bayes-Nash equilibrium of a particular incomplete-information game. The relevant "players" for this game are the different islands of our economy; their "actions" are the production levels in each island; their "types" are the local shocks and local information sets; and their "best responses" are simply the ones given by condition (7).

As evident from this condition, local output depends on the local shocks, conveniently summarized in the composite shock $f(\omega_t)$, and the local expectations of aggregate output. As anticipated in the introduction, this captures concisely in our model the broader idea that optimal behavior of an individual depends on her forecasts of aggregate economic activity. Furthermore, note that firms and workers in an island do not care *per se* about the aggregate shocks hitting the economy; any information they may have about these aggregate shocks is valuable to them only to the extent that it helps them better forecast the aggregate level of economic activity.

3.3 The equilibrium degree of strategic complementarity

The coefficient α in Proposition 1 measures how much the equilibrium level of activity in an island depends on the local expectations of the level of activity in other islands. Following our game-theoretic interpretation of this condition, α identifies the degree of strategic complementarity featured in the equilibrium of our economy.

To see this more clearly, consider a log-linear approximation—i.e., a first-order Taylor expansion in logs—to conditions (7) and (8). One then obtains the following:

$$\log q_{it} = const + (1-\alpha) f_{it} + \alpha \mathbb{E}_{it} [\log Q_t], \tag{9}$$

$$\log Q_t = const + \int \log q_{it} di, \tag{10}$$

where *const* captures second- and higher-order terms.[11] In general, these second- and higher-order terms may depend on the underlying state; treating them as constants would then introduce an approximation error; but when the underlying shocks and signals are jointly log-normal with fixed second moments (as imposed by Assumption 1 in the next section), these terms are indeed constants and the approximation error vanishes.[12] It follows that the "best response" characterizing the general equilibrium of our economy has a simple log-linear structure and the coefficient α is simply the slope of this best response with respect to aggregate activity—which is the standard definition of the degree of strategic complementarity in large games.

What we have effectively shown here is that the general equilibrium of our economy reduces to a game like the ones studied in Morris and Shin (2002) and Angeletos and Pavan (2007, 2009): in those papers, the best-response action of a player was a linear combination of an exogenous fundamental and the average action of other players, much alike condition (9) here. But whereas those papers lacked any micro-foundations and only exogenously imposed the particular form of strategic interaction, here we have obtain such a strategic interaction as a reduced-form representation of a fully micro-founded general-equilibrium economy.

We then see from Proposition 1 that, within the business-cycle context of our paper, whether employment and production choices are strategic complements ($\alpha > 0$) or substitutes ($\alpha < 0$) depends on two opposing effects. On the one hand, higher aggregate income implies a higher demand for the products of each island, which increases local returns. This "demand-side" effect—which is standard to the new-keynesian paradigm—is the source of strategic complementarity (i.e., it contributes towards a positive α). On the other hand, higher aggregate income also implies a higher demand for leisure and hence a higher real wage, which decreases local returns. This "supply-side" effect—which is standard in the neoclassical paradigm—is the source of strategic substitutability (i.e., it contributes towards a negative α).

The strength of the aforementioned demand-side effect is determined by the elasticity of substitution across commodities, here parameterized by ρ. The strength of the aforementioned supply-side effect is determined by the income elasticity of labor supply, here parameterized by γ. This explains why in our economy the equilibrium degree of complementarity depends crucially on the relation between ρ and γ. For business-cycle frequencies, one expects income effects on labor supply to

[11]The *const* term differs across the two equations; we do not make this explicit for notational simplicity.

[12]The characterization of these constants can be found in the Appendix.

be small.[13] We thus believe that the empirically relevant case is one where $\alpha > 0$. Indeed, the restriction $\alpha > 0$ is synonymous to assuming that the anticipation of high aggregate demand leads to an increase in local employment and output, which seems natural. However, we do not need to impose this restriction for any of the results that follow.

Finally, for the remainder of the paper we adopt the following convention: when we vary the degree of strategic complementarity α, we mean that we vary ρ holding all other parameters of the economy constant. This convention is motivated by the following observations. It is clear that the income and Frisch elasticities of labor supply (γ and ϵ) matter for the response of the economy to aggregate shocks no matter whether information is commonly shared or dispersed; they would matter even if the economy was populated by a single agent; and have little to do with the level of strategic interaction across different agents. In contrast, the elasticity of substitution (ρ) governs the extent to which variation in the level of economic activity in one island affects the demand for the product of another island and, in this sense, directly impacts the strength of trade links across islands—or, equivalently, the level of strategic interaction. We make this idea more precise in the sequel, first by showing how ρ (equivalently, α) is irrelevant for the response of the economy to aggregate shocks when information about these shocks is commonly shared but becomes crucial once information about these shocks is dispersed

3.4 Common-information benchmark

As a reference point, we now consider the restriction of our model to the case where all information is commonly shared. Because we have assumed that local shocks are known, imposing this restriction implies that all shocks are perfectly known. But what we wish to highlight here is only the restriction to common information, not the stronger restriction to perfect information. Thus imagine an extension of our model that allows some of the shocks to be unknown and only imposes that information is commonly shared. Because the agents' forecasts of the aggregate fundamentals are common knowledge, their forecasts of aggregate output are also common knowledge. The following is then immediate.

Corollary 1. *When information is commonly shared, the forecasts of aggregate output are pinned down by the forecasts of the fundamentals.*

[13] An extension of the model that introduces capital would most likely contribute in this direction by ensuring that labor income is only a small fraction of total wealth.

This result highlights the key insight we anticipated in the introduction: the restriction to common information imposes that any uncertainty agents may face about aggregate economic activity reduces to the uncertainty they face about the fundamentals. It is precisely this property that the dispersion of information relaxes.

We next seek to highlight that, in a certain sense, the nature of the strategic interaction underlying our economy is crucial for the business cycle only when information is dispersed. To do this in an effective way, we need to put some structure on the aggregate shocks hitting the economy: we consider *symmetric* aggregate shocks, by which we mean parallel shifts in the cross-sectional distribution of the local shocks. In other words, we keep the level of heterogeneity invariant.

Formally, let $\bar{f}_t$ and $\bar{a}_t$ denote the cross-sectional averages of the composite shock f_{it} and the productivity shock $a_{it} \equiv \log A_{it}$. When all information is commonly shared, aggregate output is also commonly known in equilibrium. Condition (7) then reduces to

$$\log q_{it} = (1-\alpha)(\bar{f}_t + \xi_{it}) + \alpha \log Q_t \tag{11}$$

where $\xi_{it} \equiv f_{it} - \bar{f}_t$ is the idiosyncratic component of the local composite shock. Note that condition (11) is exact, not an approximation, when information is common. It is then immediate that, holding constant the the cross-sectional distribution of $\xi_i t$, the entire cross-sectional distribution of $\log q_{it}$ moves one-to-one with $\bar{f}_t$. A similar result holds for employment, establishing the following.

Proposition 2. *Suppose that information is commonly shared and that the level of heterogeneity (i.e., the cross-sectional distribution of ξ_{it}) is invariant. Then the equilibrium levels of aggregate output and employment are given by*

$$\log Q_t = const + \bar{f}_t \quad \textit{and} \quad \log N_t = const + \frac{1}{\theta}(\bar{f}_t - \bar{a}_t)$$

where $\bar{f}_t$ and $\bar{a}_t$ denote the aggregate composite shock and the aggregate productivity shock.

Recall that, by its definition, the composite shock depends on ϵ and γ but not on ρ. It is then evident that the response of the economy to the underlying aggregate productivity, taste, or mark-up shocks is independent of ρ. In this sense, the following is true.

Corollary 2. *Suppose that information is commonly shared and that the level of heterogeneity is invariant. Then, the degree of strategic complementarity is irrelevant for the response of the economy to aggregate shocks.*

This result helps explain why macroeconomists are not used to think of strategic complementarities within the context of the neoclassical growth model: they are indeed irrelevant for the business cycle when information is symmetric!

This result does not hinge on the absence of private information about *idiosyncratic* shocks: we could have allowed the purely idiosyncratic components of the shocks to be private information to each island. Rather, the key is the absence of private information about the *aggregate* shocks of the economy. Furthermore, the result does not hinge on the absence of uncertainty about these shocks: it easily extends to situations as long as agents have symmetric information about these shocks. The result needs to be qualified only if aggregate shocks involve a change in the level of heterogeneity, for then ρ matters for how much the associated change in heterogeneity impacts aggregate output. We abstract from this kind of effects only for expositional simplicity.[14]

In conclusion, the degree of strategic complementarity—equivalently, the strength of trade links—is largely irrelevant when information about the aggregate shocks hitting the economy is symmetric. For it is only when agents have asymmetric information that their uncertainty about the fundamentals does not pin down their uncertainty about aggregate economic activity, and it is only then that the degree of strategic complementarity starts playing a crucial role by regulating how the residual uncertainty about aggregate economic activity impacts individual behavior. We illustrate the potential implications of this insight for the business cycle in the next section.

4 Dispersed information and the business cycle

In this section we seek to illustrate how the combination of dispersed information and strategic complementarity matters for the business cycle. To facilitate this task, we impose a log-normal specification on the shocks and the information structure. This permits a simple closed-form solution of the equilibrium, leading to transparent comparative statics.

Assumption 1. *The shocks and the available information satisfy the following properties:*

(i) The aggregate shock $\bar{f}_t$ follows a Gaussian AR(1) or random walk process:

$$\bar{f}_t = \psi \bar{f}_{t-1} + \nu_t,$$

[14] Note in particular that the effects of strategic complementarity that we document in the subsequent analysis for the case of dispersed information do not rely on any variation in the level of heterogeneity in either the fundamentals or the information; they obtain holding constant both types of heterogeneity.

where ψ parameterizes the persistence of the composite shock and ν_t is a Normal innovation, with mean 0 and variance $\sigma_\nu^2 \equiv 1/\kappa_f$, i.i.d. over time.

(ii) The local shock f_t is given by

$$f_t = \bar{f}_{it} + \xi_{it},$$

where ξ_{it} is a purely idiosyncratic shock, Normally distributed with mean zero and variance σ_ξ^2, orthogonal to $\bar{f}_t$, and i.i.d. across islands.

(iii) The private information of an island about the aggregate shock $\bar{f}_t$ is summarized in a Gaussian sufficient statistic x_{it} such that

$$x_{it} = \bar{f}_t + \varsigma_{it},$$

where ς_{it} is noise, Normally distributed with mean zero and variance $\sigma_x^2 \equiv 1/\kappa_x$, orthogonal to both $\bar{f}_t$ and ξ_{it}, and i.i.d. across islands.[15]

(iv) The public information about the aggregate shock $\bar{f}_t$ is summarized in a Gaussian sufficient statistic y_t such that

$$y_t = \bar{f}_t + \varepsilon_t,$$

where ε_t is noise, Normally distributed with mean zero and variance $\sigma_\varepsilon^2 \equiv 1/\kappa_y$, and orthogonal to all other variables.

This specification imposes a certain correlation in the underlying productivity, taste and mark-up shocks: for the composite shock f_{it} to follow a univariate process as above, it must be that all the three type of shocks are moved by a single underlying factor. However, this is only for expositional simplicity. We can easily extend our results to a situation where each of the shocks follows an independent Gaussian process, or consider a more general correlation structure among the shocks. Finally, one should not give a narrow interpretation to the signal y_t. This signal is not meant to capture per se public information about the fundamentals; rather, it is a convenient modeling device for introducing common noise in forecasts of aggregate economic activity.

Under the above univariate specification, we can identify ω_t with the vector (f_t, x_t, y_t). Because Ω_t is then a joint normal distribution with mean $(\bar{f}_t, \bar{f}_t, y_t)$ and an invariant variance-autocovariance matrix, we can also reduce the aggregate state variable from Ω_t to the more convenient vector $(\bar{f}_t, y_t)$. Next, we can guess and verify that there is always an equilibrium in which $\log q_{it}$ is linear in

[15]Note that the local fundamental f_{it} is itself a private signal of the aggregate fundamental $\bar{f}_t$. However, by the fact that we define x_{it} as a sufficient statistic of *all* the local private information, the informational content of f_{it} is already included in x_{it}.

