Measuring and Mismeasuring Aggregate Productivity Growth Using Plant-level Data

by

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Abstract. We define productivity growth as the shift in the social production possibilities frontier (PPF) holding primary inputs constant. We show this measure is exactly equal to the change in welfare that arises because of plant-level technical-efficiency shocks in a competitive setting (Hulten (1978)), and it provides a first-order approximation to welfare under imperfect competition if a representative agent model is reasonable. We describe the unique best approximation to it when using plant-level data. We note that this quantity is almost never reported in the literature, and we show that the most popular productivity index for plant-level data adds a "reallocation" term to our preferred measure. Empirically, the "reallocation" term is large and volatile relative to the social PPF index, leading to a weak relationship between the two indexes in almost every manufacturing industry in both Chile from 1987-1996 and Columbia from 1981-1991, and calling into question the literature's interpretation of the "reallocation" term as productivity growth. We provide a new definition of "reallocation" that is entirely based on decomposing changes in the social PPF measure. In contrast to current findings using the popular index, where reallocation effects vary in sign and magnitude across time and sector, our new measure suggests that reallocation effects are reasonably stable within industries and almost always positively impact the productivity growth rate, even in instances where aggregate productivity falls.

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1. Introduction

Proper measurement of productivity growth is at the heart of a wide range of fields in economics, including macroeconomics, trade, industrial organization, and regulation (for example). Historically, the shift in the social production possibility frontier (PPF) holding primary inputs constant has been a popular definition of aggregate productivity growth. When markets are competitive the shift is exactly equal to the change in welfare that arises as society's ability to consume and invest increases in response to the plant-level technology shocks. Even when markets are not perfectly competitive, if a representative consumer model is appropriate for demand this measurement provides a first-order approximation to the change in welfare. Thus, the definition provides practitioners with a measurement that is useful for cost-benefit/policy analysis. It is also readily comparable across time, industries, countries, and thus empirical studies.[1]

With the increasing availability of plant-level data, there is a large and rapidly growing body of research that estimates plant-level productivity and then aggregates to the industry level.[2]

This paper grew out of initial work in "When Industries Become More Productive, Do Firms?: Investigating Productivity Dynamics" (NBER Working Paper 6893), and appeared in an earlier form as "On the Micro-Foundations of Productivity Growth." We are grateful to the Russell Sage Foundation and the University of Chicago GSB for support and for helpful discussions from Bert Balk, Susanto Basu, Steve Davis, John Haltiwanger, Alwyn Young, the NBER Productivity meetings, the Chicago Applied Micro Lunch, and the 2005 Society for Economic Dynamics meeting.

[1] Hulten (2001) provides a thoughtful and detailed history of this index.

[2] The data sets used include U.S. data from the Longitudinal Research Database (LRD) of the U.S. Census, French data from the Declarations Annuelles des Salaires (DAS) collected by INSEE (Institut National de la Statistique et des Etudes Economiques), and several plant-level manufacturing censes from developing countries, to name but a

Typically, plant-level productivity is measured using the total factor productivity (TFP) residual ($ln\omega$), computed as (log) output (lnQ) minus the contribution of inputs ($\beta' lnX$), or

$$ln\omega_{it} = lnQ_{it} - \beta' lnX_{it}, \tag{1}$$

where (1) represents either the gross-output or the value added production function. In this paper, we ask (and answer) "How do we aggregate these residuals to get the shift in the social PPF holding primary inputs constant?"

In a competitive setting the answer is known. For the value-added residual it is given by

$$d\Omega = \sum_{i=1}^{N} s_{v_i} dln\omega_i^V, \tag{2}$$

where $dln\omega_i^V$ denotes the instantaneous change in the residual from the value added production function, and the share weights s_{v_i} are in terms of a plant's contribution to industry value added. Lemmas 1 and 2 in Section 2 formalize this proposition for the competitive case using results from Hulten (1978), who uses Domar (1961). In an imperfectly competitive setting (2) is appropriately modified, and the expression yields a first-order approximation to the change in welfare, as shown by Basu and Fernald (2002).[3] In both cases, (2) equals the growth rate in final demand (value added) at plant i holding primary inputs constant - $dln\omega_i^V$ - scaled by i's share in aggregate value added, which translates $dln\omega_i^V$ into welfare.

Some approximation with (discrete) data is needed. With no specific prior information, the unique best approximation is given by multiplying the change in productivity growth from $t-1$ to t, estimated using $\Delta ln\omega_{it}^V = ln\omega_{it}^V - ln\omega_{i,t-1}^V$, by the average share from the beginning and ending period:

$$\widehat{d\Omega} = \sum_i \frac{(s_{v_{it}} + s_{v_{i,t-1}})}{2} * \Delta ln\omega_{it}^V. \tag{3}$$

This is a Tornquist-Divisia quantity index, and Lemmas 3 and 4 in Section 3 review the reasons for most preferring (3), although it is rarely reported.

few. Examples of papers using the U.S. data include Bailey, Hulten, and Campbell (1992), Olley and Pakes (1996), Bernard and Jensen (1999), Bernard, Eaton, Jensen, and Kortum (2003), and Foster, Haltiwanger, and Krizan (2001). Abowd, Kramarz, and Margolis (1999) and Eaton, Kortum, and Kramarz (2004) use the French data while the several papers in Roberts and Tybout (1996) use the LDC data. These are but examples. A careful bibliography would include dozens of papers.

[3] See Nishida and Petrin (2004), who explore the estimation issues with imperfect competition and plant-level data.

The recent literature using plant-level data almost exclusively employs some variant of the Bailey, Hulten, and Campbell (1992) index (the "BHC Index"). The value added formulation defines the growth rate of industry productivity from period $t-1$ to period t as

$$BHC_t = \sum_i s_{it} ln\omega_{it}^V - \sum_i s_{i,t-1} ln\omega_{i,t-1}^V, \tag{4}$$

with s_{it} denoting the share of labor input or gross output (see Foster et al. (2001), and the extensive literature summarized therein).[4] (4) can be decomposed as

$$\begin{aligned} BHC_t &= \widehat{d\Omega} + \sum_i \frac{(ln\omega_{it}^V + ln\omega_{i,t-1}^V)}{2} * (s_{it} - s_{i,t-1}) \\ &= \widehat{d\Omega} + \sum_i \overline{ln\omega_i^V} * \Delta s_i, \end{aligned} \tag{5}$$

where $\overline{ln\omega_i^V}$ is the average $ln\omega_{i\cdot}^V$ (as noted in Fox (2003), who uses Bennet (1920)).[5] Thus, (4) is equal to the shift in the social PPF holding primary inputs constant (i.e. $\widehat{d\Omega}$) *plus* an additional term. This new term - $\sum_i \overline{ln\omega_i^V} * \Delta s_i$ - resembles what is often characterized as a "reallocation" effect; it is the sum across plants of the change in plant share (however defined), multiplied by the average of log-level plant productivity.

A major theme of this paper is that the addition of this "reallocation" term to the measured shift in the PPF holding primary inputs constant is neither theoretically nor empirically innocuous. We document this in Sections 6-8, and provide a brief sketch of the ideas here.

On the theory point, a simple example is illustrative. Suppose there are two firms in a competitive economy and there is no growth in technical efficiency for either firm from period 1 to period 2. Let firm 2 have higher productivity: $ln\omega_2 > ln\omega_1$. Finally, suppose firm 1 and firm 2 have equal revenues (or equal shares) in period 1, but in period 2 the price of good 1 increases in a way that causes firm 1's share to fall to 0.25 and firm 2's share to increase to 0.75 (from a tax on one good, for example). There is no shift in the social PPF because there are no shocks to technical efficiency, so $\widehat{d\Omega} = 0$, exactly as it should. The BHC index is positive because the share at the more productive

[4] Olley and Pakes (1996) suggest the use of the related form $\sum_i s_{it} exp(ln\omega_{it}^V) - \sum s_{i,t-1} exp(ln\omega_{i,t-1}^V)$.

[5] The main variant of the BHC decomposition is given by

$$\sum_i lns_{i,t-1} * (ln\omega_{it}^V - ln\omega_{i,t-1}^V) + \sum_i ln\omega_{it}^V * (s_{it} - s_{i,t-1}).$$

We provide a full discussion in Section 6, and simply note here that it is very similar in spirit to (5), and for this reason it will share the problems of (5) that we raise here plus one additional problem because it is not (5).

plant (firm 2) increases, so $\sum_i \overline{ln\omega_i^V} * \Delta s_i > 0$. Welfare unambiguously falls because the tax moves the society along the budget set but away from the utility maximizing combination of good 1 and 2. Thus, in this example, the BHC index reports positive productivity growth when welfare falls and there is no change in technical efficiency, and it does so exclusively because the "reallocation" term is positive.

Fox (2003) notes that productivity growth can decrease even when technical efficiency at every plant increases. He refers to this phenomenon as the "productivity paradox." We do not view either our example or his as paradoxes (yet), although we might if we knew the underlying model that linked the "reallocation" term to a known economic quantity. For this reason, we believe that defining productivity growth as (4), breaking out $\sum_i \overline{ln\omega_i} * \Delta s_i$, and attributing a "growth" interpretation to it is an exercise for which "growth" has not been clearly defined. Similarly, it is hard to know what to make of comparisons between the magnitude of $\widehat{d\Omega}$ and $\sum_i \overline{ln\omega_i^V} * \Delta s_i$, as is often done to determine the respective roles in "productivity growth" of "real productivity" vs. "reallocation."

