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Technological Diversification*

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Abstract

Why is GDP so much more volatile in poor countries than in rich ones? To answer this question, we propose a theory of technological diversification. Production makes use of different input varieties, which are subject to imperfectly correlated shocks. Technological progress takes the form of an increase in the number of varieties, raising average productivity. In addition, the expansion in the number of varieties in our model provides diversification benefits against variety-specific shocks and it hence lowers the volatility of output growth. Technological complexity evolves endogenously in response to profit incentives. The decline in volatility thus arises as a robust by-product of firms' incentives to increase profits and is hence an inexorable feature of the development process. We discuss the predictions of the model in light of the empirical evidence.

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1 Introduction

Economies at early stages of the development process are often shaken by abrupt changes in growth rates. In his influential paper, Lucas (1988) brings attention to this fact, noting that "within the advanced countries, growth rates tend to be very stable over long periods of time," whereas within poor countries "there are many examples of sudden, large changes in growth rates, both up and down."

Motivated by this empirical observation, this paper proposes an endogenous growth model of technological diversification. The key idea of the model is that firms using a large variety of inputs can mitigate the impact of shocks affecting the productivity of individual inputs. This takes place through two channels. First, with a larger variety of inputs, each individual input matters less in production, and productivity becomes less volatile by the law of large numbers. Second, whenever a shock hits a particular input, firms can adjust the use of the other inputs to partially offset the shock. Both channels make the productivity of firms using more sophisticated technologies less volatile.

The idea can be illustrated with an example from agriculture: Growing wheat with only land and labour as inputs renders the yield vulnerable to idiosyncratic shocks (for example, weather shocks such as a severe drought). In contrast, using land and labour together with artificial irrigation, fertilizers, pesticides, etc., can make wheat-growing not only more productive on average but also less risky, because farmers have more options to react to external shocks. Figure 1 provides a graphical illustration of this example. It displays the volatility of wheat yield (calculated as the standard deviation of percentage deviations from the country's average yield) of the 20 biggest wheat producers against their level of GDP per capita.[1] The plot shows a sharp decline of yield volatility with the level of development.[2] A second, more topical example, can be drawn from the energy sector. According to The Economist, the recent increase in oil prices has led to a growing move towards ethanol and biofuels produced from canola and soya beans and thus less exposed to fluctuations in production and political turmoil.[3]

Our model builds on the seminal contributions by Romer (1990) and Grossman and Helpman (1991) and characterizes technological progress as an expansion in the

[1]Note that agricultural technology varies substantially with development. For example, of the top 20 wheat producers, India uses 2.3 tractors per 1,000 acres of arable land; this number is 128.8 for Germany. Fertilizer use also varies hugely. India uses 21.9 tons of nitrogenous fertilizers per acre; Germany uses 183.8 tons. We take the level of development as an overall indicator of agricultural sophistication.

[2]This remains true if we control for differences in climate across countries, including the volatility of rainfall and temperature.

[3]The Economist, 05/06/06, page 52, "Alternative Energy: Canola and Soya to the Rescue."

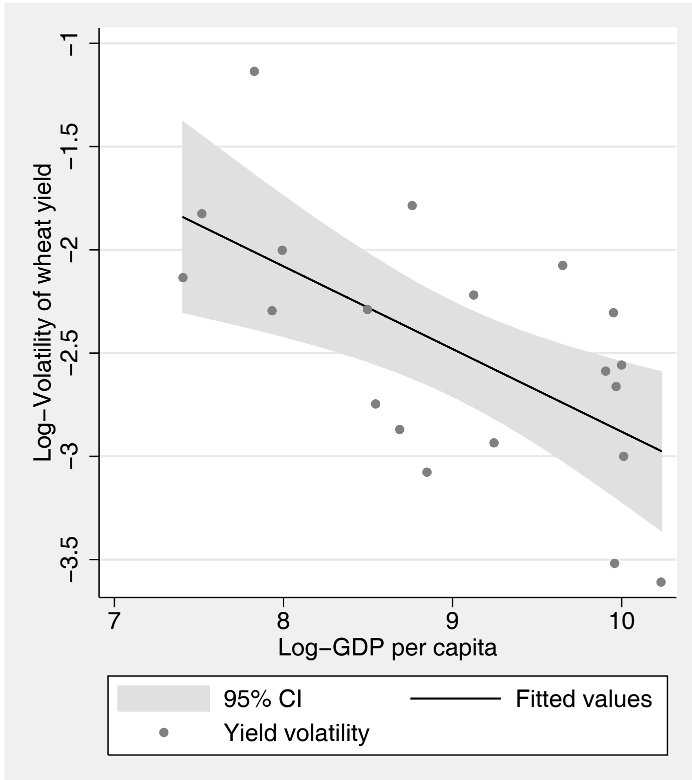

Figure 1: Wheat Yield Volatility and Development

number of input varieties.[4] The number of varieties evolves endogenously in response to profit incentives, and increases in the number of varieties raise the average level of productivity. What is new here is that the expansion in the number of varieties provides diversification benefits against variety-specific shocks and it hence reduces the level of volatility. In other words, the reduction in volatility in the model arises as a by-product of firms' incentives to increase profits. As such, our model highlights a hitherto glossed over implication of the expanding-variety growth models, which makes them suitable to explain the secular decline in volatility.

Previous theoretical work on the relationship between volatility and development, including Greenwood and Jovanovic (1990), Saint-Paul (1992), Obstfeld (1994), and Acemoglu and Zilibotti (1997), has focused on *financial*—as opposed to *technological*—diversification. These models feature an inherent trade-off between productivity and risk at the firm-level: Firms must choose between low-return but safe activities and high-return but risky ones. The key driving force of the relationship between volatility and income is financial development. Financially underdeveloped countries

[4]See also **?** for a comprehensive formalization and discussion of expanding-variety models.

do not have the facility to pool risks, so risk-averse entrepreneurs minimize firm-level risk by choosing low productivity projects. In financially developed countries, risks can be pooled and hence entrepreneurs undertake high return projects. Clearly this mechanism can generate a negative relationship between aggregate risk and aggregate productivity.

Unlike existing models, the expanding-variety model we propose posits no trade-off between productivity and risk at the firm level. Indeed our point is that there are technological reasons to expect the adoption of a new variety to concurrently lead to a decline in volatility. Hence, preferences towards risk, which are crucial in models of "financial diversification," play no role in our story, where firms are uniquely concerned with profit maximization. Similarly, and perhaps most importantly, in our story the decline in aggregate volatility with the level of development takes place independently of the level of financial development.

These theoretical differences lead to important differences in empirical implications. First, models of financial diversification predict an increase in firm-level volatility with the level of development, while our model predicts a decline in firm-level volatility. We view both margins of diversification for the firm, financial and technological, as complementary and empirically plausible.[5] Indeed, there is evidence consistent with both margins. Comin and Philippon (2005) document that, in the US, publicly traded firms have experienced an increase in volatility in the last three decades. On the other hand, and consistent with our model, Davis, Haltiwanger, Jarmin and Miranda (2006) show that, over the same period in the US, non-publicly traded firms have experienced a sharp decline in volatility; they furthermore argue that the decline in aggregate volatility in the US has been overwhelmingly driven by a decline in firm-level volatility and not by the aggregation of highly volatile firms. One important difference between these two sets of firms is the access to external finance and the opportunities to diversify risk (traded firms can share the risk with a large number of investors). Since a majority of firms in developing countries falls in the second category (non-publicly traded), given the motivation of this paper, our model focuses on the second margin.

Second, models of financial diversification predict that a critical element explaining the decline in aggregate volatility with development is the level of financial deepening. In our model, the decline in volatility takes place independently of the level of financial deepening. This implication finds support in the data: As we show later, the negative

[5]Technological diversification is also complementary to other other finance-related mechanisms emphasized in the literature. In particular, shocks can be amplified by introducing financial frictions, a task we do not undertake in the interest of clarity and simplicity. For models with financial frictions, see, among others, Bernanke and Gertler (1990), Kiyotaki and Moore (1997), Aghion, Angeletos, Banerjee and Manova (2004).

correlation between volatility and development takes place at all levels of financial development.

