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Coping with Spain's Aging: Retirement Rules and Incentives

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Abstract

This paper examines Spain's pension system and evaluates the macroeconomic and welfare effects of parametric reform to the PAYG system using a DGE framework in the Auerbach-Kotlikoff tradition. The study highlights the interactions between the averaging period—used to compute the pension benefit—and the indexation of pension contributions, evaluates the complementarities between reforms extending the averaging period and those increasing the retirement age, and between these reforms and the financing of aging-related expenditure pressures. Without reforms, the simulations suggest that age relating spending increases about 16 percentage points of GDP by 2050. Also, the simulations suggest that extending the averaging period can limit the adverse macroeconomic effects of aging as much as increasing the retirement age during the demographic transition.

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I. INTRODUCTION

Policymakers are increasingly aware that population aging poses significant macroeconomic challenges, which include safeguarding growth and fiscal sustainability, maintaining inter-generational equity, and ensuring a smooth operation of labor markets. Pension reforms—from minor adjustments of pay-as-you-go (PAYG) systems to the adoption of fully funded-schemes—have been proposed and implemented in a number of countries. These reforms have received a significant amount of attention in the economic literature, but changes in the contribution-benefit linkages have not (exceptions are Auerbach and Kotlikoff, 1987, and De Nardi, Imrohoroglu, and Sargent, 1999), particularly in the context of quantitative assessments using applied dynamic general equilibrium (DGE) models.

In this connection, Spain presents a valuable opportunity for analysis and serves as an interesting case study due to its specific pension rules, social agreements, and demographic profile. First, the Spanish public PAYG system calculates pension benefits based on average, inflation-indexed, wage earnings in the corresponding averaging period. Moreover, reforms doubled the averaging period to the last fifteen years of an individual's work life in 1997, with the view of enhancing the contribution-benefit linkage. Second, Spain reached a broad political and social consensus—known as the *Pacto de Toledo*—on the need to preserve the public PAYG system through reforms geared to ensuring its sustainability, including by aligning benefits and contributions; the *Pacto* ruled out privatization and reforms toward compulsory fully-funded schemes.[1] Third, even considering Spain's remarkable immigration

[1] The *Pacto* was established through a broad political agreement—ratified by the Spanish Congress by virtual unanimity in April, 1995—seeking to preserve the public PAYG nature of the old-age pension system through

(continued…)

phenomenon, the fiscal costs of aging are expected to be larger than elsewhere in Europe, and thus pose more substantial challenges for long-run fiscal sustainability.

This paper evaluates the macroeconomic and welfare effects of introducing parametric reforms to the Spanish PAYG system using a DGE framework in the Auerbach-Kotlikoff tradition. Consistent with Spain's social consensus, the analysis focuses on reforms that increase the retirement age and extend the averaging period under alternative tax policies to finance old-age related spending. In principle, it is not clear whether extending the averaging period can deliver welfare and aggregate labor gains. The he analysis illustrates the trade-offs involved in reforming the pension system: it increases households' labor effort, particularly, at high-skill ages by removing inter-temporal distortions, but it also discourages labor effort by reducing pension benefits as real wage rates—driven by technological progress—increase over time.[2] In pursuing this analysis, this study: (a) highlights the interactions between the averaging period and the indexation of contributions; (b) explores the complementarities between reforms extending the averaging period and those increasing the retirement age; and (c) evaluates the complementarities between these reforms and the financing of the fiscal expenditure pressures that arise during the demographic transition.

The simulations of the baseline scenario—where the retirement age and the averaging period remain unchanged in the future, and aging-related fiscal pressures are financed with consumption taxes—highlight the extent of the aging shock: pension expenditures increase 16 percentage points of output in the period 2008-2050, more than twice as much as in

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parametric reforms; labor unions joined the call for reform a year later. The *Pacto* was updated in 2003 and continues to stress the need to align pension benefits and contributions.

[2] For an analysis of Spain's pension rule, including the averaging period, see Diamond (2001).

official and European Commission projections (7.1 percentage points) and 3 percentage points more than Rojas (2005). To finance this spending, the consumption tax rate rises more than 30 percentage points, with severe macroeconomic consequences.

Simulations of pension reforms point to the macroeconomic benefits of addressing the demographic shock. When combined with specific tax and debt management policies, parametric reforms increasing the retirement age and extending the averaging period to the entire work life deliver the maximum welfare gains to future generations, but not to all current generations. In particular, the main results of the simulations are as follows:

- Increasing the retirement age attenuates expenditure pressures and the needed increase in taxes to finance the demographic shock, while boosting the aggregate capital stock and output. It also limits the increase in pension expenditure over the next 40 years by 4 percentage points of GDP and the consumption tax rate increase to about 8 percentage points. Furthermore, the reform induces a "bust-boom" cycle in the aggregate labor supply: aggregate effective labor declines over the next two decades, but increases thereafter, reflecting grandfathering clauses and households' inter-temporal labor substitution effects.

- Conditional on increasing the retirement age, extending the averaging period further limits the increase in pension expenditures over the next 4 decades by 4 percentage points of output. Thus, extending the averaging period is as important in mitigating the macroeconomic effects of aging as increasing the retirement age during the peak of the demographic shock. In contrast to increasing the retirement age, extending the averaging period—conditional on raising the retirement age—triggers a "boom-bust" cycle in aggregate effective labor, making it scarcer when the demographic storm intensifies.

- Pre-funding the demographic shock by permanently increasing the consumption tax rate in 2008 further attenuates the negative macroeconomic effects during the peak of the demographic shock. However, it shifts the tax burden from generations that are active when the dependency ratio peaks to current and future generations.

- The relative contribution of reforms to lowering tax rates varies over time. At the peak of the demographic transition, extending the averaging period accounts for half of the tax rate reduction obtained from a full pension reform. In the long-term, however, extending the averaging period accounts for just a tenth of the tax rate reduction.

The combined effects of reforms and pre-funding creates intergenerational welfare transfers, with net loses for current generations and net gains for future generations. Thus, an additional mechanism is needed to ensure a Pareto improving package of reforms. [In this connection, this paper studies two mechanisms: a system of intergenerational transfers from net winners to net losers, and a delay in the permanent increase in the consumption tax rate to avoid hurting current generations.] [pending]

The rest of the paper is organized as follows. Section II discusses the model and calibration. Section III presents the baseline results. Section IV assesses the effects of pension reforms. Section V concludes.

II. The Model

A. Model Overview

The model follows the Auerbach-Kotlikoff tradition.[3] The (closed) economy is populated by overlapping generations of finitely-lived households, atomistic firms, and an infinitely-lived government. Households consume and accumulate assets during their lifetime, work during their youth, and retire when old. Firms produce the single good in the model using labor and capital, and the government collects income, consumption and payroll taxes to finance government expenditures and pension benefits, and redeem the initial government debt.

Although the general equilibrium structure is standard, the model incorporates specific features of the Spanish pension system. Specifically, it incorporates a stylized version of the Spanish pension rule whereby the old-age benefit is calculated based on average wage earnings in the corresponding averaging period; in the baseline scenario below, the averaging period is the last 15 years of the individual's work life.[4]

In addition, to capture the effects on household behavior of demographic aging and pension reforms, the model includes the following elements. Households retire at an exogenously given age, but labor supply is endogenous as households choose the amounts of labor and leisure time during their work life. Households' labor skills (productivity) vary exogenously

[3] A survey of the literature—extending back to Auerbach and Kotlikoff (1987)—can be found in Kotlikoff (2000). The numerical solution methods involved are described in Heer and Maußner (2004) and Judd (1999).

[4] The model is real, that is, money and inflation play no explicit role. This is consistent with the paper's focus on the long term effects of aging. Also, the inflation indexation of pension contributions and benefits implies that inflation is neutral with respect to the pension system.

with age to account for the observed hump in wage rates over years of employment. Labor-augmenting productivity growth causes real wage growth over time, thus allowing the model to capture the trade-off noted in the introduction: extending the averaging period discourages labor effort by reducing average wage earnings and pension benefits, but it encourages labor effort, particularly when skills are high, by eliminating inter-temporal distortions. Life expectancy is exogenous but increases over time to match current demographic projections. Finally, the model explicitly accounts for the effects of population aging on public health-related expenditures. These features allow the model to meaningfully quantify the effects of reforming the pension rule—increasing the retirement age and extending the averaging period—on labor market incentives and macroeconomic outcomes, including complementarities with alternative tax policies to finance old-age related spending. For the reader's convenience, the model's notation is summarized in Table 1.

B. Households

The lifetime utility of a household born at time t is determined by its lifetime consumption (c) and leisure (l), and is given by[5]

$$\sum_{s=1}^{T_t+T_t^R} \beta^{s-1} \cdot \left\{ \log(c_{t+s-1}^s) + \gamma \cdot \log(l_{t+s-1}^s) \right\}, \tag{1}$$

[5] Strictly, the utility function is given by
$$\sum_{s=1}^{T_t+T_t^R} \beta^{s-1} \cdot \left\{ \log(\hat{c}_{t+s-1}^s) + \gamma \cdot \log(l_{t+s-1}^s) \right\} = \sum_{s=1}^{T_t+T_t^R} \beta^{s-1} \cdot \left\{ \log(c_{t+s-1}^s) + \gamma \cdot \log(l_{t+s-1}^s) + (t+s-1) \cdot \log(1+\xi) \right\};$$

however, the term $\sum_{s=1}^{T_t+T_t^R} \beta^{s-1} \cdot (t+s-1) \cdot \log(1+\xi)$ is a constant that can be ignored without affecting the solution of the household's optimization problem.

where the household's life is characterized by two distinct phases: a work life lasting T_t periods or years ($s = 1,...,T_t$) and a mandatory retirement lasting T_t^R years ($s = T_t + 1,...,T_t + T_t^R$). Note that the model allows the household's life expectancy and retirement age to vary across generations, and henceforth, these are assumed to be non-decreasing over time. The household is endowed with a fixed number of hours per year, which is normalized so that work (n) and leisure (l) add up to one:

$$l_{t+s-1}^s = \begin{cases} 1 - n_{t+s-1}^s & s = 1,...,T_t \\ 1 & s = T_t + 1,...T_t + T_t^R \end{cases} . \qquad (2)$$

A household accumulates assets (A) during its work life according to the following budget constraint:

$$(1+\xi)\cdot A_{t+s}^{s+1} = [1 + r_{t+s-1}\cdot(1-\tau_{t+s-1}^I)]\cdot A_{t+s-1}^s + (1-\tau_{t+s-1} - \tau_{t+s-1}^I)\cdot W_{t+s-1}\cdot e^s \cdot n_{t+s-1}^s - (1+\tau_{t+s-1}^c)\cdot c_{t+s-1}^s, \quad (3)$$

where next year's assets are determined by adding to this year's assets the household's savings, which are obtained by adding net return on assets to net wage income and subtracting consumption. As noted above, the household's labor productivity per hour varies with age according to a skill premium (e^s). The premium is defined as the relative productivity of an s-year old household to that of a 1-year old (unskilled) household; the latter is normalized to 1. Thus, W denotes the wage per unit of labor time of an unskilled worker. In equation (3), the household takes as given the payroll (τ), income (τ^I), and consumption (τ^c) tax rates, and the interest (r) and wage (W) rates.[6]

During retirement, the household's wage income is replaced by an old-age pension (b) in the budget constraint, as follows:

$$(1+\xi)\cdot A_{t+s}^{s+1} = [1+r_{t+s-1}\cdot(1-\tau_{t+s-1}^{I})]\cdot A_{t+s-1}^{s} + \frac{b_{t+T_t}^{T_t+1}}{(1+\xi)^{s-T_t-1}} - (1+\tau_{t+s-1}^{c})\cdot c_{t+s-1}^{s}\,. \tag{4}$$

Note that the old-age pension for a household born at time t and retiring at time $t+T_t$ can be expressed as follows:[7]

$$b_{t+T_t}^{T_t+1} = \psi\cdot\frac{1}{\mu}\cdot\sum_{j=T_t+1-\mu}^{T_t}\frac{W_{t+j-1}}{(1+\xi)^{T_t+1-j}}\cdot e^{j}\cdot n_{t+j-1}^{j}, \tag{5}$$

where the average (gross) wage in the averaging period (covering the last μ years before retirement) is "scaled down" by the replacement ratio (ψ).[8] Notice that pension benefits and real wage earnings are discounted by labor augmenting productivity growth in the stationary-transformed equations (4) and (5). This discounting reflects the fact that the household's

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[6] Income taxes are levied on labor income and asset earnings; for simplicity, these tax rates are assumed to be the same.

[7] The pension benefit formula, before applying stationary transformations is given by: $\hat{b}_{t+T_t}^{T_t+1} = \psi\cdot\frac{1}{\mu}\cdot\sum_{j=T_t+1-\mu}^{T_t}\hat{W}_{t+j-1}\cdot e^{j}\cdot n_{t+j-1}^{j}$. Notice that the pension benefit, once determined, remains constant in real terms throughout retirement, that is $\hat{b}_{t+T_t}^{T_t+1} = \hat{b}_{t+s-1}^{s}$ for $s = T_t+2,\ldots,T_t+T_t^{R}$.

[8] Consistent with the majority of old-age pensions in Spain, pensions are taken as not taxed.

pension benefits (in equation (4)) and the past nominal wage earnings used to compute the initial pension benefit (in equation (5)) are adjusted by inflation, but not by productivity growth.

The model assumes that there are no intergenerational bequests or inheritances: the household is born (enters the labor force) with zero assets at age $s=1$, and dies without assets at age $s = T_t + T_t^R + 1$,

$$A_t^1 = A_t^{T_t + T_t^R + 1} = 0. \tag{6}$$

The household's problem is to choose the paths of consumption, leisure and asset holdings $\left\{c_{t+s-1}^s, l_{t+s-1}^s, A_{t+s-1}^s\right\}_{s=1}^{T_t+T_t^R}$ to maximize its lifetime utility (1) subject to constraints (2)-(6).

The household's problem can be expressed as a sequence of two dynamic optimization problems, as follows:

$$\underset{\left\{c_{t+s-1}^s, l_{t+s-1}^s, A_{t+s}^{s+1}\right\}_{s=1}^{T_t}}{Max} \sum_{s=1}^{T_t} \beta^{s-1} \cdot \left\{\log(c_{t+s-1}^s) + \gamma \cdot \log(l_{t+s-1}^s)\right\} + \beta^{T_t} \cdot V(A_{t+T_t}^{T_t+1}, b_{t+T_t}^{T_t+1})$$

subject to (2), (3), (5) and (6).

where $V(A_{t+T_t}^{T_t+1}, b_{t+T_t}^{T_t+1})$ is the household's value function or discounted indirect utility when it retires at time $t+T_t$ having reached the age of T_t+1 years. Upon retirement, the household's optimization problem can be expressed recursively, and a closed-form solution for the value function (V) follows from the log utility assumption.[9]

Two sets of conditions solve the household's problem under standard dynamic optimization techniques; see Table 2 for the first order conditions, where $V_A(.)$ and $V_b(.)$ denote the partial derivatives of $V(.)$ with respect to $A_{t+T_t}^{T_t+1}$ and $b_{t+T_t}^{T_t+1}$. The first set—equations (7), (9) and (11)—refers to a household's consumption-leisure choice at specific ages (intra-temporal first order conditions). In each period, the household equates the marginal utility of consumption (scaled by wages) to the marginal utility of leisure. The second set—equations (8), (10), (12) and (13)—governs the household's consumption-saving decisions over time (inter-temporal first order conditions or Euler equations).[10] In this case, households equate the marginal utility of current consumption to the discounted marginal utility of future consumption (scaled by the net return on savings).

These sets of equations reflect the peculiarities of the Spanish pension rule—averaging period and inflation indexation of contributions and benefits—and whether a household is working or retired. Specifically, while the household is in the labor force the pension rule introduces three sub-periods in the household's optimization problem. The first comprises the initial years in the labor force prior to the averaging period (μ) $(s=1,...,T_t-\mu)$, when the

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[9] Notice that the household's value function $V(.)$ depends not only on its stock of assets at retirement ($A_{t+T_t}^{T_t+1}$) and its future annual pension benefits ($b_{t+T_t}^{T_t+1}$) but also on future interest rates, and income tax rates. Appendix I provides details of the derivation of the value function (V).

[10] When the household retires, it faces only an inter-temporal condition as it no longer supplies labor.

household's annual wage earnings do not affect its future pension benefits. The second corresponds to the first *μ-1* years of the averaging period $(s=T_t-\mu+1,\ldots,T_t-1)$, when the consumption-leisure choice (intra-temporal first-order conditions) also reflects the fact that wage earnings accrued in this sub-period provide additional utility during retirement because of their effect on the pension benefit;[11] however, the consumption-saving decision remains unchanged. In the final year of the averaging period $(s=T_t)$, the consumption-saving decision reflects, nonetheless, the retirement of the individual in the following period (V_A). When the household retires $(s=T_t+1,\ldots,T_t+T_t^R-1)$, however, there is no labor supply choice and only the consumption-saving decision remains.[12]

The stationary-transformed aggregate household consumption (C_t^h), effective labor supply (N_t^h), and assets (A_t^h) are obtained by aggregating individual household's variables at each point in time, as follows:[13]

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[11] All else equal, the household increases the supply of labor during the reference period because of this added "benefit" to work.

[12] The analysis starts with a full set of generations, $T_0+T_0^R$, at time $t=0$. Thus, during the first $T_0+T_0^R$ years a number of "truncated" optimization problems are associated with those households of ages $\tilde{s}=2,\ldots,T_0+T_0^R$ that were born before $t=0$. Notice that we are assuming that all "truncated" generations have the same work life and retirement periods.

