# Growth and Capital Flows with Risky Entrepreneurship

Damiano Sandri<sup>§</sup>
Johns Hopkins University
dsandri@jhu.edu

June 23, 2008

# PRELIMINARY AND INCOMPLETE, DO NOT QUOTE WITHOUT AUTHOR'S PERMISSION

The most updated version of the paper is available at: https://jshare.johnshopkins.edu/dsandri1/research/SandriGrowthKFlows.pdf

#### Abstract

Contrary to the prediction of standard permanent income growth models, faster growing developing countries tend to experience net capital outflows. In this paper, I show that a small open economy model featuring endogenous entrepreneurship and uninsurable idiosyncratic risk can account for the empirical evidence. I consider an economy in which agents are initially prevented from undertaking private business activities and study its dynamic evolution once this constraint is removed. The individuals with sufficiently high entrepreneurial ability start up their own firms generating a prolonged period of high growth. The key insight of the model is that since entrepreneurial activities are subject to the uninsurable risk of bankruptcy, entrepreneurs save rather than borrow not only to self-finance investment, but also to accumulate safe assets for self-insurance. This leads to the improvement of the country's net foreign asset position, thus accounting for the observed net capital outflows. The welfare costs of the lack of insurance are found to be very large especially at the beginning of the transition period.

**Keywords:** Capital flows, growth, entrepreneurship, idiosyncratic risk, heterogeneity, financial underdevelopment, saving.

JEL Classification Codes: C6, E2, F3, F43, O1

§I am grateful to my advisors Christopher Carroll and Jon Faust for insightful and stimulating discussions. I also want to thank Olivier Jeanne and participants to the SOEGW II conference for precious comments and suggestions. The *Mathematica* programs generating the numerical simulations in the paper are available from the author.

## 1 Introduction

In a well-known paper Lucas (1990) pointed out that capital flows from rich to developing countries were much smaller than suggested by the large return differentials. As discussed in Prasad, Rajan, and Subramanian (2007), the pattern of international capital flows has become even more puzzling in the last decade with low income countries actually financing more advanced economies. It is remarkable for example that in 2007 the largest borrower on international markets has been the US with a current account deficit of \$811 billion, while the major lender has been China (whose percapita income in current dollars was in 2006 still slightly less than a country as Armenia) with a current account surplus of \$238 billion.

What is even more surprising about the current patterns of capital flows is that China is a very fast growing country and according to standard models based on the Permanent Income Hypothesis (PIH) should thus be heavily borrowing in order to finance investment and smooth consumption over time. It is tempting to think about the Chinese experience as very peculiar and driven by various policy interventions, but recent empirical studies reject such a claim. Prasad, Rajan, and Subramanian (2007) show that the positive correlation between growth and net capital outflows is actually common to all developing countries. Similarly Gourinchas and Jeanne (2007) find that the capital flows predicted by the standard neoclassical growth model calibrated with country specific productivity growth are negatively correlated with the actual flows across all developing countries. Such a systematic failure of a key implication of PIH growth models therefore urges to devise an alternative theoretical framework and this is the purpose of the present paper.

At the root of the inability of standard PIH models to account for the empirical evidence lies the behavior of the saving rate: the implication of the theory is that the prospect of higher future income by increasing the permanent income relative to current income should boost consumption and reduce the saving rate. This is however totally counterfactual: as pointed out already by Modigliani (1970) and further documented by many others among who Carroll and Weil (1994), Attanasio, Picci, and Scorcu (2000) and Rodrik (2000) saving and growth are actually positively

<sup>&</sup>lt;sup>1</sup>Even without considering China and the US, recent years have witnessed a relative reduction in the GDP per capita of countries with current account surpluses and an increase for countries with current account deficits.

correlated.

A possible explanation of the empirical findings was suggested by Modigliani (1986) considering an overlapping generation model based on the Life Cycle Hypothesis: by increasing the life time income of the younger generations relative to the older ones, growth boosts the amount of saving of the young workers relative to the dissaving of the retirees, thus leading to an increase in the aggregate saving rate. This conjecture relies however on the assumption that growth leads to a higher life time income of the newly born generations but does not change the expected growth rate of income, which was proven counterfactual by Carroll and Summers (1991). Simulation results show that the boost in consumption derived by higher income growth for a given generation can easily outweigh the aggregate effect suggested by Modigliani.

Another attempt to explain the positive correlation between growth and saving has been to consider models with habits in consumption. Carroll, Overland, and Weil (2000) and Carroll (2000) have demonstrated that habits by slowing down the rate of increase in consumption following a productivity growth shock are potentially able to generate an increase in saving. The attractiveness of this approach has however been largely diminished by a large empirical literature which has rejected habits as a structural element of individual utility by using household data (Naik and Moore (1996), Meghir and Weber (1996), Dynan (2000) and Flavin and Nakagawa (2004)).<sup>2</sup>

A further reason of dissatisfaction with the current understanding of the relation between growth and saving is that it is rather uninformative about the most recent finding that also the current account appears to be positively correlated with growth, i.e. that the correlation between saving and growth is not only positive, but it is even stronger than the one between investment and growth.<sup>3</sup> Overlapping generation models or habits-augmented growth models being based on the Fisherian separation between consumption and investment do not have particularly interesting implications for the current account per se. It would be instead preferable to work with a framework which keeps intertwined saving and investment decisions, thus leading to a more meaningful discussion about the current account and international capital flows.

<sup>&</sup>lt;sup>2</sup>Another problematic issue with habits is that their implications for the saving rate are largely dependent on the assumption about the uncertainty surrounding people's expectations about future growth. For example, if we assume that agents are able to foresee growth patterns the deriving boost in consumption due to the human wealth effect is often predominant notwithstanding the strength of the habit parameter.

<sup>&</sup>lt;sup>3</sup>The current account is equal to the difference between national saving and investment.

To address this need and account for the positive correlation between growth and capital outflows, I design a small open economy model with incomplete financial markets populated by agents with heterogeneous entrepreneurial ability. The key assumption of the model is that entrepreneurial activity is subject to the uninsurable idiosyncratic risk of bankruptcy. I consider an economy in which private entrepreneurship is initially banned and study its dynamics once this constraint is progressively removed:<sup>4</sup> The most talented entrepreneurs start their own business activities which due to the bankruptcy risk and the inability of financial markets to provide risk sharing opportunities have to be self-financed. Contrary to the standard perfect foresight model which would predict entrepreneurs borrowing to scale up their firms and boost consumption, my model is based on the idea that risky, but potentially lucrative investment opportunities give the incentive to restrain today consumption in order to have the financial resources to seize them. Furthermore, the model features an optimal portfolio condition requiring a balanced allocation between capital investment and safe assets, so that entrepreneurs will not allocated all their savings to investment, but hold an increasing share in safe assets. This is the crucial mechanisms which allows the model to generate large capital outflows during a period of rapid growth.

I choose the model parameters with a combination of calibration and structural estimation. The simulation exercise wants to explore the dynamics of an initially poor economy with no entrepreneurship as it converges to the stochastic steady state under private entrepreneurship. Under the assumption that the US has already achieved such a stochastic equilibrium, I perform the structural estimation by matching relevant US moments from the 2004 Survey of Consumer Finance: in particular, I consider the mean wealth and income of entrepreneurs relative to workers and the portfolio share of entrepreneurs invested in businesses. The simulation results show that the transition to private entrepreneurship generates a prolonged period of high growth and current account surpluses, thus successfully accounting for the positive correlation between growth and capital outflows. Furthermore the results are quantitatively very close to the recent Chinese experience. I find the results largely unsensitive to a different calibration of the estimated parameters.

<sup>&</sup>lt;sup>4</sup>This exercise was inspired by the process of economic reforms followed by China with the gradual transition from publicly owned firms to private businesses. However, the key results of the model will be largely unchanged if we were to consider an economy with private entrepreneurship facing an improvement in investment opportunities.

I finally evaluate the welfare costs of the inability of financial markets to allow for risk sharing. These are found to be around 8% of annual consumption for an economy which has already completed its transition to private entrepreneurship, with entrepreneurs actually worse off under perfect risk sharing since in the lack of coordination would excessively scale up their companies thus bidding up wages. Even larger, in the order of 18% of annual consumption, are the welfare costs for an economy at the beginning of the transition process since the pace of investment and wage increase is considerably slowed down.

The structure of the paper is the following. Section 2 discusses related literature and section 3 reviews the empirical evidence on the relation between growth, saving and investment. Section 4 describes the model which is calibrated and simulated in section 5. Section 6 evaluates the welfare costs due to the lack of risk sharing. The main findings are summarized in the conclusion.