$(\bar{f}_{t-1}, f_{it}, x_{it}, y_t)$ and $\log Q_t$ is linear in $(\bar{f}_{t-1}, \bar{f}_t, y_t)$. Finally, we can use an independent argument to rule out any other equilibrium. We thereby reach the following result.

Proposition 3. *Under Assumption 1, the equilibrium level of local output is given by*

$$\log q_{it} = const + \varphi_{-1}\bar{f}_{t-1} + \varphi_f f_{it} + \varphi_x x_{it} + \varphi_y y_t, \tag{12}$$

where the coefficients $(\varphi_{-1}, \varphi_f, \varphi_x, \varphi_y)$ *are given by*

$$\begin{aligned} \varphi_{-1} &= \left\{\frac{\kappa_f}{(1-\alpha)\kappa_x + \kappa_y + \kappa_f}\right\}\alpha\psi & \qquad \varphi_f &= (1-\alpha) \\ \varphi_x &= \left\{\frac{(1-\alpha)\kappa_x}{(1-\alpha)\kappa_x + \kappa_y + \kappa_f}\right\}\alpha & \qquad \varphi_y &= \left\{\frac{\kappa_y}{(1-\alpha)\kappa_x + \kappa_y + \kappa_f}\right\}\alpha \end{aligned} \tag{13}$$

This result gives a closed-form solution of the equilibrium level of output in each island as a log-linear function of the past aggregate fundamental $\bar{f}_{t-1}$, the current local fundamental f_{it}, the local (private) signal x_{it}, and the public signal y_t. Note then that the equilibrium level of output is necessarily an increasing function of the local fundamental f_{it}: $\varphi_f > 0$ necessarily. To interpret this sign, note that higher f means a higher productivity, a lower disutility of labor, or a lower monopolistic distortion. But whether and how local output depends on $\bar{f}_{t-1}$, x_{it} and y_t is determined by the degree of strategic complementarity α. To understand why, note that local output depends on these variables only because these variables contain information about the current aggregate shocks and, in so doing, help agents forecast the aggregate level of output. But when $\alpha = 0$, the demand- and supply side effects that we discussed earlier perfectly offset each other, so that at the end economic decisions are not interdependent: local incentives depend only the local fundamentals and not on expectations of aggregate activity. It follows that the dependence of local output to $\bar{f}_{t-1}$, x_{it} and y_t vanishes when $\alpha = 0$. On the other hand, if $\alpha \neq 0$, local output depends on $\bar{f}_{t-1}$, x_{it} and y_t because, and only because, these variables help predict aggregate output. In particular, when economic decisions are strategic complements ($\alpha > 0$), the equilibrium level of output in each island responds positively to expectations of aggregate output; in this case, the coefficients φ_{-1}, φ_x, and φ_y are all positive. When instead economic decisions are strategic substitutes ($\alpha > 0$), the equilibrium level of output in each island responds negatively to expectations of aggregate output; in this case, the coefficients φ_{-1}, φ_x, and φ_y are all negative. As mentioned earlier, we view the case in which $\alpha > 0$, and hence in which economic activity responds positively to good news about aggregate fundamentals, as the empirically most relevant scenario. For this reason, our subsequent discussion will focus on this case; however, our results apply more generally.

The precise values of the coefficients $(\varphi_{-1}, \varphi_f, \varphi_x, \varphi_y)$ can be obtained by the method of undetermined coefficients. In particular, suppose that local output is given by (12) for *some* coefficients $(\varphi_{-1}, \varphi_f, \varphi_x, \varphi_y)$. Using (10) and aggregating across islands, we get

$$\log Q_t = const + \varphi_{-1}\bar{f}_{t-1} + (\varphi_f + \varphi_x)\bar{f}_t + \varphi_y y_t,$$

and therefore the local expectation of aggregate output is given by

$$\mathbb{E}_{it}[\log Q_t] = const + \varphi_{-1}\bar{f}_{t-1} + (\varphi_f + \varphi_x)\mathbb{E}_{it}[\bar{f}_t] + \varphi_y y_t.$$

where we have used the fact that $\bar{f}_{t-1}$ and y_t belong to the information set of the agents. By standard Gaussian updating, we have that the forecast of the current aggregate fundamental $\bar{f}_t$ is given by

$$\mathbb{E}_{it}[\bar{f}_t] = \mathbb{E}[\bar{f}_t|\bar{f}_{t-1}, x_{it}, y_t] = \frac{\kappa_f}{\kappa}\psi\bar{f}_{t-1} + \frac{\kappa_x}{\kappa}x_{it} + \frac{\kappa_y}{\kappa}y_t$$

where $\kappa \equiv \kappa_f + \kappa_x + \kappa_y = 1/Var_{it}(\bar{f}_t)$ is the overall precision of the local forecasts of the aggregate fundamentals. Substituting the above in to the previous expression for $\mathbb{E}_{it}[\log Q_t]$, and substituting the resulting expression into the best-response condition (9), gives us $\log q_t$ as a linear function of $(\bar{f}_{t-1}, f_{it}, x_{it}, y_t)$. Requiring that this expression coincides with our initial guess gives a system of equations that the coefficients $(\varphi_{-1}, \varphi_f, \varphi_x, \varphi_y)$ must solve. Solving this system gives the equilibrium values of the coefficients. We have reported these values in Proposition 2, but have delegated the details of their derivation to the Appendix.

4.1 Impact of fundamentals and noise

We now study how the dispersion of information and the degree of strategic complementarity affect aggregate fluctuations. Towards this goal, we aggregate condition (12) and use the fact that $\bar{f}_t = \psi\bar{f}_{t-1} + \nu_t$ to obtain the following characterization of aggregate output.

Corollary 3. *Under Assumption 1, the equilibrium level of aggregate output is given by*

$$\log Q_t = const + \psi\bar{f}_{t-1} + \varphi_\nu\nu_t + \varphi_\varepsilon\varepsilon_t, \tag{14}$$

where

$$\varphi_\nu \equiv \varphi_f + \varphi_x + \varphi_y = 1 - \frac{\alpha\kappa_f}{(1-\alpha)\kappa_x + \kappa_y + \kappa_f} \quad \textit{and} \quad \varphi_\varepsilon \equiv \varphi_y = \frac{\alpha\kappa_y}{(1-\alpha)\kappa_x + \kappa_y + \kappa_f},$$

and where $\nu_t = \bar{f}_t - \psi\bar{f}_{t-1}$ *is the innovation in the fundamentals,* ψ *is the persistence in the fundamentals,* $\varepsilon_t = y_t - \bar{f}_t$ *is the aggregate noise.*

Condition (14) gives the equilibrium level of aggregate output as a log-linear function of the past aggregate fundamentals, $\bar{f}_{t-1}$, the current innovation in the fundamentals, ν_t, and the current noise, ε_t. A similar condition holds for aggregate employment. We now use this result to study the positive properties of the business cycle.

Consider the impact effect on output of an innovation in fundamentals. This effect is measured by the coefficient φ_ν. Because the latter is a decreasing function of the precisions κ_x and κ_y, we have that the impact effect of an innovation in fundamentals decreases with the level of noise. This is essentially the same insight as the one that drives the real effects of monetary shocks in both the older macro models with informational frictions (e.g., Lucas, 1972; Barro, 1976) and their recent descendants (e.g., Mankiw and Reis, 2002): the less informed economic agents are about the underlying shocks, the less they respond to these shocks. Clearly, this is true no matter whether agents interact with one another—it is true even in a single-agent decision problem.

More interestingly, we find that φ_ν is a decreasing function of α. That is, the more economic agents care about aggregate economic activity, the weaker the response of the economy to innovations in the underlying fundamentals. At the same time, we find that φ_ε is an increasing function of α. This, the more economic agents care about aggregate economic activity, the stronger the equilibrium impact of noise. These properties originate for the interaction of strategic complementarity with dispersed information. Indeed, if the underlying shock was common knowledge (which here can be nested by taking the limit as the public signal becomes infinitely precise, $\kappa_y \to \infty$), then both φ_ν and φ_ε would cease to depend on α. But as long as information is dispersed, a higher α reduces φ_ν and raises φ_ε. This highlights how strategic complementarity becomes crucial for the business cycle once information is dispersed.

Corollary 4. *When information is dispersed, and only then, stronger complementarity dampens the impact of innovations in the fundamentals on equilibrium output and employment, while amplifying the impact of noise.*

The key intuition behind this result is the following. Public information and past fundamentals (which here determine the prior about the current fundamentals) help forecast the aggregate level of output relatively better than private information. The higher α is, the more the equilibrium level of output in any given island depends on the local forecasts of aggregate output and the less it depends on the local current fundamentals. It follows that a higher α induces the equilibrium output of each island to be more anchored to the past aggregate fundamentals, more sensitive to public information, and less sensitive to private information. The anchoring effect of past aggregate fundamentals explains why aggregate output responds less to any innovation in the fundamentals, while the heightened sensitivity to noisy public information explains why aggregate output responds more to noise.[16] However, as mentioned before, one should interpret public information more generally as a source of correlated noise in forecasts of aggregate economic activity.

As another way to appreciate the aforementioned result, consider following variance-decomposition exercise. Let $\log \hat{Q}_t$ be the projection of $\log Q_t$ on *past* fundamentals. The residual, which is given by $\log \tilde{Q}_t \equiv \log Q_t - \log \hat{Q}_t = \varphi_\nu \nu_t + \varphi_\varepsilon \varepsilon_t$, can be interpreted as the "high-frequency component" of aggregate output. Its total variance is $Var(\log \tilde{Q}_t) = \varphi_\nu^2 \sigma_\nu^2 + \varphi_\varepsilon^2 \sigma_\varepsilon^2$, where σ_ν^2 ($\equiv 1/\kappa_f$) is the variance of the innovation in the fundamentals and σ_ε^2 ($\equiv 1/\kappa_y$) is the variance of the noise. The fraction of the high-frequency variation in output that originates in noise is thus given by the following ratio:[17]

$$R_{noise} \equiv \frac{Var(\log \tilde{Q}_t | \nu_t)}{Var(\log \tilde{Q}_t)} = \frac{\varphi_\varepsilon^2 \sigma_\varepsilon^2}{\varphi_\nu^2 \sigma_\nu^2 + \varphi_\varepsilon^2 \sigma_\varepsilon^2}.$$

Since a higher α raises φ_ε and reduces φ_ν, it necessarily raises this fraction: the more agents care about the aggregate level of economic activity, the more the high-frequency volatility in output that is driven by noise. A similar result holds for employment.

This result is illustrated in Figure 1. To obtain this figure, we "calibrate" our model as follows. First, we focus on productivity shocks as the only shock to fundamentals, we interpret the time period as a quarter, and we let $\sigma_\nu = 0.02$ for the standard deviation of the productivity innovation and $\psi = 0.99$ for its persistence. Next, we set $\theta = .40$ and $\epsilon = .5$, which correspond to an income share of labor equal to 40% and a Frisch elasticity of labor supply equal to 2. These parameter values are broadly consistent with the literature. More unorthodox is our choice of γ. Recall that in our setting there is no capital, implying that labor income is the only source of wealth, the elasticity of intertemporal substitution is irrelevant, and γ only controls the income elasticity of labor supply. We accordingly set $\gamma = .2$ to ensure an empirically plausible income effect on labor supply. Next, we set the precisions of private and public information to one half of the precision of the prior: $\sigma_x = \sigma_y = 5\sigma_v$. These values are arbitrary, but they are not implausible: when the period is interpreted as a quarter, the information about the current innovations to fundamentals and/or the current level of economic activity is likely to be very limited. Finally, we do not pick

[16]The anchoring effect of the common prior underlies also the inertia effects documented in Woodford (2003a) and Morris and Shin (2006), while the high sensitivity to public information is the same as in Morris and Shin (2002).

[17]This fraction equals 1 minus the R-square of the regression of $\log \tilde{Q}_t$ on the innovation ν_t.

