Default and the Maturity Structure in Sovereign Bonds[*]

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Abstract

This paper studies the maturity composition and the term structure of interest rate spreads of government debt in emerging markets. In the data, when interest rate spreads rise, debt maturity shortens and the spread on short-term bonds is higher than on long-term bonds. To account for this pattern, we build a dynamic model of international borrowing with endogenous default and multiple maturities of debt. Short-term debt can deliver higher immediate consumption than long-term debt; large long-term loans are not available because the borrower cannot commit to save in the near future towards repayment in the far future. However, issuing long-term debt can insure against the need to roll-over short-term debt at high interest rate spreads. The trade-off between these two benefits is quantitatively important for understanding the maturity composition in emerging markets. When calibrated to data from Brazil, the model matches the dynamics in the maturity of debt issuances and its comovement with the level of spreads across maturities.

1   Introduction

Emerging markets face recurrent and costly financial crises that are characterized by limited access to credit and high interest rates on foreign debt. As crises approach, not only is debt limited but also the maturity of debt shortens, as documented by Broner, Lorenzoni, and Schmukler (2007).[1] During these periods, however, the interest rate spread on short-term bonds rises more than the spread on long-term bonds. Why do countries shorten their debt maturity during crises even though spreads appear higher for shorter maturity debt? To answer this question, this paper develops a dynamic model of the maturity composition in which debt prices reflect endogenous default risk and debt maturity responds to the prices of short- and long-term debt contracts. Our model can rationalize shorter debt maturity during crises as the result of a liquidity advantage in short-term debt contracts; although these contracts carry higher spreads than longer term debt, they can deliver larger resources to the country in times of high default risk.

We first analyze the dynamics of the maturity composition of international bonds and the term structure of interest rate spreads for four emerging market countries: Argentina, Brazil, Mexico, and Russia. We use data on prices and issuances of foreign-currency denominated bonds to estimate *spread curves* – interest rate spreads over U.S. Treasury bonds across maturity – as well as *duration*, a measure of the average time to maturity of payments on coupon paying bonds. We find that governments issue short-term debt more heavily when spreads are high and spread curves are downward sloping, and they issue long-term debt more heavily when spreads are low and spread curves are upward sloping. Across these four countries, within periods in which 2-year spreads are below their 25th percentile, the average duration of new debt is 7.1 years, and the average difference between the 10-year spread and the 2-year spread is 2.3 percentage points. But when the 2-year spreads are above their 75th percentile, the average duration shortens to 5.7 years, while the average difference between the 10-year spread and the 2-year spread is −0.5 percentage points. From this evidence we conclude that the maturity of debt shortens in times of high spreads and downward-sloping spread curves.

We then develop a dynamic model with defaultable bonds to study the choice of debt maturity and its covariation with the term structure of spreads. In our model, a risk averse borrower faces persistent income shocks and can issue long and short duration bonds. The borrower can default on debt at any point in time, but faces costs of doing so. Default

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[1]Calvo and Mendoza (1996) document in detail how in Mexico during 1994, most of the public debt was converted to 91-day *Tesobonos*. Bevilaqua and Garcia (2000) document a similar rise in short-term government debt in Brazil during the 1999 crisis.

occurs in equilibrium in low-income, high-debt times because the cost of coupon payments outweighs the costs of default when consumption is low. Interest rate spreads on long and short bonds compensate foreign lenders for the expected loss from future defaults. Thus, the supply of credit is more stringent in times of low income and high outstanding debt, because the probability of default is high. In fact, countercyclical default risk substantially limits the degree of risk sharing, and the model can generate capital outflows in recessions, when interest rate spreads are at their highest.

The model generates the observed dynamics of spread curves because the endogenous probability of default is persistent, yet mean reverting, as a result of the dynamics of debt and income. When debt is low and income is high, default is unlikely in the near future, so spreads are low. However, long-terms spreads are higher than short-term spreads because default may become likely in the far future if the borrower receives a sequence of bad shocks and accumulates debt. On the other hand, when income is low and debt is high, default is likely in the near future, so spreads are high. Long-term spreads, however, increase by less than short-term spreads because the borrower's likelihood of repaying may rise if it receives a sequence of good shocks and reduces its debt. Although cumulative default probabilities on long-term debt are always larger than on short-term debt, the long spread can be lower than the short spread because it reflects a lower average future default probability.

The model can rationalize the covariation observed in the data between the maturity structure of debt issuances and the term structure of spreads as reflecting a trade-off between *insurance* benefits of long-term debt and *liquidity* benefits of short-term debt, both due to the presence of default. Long-term debt provides insurance against the uncertainty of short-term interest rate spreads. Since short-term spreads rise during periods of low income, when default risk is high, issuing long-term debt allows the borrower to avoid rolling over short-term debt at high spreads in states when consumption is low. Moreover, long-term debt insures against future periods of limited credit availability; in particular, the borrower can avoid capital outflows in recessions by issuing long-term debt.

Even though long debt dominates short debt in terms of insurance, it is not as effective in delivering high immediate consumption; hence the liquidity benefit of short-term debt. Short-term debt allows the borrower to pledge more of his future income toward debt repayment because in each subsequent period the threat of default punishment gives him incentives for repayment before any further short debt is issued. Long-term debt contracts do not allow such large transfers because the borrower is unable to commit to saving in the near future toward repayment in the further future. Effectively, the threat of default punishment is lower with long-term debt given that it will be relevant only in the future, when the long-term debt

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is due. This greater efficacy of short-term debt in alleviating commitment problems for debt repayment is reflected in more lenient price schedules and smaller drops in short-term prices with increases in the level of debt issues. In this sense, short debt is a more liquid asset, and consumption can always be marginally increased by more with short-term debt than with long-term debt.

The time-varying maturity structure responds to a time-varying valuation of the insurance benefit of long-term debt and the liquidity benefit of short-term debt. Periods of low default probabilities and upward spread curves correspond to states when the borrower is wealthy and values insurance. Thus, the portfolio is shifted toward long debt. Periods of high default probabilities and inverted spread curves correspond to states when the borrower is poor and credit is limited. These are times when liquidity is most valuable, and thus the portfolio is shifted toward shorter-term debt. We can therefore rationalize higher short-term debt positions in times of crises as an optimal response to the illiquidity of long-term debt, and the tighter availability of its supply.

When calibrated to Brazilian data, the model quantitatively matches the dynamics of the maturity composition of new debt issuances and its covariation with spreads observed in the data. In connecting our model to the data, a methodological contribution of the paper is to develop a tractable framework with bonds that have empirically relevant duration. Bonds in our model are perpetuity contracts with non-state-contingent coupon payments that decay at different rates. Bonds with payments that decay quickly have more of their value paid early, and so have short duration. This gives a recursive structure to debt accumulation that allows the model to be characterized in terms of a small number of state variables although decisions at any date are contingent on a long sequence of future expected payments. Our findings indicate that the insurance benefits of long-term debt and the liquidity benefits of short-term debt are quantitatively important in understanding the dynamics of the maturity structure observed in Brazil. Importantly, the maturity structure in the model responds to the underlying dynamics of default probabilities reflected in spread curves, which match the data well.

Related Literature

This paper is related to the literature on the optimal maturity structure of government debt. Angeletos (2002), Buera and Nicolini (2004) and Shin (2007) show that, when debt is not state contingent, a rich maturity structure of government bonds can be used to replicate the allocations obtained with state-contingent debt in economies with distortionary taxes as in Lucas and Stokey (1983). In these closed economy models, short- and long-term interest

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rate dynamics reflect the variation in the representative agent's marginal rate of substitution, which changes with the state of the economy. Thus, having a rich enough maturity structure is equivalent to having assets with state-contingent payoffs.[2] Our paper shares with these papers the message that managing the maturity composition of debt can provide benefits to the government because of uncertainty over future interest rates. The message is particularly relevant for the case of emerging market economies. As Neumeyer and Perri (2005) have shown, fluctuations in country specific interest rate spreads play a major role in accounting for the large business cycle fluctuations in emerging markets. The lesson that our paper provides in this context is that the volatility of the maturity composition of debt in these countries is an optimal response to these interest rate fluctuations. However, in contrast to these papers, the fluctuations in interest rates in our model reflect time variation in the endogenous country's own probability of default.[3]

The maturity of debt in emerging countries is also of interest because of the general view that countries could alleviate their vulnerability to very costly crises by choosing the appropriate maturity structure. For example, Cole and Kehoe (1996) argue that the 1994 Mexican debt crisis could have been avoided if the maturity of government debt had been longer. Longer maturity debt would allow countries to better manage external shocks and sudden stops. Broner, Lorenzoni, and Schmukler (2007) formalize this idea in a model where the government can avoid a crisis in the short term by issuing long-term debt. In their model, with risk averse lenders who face liquidity shocks, long-term debt is more expensive, so the maturity composition is the result of a trade-off between safer long-term debt and cheaper short-term debt. In line with their paper, we also find that short-term debt provides larger liquidity benefits. In contrast to Broner, Lorenzoni, and Schmukler, in our model the time-varying availability of short- and long-term debt is an equilibrium response to compensate for the economy's default risk, rather than to compensate for foreign lenders' shocks. Moreover, our paper is the first to develop a dynamic framework with defaultable debt and multiple maturities with which these questions can be analyzed and assessed quantitatively.

The larger liquidity benefits of short-term debt relative to long-term debt arise in our model because short-term contracts are more effective in solving the commitment problem of the borrower in terms of future debt and default policies. In this regard, our paper is related to Jeanne's (2004) model where short-term debt gives more incentives for the government

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[2]Lustig, Sleet, and Yeltekin (2006) develop a general equilibrium model with uninsurable nominal frictions to study the optimal maturity of government debt. They find that higher interest rates on long-term debt relative to short-term debt reflect an insurance premium paid by the government, for the benefits long-term debt provides in hedging against future shocks.

[3]The idea that credit risk makes longer term debt attractive is also present in Diamond (1991) in a three period model of corporate debt where firms have private information about their future credit rating.

to implement better policies. When short-term debt needs to be rolled over, creditors can discipline the government by rolling over the debt only after desired policies are implemented.[4] Moreover, when defaulted debt is renegotiated, Bi (2007) shows that long-term debt is more expensive also to compensate for debt dilution. Absent explicit seniority clauses, issuing short-term debt can dilute the recovery of long-term debt in case of default.

The theoretical model in this paper builds on the work of Aguiar and Gopinath (2006) and Arellano (2008), who model equilibrium default with incomplete markets, as in the seminal paper on sovereign debt by Eaton and Gersovitz (1981). This paper extends this framework to incorporate long debt of multiple maturities. In recent work, Chatterjee and Eyigungor (2008) and Hatchondo and Martinez (2008) show that long-term defaultable debt allows a better fit of emerging market data in terms of the volatility and mean of the country spread as well as debt levels. All these models generate a time-varying probability of default that is linked to the dynamics of debt and income. The dynamics of the spread curve in our model reflect the time-varying default probability, in the same way that Merton (1974) derived for credit spread curves on defaultable corporate bonds. In Merton's model, when the exogenous default probability is low, the credit spread curve is upward sloping, and when the default probability is high, credit spread curves are downward sloping or hump shaped. The spread curve dynamics in this paper follow Merton's results. However, our framework differs from Merton's in that the probability of default and the level and maturity composition of debt issuances are endogenous variables.

The outline of the paper is as follows. Section 2 documents the dynamics of the spread curve and maturity composition for four emerging markets: Argentina, Brazil, Mexico, and Russia. Section 3 presents the theoretical model. Section 4 presents some examples to illustrate the mechanism for the optimal debt portfolio. Section 5 presents all the quantitative results, and Section 6 concludes.