[13] Notice the contrast between the definitions of the *aggregate effective labor supply* (N_t^h) and the *aggregate labor effort*. The *aggregate effective labor supply* is the sum of the time devoted to work by all the generations in the labor force in a given year, where each generation's working time is weighted by its skills and population size. By contrast, the *aggregate labor effort* (n_t^h) is the weighted (by population size) sum of the time devoted to work by all generations in the labor force, without accounting for skill differences: $n_t^h=\sum_{s=1}^{T_t} n_t^s\cdot\frac{P_t^s}{P_t}$.

$$N_t^h = \sum_{s=1}^{T_t} e^s \cdot n_t^s \cdot \frac{P_t^s}{P_t}, \qquad A_t^h = \sum_{s=1}^{T_t+T_t^R} A_t^s \cdot \frac{P_t^s}{P_t}, \qquad C_t^h = \sum_{s=1}^{T_t+T_t^R} c_t^s \cdot \frac{P_t^s}{P_t}.$$

C. Firms

Firms maximize a (stationary-transformed) profit function net of capital depreciation Π_t^f. They do so subject to a constant-returns-to-scale Cobb-Douglas production function with labor-augmenting technological progress,

$$\Pi_t^f = \mathrm{Z} \cdot \left(K_t^f\right)^{\alpha} \cdot \left(N_t^f\right)^{1-\alpha} - (r_t + \delta) \cdot K_t^f - W_t \cdot N_t^f,$$

where δ is the rate of capital depreciation. Both output and factor markets are perfectly competitive, and therefore, individual firms face given wages (W_t) and rental rates (r_t). The first order conditions require that W_t ($r_t + \delta$) equal the marginal product of labor (capital):

$$W_t = \mathrm{Z} \cdot (1-\alpha) \cdot \left(\frac{K_t^f}{N_t^t}\right)^{\alpha}, \qquad r_t = \mathrm{Z} \cdot \alpha \cdot \left(\frac{K_t^f}{N_t^t}\right)^{-(1-\alpha)} - \delta.$$

D. The Government

The government sets taxes to ensure long-run fiscal sustainability. As noted before, the government collects payroll, income, and consumption taxes from households. Tax revenues are used to finance public consumption (*G*), pension benefits, and redeem government debt (*D*). Public consumption has two components: health-related public consumption whose evolution is driven by changes in the population's age structure (see Appendix II for further details); and non health-related public consumption that remains constant as a share of aggregate output. Thus, the government's budget constraint is as follows:[14]

$$D_{t+1}\cdot(1+\xi)\cdot\frac{P_{t+1}}{P_t}=(1+r_t)\cdot D_t+[G_t-\tau_t^I\cdot(r_t\cdot A_t^h+W_t\cdot N_t^h)-\tau_t^c\cdot C_t^h]$$
$$+\sum_{s=T_t+1}^{T_t+T_t^R}\frac{b_{t+T_t+1-s}^{T_t+1}}{(1+\xi)^{s-T_t-1}}\cdot\frac{P_t^s}{P_t}-\tau_t\cdot W_t\cdot N_t^h,$$

where, for clarity, the (non-social security) primary deficit (term in brackets), and the social security deficit (last two terms) are shown separately.

E. Equilibrium

An equilibrium is defined as a state of affairs that simultaneously places all households and firms on their maximizing paths, establishes the solvency of the government, and clears markets. Consider an initial population of size P_0 with age structure $\left\{P_0^s\right\}_{s=1}^{T_0+T_0^R}$, a given sequence of new-born cohorts $\left\{P_t^1\right\}_{t=1}^{\infty}$ with work lives $\left\{T_t\right\}_{t=1}^{\infty}$ and life expectancies $\left\{T_t+T_t^R\right\}_{t=1}^{\infty}$, government debt $D_0\geq 0$, capital stock $K_0>0$, and distribution of assets

[14] The budget constraint, before stationary transformations, is given by

$\widehat{D}_{t+1}=(1+r_t)\cdot\widehat{D}_t+[\widehat{G}_t-\tau_t^I\cdot(r_t\cdot\widehat{A}_t^h+\widehat{W}_t\cdot\widehat{N}_t^h)-\tau_t^c\cdot\widehat{C}_t^h]+\sum_{s=T_t+1}^{T_t+T_t^R}\hat{b}_t^s\cdot P_t^s-\tau_t\cdot\widehat{W}_t\cdot\widehat{N}_t^h$. Notice that the (stationary-transformed) old-age pension for a household born at time $t-T_t$ and retiring at time t is given by $b_t^{T_t+1}=\frac{\psi}{\mu}\cdot\sum_{j=T_t+1-\mu}^{T_t}\frac{W_{t-T_t+j-1}}{(1+\xi)^{T_t+1-j}}\cdot n_{t-T_t+j-1}^j$. Using the equality $\hat{b}_t^s=\hat{b}_{t+T_t+1-s}^{T_t+1}$ for $s=T_t+1,...,T_t+T_t^R$, it can be shown that $\frac{\hat{b}_t^s}{(1+\xi)^t}=\frac{b_{t+T_t+1-s}^{T_t+1}}{(1+\xi)^{s-T_t-1}}$. The discount of the stationary-transformed pension benefit by productivity growth reflects the fact that nominal benefits are adjusted only by inflation in Spain.

$\left\{A_0^s\right\}_{s=1}^{T_0+T_0^R}$, such that $D_0 + K_0 = A_0^h = \sum_{s=1}^{T_0+T_0^R} A_0^s \cdot \frac{P_0^s}{P_0}$. The equilibrium is thus a collection of lifetime plans for both, households born during the period of analysis ($t \geq 0$),

$$\left\{c_{t+s-1}^s, l_{t+s-1}^s, A_{t+s}^{s+1}\right\}_{s=1}^{T_t+T_t^R}, \text{ for } t = 0,1,...,\infty,$$

and those born before then ($t < 0$)—households of ages 2 through $T_0 + T_0^R$ at $t = 0$—that face "truncated" lifetime plans

$$\left\{c_{s-\tilde{s}}^s, l_{s-\tilde{s}}^s, A_{1+s-\tilde{s}}^{s+1}\right\}_{s=\tilde{s}}^{T_0+T_0^R}, \text{ for } \tilde{s} = 2,...,T_0 + T_0^R,$$

a sequence of allocations for the firms $\left\{K_t^f, N_t^f\right\}_{t=0}^{\infty}$, a sequence of relative prices of labor and capital $\left\{W_t, r_t\right\}_{t=0}^{\infty}$, and a sequence of government policy variables including payroll, income, and consumption tax rates, and government consumption and debt, $\left\{\tau_t, \tau_t^I, \tau_t^c, G_t, D_t\right\}_{t=0}^{\infty}$, such that:

- the sequence of allocations $\left\{K_t^f, N_t^f\right\}_{t=0}^{\infty}$ solves the firm's optimization problem;
- the lifetime plans for households born during the period of analysis $\left\{c_{t+s-1}^s, l_{t+s-1}^s, A_{t+s}^{s+1}\right\}_{s=1}^{T_t+T_t^R}$; $t = 0,1,...,\infty$ solve their optimization problems, and the lifetime plans for households of ages $\tilde{s} = 2,...,T_0 + T_0^R$ at time $t = 0$ $\left\{c_{s-\tilde{s}}^s, l_{s-\tilde{s}}^s, A_{1+s-\tilde{s}}^{s+1}\right\}_{s=\tilde{s}}^{T_0+T_0^R}$ solve their truncated optimization problems;
- the government budget constraint is satisfied for $t \geq 0$;

- the labor market clears, $N_t = N_t^f = N_t^h$, for $t \geq 0$;

- the asset market clears, $A_t = D_t + K_t^f = A_t^h$, for $t \geq 0$; and

- the output market clears, $K_{t+1} \cdot (1+\xi) \cdot \frac{P_{t+1}}{P_t} = (1-\delta) \cdot K_t + Y_t - C_t - G_t$ for all $t \geq 0$, where $Y_t = Y_t^f$ and $C_t = C_t^h$ are the equilibrium aggregate output and consumption levels.[15]

F. Balanced Growth Equilibrium and Calibration

To calibrate the model and start off the quantitative analysis, a balanced growth equilibrium is defined. The economy is said to exhibit a balanced-growth equilibrium—assuming constant population growth rate (*p*), work life ($T_t = T$), and retirement period ($T_t^R = T^R$)—when the government implements a fiscal policy characterized by a constant government expenditure-to-output ratio, constant tax rates, and a constant debt-to-output ratio.[16] Along the balanced growth equilibrium path, all endogenous variables grow at constant rates (Table 3). The balanced-growth equilibrium can be expressed as a *steady state in*

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[15] The economy's aggregate constraint $K_{t+1} \cdot (1+\xi) \cdot \frac{P_{t+1}}{P_t} = (1-\delta) \cdot K_t + Y_t - C_t - G_t$ is obtained from the aggregate constraint of the household sector, the first-order conditions of firms, the market equilibrium conditions, and the government budget constraint. The aggregate constraint of the household sector at time t is given by

$$(1+\xi) \cdot A_t^h = [1 + r_t \cdot (1-\tau_t^I)] \cdot A_t^h + (1-\tau_t^I - \tau_t) \cdot W_t \cdot N_t^h + \sum_{s=T_t+1}^{T_t+T_t^R} \frac{b_{t+T_t+1-s}^{T_t+1}}{(1+\xi)^{s-T_t-1}} \cdot \frac{P_t^s}{P_t} - (1+\tau_t^c) \cdot C_t^h .$$

[16] The age structure of the population remains invariant over time, and thus, both components of public consumption (health-related and non health-related) are constant as a share of output.

"detrended" variables by transforming aggregate variables to eliminate the effects of technological progress and population growth.

The model is calibrated to match the stylized facts and relevant features of the Spanish economy, as follows.

- Standard parameter values in the real business cycle literature are used for the household's discount factor (β) and the depreciation rate (δ). The share of capital in production (α) is obtained from previous Spanish-specific calibrations and econometric studies. The total factor productivity parameter (Z) is set so that the capital-output ratio in the initial steady state is consistent with Spain's data. The rate of labor-augmenting technological progress is set to be consistent with long-term output per capita growth. The value of (p) matches the average population growth rate for 1900-70. The leisure parameter is calibrated so that the fraction of time worked by a representative household in the population is 0.274.[17]

- Tax rates are calibrated to match effective rates observed in 1994-2004. Specifically, the payroll tax rate (τ) matches the observed ratio of social security contributions to wage income, and the consumption (τ^c) and income tax rates (τ^I) match, respectively, the ratios of indirect tax revenues to private consumption and direct tax revenues to GDP.

- The pension replacement ratio value is set at 0.65. Because of recent changes to the social security system in Spain, the replacement ratios for new pensioners differ from those of pensioners retired under previous regimes. The parameter choice matches the ratios observed in the most recent pensioners' cohorts. The values of the work life

[17] Assuming that this household or individual sleeps 8 hours per day, the leisure-work decision is made for the remaining 16 hours. This translates into a total of 112 (=7x16) hours per week. Assuming 40 working hours per week, the individual works 35.7 (=40/112) percent of the time that he/she is awake. Adjusting the fraction of time worked by labor force participation—about 77 percent for those between the ages of 16-64—yields 0.274.

and retirement periods are set so that households enter the labor force when they are on average 22 years old, retire at age 62 and live 80 years with certainty, which is the implicit "life expectancy" at birth.

- The household's labor skills profile by age (e^s) was calibrated to match the profile of average hourly wages by age of Spanish workers (Appendix II). This profile is similar to that of the US economy—as reported by Hansen (1993)—in that skills are low at the beginning of the household's work life, peak at about 50 years of age, and decline to intermediate levels by the end of the work life.

Given these values, the calibration exercise verifies that the resulting values of the endogenous variables in the initial steady state and the fiscal ratios closely resemble those observed in the Spanish data (Table 4).

G. Household's Labor, Consumption and Asset Holdings in Initial Steady State

As anticipated, the averaging period introduces a discrete jump in the households' labor effort profile precisely fifteen years before retirement (Figure 1). Note that at the beginning of work life, households are relatively unskilled and thus work few hours; however, as they age, and labor skills improve, time devoted to work increases. Still, households' labor effort peaks before the beginning of the averaging period even though skills are still increasing.[18] This is because households reduce labor effort *before* the averaging period to compensate for the higher labor effort they will exert *during* the averaging period. Intuitively, the desire to anticipate leisure before the averaging period dominates the incentive to increase labor effort

[18] Notice in Appendix II that the household's skills peak at about 25 years of employment, whereas in Figure 1, the labor effort peaks after 12 years of employment.

provided by the gains in skills. Note further that during the averaging period the number of hours worked jumps, and remains high until retirement, because households internalize the effect of labor effort on future pensions.

Still, the labor effort implies that a households' wage earnings increase over time. Accordingly, households incur debt at the beginning of their lives to *partially* smooth consumption—which still increases over time to offset the disutility associated with the decline in leisure. During the averaging period, households intensify their asset accumulation to supplement their pension income and boost consumption during retirement. In retirement, consumption is highest and assets are depleted.

III. Baseline Simulations

The time line for the simulations corresponds to a 370-year period divided into three unequal sub-periods. In the first century, the economy is in the steady state described in Section II. The middle segment covers the demographic transition—from a high to a low fertility rate, time-varying immigration and increasing life expectancy—that takes 170 years to work itself out. In the last 100 years, the economy is in a new steady state characterized by lower population growth and higher life expectancy than in the first century.[19] The beginning of the three sub-periods corresponds to the years 1857, 1957, and 2127.

[19] Specifically, the annual rate of population growth in the last century is equal to 0.5 percent—the average observed in the decade ending in 2001—reflecting a moderate rebound from the minimum (0.3 percent per annum) observed in the decade ending in 1991.

A. Demographic Transition

Among the exogenous elements in the simulations, the demographic shock and immigration merit specific attention. The onset of the demographic transition—specifically, to higher life expectancy—is taken to be 1957, with households (22 years of age) entering the labor force that year and dying 59 years later (81 years old) in 2015 (Table 5). Life expectancy is assumed to increase one year per decade starting in 1957 so that households entering the labor force nine decades later (in 2047) die at 90 years of age (in 2114). From 2047, the life expectancy of households entering the labor force remains fixed.

The number of labor force entrants reflects the combined effects of fertility and immigration and is set so that the endogenous trajectory of the model's dependency ratio—the ratio of population aged 62 years and older over population between 22 and 61 years of age—matches that of the available official projections through 2060 (Figure 2).[20] Accordingly, the annual growth rate of labor force entrants in the period 1980-2006 is higher than in the initial steady state, declines between 2006 and 2059, and it is constant afterward. Note that the generation entering the labor force in 2060 dies in 2127, the year marking the end of the demographic transition and the beginning of the final steady state.

Following the Spanish National Statistics Institute (INE), a low- and a high-immigration scenarios are considered. In the low immigration scenario, the dependency ratio is stable at about 35 percent until 1985, declines slowly for the next twenty years due to immigration and

[20] Note that if the growth rate of labor force entrants and life expectancy are constant—as in the steady states—the growth rate of the total population is equal to that of the labor force entrants.

other temporary factors, bottoms out at 34 percent by 2004 and then rises slowly to reach about 35.5 percent by 2010. From 2010, and for the next forty years, however, the ratio increases sharply to: 42 percent in 2020, 55 percent in 2030, and 75 percent in 2040; it peaks at over 85 percent in 2048. In sum, by 2050, the dependency ratio is 2½ times higher than in 2005. In the high immigration scenario, the dependency ratio also rises sharply, but is not all that different from the low immigration scenario as, by 2050, the ratio is 2¼ times higher than in 2005.

Note that in the model after peaking, the dependency ratio declines for the next three decades (2060-90) as labor force entrance recovers. However, the ratio increases again when the growth of labor force entrance stabilizes, reflecting the continued improvements in life expectancy. Only when both labor force entrance and life expectancy stabilize does the dependency ratio gradually converge (from above) to its steady state level of 59 percent. Note that, although the exact numbers vary, the evolution of the dependency ratio is qualitatively the same across the immigration scenarios.

B. Baseline Macroeconomic Scenario

In the baseline, the following assumptions are made regarding government policies and immigration. First, the parameters of the social security system—retirement age and averaging period—remain unchanged over the 370 years of analysis. Second, as fiscal pressures arise during the demographic transition, the government implements a “tax-as-you-go policy:” consumption tax rates are adjusted so that the government budget constraint holds while other tax rates and the government non-health expenditure-to-output and debt-to-output ratios remain constant over time.[21] Immigration corresponds to the low-immigration scenario.

The baseline simulations suggest that pension expenditures increase by about 16 percentage points of output by 2050 with severe macroeconomic consequences (Figure 3 and Table 7). The consumption tax rate peaks at 51 percent in 2050—more than 30 percentage points above its 2007 level—to finance the social security deficit. As a result, output and consumption per capita (adjusted for the effects of technological progress) are 18 percent lower than in the initial steady state.[22] Moreover, as taxes start rising in 2010, output deteriorates long before the peak of the demographic shock in 2050; output growth falls below its initial rate for about 80 years and virtually stalls in the 2040s.