The lack of trade-off between productivity and volatility in our model can be substantiated by two additional pieces of evidence: First, more productive and larger firms tend to be less volatile, a result we document in the empirical section. And second, as shown in Koren and Tenreyro (2007), countries at early stages of development tend to specialize in low-productivity, high-risk activities, whereas the opposite pattern is observed at later stages; in other words, the development process is characterized by a move towards both more productive and safer sectors.

The remainder of the paper is organized as follows. Section 2 presents the model of technological diversification for the firm and derives the aggregate dynamics implied by the model. Section 3 discusses the implications in light of the empirical evidence. And Section 4 presents concluding remarks.

2 A Model of Technological Diversification

2.1 Production technology

This section introduces a production process that features technological diversification within the firm. Each firm produces output by combining a variety of technologies (or inputs) in a constant-elasticity-of-substitution (CES) production function,[6]

$$y = \left[\sum_{i=1}^{n} (\chi_i l_i)^{1-1/\varepsilon} \right]^{\varepsilon/(\varepsilon-1)}, \tag{1}$$

where l_i denotes the number of workers allocated to the operation of technology-variety i, χ_i is the productivity of this variety, n denotes the number of varieties used by the producer, and $\varepsilon \in (1, \infty)$ is the elasticity of substitution across varieties.[7]

All technologies are symmetric *ex ante* (before the realization of productivity shocks) so that $l_i = l/n$ and $\chi_i = 1$ (by normalization) for all i, with l denoting the total number of employees working at the firm. We can then rewrite (1) as

$$y = n^{1/(\varepsilon-1)} l. \tag{2}$$

[6]For convenience, we omit subindices for the firm.

[7]As usual in endogenous growth models, we assume that $\varepsilon > 1$, that is, technologies are gross substitutes. In an appendix available at request, we prove that the diversification result holds even with Leontief technologies (as in Kremer (1993)'s O-ring technology) as long as the shocks are not terminal. This is because more varieties allow the firm to adjust the use of more inputs, therefore providing more margins of adjustment, even if the inputs themselves are complements. Also note that introducing additional (scarce) factors of production would not change our qualitative results.

Labour productivity ($\frac{y}{l}$) is increasing in the number of varieties, since the varieties are imperfect substitutes ($\varepsilon < \infty$). This is the usual "love of variety" effect of many endogenous growth models (Romer 1990, Grossman and Helpman 1991). The effect is stronger the lower is ε, that is, the less substitutable varieties are. Intuitively, if varieties are highly substitutable, any additional variety is less needed. To rule out explosive growth, we assume $\varepsilon \geq 2$.[8]

Now suppose that variety-specific productivities are random, independently and identically distributed with mean $\mathrm{E}(\chi_i) = 1$ and variance $\mathrm{Var}(\chi_i) = \sigma^2$. We approximate the variance of output, a nonlinear function of productivity shocks, by linearizing (1) around the mean of each shock:

$$\hat{y} = \sum_{i=1}^{n} \frac{\mathrm{MP}_i l_i}{y} \left(\hat{\chi}_i + \hat{l}_i \right) = \frac{1}{n} \sum_{i=1}^{n} \hat{\chi}_i, \tag{3}$$

where $\hat{x} \equiv (x - \mathrm{E}\, x)/\, \mathrm{E}\, x$ denotes the infinitesimal deviation of variable x from its mean in proportional terms and MP_i denotes the marginal product of technology i. The last equality follows from Euler's theorem, the fact that varieties are *ex ante* symmetric, and full employment, which implies $\sum_i \hat{l}_i = \hat{l} = 0$.[9]

The proportional variance of output shocks is then

$$\mathrm{Var}\, \hat{y} = \frac{\sigma^2}{n}. \tag{4}$$

The variance is declining in n, the number of technologies. This is a simple application of the law of large numbers: the variance of the average of n independent random variables is proportional to $1/n$.

2.2 The dynamics of technological diversification

What determines the level of technological complexity in the long run? In this section we endogenize the firm's decision to invest in new varieties. Much as in models of endogenous growth, firm owners will be attracted by greater profit opportunities.

To spell out the dynamics of the model, we specify the stochastic properties of the productivity process, χ_i as follows. Time is continuous. Varieties have a constant productivity (normalized to 1) during their random lifetime, after which they irreversibly cease to contribute to production. The arrival of failures follows a Poisson process with

[8] Otherwise the love of variety effect would be so powerful that the aggregate return to varieties would become increasing. Higher levels of development would counterfactually imply increasing rates of return on capital, inconsistent with observed development patterns (see Caselli and Feyrer (2006)).

[9] We assume all firms are ex-ante identical; hence, full employment in the aggregate implies that there are no changes in the level of employment at the firm level: $\frac{1}{n} \sum_{i=1}^{n} \hat{L}_i = 0$.

arrival rate γ. In other words, the hazard rate of a failure is independent of the time the technology has been in use. Failures are independent across varieties.[10]

Let $\chi_i(t)$ denote the productivity of technology i at time t and T_i the (random) lifetime of this technology. Productivity equals 1 until time T_i, when it falls to 0. Because failure arrives with a Poisson process, the lifetime follows an exponential distribution with parameter γ (the expected lifetime is hence $1/\gamma$). The probability that $T_i \leq t$ is thus

$$\Pr(T_i \leq t) = 1 - e^{-\gamma t}.$$

Clearly, the distribution of $\chi_i(t)$ is given by

$$\chi_i(t) = \begin{cases} 1 & \text{with prob. } e^{-\gamma t}, \\ 0 & \text{with prob. } 1 - e^{-\gamma t}. \end{cases}$$

To illustrate how shocks to technology affect the dynamics of a firm, let us follow a firm over time. The firm hires workers in competitive labour markets; at time t it faces a wage rate $w(t)$ (taken as given by individual firms). The only state variable for the firm is the number of technologies currently in use, which takes value $n(t)$ at time t.[11] The marginal cost is given by:

$$\left[\sum_{i=1}^{n(t)} w(t)^{1-\varepsilon}\right]^{1/(1-\varepsilon)} = w(t)n(t)^{1/(1-\varepsilon)}.$$

Firms using more varieties have lower marginal costs.

Suppose the firm faces a downward-sloping demand curve. In particular, assume that its demand is iso-elastic with elasticity η: $y(t) = Y(t)p(t)^{-\eta}$, where $y(t)$ is the firm's output, $Y(t)$ is aggregate output (taken as the numeraire), and $p(t)$ is the price charged by the firm. Profit maximization hence implies that the firm charges a constant $\eta/(\eta-1)$ markup over its marginal cost:

$$p(t) = \frac{\eta}{\eta - 1} w(t)n(t)^{1/(1-\varepsilon)},$$

[10] We take the extreme assumption of independence for expositional clarity, but our argument goes through as long as failures are imperfectly correlated. Similarly, the assumption that random failures turn the input completely useless makes the model more tractable; however, technological diversification would take place with non-terminal shocks. Finally, note that, though we refer to variety failures, the shocks to χ_i can be the result of increases in the price of the variety, weather shocks that render a variety useless, and trade disruptions, among other factors.

[11] The symmetry assumptions above ensure that firms only care about the *number* of technologies but not about *which* technologies they use.

and its revenues are given by

$$p(t)y(t) = Y(t)p(t)^{1-\eta} = Y(t)\left[\frac{\eta w(t)}{\eta-1}\right]^{1-\eta} n(t)^{(1-\eta)/(1-\varepsilon)}. \tag{5}$$

Profits are a constant $1/\eta$ fraction of revenues,

$$\pi(t) = p(t)y(t)/\eta = \frac{1}{\eta}Y(t)\left[\frac{\eta w(t)}{\eta-1}\right]^{1-\eta} n(t)^{(1-\eta)/(1-\varepsilon)} = A(t)n(t)^{(1-\eta)/(1-\varepsilon)},$$

where $A(t) \equiv \frac{1}{\eta}Y(t)\left[\frac{\eta w(t)}{\eta-1}\right]^{1-\eta}$. Since an individual firm takes $Y(t)$ and $w(t)$ as given, $A(t)$ is also given from a firm's perspective.