2   Emerging Markets Bond Data

We examine data on sovereign bonds issued in international financial markets by four emerging-market countries: Argentina, Brazil, Mexico, and Russia. We look at the behavior of the interest rate spreads over default-free bonds, across different maturities, and at the way the maturity of new debt issued covaries with spreads. We find that when spreads are low, governments issue long-term bonds more heavily and long-term spreads are higher than short-term

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[4]Commitment problems have been shown to reduce the level of sustainable debt in the literature of optimal policy without commitment, as in Krusell, Martin, and Rios-Rull (2006).

spreads. When spreads rise, the maturity of bond issuances shortens and short-term spreads are higher than long-term spreads. Our findings also confirm the earlier results of Broner, Lorenzoni, and Schmukler (2007), who showed in a sample of eight emerging economies that debt maturity shortens when spreads are very high.[5]

2.1 Spread Curves

We define the n-year *spread* for an emerging market country as the difference between the yield on a defaultable, zero-coupon bond maturing in n years issued by the country and on a zero-coupon bond of the same maturity with negligible default risk (for example, a U.S. Treasury note). The spread is the implicit interest rate premium required by investors to be willing to purchase a defaultable bond of a given maturity.[6] The *spread curve* depicts spreads as a function of maturity.

We denote the annually compounded yield at date t on a zero-coupon bond issued by country i, maturing in n years, as $r_{t,i}^n$. The yield is related to the price $p_{t,i}^n$ of an n-year zero-coupon bond, with face value 1, through

$$p_{t,i}^n = (1 + r_{t,i}^n)^{-n}. \tag{1}$$

We define country i's n-year spread as the difference in zero-coupon yields between a bond issued by country i relative to a default-free bond. The n-year spread for country i at date t is given by: $s_{t,i}^n = r_{t,i}^n - r_{t,rf}^n$, where $r_{t,rf}^n$ is the yield of a n-year default-free bond.[7]

Since governments do not issue zero-coupon bonds in a wide range of maturities, we estimate a country's spread curve by using secondary market data on the prices at which coupon-bearing bonds trade. The estimation procedure, described in the Appendix, follows Svensson (1994) and Broner, Lorenzoni, and Schmukler (2007).

We compute spreads starting in March 1996 at the earliest and ending in May 2004 at the latest, depending on the availability of data for each country. Figure 1 displays the estimated spreads for 2-year and 10-year bonds for Argentina, Brazil, Mexico, and Russia.

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[5]Broner, Lorenzoni and Schmukler (2007) focus on the relationship between the term structure of *risk premia* (compensation for risk aversion) and the average *maturity* of debt. In this section we construct measures of the term structure of *yield spreads* and the average *duration* of debt because these statistics provide the basis for the quantitative assessment of our model.

[6]Yield spreads on bonds issued by emerging markets could also arise due to risk premia or liquidity differences. However, given the incidence of sovereign defaults in emerging markets, in our model we abstract from these other factors and examine the extent to which default risk can rationalize these spread dynamics.

[7]Our data include bonds denominated in U.S. dollars and European currencies, so we take U.S. and Euro-area government bond yields as default-free.
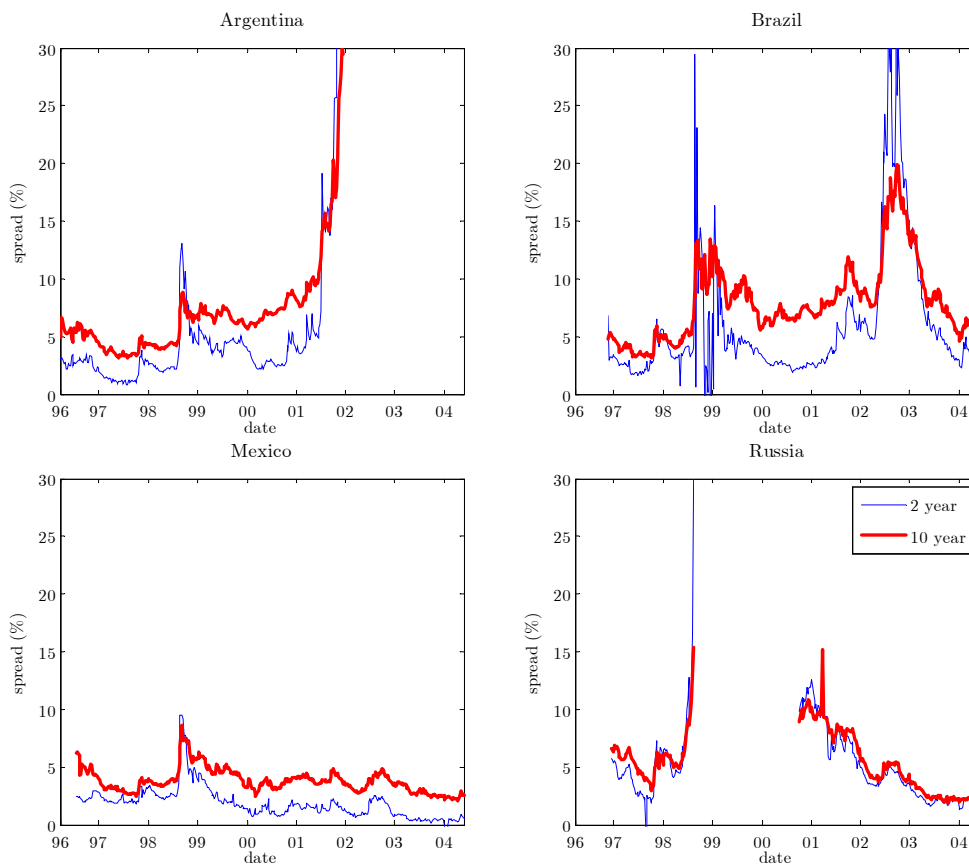
Figure 1: Time series of 2-year and 10-year spreads.

Spreads are very volatile, and the difference between long-term and short-term spreads varies substantially over time. When spreads are low, long-term spreads are generally higher than short-term spreads. However, when the level of spreads rises, the gap between long and short-term spreads tends to narrow and sometimes reverses; the spread curve is flatter or inverted. The time series in Figure 1 show sharp increases in interest rate spreads associated with Russia's default in 1998, Argentina's default in 2001, and Brazil's financial crisis in 2002.[8] The expectation that the countries would default in these episodes is reflected in the high spreads charged on defaultable bonds.

To emphasize the pattern observed in the time series that short-term spreads tend to rise more than long-term spreads, in Figure 2 we display spread curves averaged across different

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[8]For Argentina and Russia, we do not report spreads after default on external debt, unless a restructuring agreement was largely completed at a later date. We use dates taken from Sturzenegger and Zettelmeyer (2005). For Argentina, we report spreads until the last week of December 2001, when the country defaulted. The restructuring agreement for external debt was not offered until 2005. For Russia, we report spreads until the second week of August 1998 and beginning again after August 2000 when 75% of external debt had been restructured.

time periods for each country: the overall average, the average within periods with the 2-year spread below its 10th percentile, and the average within periods with the 2-year spread above its 90th percentile. When spreads are low, the spread curve is upward sloping: long-term spreads are higher than short-term spreads. When spreads are high, short-term spreads rise more than long-term spreads. For Argentina, Brazil, and Russia, the spread curve becomes downward sloping in these times. For Mexico, which had relatively smaller increases in spreads during this time period, the spread curve flattens as short spreads rise more than long spreads.[9]

2.2   The Maturity Composition of Debt and Spreads

We now examine the maturity of new debt issued by the four emerging market economies during the sample period, and relate the changes in the maturity of debt to changes in spreads.[10]

In each week in the sample, we measure the maturity of debt as a quantity-weighted average maturity of bonds issued that week. We measure the maturity of a bond using two alternative statistics. The first is simply the number of years from the issue date until the maturity date. The second is the bond's *duration*, defined in Macaulay (1938) as a weighted average of the number of years until each of the bond's future payments. A bond issued at date t by country i, paying annual coupon c at dates $n_1, n_2, \ldots n_J$ years into the future, and face value of 1 has duration $d_{t,i}(c)$ defined by

$$ d_{t,i}(c) = \frac{1}{p_{t,i}(c)} \left(\sum_{j=1}^{J} n_j c (1 + r_{t,i}^{n_j})^{-n_j} + n_J (1 + r_{t,i}^{n_J})^{-n_J} \right), \tag{2} $$

where $p_{t,i}(c)$ is the coupon bond's price, and $r_{t,i}^n$ is the zero-coupon yield curve. The time until each future payment is weighted by the discounted value of that payment relative to the price of the bond. A zero-coupon bond has duration equal to the number of years until its maturity date, but a coupon-paying bond maturing on the same date has shorter duration. We consider duration as a measure of maturity because it is more comparable across bonds

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[9]The findings are similar to empirical findings on spread curves in corporate debt markets. Sarig and Warga (1989), for example, find that highly rated corporate bonds have low levels of spreads, and spread curves that are flat or upward-sloping, while low-grade corporate bonds have high levels of spreads, and average spread curves that are hump-shaped or downward-sloping.

[10]In addition to external bond debt, emerging countries also have debt obligations with multilateral institutions and foreign banks. However, marketable debt constitutes a large fraction of the external debt. The average marketable debt from 1996 to 2004 is 56% of total external debt in Argentina, 59% in Brazil, and 58% in Mexico (Cowan et al. 2006).
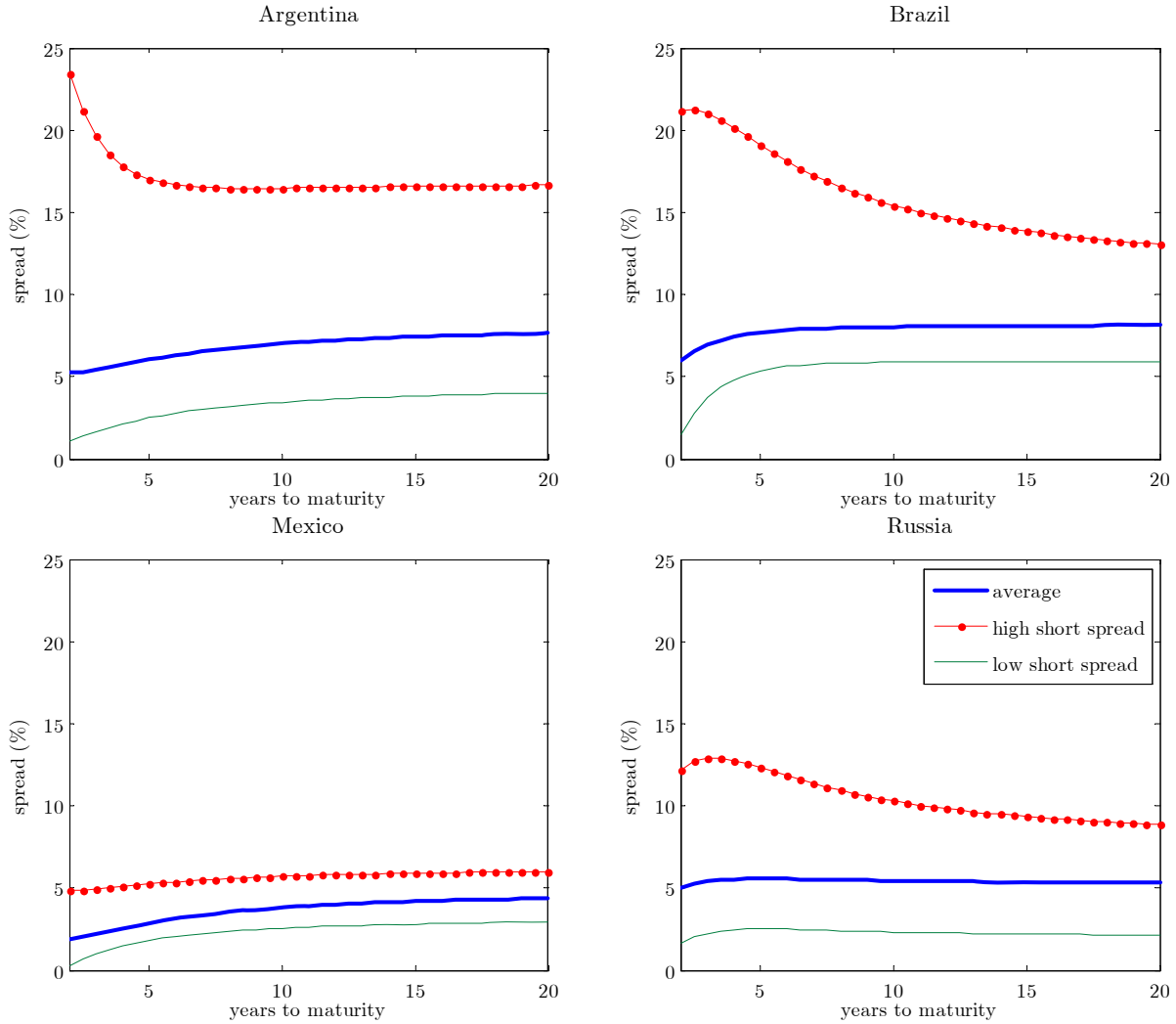
Figure 2: Average spread curves: overall, and within periods in the highest and lowest deciles of the 2-year spread.

with different coupon rates.