Paradoxically, aggregate output remains unscathed for a couple of decades (through 2025) even though tax rates and the dependency ratio increase. This reflects the fact that aggregate capital per capita increases due to the rising share of old working households—whose asset holdings peak at retirement—in the population. In addition, aggregate effective labor is also sustained by the rising share of old working households and the higher marginal productivity of labor. The change in the population's age structure also accounts for the increase in consumption per capita, which is reinforced by the anticipation of consumption by young

[21] Auerbach and Kotlikoff (1987) and De Nardi et al. (1999) show that financing the demographic shock in the US economy with consumption taxes is less distortionary than financing it with other taxes—payroll and income taxes—as the tax burden is shared by more generations. Catalán et. al. (2005) confirms this result for the Spanish economy. Note that the ratio of government consumption-to-output varies over time according to the evolution of the population's age structure, reflecting the provision of health-care services, as dscribed in Appendix II.

[22] Note that in Figure 3 and Table 7, output (consumption) is stationary-transformed as indicated in Table 1—adjusted by technological progress and population growth. Accordingly, these variables can also be interpreted as output (consumption) per capita deviations from their long term trend.

generations that foresee rising tax rates. After 2025, however, capital, labor, output and consumption per capita fall sharply until about 2050. This turnaround comes about as the share of newer generations—less able to accumulate assets because they have been taxed heavily since birth—rises relative to previous, more affluent, generations. In addition, the recently retired affluent generations start depleting their sizable asset holdings, which reinforces the downturn in output and the capital stock. Note that factor prices track the evolution of the dependency and the capital-output ratios: the return on capital falls, and the wage rate (detrended to account for technological progress) increases until about 2050.

Further insights into the baseline simulation can be gleaned from households' behavior (Figure 1). In this connection, it is useful to consider two generations of households: one entering the labor force in 1990 and the other in 2010. The former generation is among the most heavily taxed generations as it dies in 2053, when taxes peak. The latter generation is among those living through the widest tax rate swings as it dies in 2074, and thus faces both the sharp tax increase associated with the demographic shock and the subsequent decline as the shock dissipates.

Consider first the hypothetical case when factor prices are constant so that households only face changes in consumption taxes needed to finance pension expenditures. The path of consumption tax rates noted above—a sharp increase during 2010-50, and subsequent decrease through 2080—generates wealth and substitution effects affecting households' lifetime plans. In particular, an inter-temporal consumption substitution effect arises from households foreseeing higher (lower) future consumption tax rates, which thus anticipate (delay) consumption.[23] Note that both generations noted above—those entering the labor market in 1990 and 2010—shift consumption away from the period of the highest tax rates, but their consumption profiles differ. The 1990 generation anticipates consumption in response to heavy tax rates at the end of its life (around 2050), reinforcing the aggregate consumption boom prior to 2025. But the 2010 generation tends to postpone consumption until the end of its life, when tax rates have already declined. Thus, their behavior reinforces the collapse in aggregate consumption around 2050.

Changes in factor prices provide an additional source of inter-temporal substitution. Specifically, the rising wage rates before 2050 induce an inter-temporal delay of work effort in the lifetimes of the 1990 and 2010 generations—whose work lives end in the years 2030 and 2050—relative to generations born in the initial steady state[24]. Both elements, higher wage rates and higher households' labor effort in the averaging period increase individual pension benefits, and thus—in addition to the increased dependency ratio—put further pressure on public finances.

In the final steady state, the dependency ratio and the consumption tax rate are higher due to a longer retirement period—as life expectancy increases and the working life remains fixed—and a smaller rate of population growth than in the initial steady state (Figure 4). The negative wealth effect associated with higher taxes implies lower lifetime consumption, and

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[23] An increasing path of consumption tax rates would also induce households to anticipate labor effort if wage rates remained constant, as the consumption-leisure ratio in each period is inversely related to the consumption tax rate (equation (7) in Table 2). However, wage rates do change over time in our simulation.

[24] Rising wages increase pension benefits for a given labor effort—a positive wealth effect that provides an incentive to consume more but work and save less. However, this effect is weak when compared to the opposite wealth effect caused by high consumption taxes.

higher lifetime work effort (for the representative household) because of the need to accumulate more assets and enhance the pension income to finance a longer retirement period.[25] Indeed, asset holdings continue increasing during the initial years of retirement, in contrast with the initial steady state.

IV. Simulations of Pension Reforms

The partial and joint effects of two parametric reforms are considered: increasing the retirement age, and extending the pension rule's averaging period. A "partial pension reform" is defined as one that increases households' retirement age but leaves the pension rule's averaging period unchanged. A "full pension reform" is considered to be one that, in addition, extends the averaging period used to compute the pension benefit to the entire work life.

A. Effects of Pension Reform in the Final Steady State

In a nutshell, reforming the pension system improves welfare in the final steady state, as the benefits associated with lower taxes—including lower inter-temporal distortions in consumption and labor—more than offset the welfare costs of reductions in pension benefits.

[25] Notice that the longer life expectancy fully accounts for the higher work effort. Our utility assumptions imply that the wealth and substitution effects on leisure of higher consumption tax rates cancel out; thus, lifetime work effort would not change if population growth were lower but life expectancy remained constant. Also, slower population growth, *ceteris paribus*, has an interesting steady state effect in this model: it implies lower consumption, capital and output per capita levels. This contrasts starkly with the standard Solow growth model, where lower population growth rates imply higher consumption, capital and output per capita levels.

Partial Pension Reform

Consider increasing the retirement age from 47 to 49 years—and the work life from 46 to 48 years—while holding constant the averaging period (Figure 5).[26,27] This reform lowers the dependency ratio and boosts the aggregate effective labor supply by increasing the number of cohorts working each period. The greater lifetime labor income for households results in a higher capital accumulation, which increases the capital-labor ratio and the wage rate. Although the wage rate is higher, individual pensions decline because the hump-shaped skills imply a reduction in households' average labor skills (and wage incomes earned) in the averaging period. Still, welfare improves as the lower individual pensions and dependency ratio imply a reduction in consumption tax rates and an increase in consumption.

Full Pension Reform

Consider first extending the averaging period to the entire work life (Figure 5). In this case, welfare increases monotonically despite the decline in pension benefits due to the concomitant fall in consumption tax rates. The interaction of the hump-shaped skills and technological progress plays a central role in explaining why the resulting aggregate effective labor and output increase when the averaging period is shorter than 15 years, but decrease when it is longer. Specifically,

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[26] With a work life of 48 years and a retirement period of 20 years, the model's dependency ratio in the final steady state is the same as in the initial steady state (0.35). Also, with a work life of 48 years, the ratio of working-to-retirement years for a given household in the final steady state is only slightly higher (2.4) than in the initial steady state (2.2)—an unchanged ratio would result from a work life of 47 years and a retirement period of 21 years.

[27] Since households enter the labor force with 22 years of age, this corresponds to increasing the retirement age from 68 to 70 years. Compared to the initial steady state, increasing the model's retirement age from 40 to 48 years corresponds to increasing the natural retirement age from 62 to 70 years.

- If the averaging period is shorter than 15 years—as it was prior to the 1997 reform, when it was eight years long—extending it results in a higher aggregate effective labor supply, which reflects the higher average skills of households in the averaging period and enhanced labor effort during high-skill ages.[28] Pension benefits decrease, however, as this effect is more than offset by the impact of technological progress, as high wage earning years are more heavily discounted by growth in productivity.[29]

- If the averaging period is 15 years—as it is currently—or longer, extending it decreases the aggregate effective labor supply as it results in lower average skills in the averaging period. This effect and technological progress reduce the pension benefit.

In either case, the decline in the pension benefit allows a reduction in the consumption tax rate that boosts household's consumption and welfare.

These simulations also suggest that a full pension reform—increasing the averaging period to the entire work life and increasing the retirement age—increases social welfare in the final steady state. As noted above, increasing the retirement age increases welfare—the dependency ratio declines enabling consumption tax rates to fall—and so does extending the averaging period to the entire work life. Combining these partial effects improves welfare unambiguously.

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[28] Figure 4 does not show labor profiles by age for this particular reform; however, it is still useful to illustrate why extending the averaging period induces households to exert more effort when they are highly skilled, thus increasing aggregate effective labor.

[29] Labor-augmenting technological progress increases real wages over time, whereas the pension calculation is based on inflation-adjusted wage earnings during the household's averaging period. In equation (5), an increase in ξ reduces, *ceteris paribus*, the pension benefit $b_{t+T_t}^{T_t+1}$.

A word of caution regarding extending the averaging period: the monotonic improvement in welfare depends on the rate of technological progress.[30] Specifically, when there is no technological progress, welfare is maximized when the averaging period is shorter than the entire work life (see Appendix III). Intuitively, this is because when the averaging period is extended, but is already long, the pension benefit rises as the decline in effective labor induces higher wages that, in contrast to the discussion above, are not deflated by (positive) technological progress. Consumption tax rates, in turn, are higher, and reduce household's welfare.

B. Effects of Pension Reforms During the Demographic Transition

Before turning to the discussion of the simulation results, it is important to note that these are obtained by assuming that they are unanticipated—in the sense that households previously envisaged the future to be consistent with the baseline scenario—and are simultaneously announced and implemented at the *beginning* of 2008; the announcement includes details of the partial grandfathering clauses for the transitional generations as detailed below.

Grandfathering Clauses

In this study, the principle guiding the design of grandfathering clauses is to provide more grandfathering to households nearing retirement—with less time to adjust to the new pension

[30] Also, in these steady state simulations, households' welfare improves when pension benefits and the consumption tax rate fall. This is because households use tax reductions to save on their own for retirement—earning a market rate of return. We do not model myopia or self-control problems that could prevent households from saving in this rational, forward looking way. If these additional distorsions were present, the welfare analysis and the policy implications could change significantly—for instance, in order to replicate the welfare implications of our analysis, as part of the reform strategy, the government could place the tax reduction in individual retirement accounts to deal with the self-control problem.

rules—and gradually less grandfathering to households further away from retirement (Table 6); households that are 48 years of age or older in 2008—including those already retired and those that have been in the workforce for at least 27 years and entered the averaging period—are fully grandfathered.

Younger households face, to a varying degree, the parametric pension reforms discussed below. Specifically,

- The *retirement ages* of households with 34-47 years of age in 2008 are increased by one year (and their work lives extended to $T = 41$ years) relative to the baseline. The retirement ages of generations aged 22-33 years in 2008 and of those generations entering the labor force in 2009-2016 are increased by 2 years (and their work lives extended to $T = 42$ years). Since 2017, the retirement ages increase 2 years every decade until work lives reach a maximum of 48 years—generations born in 2017-2026 have work lives lasting 44 years, and all generations born after 2046 have work lives lasting 48 years.

- The *averaging period* of each generation 47 years of age or younger in 2008 is extended to the whole period between 2008 and retirement, with the averaging period of households born in 2008 and later covering their entire work lives.[31]

Note that only a partial grandfathering of the existing pension system is considered because a complete grandfathering would not yield macroeconomic benefits until the late 2040s—when the first generations of the reformed system retire, too late to mitigate the adverse macroeconomic effects of aging.

[31] Notice that the generation aged 47 years old—26 years old in the model—enters the averaging period in 2008. Before the pension reforms (in the baseline), the generation aged 47 in 2008—with 26 years in the workforce—retires in 2023 and has a 15 year averaging period. After the pension reforms, this generation retires in 2024 and has a 16 year averaging period.

Partial Pension Reform

In sum, a partial pension reform attenuates expenditure pressures and the needed increase in the consumption tax rate, while boosting the aggregate capital stock and output. It offsets the adverse effects of aging, but it limits the increase in pension expenditure in the period 2008-2050 to 12 percentage points of output, 4 percentage points less than in the baseline; tax rate increases are also limited to 44 percent, or about 8 percentage points less than in the baseline.

Comparing the paths of aggregate variables in the partial pension reform—holding the averaging period at 15 years—with those in the baseline scenario reveals the effects of increasing the retirement age (Figure 3 and Table 7):

- Pension expenditures increase by about 12 percentage points of output between 2008 and 2050—4 percentage points less than in the baseline. The consumption tax rate peaks in 2053 at 44 percent—up 25 percentage points from its 2007 level. Still, this is about 7 percentage points lower than in the baseline. In the final steady state the consumption tax rate is 19 percent, 18 percentage points lower than in the baseline.

- The capital stock is consistently higher than in the baseline, albeit significantly so after 2015. The grandfathering rules imply that the aggregate labor supply is significantly higher than in the baseline after 2023. Interestingly, the aggregate effective labor is slightly lower than in the baseline before 2023. Output is significantly higher than in the baseline only since the early 2020s. By 2050, when the dependency ratio peaks, output is 4 percent higher than in the baseline—14 percent lower than in the initial steady state.

- Capital-labor ratios and wage rates are consistently higher than in the baseline; rates of return on assets are lower than in the baseline, except for a brief period (2038-2044).

- Consumption is considerably higher than in the baseline after 2025.
- Pension benefits are lower than in the baseline, particularly after 2023. As in the baseline simulations, however, pension benefits rise until 2055, driven by rising wage rates, and fall sharply thereafter, driven by declining wage rates.

Households' behavior, underlying the macroeconomic effects, helps to account for the small decline in the effective labor supply in the years 2008-2022 (Figure 8). As households work longer (with an unchanged averaging period) they substitute labor inter-temporally: working more at older ages, when their skills have already declined, while exerting less effort during their middle work lives, when their skills are highest. At the outset of the reform, this inter-temporal substitution effect causes a small reduction in the aggregate labor supply. As the time goes by, however, more households enter the upper age ranges and the reform delivers aggregate effective labor gains. These labor gains result from the high labor effort exerted just before retirement—though lower than in the baseline—and from a larger number of working cohorts, that is, a lower dependency ratio. A noteworthy result is the "bust-boom" cycle in aggregate effective labor induced by the partial reform: aggregate effective labor declines before the 2020s, but increases thereafter, reflecting the grandfathering clauses and households' inter-temporal labor substitution effects.

Full Pension Reform

In sum, a full pension reform delivers more significant macroeconomic improvements: it further limits the increase in the pension expenditure in the period 2008-2050 to 8 percentage points of output, 8 percentage points less than in the baseline; tax rate increases are also further limited to 37 percent or about 14 percentage points lower than in the baseline. This suggests that extending the averaging period—conditional on increasing the retirement age—is as important in mitigating the macroeconomic effects of aging as increasing the retirement age during the peak of the demographic shock.

Consider the macroeconomic and welfare effects of changing the averaging period and extending the retirement age across generations. By comparing the full and partial pension reform scenarios (Figure 3 and Table 7) the "pure" effects of changing the averaging period from 15 years to the whole working life can be analyzed. Specifically, extending the averaging period

- Reduces the pension expenditure pressures: these increase 8 percentage points of output between 2008 and 2050—4 percentage points lower compared to a partial reform. The consumption tax rate peaks in 2050 at 37 percent—7 percentage points lower compared to a partial reform (baseline scenario), but still 18 percentage points higher than in 2007. In the final steady state the consumption tax rate is 17 percent, 2 percentage points lower than in the partial reform (baseline) scenarios.[32]

- Leads to consistently higher capital stock levels. Aggregate effective labor is higher before 2020, but lower thereafter. Output is higher before 2025, but lower afterwards—the higher capital stock does not offset the lower labor input. By 2050, output is 1 percent lower (15 percent lower than in the baseline).

- Increases capital-labor ratios after 2013, implying higher wage rates and lower rates of return on capital. In 2008-2013, however, the higher effective labor—caused by the extension of the averaging period—reduces the capital-labor ratio.

[32] Compared to the baseline: pension expenditure pressures are 8 percentage points lower, the peak tax rate in 2050 is 14 percentage points lower, but still 24 percentage points higher than in 2007, and in the final steady state, the consumption tax rate is 20 percentage points lower.

- Boosts consumption before 2023, but reduces it afterwards.
- Reduces pension benefits sharply from the early 2020s. Until then, the grandfathering clauses prevent sharp reductions in pension benefits.

Intuitively, extending the averaging period causes an inter-temporal labor substitution at the household level that is reflected at the aggregate level. Specifically, households intensify their labor efforts during the middle of their working lives, when their skills are highest, and exert less effort when they are close to retirement. In contrast with the delayed aggregate labor gains in the partial reform, the inter-temporal labor substitution effect in the full pension reform leads to immediate aggregate labor gains. But the costs of anticipating the aggregate labor gains shows up later. As a larger share of the population approaches retirement, aggregate effective labor declines, and thus reduces aggregate labor precisely when labor is most scarce, that is, when the dependency ratio starts begins sharply. Note further that compared with the partial reform, a “boom-bust” cycle in aggregate effective labor emerges, which makes labor more scarce as the demographic storm intensifies.

Note that the relative contributions of the pension reforms—increasing the retirement age and (further) extending the averaging period—to limiting the needed increase in the consumption tax rate vary over time. At the peak of the demographic transition, extending the averaging period accounts for half of the tax rate reduction obtained from a full pension reform (relative to the baseline). In the final steady state, however, extending the averaging period accounts for just a tenth of the tax rate reduction. Intuitively, extending the averaging period lowers pension benefits more when (detrended) wage rates rise over time, as is the case in the period

2010-2050. When (detrended) wage rates are constant, as is the case in the final steady state, the relative contribution of extending the averaging period is much smaller.

C. Effects of Tax-smoothing policies

An alternative taxing strategy deserves consideration: pre-funding the fiscal costs associated with aging—with or without pension reforms—by raising the consumption tax rate in 2008 to a new constant rate. This strategy seeks to avoid the distortions and adverse macroeconomic effects resulting from sharp changes in tax rates in the tax-as-you-go scenario. Also, tax-smoothing reduces the tax burden imposed on the younger households, during the toughest years of the demographic transition, at the cost of increasing the burden on older and future generations.