For analytical convenience, we assume the elasticity of demand (η) to be the same as the elasticity of substitution between varieties (ε). This assumption ensures that profits are linear in the number of varieties. The assumption is satisfied naturally if varieties represent different brands valued by the consumer, in which case the elasticity of demand and the elasticity of substitution are equal.

2.2.1 Technology adoption

As in (Romer 1990, Grossman and Helpman 1991), adopting new varieties is a costly activity.[12] For analytical convenience, we assume that investment in adoption pays off only after a random waiting time. Higher investment in adoption results in a shorter expected waiting time for the next variety.

The adoption of a new variety requires both a stock of knowledge (embedded in current technologies, $n(t)$) and a flow of investment. If the firm spends I units of the final good to adopt a new variety, the adoption will be successful with a Poisson arrival rate $\Lambda = f(I, n)$, where $f(.,.)$ is a standard neoclassical production function subject to constant returns to scale and satisfying the Inada conditions.[13] Let $\lambda = \Lambda/n$ denote the adoption *intensity*. By the CRS property of f, the flow cost of this adoption intensity is

$$I = g(\lambda)n,$$

where $g(.)$ is the inverse of $f(., 1)$, an increasing, convex function with $g(0) = g'(0) = 0$, $\lim_{x\to\infty} g'(x) = \infty$.

As mentioned, technological diversification in this model is not driven by risk aversion. To stress this point, we next characterize the optimal rate of technology adoption

[12] Adoption costs can be also thought as the cost of research and development of new varieties. For developing countries, however, referring to adoption (or imitation) costs seems more appropriate.

[13] This formulation follows **?**. The random, "memoryless" adoption process ensures that we do not have to track past R&D investment flows of the firm. This is a standard simplifying assumption in endogenous growth models.

in the case of risk neutral agents. In Appendix A we characterize adoption under complete financial autarky and risk averse investors. We do this to highlight that there is technological diversification in both cases and that the incentive to diversify does not hinge on the financial structure of the economy nor the degree of risk aversion (though they may affect these incentives).

Risk neutral households maximize the present value of consumption, discounted at the rate ρ:

$$\mathcal{U} \equiv \int_{t=0}^{\infty} e^{-\rho t} C(t) \, \mathrm{d}t.$$

The Euler equation pins down the riskless rate in the economy at $r(t) = \rho$. Investors maximize the expected present value of profits, discounted with the rate ρ.

To ensure non-negative growth and a finite value for the firm, we assume that the cost of adoption satisfies:

$$g(\gamma) + \rho g'(\gamma) \leq L/2 and \lim_{x \to \gamma+\rho} g(x) = \infty. \tag{6}$$

The first condition ensures that a variety is profitable enough so that it is worth replacing every failed variety. The second condition ensures that adoption is costly enough so that the growth rate of the economy will never exceed ρ, the subjective discount rate.

As we show in Section 2.3, in equilibrium, aggregate profitability, $A(t)$ follows a Markov process. Then $A(0)$ is a sufficient statistic to describe the future dynamics of profitability. Individual firms take the Markov process for $A(t)$ as exogenous.

Let $V_n(A)$ denote the expected present discounted value of profits for a firm with n varieties in an economy with profitability A.

$$V_n(A) = \mathrm{E}_0 \int_{t=0}^{\infty} e^{-\rho t} [\pi(t) - I(t)] \, \mathrm{d}t = \mathrm{E}_0 \int_{t=0}^{\infty} e^{-\rho t} \{A(t) - g[\lambda(t)]\} n(t) \, \mathrm{d}t, \tag{7}$$

and the stochastic dynamics of $n(t)$ is described as follows. In each infinitesimal time period of length h, one of the technologies fails with probability $\gamma n h$ (omitting higher order terms), decreasing n by 1, or the firm becomes successful in adopting a new technology (with probability $\lambda n h$), increasing n by 1.

The Bellman equation describing the decision problem and the value of the firm is

$$\begin{aligned} \rho V_n(A) = \max_{\lambda} \{ & An - g(\lambda) n \\ & + \lambda n \, [V_{n+1}(A) - V_n(A)] \\ & + \gamma n \, [V_{n-1}(A) - V_n(A)] \\ & + \lim_{h \to 0} \mathrm{E}_t \, [V_n[A(t+h)] - V_n(A)]/h \}. \end{aligned} \tag{8}$$

The opportunity cost of the value of the firm ($\rho V_n(A)$) equals the sum of (i) flow profits net of adoption costs ($An - g(\lambda)n$), (ii) capital gain from successful adoption of a new technology (which occurs with hazard rate λn), (iii) capital loss if any of the n variety fails (each of which has a hazard rate γ), and (iv) exogenous capital gains (due to changes in the aggregate environment affecting profitability).

Proposition 1. The optimal adoption rate is $\Lambda = \lambda(A)n$, implying that the adoption *intensity* $\lambda(A)$ is independent of n. The value of the firm is of the form $V_n(A) = v(A)n$, where $v(A)$ and $\lambda(A)$ are jointly determined by

$$g'[\lambda(A)] = v(A), \tag{9}$$

$$A - g[\lambda(A)] = \left[\rho + \gamma - \lambda(A) - \frac{\mathrm{E}_t(\mathrm{d}v/v)}{\mathrm{d}t}\right] v(A). \tag{10}$$

Adoption intensity, $\lambda(A)$, is positive and unique. It is increasing in profitability, A , decreasing in the discount rate, ρ, and decreasing in the probability of failure, γ.

The first equation is the first-order condition for optimal adoption: the marginal cost of adoption, $g'(\lambda)$, should equal the marginal value of an additional variety, v. The second equation defines the value of a variety recursively: given the optimal adoption intensity, current profits, $A - g$, should compensate for the opportunity cost of capital, ρv, as well as for the expected capital loss, $(\gamma - \lambda)v - \mathrm{E}_t(\mathrm{d}v/v)/\,\mathrm{d}t$. The term $\mathrm{E}_t(\mathrm{d}v/v)/\,\mathrm{d}t$ captures the expected capital loss due to the fact that profits per variety fall (at least do not increase) over time, as we show in Section 2.3. The proof of this and all the remaining propositions are in the appendix.

The linearity of the program ensures that the firm's problem is scale independent. The intensity of adoption, and therefore firms' growth rate is independent of n. We can now fully characterize the dynamics of a firm.

Proposition 2. Conditional on the dynamics of aggregate variables, $Y(t)$ and $w(t)$, expected growth of sales per worker for the firm is $\lambda(A) - \gamma$, and the variance of growth in sales per worker is $[\lambda(A) + \gamma]/n(t)$.[14]

Equation (5) shows that, conditional on aggregate variables, sales is a linear function of n, hence its growth rate equals the growth rate of n. The expected growth in the number of varieties equals the rate of technology adoption minus the rate of technology failure, $\lambda - \gamma$. The variance of sales growth is driven by the two shocks the firm faces: the randomness of the adoption process and variety failures. Hence the variance of an

[14] We focus on sales growth, since this is the firm-level performance variable we observe in the data.

individual variety is $\lambda + \gamma$. Total sales volatility then declines with n by the law of large numbers.[15] (The Appendix gives a formal proof.)

This proposition implies that bigger, more productive firms are less volatile,[16] a result we discuss in the empirical section.

2.3 Aggregate dynamics

So far we have taken aggregate output $Y(t)$ and the wage rate $w(t)$ as given, independent of $n(t)$. In general equilibrium, however, more varieties lead to higher productivity, resulting in higher output and wages. These in turn affect firms' profitability and their path of adoption. We close the model by considering these linkages.