We calculate the average maturity and average duration of new bonds issued in each week by each country. Table 1 displays each country's averages of these weekly maturity and duration series within periods of high (above median) and low (below median) 2-year spreads.

First, the table shows that duration tends to be much shorter than maturity. Because the yield on an emerging market bond is typically high, the principal payment at the maturity date is severely discounted, and much of the bond's value comes from coupon payments made sooner in the future. This weight on coupon payments shortens the duration measure relative

Table 1: Average Maturity and Duration of New Debt

2-year spread:	Maturity (years)		Duration (years)	
	< median	≥ median	< median	≥ median
Argentina	9.15	9.05	5.70	5.10
Brazil	14.02	6.60	6.59	4.47
Mexico	13.50	10.30	7.72	6.52
Russia	8.89	10.98	6.11	5.42

to the time-to-maturity measure.

Second, the average duration of debt is shorter when spreads are high than when they are low. Mexico, for example, issues debt that averages about 1.2 years longer in duration when the 2-year spread is below its median than when it is above its median. For all countries except Russia, this pattern also holds for the average time-to-maturity of bonds issued during periods of high spreads compared to low spreads: Mexico issues bonds that mature 3.2 years sooner when spreads are high. Our unconditional point estimates for a shorter debt duration when spreads are high mirrors the findings in Broner, Lorenzoni, and Schmukler (2007). They show that a high spread level is a statistically significant determinant for a shorter maturity of debt issuances even after controlling for selection effects due the fact that the timing of debt issuances is very irregular.

In Table 2, we emphasize the relationship between the spread curve *slopes* and average duration. The slope of the spread curve, defined here as the difference between the 10-year (long-term) and 2-year (short-term) spread, falls when the 2-year spread is high – the numbers in column 4 of Table 2 are smaller than those in column 3. During these times, however, the countries shift toward short-term debt, even though the spreads on long-term debt rise less than for short-term debt. In Brazil, for example, while the spread curve changes from depicting a 10-year spread that is 4 percentage points above the 2-year spread to one that is 1.33 percentage points below the 2-year spread, the average duration of newly issued debt reduces by more than 2 years.

The message of this section is that the spread curve and the maturity of bond issuances in emerging markets are time-varying. In particular, the slope of the spread curve covaries positively with the maturity of new debt, and negatively with the levels of spreads: when short-term spreads are low, the slope of the spread curve is higher, and the maturity of new debt is longer, than when short-term spreads are high. In what follows, we build a dynamic

Table 2: Slope of Spread Curve and Average Duration of Issuances

	Duration (years)		Spread curve slope (%) $s^{10} - s^2$	
short spread:	< 25th pct	≥ 75th pct	< 25th pct	≥ 75th pct
Argentina	6.40	5.64	2.47	-1.16
Brazil	6.80	4.63	4.01	-1.33
Mexico	8.45	6.39	2.30	1.24
Russia	6.57	6.19	0.57	-0.67

model that rationalizes this pattern, in which spreads reflect the government's likelihood of defaulting, and the average maturity of new debt endogenously varies over time.

3 The Model

Consider a dynamic model of defaultable debt that includes bonds of short and long duration. A small open economy receives a stochastic stream of output, y, of a tradable good. The output shock follows a Markov process with compact support and transition function $f(y', y)$. The economy trades two bonds of different duration with international lenders. Financial contracts are unenforceable: the economy can default on its debt at any time. If the economy defaults, it temporarily loses access to international financial markets and also incurs direct output costs.

The representative agent in the small open economy (henceforth, the "borrower") receives utility from consumption c_t and has preferences given by

$$E\sum_{t=0}^{\infty}\beta^t u(c_t), \tag{3}$$

where $0 < \beta < 1$ is the time discount factor and $u(\cdot)$ is increasing and concave.

The borrower issues debt in the form of two types of perpetuity contracts with coupon payments that decay geometrically. We let $\{\delta_S, \delta_L\} \in [0, 1]$ denote the "decay factors" of the payments for the two bonds. A perpetuity with decay factor δ_m is a contract that specifies a price q_t^m and a loan face value ℓ_t^m such that the borrower receives $q_t^m \ell_t^m$ units of goods in period t and promises to pay, conditional on not defaulting, $\delta_m^{n-1} \ell_t^m$ units of goods in every

future period $t + n$. The decay of each perpetuity is related to its duration: a bond of this type with rapidly declining payments has a larger proportion of its value paid early on, and therefore a shorter duration, than a bond with more slowly declining payments. We let $\delta_S < \delta_L$, so that δ_S is the decay of the perpetuity with short duration and δ_L is the decay of the perpetuity with long duration. We will refer to the perpetuities with decay factors δ_S and δ_L throughout as short and long bonds, respectively.

At every time t the economy has outstanding all past perpetuity issuances. Define b_t^m, the stock of perpetuities of duration m at time t, as the total payments due in period t on all past issuances of type m, conditional on not defaulting:

$$b_t^m = \sum_{j=1}^{t} \delta_m^{j-1} \ell_{t-j}^m = \ell_{t-1}^m + \delta_m \ell_{t-2}^m + \delta_m^2 \ell_{t-3}^m + ... + \delta_m^t \ell_0^m + b_0^m,$$

where b_0^m is given. Thus, the accumulation for the stocks of short and long perpetuities can be written recursively by the following laws of motion:

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\begin{aligned}
b_{t+1}^S &= \delta_S b_t^S + \ell_t^S \\
b_{t+1}^L &= \delta_L b_t^L + \ell_t^L
\end{aligned}
\tag{4}
$$

With these definitions, we can compactly write the borrower's budget constraint conditional on not defaulting. Purchases of consumption are constrained by the endowment less payments on outstanding debt, $b_t^S + b_t^L$, plus the issues of new perpetuities of short duration ℓ_t^S at price q_t^S and long duration ℓ_t^L at a price q_t^L:

$$c_t = y_t - b_t^S - b_t^L + q_t^S \ell_t^S + q_t^L \ell_t^L \tag{5}$$

The borrower chooses new issuances of perpetuities from a menu of contracts where prices q_t^S and q_t^L for are quoted for each pair (b_{t+1}^S, b_{t+1}^L).

If the economy defaults, we assume that all outstanding debts and assets $(b_t^S + b_t^L)$ are erased from the budget constraint, and the economy cannot borrow or save, so that consumption equals output. In addition, the country incurs output costs:

$$c_t = y_t^{def},$$

where $y_t^{def} = h(y_t) \leq y_t$.

3.1 Recursive Problem

We now represent the borrower's infinite horizon decision problem as a recursive dynamic programming problem. The model has two endogenous states, which are the stocks of each type of debt, b_t^S and b_t^L, and one exogenous state, the output of the economy, y_t. The state of the economy at date t is then given by $(b_S, b_L, y) \equiv (b_t^S, b_t^L, y_t)$.

At any given state, the value of the option to default is given by

$$v^o(b_S, b_L, y) = \max_{c,d} \left\{ v^c(b_S, b_L, y), v^d(y) \right\}, \tag{6}$$

where $v^c(b_S, b_L, y)$ is the value associated with not defaulting and staying in the contract and $v^d(y)$ is the value associated with default.

Since we assume that default costs are incurred whenever the borrower fails to repay its obligations in full, the model will only generate complete default on all outstanding debt, both short and long term. When the borrower defaults, output falls to y^{def}, and the economy is temporarily in financial autarky; θ is the probability that it will regain access to international credit markets each period. The value of default is then given by the following:

$$v^d(y) = u(y^{def}) + \beta \int_{y'} \left[\theta v^o(0, 0, y') + (1 - \theta) v^d(y') \right] f(y', y) dy'. \tag{7}$$

We are taking a simple route to model both costs of default that seem empirically relevant: exclusion from financial markets and direct costs in output. Moreover, we assume that the default value does not depend on the maturity composition of debt prior to default. This captures the idea that the maturity composition of defaulted debt is not relevant for the restructuring procedures that allow the economy to reenter the credit market.[11]

When the borrower chooses to remain in the contract, the value is the following:

$$v^c = \max_{\left\{ b'_S, b'_L, \ell_S, \ell_L, c \right\}} \left(u(c) + \beta \int_{y'} v^o(b'_S, b'_L, y') f(y', y) dy' \right) \tag{8}$$

subject to the budget constraint:

$$c - q^S \left(b'_S, b'_L, b_S, b_L, y \right) \ell'_L - q^L \left(b'_S, b'_L, b_S, b_L, y \right) \ell'_L = y - b_S - b_L \tag{9}$$

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[11]This is consistent with empirical evidence regarding actual restructuring processes, where the maturity composition of the new debt obligations is part of the restructuring agreement (Sturzenegger and Zettelmeyer 2005).

and to the laws of motion for the stock of perpetuities of short and long duration:

$$b'_S = \delta_S b_S + \ell_S$$

$$b'_L = \delta_L b_L + \ell_L.$$

The borrower decides on optimal debt levels b'_S and b'_L to maximize utility. The borrower takes as given that each contract $\{b'_S, b'_L\} \in B$ comes with specific prices $\{q^S, q^L\}$ that are contingent on today's states (b_S, b_L, y). The decision of whether to remain in the credit contract or default is a period-by-period decision, so that the expected value from next period forward in (8) incorporates the option to default in the future.

The default policy can be characterized by default sets and repayment sets. Let the repayment set, $R(b_S, b_L)$, be the set of output levels for which repayment is optimal when short- and long-term debt are (b_S, b_L):

$$R(b_S, b_L) = \left\{ y \in Y : v^c(b_S, b_L, y) \geq v^d(y) \right\}, \tag{10}$$

and let the complement, the default set $D(b_S, b_L)$, be the set of output levels for which default is optimal for debt positions (b_S, b_L):

$$D(b_S, b_L) = \left\{ y \in Y : v^c(b_S, b_L, y) < v^d(y) \right\}. \tag{11}$$

When the borrower does not default, optimal new debt takes the form of two decision rules mapping today's state into tomorrow's debt levels:

$$\begin{aligned}
b'_S &= \tilde{b}_S(b_S, b_L, y) \\
b'_L &= \tilde{b}_L(b_S, b_L, y)
\end{aligned} \tag{12}$$

Given this characterization of debt and default decisions, we can now define the equilibrium bond prices at which lenders are willing to offer contracts.