We use panel data for 49 manufacturing industries from Chile and Columbia to investigate the magnitude of $\sum_i \overline{ln\omega_i^V} * \Delta s_i$. In almost every 3-digit Chilean and Columbian manufacturing industry the reallocation term is large, volatile, and it dominates the BHC index of productivity growth. These findings are consistent with the large body of results from other plant-level surveys that have reported "reallocation" as the dominant and volatile component of the BHC "productivity growth" index. Furthermore, industry-by-industry regressions of (3) on the BHC index yield slope estimates that are almost entirely less than 0.5, with most being in the range of 0.2 to 0.4, meaning that (4) typically differs from (3) by several orders of magnitude. These regressions also indicate that it is not just a matter of rescaling this mismeasured index; most of the r-squareds from these regressions range from between 0.2 to 0.4.[6]

Entry and Exit

One important issue that does arise with the move from industry- to plant-level data is that industries do not enter and exit, while plants do. In Section 4 we discuss the issues raised by entrants and exiters. There we show that another attractive feature of the instantaneous growth measure from (2) is that conceptually neither entry nor exit raises any difficulties.

[6] Overall, these findings are robust to a number of different methods of estimating plant-level TFP, including ordinary least squares, Solow's approach, and a proxy approach.

Observations on plants are only made at discrete intervals, typically annually, every five years, or every ten years. Since we never observe the exact time between $t-1$ and t at which entry/exit occurs, nor do we observe the path of i's productivity growth rate during its existence in this period, we cannot account for entrants' and exiters' contribution to productivity growth without a more complete model of the unobserved factors. This "discrete data problem" will arise for any productivity measure that is based on following the path of plant-level productivity over time.

Our approach in Section 4 is twofold. We bound the error that arises from this missing information, and we provide estimators for the actual productivity contribution from entrants and errors. The error in aggregate productivity growth from truncating entrants and exiters is small if these plants contribute only a small fraction to industry value added or have stagnant productivity growth. This is true in our annually observed data, as entrants or exiters make up only a small fraction of aggregate productivity growth.[7] When we look at the five year intervals, entrants and exiters sometimes make up a much larger share of aggregate value added. Our conclusion is with annual data perhaps no additional correction needs to be made, but with data collected at 5 year intervals one of the approximations to the contribution to growth from entrants and exits may be desirable.

Real Productivity vs. Reallocation Revisited

Much of the recent interest in using plant-level data to estimate plant- and industry-level productivity arises because plant-level data allows one to investigate "real productivity" and "reallocation" effects. Lemma 1 shows that there is no "reallocation" effect in the case of the instantaneous growth rate under perfect competition. Primary inputs are reallocated in (2), but the contribution of these inputs at the plants to which they are reallocated are deducted from the change in value added to get productivity growth (by definition). The "reallocation" term in the BHC index will generally not be zero in the case of perfect competition.

Given the difficulty of interpreting the economic meaning of the BHC "reallocation" term, we address reallocation in Section 5, exploring two different formulations, each of which is tightly linked to an economic model. We discuss Basu and Fernald (2002), who show how reallocation terms can arise under imperfect competition. We also develop a decomposition that can be used in either the perfect or imperfectly competitive case.

For our decomposition we show that changes in productivity growth rates over time (i.e. changes in (3)) can be decomposed into three terms, one term representing the change in aggregate growth

[7] Not only are they typically the smallest plants, but in annual data the fraction of turnover is limited.

from changes in plant-level productivity growth, one term representing change in growth from the reallocation of value added across plants with differing productivity levels, and one term representing a net entry effect. For this decomposition the empirical results suggest that a reallocation effect so defined is almost always present, economically important, reasonably stable, and almost always works to increase the growth rate in aggregate productivity, even in the instances where the growth rate in aggregate productivity falls. These findings stand in stark contrast to the collection of results on the BHC "reallocation" term, which in plant-level data can vary dramatically in magnitude and sign both over time and across sectors.

We next turn to the economic model behind (2). Readers interested in the discussion of the BHC index and decomposition and the empirical results comparing the BHC index to the shift in the social PPF holding inputs constant can skip directly to Sections 6-8.

2. Aggregate Productivity Growth

We start with the perfectly competitive case (with no distortions), where the shift in the social PPF is exactly equal to the change in welfare that arises as society's ability to consume and invest increases in response to the plant-level technology shocks. Let the constant returns-to-scale technology be given by $Q_i = F^i(M^i, X^i, t)$, where $(M^i = M_1^i, \ldots, M_N^i)$ and $(X^i = X_1^i, \ldots, X_K^i)$ denote respectively intermediate and primary inputs used at firm i. Let P_i denote the output price and P_iY_i denote real final demand (or value added) from plant i, where $Y_i = Q_i - \sum_{j=1}^N M_j^i$ for good i. Aggregate final demand is given by $\sum_i P_iY_i$.

From Solow (1957), total differentiation of lnQ_i and optimization together yield

$$dlnQ_i = \sum_{j=1}^{N} \beta_j^i dlnM_j^i + \sum_{k=1}^{K} \beta_k^i dlnX_k^i + dln\omega_i \quad = \sum_{j=1}^{N} s_{m_j^i} dlnM_j^i + \sum_{k=1}^{K} s_{w_k^i} dlnX_k^i + dln\omega_i \quad i = 1, \ldots, N, \tag{6}$$

where $\beta_j^i = s_{m_j^i} = \frac{P_j M_j^i}{P_i Q_i}$ and $\beta_k^i = s_{w_k} = \frac{W_k X_k^i}{P_i Q_i}$, the elasticities of output with respect to the intermediate and primary inputs, which equal the revenue shares under perfect competition. Plant-level technical efficiency is

$$dln\omega_i = \frac{\partial F^i / \partial t}{F^i},$$

the well-known Solow residual. Lemma 1 shows that aggregate productivity growth $d\Omega$ is

$$d\Omega = \sum_{i=1}^{N} \frac{P_i Q_i}{\sum_i P_i Y_i} dln\omega_i. \tag{7}$$

Hulten (1978) proves (7) for sector-level production functions and Lemma 1 restates his argument when i indexes plants instead of sectors.[8]

Lemma 1

Let $(Y = Y_1, \ldots, Y_N)$, $(M = M_1, \ldots, M_N)$, and $(P = 1, \ldots, P_N)$ denote vectors of real final demand, intermediate input, and normalized output prices for the N plants in the economy. Let $(X = X_1, \ldots, X_K)$ and $(W = W_1, \ldots, W_K)$ denote vectors of primary inputs and their factor prices. Define the social production possibilities frontier implicitly using $F(Y, X, t) = 0$. Assume 1) $F(\cdot)$ is continuously differentiable and homogeneous of degree one in (Y, X), 2) that the economy is in a competitive equilibrium, and 3) the technology of each firm is characterized by constant returns to scale $Q_i = F^i(M^i, X^i, t)$ $i = 1, \ldots, N$, with $(M^i = M_1^i, \ldots, M_N^i)$ and $(X^i = X_1^i, \ldots, X_K^i)$. Then the instantaneous shift in the social production possibilities frontier is the difference between the Divisia index of aggregate final demand and the Divisia index of total primary inputs:

$$d\Omega = \sum_{i=1}^{N} \frac{P_i Y_i}{\sum_{i=1}^{N} P_i Y_i} \frac{dY_i}{Y_i} - \sum_{k=1}^{K} \frac{W_k X_k}{\sum_{k=1}^{K} W_k X_k} \frac{dX_k}{X_k}, \tag{8}$$

and (8) is equal (7).

The proof follows Hulten directly and is provided in the Appendix. Alternatively,

$$d\Omega = \sum_{i=1}^{N} s_Y^i dlnY_i - \sum_{k=1}^{K} s_{X_k}^i dlnX_k, \tag{9}$$

where $s_Y^i = \frac{P_i Y_i}{\sum_{i=1}^{N} P_i Y_i}$ and $s_{X_k}^i = \frac{W_k X_k}{\sum_{k=1}^{K} W_k X_k}$. The growth rate in aggregate final demand is given by the the sum across plants of their value-added share times their growth rate in final demand. To hold primary input use constant, aggregate productivity growth deducts the aggregate growth rate of primary inputs, which is given by the sum across plants of their primary input share times their growth rate in primary inputs. Identically, (7) shows this measure multiplies i's price times the additional quantity available from increased productivity at i and then sums these gains across plants (rescaling by aggregate value added). Lemma 1 shows (7), (8), and (9) are all equal.

Domar (1961) first proposed these aggregation weights. In his words, they are defined in order to allow one to

> be free to take the economy apart, to aggregate one industry with another, to integrate final products with their inputs, and to reassemble the economy once more and possibly over different time units without affecting the magnitude of the Residual.