There is a unit mass of identical firms, indexed by i. Firm i has $n(i,t)$ varieties. The output of the final good is a CES aggregate of firm-level outputs,

$$Y(t) = \left[\int_{i=0}^{1} y(i,t)^{(\varepsilon-1)/\varepsilon}\,\mathrm{d}i\right]^{\varepsilon/(\varepsilon-1)} = N(t)^{1/(\varepsilon-1)}L, \tag{11}$$

where we have substituted individual-firm output $y(i,t)$ with expression (2); L denotes the fixed labour supply; and $N(t) = \int_{i=0}^{1} n(i,t)\,\mathrm{d}i$ is the aggregate number of varieties.

Aggregate productivity is increasing in the aggregate number of varieties, $N(t)$. For ease of exposition, we assume here that both technology shocks and the random success of adoption hit all firms at the same time. In other words, the shocks are technology-specific, not firm-specific. This ensures that shocks have aggregate effects and there is aggregate volatility.[17] In a symmetric equilibrium, this means that all firms use the same number of varieties, $n(i,t) = N(t)$. In Appendix B we relax this working hypothesis and derive the aggregate dynamics in the case of firm heterogeneity. Our main results do not hinge on this assumption.

The specification of technology shocks and the adoption process ensure that $N(t)$ is the single state variable for the economy. Output, wage, and the adoption rate are deterministic functions of N at any point in time. The dynamics of N is as follows. With hazard rate $\lambda_N N$, adoption is successful and N jumps to $N+1$. With hazard rate γN, one of the varieties fails and N jumps to $N-1$.

[15] We focus on the behavior of sales growth, for which data are available at the firm level.

[16] As is generally the case in monopolistic competition models, n is an index of both productivity and size.

[17] So, for example, an increase in oil prices, which in the model materializes as a drop in productivity χ (more units of final output are needed to operate one unit of oil-dependent inputs), affects all firms in the same way since they are ex-ante identical.

We can then describe the volatility of the growth rate of Y as

$$\begin{aligned}\mathrm{Var}(\mathrm{d}Y/Y) &= \frac{\lambda_N N\left[(N+1)^{1/(\varepsilon-1)} - N^{1/(\varepsilon-1)}\right]^2 + \gamma N\left[(N-1)^{1/(\varepsilon-1)} - N^{1/(\varepsilon-1)}\right]^2}{N^{2/(\varepsilon-1)}}\,\mathrm{d}t \\ &\approx \left(\frac{1}{\varepsilon-1}\right)^2 \frac{\lambda_N+\gamma}{N}\,\mathrm{d}t, \end{aligned} \tag{12}$$

where the approximation uses the first-order expansion of $(N+1)^{1/(\varepsilon-1)} - N^{1/(\varepsilon-1)}$ as $N^{(2-\varepsilon)/(\varepsilon-1)}/(\varepsilon-1)$, and is exact for $\varepsilon = 2$.

Aggregate volatility depends on the randomness of adoption and technology failures. Because λ_N is non-increasing in N, volatility is decreasing in N, which we can think of as an index of economic development (equation (21)). Proposition 3 summarizes this key result.

Proposition 3. The volatility of GDP growth rates declines with the level of GDP per capita.

To derive equilibrium wages, note that each firm has a constant profit margin $(1/\varepsilon)$. The total wage bill is a fraction $1-1/\varepsilon$ of total output, which pins down the wage rate at

$$w_N = \left(1 - \frac{1}{\varepsilon}\right) N^{1/(\varepsilon-1)}. \tag{13}$$

Equations (21) and (13) together imply that the demand shifter of the firm is

$$A_N = \frac{1}{\varepsilon} N^{(2-\varepsilon)/(\varepsilon-1)} L. \tag{14}$$

Individual-firm profits per variety decrease with aggregate productivity (or remain unchanged when $\varepsilon = 2$). There are two opposing forces at play. On the one hand, since varieties are substitutes, higher productivity of competitors implies lower demand for a particular firm's product. On the other hand, since varieties are *imperfect* substitutes, there is a demand externality: more aggregate varieties raise income and hence demand for every firm's product. As long as the elasticity of substitution is not too low, the first effect dominates. If $\varepsilon = 2$, the two effects exactly cancel out, and we obtain a balanced growth path, as summarized in the following proposition.

Proposition 4. If the elasticity of substitution is $\varepsilon = 2$, the economy grows at a constant *mean* growth rate x, $\mathrm{E}(\mathrm{d}Y/Y) = x\,\mathrm{d}t$. If the elasticity of substitution is $\varepsilon > 2$, the economy has a stochastic steady state, in which N (and hence Y) has a steady state distribution.

In the balanced growth case, the expected growth rate x is implicitly defined by

$$[\rho - x]\, g'(\gamma + x) = L/2 - g(\gamma + x), \tag{15}$$

and our assumptions ensure that $x \in [0, \rho)$. The growth rate is increasing in L; decreasing in the discount rate, ρ; decreasing in the probability of a technology shock, γ; and decreasing with an upward shift in $g(.)$ (costlier adoption).

If $\varepsilon > 2$, we can characterize the steady state distribution of N with its mode, N^*, which is implicitly defined by $\lambda(N^*) = \gamma$.[18] Substituting this in equations (9), (10), and (14),

$$N^* = \left[\frac{\rho g'(\gamma) + g(\gamma)}{L/\varepsilon}\right]^{\frac{\varepsilon-1}{2-\varepsilon}}. \tag{16}$$

The mode N^* is increasing in L; decreasing in ρ; decreasing in γ; and decreasing with an upward shift of $g(.)$.

If N is below its mode, it is *expected* to increase with the rate $\mathrm{E}(\mathrm{d}N/N) = [\lambda_N - \gamma]\,\mathrm{d}t$. Similarly, if N is above its mode, it is expected to decrease. However, because of the stochastic shocks, it also has a positive probability of moving *away* from N^*.

It remains to be shown that profitability per variety in the aggregate, $A(t)$, follows a stationary Markov process. To see this, note that if $\varepsilon = 2$, $A(t)$ is a constant equal to $L/2$. If $\varepsilon > 2$, A is a monotonically decreasing function of N (see equation (14)) and it hence inherits the properties of the stochastic process for N. As shown in the proof of Proposition 4 (in Appendix), N is a stationary Markov process. Therefore, A is also stationary Markov.

3 Volatility and Development: Empirics and Discussion

The model developed in the previous sections is consistent with four empirical regularities. We discuss these regularities in conjunction with the predictions of the model.

Fact 1. GDP volatility declines with development, both in the cross section, and for a given country over time.

The decline in aggregate volatility with the level of development is one of the stylized facts in the literature and the main motivation of this paper. There are large cross-country differences in volatility. The standard deviation of annual GDP growth during the period 1970 through 2000 ranges from 1.4 percent to 21.8 percent (or a factor of 15). The cross-country variation in volatility is highly correlated with the cross-country variation in the level of development, gauged by real GDP per capita.

[18] Strictly speaking, because N^* can only be an integer, it is defined as the lowest N for which $\lambda_N \le \gamma$.

This is illustrated in the left hand-side panel of Figure 2, which plots the (log) level of volatility, measured as the standard deviation of growth rates over non-overlapping decades from 1960 through 2000, against the average (log) level of real GDP per capita over the decade. The graph also shows the linear regression line together with the 95-percent confidence-band intervals. In the model, the cross-sectional decline in volatility results naturally as countries with a higher degree of technological sophistication enjoy higher productivity and lower volatility levels.