3.2 Bond Prices, Spreads, and Duration

Lenders are risk neutral and have an opportunity cost of funds equal to the risk-free rate r. Lenders are therefore willing to purchase a defaultable bond at a price equal to the expected discounted value of payments received from the bond. Each new issue of debt $\ell_t^S > 0$ or $\ell_t^L > 0$ by the borrower is a promise to pay a coupon payment every period in the future, conditional on not defaulting up to that period. The price of a new debt issue, then, is the

sum of the value of these coupon payments, each discounted by the risk-free rate and the probability of repayment up to the date of the payment. If the borrower's state is $\left(y_t, b_t^S, b_t^L\right)$, the prices q_t^S and q_t^L for loans ℓ_t^S and ℓ_t^L given future sequences of debts $\left\{b_{t+n}^S, b_{t+n}^L\right\}_{n=1}^{\infty}$ are given by

$$q_t^m = \sum_{n=1}^{\infty} \frac{\delta_m^{n-1}}{(1+r)^n} \int_{R(b_{t+1}^S, b_{t+1}^L)} \cdots \int_{R(b_{t+n}^S, b_{t+n}^L)} f\left(y_{t+n}, y_{t+n-1}\right) \cdots f\left(y_{t+1}, y_t\right) dy_{t+n} \cdots dy_{t+1} \quad (13)$$

for $m = \{S, L\}$. In each element of the sum on the right-hand side, the term δ_m^{n-1} corresponds to the coupon rate due in period $t + n$; $(1+r)^{-n}$ is the lender's n-period discount factor; and the term under the integral calculates the probability that the borrower receives output shocks that are in the repayment set each period up to $t + n$ – that is, the borrower repays up to period $t + n$. If default never occurs, that is $\int_{R(b_{t+1}^S, b_{t+1}^L)} f\left(y_{t+1}, y_t\right) dy_{t+1} = 1$ for all t, then the price at date t is equal to the risk-free price,

$$q_t^m = \frac{1}{1+r-\delta_m}.$$

Note that the price q_t^m of new debt issuances depends on current output, y_t, as it influences expectations of future output realizations which determine future default decisions. The price also depends on the entire future sequence of debts, $\left\{b_{t+n}^S, b_{t+n}^L\right\}_{n=0}^{\infty}$, since the outstanding debt in any period determines the decision to default, given the output shock. However, we can transform the infinite sum in (13) into a recursive expression for q_t^m by assuming that the lender forecasts the future debt levels using the borrower's own decision rules for debt, defined in (12), which are functions only of the debt choice next period. The sum in (13) can then be written with recursive notation as

$$\int_{R(b_S', b_L')} \frac{f(y', y)}{1+r} dy' + \delta_m \int_{R(b_S', b_L')} \left[\int_{R(\tilde{b}_S(b_S', b_L', y), \tilde{b}_L(b_S', b_L', y))} \frac{f(y'', y')}{(1+r)^2} dy''\right] f(y', y) dy' + ...$$

Each future debt level is replaced in sequence by the optimal decision rules $\tilde{b}_S(b_S', b_L', y)$ and $\tilde{b}_L(b_S', b_L', y)$. Prices for debt then satisfy the functional equations:

$$\hat{q}^S\left(b_S', b_L', y\right) = \frac{1}{1+r} \int_{R(b_S', b_L')} \left[1 + \delta_S \hat{q}^S\left(\tilde{b}_S\left(b_S', b_L', y'\right), \tilde{b}_L\left(b_S', b_L', y'\right), y'\right)\right] f\left(y', y\right) dy' \quad (14)$$

$$\hat{q}^L\left(b_S', b_L', y\right) = \frac{1}{1+r} \int_{R(b_S', b_L')} \left[1 + \delta_L \hat{q}^L\left(\tilde{b}_S\left(b_S', b_L', y'\right), \tilde{b}_L\left(b_S', b_L', y'\right), y'\right)\right] f\left(y', y\right) dy' \quad (15)$$

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If at any state (y, b_S, b_L) the borrower chooses to save, $\ell_S < 0$ or $\ell_L < 0$, the contract constitutes a promise from the lender to the borrower to pay thereafter the coupon payment. We assume that savings rates for the borrower are risk-free, so that the effective prices the borrower faces in the budget constraint in (9) are

$$q^S(b'_S, b'_L, b_S, b_L, y) = \begin{cases} \hat{q}^S(b'_S, b'_L, y) & \text{if } b'_S \geq \delta_S b_S \\ \frac{1}{1+r-\delta_S} & \text{if } b'_S < \delta_S b_S \end{cases} \qquad (16)$$

$$q^S(b'_S, b'_L, b_S, b_L, y) = \begin{cases} \hat{q}^L(b'_S, b'_L, y) & \text{if } b'_L \geq \delta_L b_L \\ \frac{1}{1+r-\delta_L} & \text{if } b'_L < \delta_L b_L \end{cases} \qquad (17)$$

We are modeling savings contracts as risk-free because they seem the most empirically relevant for emerging markets where savings are generally done at the international interest rates (generally with T-bills), yet borrowing contracts compensate investors for default. Additionally for computational convenience we are assuming that after default any savings that the government has in international financial markets are dissipated.[12],[13]

We define the yield-to-maturity on each bond as in the data, as the implicit constant interest rate at which the discounted value of the bond's coupons equal its price. That is, given a price q^m, the yield r^m is defined from

$$q^m = \sum_{n=1}^{\infty} \frac{\delta_m^{n-1}}{(1+r^m)^n}.$$

So,

$$r^S = \frac{1}{q^S} + \delta_S - 1 \text{ and } r^L = \frac{1}{q^L} + \delta_L - 1.$$

We define spreads as the difference between the yield on a defaultable bond and the default-free rate:

$$s^S = r^S - r \text{ and } s^L = r^L - r.$$

As output and debt change, the period-by-period probability of default varies over time,

---

[12] Ideally, one could have a model with four endogenous state variables, two for short- and long-term debt issuances and two for short- and long term savings. However this specification is computationally unfeasible. Thus, under the assumption that after default any savings that the government has in international financial markets are dissipated, we can maintain risk-free savings and defaultable short- and long-term debt with only two endogenous states.

[13] We could alternatively assume that savings contracts also carry the defaultable price, i.e. interest rates on savings are higher than the risk-free rate. Results are similar with this alternative specification. However, by having savings contracts being risk-free, we avoid having cases that seem empirically implausible where the government borrows large long-term loans just to increase its default probability and be able to save at excessively high interest rates.

and therefore the prices of long-term and short-term debt differ, since they each put different weights on repayment probabilities in the future, as seen in (13). Spreads on short-term and long-term bonds therefore generally differ, and the relationship between the two spreads changes over time, so that the spread curve is time-varying.

Finally, we define as in the data, the duration of debt issued at each date as the weighted average of the time until each coupon payment, with the weights determined by the fraction of the bond's value on each payment date:

$$d_m = \frac{1}{q^m} \sum_{n=1}^{\infty} n \frac{\delta_m^{n-1}}{(1 + r^m)^n}.$$

So,

$$d_S = \frac{1 + r^S}{(1 + r^S - \delta_S)} \text{ and } d_L = \frac{1 + r^L}{(1 + r^L - \delta_L)}. \tag{18}$$

For comparison, note that if the bonds were default-free, yields, and duration would be

$$
\begin{aligned}
r_m^{rf} &= r \\
d_m^{rf} &= \frac{1 + r}{1 + r - \delta_m}.
\end{aligned}
$$

We now define equilibrium. A recursive equilibrium for this economy is (i) a set of policy functions for consumption $\tilde{c}(b_S, b_L, y)$, new issuances for short-term debt $\tilde{\ell}_S(b_S, b_L, y)$ and long-term debt $\tilde{\ell}_L(b_S, b_L, y)$, perpetuity stocks for short-term debt $\tilde{b}_S(b_S, b_L, y)$ and long-term debt $\tilde{b}_L(b_S, b_L, y)$, repayment sets $R(b_S, b_L)$, and default sets $D(b_S, b_L)$, and (ii) price functions for short debt $q^S(b'_S, b'_L, b_S, b_L, y)$ and long debt $q^L(b'_S, b'_L, b_S, b_L, y)$, such that:

1. Taking as given the bond price functions $q^S(b'_S, b'_L, b_S, b_L, y)$ and $q^L(b'_S, b'_L, b_S, b_L, y)$, the policy functions $\tilde{b}_S(b_S, b_L, y)$, $\tilde{b}_L(b_S, b_L, y)$, $\tilde{\ell}_S(b_S, b_L, y)$, $\tilde{\ell}_L(b_S, b_L, y)$ and $\tilde{c}(b_S, b_L, y)$, repayment sets $R(b_S, b_L)$, and default sets $D(b_S, b_L)$ satisfy the borrower's optimization problem.

2. The bond price functions $q^S(b'_S, b'_L, b_S, b_L, y)$ and $q^L(b'_S, b'_L, b_S, b_L, y)$ reflect the borrower's default probabilities and lenders break even in expected value: equations (14), (15), (16), and (17) hold.

4   Default and Optimal Maturity

In this section we illustrate the mechanisms that determine the optimal maturity composition of debt in two simplified example economies. We view the borrower's choice as a portfolio allocation problem, in which the benefits and costs of short-term and long-term debt determine the relative amounts of each type issued. In the first example, we show that, in the presence of lack of commitment in future debt and default policies, short-term debt is more effective than long-term debt in transferring future resources to the present. If the borrower would try to borrow a lot of long-term debt, its price would fall to zero faster than if instead the large loan would be short-term; hence, short-term debt is beneficial for liquidity. In the second example, we show that long-term debt allows the borrower to avoid the risk of rolling over short-term debt at prices that differ across future states due to differences in default risk; hence, long-term debt provides insurance.

We construct the simplest possible examples to illustrate the mechanisms clearly. The economy lasts for three periods. In period 0, income equals zero, and in periods 1 and 2 income is stochastic (with details to be specified in each example). The borrower can default at any time, in which case consumption from then on is equal to y^{def}.

In each example, we compare the allocation with only one maturity of debt – one- or two-period bonds – against the allocation with both maturities of debt.[14] In each economy, with both maturities available, in period 0 the borrower can issue one- and two-period bonds b_0^1 and b_0^2 given price schedules $q_0^1(b_0^1, b_0^2)$ and $q_0^1(b_0^1, b_0^2)$, and consumption is

$$c_0 = q_0^1(b_0^1, b_0^2)b_0^1 + q_0^1(b_0^1, b_0^2)b_0^2.$$

In period 1, conditional on not defaulting, new short bonds b_1^1 are issued given price schedule $q_1^1(b_1^1)$. Consumption is equal to income plus net debt:

$$c_1 = y_1 + q_1^1(b_1^1)b_1^1 - b_0^1.$$

In period 2, conditional on not defaulting, the borrower pays off long- and short-term debt, and consumption equals income minus the repayment:

$$c_2 = y_2 - \left(b_1^1 + b_0^2\right).$$

In the cases with only one type of debt available, the budget constraints are modified accord-

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[14]It is straightforward to extend these examples for the case where long bonds pay a coupon in period 1 in addition to the payment in period 2, as long as y_1 and y_2 are sufficiently different.

ingly.