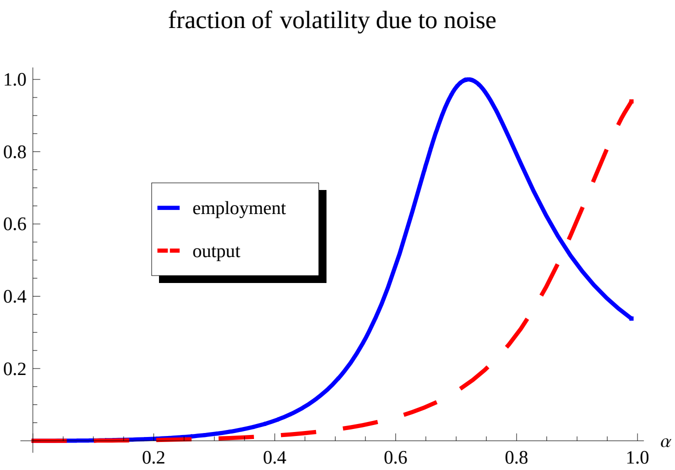

Figure 1: Contribution of noise to high-frequency components of output and employment.

any specific value for α (equivalently, ρ). Rather, we study how the variance decomposition of the high-frequency components of output and employment varies as we vary α from 0 to 1 (keeping in mind that a higher α means stronger trade links or, equivalently, a lower ρ). We then see in Figure 1 that, at least for our numerical example, noise can contribute to significant fractions of the high-frequency volatility in either output or employment when α is sufficiently high.

This numerical example assumes rather imprecise information about the underlying innovations to productivity. However, this is not what drives the result that noise can contribute to a significant fraction of the business cycle. We clarify this point with a limit result, which illustrates in a sharp way the distinct nature of dispersed information.

Proposition 4. *When information is dispersed and strategic complementarity is sufficiently strong, agents can be arbitrarily well informed about the fundamentals* ($\kappa \approx \infty$) *and, yet, the high-frequency variation in aggregate output can be driven almost exclusively by noise* ($R_{noise} \approx 1$).

Clearly, this would not be possible if information were commonly shared. In that case, the contribution of noise on the business cycle is tightly connected to the precision of available information about the fundamentals and vanishes as this precision becomes infinite. In contrast, when information is dispersed, the contribution of noise in the business cycle can be high even when the precision of information is arbitrarily high. What makes this possible is (i) that agents may still face non-trivial residual uncertainty about the underlying economic activity and (ii) that a high degree of strategic complementarity amplifies the impact of this uncertainty.

5 A dynamic extension

The preceding has focused on a setting where the underlying shocks becomes common knowledge within a period. Although this permitted a sharp theoretical analysis of the distinct implications of dispersed information, it makes it hard to map our results to real-world business cycles. We now seek to illustrate how incorporating slower learning can facilitate a clearer mapping between our analysis and the data.

We thus seek to relax the assumption that the aggregate state, Ω_t, becomes publicly revealed at the end of stage 2 each period, and instead allow for more interesting learning dynamics. To accommodate this possibility in a fully micro-founded way, we would need to remove the centralized commodity trading that we allowed in our baseline model: as long as there is centralized trading, the state will get revealed from equilibrium prices. However, allowing for more decentralized trading could complicate the analysis by introducing informational externalities and by letting the relevant state space explode as in Townsend (1983). We are currently exploring some possibilities along these lines. However, for the purposes of the present section, we opt to trade off elegance for tractability.

In particular, we assume that firms and workers do not learn from observing past aggregate economic outcomes or past prices, nor do they ever observe the underlying state. Rather, they only observe exogenous signals about the current fundamentals, as in Assumption 1, and they use these signals to update each period their beliefs about the underlying state. Think of this as follows. Each firm has two managers: one who decides the level of employment and production; and another who sells the product, receives the revenue, and sends the realized profits to the firm's shareholders. The two manages share the same objective—maximize firm valuation—but do not communicate with one another. Moreover, the first manager never receives any signals on economic activity. He only observes the exogenous local private and public signals. Similarly, the consumers, who observe all the prices in the economy, fail to communicate this information to the workers in their respective families. The workers also base their decisions solely on the exogenous signals. Clearly, this is not an elegant specification. But it is convenient and, most likely, it is largely inconsequential for our purposes: alternative formalizations of a decentralized learning process are bound to matter quantitatively, but need not impact the qualitative properties we wish highlight here.

Equilibrium behavior continues to be characterized by the same best-response-like condition as in the baseline model:

$$\log q_{i,t} = (1-\alpha) f_{i,t} + \alpha \mathbb{E}_{i,t} \left[\log Q_t \right], \tag{15}$$

where we have normalized the constant to zero. The only difference is in the underlying information structure. Finally, for concreteness, we focus on productivity shocks as the only shock to fundamentals: $f_{i,t} = \beta \log A_{i,t}$ with $\beta \equiv \frac{1+\epsilon}{1-\theta+\epsilon+\theta\gamma}$.

The procedure we follow to solve for the equilibrium dynamics is based on Kalman filtering and is similar to the one in Woodford (2003). We guess and verify that the aggregate state can be summarized in a vector X_t comprised of the aggregate fundamental and aggregate output:

$$X_t \equiv \begin{bmatrix} \bar{f}_t \\ \log Q_t \end{bmatrix}, \tag{16}$$

Firms and workers in any given island never observe the state, but instead receive the following vector of signals each period:

$$z_{it} \equiv \begin{bmatrix} x_{it} \\ y_t \end{bmatrix} = \begin{bmatrix} \bar{f}_t + \varsigma_{it} \\ \bar{f}_t + \varepsilon_t \end{bmatrix} \tag{17}$$

As emphasized before, y_t should not be taken too literally—it is a convenient modeling device for introducing common noise in the agents' forecasts of the state of the economy. Finally, we guess and verify that the state vector X_t follows a simple law of motion:

$$X_t = MX_{t-1} + m_\nu \nu_t + m_\varepsilon \varepsilon_t \tag{18}$$

where M is a 2×2 matrix, while m_ν and m_ε are 2×1 vectors. We then seek to characterize the equilibrium values of M, m_ν, and m_ε.

In each period t, firms and workers start with some prior about X_t and use the new signals that they receive in the beginning of period t to update their beliefs about X_t. Local output is then determined Condition (15) then givens local output as a function of the local belief about X_t. Aggregating across islands, we obtain the aggregate level of output. In equilibrium, the law of motion that aggregate output follows must match the one believed by the firms. Therefore the equilibrium is a fixed point between the law of motion believed by agents and used to form their forecasts of the aggregate state, and the law of motion induced by the optimal output and employment decisions that firms and workers are making following their signal extraction problem. We characterize the fixed point of this problem in the Appendix and use its solution to numerically simulate the impulse responses of output and employment to positive innovations in v_t and ε_t.

For our numerical simulations, we now interpret a period as a quarter and set the parameters of the economy similarly to those used in the previous section: $\sigma_\nu = 0.02$, $\sigma_x = \sigma_y = 5\sigma_\nu$, $\epsilon = .5$, and $\gamma = .2$, and $\theta = .4$. Finally, we once again experiment with various values for α.

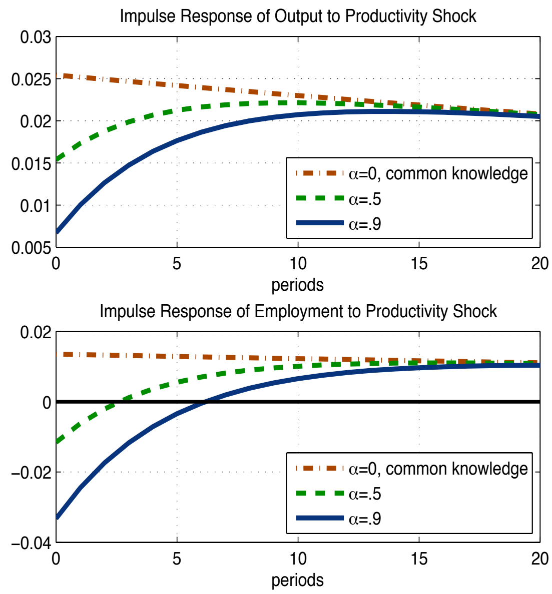

Figure 2: Impulse responses to innovation in productivity.

5.1 Impulse responses to productivity and noise shocks

Figure 2 plots the impulse responses of aggregate output and employment to a positive innovation of productivity, for various degrees of α. (The size of the innovation here, and in all other impulse responses we report, is equal to one standard deviation.) Clearly, if aggregate productivity were common knowledge, then output would follow the same AR(1) process as aggregate productivity itself. This is simply because there is no capital in our model. The same thing happens when information is dispersed but there is no strategic complementarity in output decisions ($\alpha = 0$). This is simply because when $\alpha = 0$ islands are effectively isolated from one another; but as each island knows perfectly its own productivity, the entire economy responds to the aggregate shock as if the aggregate shock had been common knowledge.

In contrast, when information is dispersed but islands are interconnected ($\alpha \neq 0$), employment and output in one island depends crucially on expectations of employment and output in other islands. As a result, even though each island remains perfectly informed about their local fundamentals, each island responds less to the shock than what it would have done had the shock been common knowledge, precisely because each island expects output in other islands to respond less. Note then that the key for the response of each island is not per se whether the island can disentangle an aggregate shock from an idiosyncratic shock. Even if a particular island was perfectly informed about the aggregate shock, as long as $\alpha > 0$ the island will respond less to this shock than under common knowledge if it expects the other island to respond less, presumably because the other island has imperfect information about the shock. Thus, the key for the inertia in the response of aggregate outcomes is the uncertainty islands face about one another's response, not necessarily the uncertainty they themselves face about the aggregate shock.

As evident in Figure 2, the equilibrium inertia is higher the higher the degree of strategic complementarity. This is because of two reasons. First, there is a direct effect: the higher α is, the less the incentive of each island to respond to the underlying shock for any given expectation of the response of other islands. But then there is also an indirect, multiplier-like, effect: as all islands are expected to respond less to the underlying shock, each island finds it optimal to respond even less.

At the same time, the inertia vanishes in the long-run: the long-run response of the economy to the shock is the same as with common knowledge. This seems intuitive: as time passes, agents become better informed about the underlying aggregate shock. However, that's only part of the story. First, note that agents are always perfectly informed about their own fundamentals, so there is no learning this dimension. Second, recall that agents do not care per se about the aggregate fundamentals, so the fact that they are learning more about them is per se inconsequential. Rather, the key is that agents in each island are revising their forecasts of the output of other islands. What then drives the result that inertia vanishes in the long-run is merely that forecasts of aggregate output eventually converge their common-knowledge counterpart.[18]

Finally, a salient property of the dynamic response of employment is that, for sufficiently high α, the short-run impact of a productivity shock on employment turns from positive to negative; this happens for parameters values for which the model would have generate a strong positive response had information been symmetric. We find this striking. The baseline RBC paradigm has long being criticized for generating a near perfect correlation between employment and output, whereas in the data this correlation is near zero. In our setting, this correlation could be close to zero or even turn negative if α is sufficiently high. Of course, correlations confound the effects of multiple shocks. Some authors in the structural VAR literature have thus sought to show that identified technology

[18]It may be hard to fully appreciate this point, because how fast output forecasts converge to their common-knowledge counterpart is itself pinned down by the speed of learning about the underlying aggregate productivity shock. However, with richer information structures, one can disentangle the speed of adjustment in output forecasts from the speed of learning about the fundamentals. It is then only the former that matters for the result. See Angeletos and La'O (2009a) for a related example within the context of a Calvo-like monetary model.

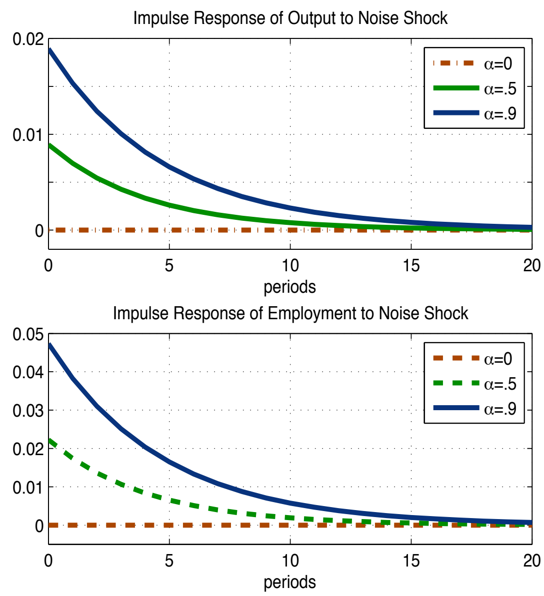

Figure 3: Impulse responses to noise.

shocks lead to a reduction in employment and have then argue that this as a clear rejection of the RBC paradigm (e.g., Galí, 1999). Here, we have shown that dispersed information may potentially help the RBC paradigm accommodate this fact without any need to invoke sticky prices.