## 2 Related Literature

The paper contributes to the recent literature emphasizing the role of financial market imperfections in the emergence of current global imbalances. Mendoza, Quadrini, and Rios-Rull (2007a) also consider a setting with exclusively idiosyncratic shocks and show in a two country model that financial globalization produces capital outflows from the country with less developed domestic financial markets. The key idea is that under autarky the country with lower risk sharing ability would have a higher demand for precautionary savings and thus a lower interest rate. With financial globalization the agents in the financially underdeveloped country witness an increase in the interest rate (which is equalized internationally) and thus desire to increase savings by purchasing foreign assets. Caballero, Farhi, and Gourinchas (2008) also attribute the current global imbalances to the heterogeneity in domestic financial development across countries and in particular to different abilities to supply financial assets. They show that financial integration or a crash in the ability to supply financial assets generate large capital outflows from the less financially developed economy. I share with these papers the idea that financial imperfections are important determinants of global imbalances, but I put at the center of my analysis the process of growth acceleration experienced by emerging markets instead than the process of financial integration: I believe that the former

issue is really the one deserving more attention since it is dramatically opposite to the predictions of standard PIH growth models.

The general structure of the model, involving a distribution of agents heterogenous in entrepreneurial ability deciding between being workers or entrepreneurs is inspired by the work of Quadrini (2000) and Cagetti and De Nardi (2006) aiming to account for the US wealth distribution. Buera (2006) also uses a similar setting to understand the extent to which agents are able to overcome by saving credit constraints which would prevent in a static setting the creation of profitable businesses. Consistently with the approach of my model which emphasizes the savings of entrepreneurs, these studies document using US data that a disproportionate share of wealth is hold by entrepreneurs, that they have higher saving rates than workers and that they save more especially at the time of entry. Similar findings are also discussed by Gentry and Hubbard (2004), while Campbell and Watanabe (2001) present evidence that both in the US and in Japan the wealthiest 20% of households are responsible for more than two thirds of aggregate saving.

The structure of the model is also similar to the one of Buera and Shin (2008) who study the role of financial constraints in slowing down the process of development in a closed economy with initial wealth misallocation. A key difference is that entrepreneurial activities are not subject in their setting to risk and borrowing is simply limited by an exogenous constraint. The lack of insurance against idiosyncratic shocks is instead the crucial element in my model to rule out borrowing and especially to generate the positive correlation between growth and the accumulation of safe assets.

The core structure of consumption decision is built upon the insights of the buffer stock consumption literature: the pioneering work of Zeldes (1989), further developed by Deaton (1991) and Carroll (1992) has shown that the presence of uncertainty can drastically change consumption and saving behavior. Particularly important for the purpose of the paper is the implication that a risk averse impatient consumer facing uncertain labor income has a stable target level of wealth, due to the desire to hold a certain amount of buffer stock savings to sustain consumption in case of negative shocks. This is the underlying reason for which the model features a well defined net foreign asset position and which drives the accumulation of safe assets holdings as entrepreneurs invest in their companies.

The idea of considering idiosyncratic shocks to understand macro dynamics relates my work to

various studies who have assessed the importance of idiosyncratic shocks on aggregate variables. Aiyagari (1994) found that idiosyncratic labor income uncertainty boosts aggregate capital, while more recently Angeletos (2007) has shown that idiosyncratic investment risk may have the opposite effect. Idiosyncratic risk has small effects in the work of Huggett (1997), studying the growth dynamics of a closed economy, and Krusell and Smith (1998) considering the extent to which wealth heterogeneity can impact the behavior of macro aggregates at the stationary stochastic equilibrium. I show that in a small open economy also a very low uninsurable risk of bankruptcy can actually lead to significant quantitative differences for growth dynamics and in the stochastic steady state.

The modeling of the investment and saving decision for the entrepreneur is based on the insights from the corporate finance literature. The seminal work of Modigliani and Miller (1958), proving under complete financial markets the irrelevance of the financing structure of the firm, led for many years scholars to study investment decisions as independent from financial factors. Fazzari, Hubbard, and Petersen (1988) have been among the first to find empirical evidence undermining this approach by detecting a positive relationship between internal finance and investment.<sup>5</sup> Similar results have been found more recently by Love and Zicchino (2006) who show that the correlation is stronger among financially underdeveloped countries. This is consistent with my model since entrepreneurs facing the risk of bankruptcy have to use their own resources to finance investment.

# 3 Empirical Evidence

It is a well established fact that growth is positively correlated with the saving rate. Some preliminary cross sectional evidence was already provided long ago by Houthakker (1961) and Modigliani (1970) who pointed out that faster growing countries tend to have higher saving rates. Strong positive correlation between saving and growth is also found by Maddison (1992) on data going back to 1870. These results have been initially interpreted as supporting the standard growth models a b b Solow or Rebelo in which higher saving rates lead to more investment and growth. This direction of causality has been however disputed by a large body of research focusing on the determinants

<sup>&</sup>lt;sup>5</sup>See Hubbard (1998) for an excellent survey of the literature on capital market imperfections and investment.

of saving which has identified growth as a key driving factor: Bosworth (1993) looking at OECD countries during the period 1960-1980 concludes that growth is the most important determinant for saving, Edwards (1996) finds similar results also for developing countries over the period 1970-92 and Loayza, Schmidt-Hebbel, and Serven (2000) in a comprehensive World Bank study confirm using panel instrumental-variable techniques that higher growth leads to higher saving.

The most compelling evidence about the sense of causality has come however from studies focusing on the time variation of growth and saving within countries. Carroll and Weil (1994) perform Granger causality tests and find that growth positively Granger causes saving not only when considering country aggregate variables, but also on household level data. Similar results are found by Attanasio, Picci, and Scorcu (2000) on a larger country dataset and under several alternative econometric techniques. Particularly convincing is also the evidence provided by Rodrik (2000) who shows that while periods of sustained increase in saving rates do not lead systematically to higher growth, transitions into higher growth are followed by persistent higher saving rates.

Recent empirical work has documented another interesting related regularity, such as that the correlation between saving and growth is even higher than the correlation between investment and growth. Gourinchas and Jeanne (2007), considering the period 1980-2000, have indeed shown that faster growing developing countries have experienced larger net capital outflows. Similarly, Prasad, Rajan, and Subramanian (2007) confirm that for non industrial countries growth tends to be positively correlated with the current account also when looking at different time samples. China is of course a clear example of a faster growing country exporting capital to the rest of the world, but its experience is far from unique. While there is a certain tendency to explain the high Chinese saving rate and large current account surplus through various forms of government intervention, first and for most with exchange rate manipulation, the above studies clearly show that many other countries have experienced similar dynamics.

An effective way to get a quick grasp of the empirical evidence, is to look at the time series behavior of saving, investment and the current account of countries during high growth periods.<sup>6</sup> More specifically I identify the year in which a country experiences a growth acceleration episode using the criteria proposed by Hausmann, Pritchett, and Rodrik (2005): GDP growth has to be

 $<sup>^6\</sup>overline{\text{A}}$  similar exercise is also included in Prasad, Rajan, and Subramanian (2007).

rapid (at least 3% over the next seven years), GDP growth has to accelerate (the growth rate over the next seven years has to be at least 2% higher than over the previous seven years) and growth should not be driven by a recovery phase out of a recession (output seven years later has to be at least as large as the pre-growth peak). In case of multiple years for a given country satisfying the above criteria (and less than 5 years apart), the relevant episode is selected by considering the year with the highest explanatory power as the breaking point of a spline regression of growth over time. Using data from the World Development Indicators on developing countries between 1960 and 2004, I detect 34 growth acceleration episodes and I consider the time series dynamics of the relevant aggregate variables in the seven years before and after the episode.<sup>7</sup>

A clear case confirming that the positive correlation between growth, saving and the current account is not a peculiar Chinese feature, is the one of Korea. According to the above criteria, Korea experienced a growth acceleration episode in 1983 and as shown in figure 1 this has come together with a steep increase in the saving rate. The increase has been much larger than the rise in the investment rate, thus leading to a sharp improvement in the current account displayed on the right side of the figure. I plot both the current account and the difference between saving and investment in order to evaluate the importance of possible statistical discrepancies.