The risk neutral lenders discount time at rate r and offer debt contracts that compensate them for the risk of default and give them zero expected profits.

4.1 Example 1: Short-Term Debt Provides Liquidity

For this example we consider the following income process. Income in period 0 is equal to 0. Income in period 1 is equal to y. Income in period 2 can take 2 values, y^H or y^L with $y^H > y^L = 0$, and the probability of y^H is equal to g with $0 < g < 1$. Also, consumption in default, y^{def}, is equal to 0. To abstract from any insurance properties of debt, we assume that preferences are linear in consumption and given by

$$U = E[c_0 + \beta c_1 + \beta^2 c_2].$$

We assume that the borrower likes to front-load consumption, while lenders do not discount the future: $\beta < \frac{1}{1+r} = 1$, and we impose that consumption must be non-negative: $c_t \geq 0$ for $t = 0, 1$, and 2.

4.1.1 Only Two-Period Bonds

First, consider the borrower's problem when only two-period bonds are available in period 0, and one-period bonds are available in period 1. Under the assumption that $\beta < (gy^H - y^L)/(y^H - y^L)$, the solution to the borrower's problem is the following. In period 2, the borrower defaults when income equals y^L. In period 0, the borrower borrows against all his period 2 income, at price g, and in period 1 the borrower consumes his period 1 income, so consumption is

$$c_0 = g y_H$$

$$c_1 = y$$

$$c_2 \left(y^H \right) = 0, \; c_2 \left(y^L \right) = y^{def} = 0.$$

Although the borrower does not have preferences for smoothing consumption over time, and would prefer to consume everything up front, it is not possible to consume everything in period 0, because none of the income in period 1 can be borrowed against using two-period debt. This is because such a contract would require a two-period loan with face value larger than y^H, so that the borrower would have to save part of the period 1 income to repay the loan in period 2. Since the borrower cannot commit to this policy in period 0, however, the

optimal choice in period 1 would be not to save, and then to default in period 2 regardless of the level of income. That is, a debt contract that offered $q_0^2 b_0^2 = a + g y^H$, for any $a > 0$, is not possible, because the probability of default on the loan would be equal to one, and hence the price q_0^2 would be zero. Effectively, the threat of punishment for default in period 2 when the two-period loan is due does not induce the borrower to repay, because the borrower discounts the future, so that reducing consumption in period 1 is worse than facing the punishment for default in period 2. At the same time, the threat of punishment for default in period 1 is irrelevant, because none of the debt is due in period 1, and the threat of punishment cannot be used to induce savings.

4.1.2 One- and Two-Period Bonds

Now, if the borrower were able to issue one-period debt in period 0, consumption would be

$$c_0 = y + g y^H$$

$$c_1 = 0$$

$$c_2\left(y^H\right) = 0, \ c_2\left(y^L\right) = y^{def} = 0.$$

Multiple possible portfolios allow this consumption pattern. The borrower could use short-term debt to borrow against all period 1 income and long-term debt to borrow against all period 2 income ($b_0^1 = y$ with $q_0^1 = 1$, $b_0^2 = y^H$ with $q_0^2 = g$, $b_1^1 = 0$); or, the borrower could use only short-term debt, issuing bonds in period 0 and period 1 ($b_0^1 = y + g y^H$ with $q_0^1 = 1$, $b_1^1 = y^H$ with $q_1^1 = g$). Since all consumption occurs in the first period, utility in this case is higher than in the case with long-term debt only. With one-period bonds, the threat of punishment for default is being used in both periods to induce repayment.

In this example, long-term debt is illiquid in the sense that a loan that would provide the same level of consumption in the first period does not exist, because the price of long-term debt falls to zero. This example illustrates that in the presence of lack of commitment in debt policies and default risk, short-term debt is more liquid due to more lenient bond prices, and thus it is a superior instrument to provide up-front resources.[15]

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[15]It is easy to extend this example to an infinite horizon environment with deterministic and time varying output. A one-period bond economy can deliver higher initial consumption than a longer-term bond – two-period or perpetuity – economy. The main idea is again that the threat of punishment can be used more effectively with one-period bonds because longer-term contracts might require savings in the future which are impossible to induce with default punishments.

4.2 Example 2: Long-Term Debt Provides Insurance

For the second example, we focus on the motive for insurance by assuming that the borrower's preferences are given by

$$U = E[u(c_0) + \beta u(c_1) + \beta^2 u(c_2)]$$

with $u(\cdot)$ strictly concave and $\beta = 1$. We also now consider a different income process. Income in period 0 is equal to 0, income in period 1 is equal to y, and income in period 2 can take two values: y^H or y^L with $y^H > y^L$. The probability of y^H is learned in period 1 and can be either g or p with $0 < g < 1$ and $0 < p < 1$.

4.2.1 Only One-Period Bonds

First, consider the borrower's choice under the assumption that only one-period bonds are available. Under the assumption that $\frac{y + \frac{p+g}{2} y^H}{2 + \frac{p+g}{2}} > y^{def} > y^L - \frac{2y^H - y}{2 + \frac{p+g}{2}}$, the solution to the borrower's problem is the following. The borrower defaults in period 2 if income is y^L and does not default in all other states. Hence, $c_2^L(p) = c_2^L(g) = y^{def}$. Contingent on the realization of the probability p or g, consumption is equalized between period 1 and the high-income state in period 2:

$$
\begin{aligned}
c_1(p) &= c_2^H(p) \\
c_1(g) &= c_2^H(g)
\end{aligned}
$$

Finally, consumption in period 0 is set to equalize expected marginal utility in period 1 to marginal utility in period 0:

$$u'(c_0) = \frac{1}{2}\left(u'(c_1(p)) + u'(c_1(g))\right)$$

Importantly, $c_1(p) \neq c_1(g)$, so that consumption is not equalized across states within a period. With only short-term debt available, the borrower borrows in period 0, then borrows again in period 1. Debt issues are $b_0^1 = c_0$, $b_1^1(p) = \frac{y_H + c_0 - y}{1 + p}$, and $b_1^1(g) = \frac{y_H + c_0 - y}{1 + g}$. The price of debt issued in period 1 depends on the state realized: $q_1^1(p) = p$ and $q_1^1(g) = g$. Therefore, as long as $p \neq g$, the price at which debt is rolled over in period 1 differs across states, and consumption differs as well.

4.2.2   One- and Two-Period Bonds

Now, if the borrower has access to both one-and two-period bonds, it is possible to equalize consumption across all states in which the borrower does not default:

$$c_0 = c_1\left(p\right) = c_1\left(g\right) = c_2^H\left(p\right) = c_2^H\left(g\right) = \frac{\frac{p+g}{2}y_H + y}{\frac{p+g}{2} + 2}$$

The portfolio required involves using long-term and short-term debt in period 0, while borrowing nothing in period 1:

$$
\begin{aligned}
b_0^2 &= \frac{2y_H - y}{\left(\frac{p+g}{2} + 2\right)} \\
b_0^1 &= \frac{\left(1 + \frac{p+g}{2}\right)y - \left(\frac{p+g}{2}\right)y_H}{\left(\frac{p+g}{2} + 2\right)} \\
b_1^1 &= 0
\end{aligned}
$$

In this example the borrower faces risk because of the variation in bond prices across states in period 1 due to differences in default risk in period 2. Using long-term debt in period 0 allows the borrower to avoid the risk involved with rolling over short-term debt in period 1. The borrower benefits from this insurance with smoother consumption and higher utility.

Note that in period 0 short debt has a higher price than long debt, $q_0^1 = 1 > \frac{p+g}{2} = q_0^2$, yet the borrower issues long-term debt. The lower discount price on long debt is the insurance premium the borrower is willing to pay for insurance against the variation in bond prices in period 1. This insurance mechanism is the same as that emphasized in Kreps (1982), Angeletos (2002) and Buera and Nicolini (2004) in their models of the optimal maturity structure of debt with incomplete markets. The difference in our model is that the variation in bond prices comes from the government's inability to commit to repaying, rather than from variation in the lender's marginal rate of substitution.

4.3   Summary

In a standard incomplete markets model with fluctuating output and without default, a borrower would find the portfolio of long and short debt indeterminate if the risk-free rate were constant across time; the two assets would have payoffs that make them equivalent. However, in our model, the risk of default makes the two assets distinct. The first example illustrated that long-term debt is more illiquid than short-term debt due to the inability of

the borrower to commit to future debt and default policies. However, the second example illustrated that long-term is beneficial because it hedges against variations in short rates and provides insurance for default risk.

Insurance and liquidity shape the optimal maturity structure of debt for a borrowing government. The quantitative relevance of each of these forces depends on the specifics of preferences and the income process. Thus, in the next section we quantify these two sources by calibrating our general model to an actual emerging market economy.

5 Quantitative Analysis

5.1 Calibration

We solve the model numerically to evaluate its quantitative predictions regarding the dynamic behavior of the optimal maturity composition of debt and the spread curve in emerging markets. We calibrate an annual model to the Brazilian economy.

The utility function of the borrower is $u(c) = \dfrac{c^{1-\sigma}}{1-\sigma}$. The risk aversion coefficient is set to 2, which is a common value used in real business cycle studies. The risk-free interest rate is set to 4.0% annually, which equals the average annual yield of a two year U.S. bond from 1996 to 2004. The stochastic process for output is assumed to be a log-normal AR(1) process $\log(y_t) = \rho \log(y_{t-1}) + \varepsilon$ with $E[\varepsilon^2] = \eta_y^2$. Shocks are discretized into a seven-state Markov chain using a quadrature-based procedure (Tauchen and Hussey 1991). We use annual series of GDP growth for 1960–2004 taken from the World Development Indicators to calibrate the volatility of output. Due to the short sample, rather than estimating the autocorrelation coefficient we choose an autocorrelation coefficient for the output process of 0.9, which is in line with standard estimates for developed countries. The decay parameters of the short and long bonds, δ_S and δ_L, are set such that the default-free durations equal 2 and 10 years.

Following Arellano (2008) we assume that after default, output before reentering financial markets remains low and below some threshold, according to the following:

$$
h(y) = \begin{cases} y & \text{if } y \leq (1-\lambda)\bar{y} \\ (1-\lambda)\bar{y} & \text{if } y > (1-\lambda)\bar{y} \end{cases},
$$

where $\bar{y}$ is the mean level of output.

The output cost after default λ, the time preference parameter β, and the probability of reentering financial markets after default θ are calibrated jointly to match three moments in Brazil: the average 2-year spread of 6%, the volatility of the 2-year spread of 5.3 and the

24

average duration of debt issuances in Brazil of 5.5 years. Table 3 summarizes the parameter values.

Table 3: Parameters

	Value	Target
Discount factor lender	$r = 4\%$	U.S. annual interest rate 4%
Risk aversion	$\sigma = 2$	Standard value
Perpetuity decay factors	$\delta_S = 0.52$	Default-free durations of 2 and 10 years
	$\delta_L = 0.936$	
Stochastic structure	$\rho = 0.9, \eta = 0.022$	Brazil output
Probability of reentry	$\theta = 0.24$	Mean 2-year spread of 6%
Output after default	$\lambda = 0.025$	Volatility of 2-year spread of 5.3
Discount factor borrower	$\beta = 0.935$	Average bond duration of 5.5 years

5.2 Results

We simulate the model, and in the following subsections we report statistics on the dynamic behavior of spreads and the maturity composition of debt from the limiting distribution of debt holdings. The model contains a dynamic portfolio problem where the borrower chooses holdings of two defaultable bonds of shorter and longer duration. Below, we show how movements in the probability of default generate time-varying differences in the prices, and in the liquidity and insurance benefits of these two assets, which rationalize the movements in spread curves and maturity composition observed in the data.