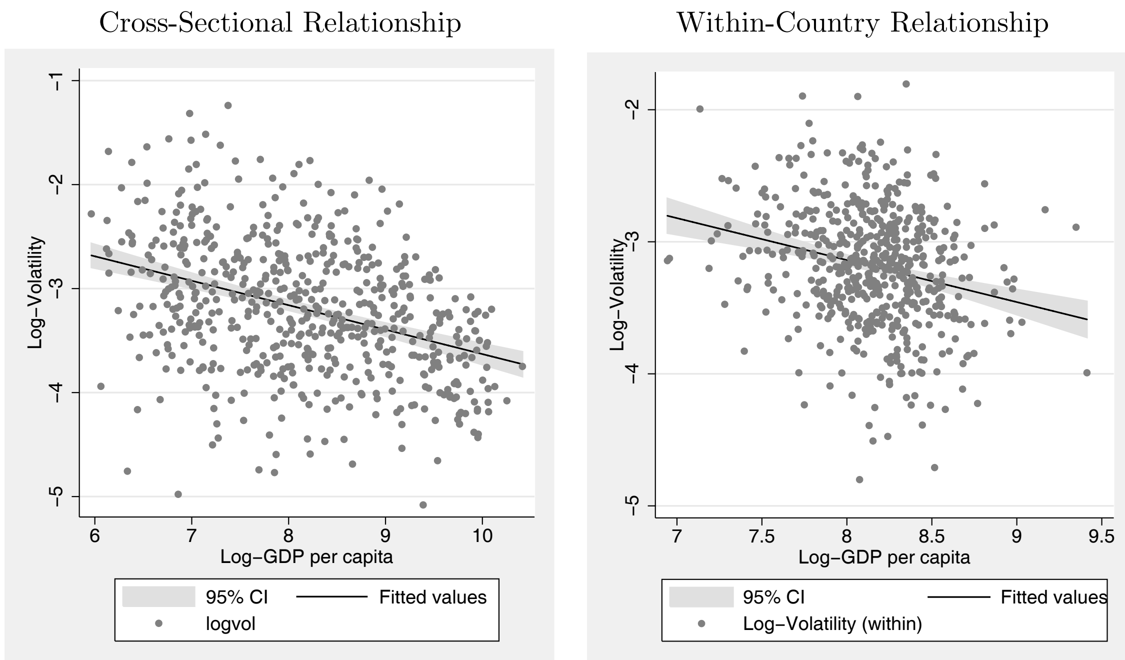

Figure 2: Volatility and Development

The model also predicts that, for a given country over time, volatility declines with development. This is illustrated in the second panel of Figure 2, which plots the same variables after controlling for country-specific effects. In other words, keeping country characteristics (e.g., geography, institutions) constant, growth and changes in volatility are negatively correlated.This negative correlation holds at different levels of financial development, as illustrated in Figure 3. In this Figure, we split the level of financial development, measured as is standard, by the (log) ratio of private credit to GDP, into four quartiles. Similar results obtain by splitting financial development in narrower quantiles.[19]

[19]In related work, Ramey and Ramey (1995) study the link between volatility and growth. Here, as suggested by the model, we study the links between volatility and *productivity* or between *changes* in volatility and growth.

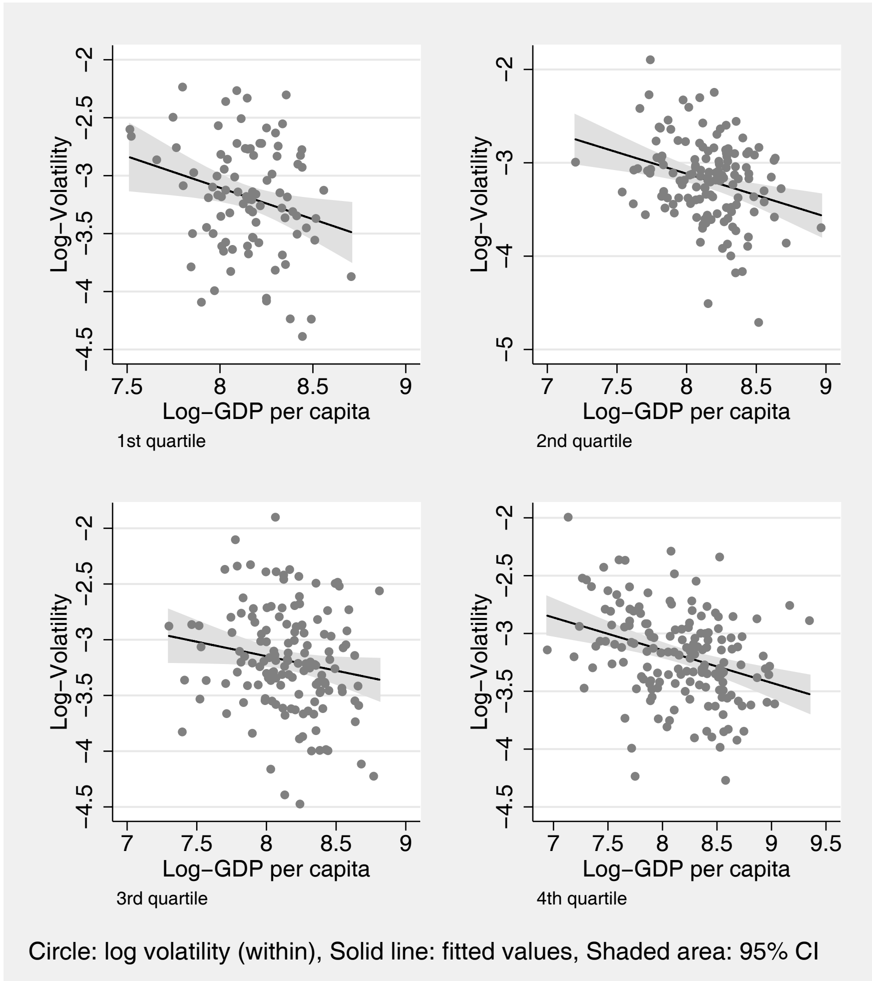

Figure 3: Volatility and Development by Financial Development Quartile

The graphs indicate that the decline of volatility with development is not sensitive to the level of financial development of the country. Even countries with very limited financial infrastructure experience a decline in volatility when they grow.

For completeness, we show in Figure 4 the cross-sectional and within-country relationship between volatility and financial development. We observe a strong negative correlation between volatility and financial development in the cross-section. However, the correlation vanishes once we control for country-specific effects. This result is interesting as it suggests that the decline in volatility for a given country over time cannot be explained in a statistical sense by higher levels of financial development.

We summarize these correlations in Table 1. The first two columns shows the coefficients from a regression of (log) volatility on real GDP per capita, excluding and including fixed effects. The coefficients are statistically significant at the 1 percent