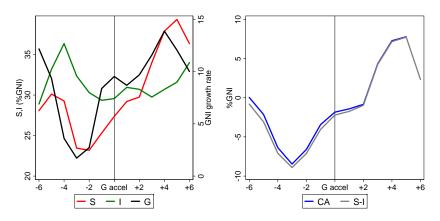


Figure 1: Korea (1983)

Other countries too experienced similar trends in saving rates and the current account during periods of higher growth, among which Pakistan in 1976 and of Uruguay in 1976, respectively in figure 2 and 3.

<sup>&</sup>lt;sup>7</sup>See appendix A for a detailed description of the data.

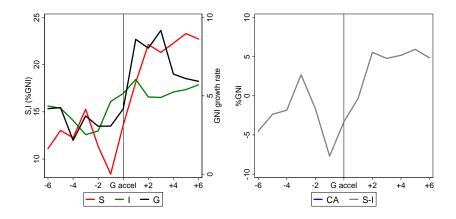


Figure 2: Pakistan (1976)

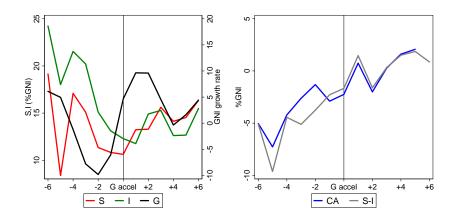


Figure 3: Uruguay (1976)

In order to summarize the dynamics of all the detected acceleration episodes, figure 4 shows the mean and median across countries of saving, investment and growth confirming the stronger increase in saving relative to investment.<sup>8</sup> The current account mean and median dynamics is shown in figure 5.

<sup>&</sup>lt;sup>8</sup>I am considering all the countries with no missing data on saving, investment, growth and the current account for the 14 years around the growth acceleration year.

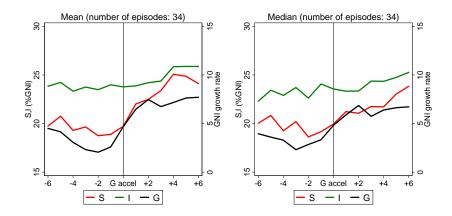


Figure 4: Saving, investment and growth (all growth episodes)

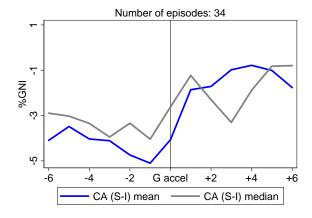


Figure 5: Current account and saving less investment (all growth episodes)

# 4 The Model

I consider a small open economy populated by infinite-lived impatient individuals with heterogenous endowments of market resources (or wealth) m and entrepreneurial ability  $\eta$ . Agents can decide to earn their living as workers or entrepreneurs and for the sake of simplicity I assume that the choice of entering entrepreneurship is irreversible. The optimization problems of the workers and entrepreneurs are discussed in details in the following two subsections. Agents maximize the

<sup>&</sup>lt;sup>9</sup>The impatience condition of the model corresponds to  $\beta R < 1$  and it is needed to avoid degenerate solutions under precautionary motives as discussed in section 4.5.

<sup>&</sup>lt;sup>10</sup>This assumption simplifies significantly the numerical solution of the consumption functions, but the key implications of the model would not change if we were to relax it.

expected present discounted utility of consumption:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\rho}}{1-\rho} \tag{1}$$

where  $\beta$  is the intertemporal discount factor and  $\rho$  is the relative risk aversion coefficient of the CRRA utility function. The economy features two sectors: a traditional farming sector in which workers earn as self-employed the exogenous wage  $\Omega$  and the entrepreneurial sector in which entrepreneurs employ workers paying the endogenous wage  $\Omega_t$ . Financial markets are incomplete in so far as they do not allow borrowing against future labor income and do not provide risk sharing instruments.<sup>11</sup> I see this as a plausible characterization of underdeveloped financial markets in emerging economies.

#### 4.1 Workers

At each time workers decide whether to earn their income as self-employed farmers or to offer their labor unit to entrepreneurs. They also choose in each period how much to consume and save and are allowed to invest their savings only in a risk free asset a earning the return R, i.e. the exogenous world interest rate. Formally they solve the following optimization problem expressed in Bellman form:

$$V_{t}^{w}(m_{t}) = \max_{a_{t+1}^{w} \geq 0} \{ u(\overline{m_{t} - a_{t+1}^{w}}) + \overbrace{\beta V_{t+1}^{w}(m_{t+1}^{w})}^{v_{t+1}^{w}(a_{t+1}^{w})} \}$$

$$s.t.$$

$$m_{t+1}^{w} = Ra_{t+1}^{w} + \bar{\Omega}_{t+1}$$

$$(2)$$

where  $\bar{\Omega}_{t+1} = \max\{\underline{\Omega}, \Omega_{t+1}\}$ . Note that we restrict the end of period savings to be positive  $a_{t+1}^w \geq 0$ , since we rule out the possibility of borrowing against future labor income. The first order condition simply involves equating in the unconstrained region the marginal utility of today consumption to

<sup>&</sup>lt;sup>11</sup>These market imperfections can be derived as the outcome of several different forms of information asymmetries and poor legal enforcement. Appendix B presents one of them.

the marginal utility of saving:

$$\frac{\partial u(c_t^w)}{\partial c_t^w} = u^c(c_t^w) \ge \mathfrak{v}_{t+1}^{w,a}(a_{t+1}^w) = \frac{\partial \mathfrak{v}_{t+1}^w(a_{t+1}^w)}{a_{t+1}^w}$$
(3)

#### 4.2 Entrepreneurs

Entrepreneurs run their own business activities which similarly to Lucas (1978) and Buera (2006) is based on a nested Cobb Douglas production function in entrepreneurial ability, invested capital and employed labor:

$$\hat{f}(\eta, k_t, l_t) = \eta^{\nu} (k_t^{\alpha} l_t^{1-\alpha})^{1-\nu} \qquad 0 < \alpha < 1, \quad 0 < \nu < 1$$
(4)

In order to limit the number of state variables in the dynamic maximization problem, I rule out adjustment costs to capital or labor so that the optimal input composition is freely chosen at each time.<sup>12</sup> Solving the maximization problem:

$$\max_{l_t} \left\{ \eta^{\nu} (k_t^{\alpha} l_t^{1-\alpha})^{1-\nu} - \Omega_t l_t \right\} \tag{5}$$

we obtain that the optimal labor demand is given by: $^{13}$ 

$$l_t^* = \left(\frac{(1-\nu)(1-\alpha)\eta^{\nu}k_t^{\alpha(1-\nu)}}{\Omega_t}\right)^{\frac{1}{\nu(1-\alpha)+\alpha}}$$
(6)

and thus the production function can be more compactly written as:

$$\Upsilon_t \eta^{1-\phi} k_t^{\phi} \tag{7}$$

where 
$$\phi = \frac{(1-\nu)\alpha}{1-(1-\nu)(1-\alpha)}$$
 and  $\Upsilon_t = \left(\frac{(1-\nu)(1-\alpha)}{\Omega_t}\right)^{\frac{(1-\nu)(1-\alpha)}{1-(1-\nu)(1-\alpha)}} (1-(1-\nu)(1-\alpha)).$ 

The crucial element of the model is to assume that the entrepreneurial activity is subject to

<sup>&</sup>lt;sup>12</sup>One of the key justifications for the introduction of adjustment costs is to avoid sudden large variations in the stock of capital. As shown in section 4.5, the presence of bankruptcy shocks automatically leads to smooth investment dynamics.

<sup>&</sup>lt;sup>13</sup>For simplicity I do not impose  $l_t^*$  to be an integer, so that a worker may be supplying his labor endowment to various entrepreneurs in the same period.

the risk of bankruptcy: with probability bethe company is hit by a shock causing the total loss of the invested capital. 14 Since the CRRA utility function leads to infinite negative utility for zero consumption, the entrepreneur will have to hold always some safe assets in order to be able to finance positive consumption (and investment) even if the company goes suddenly bust.