Turning to the effects of noise, in Figure 3 we consider the impulse responses of output and employment in response to a positive innovation in ε_t. As emphasized before, this should be interpreted as a positive error in expectations of aggregate output, rather than as an error in expectations of aggregate fundamentals. When $\alpha = 0$, such forecast errors are irrelevant, simply because individual incentives do not depend on forecasts of aggregate activity. But when $\alpha = 0$, they generate a positive response in output and employment, thus becoming partly self-fulfilling. Furthermore, the stronger the complementarity, the more pronounced the impact of these errors on aggregate employment and output.

The figure considers a positive noise shock, which means a positive shift in expectations about economic activity. The impact of a negative shift in expectations is symmetric. Note that when these shocks occur, output, employment and consumption move in the same direction, without any movement in TFP. The resulting booms and recessions could thus be (mis)interpreted as a certain type of demand shocks. We will return to this point in a moment. Finally, note that the impact of these noise shocks on output and employment can be quite persistent, even though the noise itself

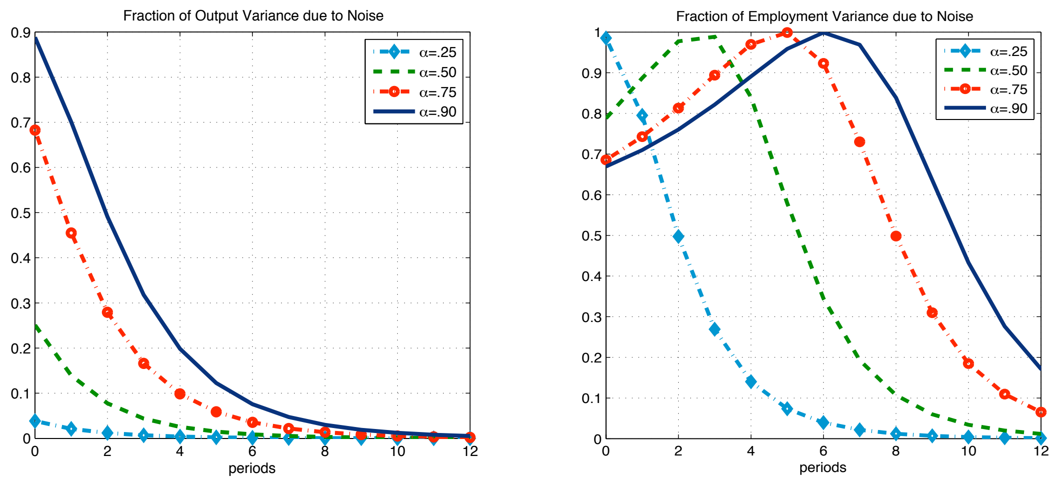

Figure 4: Variance decomposition.

is not. This is simply because the associated forecast errors are themselves persistent.

5.2 Variance decomposition and forecast errors

Comparing the responses of employment with those of output to the two shocks, we see that the former is smaller than the latter in the case of productivity shocks but quite larger in the case of noise. This is simply because productivity shocks have a double effect on output, both directly and indirectly through employment, while the noise impacts output only through employment. But then the response of employment to noise is bound to be stronger than that of output as long as there are diminishing returns to labor ($\theta < 1$), and the more show the lower θ. It follows that noise contributes to a higher relative volatility for employment, while productivity shocks contribute in the opposite direction. In the standard RBC framework, employment may exhibit a higher volatility than output to the extent that there are powerful intertemporal substitution effects (which here we have ruled out since we have also ruled out capital). However, the RBC framework is known to lack in this dimension. Our results here indicate how noise could help improve the performance of the RBC framework in this dimension.

Comparing Figures 2 and 3, it is evident that low-frequency movements in employment and output are dominated by the productivity shocks, while noise contributes relatively more to high-frequency movements. To further illustrate this property, in Figure 4 we plot the variance decomposition of output and employment at different time horizons. Much alike in our baseline model, for sufficiently strong strategic complementarity, noise can contribute to a significant fraction of the

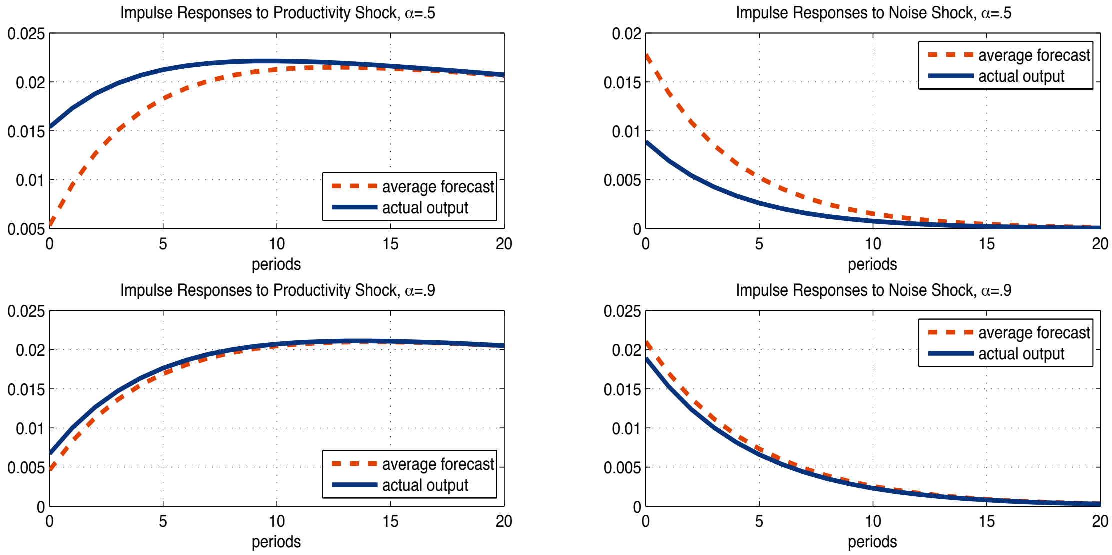

Figure 5: Forecast errors in response to productivity and noise shocks.

high-frequency variation in output. As for employment, the contribution of noise is quite dramatic.

Finally, Figure 5 plots the dynamics of the average forecast of aggregate output and the true level of aggregate output in response to a productivity or noise shock. The average forecast error is the distance between the two aforementioned variables. A salient feature of this figure is that for the forecast errors are *smallest* when the degree of strategic complementarity is highest.

This is crucial. We earlier showed that a higher α leads to both more inertia in the response of output and employment to productivity shock, and to a bigger impact of noise. In this sense, the deviation from the common-knowledge benchmark is highest when α is highest. However, one should not expect that these large deviations will show up in large forecast errors. To the contrary. A higher α implies that actual economic activity is more driven by forecasts of economic activity, so that at the end a higher α guarantees that the forecast errors are smaller. It follows that, as we vary α, the magnitude of the deviations of actual outcomes from their common-knowledge counterparts is *inversely* related to the magnitude of the associate forecast errors. of output. Indeed, both the inertia and the impact of noise become nearly self-fulfilling as α gets closer to 1.

In conclusion, the instantaneous impact of the response of output and employment to productivity and noise shocks behave very much like in the baseline model: complementarity dampens the effect of productivity shocks, while amplifying the response to noise. However, these effects now persist for more than a period. Finally, provided that α is high enough, the inertia can be quite strong, and the contribution of noise to high-frequency variation can be quite high, while at the same time the associated forecast errors are very small.

5.3 Demand shocks, new-Keynesian models, and structural VARs

We now discuss how our results may offer a certain explanation for some of the prevalent empirical properties of business cycles.

The noise-driven fluctuations we have documented here resemble"demand" shocks in the following sense: they contribute to positive co-movement in employment, output and consumption; they are orthogonal to the underlying productivity shocks; they are closely related to shifts in expectations about aggregate demand; and they explain a large portion of the high-frequency variation in employment and output while vanishing at low frequencies.

To better appreciate this, suppose that we generate date from our model and let an applied macroeconomist—preferably of the new-keynesian type— to run a structural VAR as in Blanchard and Quah (1989) or Galí (1999). One would then correctly identify the underlying innovations to productivity by the shock that is allowed to have a long-run effect on output or labor productivity, and the underlying noise shocks by the residual. In the language of Blanchard and Quah, the productivity shocks would be interpreted as "supply shocks" and the noise shocks as "demand shocks". however, the latter would have no relation to sticky prices and the like; to the contrary, both type of shocks emerge from a purely supply-side mechanism. In the language of Galí (1999), on the other hand, the productivity shocks would be interpreted as "technology shocks". Furthermore, as already noted, the short-run response of employment to these identified shocks would be negative for high enough α; but this would no favor a sticky-price interpretation.

As mentioned in the introduction, Beaudry and Portier (2004, 2006), Christiano et al. (2008), Jaimovich and Rebelo (2008), and Lorenzoni (2008) have explored the idea that noisy news about future productivity contribute to short-run fluctuations. Furthermore, Lorenzoni (2008) interprets the resulting fluctuations as "demand shocks" and discusses how they help match related facts. However, all these papers focus on fluctuations that originate from uncertainty about a certain type of fundamentals (namely future productivity), not on the distinct type of uncertainty we highlight in this paper.[19] Second, as often the case with new-keynesian models, Lorenzoni's "demand shocks" confound real shocks with monetary shocks. By this we mean the following. Since there is no capital

[19] In his baseline model, Lorenzoni considers a representative-agent model with symmetric information. In an extension, he allows for dispersed information, but only to facilitate a more plausible calibration of the model.

in his model (as in ours), expectations of future productivity would have been irrelevant for current macroeconomic outcomes had nominal prices been flexible; the only reason then that news about future productivity cause demand-like fluctuations is that they cause an expansion in monetary policy away from the one that would replicate flexible-price allocations. In contrast, our "demand shocks" obtain in an RBC setting and are completely unrelated to monetary policy.

Finally, note that a positive productivity shock in our model induces a small initial impact on output and then a slow convergence to a permanently higher level. Again these qualitative properties are consistent with the estimated dynamics of “technology” shocks.

More generally, it is interesting to note that in many empirical new-keynesian models sticky prices dampen the response of output to productivity shocks relative to the RBC framework and help get a negative response for employment. According to some researchers, these properties seem to be more consistent with the data than their RBC counterparts. However, what is a success for these models is only a failure for monetary policy: the only reason that the response of the economy to productivity shocks in the baseline new-keynesian model differs from that in the baseline RBC model is that monetary policy fails to replicate flexible-price allocations, which is typically the optimal thing to do. Here, instead, we obtain the same empirical properties without introducing sticky properties and without presuming any suboptimality for policy.

Perhaps as interestingly, our approach may have intriguing implications for the identification of monetary shocks. One of the standard identification strategies is based on the idea that monetary policy often reacts to measurement error in the level of aggregate economic activity (Bernanke and Mihov, 1995; Christiano, Eichenbaum and Evans, 1999). In particular, the idea is that measurement error justifies the existence of random shocks to monetary policy, which are orthogonal to the true underlying state of the economy. If one then traces the impact of these particular shocks on subsequent aggregate outcomes, one can escape the endogeneity problem and identify the impact of monetary shocks. However, these measurement errors, or more generally any forecast errors that the central bank makes about current and future economic activity, are likely to be correlated with the corresponding forecast errors of the private sector. But then the so-identified monetary shocks may actually be proxying for the real effects of the forecast errors of the private sector, which unfortunately are not observed by the econometrician.