This greatly facilitates policy analysis and the comparability of results across studies. For example, subsets of plants in the economy - like industries - can be aggregated with shares normalized to

[8] Because of the way our plant-level data are reported, we treat plants as firms, although there are probably multi-plant firms in the sample.

industry output, and the transformation to the aggregate economy is immediate via the scale factor given by the ratio of industry output to aggregate value added.

(7) must account for the use of intermediate inputs in production. Mathematically, this means that the plant-level efficiency shocks must be grossed up by the ratio of plant gross output to aggregate value added. The story is as follows. When i's output is part intermediate input, some of plant i's output is used as input at other plants, so $dln\omega_i > 0$ leads both to an increase in final demand for i and to an increase in i's intermediate deliveries. These new intermediate input deliveries are used in production at the plants to which they are delivered, where they increase output. This new output will both increase final demand and fulfill more intermediate deliveries elsewhere. The process continues. In the end, the greater the role of intermediate inputs used in the economy, the larger $\frac{\sum_{i=1}^{N} P_i Q_i}{\sum_i P_i Y_i}$ is, and the larger the impact of increases in plant-level technical efficiency on final demand.

If the gross-output production function is functionally separable in intermediate and primary inputs, Bruno (1978) shows that a value-added production technology exists. In this case, (7) can be directly expressed in terms of the share of firm value added, denoted $s_{v_i} = \frac{P_i Y_i}{\sum_i P_i Y_i}$, and the residual from the "value added" production technology. Let $S_{M^i} = \frac{\sum_{j=1}^{N} P_j M_j^i}{P_i Q_i}$ be the revenue share of intermediate inputs.

Lemma 2

If $Q_i = F^i(M^i, X^i, t)$ $i = 1, \ldots, N$, *is functionally separable in* M^i *and* X^i,

$$\sum_{i=1}^{N} s_{v_i} dln\omega_i^V = d\Omega, \tag{10}$$

where $dln\omega_i^V = \frac{dln\omega_i}{(1 - S_M^i)}$.

See Appendix for proof.

3. Estimating Productivity Growth

There are two steps to estimating aggregate productivity growth. The plant-level productivity residuals must be estimated. Then they must be aggregated. We discuss each in turn.

Two prominent and complementary methods are frequently used to estimate plant-level technical efficiency. One method begins with an assumed functional form for the production function and then directly estimates its parameters using monotonic transformations of input levels.[9] The second approach uses the Solow (1957) insight that optimizing behavior implies observed revenue shares are a consistent estimator for the elasticity of output with respect to any input.

The aggregate growth measure is defined in terms of instantaneous changes. The value added formulation (for example) is given as

$$d\Omega = \sum_i s_{v_{it}} dln\omega_{it}^V.$$

One representation of aggregate productivity growth from period $t-1$ to period t is then given by the integral

$$\Omega_{[t-1,t]} = \int_{t-1}^{t} \sum_i s_{v_{it}} dln\omega_{it}^V. \tag{11}$$

Data is observed discretely, requiring some discrete time approximation to the integral in (11). For productivity measurement the most popular discrete-time approximations are probably given by log-change indexes.[10] In this case, $ln\omega_{it}^V - ln\omega_{i,t-1}^V$ is used as an approximation to the growth rate in value added productivity over the time period. In the calculation of (11) a weight α_i is applied to the estimated average growth rate at each firm, and then aggregated across plants, yielding the approximation:

$$\sum_i \alpha_i (ln\omega_{it}^V - ln\omega_{i,t-1}^V).$$

For α_i Tornquist (1936) advocates averaging beginning and ending period shares. In the context of the value added residual $dln\omega_i^V$, the Tornquist weight is given by

$$\alpha_i = \frac{s_{v_{it}} + s_{v_{i,t-1}}}{2}.$$

In the context of $dln\omega_i$, the residual from the full production function,

$$\alpha_i = 1/2 * \left[\frac{P_{it} Q_{it}}{\sum_i P_{it} Y_{it}} + \frac{P_{i,t-1} Q_{i,t-1}}{\sum_i P_{i,t-1} Y_{i,t-1}} \right].$$

Theil (1967), Hulten (1973), Diewert (1976), Star and Hall (1976), Trivedi (1981), and Balk (2005) (among others) have all argued for the Tornquist approximation.

[9] One can use ordinary least squares or one of many alternatives that attempt to address the simultaneity of input choices and productivity raised by Marschak and Andrews (1944). Alternatives include (for example) instrumental variables, fixed effects, and the proxy methods of Olley and Pakes (1996) and Levinsohn and Petrin (2003).

[10] Sato (1987) in New Palgrave describes the properties of these approximations.

Lemma 3

The Tornquist approximation to the Divisia index is superlative (or without error) if the underlying function has the homogeneous translog form. With constant returns to scale the Tornquist approximation to the Divisia index is uniquely superlative.

See Diewert (1976), who uses the quadratic approximation lemma to single out the Tornquist approximation as most preferred. When the underlying function is not translog, the Tornquist approximation remains attractive because the translog form provides a second-order Taylor series approximation.

Trivedi develops approximation results for a more general class of functions.

Lemma 4

The error in the Tornquist approximation to the Divisia index is on the order of the square of the length of the time interval.

See Trivedi (1981). Note that Paasche and/or Laspeyres log change approximations use $\alpha_i = s_{v_{it}}$, the ending period share (Paasche), or $\alpha_i = s_{v_{i,t-1}}$, the beginning period share (Laspeyres). They have larger magnitudes of approximation error because they both ignore one of the two available pieces of information on the path from $t-1$ to t of the share-weight (or price). Table 4 reports estimated growth using log-changes with average share (Tornquist) and base-period share (Laspeyres), and shows they are quite different in our annual plant-level data. This can only be true if shares are changing in a substantial way. Lemmas 3 and 4 and our findings are the reasons we prefer the Tornquist approximation to the Divisia index.[11]

4. Accounting for Entry and Exit

One important issue that does arise with the move from industry- to plant-level data is that industries do not enter and exit, while plants do. An attractive feature of the aggregate productivity growth measure from Lemma 1 is that conceptually neither entry nor exit raises any difficulties. Let $1_{i,Continue}, 1_{i,Enter}$, and $1_{i,Exit}$ be variables that indicate whether firm i is either a continuing

[11] Before moving on we note that we have not addressed a more fundamental (although perhaps less well-known) question. (11) is a line integral over a path from $t-1$ to t. To be well-defined this integral must be path independent, or the index may suffer from cycling, where arbitrarily large or small values can be associated with (11). Hulten (1973) explores the necessary and sufficient conditions for (11) to satisfy path independence. Petrin (2004) discuss the implications of these conditions for using plant-level data. See also Balk (2003). Any measure of productivity growth based on aggregating over the (continuous) time-series path of plant-level productivity is likely to be a line integral, meaning that path independence will be a necessary condition for these types of indexes.

firm, an entrant, or an exiter respectively for the period $t-1$ to t. Firm i's contribution to aggregate productivity growth from period $t-1$ to t is then given as

$$1_{i,Continue} \int_{t-1}^{t} s_{v_{it}} \; dln\omega_{it}^{V} + 1_{i,Exit} \int_{t-1}^{t^*} s_{v_{it}} \; dln\omega_{it}^{V} + 1_{i,Enter} \int_{t^*}^{t} s_{v_{it}} \; dln\omega_{it}^{V} \tag{12}$$

Specifically, a continuing firm contributes $\int_{t-1}^{t} s_{v_{it}} \; dln\omega_{it}^{V}$, an exiting firm contributes $\int_{t-1}^{t^*} s_{v_{it}} \; dln\omega_{it}^{V}$, where t^* denotes the time at which the firm exits and $s_{v_{it^*}} = 0$, and an entrant contributes and $\int_{t^*}^{t} s_{v_{it}} \; dln\omega_{it}^{V}$, where t^* denotes the time of entry and again $s_{v_{it^*}} = 0$. These are all well-defined quantities.[12]

Since observations on plants are only made at discrete intervals, for plants that enter and exit the choice of discrete approximation must be revisited because we do not have enough information to calculate the Tornquist-Divisia quantity. In particular, while we know that $s_{v_{it^*}} = 0$ at the time of entry/exit and from that time backward/forward, we do not observe the exact time between $t-1$ and t at which entry/exit occurs, nor do we observe the path of i's productivity growth rate during its existence in this period. This will be a problem for any measure that is based on following the path of plant-level productivity.

Using (12) it is possible to bound the error that arises from this missing information. Let the contribution of all continuing plants to aggregate productivity growth be given by:

$$C = \sum_{i=1}^{N} 1_{i,Continue} \int_{t-1}^{t} s_{v_{it}} \; dln\omega_{it}^{V}.$$

Similarly, let exiters' contribution be

$$X = \sum_{i=1}^{N} 1_{i,Exit} \int_{t-1}^{t^*} s_{v_{it}} \; dln\omega_{it}^{V} \tag{13}$$

and entrants

$$E = \sum_{i=1}^{N} 1_{i,Enter} \int_{t^*}^{t} s_{v_{it}} \; dln\omega_{it}^{V}. \tag{14}$$

Then the true value for aggregate productivity growth over the period [t-1,t] is bounded:

$$\Omega_{[t-1,t]} \in [C - |X| - |E|, C + |X| + |E|]. \tag{15}$$

The error in aggregate productivity growth from truncating entrants and exiters will be small if $|X|$ and $|E|$ are small. $|X|$ and $|E|$ are likely to be small when these plants contribute only a small fraction to industry value added or have stagnant productivity growth.