Similarly to the case of the worker, the entrepreneurial problem consists in choosing at each time how much to consume and how much to save. The entrepreneur has however also to decide how to allocate savings between the risk free asset a, earning a return R, and the invested capital k with a potential rate of return  $\Re$  directly proportional to the entrepreneurial ability and the inverse of the invested capital. Formally the entrepreneur solves the following problem:

$$V_{t}^{e}(m_{t}, \eta) = \max_{\substack{a_{t+1}^{e}, k_{t+1} \\ t+1}} \left\{ u(m_{t} - a_{t+1}^{e} - k_{t+1}) + \beta \left[ (1 - \flat)V_{t+1}^{e}(m_{t+1}^{e}, \eta) + \flat V_{t+1}^{e}(m_{t+1}^{\flat}, \eta) \right] \right\}$$

$$s.t.$$

$$m_{t+1}^{e} = Ra_{t+1}^{e} + \Upsilon_{t+1}\eta^{1-\phi}k_{t+1}^{\phi} + (1 - \delta)k_{t+1}$$

$$= Ra_{t+1}^{e} + \left( 1 - \delta + \Upsilon_{t+1}\phi \left( \frac{\eta}{k_{t+1}} \right)^{1-\phi} \right) k_{t+1} + \eta \Upsilon_{t+1}(1 - \phi) \left( \frac{k_{t+1}}{\eta} \right)^{\phi}$$

$$m_{t+1}^{\flat} = Ra_{t+1}^{e}$$

where  $0 < \delta < 1$  is the capital depreciation rate,  $\Omega_{t+1}^e$  is the entrepreneur wage for unit of entrepreneurial ability and  $m_{t+1}^e$  and  $m_{t+1}^b$  are the beginning of next period market resources for the entrepreneur respectively with a successful and bankrupt business. There are two optimality conditions:<sup>15</sup>

$$\frac{\partial \mathfrak{v}_{t+1}^{e}(a_{t+1}^{e}, k_{t+1})}{\partial k_{t+1}} = \mathfrak{v}_{t+1}^{e,k}(a_{t+1}^{e}, k_{t+1}) = \mathfrak{v}_{t+1}^{e,a}(a_{t+1}^{e}, k_{t+1}) = \frac{\partial \mathfrak{v}_{t+1}^{e,a}(a_{t+1}^{e}, k_{t+1})}{\partial a_{t+1}^{e}} \qquad (9)$$

$$u^{c}(c_{t}) = \mathfrak{v}_{t+1}^{e,a}(a_{t+1}^{e}, k_{t+1}) \qquad (10)$$

$$u^{c}(c_{t}) = \mathfrak{v}_{t+1}^{e,a}(a_{t+1}^{e}, k_{t+1}) \tag{10}$$

<sup>&</sup>lt;sup>14</sup>We could have consider the case in which the bankruptcy shock only leads to the loss of a proportion  $\chi$ of capital. In this case the entrepreneur with positive market resources may have the incentive to contract some debt to speed up investment. The amount of borrowing would be constrained by  $(1-\chi)k+a\geq 0$ , i.e. the amount of market resources in case of bankruptcy cannot be negative. The process of transition to entrepreneurship will thus likely be characterized by a current account deficit in the short run followed by capital outflows in the medium term.

<sup>&</sup>lt;sup>15</sup>The absence of exogenous borrowing constraints rules out corner solutions and thus the fist order conditions hold with equality.

Equation 9 is the arbitrage condition which requires equality between the expected marginal value of investing in the safe asset and in the firm's capital. This is the key equation of the model which requires under idiosyncratic shocks both invested capital and safe asset holdings to increase as the entrepreneur becomes wealthier. The second optimality condition 10 equates, as in the case of the worker, the marginal utility of today consumption with the expected marginal utility of savings.

### 4.3 The net foreign asset position

The net foreign asset position of the country, A, is given by the sum of financial assets hold by workers and entrepreneurs:

$$A_t = \int_{i \in W_t} a_{i,t}^w + \int_{i \in E_t} a_{i,t}^e$$

where  $W_t$  and  $E_t$  are respectively the set of workers and entrepreneurs at time t. Note that while the net foreign asset position of small open economy models under perfect foresight is indeterminate, it is here well defined as an implication of the existence of a target level of wealth under precautionary motives as discussed in section 4.5.<sup>16</sup> Given that the current account, and thus the amount of net capital flows, is by definition equal to the change in the net foreign asset position, the key purpose of the present analysis thus boils down to understand the relationship between growth and the dynamics of A.

# 4.4 Competitive equilibrium

Let  $\epsilon_i$  be the employment choice of agent i. Given the world interest rate R and the initial distribution of wealth and employment choices  $\mathbb{H}_0(m, \epsilon)$ , a competitive equilibrium is given by the sequence of distributions  $\{\mathbb{H}_t(m, \epsilon)\}_{t=0}^{\infty}$ , policy functions  $\{c_t(m_t, \eta, \epsilon_t), a_{t+1}(m_t, \eta, \epsilon_t), k_{t+1}(m_t, \eta, \epsilon_t), l_{t+1}(m_t, \eta, \epsilon_t)\}_{t=0}^{\infty}$ , value functions  $\{V_t(m_t, \eta, \epsilon_t)\}_{t=0}^{\infty}$  and wage rates  $\{\Omega_t\}_{t=0}^{\infty}$  such that:

- given  $\{\Omega_t\}_{t=0}^{\infty}$ , the policy rules solve the worker and entrepreneur's problems
- the labor and capital markets clear for all  $t \geq 0$

<sup>&</sup>lt;sup>16</sup>The net foreign asset position of a small open economy model under perfect foresight is also not stationary. Schmitt-Grohé and Uribe (2003) discuss various *ad hoc* mechanisms to induce stationarity.

•  $\{\mathbb{H}_t(m,\epsilon)\}_{t=0}^{\infty}$  is consistent with  $\mathbb{H}_0(m,\epsilon)$ , the policy rules and the idiosyncratic shocks

Note that while the interest rate is exogenous, the wage is endogenous and plays an important role in the evaluation of the welfare costs of the lack of insurance. The algorithm used to solve for the equilibrium dynamics is described in appendix D.

### 4.5 Policy functions

The main insights of the model are best understood by looking at the policy functions which are solved numerically as described in appendix C. The left plot of figure 6 shows the consumption function for the worker  $(c^w)$  and the entrepreneur  $(c^e)$  under incomplete financial markets. The consumption function for the impatient and borrowing constrained worker is concave and the wealth level converges towards  $\bar{m}^w$ . As typical of a setting with precautionary motives and labor income uncertainty, the consumption function for the entrepreneur is also concave in the level of market resources and there exists a target level of wealth  $\bar{m}^e$  around which the stochastic steady state will fluctuate.<sup>17</sup>

The underlying reason for the existence of the target level is given by the tension between precautionary motives and impatience: uncertainty over future labor income provides an incentive to accumulate precautionary savings which would lead to permanent wealth growth. Since the precautionary motive goes to infinity as m shrinks to zero and vanishes as wealth goes to infinity, there has to exist a level of wealth at which the precautionary desire to build up wealth is exactly counterbalanced by the impatience motive. This is what I refer to as the target level of wealth and the stochastic steady state would involve stationary movements around it.

The left plot of figure 6 shows that the target level of wealth for the entrepreneur is larger than for the worker. In particular it is crucial to note the transition dynamics of a worker who decides to become entrepreneur: from the initial wealth  $\bar{m}^w$  the new entrepreneur wants to restrain consumption in order to have more resources to scale up the firm quickly enough. This is in stark contrast with the standard model under complete financial markets whose consumption functions are plotted on the right side of the figure.<sup>18</sup> In this case the entrepreneur would immediately borrow

<sup>&</sup>lt;sup>17</sup> A formal proof of the existence of the target level of wealth is provided by Carroll (2004).

<sup>&</sup>lt;sup>18</sup>In the absence of uninsurable risk the impatient condition would lead wealth to asymptote towards its

as much as needed to bring the firm at the optimal scale equating the marginal return to capital to the interest rate on debt:

$$\hat{k} = \eta \left( \frac{\Upsilon \phi}{\mathsf{R} - 1 + \delta} \right)^{\frac{1}{1 - \phi}}, \quad \hat{\Omega} = \Upsilon (1 - \phi) \left( \frac{\hat{k}}{\eta} \right)^{\phi} \tag{11}$$

and the consumption functions will be analytically defined as:

$$\hat{c}_t^w = \frac{\mathsf{R} - (\mathsf{R}\beta)^{1/\rho}}{\mathsf{R}} \left( m_t + \frac{\hat{\Omega}}{\mathsf{R} - 1} \right), \quad \hat{c}_t^e = \frac{\mathsf{R} - (\mathsf{R}\beta)^{1/\rho}}{\mathsf{R}} \left( m_t + \frac{\eta \hat{\Omega}}{\mathsf{R} - 1} \right)$$
(12)

As soon as the worker is allowed to become entrepreneur, the higher present discounted value of labor income would lead to a sudden increase in consumption and there would be no incentive to accumulate more wealth since the return to invested capital is immediately equalized to the interest rate.