If no pension reform is implemented, the consumption tax rate must increase to 25.4 percent—up 6.4 percentage points from 19 percent in 2007—to pre-finance the demographic shock (Figure 6 and Table 7). From that level, the partial pension reform—increasing the retirement age—reduces the tax rate by 1.2 percentage points, that is, to 24.2 percent. A full pension reform—that also extends the averaging period—reduces further the tax rate by 0.8 percentage points, that is to 23.4 percent. Note that regardless of the pension reform, the government debt-output ratio declines rapidly before the peak of the dependency ratio, and the government becomes a net creditor as public reserve funds are accumulated.[33]

The simulations suggest that compared to tax-as-you-go,

[33] Notice that the resulting changes in debt-output ratios—between the 2008 and minimum levels—are very large—about 70 percentage points under a full pension reform.

- Labor, output, and consumption all increase during the worst period of the demographic transition (2025-2055), but decline before 2025 and after 2055.
- Capital and capital-labor ratios are higher before 2060, and lower thereafter.
- Pension expenditure-output ratios do no vary significantly.

Intuitively, tax smoothing reduces the inter-temporal distortions on asset accumulation relative to the baseline scenario, but also affects the inter-generational welfare distribution. Pension reforms combined with tax-smoothing policies would result in a Pareto improvement only if the negative welfare effects of tax-smoothing on some generations is avoided or (partially) undone by some other mechanism. Such alternatives are discussed below.

D. Welfare Analysis

To complete the discussion of the simulation results, cross generational welfare is computed for the baseline and reform scenarios noted above, under the two taxing strategies (Figure 7).

Tax-as-you-go

In a nutshell, under tax-as-you-go policies both the partial and full pension reforms reduce the welfare of generations entering the labor force in the period 1983-2002; losses for these generations are larger with a full pension reform. All generations entering the labor force before 1983, and after 2002, are either unaffected or benefit from the reforms; gains for these generations are larger under a full pension reform.

Intuitively, the generations that entered the labor before 1983 are already retired or have entered the averaging period before 2008, and thus, are completely grandfathered (Table 6). Although their pension rule does not change, they benefit from the lower consumption tax rates and higher returns on asset holdings made possible by the reform.

However, generations that entered the labor force between 1983 and 2002 are directly affected by the reform. Specifically, their retirement age increases by one year if they entered the labor force between 1983 and 1994, and by two years if they entered the labor force after 1994; also, their averaging period is extended to the end of their work life starting in 2008. Thus, labor-leisure plans made by these generations before the reform are no longer optimal, as these were based on the assumption of no pension reform. Since these generations are forced to re-assess their plans, their resulting labor effort, consumption and asset accumulation profiles are not as smooth had they had anticipated the reforms. Moreover, as these are the first generations retiring under the new system, they face a shorter retirement period and lower pension benefits, but hardly benefit from reduced tax rates, which occur at the end of their lives.

Generations that entered the labor force between 2002 and 2007 benefit from the reform as the reduction in tax rates in the second half of their lives more than offsets the suboptimal allocations made before the reforms are implemented. Likewise, those generations entering the labor force on and after 2008 are net winners as they did not make suboptimal choices and face lower tax rates during most of their lives.

Tax smoothing

Conditional on full pension reform, tax-smoothing policies deliver welfare gains to generations that enter the labor force in 1992-2055, and cause welfare losses to generations that enter the labor force in the period 1951-1991, and after 2055.

This reflects the fact that the tax burden shifts from generations that are net winners to those that fair unfavorably under the reform. The generations that entered the labor market before 1992 (40 years of age and older) were not going to face any major tax rate increase in the tax-as-you-go scenario, and thus they are worse off when taxes are increased in 2008. Those generations entering the labor force between 1992 and 2064 were going to face large tax hikes in the tax-as-you-go scenario (above the tax smoothing level), thus benefiting from a lower and constant consumption tax rate, which results in positive wealth effects and undistorted saving decisions. Finally, those generations entering the labor force after 2064 are worse off as they face a lower tax rate than in the tax-smoothing scenario.

In sum, the inter-generational welfare transfers posits the question of whether a mechanism can be designed to achieve a Pareto improving situation—transferring resources from future generations to present generations—such that none of the existing nor future generations lose relative to the baseline. Two specific mechanisms are studied below: lump sum transfers to current generation financed through debt, and delaying the tax increase in the tax-smoothing scenario.

Pareto improving pension reforms (with intergenerational welfare transfers)

[To be completed]

V. Conclusions

Using a DGE framework in the Auerbach-Kotlikoff tradition, this paper evaluates the macroeconomic and welfare effects of parametric reforms of the Spanish PAYG system needed to address the challenges posed by aging. Consistent with the broad political and social consensus in Spain, the analysis focuses on reforms extending the averaging period—a central element of the pension rule used to compute the pension benefit based on inflation-indexed wage earnings—and the complementarities of these reforms with increases in the retirement age under alternative tax policies to finance old-age related spending.

Under current policies, and assuming that consumption taxes increase to finance age-related spending, the baseline scenario shows pension expenditures increasing by about 16 percentage points of output by 2050, more than twice as much as in official and European Commission projections (7.1 percentage points). To finance this spending, the consumption tax rate must rise by more than 30 percentage points, with severe macroeconomic consequences.

Pension reforms can limit significantly the adverse effects of Spain's aging population, particularly during the demographic transition. The simulations from the model, which explicitly account for the Spanish pension rule, suggest that during the demographic transition:

- Increasing the retirement age gradually from 62 to 70 years (starting in 2008) limits the increase in pension expenditures to 12 percentage points of output (4 percentage points less than otherwise) over the period 2008-2050, and thus reduces the need to increase tax rates; compared to the baseline, the aggregate capital stock and output are boosted. Of note, the reform induces a "bust-boom" cycle in aggregate effective labor—lowering labor supply before the 2020s and increasing it thereafter—reflecting households' inter-temporal substitution effects and the impact of grandfathering clauses.
- Extending the averaging period—conditional on raising the retirement age—delivers comparable macroeconomic gains at the peak of the demographic transition. It further limits the increase in pension expenditures to 8 percentage points of output (8

> percentage points less than without reforms) over the period 2008-2050. The smaller tax rate increases serve to attenuate further the adverse macroeconomic effects at the peak of the demographic shock. In contrast to increasing the retirement age, extending the averaging period—conditional on raising the retirement age—triggers a "boom-bust" cycle in the aggregate effective labor, reducing it after the 2020s, when the demographic storm intensifies and labor supply is most scarce.

Parametric reforms also improve the long run outlook. In the final steady state, when population growth and the population's age structure stabilize, increasing the retirement age and extending the averaging period (from its current level to the whole work life) results in the highest welfare gains. Although extending the averaging period reduces pension benefits, households are still better off because of the indirect (general equilibrium) effects associated with lower taxation. With no technological progress, or with progress significantly lower than its historical average, however, long run welfare gains are highest for an averaging period that is longer than the current period, but shorter than the entire work life.

A final word on the relative merits of increasing the retirement age and extending the averaging period during the demographic transition and in the long-term. At the peak of the demographic transition, extending the averaging period accounts for half of the tax rate reduction obtained from a full pension reform (relative to the baseline). In the final steady state, however, extending the averaging period accounts for just a tenth of the tax rate reduction. This reflects the fact that extending the averaging period when wage rates as rising lowers pension benefits more than when wage rates are constant.

APPENDIX I: VALUE FUNCTION AT RETIREMENT

The value function $V(A_{t+T_t}^{T_t+1}, b_{t+T_t}^{T_t+1})$ is the solution of the following problem:

$$V(A_{t+T_t}^{T_t+1}, b_{t+T_t}^{T_t+1}) = \underset{\left\{c_{t+s-1}^{s}, A_{t+s}^{s+1}\right\}_{s=T_t+1}^{T_t+T_t^R}}{Max} \sum_{s=T_t+1}^{T_t+T_t^R} \beta^{s-1} \cdot \log(c_{t+s-1}^{s})$$

subject to (4), (6) and given $A_{t+T_t}^{T_t+1}$ and $b_{t+T_t}^{T_t+1}$.

Notice that the household's asset holdings at retirement ($A_{t+T_t}^{T_t+1}$), and the annual pension benefit ($b_{t+T_t}^{T_t+1}$) are given, as they are determined by household's past decisions.

Let $\tilde{r}_t$ denote the year t rate of return on assets holdings net of the income tax, $\tilde{r}_t = r_t \cdot (1-\tau_t^I)$. We use the budget constraint (4) to solve for c_{t+s-1}^{s} and to express the value function recursively, in a Bellman's equation form (for $s = T_t+1,...,T_t+T_t^R$), as follows:

$$V\left(A_{t+s-1}^{s}, b_{t+T_t}^{T_t+1}\right) = \underset{A_{t+s}^{s+1}}{Max} \log\left\{(1+\tilde{r}_{t+s-1}) \cdot A_{t+s-1}^{s} + \frac{b_{t+T_t}^{T_t+1}}{(1+\gamma)^{s-T_t-1}} - (1+\gamma) \cdot A_{t+s}^{s+1}\right\} + \beta \cdot V\left(A_{t+s}^{s+1}, b_{t+T_t}^{T_t+1}\right).$$

We obtain the value function by backward induction, that is, we start with the household's problem in its last year of life, and proceed backwards.

1. The household's problem at date $t+T_t+T_t^R-1$ (household's age is $s = T_t+T_t^R$) is given by

$$V\left(A_{t+T_t+T_t^R-1}^{T_t+T_t^R}, b_{t+T_t}^{T_t+1}\right) = \underset{A_{t+T_t+T_t^R}^{T_t+T_t^R+1}}{Max} \log\left\{\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right) \cdot A_{t+T_t+T_t^R-1}^{T_t+T_t^R} + \frac{b_{t+T_t}^{T_t+1}}{(1+\gamma)^{T_t^R-1}} - (1+\gamma) \cdot A_{t+T_t+T_t^R}^{T_t+T_t^R+1}\right\}.$$

The household consumes all its remaining assets in its last period of life, as it leaves no bequests and the no-Ponzi condition ($A_{t+T_t+T_t^R}^{T_t+T_t^R+1}=0$) is satisfied. Thus, the solution is given by

$$V\left(A_{t+T_t+T_t^R-1}^{T_t+T_t^R}, b_{t+T_t}^{T_t+1}\right)=\log\left\{\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right)\cdot A_{t+T_t+T_t^R-1}^{T_t+T_t^R}\right\}+\frac{b_{t+T_t}^{T_t+1}}{\left(1+\gamma\right)^{T_t^R-1}}.$$

2. The household's problem at date $t+T_t+T_t^R-2$ (household's age is $T_t+T_t^R-1$) is given by

$$V\left(A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}, b_{t+T_t}^{T_t+1}\right)=\underset{A_{t+T_t+T_t^R-1}^{T_t+T_t^R}}{Max}\ \log\left\{\left(1+\tilde{r}_{t+T_t+T_t^R-2}\right)\cdot A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}+\frac{b_{t+T_t}^{T_t+1}}{\left(1+\gamma\right)^{T_t^R-2}}-\left(1+\gamma\right)\cdot A_{t+T_t+T_t^R-1}^{T_t+T_t^R}\right\}$$
$$+\beta\cdot V\left(A_{t+T_t+T_t^R-1}^{T_t+T_t^R}, b_{t+T_t}^{T_t+1}\right).$$

Plug the solution of $V\left(A_{t+T_t+T_t^R-1}^{T_t+T_t^R}, b_{t+T_t}^{T_t+1}\right)$ found in 1 to obtain the following expression:

$$V\left(A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}, b_{t+T_t}^{T_t+1}\right)=\underset{A_{t+T_t+T_t^R-1}^{T_t+T_t^R}}{Max}\ \log\left\{\left(1+\tilde{r}_{t+T_t+T_t^R-2}\right)\cdot A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}+\frac{b_{t+T_t}^{T_t+1}}{\left(1+\gamma\right)^{T_t^R-2}}-\left(1+\gamma\right)\cdot A_{t+T_t+T_t^R-1}^{T_t+T_t^R}\right\}$$
$$+\beta\cdot\log\left\{\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right)\cdot A_{t+T_t+T_t^R-1}^{T_t+T_t^R}+\frac{b_{t+T_t}^{T_t+1}}{\left(1+\gamma\right)^{T_t^R-1}}\right\}.$$

Find the first order condition of this optimization problem and solve for $A_{t+T_t+T_t^R-1}^{T_t+T_t^R}$,

$$A_{t+T_t+T_t^R-1}^{T_t+T_t^R}=\frac{\beta\cdot\prod_{i=1}^{2}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\cdot A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}}{\left(1+\gamma\right)\cdot\left(1+\beta\right)\cdot\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right)}-\frac{\left[1-\beta\cdot\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right)\right]\cdot\frac{b_{t+T_t}^{T_t+1}}{\left(1+\gamma\right)^{T_t^R-2}}}{\left(1+\gamma\right)\cdot\left(1+\beta\right)\cdot\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right)};$$

plug this expression into the value function $V\left(A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}, b_{t+T_t}^{T_t+1}\right)$ and solve, as follows:

$$V\left(A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}, b_{t+T_t}^{T_t+1}\right)=\left(1+\beta\right)\cdot\log\left\{\prod_{i=1}^{2}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\cdot A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}+\left(2+\tilde{r}_{t+T_t+T_t^R-1}\right)\cdot\frac{b_{t+T_t}^{T_t+1}}{\left(1+\gamma\right)^{T_t^R-2}}\right\}-\Omega_1,$$

where Ω_1 is a constant: $\Omega_1=\log\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right)+\left(1+\beta\right)\cdot\log\left(1+\beta\right)+\beta\cdot\log\left(1+\gamma\right)-\beta\cdot\log\left(\beta\right)$.

3. The household's problem at date $t+T_t+T_t^R-3$ (household's age is $s=T_t+T_t^R-2$):

$$V\left(A_{t+T_t+T_t^R-3}^{T_t+T_t^R-2}, b_{t+T_t}^{T_t+1}\right) = \underset{A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}}{Max} \log\left\{\left(1+\tilde{r}_{t+T_t+T_t^R-3}\right)\cdot A_{t+T_t+T_t^R-3}^{T_t+T_t^R-2} + \frac{b_{t+T_t}^{T_t+1}}{\left(1+\gamma\right)^{T_t^R-3}} - \left(1+\gamma\right) A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}\right\}$$
$$+\beta\cdot V\left(A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}, b_{t+T_t}^{T_t+1}\right).$$

Replacing $V\left(A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}, b_{t+T_t}^{T_t+1}\right)$ from 2, we can write the previous expression as follows:

$$V\left(A_{t+T_t+T_t^R-3}^{T_t+T_t^R-2}, b_{t+T_t}^{T_t+1}\right) = \underset{A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}}{Max} \log\left\{\left(1+\tilde{r}_{t+T_t+T_t^R-3}\right)\cdot A_{t+T_t+T_t^R-3}^{T_t+T_t^R-2} + \frac{b_{t+T_t}^{T_t+1}}{\left(1+\gamma\right)^{T_t^R-3}} - \left(1+\gamma\right)\cdot A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}\right\}$$
$$+\beta\cdot\left(1+\beta\right)\cdot\log\left\{\prod_{i=1}^{2}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\cdot A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1} + \left(2+\tilde{r}_{t+T_t+T_t^R-1}\right)\cdot\frac{b_{t+T_t}^{T_t+1}}{\left(1+\gamma\right)^{T_t^R-2}}\right\} - \beta\cdot\Omega_1.$$

Find the first order condition and solve for $A_{t+T_t+T_t^R-2}^{T_t+T_t^R}$,

$$A_{t+T_t+T_t^R-2}^{T_t+T_t^R} = \frac{\beta\cdot(1+\beta)\cdot\prod_{i=1}^{3}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\cdot A_{t+T_t+T_t^R-3}^{T_t+T_t^R-2}}{\left(1+\gamma\right)\cdot\left(1+\beta+\beta^2\right)\cdot\prod_{i=1}^{2}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)} - \frac{\left[\beta\cdot(1+\beta)\cdot\prod_{i=1}^{2}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)-\left(2+\tilde{r}_{t+T_t+T_t^R-1}\right)\right]\cdot\frac{b_{t+T_t}^{T_t+1}}{\left(1+\gamma\right)^{T_t^R-3}}}{\left(1+\gamma\right)\cdot\left(1+\beta+\beta^2\right)\cdot\prod_{i=1}^{2}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)}.$$

Plug this previous expression into the value function $V\left(A_{t+T_t+T_t^R-3}^{T_t+T_t^R-2}, b_{t+T_t}^{T_t+1}\right)$ and solve,

$$V\left(A_{t+T_t+T_t^R-3}^{T_t+T_t^R-2}, b_{t+T_t}^{T_t+1}\right) = \left(1+\beta+\beta^2\right)\cdot\log\left\{\prod_{i=1}^{3}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\cdot A_{t+T_t+T_t^R-3}^{T_t+T_t^R-2} + \left[1+\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right)+\prod_{i=1}^{2}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\right]\cdot\frac{b_{t+T_t}^{T_t+1}}{\left(1+\gamma\right)^{T_t^R-3}}\right\} - \Omega_2,$$

where Ω_2 is a constant: $\Omega_2 = \log\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right)+\log\left(1+\tilde{r}_{t+T_t+T_t^R-2}\right)+\left(1+\beta+\beta^2\right)\cdot\log\left(1+\beta+\beta^2\right)$

$$+\beta\cdot\left(1+\beta\right)\cdot\log\left(1+\gamma\right)-\beta\cdot\left(1+\beta\right)\cdot\log\left[\beta\cdot\left(1+\beta\right)\right].$$