[12] This is true as long as path independence is satisfied, as described in the previous footnote.

The fraction of value added that is attributable to these plants can be directly calculated from the data. In the two annual censuses from Chile and Columbia that we use in the empirical work continuing plants account for between 94% and 98% of industry value added. When data are observed at 5 year intervals, the fraction of value added accounted for by entrants and exiters ranges from 10% to 40%. Thus, with annual data no additional correction may be necessary, but with data collected at 5 year intervals some approximation to the contribution to growth from entrants and exits may be desirable. We turn to this question next.

Approximations

We develop the approximations using the Tornquist-Divisia measure as our estimation objective because of its attractive properties. There are two related issues for any approximation to an individual plant's contribution to $|X|$ or $|E|$: what is the appropriate estimator for 1) the share-weight and 2) the growth rate of productivity. A complete model of industry evolution may provide additional structure useful in inferring the unobserved paths of technical efficiency and value added for entrants and exiters. We suggest estimators for the share-weight and productivity path when a full model is not specified, leaving for future work the development of more encompassing models.

We first focus on the share-weight and consider the case of exit; the case of entry is symmetric. Our goal is an estimate of the average share between time $t-1$ and time t. We know for exiters that the share weight decreases (possibly monotonically) from the observed share at time $t-1$ to zero some time after $t-1$, and remains at zero until t. This means we know that the true Tornquist weight lies somewhere on the interval $[0, \frac{s_{v_{i,t-1}}}{2}]$. $\frac{s_{v_{i,t-1}}}{2}$ provides an upper bound that approaches the Tornquist weight as the time of exit approaches t, and we suggest using $\frac{s_{v_{i,t-1}}}{2}$ as the weight. The bound is also helpful because it can be used in a resampling procedure we describe momentarily that estimates the variability in the estimated values of $|X|$ or $|E|$.

Next we consider possible estimators of the average growth rate of productivity. For the exit case we want an estimate of $ln\omega_{it^*} - ln\omega_{i,t-1}$. The missing piece of information is $ln\omega_{it^*}$, the productivity level at the time of exit. We suggest forecasting $ln\omega_{it}$ using past values of $ln\omega$ and any other relevant state variables. Given an estimate $\widehat{ln\omega_{it}}$, one can use $\widehat{ln\omega_{it}} - ln\omega_{i,t-1}$ as the approximation to $ln\omega_{it^*} - ln\omega_{i,t-1}$. The approximation converges to the Tornquist index as t^* approaches t and $\widehat{ln\omega_{it}}$ converges to $ln\omega_{it}$.

Some examples may be helpful. If productivity is posited as a first-order Markov process, a regression of $\ln\omega_{it}$ on previous log-levels of productivity provides an estimate of

$$E[\ln\omega_{it} \,|\, ln\omega_{i,t-1}].$$

A second-order Markov process would include $ln\omega_{i,t-2}$ in the conditioning set. If the other state variables are co-determined with productivity, then the regression might include these state variables. For example, a first order Markov process with state variables productivity and capital k would point to

$$E[\ln \omega_{it} \mid ln\omega_{i,t-1}, k_{i,t-1}]$$

as the key equation of interest in forecasting the missing productivity level. In the case of entry, the conditional mean will use future outcomes to project the current state at the time of entry.

An important question in the construction of the estimator is what set of plants to use when estimating this process. If entrants and exiters are assumed to be similar to continuing plants, all plants can be pooled in an estimation routine. Alternatively, if the process for recent entrants and recent exiters is thought to be different, we could select on the appropriate set of plants and estimate on this subset.

Robustness analysis will be important, especially as the time interval between plant-level observations grows. Many resampling schemes are available that will illustrate the possible variability in $\widehat{|X|}$ and $\widehat{|E|}$. In the case of entry, random draws can be taken from the (conditional) distribution of $ln\omega_{it}$ and from the observed interval $[0, \frac{s_{v_{i,t-1}}}{2}]$ to generate a range of possible estimates of the contribution to aggregate productivity growth of the entering firm i in period t. Repeating this with all entrants and aggregating provides an estimate of the variability introduced into aggregate productivity from not observing the path of the entrant. The estimator is easily modified for the exiters. Other sources of sampling error, like that in the estimated equation for productivity, are easily incorporated into this resampling scheme in the usual way.

We close this section with the observation that many economic questions relate to the performance of entrants or exiters. The truncation problem does not affect the ability to track entrants/exiters in higher frequency data. Even in the case where all entrants (exiters) are truncated in their entering (exiting) year, one can compute these plants' contribution to aggregate productivity growth in the other years of their existence.[13] In fact, one principle of our preferred measure of productivity growth is that we are free to group plants in any subaggregates that we desire without affecting the measure of aggregate productivity growth.

[13] Categories for recent entrants and/or upcoming exiters can be defined separate from continuing plants, their fraction of productivity growth being then directly comparable to continuing plants.

5. Real Productivity vs. Reallocation Revisited

Much of the recent interest in using plant-level data to estimate plant- and industry-level productivity arises because plant-level data allows one to investigate "real productivity" and "reallocation" effects. The literature defines "real productivity" growth as the change in aggregate output arising because of increases in plant-level technical efficiency, holding primary inputs constant.[14] There is less agreement on the exact specifics of what is meant by "reallocation" in relation to aggregate productivity growth. Broadly speaking, most practitioners would agree that changes in aggregate productivity that occur because inputs are reshuffled among plants with differing productivities constitutes a "reallocation" effect.

We explore several different cases when aggregate productivity growth is defined as the shift in the social production possibilities frontier holding primary inputs constant.

Decomposing Aggregate Productivity Growth: Competitive Case

Lemma 1 shows that there is no "reallocation" effect in the case of the instantaneous growth rate under perfect competition. Alternatively, in the competitive case there is no decomposition for aggregate productivity growth; it is all "real" productivity growth.

Decomposing Changes in Aggregate Productivity Growth Rates: Competitive Case

While the levels of growth rates cannot be decomposed, we now show that one can decompose the *changes* in the instantaneous growth rates using a Bennet decomposition. The approximation to the change in the growth rate uses data from three juxtaposed time periods, which we denote as t, $t+1$, and $t+2$. By construction, our estimated change in the growth rate is given by the difference in two "average-share log-change" indexes (i.e. Tornquist-Divisia indexes), which using the value added formulation gives:

$$\sum_{i=1}^{N_2} \frac{s_{v_{i,t+2}} + s_{v_{i,t+1}}}{2}(ln\omega_{i,t+2}^{V} - ln\omega_{i,t+1}^{V}) - \sum_{i=1}^{N_1} \frac{s_{v_{i,t+1}} + s_{v_{it}}}{2}(ln\omega_{i,t+1}^{V} - ln\omega_{it}^{V}). \qquad (16)$$

[14] Among other reasons, plants might become more productive because they learn by doing or because they are exposed to new and better methods of production. Examples of the former range from the low-tech sewing machine operator at an apparel firm to the high-tech process of increasing yields on silicon chip production. An example of the latter is the set of manufacturing practices known as "lean production." U.S. manufacturers, initially auto producers but later others, adopted these methods after observing Japanese success (see Biesebroeck (2001)). In both the learning-by-doing and the learning-by-watching cases, firm productivity increases and with that comes increases in industry productivity. This is an uplifting explanation of the mechanism for productivity increases, as there are no obvious bounds to learning and ingenuity.

Equation (16) decomposes into a "productivity" term, a "reallocation" term, and a "net entry" term. The set of plants that exist in t, $t+1$, and $t+2$ is denoted C (for continuers), and each of these plants contributes a productivity component and a reallocation component to the growth in aggregate productivity. Plants that exist in t and $t+1$, or $t+1$ and $t+2$, contribute to the overall change in (16), but only through a net entry term.

One term is the productivity term. Aggregated across plants it is written as

$$\sum_{i \in C} \frac{s_{v_{i,t+2}} + 2 * s_{v_{i,t+1}} + s_{vit}}{4} * (ln\frac{\omega^V_{i,t+2}}{\omega^V_{i,t+1}} - ln\frac{\omega^V_{i,t+1}}{\omega^V_{it}}).$$

Each continuer contributes the change in the rate of their productivity growth (the term on the right). The weight in the aggregation to the industry level is an average over the three time periods of the plant's share in value added, where period $t+1$ gets twice the weight of period t and period $t+2$.

The second term is the reallocation term, given as

$$\sum_{i \in C}(ln\frac{\omega^V_{i,t+2}}{\omega^V_{i,t+1}} + ln\frac{\omega^V_{i,t+1}}{\omega^V_{it}})/2 * (s_{v_{i,t+2}} - s_{vit})/2.$$

The term on the right gives the change in the share of value added from period t to period $t+2$. This multiplies the average rate of productivity growth over the periods for the plant (i.e. $ln(\frac{\omega^V_{i,t+2}}{\omega^V_{it}})/2$).