5.2.1 Prices and Spreads

In the model all decision rules are functions of three state variables (b_S, b_L, y). However, for the purpose of illustration, we consider a single artificial state variable, the wealth of the economy: $w = y - b_S - b_L$. This variable is informative because it is highly correlated with the true state variables: the correlations between wealth and income, short debt and long debt equal 0.99, -0.56, 0.65 respectively. In what follows we analyze decision rules as functions of wealth, constructed as scatter plots from the model simulation.

We first analyze the default decision and spreads, and their relationship to wealth. Default happens when the economy has a low level of wealth, as the left panel of Figure 3 indicates. In the right panel of Figure 3, we see that, conditional on not defaulting, spreads are higher for relatively lower levels of wealth. However in equilibrium, for very low wealth levels the spread
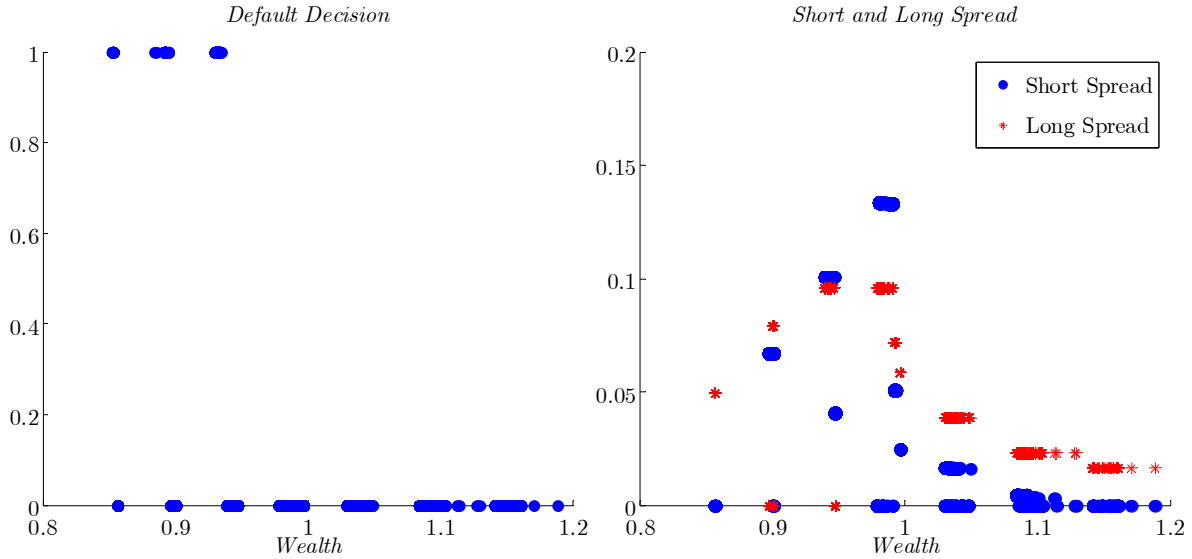
Figure 3: Default decision (left panel) and spreads (right panel) as a function of wealth.

is not as high because the borrower actually prefers to default than borrow at excessively low prices.

We now compare the model and data in terms of price and spread dynamics. The spread and price series for the data are for 2- and 10-year bonds of Brazil from Section 2.[16] For this comparison, we organize the data into quantiles based on the level of the short spread. Table 4 presents average spreads and prices for short and long debt across periods when short spreads are below their 25th and 50th percentile and above their 50th and 75th percentile.

The model generates spread curve dynamics that match the Brazilian data well. The first two columns of Table 4 present the model's short and long spreads, and the fifth and sixth columns present the data counterparts. In the model when default is unlikely, both spreads are low, and the spread curve is upward-sloping: when the short spread is below its 25th percentile, for example, the average short spread is 1.04%, and the average long spread is 3.83%. In contrast, when the probability of default is higher, both spreads rise, and the spread curve becomes downward-sloping: when the short spread is above the 75th percentile,

---

[16]As explained in Section 2, spread curves for Brazil are estimated using equation (21) in the Appendix. In the model for simplicity we are directly applying the yield to maturity formulas to compute spreads. However, we could also estimate in the model spread curves using a reduced equation with only two parameters. Results under this alternative estimate are similar, although the spread curves are slightly steeper.

Table 4: Spread Curves

s^S pct	MODEL				DATA			
	s^S	s^L	q^S/q^S_{rf}	q^L/q^L_{rf}	s^S	s^L	q^S/q^S_{rf}	q^L/q^L_{rf}
< 25	1.04	3.83	0.98	0.73	2.12	6.12	0.96	0.58
< 50	1.37	3.85	0.97	0.74	2.75	6.07	0.95	0.58
≥ 50	11.68	8.68	0.81	0.56	9.25	9.99	0.85	0.42
≥ 75	13.30	9.57	0.79	0.52	13.55	12.23	0.79	0.34
Overall Mean	6.58	6.25	0.89	0.65	6.01	8.04	0.90	0.50
Standard Deviation	5.53	3.39	0.08	0.15	5.85	3.39	0.08	0.13

the average short spread is 13.3%, and the average long spread is 9.57%. The fact that short spreads rise more than long spreads is also reflected in the difference in the volatilities of the two spread series: the standard deviation of the long spread is lower than that of the short spread. Compared to the data for Brazil, the model captures the difference observed in the slope of the spread curve associated with periods of high and low short spreads, as well as the difference in volatilities of the two spreads. The model also matches quantitatively the volatility of the long spread. The model's overall average short and long spreads, however, are both pinned down by the average probability of default, so the average spread curve is quite flat.

Underlying the time-varying spreads is the interaction of the dynamics of income and debt with the price schedules for short and long debt. (Figure 6, in the Appendix, illustrates the equilibrium price schedules for short debt $\hat{q}^S(b'_S, b'_L, y)$ and long debt $\hat{q}^L(b'_S, b'_L, y)$.) However, the mapping from discount prices to spreads is not linear (eq. 1). Thus, to understand the total default probabilities of each bond, it is informative to analyze *price ratios* defined as defaultable discount prices relative to default-free prices for a bond with duration m: q^m/q^m_{rf}. The price ratio of each bond is the total repayment probability over the lifetime of the bond. Table 4 presents statistics for these price ratios in the model and the data. The table shows that contrary to spreads, price ratios for short-term debt are always higher than for long-term debt both in the model and in the data. Moreover, price ratios are disproportionately lower for short-term debt when short spreads are high both in the model and in the data.[17]

The distinct dynamics of price ratios and spreads can be understood as follows. Price ratios reflect cumulative repayment (and default) probabilities, whereas spreads reflect average default probabilities. Cumulative default risk for long-term debt is always larger than for short-term debt both in the data and the model. However, annualized (average) default risk

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[17]For Argentina, Mexico, and Russia price ratios for short-term debt are also always higher than for long-term debt, and the difference is accentuated in times of high spreads.

can be lower on long-term debt during times when the annual default probability in the short run is larger than the annual default probability in the long run. Thus, contrary to common belief in sovereign debt markets, the spread is not a comprehensive measure of the relative cost of borrowing in different maturities of debt. In particular, in times when the probability of default is high, short-term debt may appear to be more expensive for the borrower than long-term debt, in the sense that it has a higher spread, although long-term debt is worse in the sense that it has a lower price, relative to the risk-free price. The connection between the dynamic behavior of prices and spreads in our model is borne out in the data as well.

The preceding discussion also indicates that the important feature of our model for generating the observed dynamics of prices and the spread curve is that the probability of default is mean-reverting: a period with high probability of default is followed by a period with lower probability of default, and vice versa. The effects of mean-reverting default probabilities on the spread curve are the same as those highlighted by Merton (1974) in the case of credit spreads for corporate debt. In our model the probability of default is endogenously mean-reverting as a result of the dynamics of the output process and debt accumulation. When output is high, it is also expected to be high in the near future, so the probability of default in the next period is low. The economy borrows a large amount at low interest rate spreads, so that in states where the economy is hit by a bad shock, default becomes more likely further in the future. In contrast, when the likelihood of imminent default is high, the economy avoids default in the next period only in states with high output. Conditional on not defaulting, then, output is expected to remain high, and the probability of default further in the future falls. The persistence and mean reversion of default and repayment probabilities driven by the dynamics of debt and income therefore rationalize the dynamic behavior of the spread curve observed in the data.

5.2.2 Maturity Composition

We now present the quantitative predictions for the maturity composition of debt. It is important to note that we analyze the optimal maturity composition of debt in a framework that generates the empirically observed dynamics of debt prices. As discussed in Section 4, two forces in the model shape the dynamic behavior of the maturity composition. First, long-term bonds insure against future price fluctuations; we find that the insurance motive is more valuable in times of high wealth. Second, short-term bonds are more liquid and allow larger transfers of resources to the present with a smaller change in price; we find that the liquidity advantage for short debt is more valuable in times of low wealth. Given the negative correlation between wealth and spreads, these two forces lead the borrower to use long-term

debt more heavily in times when spreads are low and shift toward shorter term debt when spreads are high.

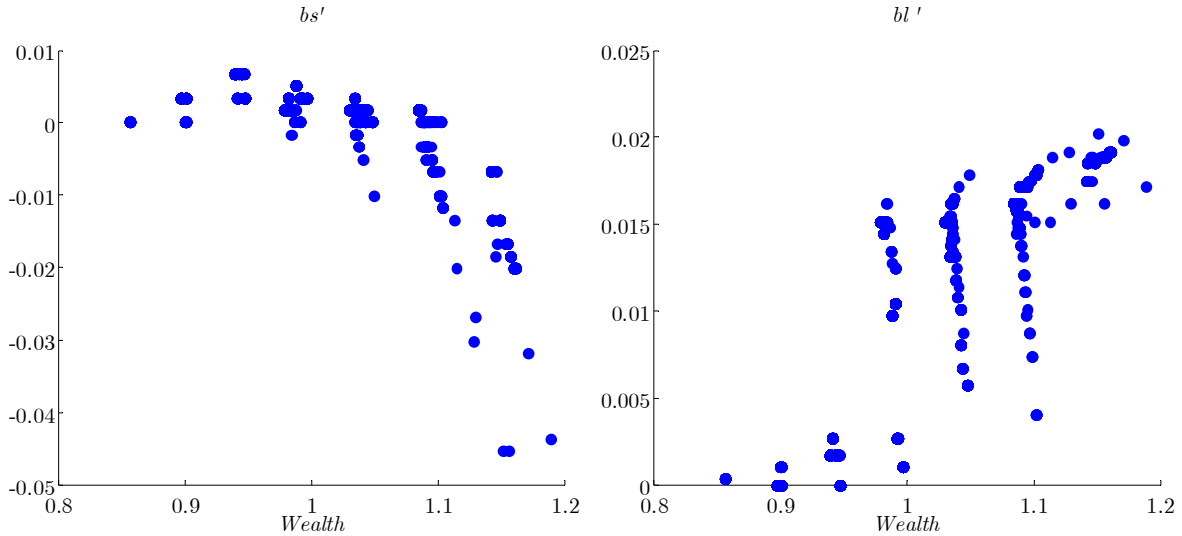Figure 4 plots the equilibrium choices of the perpetuity stocks b'_S and b'_L for different

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w

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Figure 4: Short-term debt (left panel) and long-term debt (right panel) as a function of wealth.