4. Repeating the procedure backwards, the value function at date $t+T_t$ (household's age is T_t+1) is given by

$$V\left(A_{t+T_t}^{T_t+1}, b_{t+T_t}^{T_t+1}\right)=\left(\sum_{j=1}^{T_t^R}\beta^{j-1}\right)\cdot\log\left\{\prod_{i=1}^{T_t^R}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\cdot A_{t+T_t}^{T_t+1}+\left\{1+\sum_{j=1}^{T_t^R-1}\left[\prod_{i=1}^{j}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\right]\right\}\cdot b_{t+T_t}^{T_t+1}\right\}-\Omega,$$

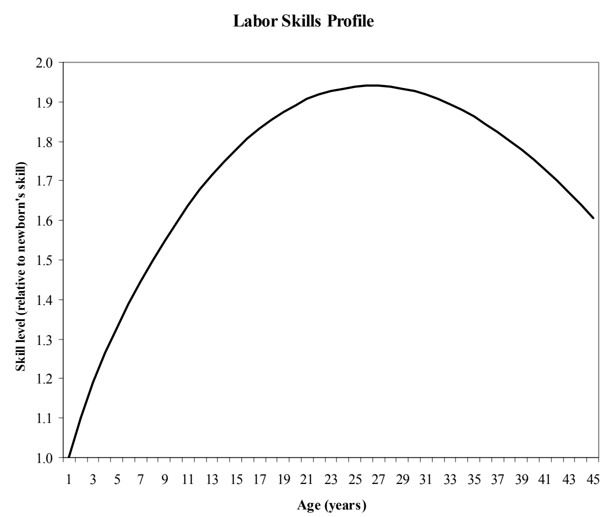
where Ω is a constant. The derivatives of the value function with respect to changes in asset holdings $\left(V_A\right)$ and pension benefits $\left(V_b\right)$ are given by

$$V_A(.)=\frac{\left(\sum_{j=1}^{T_t^R}\beta^{j-1}\right)\cdot\prod_{i=1}^{T_t^R}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)}{\prod_{i=1}^{T_t^R}\left(1+r_{t+T_t+T_t^R-i}\right)\cdot A_{t+T_t}^{T_t+1}+\left\{1+\sum_{j=1}^{T_t^R-1}\left[\prod_{i=1}^{j}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\right]\right\}\cdot b_{t+T_t}^{T_t+1}},$$

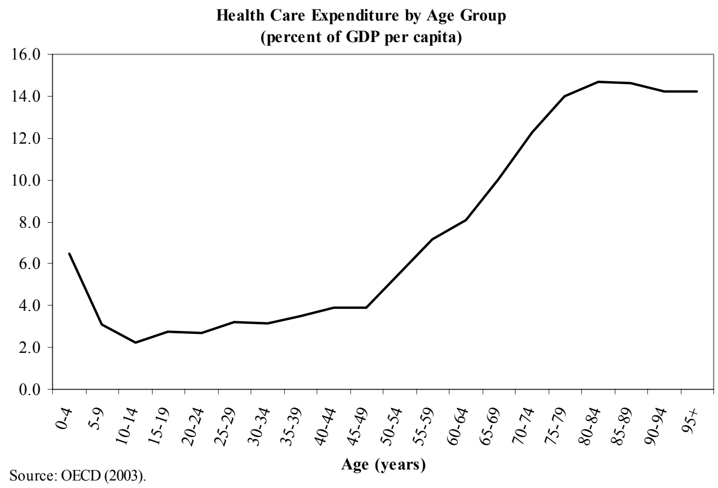
$$V_b(.)=\frac{\left(\sum_{j=1}^{T_t^R}\beta^{j-1}\right)\cdot\left\{1+\sum_{j=1}^{T_t^R-1}\left[\prod_{i=1}^{j}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\right]\right\}}{\prod_{i=1}^{T_t^R}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\cdot A_{t+T_t}^{T_t+1}+\left\{1+\sum_{j=1}^{T_t^R-1}\left[\prod_{i=1}^{j}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\right]\right\}\cdot b_{t+T_t}^{T_t+1}}.$$

APPENDIX II: CALIBRATION-DATA SOURCES

Household's labor skills by age (figure below): the labor skills profile by age was calibrated to match the relative wage rates (per hour) earned by households in different age groups in Spain, according to data from the Spanish National Statistics Institute (INE). The calibration of skills for households with more than 30 years in the workforce is based on Hansen (1993).



Government health expenditure profile by age: the figure below shows the private and public health-related expenditure by age group as a share of GDP per capita in Spain in the year 1998. Dividing by the total population in that year, we obtained the sum of the private and public health-related expenditures by age group as a share of GDP. Using the fact that sum of public health-related consumption over all age groups was 5.5 percent of GDP in the year 1998, and assuming that private and public health-related expenditures exhibit the same age profiles, we obtained the public health-related expenditure profile by age group as a share of output. Dividing by the population of each age group and multiplying by output we obtained the expenditure per individual of a given age group and assumed that it grows over time at the rate of technological progress.[34] Having obtained the public expenditure per individual of each age group, it was straightforward to track the total public health-related expenditures at each point in time.



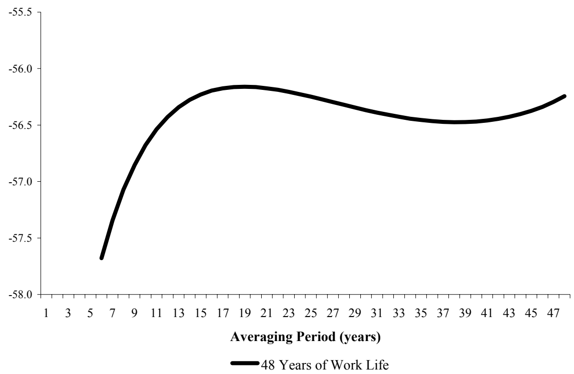
Source: OECD (2003).

[34] In recent years, health related expenditures per individual have grown faster than output. Therefore, our assumptions may be underestimating future health-related expenditure pressures arising from population aging.

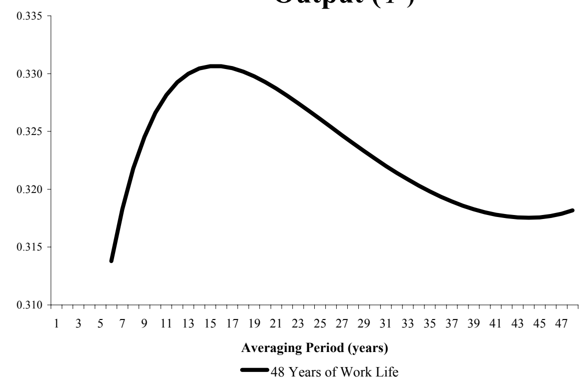
APPENDIX III: COMPARATIVE STATICS IN THE FINAL STEADY STATE:

We evaluate the welfare and macroeconomic effects of extending the averaging period when the rate of labor-augmenting technological progress is zero ($\xi = 0$). We find that welfare is maximized for an averaging period of 19 years.

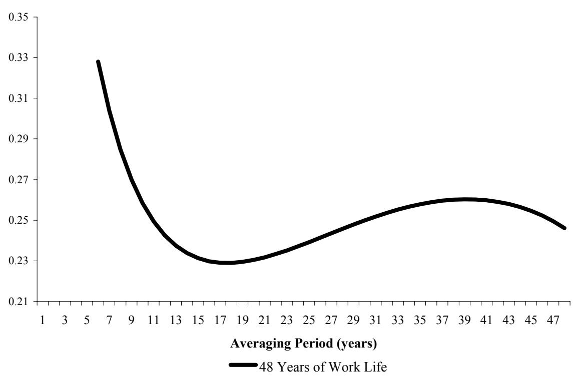
Welfare



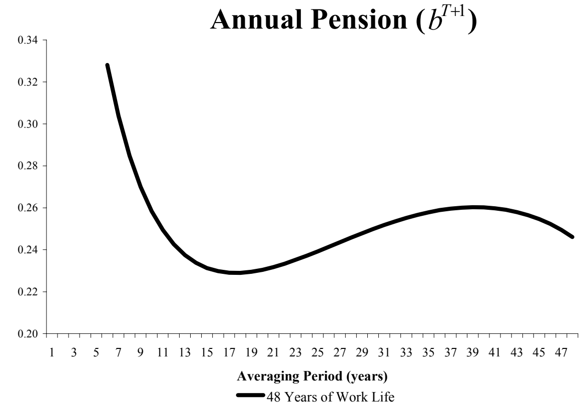
Output (Y)



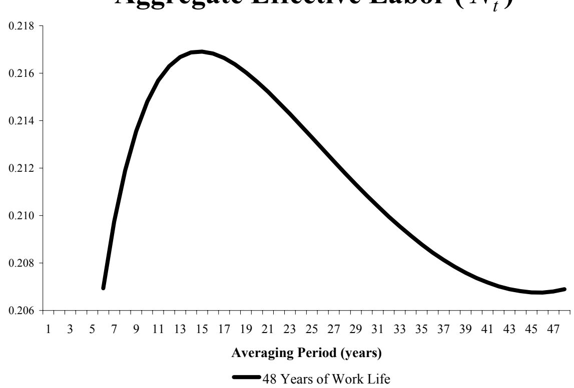
Consumption Tax Rate (τ^c)



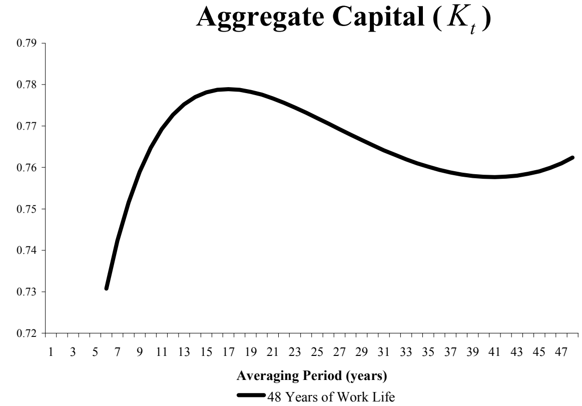
Annual Pension (b^{T+1})



Aggregate Effective Labor (N_t)



Aggregate Capital (K_t)



REFERENCES

Auerbach, Alan and Laurence Kotlikoff, 1987. "Dynamic Fiscal Policy" - Cambridge University Press.

Bailén José M., and Joan Gil, 1998, “Transitional Effects of a System Change in Spain,” in Marmor, T. and P. De Jong (Eds), “Ageing, Social Security and Affordability”.

Balmaseda, Manuel, and Patrocinio Tello, 2003, “The impact of legislative changes on contributory pensions,” *Situación Spain*, pp. 29-40.

Blinder, Alan, Gordon, Roger, and Donald Wise, 1980, “Reconsidering the Work Disincentive Effects of Social Security,” *National Tax Journal*, Vol. 33, Iss.4, pp 431-33.

Catalán, Mario, Alexander Hoffmaister and Jaime Guajardo, 2005, “Pension Reform in Spain: Macroeconomic Impact,” in *Spain: Selected Issues*, IMF Country Report No. 05/57.

Conde-Ruiz J. Ignacio and Javier Alonso, 2004. “El futuro de las pensiones en España: perspectivas y lecciones”. FEDEA.

De Nardi, Mariacristrina, Imrohoroglu, Selahattin, and Thomas Sargent, 1999, “Projected US Demographics and Social Security,” *Review of Economic Dynamics*, Vol. 2, 575-615.

Diamond, Peter, 2001, “Issues in Social Security Reform with a Focus on Spain,” mimeo.

European Commission, 2001, “*Budgetary Challenges Posed by Aging Populations: The Impact on Public Spending on Pensions, Health, and Long-Term Care for the Elderly and Possible Indicators of the Long-Term Sustainability of Public Finances*,” (Brussels).

Estrada, Angel, Fernandez, Jose Luis, Moral Esther and Ana Regil, 2004, “A quarterly macroeconometric model of the Spanish economy,” Bank of Spain, Working Paper #413.

Fernandez de Cordoba, Gonzalo and Timothy Kehoe, 2000, “Capital flows and real exchange rate fluctuations following Spain’s entry into the European Community,” Journal of International Economics, 51, pp 49-78.

Hansen, G. D. (1993), “The Cyclical and Secular Behavior of the Labor Input: Comparing Efficiency Units and Hours Worked,” *Journal of Applied Econometrics*, Vol. 8, No. 1, pp. 71-80.

Heer, Burkhard and Alfred Mauβner, 2004, "Dynamic General Equilibrium Modelling: Computational Methods and Applications" - Springer-Verlag.

Herce, José, and Javier Alonso Meseguer, 2000, “La reforma de las pensiones ante la revisión del Pacto de Toledo,” Colección Estudios Económicos, No. 19, Servicio de Estudios, la Caixa.

Instituto Nacional de Estadística, 2004, “Evolución futura de la población,” Chapter 6, in *Tendencias demográficas durante el siglo XX en España*, pp. 257-94.

Jimeno, Juan F., 2000, “El sistema español de pensiones: previsiones de gasto y medidas de reforma”, Hacienda Pública Española, 1/2000.

Judd, Kenneth, 1999, "Numerical Methods in Economics" - MIT Press.

Kotlikoff, Laurence, 2000, "The A-K OLG Model: Its Past, Present, and Future," in Harrison G.W., S. Jensen, and T.F. Rutherford (Eds), “Using Dynamic General Equilibrium Models for Policy Analysis, Amsterdam, Elsevier.

Kotlikoff, Laurence, Smetters, Kent, and Jan Walliser, 1999, “Privatizing Social Security in the US—Comparing the Options,” *Review of Economic Dynamics*, Vol. 2, pp 532-74.

Newhouse, Joseph P., 1977, "Medical-Care Expenditure: A Cross-National Survey." *Journal of Human Resources* 12(1):115-125

Organization for Economic Cooperation and Development, 2001, “Fiscal Implications of Aging: Projections of Age-Related Spending,” *Economic Outlook 2001/1*, Vol. 69 (June), pp. 145-67

Organization for Economic Cooperation and Development, 2003, “Spending on Health and Long-Term Care,” Working Party No. 1 on Macroeconomic and Structural Policy Analysis, (March).

Rojas, Juan, (2005), “Life-cycle Earnings, Cohort Size Effects and Social Security: A Quantitative Exploration,” *Journal of Public Economics*, vol. 89, pp. 465-85.

Sandell, Steven, Iams, Howard, and Daniel Fanaras, 1999, “The Distributional Effects of Changing the Averaging Period and Minimum Benefit Provisions,” *Social Security Bulletin*, Vol. 62, No. 2, pp 4-13.

Serrano, Felipe, Miguel Angel García, and Carlos Bravo, 2004, “*El Sistema Español de Pensiones: Un Proyecto Viable desde un Enfoque Económico*,” Ariel Publisher.

World Bank, 2004, “World Development Indicators.”

Table 1. Variable Definition and Notation

Variable	Notation	Stationary Transformation	Variable	Notation	Stationary Transformation
			Parameters		
Discount factor (utility)	β		Rate of labor augmenting technological progress	ξ	
Leisure preference (utility)	γ		Replacement ratio (pension rule)	ψ	
Capital share (production)	α		Averaging period (pension rule)	μ	
Capital depreciation rate	δ		Constant rate of population growth 1/	p	
Labor skill	e^s		Total factor productivity	Z	
			Population		
s -year old population	P_t^s		Total population	P_t	
			Households		
Labor effort	n_t^s		Aggregate effective labor supply	$\hat{N}_t^h$	$N_t^h = \frac{\hat{N}_t^h}{P_t}$
Leisure	l_t^s		Aggregate labor effort	$\hat{n}_t^h$	$n_t^h = \frac{\hat{n}_t^h}{P_t}$
Consumption	$\hat{c}_t^s$	$c_t^s = \frac{\hat{c}_t^s}{(1+\xi)^t}$	Aggregate consumption	$\hat{C}_t^h$	$C_t^h = \frac{\hat{C}_t^h}{(1+\xi)^t \cdot P_t}$
Asset holdings	$\hat{A}_t^s$	$A_t^s = \frac{\hat{A}_t^s}{(1+\xi)^t}$	Aggregate asset holdings	$\hat{A}_t^h$	$A_t^h = \frac{\hat{A}_t^h}{(1+\xi)^t \cdot P_t}$
Annual pension	$\hat{b}_{t+T_t}^{T_t+1}$	$b_{t+T_t}^{T_t+1} = \frac{\hat{b}_{t+T_t}^{T_t+1}}{(1+\xi)^{t+T_t}}$			
			Firms		
Aggregate capital demand	$\hat{K}_t^f$	$K_t^f = \frac{\hat{K}_t^f}{(1+\xi)^t \cdot P_t}$	Aggregate labor demand	$\hat{N}_t^f$	$N_t^f = \frac{\hat{N}_t^f}{P_t}$
Aggregate output	$\hat{Y}_t^f$	$Y_t^f = \frac{\hat{Y}_t^f}{(1+\xi)^t \cdot P_t}$	Profits (net) 2/	$\hat{\Pi}_t^f$	$\Pi_t^f = \frac{\hat{\Pi}_t^f}{(1+\xi)^t \cdot P_t}$
			Factor Prices		
Gross rate of return on assets	r_t		Wage rate (unskilled labor)	$\hat{W}_t$	$W_t = \frac{\hat{W}_t}{(1+\xi)^t}$
			Tax Rates		
Social security contribution	τ_t		Consumption tax	τ_t^c	
Income tax	τ_t^I				
			Government		
Debt	$\hat{D}_t$	$D_t = \frac{\hat{D}_t}{(1+\xi)^t \cdot P_t}$	Expenditure	$\hat{G}_t$	$G_t = \frac{\hat{G}_t}{(1+\xi)^t \cdot P_t}$

Note: Superscripts (subscripts) indicate the age of the household (time period); stock variables are dated at the beginning of the corresponding year. 1/ Population growth rates are constant only along balanced growth equilibrium paths. 2/ Profits are net of capital depreciation.