The third term is the net entry term and is given as

$$\sum_{i \in t+1,t+2,not\ t} \frac{s_{v_{i,t+2}} + s_{v_{i,t+1}}}{2}(ln\omega^V_{i,t+2} - ln\omega^V_{i,t+1}) - \sum_{i \in t,t+1,not\ t+2} \frac{s_{v_{i,t+1}} + s_{vit}}{2}(ln\omega^V_{i,t+1} - ln\omega^V_{it}).$$

It gives the difference in growth rates between those plants absent in period t but present in $t+1$ and $t+2$ and those plants absent in period $t+2$ but present in t and $t+1$. Since the share-weights on either side of the difference do not sum to one, this term can be driven both by the prevalent growth rates of entrants and exiters and by their mass in the population.

The sum of the real productivity, reallocation, and net entry terms is equal to (16). Ignoring the net entry term, if there is no change in the growth rate of productivity at any plant, the productivity term is zero, and the aggregate rate of productivity growth can only increase (decrease) if there is reallocation of the share in value added from period t to period $t+2$ towards (away from) plants with higher average productivity growth rates. Similarly, if there is no change in the shares of value added from the start (t) to the end ($t+2$), the reallocation term is zero, and aggregate productivity growth can only increase if the (share-weighted) average growth rate of productivity increases. Net entry contributes a term that is positive (negative) if the share weighted sum of the growth rates of entrants exceeds (falls below) the share weighted sum of the growth rates of the exiters.

Decomposing Aggregate Productivity Growth: Imperfect Competition

With imperfect competition, Hall (1990) and Basu and Fernald (2002) show that the productivity residual measures more than just the effects of technical progress on final aggregate demand. There are four different types of effects in the residual that relate the reshuffling of inputs across plants to changes in aggregate productivity growth. Holding aggregate inputs constant, productivity increases if there is a reallocation of inputs from low markup plants to high markup plants. Similarly, holding plant-level inputs constant, if the technology shocks cause intermediate inputs to be used more intensively at plants with markups, this too leads to an increase in aggregate productivity. Finally, with the two primary inputs labor and capital, aggregate productivity growth increases if there is a reallocation of inputs from plants with lower shadow values to plants with higher shadow values for these inputs. Basu and Fernald (2002) show that even though productivity growth does not exclusively measure the effects of technical efficiency, aggregate productivity change provides a first-order approximation to the change in welfare if a representative consumer model is reasonable.[15]

Let the value-added input index be defined as

$$dX_i^V = \frac{s_{k^i}}{1 - s_{m^i}} dlnK_i + \frac{s_{l^i}}{1 - s_{m^i}} dlnL_i,$$

where s_l and s_k denote the revenue shares of the primary inputs labor and capital. Under cost minimization Basu and Fernald (2002) show that plant-level value added can be written as

$$dlnY_i = \mu_i^V dX_i^V + (\mu_i^V - 1)\frac{s_{m^i}}{1 - s_{m^i}}(dlnM^i - dlnQ_i) + \frac{dt_i}{1 - \mu_i s_{m^i}}$$

where $\mu_i^V = \mu_i \frac{1-s_{m^i}}{1-\mu_i s_{m^i}}$ is the "value added markup", an increasing function of the markup $\mu_i = P_i/MC_i$, where MC_i denotes marginal cost. The productivity residual deducts primary input growth from value added growth, and is given by

$$\begin{aligned} dln\omega_i^V &= dlnY_i - dX_i^V \\ &= (\mu_i^V - 1)dX_i^V + (\mu_i^V - 1)\frac{s_{m^i}}{1 - s_{m^i}}(dlnM^i - dlnQ_i) + \frac{dt_i}{1 - \mu_i s_{m^i}} \end{aligned}$$

This plant-level residual has three terms. The last term is the impact of technical progress on final demand under imperfect competition. It is increasing in the plant-specific markup; the higher the markup, the more valuable the output to society. The first term arises because only dX_i^V is

[15] The intuition for the result is that the ratio of market prices reflects consumers' marginal rate of substitution between goods, so changes in value added reflect (to first order) the change in consumer well-being.

deducted from $dlnY_i$ (instead of $\mu_i^V dX_i^V$). This term increases (along with productivity growth) if inputs are reshuffled to plants with higher markups. The second term arises because the productive contribution of intermediate inputs exceeds the revenue share because of the markup. Since the revenue share is used to deduct intermediates from gross output in the move to value added, productivity increases as plants that use intermediate inputs more intensively increase their output. Note that, in the case when markets are competitive, μ_i and μ_i^V both equal one, and $dln\omega_i^V = \frac{dt_i}{1-s_{m^i}}$, the effective rate of productivity growth under perfect competition. Thus the residual from the perfect competitively case is a special case of this more general framework.

Aggregate productivity growth can be written as the share-weighted sum of these newly-defined productivity residuals, plus two new terms:

$$d\Omega = \sum_{i=1}^{N} s_{v_i} dln\omega_i^V + R_L + R_K.$$

Let P_{l^i} be defined as the shadow value of labor at i and P_l the average across i. Then R_L is given as:

$$R_L = \sum_{i=1}^{N} s_{v_i} \frac{s_{l^i}}{1 - s_{m^i}} \frac{P_{l^i} - P_l}{P_{l^i}} dlnL_i$$

and R_K is similarly defined with labor replaced by capital:

$$R_K = \sum_{i=1}^{N} s_{v_i} \frac{s_{k^i}}{1 - s_{m^i}} \frac{P_{k^i} - P_k}{P_{k^i}} dlnK_i$$

The two new terms reflect differences across plants in the shadow values of the primary inputs labor and capital (from the average shadow value in the economy). As inputs shift from plants with low shadow values to plants with high shadow values, aggregate productivity increases. Again, if markets were competitive, $d\Omega$ would reduce to (2), because the shadow values would be identical across plants and exactly equal to wages and rental rates, so R_L and R_K would equal zero.

6. The Bailey-Hulten-Campbell Index and Decomposition

The value added formulation of the BHC index defines aggregate productivity growth as the difference in share-weighted log-levels of productivity.[16] BHC show that we can write (4) in terms of those plants that are present in both periods (continuers), and the plants that enter and exit, where the indicators denote these three types:

$$\begin{aligned} BHC_t = &\sum_i 1_{i,Continue} * (s_{it} ln\omega_{it}^V - s_{i,t-1} ln\omega_{i,t-1}^V) \\ &+ \sum_i 1_{i,Enter} * s_{it} ln\omega_{it}^V - \sum_i 1_{i,Exit} * s_{i,t-1} ln\omega_{i,t-1}^V. \end{aligned} \tag{17}$$

[16] The discussion of the gross output formulation would be very similar.

We describe the contribution to BHC productivity growth of continuing plants first, and then turn to the contribution of entrants and exiters.

Consider a single plant i's contribution to productivity growth:

$$s_{it} ln\omega_{i,t}^{V} - s_{i,t-1} ln\omega_{i,t-1}^{V}.$$

As we noted in the introduction, a Bennet decomposition of this difference is given as

$$\frac{s_{it} + s_{i,t-1}}{2} * \Delta ln\omega_{i}^{V} + \frac{ln\omega_{it}^{V} + ln\omega_{i,t-1}^{V}}{2} * \Delta s_{i}.$$

The first term multiplies the average share times the log-change in plant-level technical efficiency. If the share of aggregate value added is used, this term is an approximation to i's contribution to the shift in the PPF holding primary inputs constant (Lemmas 1 and 2). The second term - the "reallocation" term - has yet to be linked to an economic model, and, as noted in the introduction, can yield positive productivity growth when welfare falls. Aggregating these plant-level changes gives the contribution of continuers to BHC productivity growth, and is given by the top line in (17).

The decomposition of productivity growth that BHC use is slightly different from the decomposition given above. In one form BHC decompose their growth measure for a continuer in the following manner:

$$s_{i,t-1} \Delta ln\omega_{i}^{V} + ln\omega_{it}^{V} \Delta s_{i}. \tag{18}$$

We discuss each component in turn.

The first term is the "real productivity" term. If the share of value added is used, Lemmas 1 and 2 show that this term provides a log-change approximation to the shift in the social PPF holding primary inputs constant. However, instead of using the average share, this approximation uses the base period share (Laspeyres), so Lemmas 3 and 4 do not apply.

The second term measures the "reallocation" effect.[17] It multiplies the change in the share with the log-level of productivity in the current period, and thus somewhat resembles a Paasche-type index, although it is not a log-change index.

Before turning to entrants and exiters we comment briefly on the choice of the share weight. As Lemmas 1 and 2 show, if the desired measurement is the shift in the social PPF holding primary inputs constant, then the choice of the share-weight is entirely determined by which productivity

[17] Sometimes this term is further divided into two terms.

residual - gross output or value-added - is used in the calculation. If the value-added residual is employed, then the share of value-added is the correct weight. If i's gross output residual is used, then i's revenue divided by aggregate value-added is the correct weight. In neither of these cases is the labor share or the gross output share the correct weight, although these shares are almost exclusively used in the literature.

Assuming the correct share is used, the "real productivity" term can be thought of as approximating the shift in the social PPF using the base-share approximation. This gives zero weight to the share in current period t. Lemmas 3 and 4 show that the average share is generally preferred to the base-period share, especially if the shares change substantially over time. In results we present later (Table 4) we show that even in the case when the correct share-weight is used, ignoring one of the two pieces of observable information on the path of the share - by using only the base-period share - results in an the index value that differs substantially from the average-share approximation, which we interpret as approximation error.