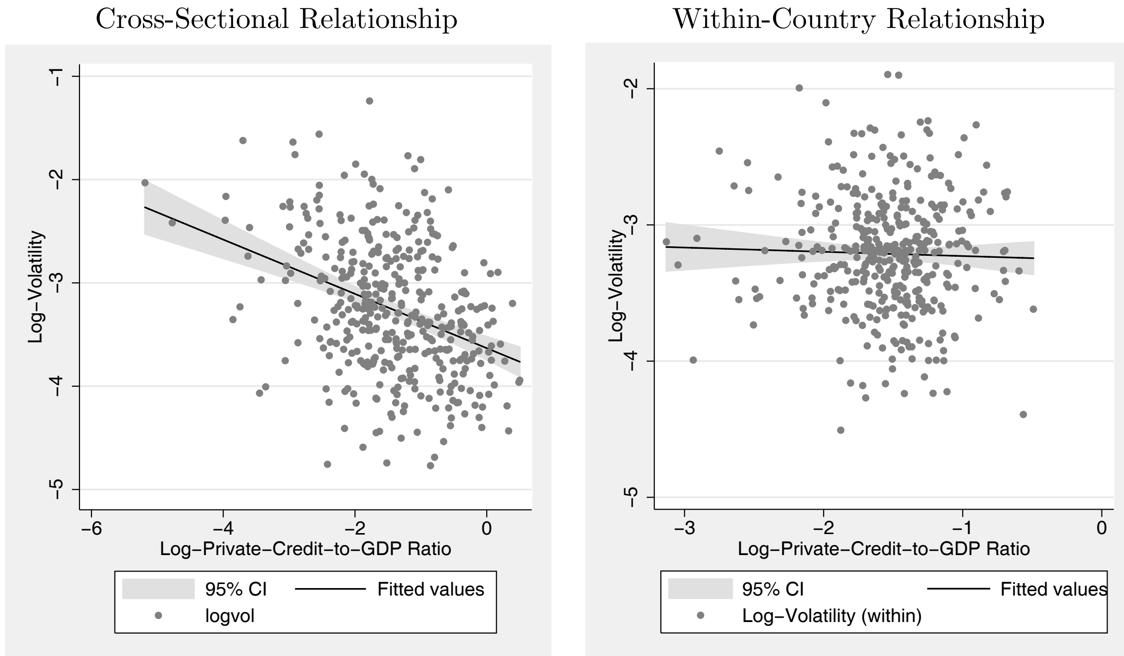

Figure 4: Volatility and Financial Development

Table 1: Volatility, Development, and Finance

	Dependent Variable: Standard Deviation of Growth Rates					
GDP per capita (constant PPP $)	-0.2319***	-0.3008***			-0.1689***	-0.3153***
	[0.0318]	[0.0622]			[0.0454]	[0.1169]
Private Credit / GDP			-0.1468***	-0.0198	-0.0711*	0.0124
			[0.0542]	[0.0489]	[0.0380]	[0.0539]
Country Fixed Effects	No	Yes	No	Yes	No	Yes
Observations	585	585	403	403	403	403
R-squared	0.13	0.56	0.09	0.60	0.13	0.62

Note: All variables are in logs. The equations use the 10-year standard deviation of annual growth rates from 1960 to 2000. The regressors are computed at their mean values over the decade. Clustered standard errors in brackets. * significant at 10%; ** significant at 5%; *** significant at 1%.

level. The third and fourth columns show the corresponding results when volatility is regressed on the (log) ratio of private credit to GDP. As anticipated earlier, the cross sectional relationship is strongly negative; however, once fixed effects are included, the estimated elasticity is both statistically and economically insignificant. Finally the last two columns show the regression results when both variables are included in the regression. Volatility is strongly (and negatively) associated with the level of per capita GDP, while there is little or no (partial) correlation with the level of financial development.

In the model, the high volatility at early stages of development results from the relatively low number of varieties used in the production process. Various empirical studies document the low or delayed adoption of varieties in developing countries. For example, Caselli and Coleman (2001) find that the adoption of computers depends crucially on the level of development of the country. Caselli and Wilson (2004) show that this result extends to a broader set of high-technology equipment (where the extent of technology embodied in capital equipment is measured as the R&D content). Comin and Hobijn (2004) provide additional support for this observation: They document how specific technological innovations have spread across countries, showing that most innovations originated in developed countries and spread gradually to less-developed countries. This implies that at any point in time poor countries use fewer varieties than rich ones.

Fact 2. More productive sectors are also less volatile.

In broad terms, manufacturing is both more productive and less volatile than agriculture. To the extent that manufacturing uses more complex production technologies than agriculture, the model predicts that manufacturing sectors should be both more productive and less volatile than agriculture. This is indeed consistent with a strong regularity in the data: On average, volatility of value-added per worker in agriculture is around 50 percent higher than that in manufacturing. At the same time, value added per worker is around twice as high in manufacturing than in agriculture. These figures are computed from the OECD-STAN database. In Table 2 we report the summary statistics by country. The table shows the average of labor productivity in manufacturing relative to labor productivity in agriculture from 1970 through 2003 and the corresponding ratio of volatilities over the same period. In all countries, manufacturing is significantly more productive, as predicted by the model. Moreover, manufacturing is also less volatile, with the only exception of Italy, where volatility is slightly higher in manufacturing.

At a more disaggregated level, Koren and Tenreyro (2007), show that this result holds within manufacturing for a broad sample of developing and developed countries. In particular, countries at early stages of development tend to specialize in low-productivity, high-volatility activities, whereas the opposite pattern is observed at later stages. The development process is characterized by a move towards both more productive and safer sectors.

Fact 3. More productive firms are less volatile. Firm-level volatility (for non-listed firms) declines with development.

Table 2: Ratio of Manufacturing-to-Agriculture Productivity and Volatility

Country	Relative Productivity in Manufacturing	Relative Volatility in Manufacturing
Australia	1.41	0.20
Austria	6.82	0.42
Belgium	2.08	0.45
Canada	1.72	0.47
Denmark	1.19	0.36
Finland	2.17	0.84
France	1.66	0.31
Germany	2.34	0.43
Greece	1.58	0.97
Italy	1.77	1.01
Japan	4.28	0.50
Korea	2.51	0.45
Luxembourg	1.78	0.35
Netherlands	1.37	0.37
Norway	1.54	0.73
Poland	4.23	0.36
Portugal	2.18	0.43
Spain	1.84	0.33
Sweden	1.46	0.65
United Kingdom	1.52	0.42
United States	2.24	0.23

Note: Column 2 shows the ratio of average labor productivity in manufacturing over labor productivity in agriculture from 1970 to 2003. Column 3 shows the corresponding ratio for standard deviation of labor productivity growth during the period.

A crucial point in the model is that, at the firm level, average productivity and volatility are negatively correlated. Table 3 shows the coefficients from a regression of (log) volatility of sales growth on average size (employment) and productivity (sales per worker) for 9000 Compustat firms in the US. Volatility is calculated for non-overlapping decades from 1950 through 2000. Both productivity and size are negatively correlated with firm-level volatility.[20] This remains true if we include firm-fixed effects to consider within-firm variation only: firms becoming more productive also become more stable. This is consistent with our result presented in Proposition 2: Productivity (and employment) growth is associated with the adoption of new varieties, technological diversification, and hence lower volatility.

[20]The negative correlation between firm-level volatility and size has been documented in an early study by (Hymer and Pashigian 1962).

The time-series behavior of firm-level volatility in the US has been recently studied in Comin and Philippon (2005) and Davis, et al. (2006). The first paper documents that publicly traded firms have experienced an increase in volatility during the last three decades. The second paper shows that non-publicly traded firms have experienced a sharp decline in volatility; furthermore, Davis et al. (2006) argue that the decline in aggregate volatility experienced by the US in the last decades has been overwhelmingly driven by a decline in firm-level volatility and not by the aggregation of highly volatile firms. These two pieces of evidence can be reconciled by noting that publicly traded firms have likely much better access to external finance and more opportunities for financial diversification than non-publicly traded ones. In other words, the evidence is consistent with both a financial-development channel (Comin-Philipon) and a *within-firm* technological-diversification channel (Davis et al.) Since a majority of firms in developing countries are non-publicly traded, our model has sought to highlight the second channel as a promising (and neglected) source of difference in firm-level and aggregate volatility between poor and rich countries.

Table 3: Firm Productivity and Volatility

	Dependent Variable: Standard Deviation of Growth Rates			
Sales per worker	-0.115***		-0.137***	-0.136***
	[0.008]		[0.007]	[0.017]
Employment		-0.192***	-0.194***	-0.198***
		[0.002]	[0.002]	[0.009]
Firm Fixed Effects	No	No	No	Yes
Decade Fixed Effects	Yes	Yes	Yes	Yes
Observations	25408	25408	25408	25408
R-squared	0.06	0.24	0.26	0.26

Note: All variables are in logs. The equations use the 10-year standard deviation of annual sales growth rates from 1950 to 2000. The regressors are computed at their mean values over the decade. Clustered standard errors in brackets. * significant at 10%; ** significant at 5%; *** significant at 1%.

4 Conclusion

This paper proposes a model in which the production process makes use of different input varieties subject to imperfectly correlated shocks. As in other growth models, technological progress takes place as an expansion in the number of input varieties, increasing productivity. The new insight in the model is that the expansion in varieties also leads to lower volatility of production via two channels. First, as each individual variety matters less and less in production, the contribution of idiosyncratic fluctuations to overall volatility declines. Second, each additional input provides a new adjustment margin in response to external shocks, making productivity less volatile.

In the model, the number of varieties evolves endogenously in response to profit incentives and the decrease in volatility comes out as a powerful by-product of firms' incentives to increase profits.