Table 2. First Order Conditions—Household's Optimization Problem

	Consumption-Leisure Decision (Intra-temporal condition)	Consumption-Saving Decision (Inter-temporal condition)
Working Age 1) Before averaging period $(s=1,\ldots,T_t-\mu)$	$\frac{\gamma}{l^s_{t+s-1}} = \frac{W_{t+s-1}\cdot e^s\cdot(1-\tau_{t+s-1}-\tau^I_{t+s-1})}{c^s_{t+s-1}\cdot(1+\tau^c_{t+s-1})}$ (7)	$\frac{(1+\xi)}{c^s_{t+s-1}\cdot(1+\tau^c_{t+s-1})} = \beta\cdot\frac{[1+r_{t+s}\cdot(1-\tau^I_{t+s})]}{c^{s+1}_{t+s}\cdot(1+\tau^c_{t+s})}$ (8)
2) During averaging period $(s=T_t-\mu+1,\ldots,T_t-1)$	$\frac{\gamma}{l^s_{t+s-1}} = \frac{W_{t+s-1}\cdot e^s\cdot(1-\tau_{t+s-1}-\tau^I_{t+s-1})}{c^s_{t+s-1}\cdot(1+\tau^c_{t+s-1})} + W_{t+s-1}\cdot e^s\cdot\frac{\psi}{\mu}\cdot\frac{\beta^{T_t+1-s}}{(1+\xi)^{T_t+1-s}}\cdot V_b(A^{T_t+1}_{t+T_t}, b^{T_t+1}_{t+T_t})$ (9)	$\frac{(1+\xi)}{c^s_{t+s-1}\cdot(1+\tau^c_{t+s-1})} = \beta\cdot\frac{[1+r_{t+s}\cdot(1-\tau^I_{t+s})]}{c^{s+1}_{t+s}\cdot(1+\tau^c_{t+s})}$ (10)
3) Last year before retirement $(s=T_t)$	$\frac{\gamma}{l^{T_t}_{t+T_t-1}} = \frac{W_{t+T_t-1}\cdot e^s\cdot(1-\tau_{t+T_t-1}-\tau^I_{t+T_t-1})}{c^{T_t}_{t+T_t-1}\cdot(1+\tau^c_{t+T_t-1})} + W_{t+T_t-1}\cdot e^s\cdot\frac{\psi}{\mu}\cdot\frac{\beta}{(1+\xi)}\cdot V_b(A^{T_t+1}_{t+T_t}, b^{T_t+1}_{t+T_t})$ (11)	$\frac{(1+\xi)}{c^{T_t}_{t+T_t-1}\cdot(1+\tau^c_{t+T_t-1})} = \beta\cdot V_A(A^{T_t+1}_{t+T_t}, b^{T_t+1}_{t+T_t})$ (12)
Retirement $(s=T_t+1,\ldots,T_t+T^R_t-1)$		$\frac{(1+\xi)}{c^s_{t+s-1}\cdot(1+\tau^c_{t+s-1})} = \frac{\beta\cdot[1+r_{t+s}\cdot(1-\tau^I_{t+s})]}{c^{s+1}_{t+s}\cdot(1+\tau^c_{t+s})}$ (13)

Table 3. Balanced Growth Path

Variable	Growth Rate
$\widehat{Y}_t,\ \widehat{C}_t,\ \widehat{K}_t,\ \widehat{D}_t,\ \widehat{G}_t,\ \sum_{s=T+1}^{T+T^R} \hat{b}_t^s \cdot P_t^s$	$p+\xi+p\cdot\xi$
$\widehat{W}_t,\ \hat{b}_{t+T}^{T+1},\ (\hat{c}_t^1,...,\hat{c}_t^{T+T^R}),\ (\hat{A}_t^2,...,\hat{A}_t^{T+T^R})$	ξ
$\widehat{N}_t$	p
$r_t,\ (n_t^1,...,n_t^T)$	0

Table 4. Calibration of the Baseline Model (Initial Steady State)

Symbol	Definition	Value	Source
α	Share of capital	0.3300	Fernandez de Cordoba and Kehoe (2000) and Estrada et al. (2004). 1/
γ	Leisure preference	1.8700	Value set so that the fraction of working time for the representative household is 0.274.
β	Discount factor	0.9500	From the real business cycle literature.
δ	Depreciation rate	0.0600	From the real business cycle literature.
Z	Total factor productivity	0.6100	Value set so that the capital-output ratio is 2.01. 2/
τ	Social security payroll tax rate	0.1950	Social security contributions over wage income. 3/
τ^c	Consumption tax rate	0.1890	Indirect tax revenues as percentage of private consumption (average 1994-2004). 4/
τ^I	Capital-income tax rate	0.1360	Direct tax revenues and other current revenues as percentage of GDP (average 1994-2004).
p	Rate of population growth	0.0085	Average 1900-1970.
G/Y	Government consumption (fraction of total output)	0.2280	Average 1994-2004. 5/
D/Y	Government debt (fraction of total output)	0.4100	General government (includes regional governments), 2004.
ψ	Replacement ratio	0.5438	Value set so that the pension at retirement over the (net) average wage income for the working population is 0.65. 6/
μ	Averaging period	15.0000	15 years is the reference period since the reform of 1997.
ξ	Rate of labor-augmenting technological progress	0.0150	Set to result in a 1.5 percent annual rate of output per capita growth (average).
T	Work life (years)	40.0000	Set to match individuals' entry to the labor force at age 22 and retirement at age 62.
T^R	Retirement life (years)	18.0000	Households live 80 years with certainty.

Notes: 1/ Fernandez de Cordoba and Kehoe (2004) calibrate their model using a share-of-capital parameter value of 0.302, whereas Estrada et al. (2004) estimate the parameter value at 0.36 using econometrics.
2/ Fernandez de Cordoba and Kehoe (2004) set the capital-output ratio value at 2.03.
3/ Social security contributions are 13 percent of GDP, and the labor share is 0.67.
4/ Indirect tax revenues are 11.3 percent of GDP, and the share of consumption in GDP is 0.59.
5/ Government consumption (0.228) = Current expenditures (0.359) - Social transfers (0.128) - Interest payments (0.037) + Capital expenditures (0.034). Also, Government consumption (0.228) = Health-related public expenditures (0.055) + Non-health public expenditures (0.173).
6/ The wage income is net of payroll taxes. According to Serrano et al. (2004), the average replacement ratio (average pension income over the net average wage income) in 2002 was 0.625 (0.517) for new (old) pensioners, increasing about 1.25 percentage points per year.

Sources: National Accounts and Labor Statistics: Instituto Nacional de Estadistica (INE) and AMICO; Fiscal Accounts: Ministerio de Economia y Hacienda, IGAE; Population: INE; Life expectancy: World Health Organization and World Development Indicators (World Bank).

Figure 1. Household's Asset Holdings (A^s), Labor Effort (n^s), and Consumption (c^s) Profiles by Age (Initial Steady State and Selected Generations)

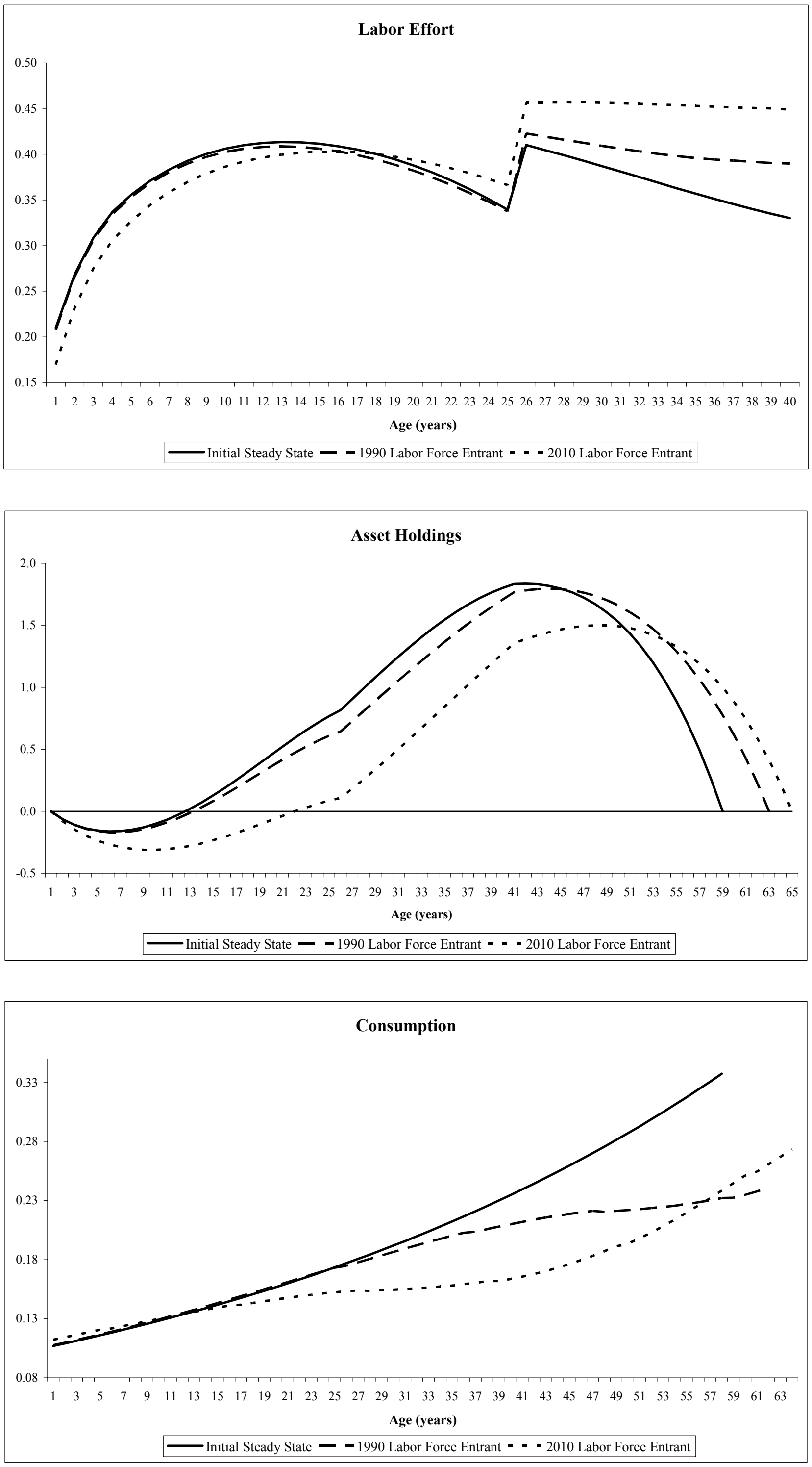


Table 5. Demographic Transition: Model's Time Line[1/]

	Period			
	First Century	Transitional		Last Century
Calendar year	1857-1956	1957-2059	2060-2126	2127-2227
Growth of labor force entrants	0.85	variable	0.5	0.5
Life expectancy of labor force entrants 2/	80 (58)	increases one year per decade	90 (68)	90 (68)
Population growth	0.85	variable	variable	0.5

1/ Annual percentage rates of growth unless otherwise indicated.
2/ Natural life expectancy at birth of the cohort entering the labor force in a given year. Numbers in parentheses indicate remaining life time upon entry to the labor force in the model. Strictly, life expectancy increases one year per decade between 1957 and 2047, and is constant thereafter.

Figure 2. Model's Dependency Ratio

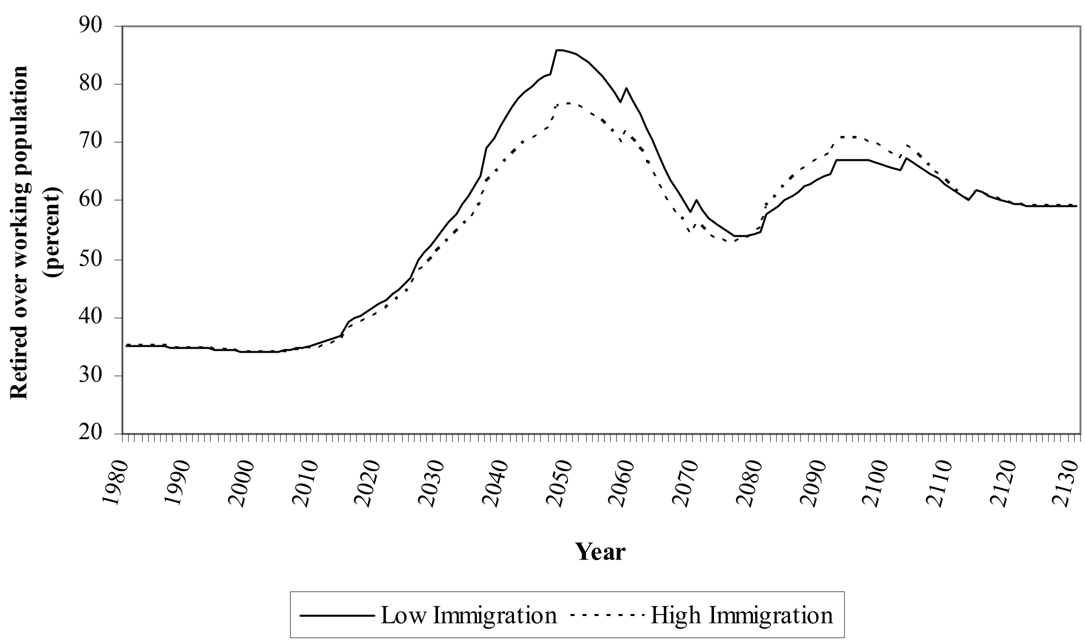


Figure 3. Baseline and Pension Reforms Macroeconomic Scenarios
(Tax-as-you-go and Low Immigration)

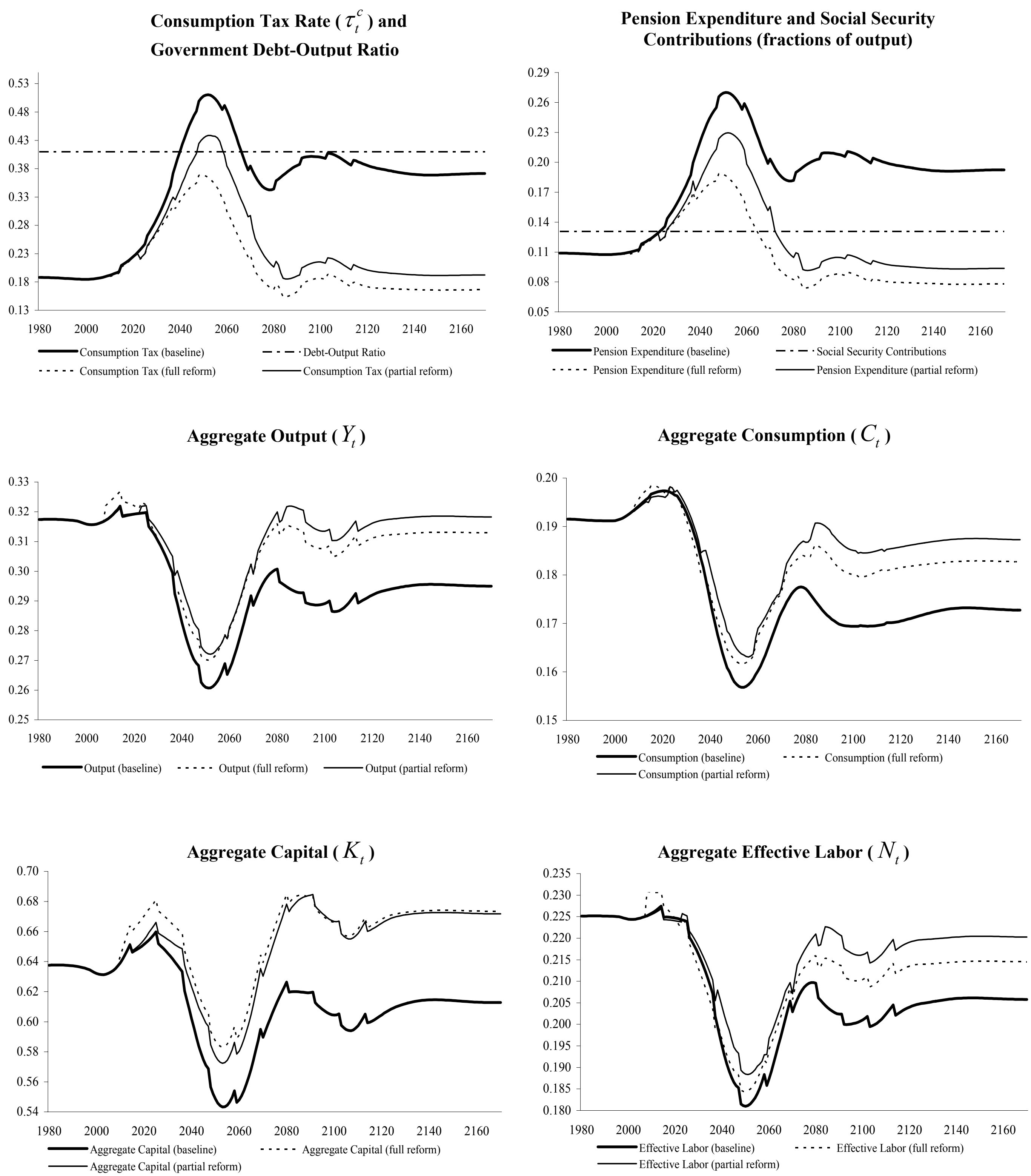


Figure 4. Household's Asset Holdings (A^s), Labor Effort (n^s), and Consumption (c^s) Profiles by Age (Final Steady State Under Alternative Reform Scenarios)