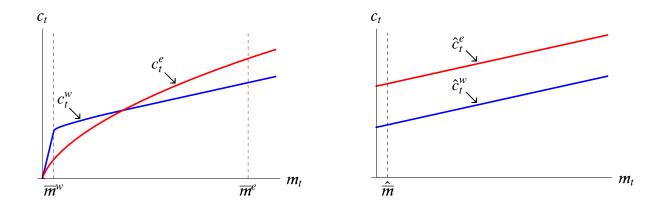


Figure 6: Consumption functions under incomplete (left) and complete (right) financial markets

Let me also point out that the decision of becoming an entrepreneur is under incomplete markets not only depending on the level of entrepreneurial ability, but also on wealth. This is illustrated in figure 8 which plots the value functions under complete and incomplete markets. In the latter case we observe that while for a relative rich agent becoming an entrepreneur may be the best

minimum bound, given by the negative of the present discounted value of future income. To prevent this degenerate behavior I assume for the perfect foresight version of the model  $R\beta = 1$ , so that each wealth level is stable.

career choice, this would no longer be true if the same individual were endowed with lower wealth. The reason for this is that becoming an entrepreneur involves giving up the worker wage and thus it is going to be profitable only if the agent can scale up rapidly enough her firm. As previously explained, due to the presence of bankruptcy risk the agent will have to rely on her own savings to finance capital investment and this makes entrepreneurship more attractive to wealthier individuals. The level of wealth above which agents find convenient to become entrepreneurs is labeled as  $m^*$  and is directly proportional to entrepreneurial ability, since individuals with higher  $\eta$  can earn a better return on the invested capital.

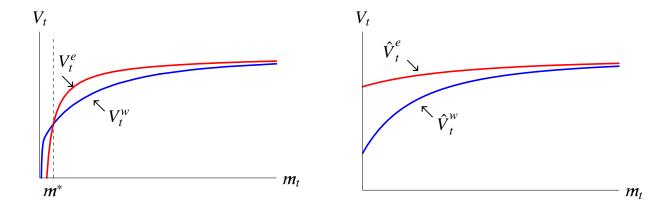


Figure 7: Value functions under incomplete (left) and complete (right) financial markets

Finally figure 8 shows the saving and investment functions. As previously discussed the worker is allowed to invest her savings exclusively in the risk free asset  $a^w$ , while the entrepreneur optimally decides between invested capital  $k^e$  and the safe asset  $a^e$ . The key result of the model is that the target wealth level of the entrepreneur  $\bar{m}^e$  implies a higher target level of safe assets  $\bar{a}^e$  than the one of the worker  $\bar{a}^w$ . The transition to entrepreneurial activity by increasing the return to capital provides the incentive to wealth accumulation which given the risk associated with the entrepreneurial activity also implies an increase in the safe asset holdings driven by the optimal portfolio condition (9). Under complete markets the entrepreneur would instead borrow to finance investment and the increase in consumption.

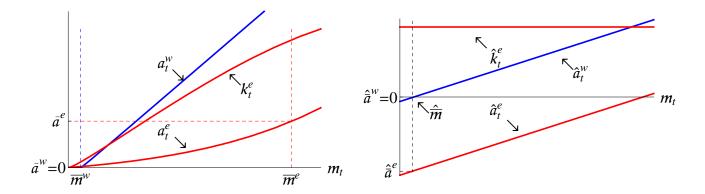


Figure 8: Investment functions under incomplete (left) and complete (right) financial markets

While it is easy to see that the inclusion of idiosyncratic shocks into the model leads to very different predictions that under perfect foresight, it is harder to understand if similar results would be obtained by exclusively considering borrowing constraints. As formally shown by Carroll and Kimball (2001), both uncertainty and liquidity constraints are able to generate concavity in the consumption function and this leads to various similar implications. For example the entry decision would still be also a function of wealth and more importantly the entrepreneur may still want to restrain initial consumption to finance investment. The key difference is however that in the absence of uncertainty there is no reason for the entrepreneur to hold any safe asset: the transition to entrepreneurship could thus be characterized by higher saving rates, but all the available financial resources will be invested in the firm until to the point at which the return to capital is equal to the interest rate. The current account being the difference between investment and saving would thus possibly not go into deficit, but also not turn to positive.

Under uncertainty, borrowing constraints are however potentially able to boost the precautionary motive thus strengthening the model results. I want however to dispense from them in order to highlight that the crucial financial market imperfection for the results of the paper is the inability of insuring against idiosyncratic shocks and not limited credit per se.

## 5 Calibration and simulation

#### 5.1 Calibration and structural estimation

Some of the model parameters are easily chosen since rather uncontroversial and commonly used in the literature. I assume  $\rho=2$ , R=1.04,  $\beta=0.96$  and  $\delta=0.06$ . I then impose a capital share ratio of 0.33 so that the inner capital share  $\alpha$  of the entrepreneurial production technology becomes exclusively a function of  $\nu$ . The remaining parameters are the production function parameter  $\nu$ , the entrepreneurial ability  $\eta$  and the bankruptcy risk  $\flat$ .

Since the simulation exercise I will consider aims to describe the convergence dynamics of an economy in which initially all agents are constrained to be workers towards the stochastic equilibrium with private entrepreneurship, I want to base the estimation on a country which is likely to have already achieved such a stochastic equilibrium. I therefore structurally estimate the remaining parameters using US data from the 2004 Survey of Consumer Finance (SCF). The sensitivity analysis in section 5.3 shows that the key model results are however preserved under alternative parameter specifications.

I decide to match the moments for which the model has very clear predictions such as the ratio of the average wealth of entrepreneurs to the average wealth of workers, the ratio of the average income of entrepreneurs to the average income of workers and the share of entrepreneurs' wealth invested in private businesses. In order to make the model fitting more successfully the US data I include two additional features.<sup>19</sup> It is for example important to consider that side to many private entrepreneurs the US has also big corporations owned by diversified shareholders and therefore I add to the model a financially unconstrained corporate sector in which idiosyncratic shocks are perfectly insured. To pin down the relative size of the entrepreneurial and corporate sector I also match the average number of workers employed by each entrepreneur which is going to allow for the identification of the equilibrium wage.<sup>20</sup> Moreover, in order to quantitatively better characterize the wealth holdings of workers, I assume that they are subject to a 5.5% unemployment risk which

<sup>&</sup>lt;sup>19</sup>The version of the model used for the structural estimation is presented in appendix E.

<sup>&</sup>lt;sup>20</sup>Note that in a partial equilibrium setting in which the wage paid by the entrepreneurs is exogenous, the model is homogenous of degree 1 in the individual specific wage. This is however no longer true in the present general equilibrium setting in which the wage earned by the workers have to equal the wage paid by the entrepreneurs.

corresponds to the US unemployment rate in 2004. The share of entrepreneurs in the population is assumed to equal 11.5% which corresponds to the proportion in the SCF data.

The calibrated and structurally estimated parameters are reported in table 1. The bankruptcy risk may appear at a first glance too small but it has to be noted that the we are considering the risk of a very dire shock involving the total loss of the invested capital.

Parameter description	Parameter label	Value	Standard errors
CRRA coefficient	ρ	2	calibrated
Intertemporal discount	eta	0.96	calibrated
World interest rate	R	1.04	calibrated
Depreciation rate	$\delta$	0.06	calibrated
Bankruptcy probability	þ	0.0016	()
Entrepreneurial ability	$\eta$	9	()
Inner capital share	$\alpha$	$.3/(1-\nu)$	()
Outer entrepreneurial share	$\nu$	.3	()

Table 1: Model parameters

#### 5.2 Simulation

I now want to consider the dynamics of an economy as it progressively moves to the equilibrium under private entrepreneurship.<sup>21</sup> I initialize the model to a condition in which agents are prevented from starting private business and are therefore forced to work as self-employed farmers. The initial wage rate is chosen so that the aggregate GDP per capita is around 50 times smaller that the GDP per capita computed for the model estimated on the US data. This roughly corresponds to the gap in PPP GDP per capita between the US in 2004 and China in 1978 at the time of the first economic reforms, the scope being to see if the model is able to somewhat match the Chinese experience also quantitatively.