Our model yields empirical predictions concerning the relationships between volatility and productivity at the aggregate and firm levels. We discuss these predictions in light of the empirical evidence.

Appendix

A Technology Adoption under Risk Aversion and Financial Autarky

In this Appendix we discuss technology adoption when agents are risk-averse and risk pooling is not possible. Each firm is owned by a risk-averse individual, whose only source of income is the profit of the firm. Utility exhibits risk aversion with $u' > 0$, $u'' < 0$, $u(0) > -\infty$, $u'(0) < \infty$. These latter assumptions ensure the finiteness of the value of the firm even if there is a positive probability that the firm profits (and hence consumption) eventually become zero.

The value of the firm is defined as lifetime expected utility,

$$V_n(A) \equiv \mathrm{E}_t \int_{s=t}^{\infty} e^{-\rho s} u\{A(t)n - g[\lambda(t)]n\} \,\mathrm{d}t. \tag{17}$$

The Bellman equation characterizing the firm's problem is

$$\begin{aligned} \rho V_n(A) = & \max_{\lambda} \{u[An - g(\lambda)n] \\ & + \lambda n \left[V_{n+1}(A) - V_n(A)\right] \\ & + \gamma n \left[V_{n-1}(A) - V_n(A)\right]\} \\ & + \lim_{h \to 0} \mathrm{E}\left[V_n[A(t+h)] - V_n(A)\right]/h\}, \end{aligned} \tag{18}$$

$$\text{s.t. } A - g(\lambda) \geq 0. \tag{19}$$

This is the same as (8) with the exceptions that (i) flow utility is a concave function of firm profits, and (ii) we rule out borrowing so that adoption has to be financed from current profits.

Proposition 5. Optimal technology adoption intensity, $\lambda(n, A)$ is strictly positive for all $n > 0$ and A.

Proof. Because $g(0) = 0$, the non-negative profit constraint provides a *positive* upper bound on λ. If the constraint is binding, λ is positive. Otherwise we can use the first-order-condition for optimal adoption,

$$u'[An - g(\lambda)n]g'(\lambda) = V_{n+1}(A) - V_n(A). \tag{20}$$

The properties of u' and g' ensure that there will be a unique positive λ for each n as long as $V_{n+1} - V_n > 0$. This condition is easy to verify. It is obvious that $V_{n+1} \geq V_n$, because the firm can always throw away the additional variety and replicate its profits with n varieties. We can also show that it is strictly better off with more varieties.

The value of a firm with n_0 products is V_{n_0} defined by (17). Now calculate a lower bound for the expected discounted utility if the firm adds a variety. Suppose the firm does not change its adoption efforts but keeps them at $\lambda(n_0)$. Let us denote the value of this strategy by $\tilde{V}_{n+1}$. It is clear that $V_{n+1} \geq \tilde{V}_{n+1}$, because the firm cannot lose by adjusting its adoption intensity optimally.

Now suppose that the additional variety is useless, $\tilde{V}_{n+1} = V_n$. In this case the firm does not innovate, and is making profits $A(t)$ per variety. The flow profits the additional variety generates while working is strictly positive, which ensures $\tilde{u}(t) > u(t)$ for all $t \leq T_{n+1}$, because $u' > 0$ even if the consumer is risk averse. Because the new variety is expected to have a positive lifetime ($T_i > 0$ with probability 1), we have that $\tilde{V}_{n+1} > V_n$, a contradiction. Hence $\tilde{V}_{n+1} > V_n$ and $V_{n+1} > V_n$. □

The proof relies on the property that new varieties lead to higher profits. This is why firms have an incentive for technological diversification even in the complete absence of financial markets. Of course, the *magnitudes* may vary with the degree of financial development and technology adoption may be faster or slower in financial developed economies. However, we demonstrated that financial deepening is not *required* for technological diversification to work.

The result that the adoption intensity is positive for *all* n depends on the functional form assumptions about the cost of adoption. In particular, the Inada conditions ensure that it is always optimal to devote some resource to adoption as long as the marginal benefit is positive. Of course, if the marginal cost of adoption is bounded away from zero, there is a range of positive but small marginal benefits for which adoption intensity will be zero. This does not alter the result that financial development is not a *necessary condition* for technological diversification.

B Heterogeneous Firms and Aggregate Dynamics

Section 2.2.1 analyzed the technology adoption problem of a single firm. We described the dynamics of the firm as a function of the number of technologies used by the firm and aggregate variables. Here we study how an economy comprised of firms with potentially different varieties evolves over time.

Just as in Section 2.3, there is a unit mass of identical firms, indexed by i. Firm i has $n(i,t)$ varieties. The output of the final good is a CES aggregate of firm-level outputs,

$$Y(t) = \left[\int_{i=0}^{1} y(i,t)^{(\varepsilon-1)/\varepsilon}\,\mathrm{d}i\right]^{\varepsilon/(\varepsilon-1)} = N(t)^{1/(\varepsilon-1)}L, \tag{21}$$

where L denotes the fixed labor supply, $N(t) = \int_{i=0}^{1} n(i,t)\,\mathrm{d}i$ is the aggregate number of varieties and individual firm output comes from equation (2).

To understand aggregate dynamics, we need to characterize the dynamics of $N(t)$, the overall number of varieties. By a change of variables, this can be written as $N(t) = \sum_{i=1}^{K} i m_i$, where m_i is the mass of firms that have exactly i varieties, and $K < \infty$ is an upper bound on the number of varieties firms can adopt (the world technology frontier, for example).

There are two types of shocks affecting $N(t)$. First, successful adoption of some firms will move them from n varieties to $n+1$ varieties. By Proposition 1, all firms adopt new varieties with intensity λ, independently of n. As a tie-breaking rule, we assume that firms try to adopt technologies with lower indexes first. A firm of size n has thus access to technologies $1, 2, ..., n$ and would, upon success, adopt technology $n+1$ next. We assume that the success of adoption is completely idiosyncratic, that is, independent across firms. Because there is a continuum of firms, a non-stochastic fraction of them is going to become successful in adoption at any point in time. This means that, in this setup, adoption does not contribute to aggregate uncertainty.

The second type of shock is the failure of a particular technology k. This decreases the number of varieties by 1 for all firms that use variety k. Because there is a positive mass of these firms, this shock induces an instantaneous *jump* in N. The aggregate impact of the shock (and, ultimately, aggregate volatility) will depend on the measure of firms using technology k. By the tie-breaking rule, these are the firms that have at least k varieties, $m_k + m_{k+1} + ... + m_K$.

Formally, we can describe the dynamics of N as follows:

$$N(t+h) = \begin{cases} N(t) + \lambda N(t)h & \text{with } \Pr = 1 - O(h) \\ N(t) + \lambda N(t)h - (m_1 + ... + m_K)1 & \text{with } \Pr = \gamma h + o(h) \\ \vdots & \vdots \\ N(t) + \lambda N(t)h - m_K 1 & \text{with } \Pr = \gamma h + o(h) \end{cases}$$

Over an infinitesimal h period of time, a non-stochastic λN measure of firms expand their varieties by 1, and with probability γh, variety k ($k = 1, ..., K$) fails, affecting a measure $m_k + ... + m_K$ of firms.

The expected growth rate

$$\mathrm{E}(\mathrm{d}N/N) = (\lambda - \gamma)\,\mathrm{d}t,$$

is the same as in Section 2.3. What is different is the volatility of N, which now depends on the whole distribution of varieties used by firms. The aggregate number of varieties, N, is no longer a sufficient statistic to describe aggregate dynamics.

Letting s_k denote the share of expenditure spent on technology k, $s_k = (m_k + ... + m_K)/N$, we can use Lemma 1 (in Appendix C) to express aggregate variance as

$$\mathrm{Var}(\mathrm{d}N/N) = \gamma \left[\sum_{k=1}^{K} s_k^2\right] \mathrm{d}t.$$

Volatility depends on the size of the technology shock, γ, as well as an index of concentration, $\sum_{k=1}^{K} s_k^2$.