New issuances of short and long debt ℓ_S and ℓ_L are closely correlated with the perpetuity stocks; thus, in high wealth states debt issuances are mostly long term and debt issuances shift to shorter term in low wealth states. To compare issuances of long and short debt between model and data, we now compute conditional averages of the duration of new debt issuances, based on the level of the short spread. Average duration in the model is the sum of the duration (equation 18) of each new bond issuance weighted by its share in total new debt issued. Moreover, given that in the data we only have information on debt issuances ($\ell_S > 0$ and $\ell_L > 0$), in the model we compute average duration of the debt component of the portfolio. Table 5 reports the average duration of new debt issuances when spreads are above their median relative to when spreads are below their median in the model and in the Brazilian data. Debt duration in the model mirrors the dynamics of duration in the bond data of Brazil. In the model, average duration when spreads are low is longer and equals 5.30 years, whereas it shortens to 3.44 when spreads are high. In Brazil, the average duration of bonds issued when spreads are high equals 6.03 years and shortens to 4.47 years when

spreads are low.

Table 5: Average Duration of New Debt Issuances

	MODEL	DATA
s^S pct		
< 50	5.30	6.03
≥ 50	3.44	4.47
Overall	4.38	5.5

In Figure 5, we illustrate the trade-off between liquidity and insurance that determines the decision of the debt portfolio. In the left panel of the figure, we show the liquidity benefits of short-term debt by plotting the increase in consumption that would be possible by marginally increasing short-term debt, relative to the increase in consumption that is possible by issuing more long-term debt. Specifically, define $QB(b'_S, b'_L, b_S, b_L, y) \equiv q^S(b'_S, b'_L, y)\ell_S + q^L(b'_S, b'_L, y)\ell_L$ as the quantity of consumption that is attained with a certain debt policy b'_S, b'_L, given the state (b_S, b_L, y). In the figure's left panel, we plot the ratio of small deviations from the equilibrium debt policy for short-term debt relative to long-term debt,

$$\frac{\Delta_S}{\Delta_L} \equiv \frac{QB(\tilde{b}_S(b_S, b_L, y) + \varepsilon_S, \tilde{b}_L(b_S, b_L, y), b_S, b_L, y) - QB(\tilde{b}_S(b_S, b_L, y), \tilde{b}_L(b_S, b_L, y), b_S, b_L, y)}{QB(\tilde{b}_S(b_S, b_L, y), \tilde{b}_L(b_S, b_L, y) + \varepsilon_L, b_S, b_L, y) - QB(\tilde{b}_S(b_S, b_L, y), \tilde{b}_L(b_S, b_L, y), b_S, b_L, y)}$$

where ε_S and ε_L are small, and are chosen so that if debt prices were always the default-free prices, the ratio plotted would be exactly equal to 1. As the figure shows, this ratio in our model is always above 1 and on average it equals 1.33. Thus, short-term debt is more liquid because consumption can always be marginally increased more with short-term debt than with long-term debt. The reason is that price schedules for short-term debt are more lenient by having higher prices –lower default premia– that decrease by less as debt increases. Looking across wealth levels, this difference is especially large in lower wealth states. Thus, short-term debt is particularly useful for increasing consumption when wealth is low.

In our model, short- and long-term debt prices are actuarially fair for the lender. Thus, if the schedules of short-term debt are more lenient, this means that the borrower will repay in more future states. However, this does not mean that the borrower is indifferent to acquiring a certain level of resources with a small safer short-term loan, versus a large risky long-term loan. In fact, we know that if the borrower chooses to default in some future states with the long-term loan while choosing to repay in those same states with the short-term loan, he

must be better off by repaying the short loan because he always have the option to default. Moreover, default risk in our model limits the maximum level of resources that the borrower can get.[18] The key is that in our model these endogenous limits and price schedules are tighter for long term debt relative to short term debt. The average ratio of borrowing in each state to the short-term debt limit versus to the long-term debt limit equals 1.84. Thus, the potential increase in consumption from exhausting short-term debt is 84% larger than from exhausting long-term debt. Figure 5 also illustrates the tighter price schedules for long-term debt as increases in short debt result in higher consumption because of more lenient prices. As discussed in the examples in Section 4, short-term debt can deliver larger absolute consumption levels and larger consumption with smaller loans, because of the inability of the borrower to commit to saving sufficiently to repay long-term debt. Effectively, the threat of default punishment is more effective to induce repayment of shorter-term debt because repayment of short debt does not require future savings.

Although short-term debt is more liquid, long-term debt provides more insurance for price fluctuations that can lead to capital outflows in recessions. The right panel of Figure 5 plots the correlation of the trade balance $tb' = y' - \tilde{c}(b'_S, b'_L, y')$ and output y' the following period conditional on not defaulting for each wealth level today. The correlations are computed using the borrower's optimal consumption decision rules the following period. When wealth is large and the portfolio is mostly long term, the correlation between the equilibrium trade balance and output tomorrow is positive, i.e. capital outflows in booms and capital inflows in recessions. However, when wealth is small and the portfolio is mostly short term, the correlation is negative, i.e. larger capital outflows in recessions than in booms. The reason why the model delivers capital outflows in recessions is that the price schedules for debt are more stringent in recessions than in booms due to countercyclical default risk. The correlation between output and the short spread in the model is -0.54. However, by issuing long-term debt the borrower can avoid being forced to save in recessions due to excessively adverse price schedules.

Table 6 provides more details about the maturity composition and the forces underlying its determination. The first two columns of Table 6 show the model's portfolio, conditional on different levels of wealth. When wealth is high the borrower issues on average 50% of his debt in long-term bonds, and 50% in short-term bonds. When wealth is low the average maturity composition shifts to 39% in long-term bonds, and 61% in short-term bonds. As illustrated above, the optimal portfolio depends on the valuations of the insurance benefits

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[18] Arellano (2008) shows that a one short-term asset version of our model generates an endogenous Laffer Curve for borrowing which features a debt limit.
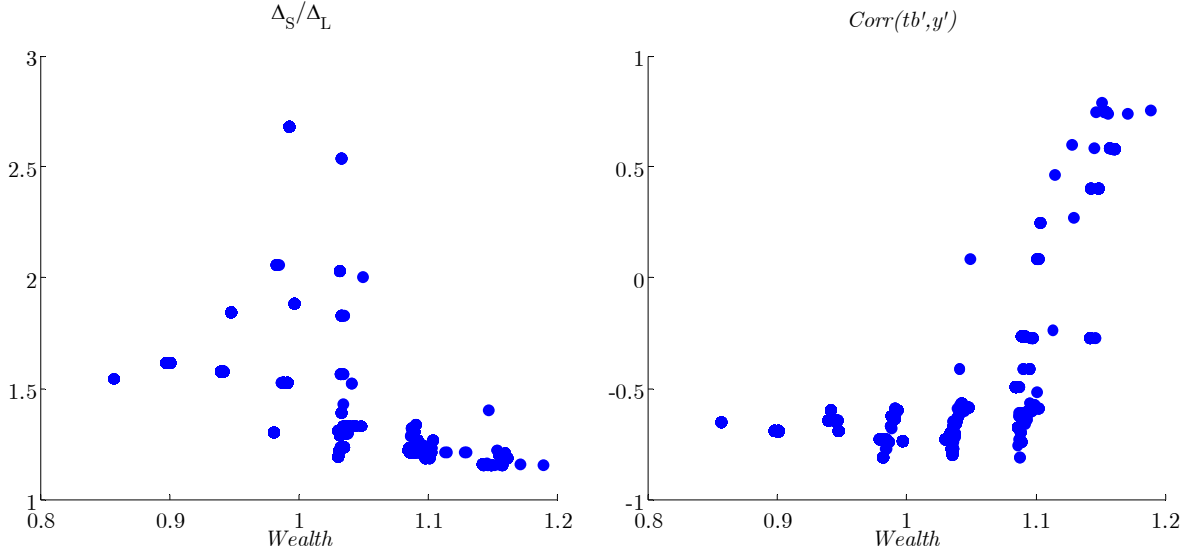
Figure 5: Liquidity benefit of short debt (left panel) and insurance benefit of long debt (right panel)

of long debt relative to the liquidity and cost advantage of short debt. The table reports two alternative metrics to evaluate these benefits.

The insurance benefits of long-term debt can be measured by the comovement between the borrower's intertemporal marginal rate of substitution, $\beta u'(c')/u'(c)$, and the short-term bond price next period, $q^{S\prime}$. As the table shows, this covariance is negative: in states with high marginal utility of consumption, the short bond price is expected to be low. Issuing long debt today allows the borrower to avoid having to issue short-term debt tomorrow in states when prices are low. The insurance benefit is stronger in high wealth periods, as this covariation is -0.21 relative to -0.16.

To measure the cost advantage of short-term debt, we compute the slope of the price ratios of the two debt classes: $\frac{q^L(1+r-\delta_L)}{q^S(1+r-\delta_S)}$. As the table shows, long-term debt is always more costly in terms of carrying lower total repayment probabilities, $\frac{q^L(1+r-\delta_L)}{q^S(1+r-\delta_S)} < 1$. Increasing consumption using short-term debt is cheaper in that it contains lower default risk. And short-term debt is disproportionately cheaper in low wealth times, as the slope of price ratios is lower, 0.70 relative to 0.75. Thus, a larger share of short-term debt in low wealth times can be understood as a reaction to the more expensive long-term debt.

Moreover, as in emerging markets data, periods of longer-term debt issuance correspond to periods with lower spreads, and upward-sloping spread curves. Specifically, when wealth is above its median, the average short spread equals 2.62%, and the long spread is on average

<div align="center">Table 6: Model Maturity Composition</div>

Wealth	$\ell^S/(\ell^S+\ell^L)$	$\ell^L/(\ell^S+\ell^L)$	spr^S	spr^L-spr^S	$\frac{q^L(1+r-\delta_L)}{q^S(1+r-\delta_S)}$	$cov\left(\frac{\beta u'(c')}{u'(c)},q^{S'}\right)$
< 50 pct	0.61	0.39	10.84	-2.43	0.70	-0.16
≥ 50 pct	0.50	0.50	2.62	1.78	0.75	-0.21

1.78% above the short spread. On the other hand, when wealth is low, the short spread is on average 10.84% and the long spread is on average 2.43% below the short spread.

In summary, through the lens of our model, the maturity structure of defaultable debt in emerging markets and its covariation with spread curves and levels can be rationalized by two factors: hedging advantage of long-term debt for insuring against fluctuations in future default risk, and a liquidity advantage of short-term debt for providing higher resources with more lenient prices.

6   Conclusion

In this paper, we have developed a dynamic model to study the maturity composition of sovereign bonds. In emerging markets data, changes in the maturity composition of debt comove with changes in the term structure of spreads: when spreads on short-term debt are low, long-term spreads are higher than short-term spreads, and the maturity of debt issued is long. When short-term spreads rise, long-term spreads rise less, and the maturity of debt shortens. Our model simultaneously reproduces the patterns observed in the term structure of spreads and bond prices, and the maturity composition of debt. Changes in the spread curve, which reflects the average default probability at different time horizons, result from the output dynamics and the endogenous dynamics of debt. Issuing long-term debt insures against future fluctuations in short-term spreads that come from changes in default risk. Short-term debt provides more liquidity because it allows the borrower to avoid the more severe commitment problem in repaying long-term debt. With these two forces, the model generates the pattern of issuances observed in the data. Long-term debt is issued mostly in times of high wealth and low spreads, when the insurance motive is the strongest. Short-term bonds are used more heavily in times when wealth is low and spreads are high, because expectations of the borrower's future debt and default choices restrict the availability of long-term debt more heavily than of short-term debt.