I then progressively allow agents to start entrepreneurial activities at a constant rate over a period of 50 years. As for any setting featuring forward looking optimal behavior, it is crucial to clearly specify which variables are forecasted by the agents. I assume that each agent can foresee the optimal wage pattern, but let agents be unaware about the possibility of becoming entrepreneurs. This simplifies a bit the numerical solution procedure and plays against the scope of the model to

 $<sup>^{21}\</sup>mathrm{The}$  simulation algorithm is described in appendix D.

generate capital outflows: if agents could foresee their entry they may want to save in advance, thus boosting the aggregate saving rate in the first transition periods. While this can certainly be one factor behind the surge in saving experienced by fast growing countries, I want to leave it aside to show that it is not a crucial element.<sup>22</sup>

The model aggregate dynamics are shown in figure 9. The economy enters a period of prolonged high growth reaching a 10% growth rate of GDP driven by the investment of private entrepreneurs. Both the saving and investment rates increase but the growth of the former is faster as can be seen from the fact that the current account improves. The wage rate is initially flat as the entrepreneurial sector progressively absorbs workers from the traditional farming sector. The current account grows until around 10% of GDP. The transition dynamics continues well after the time at which private entrepreneurship has been fully permitted, since the new entrepreneurs gradually scale up their firms while those who entered in the first years have to downsize their companies and reduce their safe asset holdings to be consistent with the new higher wage. The latter effect is very slow and leads to a moderate overshooting of the net foreign asset position which eventually shrinks to around 90% of GDP brought about by very small prolonged current account deficits (less than 1%).

<sup>90</sup> 

<sup>&</sup>lt;sup>22</sup>To simplify the solution procedure I am also ruling out the possibility for the entrepreneur to defer her entry once they are given permission: this may be possibly convenient since the agent could continue to earn labor wage for a few more years and accumulate additional capital so that she could scale up more rapidly the firm once quitting her job. This is not a valid alternative for the entrepreneurs entering at the beginning of the transition since the wage earned as a worker and paid as an entrepreneur is so low that is convenient for agents to become entrepreneurs as soon as possible. It may however be a viable opportunity for those entering in the last few years, but won't change the main predictions of the model.

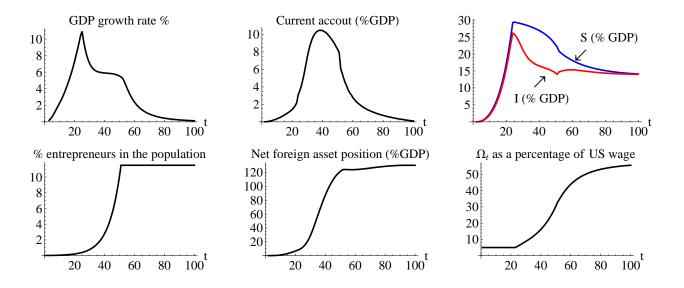


Figure 9: Simulated time series dynamics of key aggregate variables

The simulation results show therefore that the model is successfully able to generate positive correlation between growth and the current account as empirically observed. It is interesting to note that the model can also match very well the magnitude of the current account as a percentage of GDP that China has recently experienced: the simulation shows around 30 years after the beginning of the transition process a peak in the current account as a percentage of GDP of around 10% which is very similar to the Chinese 11% in 2007, 29 years after the beginning of the reform phase. As discussed in section 5.3, this results is pretty robust to alternative value of the estimated parameters, but it depends of course on the speed at which entrepreneurship is allowed in the simulation. Let me finally remark that the spike in the current account happens in the simulation at the time in which all the labor force has been absorbed and the wage rate begins to increase: this is exactly what has happened in China in the last few years which have seen the first significant increase in the wage rate of blue collars.

# 5.3 Sensitivity analysis

Table 2 shows the sensitivity to the estimated parameters of the transition dynamics and steady state under entrepreneurship. Regarding the latter, I consider the equilibrium values under private entrepreneurship of the net foreign asset position which gives us a measure of the total net capital

outflows experience by the country during the transition and the worker wage which is important to discuss the welfare implications of the model in section 6. For the transition dynamics, I focus on the years at which the GDP is half way through its transition and at which the wage rate starts to grow, and on the maximum level (and relative years) attained by the GDP growth rate and the current account (CA), investment (I) and saving (S) as a percentage of GDP.

	Entrepreneurial		Entrepreneurial		Bankruptcy				
Variables	ability product		uction :	ction share		probability			
	$\eta=5$	$\eta$ =10	$\eta$ =15	$\nu$ =.2	$\nu=.4$	$\nu$ =.6	b=.001	b = .005	b = .01
Final NFA	??	??	??	??	??	??	??	??	??
Final wage	??	??	??	??	??	??	??	??	??
Half GDP (t)	??	??	??	??	??	??	??	??	??
Wage take off (t)	??	??	??	??	??	??	??	??	??
Max GDP growth	??	??	??	??	??	??	??	??	??
Max GDP growth (t)	??	??	??	??	??	??	??	??	??
Max CA (%GDP)	??	??	??	??	??	??	??	??	??
Max CA (%GDP) (t)	??	??	??	??	??	??	??	??	??
Max I (%GDP)	??	??	??	??	??	??	??	??	??
Max I (%GDP) (t)	??	??	??	??	??	??	??	??	??
Max S (%GDP)	??	??	??	??	??	??	??	??	??
Max S (%GDP) (t)	??	??	??	??	??	??	??	??	??

Table 2: Sensitivity analysis

# 6 Welfare analysis

In this section I compute the welfare costs associated with the inability to insure against idiosyncratic shocks.<sup>23</sup> The individual welfare of each agent is given by her value function and I express the welfare costs as the permanent percentage increase in annual consumption  $\Lambda$  that would make the agent in the economy without insurance indifferent to move to the economy with actuarially fair insurance. Formally:

$$(1+\Lambda)^{1-\rho}V_t^{\epsilon}(m_t,\eta) = \tilde{V}_t^{\epsilon}(m_t,\eta)$$
(13)

<sup>&</sup>lt;sup>23</sup>I still rule out the possibility of borrowing against future labor income.

where  $\sim$  denotes variables in the economy with insurance. The individual welfare is going to be a function of wealth m and for entrepreneurs also of  $\eta$ .

Similarly to Mendoza, Quadrini, and Rios-Rull (2007b), the aggregate welfare function  $\mathfrak{V}$  weights equally each individual so that aggregate welfare for the workers, entrepreneurs and the overall population is computed respectively as:

$$\begin{split} \mathfrak{V}^w_t(\mathfrak{m}^w_t) &= \int_{i \in W_t} V^w_t(m_{t,i}) \\ \mathfrak{V}^e_t(\mathfrak{m}^e_t, \eta) &= \int_{i \in E_t} V^e_t(m_{t,i}, \eta) \\ \mathfrak{V}_t(\mathfrak{m}_t, \eta) &= \mathfrak{V}^w_t(\mathfrak{m}^w_t) + \mathfrak{V}^e_t(\mathfrak{m}^e_t, \eta) \end{split}$$

where  $\mathfrak{m}_t^e$  represent the wealth distribution.

To understand the welfare implications of the lack of insurance it is crucial to consider the dynamics of the wage rate under the alternative scenarios. Figure 10 shows that the wage rate grows faster and plateaus at a higher level for the economy with insurance. This is due to the fact that if entrepreneurs can insure against the risk of bankruptcy, they will behave as predicted by the standard perfect foresight model, i.e. borrowing to scale up immediately their companies to their optimal scale. Furthermore the scale at which the return to invested capital in the riskless economy equals the world interest rate is larger than in the economy without insurance since in the latter the return to investment is reduced by the risk premium.

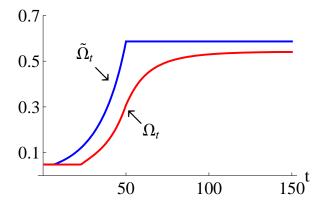


Figure 10: Wage rate with (red) and without (blue) insurance

Regarding the welfare implications of the model at the equilibrium with entrepreneurship,

Things are however less obvious for the entrepreneurs since while they are no longer exposed to the risk of bankruptcy they are hurt by the higher wage they have to pay. The balance of these two effects crucially depends on the wealth endowment as shown in figure 11 which displays on the left side the value functions for the entrepreneur in the economy with and without insurance and on the right the welfare costs over the range of the actual equilibrium distribution of entrepreneurial wealth. We observe that while the entrepreneurs at a level of wealth still far enough below the target are better off moving to the economy with insurance this is no longer true for those at the target level. The reason is that entrepreneurs with lower resources benefit substantially from the opportunity of immediately scaling up their company to the optimal scale instead of going through a period of restrained consumption to self-finance investment. Wealthier entrepreneurs are instead mainly hurt by the increase in the wage rate: under the lack of coordination, entrepreneurs under perfect risk sharing do not internalize the impact of scaling up their businesses on the equilibrium wage rate and under the parameters of the model the net welfare outcome is negative.