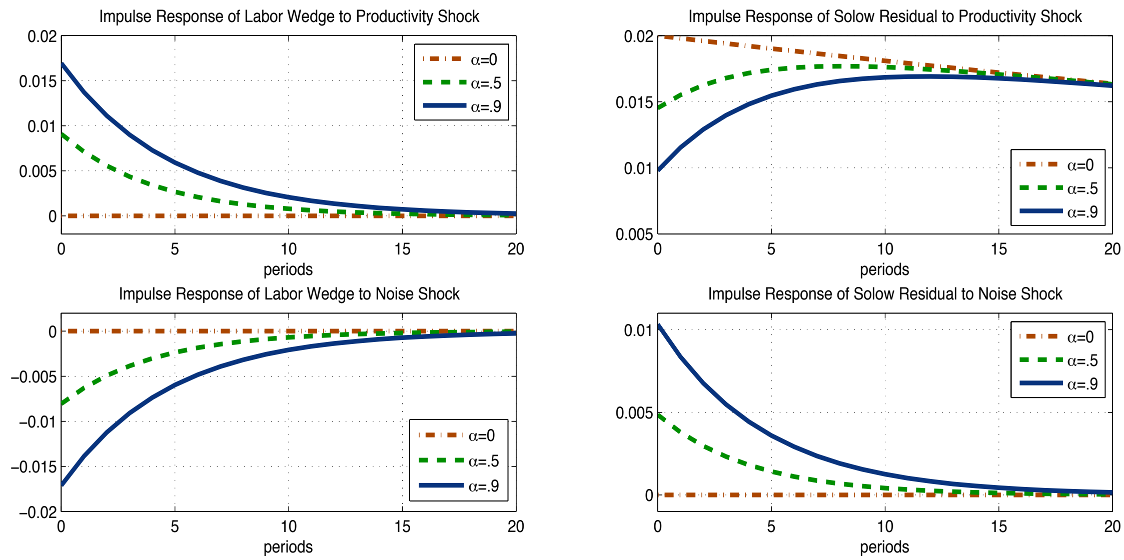

Figure 6: Labor wedges and Solow residuals.

5.4 Labor wedges and Solow residuals

We now consider the implications of our model for two other characteristics of the business cycle: labor wedges and Solow residuals.

Following the literature (e.g., Hall, 1997; Rotemberg and Woodford, 1999; Chari, Kehoe, and McGrattan, 2007; Shimer, 2009), we define the labor wedge $\tau_{n,t}$ implicitly by

$$\frac{N_t^{\varepsilon-1}}{C_t^{-\gamma}} = (1-\tau_{n,t})\,\theta\frac{Q_t}{N_t}.$$

The left panel of Figure 6 plots the impulse response of the labor wedge to a positive productivity and a positive noise shock. The labor wedge follows very different dynamics in response to the two types of shocks. In particular, a positive productivity shock induces a positive response in the labor wedge, implying positive comovement of the labor wedge with output. On the other hand, a positive noise shock produces a negative response in the observed labor wedge, implying a negative comovement with output.

Multiple authors have documented that variation in the labor wedge plays a large role in accounting for business-cycle fluctuations during the post-war period. Importantly, the labor wedge is highly countercyclical, exhibiting sharp increases during recessions. Shimer (2009) surveys the facts and the multiple explanations that have been proposed for the observed countercyclicality of the labor. These include taxes, shocks to the disutility of labor, mark-up shocks, fluctuations in wage-setting power, and Shimer's preferred explanation, search frictions in the labor market. Here, we have found that noise offers another possible explanation for the same fact.

We finally consider the potential implications of our results for observed Solow residuals. Towards this goal, we now introduce a variable input in the production function; the optimal use of this input responds to shocks, but is unobserved by the econometrician and is thus absorbed in the Solow residual. As in King and Rebelo (2000), our preferred interpretation of this input is capital utilization. The only caveat is that here we keep capital exogenously fixed. However, we could introduce capital following the same approach as Angeletos and La'O (2009b), without affecting the qualitative points we seek to make here.

We denote the unobserved input by χ_{it}; we let the gross product of a firm be $\tilde{q}_{it} = \tilde{A}_{it}\chi_{it}^{1-\tilde{\theta}} n_{it}^{\tilde{\theta}}$; and we specify the cost of this input in terms of final product as $\delta\chi_{it}^{1+\xi}$, where $\xi, \delta > 0$. The net product of a firm is then $q_{it} = \tilde{q}_{it} - \delta\chi_{ut}^{1+\xi}$. Solving out for the optimal level of this input, The optimal level of this input is given by equating its marginal product with its marginal cost: $(1-\tilde{\theta})\frac{q_{it}}{\chi_{it}} = \delta\left(1+\xi\right)\chi_{it}^{\xi}$. We thus obtain obtain the following reduced-form production function:

$$q_{it} = A_{it} n_{it}^{\theta} \tag{19}$$

where $\theta \equiv \left(\frac{1+\xi}{\tilde{\theta}+\xi}\right)\tilde{\theta}$ and $A_{it} \equiv \left(\frac{1+\xi}{\tilde{\theta}+\xi}\right)\tilde{A}_{it}^{\frac{1+\xi}{\tilde{\theta}+\xi}}$. Our analysis then remains intact, provided we reinterpret the production function in the above way. Accordingly, we now set $\tilde{\theta} = .4$ and $\xi = .1$ (a preferred value in King and Rebelo), which implies $\theta = .88$. We also re-calibrate the underlying aggregate productivity shocks so that the observed Solow residual ($SR_t \equiv \log Q_t - \theta \log N_t$) implied by the common-knowledge version of the model continues to have a standard deviation of 0.02 and a persistence of 0.99.

The right panel of Figure 6 plots the dynamic response of the Solow residual to a productivity or a noise shock. Both shocks raise the measured Solow residual, but only the innovation in productivity has a persistent effect. Moreover, these responses of the Solow residual mirror those of output. It follows that the Solow residual and output move tightly together, much alike in a standard RBC model, although employment has the more distinct behavior we mentioned earlier.[20]

[20]It is worth noting that additional variation in measured Solow residuals could obtain from variation in the dispersion of information, simply because the dispersion of information affects the cross-sectional allocations to resources. Note in particular that the observed heterogeneity in forecast surveys is highly countercyclical, suggesting that the dispersion of information is also countercyclical.

6 Higher-order beliefs

While the equilibrium characterization in Section 3 allowed for arbitrary information structures, the more concrete properties documented in Sections 4 and 5 relied on a convenient, but specific, information structure. We now explain the more general forces lying behind these properties.

Towards this goal, we combine conditions (9) and (10) to obtain the following fixed-point relation for the equilibrium level of aggregate output:

$$\log Q_t = const + (1-\alpha)\bar{f}_t + \alpha \bar{\mathbb{E}}_t[\log Q_t] \tag{20}$$

where $\bar{f}_t$ is the aggregate shock and $\bar{\mathbb{E}}_t[\log Q_t]$ is the average forecast of aggregate output. Once again, this condition highlights the idea that actual economic activity is pinned down by the fundamentals and by the forecasts of economic activity. Next, iterate condition (20) to obtain the following characterization of the average forecast of aggregate output:

$$\bar{\mathbb{E}}_t[\log Q_t] = const + \sum_{m=1}^{\infty} \alpha^m \bar{E}_t^m[\bar{f}_t] \tag{21}$$

where the sequence $\left\{\bar{E}_t^m[\bar{f}_t]\right\}_{m=1}^{\infty}$ is defined recursively by letting $\bar{E}_t^1[\bar{f}_t]$ be the average forecast of the aggregate shock $\bar{f}_t$ (a.k.a. the average first-order belief), $\bar{E}_t^2[\bar{f}_t]$ be the average forecast of $\bar{E}_t^1[\bar{f}_t]$ (a.k.a. the average second-order belief), and so on. Finally, combining (20) and (21) we obtain actual output as a linear combination of the fundamentals and the entire hierarchy of beliefs.[21]

The above characterization is similar to the one in Morris and Shin (2002), who were the first to highlight the potential relevance of higher-order beliefs for macroeconomic applications, albeit within a different, and more abstract, context; see also the extension in Angeletos and Pavan (2007, 2009) for a more general class of linear-quadratic games. Building on the insights of this earlier,

[21] If some or all of the local shocks where not known to each island, we would need to replace f_{it} in condition (9) with a linear combination of the realized local fundamentals and their forecasts. By implication, $\bar{f}_t$ in condition (20) would have to be replaced with a linear combination of the cross-sectional averages of the realized local fundamentals and the forecasts of these fundamentals. However, the rest of the analysis would be unaffected. Also, the first-order belief of an agent is, strictly speaking, the entire probability distribution that characterizes his posterior about the fundamentals. Accordingly, the second-order belief of an agent is a probability distribution over a probability distribution, and so on. In a game with linear best responses, these beliefs matter only through the implied first moments, namely the sequence $\left\{\bar{E}_t^m[\bar{f}_t]\right\}_{m=1}^{\infty}$. But in a game with non-linear best responses, higher moments also matter. In our context, the linear condition (20) is exact in the case of a Gaussian information structure but only approximate in general. In this sense, the result that the impact of higher-order beliefs is summarized by the sequence $\left\{\bar{E}_t^m[\bar{f}_t]\right\}_{m=1}^{\infty}$ is also approximate. Nevertheless, this approximation as an excellent starting point.

more abstract, work, we can now recast our results as follows. First, higher-order beliefs are more sensitive to the initial common prior, to public signals, and to signals with strongly correlated errors, than lower-order beliefs, simply because these pieces of information are relatively better predictors of the forecasts of others. It follows that higher-order beliefs are less sensitive to innovations in the fundamentals and more sensitive to common sources of noise than lower-order beliefs. Finally, note that stronger complementarity (higher α) increases the relative contribution of higher-order beliefs to forecasted and actual output. Combined, these observations explain why stronger complementarity dampens the response of the economy to innovations in fundamentals while amplifying the impact of noise. These effects do not appear to be unduly sensitive to the details of the underlying information structure; rather, they obtain from more robust properties of higher-order beliefs.

At this point, it is worth emphasizing why all these effects vanish when information is commonly shared: it is because, and only because, higher-order beliefs then collapse to first-order beliefs. Indeed, as long as agents share the same information, they cannot face any uncertainty whatsoever about one another's beliefs about the fundamentals: their first-order beliefs are common knowledge. But then second-order beliefs collapse to first-order beliefs, by implication third-order beliefs also collapse to first-order beliefs, and so on. This highlights once again the distinct nature of dispersed information. With symmetric information, forecasts of aggregate economic activity are pinned down by forecasts of the underlying fundamentals; with dispersed information, this is not the case. This point is further emphasized in Angeletos and La'O (2009b), where it is shown that there can be shocks that are orthogonal to *both* the fundamentals and the agents' first-order beliefs but nevertheless move higher-order beliefs.

It is also worth emphasizing that our results are immune to the point made in by Hellwig and Venkateswaran (2009). That paper considers a certain variant of Woodford (2003a) in which firms mistake aggregate monetary shocks for firm-specific real shocks, and in which firms end up adjusting prices to a monetary shock as a result of this confusion. Although this finding offers an intriguing possibility, it is sensitive to the particular framework and the particular quantitative exercise considered in that paper. What is more, it appears to be irrelevant for our own context. First, in our model, firms need not be confused between aggregate and idiosyncratic shocks—they can be perfectly well informed about their idiosyncratic shocks. And second, the possibility that firms mistake an aggregate shock for an idiosyncratic one appears only to reinforce our results. To see this, recall from Proposition 3 and Corollary 3 that the response of equilibrium output to an idiosyncratic shock in fundamentals is given by $\varphi_f = 1 - \alpha$, while its response to an aggregate shock is given by $\varphi_\nu = 1 - \alpha \frac{\kappa_f}{(1-\alpha)\kappa_x + \kappa_y + \kappa_f}$. As long as $\alpha > 0$, φ_f is smaller than φ_ν, which means that mistaking an aggregate shock for an idiosyncratic shock only helps dampen the response of the economy to the aggregate shock.

We conclude this section by warning the reader, and future researchers, not to get lost in either the wilderness of higher-order beliefs or the details of specific information structures. Rather, the key issue is merely the additional uncertainty about aggregate economic activity that agents can face when information is dispersed—or, in more concrete terms, the size of the forecast errors they make in predicting economic activity. We have already shown that significant macroeconomic effects are possible even when these errors are small, provided that the degree of strategic complementarity is high enough. More generally, we propose that this is the main metric that should be used for any quantitative study of the business-cycle implications of dispersed information.