We now turn to entrants and exiters. The aggregate contribution to productivity growth from entrants and exiters for BHC is given by

$$\sum_i 1_{it,Enter} * s_{it} ln\omega_{it} - \sum_i 1_{i,t-1,Exit} * s_{i,t-1} ln\omega_{i,t-1}.$$

Thus, the contribution of an exiter to productivity growth in the BHC index is given by $s_{i,t-1} ln\omega^V_{i,t-1}$, its share at $t-1$ times its log-level of productivity. Similarly, the contribution of an entrant is given by $s_{it} ln\omega^V_{it}$. As noted in Section 4, the shift in the social PPF from an exiting firm is $\int_{t-1}^{t^*} s_{v_{it}}\ dln\omega^V_{it}$, and an entrant contributes and $\int_{t^*}^{t} s_{v_{it}}\ dln\omega^V_{it}$, where t^* denotes the time of exit or entry, and $s_{v_{it^*}} = 0$. It remains to be shown that the approximation error is small if the BHC index for entrants and exiters is intended to measure the shift in the social PPF holding primary inputs constant for these firms.

7. Data and Estimation

We turn to two manufacturing censuses to explore the empirical issues that we raise. One census is from Chile's Instituto Nacional de Estadistica (INE), and the second is from Columbia's Departamento Administrativo Nacional de Estadistica (DANE.) The Chilean data span the period 1987 through 1996 and the Columbian data span the years 1981-1991. We focus on 3-digit industries with more than 200 observations, of which there are 23 in Chile and 26 in Columbia. Here, we provide a brief overview of these data. They have been used in numerous other productivity studies, and we refer the interested reader to those papers for a more detailed data description.[18]

The data are unbalanced panels and cover all manufacturing plants with at least ten employees. Plants are observed annually and they include a measure of output, two types of labor, capital, and intermediate inputs. Real value-added is nominal value added adjusted by the 3-digit industry price index. Labor is the number of man-years hired for production, and plants distinguish between their blue- and white- collar workers (we include two labor types in the production function). The method for constructing the real value of capital is documented in Lui (1991) for the Chilean data, and a similar approach is adopted for the Columbian data.[19] A data problem for the Chilean census is that approximately 3% of the plant-year observations appear to be "missing"; a plant id number is present in year $t-1$, absent in year t, and then present again in year $t+1$. We impute the values for these observations using $t-1$ and $t+1$ information (see the Appendix). Due to the way that the data are reported, we treat plants as firms, although there are probably multi-plant firms in the sample.

We estimate value added production function parameters for each of the 3-digit industries and use the parameters to estimate the plant-level TFP residuals. For any industry, the production function coefficients are assumed to be constant over time and across plants, although our findings are robust to loosening this assumption. We employ three different approaches to estimating the coefficients: ordinary least squares, revenue shares, and the proxy method from Levinsohn and Petrin (2003), which includes controls to address the correlation of productivity with input choices. For the revenue shares, for each industry we use the average over plants and time of the shares. Details on the Levinsohn and Petrin (2003) estimator are relegated to the Appendix.

8. Results

We compare the shift in the social PPF from (3) with the popular BHC index given in (4) using 49 3-digit manufacturing industries from Chile and Colombia. In an attempt to keep the analysis manageable, we start with a detailed description of results for the largest Chilean manufacturing

[18] See Lui (1991), Lui (1993), Lui and Tybout (1996), Tybout, de Melo, and Corbo (1991), Pavcnik (2002), Levinsohn (1999), and most recently Levinsohn and Petrin (2003).

[19] It is a weighted average of the peso value of depreciated buildings, machinery, and vehicles, each of which is assumed to have a depreciation rate of 5%, 10%, and 20% respectively. No initial capital stock is reported for some plants, although investment is recorded. When possible, we used a capital series that was reported for a subsequent base year. For a small number of plants, capital stock is not reported in any year. We estimated a projected initial capital stock based on other reported plant observables for these plants. We then used the investment data to fill out the capital stock data.

industries. We then describe how these findings generalize. The main result is that the micro patterns observed in the largest industries in Chile are indicative of the findings for the entire 49 3-digit industries from both countries.

Table 1 reports the annual estimates of growth rates in productivity for the two measures for ISIC 311, the Food Products industry (the largest in Chile). The production function coefficients are estimated using ordinary least squares, and the calculations are done using only plants that exist in period $t-1$ and period t, which account for 94.4% of plant-year observations and 96.4% of industry value added over the sample period. Column 1 is industry value added for 1988 to 1996, column 2 is the social PPF measure of productivity, column 3 is the BHC productivity measure, and column 4 is the difference between these two terms, which is equal to the BHC "reallocation" term described earlier.

The PPF measure averages 3.64% per annum, with standard deviation of 4.72% across the nine years. On average it accounts for slightly less than one-half of the growth rate in value added, which is consistent with the findings reported in Basu and Fernald (2002).[20] The BHC index averages – 2.93% per annum with a standard deviation of 13.48%. The divergence in these summary statistics arises because the two indexes themselves are widely divergent, as is evident from a comparison of columns 2 and 3. Column 4 is the difference and has a mean of -6.57%, with a standard deviation of 10.74%. Its volatility across the sample period is consistent with the general findings in the literature that "reallocation" can be large and volatile.

For ISIC 311, table 2 compares estimates of productivity growth across different estimators for production function parameters. The top half of the table is the social PPF measure and the bottom half is the BHC index. For the top half, column 2 is the same as column 2 from table 1, which uses ordinary least squares to obtain production function estimates. Column 3 uses the proxy approach from Levinsohn and Petrin (2003), and column 4 uses revenue shares. While the production function estimates (not reported here) do differ somewhat across the three approaches, the productivity growth numbers using the social PPF index are reasonably similar across the three estimators. Only in 1995 does there seem to be substantial divergence between the revenue share estimate and the two alternatives.

For the BHC index, the signs tend to be common across the three sets of production function estimates, but the magnitudes are quite different, with the LP and OLS estimates systematically the largest in absolute value terms. The main reason for the volatility is reflected in the "reallocation" terms, which are systematically more volatile for LP and OLS relative to revenue shares.

[20] Their approach differs principally in their use of U.S. data that has been aggregated to the industry level.

Table 3 summarizes the relationship between the social PPF index and the BHC index for the eight largest industries in Chile. The industries (along with their ISIC codes) are Food Products (311), Metals (381), Textiles (321), Wood Products (331), Apparel (322), Plastics (356), Non-electric Machinery (382), and Other Chemicals (352). The index numbers use only plants that exist in $t-1$ and t, which account for between 94% and 98% of industry value added (in this annual data, truncation due to entry and exit is not a severe problem). For each industry and each estimator of production function coefficients, the appropriate row reports the intercept, slope, and r-squared from a regression of the 9 annual growth rates using the BHC index as the independent variable and the social PPF index as the dependent variable. For example, the first row for ISIC 311 is the regression of column 2 on column 3 from table 1. An r-squared of 1 and an intercept of 0 and slope of 1 would indicate that the measures are identical. An r-squared of 1 would indicate that the BHC measure is a linear transformation of the Social PPF Index.

For industry 311, the r-squared's range from 0.22 to 0.47. The intercept is significantly different from zero and the slope is significantly different from one.[21] Overall, these results suggest that the BHC index is a poor proxy for the social PPF measure for ISIC 311. The slope terms range from 0.12 to 0.31, so the BHC index differs from the social PPF index by a factor of 8 using the proxy approach and 3 using revenue shares.

These results are similar across all of these eight industries. R-squareds are typically low, ranging between 0.2 and 0.4. Intercept and slope coefficients are significantly different from zero and one respectively. The point estimates for the slope coefficients suggests that the BHC index differs from the social PPF index by several magnitudes.

The messages that come out of the results from tables 1-3 are confirmed by the other 15 manufacturing industries in Chile and the 26 manufacturing industries from Colombia. Overall, for Chile, using OLS only 4 of 23 industries had r-squareds above 0.5, using revenue shares only 5 of 23 had r-squareds above 0.5, and using the proxy approach only 1 in 23 industries had an r-squared over 0.5. Most slope coefficients across estimators and industries varied between 0.1 and 0.3.[22] For Colombia, using OLS only 4 of 26 industries had r-squareds above 0.5, using revenue shares 8 of 26 had r-squareds above 0.5, and using the proxy approach only 5 of 26 industries had an r-squared over 0.5. Slope coefficients across estimators and industries also varied between 0.1

[21] These results do not correct for error in the parameter estimates. For this industry, with almost 10,000 observations, the parameter estimates are very precisely estimated.

[22] The industries that had higher r-squareds were also the industries where the slope coefficients tended to be larger.

and 0.3, suggesting the BHC index differs by several magnitudes from the shift in the social PPF. In summary, the results demonstrate that the BHC index adds a "reallocation" term to the social PPF index that is large and volatile, making it a very noisy indicator of the social PPF index. This raises questions about the large discussion in the literature that revolves around BHC-type indexes of productivity growth.