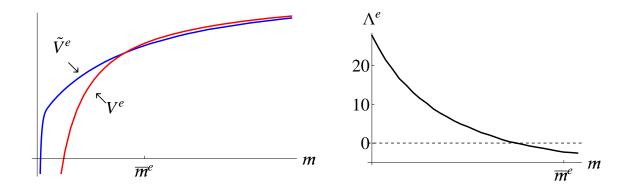


Figure 11: Entrepreneur's value function with (red) and without (blue) insurance and welfare costs at the stochastic equilibrium under entrepreneurship

The aggregate welfare costs of the lack of insurance for workers, entrepreneurs and the overall population are shown in table 3. The welfare costs for workers are found to be 8.4% of annual consumption, while the availability of insurance would make entrepreneurs in aggregate worse off by 1.16% since most of them are distributed nearby the target level. The overall population welfare costs are driven by the workers since more numerous and especially because being at a low wealth

endowment at which the value function is highly concave, they play a key role in the aggregate value function.

Welfare costs for workers	8.40%
Welfare costs for entrepreneurs	-1.16%
Welfare costs for all population	8.38%

Table 3: Aggregate welfare costs at the equilibrium with entrepreneurship

The welfare costs are much higher if computed at the beginning of the transition period since the possibility of insuring against the risk of bankruptcy allows entrepreneurs to expand their firms immediately to the optimal scale by borrowing and speeds up the growth in the wage rate. Note that the welfare costs for entrepreneurs with the same entrepreneurial ability are shrinking with respect to the year in which they are allowed to enter. A little complication in computing the welfare costs is that we have assumed that future entrepreneurs cannot foresee their entry and thus their value function at the beginning of the transition process is simply equal to the one of permanent workers. I thus re-compute for each of them a new value function which incorporates the future transition to entrepreneurship but still being based on the optimal consumption choices of unaware agents. Table 4 shows that both entrepreneurs and workers would benefit similarly by the availability of insurance and these gains would be very substantial, around 18% of annual consumption.

Welfare costs for workers	18.05%
Welfare costs for entrepreneurs	17.98%
Welfare costs for all population	18.04%

Table 4: Aggregate welfare costs at the beginning of the transition

Note that in this setting the welfare costs of financial underdevelopment dwarf those detected in other models. There are 2 reasons for this substantial difference. The first is that I am considering a very powerful shock involving the total loss of the invested capital; even if the probability of such an event is fairly remote, it is a able to dramatically change behavior since it rules out the possibility of borrowing. The second reason, especially relevant for the analysis of the transition dynamics, is that instead of evaluating the costs at the stochastic equilibrium I focus on the process of growth which as previously discussed can be substantially slowed down by the inability to borrow.

A final remark is on the benefits of financial globalization. It has been until now rather difficult to clearly identify the advantages of financial openness for developing countries mostly because scholars have been looking essentially for an elusive positive association between capital inflows and growth. The model shows that a potential benefit of financial globalization is instead to allow for the reverse effect, such as to let emerging markets balance risky domestic investment by holding safe assets abroad. It also points out that the integration with international financial markets can be extremely beneficial if it allows for risk-sharing opportunities as for example those created between local businesses and FDIs in many developing countries.

## 7 Conclusion

# Appendix

#### A Data

The data to produce figures from 1 to 5 have been downloaded from the World Development Indicators on November 19th 2007. [GIVE EXACT VARIABLE NAMES]

The method to identify the growth acceleration episodes is exactly the one suggested by Hausmann, Pritchett, and Rodrik (2005).

## B Financial markets

# C Solving numerically for the policy functions

This section describes the numerical solution procedure used to solve for the policy functions at the stochastic steady state under entrepreneurship and during the transition period.<sup>24</sup> Consider first the problem of the liquidity constrained worker whose first order condition is given by:<sup>25</sup>

$$u^{c}(c_{t}^{w}) \geq \mathfrak{v}_{t+1}^{w,a}(a_{t+1}^{w})$$
 (14)

$$a_{t+1}^w \ge 0 \tag{15}$$

i.e. the marginal utility of consumption has to equal the marginal value of savings in the unconstrained region. By using the Envelope theorem and the CRRA definition of the utility function the first order condition can be rewritten as:

$$(c_t^w(m_t^w))^{-\rho} \ge \beta \mathsf{R}(c_{t+1}^w(\overline{\mathsf{R}a_{t+1}^w + \Omega_{t+1}}))^{-\rho} \tag{16}$$

<sup>&</sup>lt;sup>24</sup>For a detailed discussion of numerical solution methods for microeconomic dynamic stochastic optimization problems see Carroll (2006a).

<sup>&</sup>lt;sup>25</sup>The specification used to perform the structural estimation of the US data involving the presence of the unemployment risk  $\mho$  is similar to the procedure used to solve for the entrepreneur's consumption function described later.

The consumption function is solved by backward iteration until convergence using the Method of Endogenous Gridpoints developed by Carroll (2006b). I initialize the consumption function to the one of the worker at a hypothetical last period of life in which all the available resources would be consumed. I then define a grid of end of period savings  $a_{t+1}^w \geq 0$  (being negative savings rule out by the borrowing constraint) and by using the optimality intertemporal condition (16) I can easily recover the value of  $c_t^w$  consistent with each of them:

$$c_t^w \ge \left(\beta R(c_{t+1}^w(Ra_{t+1}^w + \Omega_{t+1}))^{-\rho}\right)^{-1/\rho} \tag{17}$$

Using the budget constraint, it is then straightforward to compute the endogenous gridpoints of  $m_t^w = c_t^w + a_{t+1}^w$  which are consistent with the considered combinations of consumption and end of period savings. By linearly interpolating the pairs  $(c_t^w, m_t^w)$ , we are thus able to approximate the consumption function of the second to last period of life in the unconstrained region. The solution for the constrained region simply involves consuming all the available resources and can be integrated in the solution procedure by simply adding to the  $(c_t^w, m_t^w)$  list the pair (0,0). In order to improve the approximation of the consumption function at values of m above the highest endogenous gridpoint, we can add to the  $(c_t^w, m_t^w)$  list a bigger m value and evaluate the corresponding consumption value at the one predicted by the analytically solved consumption function under perfect risk sharing. We indeed know that as the m grows to infinity, the precautionary motive shrinks and optimal behavior converges to the one under full insurance.

In order to compute the welfare costs of the lack of insurance and assess the convenience for a worker to become entrepreneur, it is necessary to solve also for the value function. This can be done during the process of iteration for the consumption function by considering that:

$$V_t^w(m_t) = u(c_t^w(m_t)) + \beta V_{t+1}^w(m_{t+1}^w)$$

an initializing the value function to the utility function since in the last period of life all resources are consumed. Since the value function is highly concave for low levels of wealth and especially goes to minus infinity as wealth shrinks to 0, it is wise to make a linear interpolation of a transformation of it: given that under perfect foresight the value function is itself a particular CRRA function,

much of its non linearity can be taken away by multiplying by  $(1 - \rho)$  and raising everything to the  $1/(1 - \rho)$  power. This also prevents the transformed value function going to negative infinity.

The backward iteration is continued until convergence, defined as a maximum gap between the contiguous consumption and value function function iterations of less than 0.01%.