7 Efficiency

The positive properties we have documented so far are intriguing and, at least in our view, highlight the potential returns of incorporating dispersed information in more quantitatively-oriented business cycle models. However, their normative content is unclear. Is the potentially high contribution of noise to business-cycle fluctuations, or the potentially high inertia in the response of the economy to innovations in the underlying fundamentals, symptom of inefficiency?

More generally, it is obvious that a planner could improve welfare if he could centralize all the information that is dispersed in society and then dictate allocations on the basis of all this information. But this would endow the planner with a power that seems far remote from the powers that policy maker may have in reality. Furthermore, the resulting superiority of centralized allocations over their decentralized equilibrium counterparts would not be particularly insightful, since it would be driven mostly by the assumption that the planner has the supernatural power to overcome the information frictions imposed on the market. Following Angeletos and Pavan (2007, 2009) and Angeletos and La'O (2008), we thus contend that a more interesting question—on both practical and theoretical grounds—is to understand whether a planner could improve upon the equilibrium while being subject to the same informational frictions as the market.

This motivates us to consider a constrained efficiency concept that permits the planner to choose any resource-feasible allocation that respects the geographical segmentation of information in the economy—by which we simply mean that the planner cannot make the production and employment choices of firms and workers in one island contingent on the private information of another island. A formal definition of this constrained efficiency concept and a more detailed analysis can be found in a companion paper, Angeletos and La'O (2008). That paper retains the simplifying assumption that aggregate shocks becomes common knowledge at the end of each period and, for concreteness, focuses on productivity shocks as the only real shock in the economy. At the same time, it generalizes the framework of this paper by allowing for prices to be sticky and for firms and workers to make choices in multiple stages within each period, under evolving information. It also allows information to be partly aggregated through certain prices and macroeconomic statistics. It finally derives an number of implications for optimal fiscal and monetary policy. Here, we abstract from all these additional features and focus on explaining the normative content of the equilibrium properties we have documented so far.

Because of the concavity of preferences and technologies, efficiency dictates symmetry in consumption across households, as well as symmetry across firms and workers within any given island. Using these facts, we can represent the planning problem we are interested in as follows.

Planner's problem. *Choose a pair of local production and employment strategies, $q : \mathcal{S}_\omega \times \mathcal{S}_\Omega \to \mathbb{R}_+$ and $n : \mathcal{S}_\omega \times \mathcal{S}_\Omega \to \mathbb{R}_+$, and an aggregate output function, $Q : \mathcal{S}_\Omega^2 \to \mathbb{R}_+$, so as to maximize*

$$\int_{\mathcal{S}_\Omega} \left[U(Q(\Omega_t, \Omega_{t-1})) - \int_{\mathcal{S}_\omega} \tfrac{1}{1+\epsilon} S(\omega) n(\omega, \Omega_{t-1})^{1+\epsilon} d\Omega_t(\omega) \right] d\mathcal{P}(\Omega_t | \Omega_{t-1}) \tag{22}$$

subject to

$$q(\omega, \Omega_{t-1}) = A(\omega) n(\omega, \Omega_{t-1})^\theta \;\; \forall \omega, \Omega_{t-1} \tag{23}$$

$$Q(\Omega_t, \Omega_{t-1}) = \left[\int q(\omega, \Omega_{t-1})^{\frac{\rho-1}{\rho}} d\Omega_t(\omega) \right]^{\frac{\rho}{\rho-1}} \;\; \forall \Omega_t, \Omega_{t-1} \tag{24}$$

where $\mathcal{P}(\Omega_t|\Omega_{t-1})$ denotes the probability distribution of Ω_t conditional on Ω_{t-1}.

This problem has a simple interpretation. $U(Q(\Omega_t, \Omega_{t-1})$ is the utility of consumption for the representative household; $\frac{1}{\epsilon} S(\omega) n(\omega, \Omega_{t-1})^\epsilon$ is the marginal disutility of labor for the typical worker in a given island; and the corresponding integral is the overall disutility of labor for the representative household. Furthermore, note that, once the planner picks the production strategy q, the employment strategy n is pinned down by (23) and the aggregate output function Q is pinned down by (23). The reduced-form objective in (22) is thus a functional that gives the level of welfare implied by any arbitrary production strategy that the planner dictates to the economy.

Because this problem is strictly concave, it has a unique solution and this solution is pinned down by the following first-order condition:[22]

$$S_{it} n_{it}^{\epsilon} = \mathbb{E}_{it}\left[U'\left(Q_t\right)\left(\frac{q_{it}}{Q_t}\right)^{-\frac{1}{\rho}}\right]\left(\theta A_{it} n_{it}^{\theta-1}\right). \tag{25}$$

This condition simply states that the planner dictates the agents to equate the social cost of employment in their island with the *local* expectation of the social value of the marginal product of that employment. Essentially the same condition characterizes (first-best) efficiency in the standard, symmetric-information paradigm. The only difference is that there expectations are conditional on the commonly-available information set, while here they are conditional on the locally-available information sets.

As with equilibrium, we can use $q_{it} = A_{it} n_{it}^{\theta}$ to eliminate n_{it} in the above condition, thereby reaching the following result.

Proposition 5. *Let*

$$f^*(\omega) \equiv \log\left\{\theta^{\frac{1}{\frac{\epsilon}{\theta}+\gamma-1}}\left(\frac{A(\omega)}{S(\omega)}\right)^{\frac{\frac{\epsilon}{\theta}}{\frac{\epsilon}{\theta}+\gamma-1}}\left(\frac{A(\omega)}{S(\omega)}\right)^{\frac{\frac{\epsilon}{\theta}}{\frac{\epsilon}{\theta}+\gamma-1}}\right\}$$

be a composite of the local productivity and taste shocks. The efficient strategy $q : \mathcal{S}_\omega \times \mathcal{S}_\Omega \to \mathbb{R}_+$ is the fixed point to the following:

$$\log q\left(\omega_t, \Omega_{t-1}\right) = (1-\alpha) f^*(\omega_t) + \alpha \log\left\{\mathbb{E}\left[Q(\Omega_t, \Omega_{t-1})^{\frac{1}{\rho}-\gamma}\middle|\, \omega_t, \Omega_{t-1}\right]^{\frac{1}{\frac{1}{\rho}-\gamma}}\right\} \;\forall(\omega_t, \Omega_{t-1}), \tag{26}$$

$$Q(\Omega_t, \Omega_{t-1}) = \left[\int q(\omega, \Omega_{t-1})^{\frac{\rho-1}{\rho}} d\Omega_t(\omega)\right]^{\frac{\rho}{\rho-1}} \;\forall(\Omega_t, \Omega_{t-1}). \tag{27}$$

A number of remarks are worth making. First, note that the composite shock f_t^* plays a similar role for the efficient allocation as the composite shock f_t played for the equilibrium: it identifies the fundamentals that are relevant from the planner's point of view. This is evident, not only from the above result, but also directly from the planner's problem: using $q_t = A_t n_t^{\theta}$ to eliminate n_t in the expression for welfare given in the planner's problem, we can express welfare as a simple function of the production strategy and the composite shock f_t^* alone.

Second, note that Proposition 5 permits a game-theoretic interpretation of the efficient allocation, much alike what Proposition 1 did for equilibrium: the efficient allocation of the economy

[22]Because of the continuum, the efficient allocation is determined only for *almost* every ω. For expositional simplicity, we bypass the *almost* qualification throughout the paper.

coincides with the Bayes-Nash equilibrium of a game in which the different players are the different islands of the economy and their best responses are given by (26).

Third, note that, apart for the different composite shock, the structure of the fixed point that characterizes the efficient and the equilibrium allocation is the same: once we replace $f^*(\omega_t)$ with $f(\omega_t)$, condition (26) coincides with its equilibrium counterpart, condition (7). And because $f^*(\omega_t) = f(\omega_t)$ for every ω_t if and only if there is no monopoly power, the following is immediate.

Corollary 5. *In the absence of monopoly distortions, the equilibrium is efficient, no matter the information structure.*

This result thus establishes that the dispersion of information per se is not a source of inefficiency. This is intuitive, because the geographical segmentation of information is alike a technological constraint that impacts equilibrium and efficient allocations in a completely symmetric way. Accordingly, we can generalize the result for situations where firms have monopoly power, but there are no aggregate shocks to monopoly power, as follows.

Corollary 6. *Suppose that information is Gaussian (Assumption 1 holds) and there are no aggregate mark-up shocks ($\bar{f}_t^* - \bar{f}_t$ is fixed). Then, the the business cycle is efficient in the sense the gap $\log Q_t - \log Q_t^*$ between the equilibrium and the efficient level of output is invariant.*

In other words, the equilibrium level of activity may be inefficiently low because of monopoly power, but its response to either the underlying productivity and taste shocks or any noise in the agents' information about these shocks is efficient.[23] It follows that all the intriguing equilibrium properties we documented in Section 4 and 5—the inertia with respect to productivity shocks, the large contribution of noise in the business cycle, the presence of demand-like shocks, and so on—are symptoms of the efficient decentralized use of information. No matter how "perverse" these inertia and volatility effects may look like if seen in comparison to the common knowledge benchmark, they do not per se open the door for policy intervention.

[23]If we allow for aggregate mark-up shocks, it is clear that the response of the equilibrium to these shocks, or to any information about them, is inefficient. Whether then the response of the economy to the underlying aggregate productivity and taste shocks remains efficient depends on whether the information agents have about these shocks is orthogonal to the one they have about the aggregate mark-up shocks. In particular, if the underlying shocks are themselves orthogonal and the signals agents observe for each of the shocks have uncorrelated errors, then the equilibrium response to all the signals about productivity and tastes is efficient. But if the errors are correlated, then this efficiency breaks because each signal now contains information for all shocks.

In closing this section, we wish to iterate an important qualification: the aforementioned efficiency results rely on the assumption that the government does not have the power to impact the information structure. In practice, the government may well have this power. This is not merely because the government can collect and publicize relevant information about the economy, but also because its actions can impact the endogenous aggregation of information through prices and other economic indicators—a possibility that has been ruled out in the present paper but is considered in Angeletos and Pavan (2009) and Angeletos and La'O (2008).

8 Concluding remarks

The macroeconomics literature has used informational frictions to motivate why economic agents may happen to be—or perhaps choose to be—partly unaware about the shocks hitting the economy. Sometimes the shocks of interest are monetary, sometimes they are real. Sometimes the informational friction is completely exogenous, sometimes it is partly endogenized. Invariably, though, the main modeling role of informational frictions seems to remain a simple and basic one: to limit the knowledge that agents have about the underlying shocks to economic fundamentals.

In this paper we sought to highlight a distinct aspect of the general equilibrium implications of dispersed information. We first noted that macroeconomic models that impose symmetric information along with a unique equilibrium also impose that any uncertainty that agents may face about the level of economic activity is pinned down by the uncertainty that they face about the fundamentals. We then highlighted that this is not the case when information is asymmetric: agents can then face an additional type of uncertainty when trying to forecast economic activity, one that goes far beyond any uncertainty that may face about the exogenous fundamentals. We thus proposed that dispersed information should be viewed primarily as a modeling device for introducing this distinct type of uncertainty.

We next sought to illustrate the potential business-cycle implications of this additional type of uncertainty within the context of an RBC model. We showed how this uncertainty can dominate the business cycle even if agents are well informed about the fundamentals. We further showed how the response of the economy to the underlying aggregate productivity shocks can exhibit significant inertia; how this inertia could induce a negative response for employment; how the noise-driven fluctuations can resemble demand shocks; and how they can involve countercyclical variation in labor wedges and procyclical variation in Solow residuals.

Our results can easily be extended to new-Keynesian variants of the type of economies we have studied here. In such a context, the dispersion of information can then lead to inertia in the response of prices to innovations in monetary policy (e.g., Woodford, 2003a; Hellwig, 2002; Angeletos and La'O; 2009a), but only to the extent that there is a significant departure from common knowledge about these innovations. However, at least for some it is hard to see how reality relates to models that impose such significant lack of common knowledge about the underlying innovations to monetary policy—nowadays, information about the conduct of monetary policy is widely and readily available, this fact is commonly understood, and at least financial markets react to innovations in monetary policy within seconds. For this reason, some may feel that assuming common knowledge about monetary policy is not a bad benchmark.