We show that, under some regularity conditions, as the economy develops, it both increases the aggregate number of varieties N, and lowers the concentration of technologies, $\sum_{k=1}^{K} s_k^2$, and hence lowers volatility. (Note that this is correct if shocks to varieties are homoskedastic. If shocks are heteroskedastic, the formula will be a weighted sum of the squared shares[21]).

Proposition 6. Assume that the firm size distribution $\{m_1, ..., m_K\}$ satisfies the following property: $k \Pr(n = k)/\Pr(n > k)$ is monotonically increasing in k. Then successful adoption results in higher N and lower $\mathrm{Var}(\mathrm{d}N/N)$.

1. The regularity condition is quite weak and it is easy to verify that it is satisfied by Pareto, exponential, and uniform distributions.

C Proofs

Proof of Proposition 1. Since profits are linear in n, guess that the form of value function is $V_n(A) = v(A)n$. Equation (9) is then the first-order condition for optimal λ. Equation (10), in turn, results from substituting the guess function into the Bellman equation. From (9), λ is independent of n.

[21] Note also that if there is correlation among shocks, the formula should also be modified to account for the correlations.

Let ϕ denote expected capital gain per variety, $\mathrm{E}_t(\mathrm{d}v/v)/\mathrm{d}t$. Substituting (9) into (10),

$$[\rho + \gamma - \phi]\, g'(\lambda) = A - g(\lambda) + \lambda g'(\lambda).$$

Both sides are continuously differentiable with respect to λ, the LHS is 0 at $\lambda = 0$, the RHS is positive.

(i) Suppose first that $\phi < 0$. Then at $\lambda = \gamma + \rho$, the LHS is lower than the RHS by Assumption 6. In between, the LHS is growing faster (declining slower) than the RHS, so there is a unique $\lambda \in (0, \gamma + \rho)$.

(ii) Now suppose that $0 \leq \phi < \bar{\phi}(A)$, where $\bar{\phi}(A)$ is implicitly defined by $A = g[\gamma + \rho - \bar{\phi}(A)]$. This condition means that if the firm adopts at a rate lower than $\bar{\lambda} = \gamma + \rho - \bar{\phi}(A)$, it is making positive profits. Then at $\bar{\lambda}$, the LHS is lower than the RHS, so again we have a unique $\lambda \in (0, \gamma + \rho - \bar{\phi}(A))$.

(iii) Now suppose that $\phi \geq \bar{\phi}(A)$. Then the LHS is always lower than the RHS. For any λ, the value of adopting at that rate is higher than the marginal cost of adoption, $g'(\lambda)$. The firm would be willing to devote infinite resource to adoption. This cannot be an optimum because it would violate the transversality condition. The value of the firm will jump to a level such that the expected further increase, ϕ, is below $\bar{\phi}(A)$.

Note that as a consequence of ruling out case (iii), the firm always makes positive profits in equilibrium.

The comparative statics can be shown as follows. From (9) and the strict convexity of g, innovation intensity is a positive function of $v(A)$, the value of a given variety. The value is, in turn, ...

(i) ...increasing in A. An economy that starts from a higher $A' < A$ has greater profitability per variety today, and greater expected profitability in future periods by Assumption **??**. The present discounted value of the higher profits is higher even if the firm does not adjust its adoption policy and potentially even higher thereafter, $V(A', n) > V(A, n)$. This implies that $v(A') > v(A)$ for all $A' > A$, so that $v(A)$ is increasing in A.

(ii) ...decreasing in the discount rate ρ. We have shown above that the firm always makes positive profits. Discounting positive profits at a higher rate obviously makes the value of the firm lower.

(iii) ...decreasing in the failure rate γ. The failure rate acts as a depreciation rate, introducing additional discounting into the present value problem. As shown in equation (10), it enters the problem completely symmetric with ρ, so we can apply the same reasoning.

□

Proof of Proposition 2. The proof follows directly from the following lemma.

Lemma 1. Let x_t follow a discrete-state Markov process with Poisson jumps between states $\{E_1, E_2, ..., E_N\}$, with transition probabilities $\Pr(x_{t+h} = E_n | x_t = E_k) = \xi_{n,k} h + o(h)$. Then (i) the expected change in x_t is

$$\mathrm{E}(\mathrm{d}x_t | x_t = E_k) = \sum_{i=1}^{N} \xi_{i,k}(E_i - E_k)\,\mathrm{d}t,$$

and (ii) its instantaneous variance is

$$\mathrm{Var}(\mathrm{d}x_t | x_t = E_k) = \sum_{i=1}^{N} \xi_{i,k}(E_i - E_k)^2\,\mathrm{d}t.$$

(i) The first jump arrives with arrival rate $\sum_{i=1}^{N} \xi_{i,k}$. Conditional on a jump occurring, the probability of state n is $\xi_{n,k} / \sum \xi$, and the jump size is $(E_n - E_k)$ in this case. Taking expectation over all possible states, we get the result. (ii) Note that the instantaneous volatility equals the instantaneous second moment, $\mathrm{E}[(\mathrm{d}x_t)^2 | x_t = E_k]$, because $\mathrm{E}(\mathrm{d}x_t | x_t = E_k)^2$ is of order $O(\mathrm{d}t^2)$. Then applying (i) to jumps of size $(E_n - E_k)^2$, we obtain the result.

Substitute in $E_n = n$ and $\xi_{i,k} = \lambda k$ if $i = k + 1$, $\xi_{i,k} = \gamma k$ if $i = k - 1$, and $\xi_{i,k} = 0$ otherwise. Then $\mathrm{E}(\mathrm{d}n) = (\lambda - \gamma) n\,\mathrm{d}t$ and $\mathrm{Var}(\mathrm{d}n) = (\lambda + \gamma) n\,\mathrm{d}t$. Divide by n to obtain the result. □

Proof of Proposition 3. Equation (12) expresses the volatility of GDP growth rate as $[\lambda_N + \gamma]/N$. We show that this is declining in N because λ_N is nonincreasing in N. Because N is positively related to GDP per capita, Y/L, this will complete the proof.

Equation (14) shows that profitability per variety is non-increasing in N. By Proposition 1, adoption is increasing in profitability, so λ_N is non-increasing in N. □

Proof of Proposition 4. (i) With $\varepsilon = 2$, aggregate output is $Y = NL$, and profitability does not depend on N, $A = L/\varepsilon$. Then from (9) and (10), we have that $v(t) = v$ constant, which makes $\lambda(t) = \bar{\lambda}$ constant. Aggregate dynamics then looks the same as firm-level dynamics described in Proposition 2.

(ii) The dynamics of aggregate N is characterized as a birth and death process with birth rate $\xi_N = \lambda_N N$ and death rate $\mu_N = \gamma N$. A birth and death process has a steady state distribution if the series

$$\sum_{k=1}^{\infty} \frac{\xi_1 \xi_2 \cdots \xi_k}{\mu_2 \mu_3 \cdots \mu_{k+1}}$$

converges. Substituting in ξ_k and μ_k, this condition becomes

$$\sum_{k=1}^{\infty} \gamma^{-k} \left[\lambda(1)\lambda(2) \cdots \lambda(k)\right] < \infty.$$

The proof of Proposition (3) shows that λ_N is a non-increasing function of N. In fact, because profits tend to zero as N tends to infinity, we have $\lim_{N\to\infty} \lambda_N = 0$. This implies that for a $\delta < 1$ there exists a $K < \infty$ such that $\lambda_N < \delta\gamma$ for all $N > K$.

The product $\lambda(1)\lambda(2)\cdots\lambda(k)$ is then bounded from above by $\lambda(1)\lambda(2)\cdots\lambda(K)\delta^{k-K}\gamma^{k-K}$ for $k < K$. The series is bounded by

$$\lambda(1)\lambda(2)\cdots\lambda(K)\delta^{k-K}\gamma^{-K},$$

which is convergent as $k \to \infty$ because $\delta < 1$. Hence the series is convergent, and a steady state distribution exists. □

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