Regarding the entrepreneur's problem, we have the following two first order conditions:

$$\mathfrak{v}_{t+1}^{e,k}(a_{t+1}^e, k_{t+1}) = \mathfrak{v}_{t+1}^{e,a}(a_{t+1}^e, k_{t+1}) 
u^c(c_t) = \mathfrak{v}_{t+1}^{e,a}(a_{t+1}^e, k_{t+1})$$

which by using the Envelop theorem and making explicit the expectation operator reduce to:

$$(1 - \flat)\Re(c^e_{t+1}(m^{\flat}_{t+1}, \eta))^{-\rho} = (1 - \flat)\mathsf{R}(c^e_{t+1}(m^e_{t+1}, \eta))^{-\rho} + \flat\mathsf{R}(c^e_{t+1}(m^{\flat}_{t+1}, \eta))^{-\rho}$$
 
$$(c^e_t)(m_t, \eta)^{-\rho} = \beta\mathsf{R}\left((1 - \flat)(c^e_{t+1}(m^e_{t+1}, \eta))^{-\rho} + \flat(c^e_{t+1}(m^{\flat}_{t+1}, \eta))^{-\rho}\right)$$

with the intertemporal budget constraints:

$$\begin{array}{lcl} m^e_{t+1} & = & \mathsf{R} a^e_{t+1} + \Upsilon_{t+1} \eta^{1-\phi} k^\phi_{t+1} + (1-\delta) k_{t+1} \\ \\ m^\flat_{t+1} & = & \mathsf{R} a^e_{t+1} \end{array}$$

[TO BE CONTINUED]

# D Simulation algorithm

# E Structural estimation

## References

- AIYAGARI, S. RAO (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," Quarterly Journal of Economics, 109(3), p659 684.
- ANGELETOS, GEORGE-MARIOS (2007): "Uninsured Idiosyncratic Investment Risk and Aggregate Saving," Review of Economic Dynamics, 10(1), p1 30.
- Attanasio, Orazio P., Lucio Picci, and Antonello E. Scorcu (2000): "Saving, Growth, and Investment: A Macroeconomic Analysis Using a Panel of Countries," *Review of Economics and Statistics*, 82(2), p182 211.
- Bosworth, Barry P. (1993): Saving and investment in a global economy. Washington, D.C.:.
- Buera, Francisco (2006): "Persistency of Poverty, Financial Frictions, and Entrepreneurship,".
- Buera, Francisco, and Yongseok Shin (2008): "Financial Frictions and the Persistence of History: A Quantitative Ecploration,".
- Caballero, Ricardo J., Emmanuel Farhi, and Pierre-Olivier Gourinchas (2008): "An Equilibrium Model of 'Global Imbalances' and Low Interest Rates," *American Economic Review*, 98(1), p358 393.
- CAGETTI, MARCO, AND MARIACRISTINA DE NARDI (2006): "Entrepreneurship, Frictions, and Wealth," *Journal of Political Economy*, 114(5), p835 870.
- Campbell, David W., and Wako Watanabe (2001): "Household Saving in Japan,"

  The Japanese Economic Review, 52(2), p243 250.
- Carroll, Christopher D. (1992): "The Buffer-Stock Theory of Saving: Some Macroe-conomic Evidence," *Brookings Papers on Economic Activity*, (2), p61 135.

- ———— (2000): "Risky Habits' and the Marginal Propensity to Consume Out of Permanent Income, or, How Much Would a Permanent Tax Cut Boost Japanese Consumption?," International Economic Journal, 14(4), p1 – 40.
- ——— (2006a): "Lecture Notes on Solving Microeconomic Dynamic Stochastic Optimization Problems," *Manuscript, Johns Hopkins University*, Available at http://econ.jhu.edu/people/ccarroll/solvingmicrodsops.zip.
- ——— (2006b): "The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems," *Economics Letters*, 91(3), p312 320.
- Carroll, Christopher D., and Miles S. Kimball (2001): "Liquidity Constraints and Precautionary Saving," *National Bureau of Economic Research*, *Inc, NBER Working Papers:* 8496.
- CARROLL, CHRISTOPHER D., JODY OVERLAND, AND DAVID N. WEIL (2000): "Saving and Growth with Habit Formation," *American Economic Review*, 90(3), p341 355.
- Carroll, Christopher D., and Lawrence H. Summers (1991): Consumption Growth Parallels Income Growth: Some New Evidencepp. p305 343. A National Bureau of Economic Research Project Report, ed. by B. Douglas Bernheim and John B. Shoven. Chicago University Press, Chicago.
- CARROLL, CHRISTOPHER D., AND DAVID N. WEIL (1994): "Saving and Growth: A Reinterpretation," Carnegie-Rochester Conference Series on Public Policy, 40, p133 192.
- DEATON, ANGUS (1991): "Saving and Liquidity Constraints," *Econometrica*, 59(5), p1221 1248.

- DYNAN, KAREN E. (2000): "Habit Formation in Consumer Preferences: Evidence from Panel Data," *American Economic Review*, 90(3), p391 406.
- EDWARDS, SEBASTIAN (1996): "Why Are Latin America's Savings Rates So Low? An International Comparative Analysis," *Journal of Development Economics*, 51(1), p5 44.
- FAZZARI, STEVEN M., ROBERT GLENN HUBBARD, AND BRUCE C. PETERSEN (1988): "Financing Constraints and Corporate Investment," *Brookings Papers on Economic Activity*, (1), p141 195.
- FLAVIN, MARJORIE, AND SHINOBU NAKAGAWA (2004): "A Model of Housing in the Presence of Adjustment Costs: A Structural Interpretation of Habit Persistence," *Manuscript*, *UCSD*.
- Gentry, William M., and Glenn R. Hubbard (2004): "Entrepreneurship and Household Saving," Advances in Economic Analysis & Policy, 4(1), p1053 1053.
- Gourinchas, Pierre-Olivier, and Olivier Jeanne (2007): Capital Flows to Developing Countries: The Allocation Puzzle. National Bureau of Economic Research, Inc, NBER Working Papers: 13602.
- HAUSMANN, RICARDO, LANT PRITCHETT, AND DANI RODRIK (2005): "Growth Accelerations," *Journal of Economic Growth*, 10(4), p303 329.
- HOUTHAKKER, HENDRIK S. (1961): "An International Comparison of Personal Saving," Bulletin of the International Statistical Institute, 38, p55–70.
- Hubbard, R. Glenn (1998): "Capital-Market Imperfections and Investment," *Journal of Economic Literature*, 36(1), p193 225.
- HUGGETT, MARK (1997): "The One-Sector Growth Model with Idiosyncratic Shocks: Steady States and Dynamics," *Journal of Monetary Economics*, 39(3), p385 403.

- KRUSELL, PER, AND JR. SMITH, ANTHONY A. (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106(5), p867 896.
- LOAYZA, NORMAN, KLAUS SCHMIDT-HEBBEL, AND LUIS SERVEN (2000): "What Drives Private Saving across the World?," Review of Economics and Statistics, 82(2), p165 181.
- Love, Inessa, and Lea Zicchino (2006): "Financial Development and Dynamic Investment Behavior: Evidence from Panel VAR," Quarterly Review of Economics and Finance, 46(2), p190 210.
- Lucas, Robert E., Jr. (1978): "On the Size Distribution of Business Firms," *Bell Journal of Economics*, 9(2), p508 523.
- Maddison, Angus (1992): A Long-Run Perspective on Savingpp. p27 42. Scandinavian Journal of Economics series.
- MEGHIR, COSTAS, AND GUGLIELMO WEBER (1996): "Intertemporal Nonseparability or Borrowing Restrictions? A Disaggregate Analysis Using a U.S. Consumption Panel," Econometrica, 64(5), p1151 – 1181.
- Mendoza, Enrique G., Vincenzo Quadrini, and Jose-Victor Rios-Rull (2007a): "Financial Integration, Financial Deepness and Global Imbalances," *National Bureau of Economic Research, Inc, NBER Working Papers: 12909.*
- ———— (2007b): "On the Welfare Implications of Financial Globalization without Financial Development," *National Bureau of Economic Research, Inc, NBER Working Papers:* 13412.
- Modigliani, Franco (1970): "The Life Cycle Hypothesis of Saving and Intercountry

- Differences in the Saving Ratio," Induction, Growth and Trade: Essays in Honour of Sir Roy Harrod.
- MODIGLIANI, FRANCO, AND MERTON H. MILLER (1958): "The Cost of Capital, Corporation Finance and the Theory of Investment," *The American Economic Review*, 48(3), p261 297.
- NAIK, NARAYAN Y., AND MICHAEL J. MOORE (1996): "Habit Formation and Intertemporal Substitution in Individual Food Consumption," Review of Economics and Statistics, 78(2), p321 328.
- Prasad, Eswar S., Raghuram G. Rajan, and Arvind Subramanian (2007): "Foreign Capital and Economic Growth," *Brookings Papers on Economic Activity*, (1), p153 230.
- QUADRINI, VINCENZO (2000): "Entrepreneurship, Saving, and Social Mobility," Review of Economic Dynamics, 3(1), p1 40.
- Rodrik, Dani (2000): "Saving Transitions," World Bank Economic Review, 14(3), p481 507.
- SCHMITT-GROHÉ, STEPHANIE, AND MARTIN URIBE (2003): "Closing small open economy models," *Journal of International Economics*, 61(1), 163–185.
- Zeldes, Stephen P. (1989): "Optimal Consumption with Stochastic Income: Deviations from Certainty Equivalence," *Quarterly Journal of Economics*, 104(2), p275 298.