Our main innovation has been to introduce multiple, long-term assets into a dynamic

model with endogenous default. We view the resulting framework as useful for addressing a variety of other questions for which it is important to analyze a trade-off in maturity choice with defaultable debt. Natural applications are the maturity structure of consumer and corporate debt. The literature on consumer bankruptcy thus far has focused on modeling very short-term unsecured credit (Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) and Livshits, MacGee, and Tertilt (2007)). However, it would be interesting to analyze both long-term and short-term defaultable loans, such as mortgages and credit card debts. In addition, the mechanisms in our model are likely to be relevant in corporate debt given the similarity between our facts on emerging market spread curves and the cross section of corporate debt spread curves. Default risk has been shown to have important implications on firm's dynamics (Cooley and Quadrini (2001) and Arellano, Bai, and Zhang (2007)). The model of this paper can be used to further understand how the maturity choice can influence the entry, exit, and growth of firms. Overall, our paper provides a tractable framework to study defaultable debt of multiple maturities appropriate for these questions, and has highlighted the relevant economic trade-offs important for understanding maturity choice in the presence of default.

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Appendix

Data Description

All the sovereign bond data are from Bloomberg. For the four countries we examine, we use all bonds with prices quoted at some point between March 1996 and May 2004, with the following exceptions. We exclude all bonds with floating-rate coupon payments, and at every date, we exclude bonds that are less than three months to maturity, following Gurkaynak, Sack, and Wright (2006). For each country, we estimate spreads starting from the first week for which at least four bond prices are available every week through the end of the sample. We use data from 110 bonds for Argentina, 71 for Brazil, 63 for Mexico, and 25 for Russia. To estimate default-free yield curves, we use data on U.S. and European government bond yields. The U.S. data are from the Federal Reserve Board, and the European data are from the European Central Bank.[19] For constructing the quarterly maturity and duration statistics, we also include bonds issued during the sample period that did not have prices quoted, and use the estimated spread curve to construct their prices according to equation (19).

Spread Curve Estimation

We use a method proposed by Svensson (1994), and used recently by Gurkaynak, Sack, and Wright (2006) for the United States, and Broner, Lorenzoni, and Schmukler (2007) for a sample of emerging markets, to fit a spread curve to this data using a simple functional form suggested by Nelson and Siegel (1987).

A coupon bond is priced as a collection of zero-coupon bonds, each with maturity given by a coupon payment date, and face value given by the cash flow on that payment date. The price at date t of a bond issued by country i, paying an annual coupon rate c at dates $n_1, n_2, \ldots n_J$ years into the future, is

$$p_t^i(c, \{n_j\}) = \sum_{j=1}^{J} c(1 + r_t^i(n_j))^{-n_j} + (1 + r_t^i(n_J))^{-n_J} \tag{19}$$

with the face value of the bond paid on the last coupon date.

Spreads are defined as $s_t^i(n) = r_t^i(n) - r_t^*(n)$, where $r_t^*(n)$ is a default-free yield curve. We

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[19]The U.S. data are the Treasury constant maturities yields, available at http://www.federalreserve.gov/releases/h15/data.htm.

The European data are Euro area benchmark government bond yields, which is an average of European national government bond yields available at http://sdw.ecb.europa.eu.

introduce another measure of a bond's price, the *yield to maturity*, that is useful in estimating spreads. For a bond with coupon c and payments in $n_1, n_2, \ldots n_J$ years, the yield to maturity is the rate $y(c, \{n_j\})$ that solves

$$p_t^i(c, \{n_j\}) = \sum_{j=1}^{J} c(1+y)^{-n_j} + (1+y)^{-n_J} \tag{20}$$

with $p_t^i(c, \{n_j\})$ given by (19). That is, the yield to maturity is the constant rate of interest at which the bond's price equals the discounted value of its payments.

We define spreads as a parametric function of maturity following Nelson and Siegel (1987)

$$s_t^i(n; \beta_t^i) = \beta_{1t}^i + \beta_{2t}^i \left(\frac{1 - e^{-\lambda n}}{\lambda n} \right) + \beta_{3t}^i \left(\frac{1 - e^{-\lambda n}}{\lambda n} - e^{-\lambda n} \right) \tag{21}$$

for each country i, where $\beta_t^i = (\beta_{1t}^i, \beta_{2t}^i, \beta_{3t}^i)$ and λ are parameters. For default-free bonds, we define

$$r_t^{\$}(n; \beta_t) = \beta_{1t}^{\$} + \beta_{2t}^{\$} \left(\frac{1 - e^{-\lambda n}}{\lambda n} \right) + \beta_{3t}^{\$} \left(\frac{1 - e^{-\lambda n}}{\lambda n} - e^{-\lambda n} \right) \tag{22}$$

and

$$r_t^{\euro}(n; \beta_t) = \beta_{1t}^{\euro} + \beta_{2t}^{\euro} \left(\frac{1 - e^{-\lambda n}}{\lambda n} \right) + \beta_{3t}^{\euro} \left(\frac{1 - e^{-\lambda n}}{\lambda n} - e^{-\lambda n} \right) \tag{23}$$

for US (\$) and Euro (€) bonds.

As described by Nelson and Siegel (1987) and Diebold and Li (2006), the three components of this curve correspond to a "long-term," or "level" factor (the constant), a "short-term," or "slope" factor (the term multiplying β_2) and a "medium-term," or "curvature" factor (the term multiplying β_3). Linear combinations of these factors can capture a broad range of shapes for the spread curve.

We first estimate the parameters $\beta_t^{\$}$ and $\beta_t^{\euro}$ by OLS, using U.S. and Euro bond yields. Throughout, we follow Diebold and Li (2006) by setting the parameter $\lambda = 0.714$, so that the term multiplying β_3 in all countries' spread curves is maximized when $n = 2\frac{1}{2}$ years.

Then, given a set of parameters β_t^i, we use equation (19) to price each of country i's bonds at date t using the risk-free yield given by (22) or (23) and the spread given by (21):

$$p_t^i(c, \{n_j\}; \beta_t^i) = \sum_{j=1}^{J} c(1 + s_t^i(n_j; \beta_t^i) + r_t^*(n_j))^{-n_j} + (1 + s_t^i(n_J; \beta_t^i) + r_t^*(n_J))^{-n_J},$$

where r_t^* refers to $r_t^{\$}$ if the bond is denominated in U.S. dollars, or $r_t^* = r_t^{\euro}$ if the bond is denominated in a European currency. We use equation (20) to compute a yield-to-maturity

for each bond, given the parameters β_t^i, solving the following for $y(c, \{n_j\}; \beta_t^i)$:

$$p_t^i(c, \{n_j\}; \beta_t^i) = \sum_{j=1}^{J} c(1 + y(c, \{n_j\}; \beta_t^i))^{-n_j} + (1 + y(c, \{n_j\}; \beta_t^i))^{-n_J}.$$

We estimate the parameters β_t^i nonlinearly by GLS to minimize the sum of squared deviations of the predicted yields-to-maturity, $y(c, \{n_j\}; \beta_t^i)$ from their actual values. That is, our estimated parameters solve

$$\min_{\beta_t^i} \sum (y(c, \{n_j\}; \beta_t^i) - y(c, \{n_j\}))^2,$$

where the summation is taken over all bonds issued by country i with prices available at date t. As discussed in Svensson (1994), minimizing yield to maturity errors rather than price errors gives a better fit for short-term yields to maturity, because short-term bond prices are less sensitive to their yields to maturity than long-term bond prices.

The following features present in the data require modification of the basic bond pricing equation (19):

1. Between coupon periods, the quoted price of a bond does not include accrued interest, so we subtract from the bond price the portion of the next coupon's value that is attributed to accrued interest.

2. For bonds with principal payments guaranteed by U.S. Treasury securities, we discount the payment of principal by the risk-free yield only, without the country spread.

3. For bonds with coupon payments that increase or decrease over time with certainty ("step-up" and "step-down" bonds, respectively), we modify the sequence of payments in equation (19) accordingly.

Further Statistics on Spread Curves

Tables 7 reports further spread curves and spread volatilities for all countries.

Table 7: Average Spreads and Volatility

	Maturity (years)	Overall (%)	Std. Dev	When 2-year spread is above/below nth percentile					
				< 10th	< 25th	< 50th	≥ 50th	≥ 75th	≥ 90th
Argentina	2	5.23	7.92	1.11	1.63	2.16	8.30	12.64	23.41
	5	6.03	4.46	2.50	3.08	3.76	8.30	11.06	17.02
	10	7.02	4.15	3.45	4.10	4.95	9.08	11.49	16.48
	15	7.43	4.24	3.82	4.49	5.41	9.44	11.78	16.58
Brazil	2	6.01	5.85	1.47	2.12	2.75	9.25	13.55	21.19
	5	7.69	4.80	5.36	5.03	5.11	10.27	13.55	19.11
	10	8.04	3.39	5.89	6.12	6.08	9.99	12.23	15.43
	15	8.10	2.97	5.92	6.47	6.40	9.80	11.61	13.85
Mexico	2	1.87	1.37	0.31	0.57	0.95	2.78	3.51	4.85
	5	2.87	1.10	1.81	2.01	2.27	3.47	4.01	5.24
	10	3.81	1.05	2.55	2.86	3.30	4.33	4.76	5.72
	15	4.19	1.09	2.81	3.18	3.70	4.68	5.07	5.91
Russia	2	5.04	3.45	1.66	2.00	2.69	7.37	9.56	12.22
	5	5.57	3.18	2.51	2.66	3.30	7.82	10.13	12.34
	10	5.45	2.59	2.33	2.56	3.46	7.43	8.90	10.30
	15	5.37	2.44	2.20	2.48	3.50	7.22	8.29	9.35

Model's Debt Price Schedules

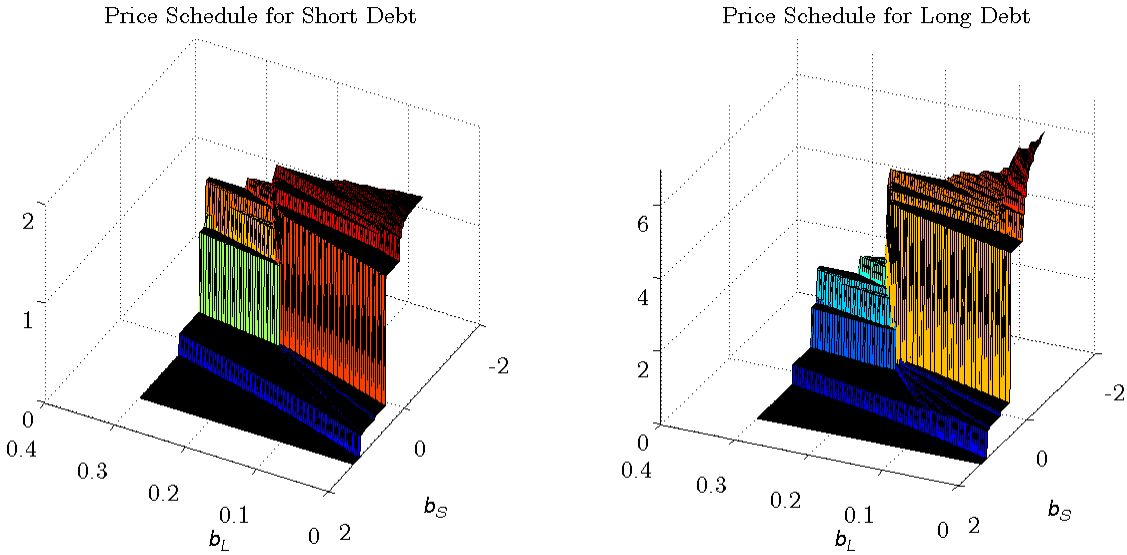Price Schedule for Short Debt

Price Schedule for Long Debt

Figure 6: Price schedules for short- and long-term debt when income is at its mean