Before turning to the decomposition that we propose, we ask whether it matters empirically if we use the base-period share (Laspeyres) or the average-share (Tornquist) when we approximate the shift in the social PPF holding primary inputs constant: $\int_{t-1}^{t} \sum_i s_{v_{it}} dln\omega_{it}^V$. As we noted earlier, if the shares are changing over time at the plant-level, then using the base-period share ignores an important piece of observed information on the path of the share (the end-period share).

Table 4 compares estimated growth rates for ISIC 311. They differ substantially, as the base-period share yields an average annual rate of growth of -9.73%, while the average-share approach yields an annual growth rate of 3.64%. In particular, it seems clear from the table that the shares do vary quite a bit in a manner that is important for the approximation to the integral given in (3). The fact that the attractive results from Lemmas 3 and 4 are not available for the base-period (Laspeyres) index, and that it differs from the average-share index so dramatically suggest that researchers should regularly report the average-share approximation as one of their results. Despite its uniquely attractive features, the average share approximation is almost never reported, making it hard to compare results across studies on related questions.

Reallocation vs. Real Productivity

We now turn to the decomposition of productivity growth. As we showed earlier, there is no "reallocation" component in the instantaneous change in productivity. However, in Section 5 we showed how to decompose the change in the growth rate of productivity into a term that is increasing if growth rates at the firm level are increasing (the "real productivity" term), and a term that is increasing when plants with higher average growth rates gain larger shares of value added (the "reallocation" term).

Table 5 provides this decomposition for ISIC 311 in Chile. The change in the growth rate from year to year is reported in column 2. Note that it does not exactly equal change in the growth rate reported in column 1; the difference is due to the entry/exit plants, (those plants that do not exist in $t-2$, $t-1$, and t), and it is equal to the change in the growth rate that is attributable to entry/exit. As defined in Section 5, for those plants that exist in the three periods, the change in column 2 can be decomposed into the "real productivity" term, column 3, and the "reallocation" term, column 4.[23] The real productivity component is quite volatile, averaging −5.95% with a standard deviation of 7.33%, with both positive and negative outcomes. The reallocation term is positive, very stable, and always contributes to increases in the rate of productivity growth, even in periods when the overall growth rate falls.

Overall, the reallocation series for ISIC 311 in Chile is remarkably similar in spirit to the reallocation series from the other 48 manufacturing industries. Regardless of the estimator for production function coefficients, the industry, or the country, the annual reallocation terms are almost universally positive, and within an industry over time they vary very little, with a typical standard deviation less than 1.[24] Thus, in contrast to current findings that reallocation effects vary in sign and magnitude across time within industries, our measure - which is based on a clearly defined economic model - suggests reallocation effects are very stable and almost always contribute positively to industry growth, even when the overall growth rate is falling.

9. Conclusions

Proper measurement of productivity growth is important for many economic questions across a broad range of fields. Using Hulten (1978) we show when markets are competitive the shift in the social production possibility frontier (PPF) holding primary inputs constant is exactly equal to the change in welfare that arises as society's ability to consume and invest increases in response to the plant-level technology shocks. Even when markets are not perfectly competitive, this measurement provides a first-order approximation to welfare change if a representative consumer model is appropriate for demand. Although it is rarely reported, we adopt this definition because it provides practitioners with a measurement useful for cost-benefit/policy analysis, and is also readily comparable across time, industries, countries, and empirical studies.

We show that the most popular index for measuring productivity growth with plant-level data deviates from our preferred measure along two major dimensions: it adds a "reallocation" term to the social PPF index, and it fails to use the correct weights in the aggregation. Empirically, even when the correct weights are used, the "reallocation" term is substantial, leading to a weak relationship between the popular measure and the social PPF measure for almost every manufacturing industry in both Chile from 1987-1996 and Columbia from 1981-1991. These findings are robust to

[23] It is possible to use the techniques suggested in the discussion on entry and exit to approximate the left out entry and exit terms. We leave this for later work.

[24] They do vary across industries, with most industries falling in between 3% to 8%.

many different estimation approaches for plant-level productivity, and they call into question the literature's interpretation of the "reallocation" term as productivity growth.

We provide a new method for separating real productivity growth from reallocation effects that is entirely based on decomposing changes in the social PPF index. In contrast to current findings that reallocation effects vary in sign and magnitude across time and sector, our new measure suggests that reallocation effects are reasonably stable within industries and almost always positively impact the productivity growth rate, even in instances where aggregate productivity falls.

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Appendix A

Proof of Lemma 1

The proof mimics Hulten. $F(\cdot)$ is differentiable and homogeneous of degree zero in (Y, X), which implies

$$\sum_{i=1}^{N} \frac{\partial F}{\partial Y_i} Y_i = -\sum_{k=1}^{K} \frac{\partial F}{\partial X_k} X_k. \tag{19}$$

By definition $F(Y, X, t) = 0$, so total differentiation of $F(\cdot)$ yields

$$-dF = \sum_{i=1}^{N} \frac{\partial F}{\partial Y_i} dY_i + \sum_{k=1}^{K} \frac{\partial F}{\partial X_k} dX_k.$$

where dF, dY, and dX denote the instantaneous change with respect to time. Divide through by the left and right hand side of (19) and substitute in the competitive equilibrium conditions $-\frac{\partial F/\partial Y_i}{\partial F/\partial Y_1} = P_i,\ i = 2, \ldots, N$ and $\frac{\partial F/\partial X_k}{\partial F/\partial Y_1} = W_k,\ k = 1, \ldots, K$, Then, the shift in the social PPF holding primary inputs constant is given as the rate of change of $F(\cdot)$ divided by $\sum_{i=1}^{N} \frac{\partial F}{\partial Y_i} Y_i$:

$$\frac{-dF}{\sum_{i=1}^{N} \frac{\partial F}{\partial Y_i} Y_i} = \sum_{i=1}^{N} \frac{P_i Y_i}{\sum_i P_i Y_i} \frac{dY_i}{Y_i} - \sum_{k=1}^{K} \frac{W_k X_k}{\sum_k W_k X_k} \frac{dX_k}{X_k}.$$

This establishes the first claim.

The second claim of the Lemma asserts that this quantity is equal to $\sum_{i=1}^{N} \frac{P_i Q_i}{\sum_i P_i Y_i} dlnF^i$, where $Q_i = F^i(M^i, X^i, t)$ and $dlnF^i = dln\omega_i$ from (7) (we write it in this way to be consistent with the notation here). To show this, note that in equilibrium supply and demand in the product and factor markets are equated:

$$Q_i = Y_i + \sum_{j=1}^{N} M_i^j \quad i = 1, \ldots, N$$

and

$$X_k = \sum_{j=1}^{J} X_k^j \quad k = 1, \ldots, K.$$

Total differentiation of these equations gives

$$\frac{dQ_i}{Q_i} = \frac{dY_i}{Q_i} + \frac{\sum_{j=1}^{N} dM_i^j}{Q_i} = \frac{P_i Y_i}{P_i Q_i} \frac{dY_i}{Y_i} + \sum_{j=1}^{N} \frac{P_i M_i^j}{P_i Q_i} \frac{dM_i^j}{M_i^j} \quad i = 1, \ldots, N \tag{20}$$

and

$$\frac{dX_k}{X_k} = \sum_{i=1}^{N} \frac{W_k X_k^i}{W_k X_k} \frac{dX_k^i}{X_k^i} \quad k = 1, \ldots, K. \tag{21}$$

Necessary conditions for the equilibrium are: $\frac{\partial Q_i}{\partial M_j^i} = \frac{P_j}{P_i}$ and $\frac{\partial Q_i}{\partial X_k^i} = \frac{W_k}{P_i}$, $i, j = 1, \ldots, N$ and $k = 1, \ldots, K$. Substituting these equalities into the logarithmic derivative of $Q_i = F^i(M^i, X^i, t)$ and dividing through by Q_i yields

$$\frac{dQ_i}{Q_i} = \sum_{j=1}^{N} \frac{P_j M_j^i}{P_i Q_i} \frac{dM_j^i}{M_j^i} + \sum_{k=1}^{K} \frac{W_k X_k^i}{P_i Q_i} \frac{dX_k^i}{X_k^i} + dlnF^i \quad i = 1, \ldots, N, \tag{22}$$

where $dlnF^i$ is the well-known Solow residual. Solving (20) for $\frac{P_i Y_i}{P_i Q_i} \frac{dY_i}{Y_i}$ and plugging in (22) yields

$$\frac{P_i Y_i}{P_i Q_i} \frac{dY_i}{Y_i} = \sum_{j=1}^{N} \frac{P_j M_j^i}{P_i Q_i} \frac{dM_j^i}{M_j^i} + \sum_{k=1}^{K} \frac{W_k X_k^i}{P_i Q_i} \frac{dX_k^i}{X_k^i} + dlnF^i - \sum_{j=1}^{N} \frac{P_i M_i^j}{P_i Q_i} \frac{dM_i^j}{M_i^j} \quad i = 1, \ldots, N \tag{23}$$