While we may largely concur with this position, we wish to highlight that this does not imply that there is no interesting interaction between monetary policy and dispersed information. To the contrary. If there is dispersed information about the underlying *real* shocks hitting the economy, the response of monetary policy—or any other macroeconomic policy—to any information that becomes available about these shocks may be crucial for how the economy responds to these shocks in the first place. This point was first emphasized by Angeletos and Pavan (2009) in an abstract class of economies and further explored by Lorenzoni (2009) and our own companion paper (Angeletos and La'O, 2008) within new-Keynesian variants of the economies we have studied in this paper.

Our results have assumed that all information is exogenous. Hellwig and Veldkamp (2008) study the implications of endogenizing the collection of information. They highlight how the type of information agents may collect in the first place may depend crucially on form of their strategic interaction, and how this feedback may in certain cases lead to multiple equilibria. Our own companion paper, on the other hand, focuses on the fact that the information that is dispersed in the economy gets aggregated through prices and various economic indicators. We then explore how the central bank can affect the extent of such information aggregation, and thereby affect the information that is available to either the market or itself, by appropriately designing its response to realized economic activity.

Our model also abstracted from capital accumulation to simplify the analysis. We do not expect any of our results to hinge on this simplification. We nevertheless find an extension with capital important for two reasons. First, it will facilitate a quantitative assessment of our insights. And second, while in our model individual incentives depended only on forecasts of *current* economic activity, intertemporal linkages such as those introduced by capital make individual incentives depend also on forecasts of *future* economic activity. This will in turn permit one to study how the distinct type of uncertainty we have identified here may operate through forecasts of future economic activity, as opposed to forecasts of current economic activity.

We would like to conclude with a comment on the alternative formalizations of informational frictions. For certain purposes, one formalization might be preferable to another. For example, if one wishes to understand which particular pieces of information agents are likely to pay more attention to, Sims (2003) offers an elegant methodology for addressing this type of questions, and Mackowiak and Wiederholt (2008) offer an excellent example of how such a methodology could inform important macro issues. Alternatively, if one has a strong prior that agents collect and process information only infrequently, probably because doing so involves significant fixed costs, then the approaches taken in Mankiw and Reis (2002), Reis (2006), or Woodford (2009) look promising. For other purposes, however, these particular formalizations may prove unnecessary, or even distracting. The results we have emphasized in this paper hinge on the asymmetry of information and the general-equilibrium interaction of agents, not on the details of the information structure.

More generally, we would invite the reader, and future researchers, to take a more flexible approach to the modeling of informational frictions. The data do not provide us with sufficient guidance on how to model the details of the information structure. It would thus be naive to commit to any specific formalization. At the same time, this does not mean that the theory is free. To the contrary, the data may provide us with discipline in the dimensions that matter the most. In a broad class of macro models, including the one we have employed here, the information structure impacts equilibrium outcomes only through two channels: by affecting the individuals' information about their own fundamentals; and by affecting their forecasts of aggregate economic activity. Micro-evidence on the response of individual actions to individual fundamentals can provide us with discipline in the first dimension. Survey data on forecasts of aggregate economic activity could provide us with discipline in the second dimension.

Appendix

Proof of Proposition 1. The characterization of the equilibrium follows directly from the discussion in the main text. Its existence and uniqueness can be obtained by showing that the equilibrium coincides with the solution to a concave planning problem. For the case that there is no monopoly power ($\eta = \infty$), this follows directly from our analysis in Section 6 and in Proposition 5. A similar result can be obtained for the case with monopoly power.

Proof of Proposition 2. This follows from the discussion in the main text.

Proof of Proposition 3. Suppose that, conditional on ω_t and Ω_{t-1}, $Q(\Omega_t, \Omega_{t-1})$ is log-normal, with variance independent of ω_t; that this is true under the log-normal structure for the underlying shocks and signals we will prove shortly. Using log-normality of Q in condition (7), we infer that the equilibrium production strategy must satisfy condition (9) with

$$const = \frac{\alpha}{2}\left(\frac{1}{\rho} - \gamma\right) \mathrm{Var}\left[\log Q(\Omega_t, \Omega_{t-1})|\omega_t, \Omega_{t-1}\right]$$

and $\mathrm{Var}\left[\log Q(\Omega_t, \Omega_{t-1})|\omega_t, \Omega_{t-1}\right] = \mathrm{Var}\left[\log Q(\Omega_t, \Omega_{t-1})|\Omega_{t-1}\right]$.

We now guess and verify a log-linear equilibrium under the log-normal specification for the shock and information structure. Suppose the equilibrium production strategy takes a log-linear form: $\log q_t = \varphi_0 + \varphi_{-1}\bar{f}_{t-1} + \varphi_f f_t + \varphi_x x_t + \varphi_y y_t$, for some coefficients $(\varphi_{-1}, \varphi_f, \varphi_x, \varphi_y)$. Aggregate output is then given by

$$\log Q(\Omega_t, \Omega_{t-1}) = \varphi_0' + \varphi_{-1}\bar{f}_{t-1} + (\varphi_f + \varphi_x)\bar{f}_t + \varphi_y y_t \tag{28}$$

where $\varphi_0' \equiv \varphi_0 + \frac{1}{2}\left(\frac{\rho-1}{\rho}\right)\left[\frac{\varphi_f^2}{\kappa_\xi} + \frac{\varphi_x^2}{\kappa_x} + 2\frac{\varphi_f\varphi_x}{\kappa_x}\right]$. It follows that $Q(\Omega_t, \Omega_{t-1})$ is indeed log-normal, with

$$\begin{aligned} \mathbb{E}\left[\log Q(\Omega_t, \Omega_{t-1})|\omega_t, \Omega_{t-1}\right] &= \varphi_0' + \varphi_{-1}\bar{f}_{t-1} + (\varphi_f + \varphi_x)\mathbb{E}\left[\bar{f}_t|\omega_t, \Omega_{t-1}\right] + \varphi_y y_t && (29) \\ Var\left[\log Q(\Omega_t, \Omega_{t-1})|\omega_t, \Omega_{t-1}\right] &= (\varphi_f + \varphi_x)^2\left(\frac{1}{\kappa_f + \kappa_x + \kappa_y}\right) && (30) \end{aligned}$$

where $\mathbb{E}\left[\bar{f}_t|\omega_t, \Omega_{t-1}\right] = \frac{\kappa_f}{\kappa_f+\kappa_x+\kappa_y}\psi f_{t-1} + \frac{\kappa_x}{\kappa_f+\kappa_x+\kappa_y}x_t + \frac{\kappa_y}{\kappa_f+\kappa_x+\kappa_y}y_t$. Substituting these expressions into (9) gives us

$$\begin{aligned} \log q\left(\omega_t, \Omega_{t-1}\right) &= const + (1-\alpha)\, f\left(\omega\right) + \alpha\left({\varphi_0}' + \varphi_{-1}\bar{f}_{t-1} + \varphi_y y_t\right) \\ &\quad + \alpha(\varphi_f + \varphi_x)\left(\frac{\kappa_f}{\kappa_f + \kappa_x + \kappa_y}\psi f_{t-1} + \frac{\kappa_x}{\kappa_f + \kappa_x + \kappa_y}x_t + \frac{\kappa_y}{\kappa_f + \kappa_x + \kappa_y}y_t\right) \end{aligned}$$

For this to coincide with $\log q(\omega) = \varphi_0 + \varphi_{-1}\bar{f}_{t-1} + \varphi_f f + \varphi_x x + \varphi_y y$ for every (f, x, y), it is necessary and sufficient that the coefficients $(\varphi_0, \varphi_{-1}, \varphi_f, \varphi_x, \varphi_y)$ solve the following system:

$$\begin{aligned}
\varphi_0 &= const + \alpha\varphi_0' \\
\varphi_f &= 1 - \alpha \\
\varphi_x &= \alpha(\varphi_f + \varphi_x)\left(\frac{\kappa_x}{\kappa_f + \kappa_x + \kappa_y}\right) \\
\varphi_{-1} &= \alpha\varphi_{-1} + \alpha(\varphi_f + \varphi_x)\left(\frac{\kappa_f}{\kappa_f + \kappa_x + \kappa_y}\right)\psi \\
\varphi_y &= \alpha\varphi_y + \alpha(\varphi_f + \varphi_x)\left(\frac{\kappa_y}{\kappa_f + \kappa_x + \kappa_y}\right)
\end{aligned}$$

The unique solution to this system for $(\varphi_{-1}, \varphi_f, \varphi_x, \varphi_y)$ is the one given in the proposition; φ_0 is then uniquely determined from the first equation of this system along with the definition of *const* and φ_0'.

Proof of Proposition 4. The result follows by a triple limit. First, take $\alpha \to 1$; next, take $\kappa_y \to 0$; and finally, take $\kappa_x \to \infty$. It is easy to check that this triple limit implies $\kappa \to \infty$ and $R \to 1$. That is, the precision of the agents posterior about the fundamentals (the mean squared forecast error) converges to zero, while the fraction of the high-frequency variation in output that is due to noise converges to 100%.

Kalman filtering for dynamic extension. The method we use in solving this equilibrium is similar to that found in Woodford (2003b).

State Vector and Law of Motion. We guess and verify that the relevant aggregate state variables of the economy at time t are $\bar{f}_t$ and $\log Q_t$ and thus define state vector X_t in (16) accordingly.

Claim. *The dynamics of the economy are given by the following law of motion*

$$X_t = MX_{t-1} + m_v v_t + m_\varepsilon \varepsilon_t \tag{31}$$

with

$$M \equiv \begin{bmatrix} \psi & 0 \\ M_{21} & M_{22} \end{bmatrix}, m_v \equiv \begin{bmatrix} 1 \\ m_{v2} \end{bmatrix}, m_\varepsilon \equiv \begin{bmatrix} 0 \\ m_{\varepsilon 2} \end{bmatrix}. \tag{32}$$

The coefficients $(M_{21}, M_{22}, m_{v2}, m_{\varepsilon 2})$ *are given by*

$$M_{21} = \psi \left(K_{21} + K_{22}\right) \tag{33}$$

$$M_{22} = \psi \left(1 - K_{21} - K_{22}\right) \tag{34}$$

$$m_{u2} = 1 - \alpha \left(1 - K_{21} - K_{22}\right) \tag{35}$$

$$m_{\eta 2} = \alpha K_{22} \tag{36}$$

and

$$K \equiv \begin{bmatrix} K_{11} & K_{21} \\ K_{21} & K_{22} \end{bmatrix}$$

is the matrix of kalman gains, defined by

$$K \equiv \mathbb{E}\left[\left(X_t - \mathbb{E}_{i,t-1}\left[X_t\right]\right)\left(z_{i,t} - \mathbb{E}_{i,t-1}\left[z_{i,t}\right]\right)'\right] \mathbb{E}\left[\left(z_{i,t} - \mathbb{E}_{i,t-1}\left[z_{i,t}\right]\right)\left(z_{i,t} - \mathbb{E}_{i,t-1}\left[z_{i,t}\right]\right)'\right]^{-1} \tag{37}$$

We verify this claim in the following and describe the procedure for finding the fixed point.

Observation Equation. In each period t, firms and workers on island i observe vector $z_{i,t}$, as in (17), of private and public signals. In terms of the aggregate state and error terms, island i's observation equation takes the form

$$z_{i,t} \equiv \begin{bmatrix} e_1' \\ e_1' \end{bmatrix} X_t + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \varsigma_{it} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \varepsilon_t \tag{38}$$

where e_j is defined as a column vector of length two where the j-th entry is 1 and all other entries are 0.

Forecasting and Inference. Island i's $t-1$ forecast of z_t^i is given by

$$\mathbb{E}_{i,t-1}\left[z_{i,t}\right] = \begin{bmatrix} e_1' \\ e_1' \end{bmatrix} \mathbb{E}_{i,t-1}\left[X_t\right]$$

where $\mathbb{E}_{i,t-1}\left[X_t\right]$ is island's i's $t-1$ forecast of X_t. Combining this with the law of motion (31), it follows that $\mathbb{E}_{i,t-1}\left[X_t\right] = M\mathbb{E}_{i,t-1}\left[X_{t-1}\right]$.