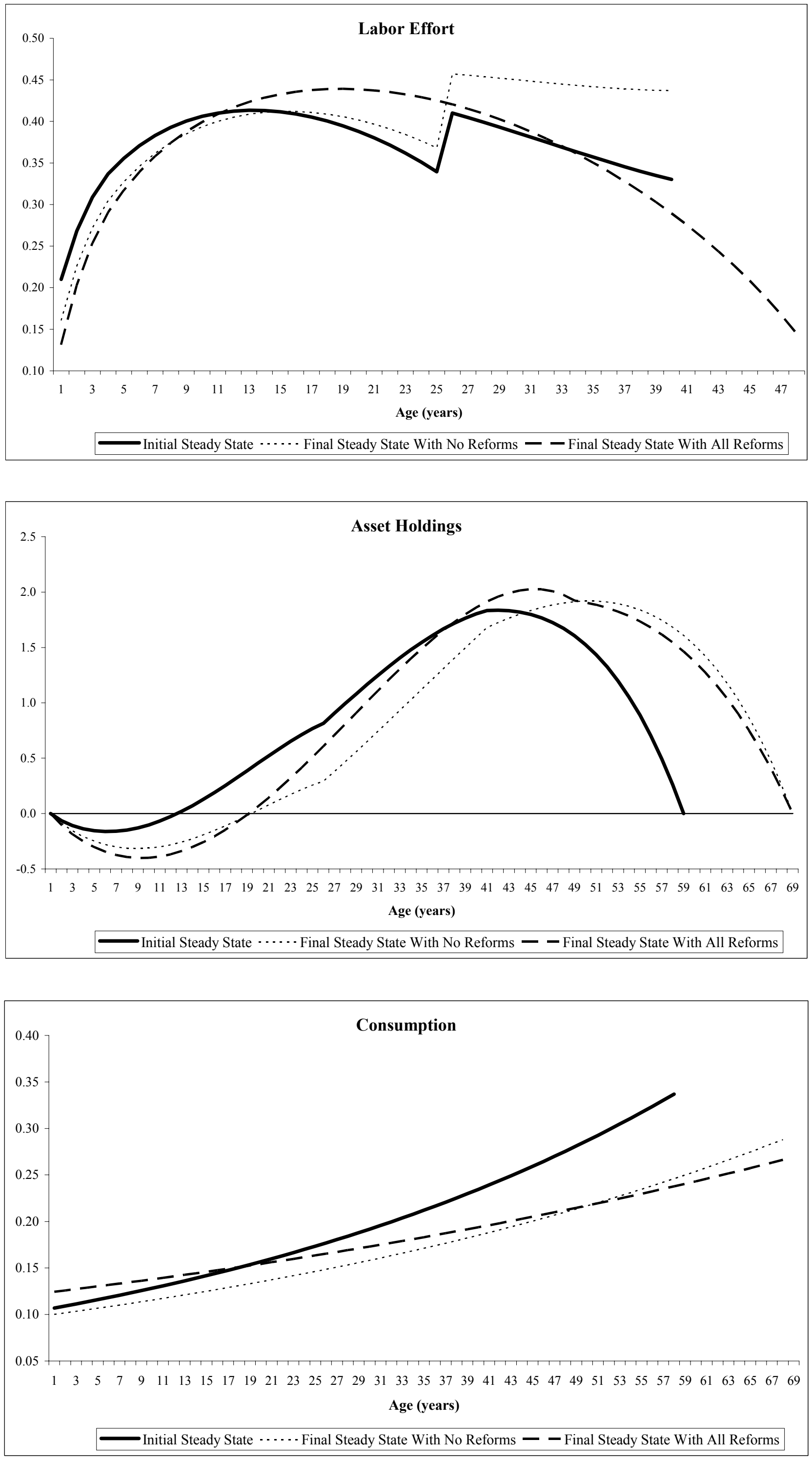
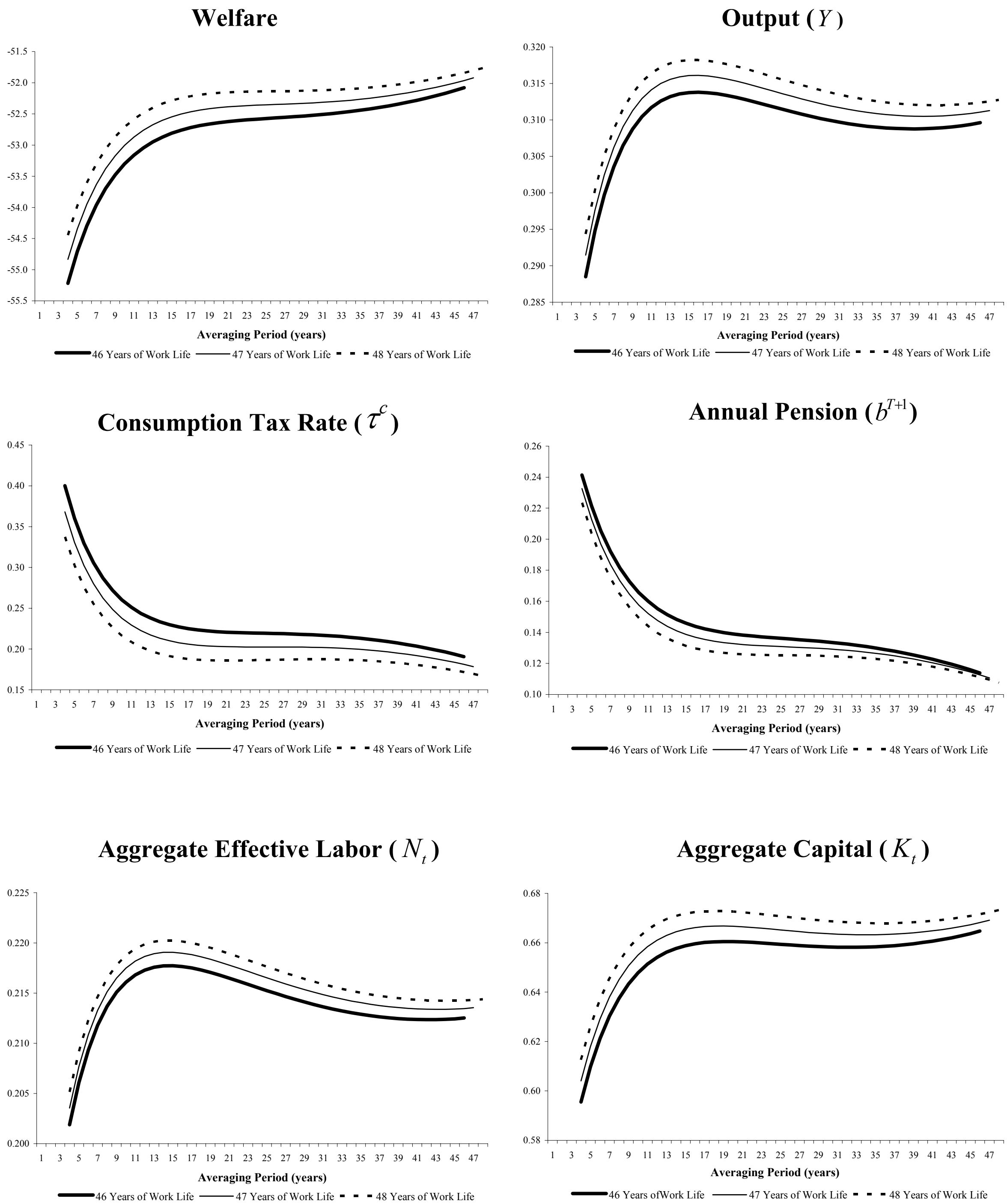


Figure 5. Welfare and Macroeconomic Effects of Pension Reforms in the Final Steady State[1/]



1/ Welfare is calculated as in equation (1), for $T = 68$ and $T^R = 46, 47, 48$ respectively.

Table 6. New Parameters of the 2008 Pension Reforms
(Generations Subject to Partial Grandfathering)

Age	Life expectancy 1/	Work life	Retirement life	Averaging period
1-2	86 (64)	42	22	Work life - Age + 1
3-12	85 (63)	42	21	Work life - Age + 1
13-22	84 (62)	41	21	Work life - Age + 1
23-26	83 (61)	41	20	Work life - Age + 1

1/ Natural life expectancy at birth of the cohort of indicated age in 2008. Numbers in parentheses indicate the corresponding life time (remaining after entry to the labor force) in the model.

Table 7. Macroeconomic Effects of Pension Reforms and Fiscal Policies During the Demographic Transition

Tax-as-you-go

Period	Aggregate Capital	Aggregate Effective Labor	Aggregate Capital-Labor Ratio	Aggregate Output	Output Growth	Net Wage Rate	Net Rate of Return on Assets	Consumption Tax Rate	Aggregate Consumption	Pension Benefit	Government Consumption-Output Ratio	Pension Expenditure-Output Ratio	Government Debt-Output Ratio
							No pension reform						
2001-07	100.0	100.0	100.0	100.0	0.022	100.0	0.091	0.186	100.0	100.0	0.228	0.108	0.410
2008-10	100.8	100.4	100.4	100.5	0.020	100.1	0.090	0.190	100.9	100.5	0.227	0.109	0.410
2011-15	102.2	100.8	101.4	101.3	0.018	100.5	0.089	0.198	101.8	100.7	0.225	0.113	0.410
2016-20	102.7	100.1	102.6	101.0	0.013	100.8	0.088	0.218	102.6	102.4	0.226	0.122	0.410
2021-25	103.8	99.8	104.0	101.1	0.011	101.3	0.087	0.237	102.6	104.6	0.225	0.131	0.410
2026-30	102.6	97.2	105.6	98.9	0.007	101.8	0.086	0.276	101.3	108.2	0.230	0.151	0.410
2031-35	101.2	94.4	107.2	96.6	0.005	102.3	0.084	0.318	98.2	112.0	0.236	0.172	0.410
2036-40	97.4	89.1	109.3	91.7	0.002	103.0	0.082	0.383	93.2	116.8	0.248	0.205	0.410
2041-45	92.5	84.3	109.6	86.9	0.003	103.1	0.082	0.445	87.4	121.1	0.262	0.238	0.410
2046-50	88.5	81.4	108.7	83.7	0.007	102.8	0.083	0.493	83.3	124.8	0.272	0.262	0.410
2051-55	86.1	81.2	106.0	82.8	0.009	102.0	0.085	0.508	81.7	126.3	0.275	0.268	0.410
2056-60	86.8	83.1	104.5	84.3	0.012	101.5	0.087	0.490	82.6	125.9	0.270	0.257	0.410
2061-65	88.4	86.0	102.9	86.8	0.013	100.9	0.088	0.448	85.1	123.6	0.263	0.236	0.410
2066-70	92.6	90.1	102.8	90.9	0.017	100.9	0.088	0.392	88.5	120.8	0.250	0.207	0.410
2071-75	95.2	92.0	103.4	93.1	0.016	101.1	0.088	0.361	91.2	118.1	0.245	0.191	0.410
2076-80	98.1	93.3	105.1	94.8	0.016	101.7	0.086	0.344	92.3	116.7	0.240	0.182	0.410
							Partial pension reform						
2001-07	100.0	100.0	100.0	100.0	0.022	100.0	0.091	0.186	100.0	100.0	0.228	0.108	0.410
2008-10	100.9	100.3	100.6	100.5	0.020	100.2	0.090	0.191	100.8	100.5	0.227	0.109	0.410
2011-15	102.3	100.6	101.7	101.1	0.018	100.6	0.089	0.199	101.5	100.8	0.225	0.113	0.410
2016-20	103.0	99.8	103.2	100.9	0.013	101.0	0.088	0.219	102.2	102.6	0.226	0.122	0.410
2021-25	104.6	100.1	104.5	101.6	0.013	101.5	0.087	0.227	102.7	103.1	0.224	0.125	0.410
2026-30	103.9	97.9	106.1	99.8	0.008	102.0	0.085	0.256	102.0	104.6	0.228	0.139	0.410
2031-35	103.0	95.4	108.0	97.9	0.006	102.6	0.084	0.294	99.0	108.4	0.233	0.157	0.410
2036-40	100.3	92.1	108.9	94.7	0.008	102.9	0.083	0.331	95.7	110.1	0.241	0.176	0.410
2041-45	96.5	88.3	109.2	91.0	0.004	103.0	0.083	0.374	91.4	112.0	0.250	0.197	0.410
2046-50	93.0	84.9	109.5	87.5	0.007	103.0	0.082	0.419	87.4	115.7	0.260	0.220	0.410
2051-55	90.7	84.2	107.7	86.3	0.008	102.5	0.084	0.438	85.3	118.0	0.264	0.228	0.410
2056-60	91.9	85.9	106.9	87.9	0.015	102.2	0.084	0.413	86.0	115.4	0.259	0.215	0.410
2061-65	93.9	89.5	105.0	90.9	0.012	101.6	0.086	0.357	89.3	108.0	0.250	0.184	0.410
2066-70	98.8	92.5	106.8	94.5	0.016	102.2	0.085	0.308	91.5	106.5	0.241	0.159	0.410
2071-75	102.0	95.5	106.8	97.6	0.019	102.2	0.085	0.250	95.3	95.3	0.233	0.128	0.410
2076-80	105.8	97.8	108.2	100.3	0.019	102.6	0.083	0.215	97.1	93.7	0.227	0.109	0.410
							Full pension reform						
2001-07	100.0	100.0	100.0	100.0	0.022	100.0	0.091	0.186	100.0	100.0	0.228	0.108	0.410
2008-10	101.0	102.7	98.3	102.2	0.025	99.4	0.092	0.189	101.5	100.4	0.223	0.108	0.410
2011-15	103.9	102.4	101.5	102.9	0.018	100.5	0.089	0.196	102.7	100.4	0.221	0.111	0.410
2016-20	105.4	100.5	104.8	102.1	0.012	101.6	0.086	0.216	103.1	102.4	0.223	0.121	0.410
2021-25	107.1	99.5	107.6	101.9	0.011	102.4	0.084	0.225	102.7	102.2	0.223	0.125	0.410
2026-30	106.1	96.1	110.5	99.3	0.006	103.3	0.081	0.254	100.9	97.3	0.229	0.137	0.410
2031-35	104.9	92.7	113.2	96.6	0.005	104.2	0.079	0.285	97.0	95.4	0.236	0.152	0.410
2036-40	101.8	88.8	114.7	92.9	0.007	104.6	0.078	0.314	93.1	93.1	0.245	0.165	0.410
2041-45	97.8	85.3	114.6	89.3	0.005	104.6	0.078	0.340	88.8	89.9	0.255	0.176	0.410
2046-50	94.4	82.8	113.9	86.5	0.008	104.4	0.079	0.362	85.4	85.9	0.263	0.185	0.410
2051-55	92.4	82.7	111.7	85.8	0.009	103.7	0.080	0.360	84.2	84.7	0.265	0.182	0.410
2056-60	93.5	85.1	109.9	87.8	0.015	103.2	0.082	0.327	85.5	85.0	0.260	0.164	0.410
2061-65	95.6	88.4	108.2	90.8	0.012	102.6	0.083	0.277	88.7	81.4	0.251	0.137	0.410
2066-70	100.3	91.7	109.4	94.4	0.016	103.0	0.082	0.235	91.2	80.7	0.241	0.117	0.410
2071-75	103.4	94.1	110.0	97.1	0.017	103.2	0.082	0.195	94.3	76.9	0.235	0.096	0.410
2076-80	107.0	95.7	111.8	99.3	0.018	103.7	0.080	0.172	95.6	74.7	0.229	0.085	0.410