To form minimum mean-squared-error estimates of the current state, firms and workers on each island use the kalman filter to update their forecasts. Updating is done via

$$\mathbb{E}_{i,t}\left[X_t\right] = \mathbb{E}_{i,t-1}\left[X_t\right] + K\left(z_{i,t} - \mathbb{E}_{i,t-1}\left[z_{i,t}\right]\right), \tag{39}$$

where K is the 2×2 matrix of Kalman gains, defined in (37). Substitution of island i's $t-1$ forecast of z_t^i into (39) gives us

$$\mathbb{E}_{i,t}\left[X_t\right] = \left(I - K\begin{bmatrix} e_1' \\ e_1' \end{bmatrix}\right) M\mathbb{E}_{i,t-1}\left[X_{t-1}\right] + Kz_{i,t} \tag{40}$$

Let $\bar{\mathbb{E}}_t\left[X_t\right] \equiv \int_I \mathbb{E}_{i,t}\left[X_t\right] di$ be the time t average expectation of the current state. Aggregation over (40) implies

$$\bar{\mathbb{E}}_t\left[X_t\right] = \left(I - K\begin{bmatrix} e_1' \\ e_1' \end{bmatrix}\right) M\bar{\mathbb{E}}_{t-1}\left[X_{t-1}\right] + K\int z_{i,t}di$$

Finally, using the fact that aggregration over signals yields $\int z_{i,t}di = \begin{bmatrix} e_1' \\ e_1' \end{bmatrix} X_t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \varepsilon_t$, it follows that the average expectation evolves according to

$$\begin{aligned}\bar{\mathbb{E}}_t\left[X_t\right] &= K\begin{bmatrix} e_1' \\ e_1' \end{bmatrix} MX_{t-1} + \left(I - K\begin{bmatrix} e_1' \\ e_1' \end{bmatrix}\right) M\bar{\mathbb{E}}_{t-1}\left[X_{t-1}\right] \\ &\quad + K\begin{bmatrix} e_1' \\ e_1' \end{bmatrix} m_v v_t + K\left(\begin{bmatrix} e_1' \\ e_1' \end{bmatrix} m_\varepsilon + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)\varepsilon_t\end{aligned} \tag{41}$$

where M,m_v, m_ε are given by (32).

Characterizing Aggregate Output. Local output in each island is determined by the best-response-like condition in (15), which may be rewritten as $\log q_{i,t} = (1-\alpha) f_t + \alpha e_2'\mathbb{E}_{i,t}\left[X_t\right]$. Aggregating over this condition, we find that aggregate output must satisfy

$$\log Q_t = (1-\alpha)\bar{f}_t + \alpha e_2'\bar{\mathbb{E}}_t\left[X_t\right] \tag{42}$$

Substituting our expression for $\bar{\mathbb{E}}_t\left[X_t\right]$ from (41) into (42), gives us

$$\begin{aligned}\log Q_t &= \left[(1-\alpha)\psi + \alpha\psi\left(K_{21}+K_{22}\right)\right]\bar{f}_{t-1} + \left[\alpha M_{21} - \alpha\psi\left(K_{21}+K_{22}\right)\right]\bar{\mathbb{E}}_{t-1}\left[\bar{f}_{t-1}\right] \\ &\quad + \alpha M_{22}\bar{\mathbb{E}}_{t-1}\left[\log Q_{t-1}\right] + \left[(1-\alpha) + \alpha\left(K_{21}+K_{22}\right)\right]v_t + \alpha K_{22}\varepsilon_t\end{aligned}$$

Moreover, rearranging condition (42), we find that $\bar{\mathbb{E}}_t\left[\log Q_t\right] = \frac{1}{\alpha}\left(\log Q_t - (1-\alpha)\bar{f}_t\right)$. Finally, using this condition in the above equation gives us

$$\begin{aligned}\log Q_t &= \left[(1-\alpha)\psi + \alpha\psi\left(K_{21}+K_{22}\right) - M_{22}(1-\alpha)\right]\bar{f}_{t-1} + M_{22}\log Q_{t-1} \\ &\quad + \left[\alpha M_{21} - \alpha\psi\left(K_{21}+K_{22}\right)\right]\bar{\mathbb{E}}_{t-1}\left[\bar{f}_{t-1}\right] + \left[1-\alpha+\alpha\left(K_{21}+K_{22}\right)\right]v_t + \alpha K_{22}\varepsilon_t\end{aligned}$$

For this to coincide with the law of motion conjectured in (31) and (32) for every $(\bar{f}_{t-1}, \log Q_{t-1}, v_t, \varepsilon_t)$, it is necessary and sufficient that the coefficients $(M_{21}, M_{22}, m_{v2}, m_{\varepsilon 2})$ solve the following system:

$$\begin{aligned}
M_{21} &= (1-\alpha)\psi + \alpha\psi(K_{21}+K_{22}) - M_{22}(1-\alpha) \\
m_{v2} &= 1-\alpha+\alpha(K_{21}+K_{22}) \\
m_{\varepsilon 2} &= \alpha K_{22} \\
0 &= \alpha M_{21} - \alpha\psi(K_{21}+K_{22})
\end{aligned}$$

The unique solution to this system for $(M_{21}, M_{22}, m_{v2}, m_{\varepsilon 2})$ is the one given in the proposition. Therefore, given the kalman gains matrix K, we can uniquely identify the coefficients of the law of motion of X_t.

Kalman Filtering. Let us define the variance-covariance matrices of forecast errors as

$$\begin{aligned}
\Sigma &\equiv \mathbb{E}\left[(X_t - \mathbb{E}_{i,t-1}[X_t])(X_t - \mathbb{E}_{i,t-1}[X_t])'\right] \\
V &\equiv \mathbb{E}\left[(X_t - \mathbb{E}_{i,t}[X_t])(X_t - \mathbb{E}_{i,t}[X_t])'\right]
\end{aligned}$$

These matrices will be the same for all islands i, since their observation errors are assumed to have the same stochastic properties. Using these matrices, we may write K as the product of two components:

$$\mathbb{E}_i\left[(X_t - \mathbb{E}_{i,t-1}[X_t])(z_{i,t} - \mathbb{E}_{i,t-1}[z_{i,t}])'\right] = \Sigma \begin{bmatrix} e_1 & e_1 \end{bmatrix} + \sigma_\varepsilon^2 m_\varepsilon \begin{bmatrix} 0 & 1 \end{bmatrix}$$

and

$$\begin{aligned}
\mathbb{E}_i\left[(z_{i,t} - \mathbb{E}_{i,t-1}[z_{i,t}])(z_{i,t} - \mathbb{E}_{i,t-1}[z_{i,t}])'\right] = {} & \begin{bmatrix} e_1' \\ e_1' \end{bmatrix} \Sigma \begin{bmatrix} e_1 & e_1 \end{bmatrix} + \sigma_v^2 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \\
& + \sigma_\varepsilon^2 \left(\begin{bmatrix} e_1' \\ e_1' \end{bmatrix} m_\varepsilon \begin{bmatrix} 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} m_\varepsilon' \begin{bmatrix} e_1 & e_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right)
\end{aligned} \tag{43}$$

Therefore, K is given by

$$K = \left(\Sigma \begin{bmatrix} e_1 & e_1 \end{bmatrix} + \sigma_\varepsilon^2 m_\varepsilon \begin{bmatrix} 0 & 1 \end{bmatrix}\right)\left(\sigma_z^2\right)^{-1} \tag{44}$$

where $\sigma_z^2 \equiv \mathbb{E}_i\left[(z_{i,t} - \mathbb{E}_{i,t-1}[z_{i,t}])(z_{i,t} - \mathbb{E}_{i,t-1}[z_{i,t}])'\right]$ is given by (43).

Finally, what remains to determine is the matrix Σ. The law of motion implies that matrices Σ and V satisfy

$$\Sigma = MVM' + \sigma_v^2 m_v m_v' + \sigma_\varepsilon^2 m_\varepsilon m_\varepsilon',$$

In addition, the forecasting equation (40) imply these matrices must further satisfy

$$V = \Sigma - \left(\Sigma \begin{bmatrix} e_1 & e_1 \end{bmatrix} + \sigma_\varepsilon^2 m_\varepsilon \begin{bmatrix} 0 & 1 \end{bmatrix}\right) \left(\sigma_z^2\right)^{-1} \left(\begin{bmatrix} e_1' \\ e_1' \end{bmatrix} \Sigma + \sigma_\varepsilon^2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} m_\varepsilon'\right)$$

Combining the above two equations, we obtain the stationary *Ricatti Equation* for Σ:

$$\begin{aligned} \Sigma &= M\Sigma M' - M\left(\Sigma \begin{bmatrix} e_1 & e_1 \end{bmatrix} + \sigma_\varepsilon^2 m_\varepsilon \begin{bmatrix} 0 & 1 \end{bmatrix}\right) \left(\sigma_z^2\right)^{-1} \left(\begin{bmatrix} e_1' \\ e_1' \end{bmatrix} \Sigma + \sigma_\varepsilon^2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} m_\varepsilon'\right) M' \\ &\quad + \sigma_v^2 m_v m_v' + \sigma_\varepsilon^2 m_\varepsilon m_\varepsilon' \end{aligned} \tag{45}$$

where M, m_v, m_ε are functions of the kalman gains matrix K, and K is itself a function of Σ and m_ε. The variance-covariance matrix Σ, the kalman gains matrix K, and the law of motion matrices M, m_v, m_ε are thus obtained by solving the large non-linear system of equations described by (33)-(36), (44), and (45). This system is too complicated to allow further analytical results; we thus solve for the fixed point numerically.

Proof of Proposition 5. The planner's problem is strictly convex, guaranteeing that its solution is unique and is pinned down by its first-order conditions. The Lagrangian of this problem can be written as

$$\begin{aligned} \Lambda &= \int_{\mathcal{S}_\Omega} \left[U(Q(\Omega_t, \Omega_{t-1})) - \int_{\mathcal{S}_\omega} \tfrac{1}{1+\epsilon} S(\omega) e^{-\frac{1+\epsilon}{\theta} a} q(\omega, \Omega_{t-1})^{\frac{1+\epsilon}{\theta}} d\Omega_t(\omega) \right] d\mathcal{F}(\Omega_t | \Omega_{t-1}) \\ &\quad + \int_{\mathcal{S}_\Omega} \lambda(\Omega_t) \left[Q(\Omega_t, \Omega_{t-1})^{\frac{\rho-1}{\rho}} - \int_{\mathcal{S}_\omega} q(\omega, \Omega_{t-1})^{\frac{\rho-1}{\rho}} d\Omega_t(\omega) \right] d\mathcal{F}(\Omega_t | \Omega_{t-1}) \end{aligned}$$

The first-order conditions with respect to $Q(\Omega)$ and $q\left(\omega\right)$ are given by the following:

$$U'\left(Q(\Omega_t, \Omega_{t-1})\right) + \lambda(\Omega_t) \left(\frac{\rho-1}{\rho}\right) Q(\Omega_t, \Omega_{t-1})^{-\frac{1}{\rho}} = 0 \tag{46}$$

$$\int_{\mathcal{S}_\Omega} \left[-\frac{1}{\theta} S(\omega) e^{-\frac{1+\epsilon}{\theta} a} q(\omega, \Omega_{t-1})^{\frac{1+\epsilon}{\theta}-1} - \lambda(\Omega_t) \left(\frac{\rho-1}{\rho}\right) q\left(\omega, \Omega_{t-1}\right)^{-\frac{1}{\rho}} \right] \mathcal{F}\left(\Omega_t | \omega, \Omega_{t-1}\right) = 0 \tag{47}$$

where $\mathcal{F}\left(\Omega_t | \omega, \Omega_{t-1}\right)$ denotes the posterior about Ω_t (or, equivalently, about $\bar{f}_t$ and y_t) given ω_t. Restating condition (46) as $\lambda(\Omega_t)\left(\frac{\rho-1}{\rho}\right) = -U'\left(Q(\Omega_t, \Omega_{t-1})\right) Q(\Omega_t, \Omega_{t-1})^{\frac{1}{\rho}}$ and substituting this into condition (47), gives condition (26), which concludes the proof.

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