Tax-smoothing

Period	Aggregate Capital	Aggregate Effective Labor	Aggregate Capital-Labor Ratio	Aggregate Output	Output Growth	Net Wage Rate	Net Rate of Return on Assets	Consumption Tax Rate	Aggregate Consumption	Pension Benefit	Government Consumption-Output Ratio	Pension Expenditure-Output Ratio	Government Debt-Output Ratio
							No pension reform						
2001-07	100.0	100.0	100.0	100.0	0.022	100.0	0.091	0.186	100.0	100.0	0.228	0.108	0.410
2008-10	101.1	97.1	104.1	98.4	0.013	101.3	0.087	0.254	98.4	100.7	0.231	0.112	0.379
2011-15	102.4	98.2	104.3	99.5	0.020	101.4	0.087	0.254	99.1	101.6	0.229	0.115	0.240
2016-20	103.7	98.4	105.3	100.1	0.015	101.7	0.086	0.254	100.5	103.9	0.227	0.124	0.038
2021-25	106.5	99.4	107.1	101.7	0.015	102.3	0.084	0.254	101.0	106.8	0.224	0.132	-0.203
2026-30	108.0	98.3	109.8	101.4	0.011	103.1	0.082	0.254	101.6	111.9	0.225	0.149	-0.476
2031-35	109.7	97.1	112.9	101.1	0.009	104.1	0.080	0.254	100.2	117.9	0.225	0.169	-0.761
2036-40	108.7	93.1	116.7	98.0	0.006	105.2	0.077	0.254	97.9	125.5	0.232	0.200	-1.017
2041-45	104.6	88.9	117.7	93.8	0.005	105.5	0.076	0.254	94.0	133.4	0.243	0.234	-1.182
2046-50	99.9	86.0	116.2	90.4	0.006	105.1	0.077	0.254	90.9	140.5	0.252	0.262	-1.217
2051-55	95.2	85.3	111.6	88.5	0.006	103.7	0.081	0.254	89.1	143.2	0.257	0.275	-1.132
2056-60	93.6	86.6	108.1	88.9	0.009	102.6	0.083	0.254	88.8	142.3	0.256	0.271	-0.983
2061-65	92.9	88.5	104.9	89.9	0.010	101.6	0.086	0.254	89.7	138.4	0.253	0.255	-0.816
2066-70	96.0	91.9	104.4	93.2	0.016	101.4	0.087	0.254	91.0	133.2	0.244	0.227	-0.705
2071-75	98.7	93.5	105.6	95.2	0.016	101.8	0.086	0.254	92.7	128.3	0.239	0.209	-0.670
2076-80	103.0	94.9	108.5	97.5	0.018	102.7	0.083	0.254	93.4	125.5	0.234	0.196	-0.712
							Partial pension reform						
2001-07	100.0	100.0	100.0	100.0	0.022	100.0	0.091	0.186	100.0	100.0	0.228	0.108	0.410
2008-10	101.1	98.0	103.2	99.0	0.015	101.0	0.088	0.241	98.8	100.7	0.230	0.111	0.386
2011-15	102.8	98.8	104.0	100.1	0.020	101.3	0.087	0.241	99.6	101.4	0.228	0.114	0.281
2016-20	104.1	98.8	105.3	100.5	0.015	101.7	0.086	0.241	101.0	103.7	0.227	0.123	0.138
2021-25	106.6	99.9	106.7	102.1	0.015	102.2	0.085	0.241	101.6	104.7	0.223	0.126	-0.021
2026-30	108.2	99.0	109.3	102.0	0.011	103.0	0.082	0.241	102.3	107.5	0.223	0.138	-0.219
2031-35	109.6	97.8	112.0	101.5	0.009	103.8	0.080	0.241	101.1	112.8	0.224	0.155	-0.411
2036-40	108.4	95.4	113.6	99.5	0.010	104.3	0.079	0.241	99.3	116.5	0.229	0.173	-0.555
2041-45	105.9	92.4	114.7	96.7	0.005	104.6	0.078	0.241	96.6	120.8	0.236	0.194	-0.654
2046-50	102.2	89.2	114.6	93.3	0.006	104.6	0.078	0.241	94.0	126.8	0.244	0.220	-0.647
2051-55	97.8	88.1	111.0	91.2	0.006	103.5	0.081	0.241	92.3	130.1	0.250	0.233	-0.501
2056-60	95.8	89.0	107.6	91.2	0.011	102.5	0.084	0.241	91.8	126.7	0.250	0.225	-0.249
2061-65	95.3	91.5	104.2	92.7	0.008	101.4	0.087	0.241	92.8	117.9	0.246	0.197	0.023
2066-70	98.1	93.2	105.3	94.8	0.012	101.7	0.086	0.241	93.4	113.2	0.240	0.173	0.266
2071-75	99.6	94.9	105.0	96.4	0.016	101.6	0.086	0.241	94.7	99.4	0.236	0.141	0.467
2076-80	103.0	96.4	106.9	98.5	0.018	102.2	0.084	0.241	95.2	94.8	0.231	0.119	0.578
							Full pension reform						
2001-07	100.0	100.0	100.0	100.0	0.022	100.0	0.091	0.186	100.0	100.0	0.228	0.108	0.410
2008-10	101.3	100.9	100.4	101.0	0.022	100.1	0.090	0.234	99.8	100.5	0.225	0.109	0.390
2011-15	104.6	101.1	103.5	102.2	0.020	101.1	0.087	0.234	101.1	100.9	0.223	0.112	0.298
2016-20	106.9	100.0	106.9	102.2	0.013	102.2	0.084	0.234	102.4	103.4	0.223	0.121	0.174
2021-25	109.6	99.8	109.8	102.9	0.013	103.1	0.082	0.234	102.2	103.7	0.221	0.124	0.048
2026-30	110.6	97.5	113.5	101.6	0.009	104.3	0.079	0.234	101.9	99.0	0.224	0.136	-0.094
2031-35	111.0	95.1	116.8	100.1	0.007	105.3	0.077	0.234	99.5	97.5	0.228	0.149	-0.207
2036-40	108.6	91.8	118.3	97.0	0.008	105.7	0.075	0.234	96.8	95.5	0.235	0.161	-0.263
2041-45	104.8	88.6	118.3	93.7	0.005	105.7	0.076	0.234	93.4	92.8	0.243	0.171	-0.264
2046-50	100.6	86.1	116.9	90.6	0.007	105.3	0.077	0.234	90.8	89.3	0.251	0.182	-0.183
2051-55	96.4	85.4	112.9	88.9	0.006	104.1	0.080	0.234	89.3	88.1	0.256	0.182	-0.009
2056-60	94.8	86.8	109.2	89.3	0.011	103.0	0.083	0.234	89.1	88.1	0.255	0.167	0.214
2061-65	94.7	89.0	106.4	90.9	0.009	102.1	0.085	0.234	90.2	84.1	0.251	0.142	0.429
2066-70	97.8	91.1	107.4	93.2	0.014	102.4	0.084	0.234	91.1	82.5	0.244	0.122	0.602
2071-75	99.8	92.4	108.0	94.8	0.015	102.6	0.084	0.234	92.4	77.8	0.240	0.101	0.724
2076-80	103.1	93.5	110.2	96.6	0.017	103.3	0.082	0.234	92.8	74.6	0.236	0.089	0.778

Notes: Numbers indicate period averages and all variables are defined as in Table 1 and expressed as stationary-transformations. Capital, effective labor, the capital-labor ratio, output, the net wage rate, consumption, and the pension benefit are normalized to 100 in the period 2001-2007.
The pension benefit is the annual pension received by the generation that retires in the indicated year.

Figure 6. Baseline and Pension Reforms Macroeconomic Scenarios
(Tax-smoothing and Low Immigration)

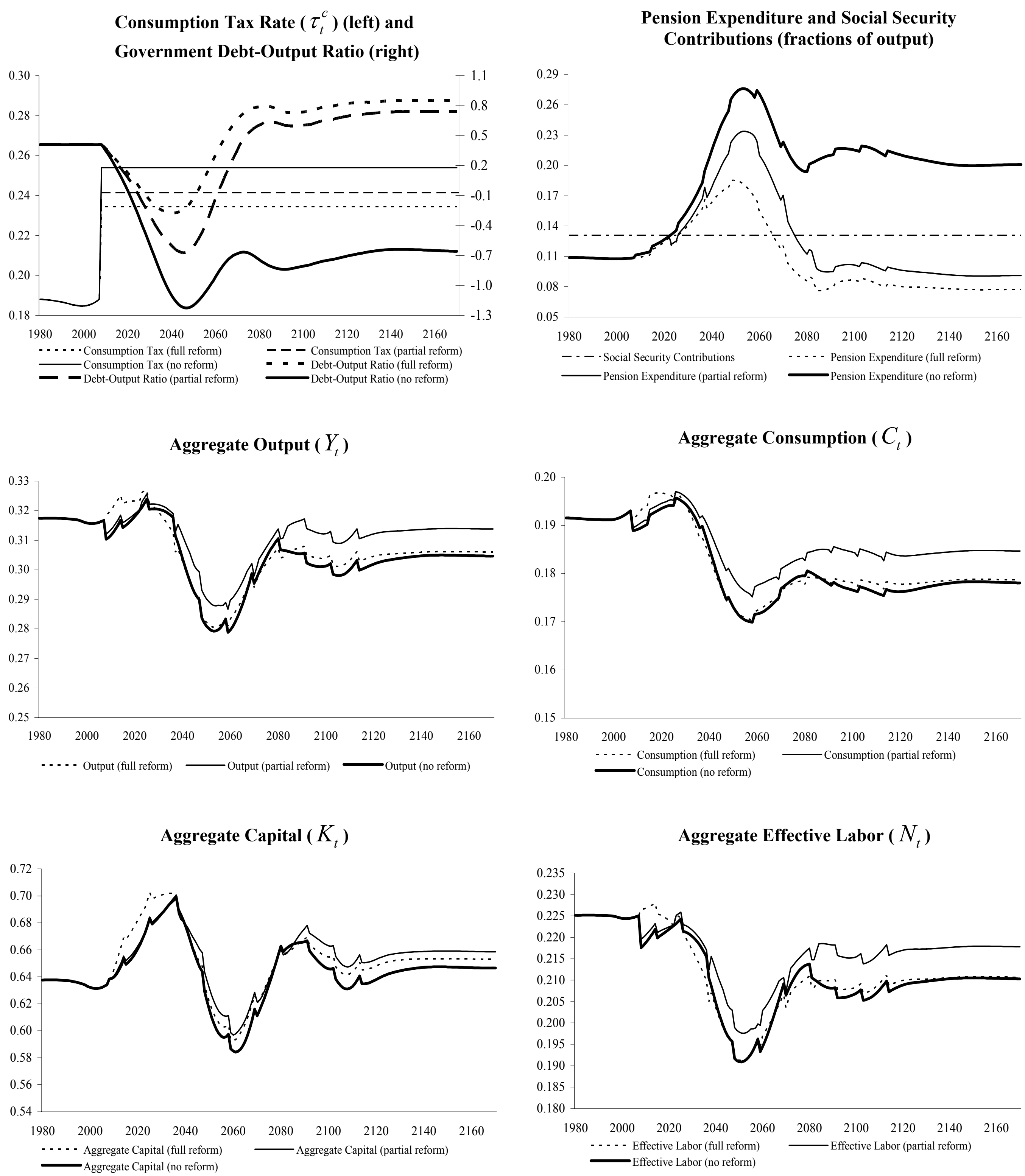
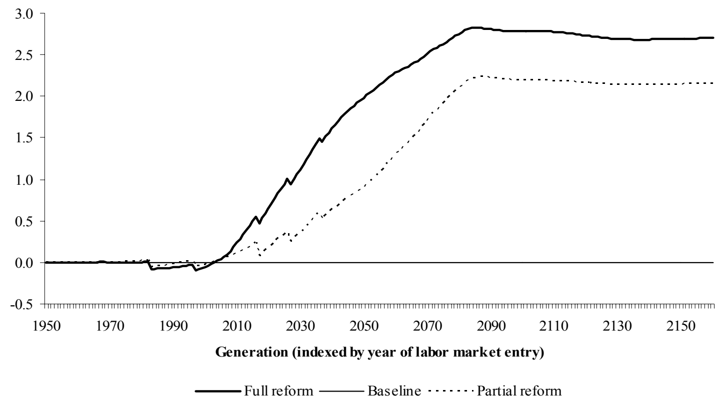
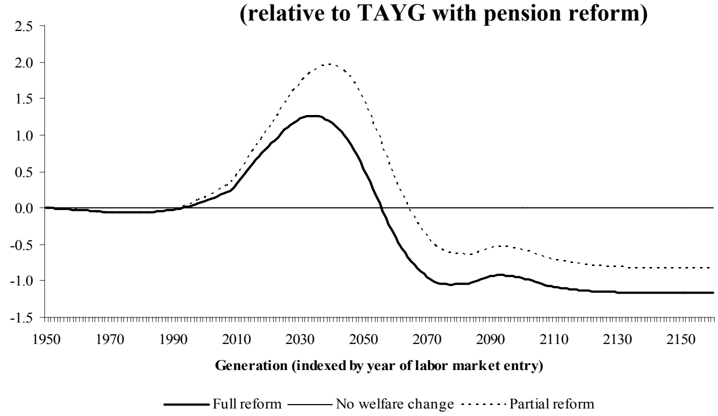


Figure 7: Welfare Effects of Pension Reforms and Fiscal Policies During the Demographic Transition

Cross-generational welfare changes due to pension reforms (relative to TAYG baseline)



Cross-generational welfare changes due to tax smoothing policies (relative to TAYG with pension reform)



Cross-generational welfare changes due to pension reforms and tax smoothing policies (relative to TAYG baseline)

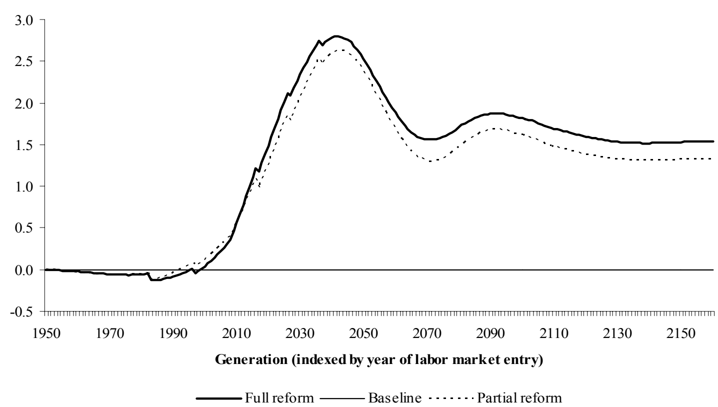
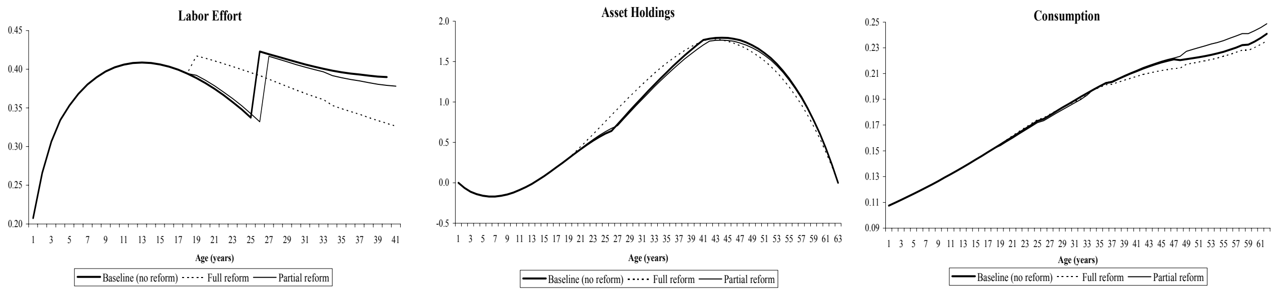
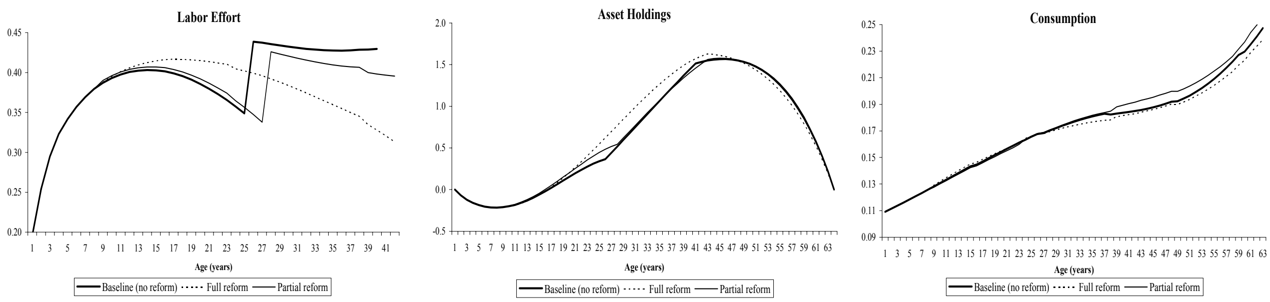


Figure 8. Household's Asset Holdings (A^s), Labor Effort (n^s), and Consumption (c^s) Profiles by Age (During Demographic Transition Under Tax-as-you-go)

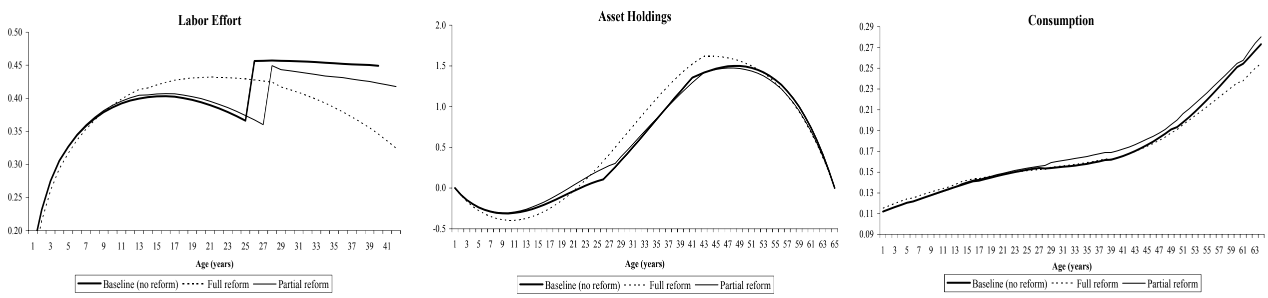
1990 Labor Force Entrant



2000 Labor Force Entrant



2010 Labor Force Entrant



2020 Labor Force Entrant