Multiplying (23) through by $\frac{P_i Q_i}{\sum_{i=1}^{N} P_i Y_i}$ and noting from the accounting identity $\sum_{i=1}^{N} P_i Y_i = \sum_{k=1}^{K} W_k X_k$ we have

$$\frac{P_i Y_i}{\sum_i P_i Y_i} \frac{dY_i}{Y_i} = \sum_{j=1}^{N} \frac{P_j M_j^i}{\sum_i P_i Y_i} \frac{dM_j^i}{M_j^i} + \sum_{k=1}^{K} \frac{W_k X_k^i}{\sum_k W_k X_k} \frac{dX_k^i}{X_k^i} + \frac{P_i Q_i}{\sum_i P_i Y_i} dlnF^i - \sum_{j=1}^{N} \frac{P_i M_i^j}{\sum_i P_i Y_i} \frac{dM_i^j}{M_i^j}. \tag{24}$$

Aggregating across i gives

$$\sum_{i=1}^{N} \frac{P_i Y_i}{\sum_i P_i Y_i} \frac{dY_i}{Y_i} = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{P_j M_j^i}{\sum_i P_i Y_i} \frac{dM_j^i}{M_j^i} + \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{W_k X_k^i}{\sum_k W_k X_k} \frac{dX_k^i}{X_k^i} + \sum_{i=1}^{N} \frac{P_i Q_i}{\sum_i P_i Y_i} dlnF^i - \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{P_i M_i^j}{\sum_i P_i Y_i} \frac{dM_i^j}{M_i^j}. \tag{25}$$

The intermediate inputs cancel out because

$$\sum_{i=1}^{N} \sum_{j=1}^{N} P_j dM_j^i = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i dM_i^j.$$

Rearranging (25) then yields

$$\sum_{i=1}^{N} \frac{P_i Q_i}{\sum_i P_i Y_i} dlnF^i = \sum_{i=1}^{N} \frac{P_i Y_i}{\sum_i P_i Y_i} \frac{dY_i}{Y_i} - \sum_{k=1}^{K} \frac{W_k X_k}{\sum_k W_k X_k} \frac{dX_k}{X_k}$$

This establishes the second claim. #

Proof of Lemma 2

Let $S_M^i = \frac{\sum_{j=1}^{N} P_j M_j^i}{P_i Q_i}$, the payments to intermediates divided by gross output. If, in addition to the assumptions from Lemma 1, $Q_i = F^i(M^i, X^i, t)$ $i = 1, \ldots, N$, is functionally separable in M^i, then it is well-known that the residual from the value added production function is given by $dlnF^{iV} \equiv \frac{dlnF^i}{(1-S_M^i)}$; the change in plant-level efficiency $dlnF^i$ is grossed up by $\frac{1}{(1-S_M^i)}$ to account for the role of intermediates on final demand.[25] Because $P_i Y_i = P_i Q_i - \sum_j P_j M_j^i$, it follows directly that

$$\sum_{i=1}^{N} s_{v_i} dlnF^{iV} = \sum_{i=1}^{N} \frac{P_i Y_i}{\sum_{i=1}^{N} P_i Y_i} \frac{dlnF^i}{(1-S_M^i)} = \sum_{i=1}^{N} \frac{P_i Y_i}{\sum_{i=1}^{N} P_i Y_i} \frac{dlnF^i}{\frac{(P_i Q_i - \sum_j P_j M_j^i)}{P_i Q_i}} = \sum_{i=1}^{N} \frac{P_i Q_i}{\sum_i P_i Y_i} dlnF^i.$$

#

Appendix B

Estimation of the Production Function: The Proxy Approach

Our estimator that proxies for correlation between productivity and inputs choices follows Levinsohn and Petrin (2003). We start with a production function given by

$$y_t = \beta_s l_t^s + \beta_u l_t^u + \beta_k k_t + \omega_t + \eta_t, \tag{26}$$

with inputs skilled labor, unskilled labor, capital, a Hicks-neutral productivity shock ω_t, and an i.i.d. error η_t. We assume the intermediate input m_t is a strictly increasing function of ω_t. That is:

$$m_t = m_t(\omega_t, k_t), \tag{27}$$

and we then invert (27) and express the unobservable productivity as a function of the intermediate input and capital, or

$$\omega_t = h_t(m_t, k_t). \tag{28}$$

This inversion plays a very important role, since it permits us to control for ω_t. To see how this is done, substitute (28) into (26) to obtain:

$$y_t = \beta_s l_t^s + \beta_u l_t^u + \phi_t(m_t, k_t) + \eta_t, \tag{29}$$

[25] See, for example, Basu and Fernald (1995).

where,

$$\phi_t(m_t, k_t) = \beta_0 + \beta_k k_t + h_t(m_t, k_t). \tag{30}$$

(29) is partially linear; it is linear in skilled and unskilled labor, and non-linear in the intermediate input and capital. We use data on electricity usage for the intermediate input m_t.[26] We proceed by regressing y_t on l_t^u, l_t^s, and a third order polynomial in electricity (m_t) and k_t, i.e. we use a polynomial series to approximate the function $\phi_t(m_t, k_t)$.[27] Thus the first stage is as simple as OLS, and it yields estimates of β_l^u and β_s^u which are not contaminated by labor's responsiveness to the current period's productivity term; including $\phi_t(\cdot)$ controls for the correlation between labor and the error term.

We now describe how β_k is identified. From (30), capital appears twice in the equation and thus β_k is not identified without some further restriction. Next period's output is written as

$$y_{t+1} = \beta_0 + \beta_s l_{t+1}^s + \beta_u l_{t+1}^u + \beta_k k_{t+1} + \omega_{t+1} + \eta_{t+1}. \tag{31}$$

Define the function $g(\omega_t)$ as

$$g(\omega_t) = \beta_0 + E[\omega_{t+1}|\omega_t].$$

The function $g(\omega_t)$ gives, up to an additive constant, the expectation of next period's productivity, ω_{t+1}, conditional on this period's productivity shock. We can rewrite $\omega_{t+1} = E[\omega_{t+1}|\omega_t] + \xi_{t+1}$, where ξ_{t+1} is the innovation in productivity. The important identification assumption for capital is that k_{t+1} does not respond to this innovation (although it can freely covary $E[\omega_{t+1}|\omega_t]$). In practice, we estimate $g(\omega_t)$ non-parametrically, substituting it into (31) to provide the population moment

$$\begin{aligned} &E[y_{t+1} - \beta_s l_{t+1}^s - \beta_u l_{t+1}^u - \beta_k k_{t+1} - g(\omega_t)|k_{t+1}] = \\ &E[\xi_{t+1} + \eta_{t+1}|k_{t+1}] = 0. \end{aligned} \tag{32}$$

This moment identifies β_k.

It is perhaps helpful to note in less technical terms what this moment condition represents. The expectation of output less inputs equals the error, or productivity plus another additive independent error. This error cannot be used as the basis for a moment condition that will identify β_k, since productivity is not orthogonal to capital. We can solve for an error term, $(\xi_{t+1} + \eta_{t+1})$, that

[26] See Levinsohn and Petrin (2003) and Levinsohn and Petrin (1999), where we show that results for production function estimates are robust across other intermediate inputs, including fuels and materials.

[27] In this and all future polynomial series approximations we experimented with a fourth order expansion and found that it had a negligible effect on our final parameter estimates.

is uncorrelated with capital by conditioning out the expectation of ω_{t+1}. It is the inclusion of the function $g(\omega_t)$ which controls for this expectation and allows for identification of the capital coefficient (via the restriction from (32).)

The second stage of the estimation uses $\hat{\beta}_l^u$, $\hat{\beta}_s^u$, and $\hat{\phi}_t(\cdot)$ to construct the sample analog to the moment restriction from (32) that identifies the capital coefficient. Given $\hat{\beta}_l^u$, $\hat{\beta}_s^u$, and $\hat{\phi}_t(\cdot)$, and any candidate value for β_k, say β_k^*, we can estimate the function $g(\omega_t)$ using a polynomial approximation with argument $\hat{\omega}_t(\beta_k^*) = \hat{\phi}_t(\cdot) - \beta_k^* k_t$. Alternatively, for any candidate value β_k^* we can compute the residual

$$[\xi_{i,t+1} + \eta_{i,t+1}](\beta_k^*)$$

for any plant i at time t (see equation (31).) We then use a non-linear least squares routine to locate the minimizer $\hat{\beta}_k$ which solves

$$min_\beta \sum_i \sum_{t=T_{i0}}^{T_{i1}} \left([\xi_{i,t+1} + \eta_{i,t+1}](\beta)\right)^2,$$

where T_{i0} and T_{i1} index the second and last period a plant is observed.

In an effort to make our estimation algorithm more readily available, exactly duplicable, and more user-friendly, the estimation algorithm has been adapted to run entirely in STATA and the program (in .do file format) is available from STATA (see Petrin, Poi, and Levinsohn (2004)) or at either author's website.

Appendix C

Imputing Missing Values

Approximately 3% of the plant-year observations in Chile are "missing"; a plant id number is present in year $t-1$, absent in year t, and then present again in year $t+1$. We impute the values for these observations using $t-1$ and $t+1$ information and the structure of the estimated production function. We use the simple average of the $t-1$ and $t+1$ (log) productivity estimates for the period t productivity estimate. Similarly, we use the simple average of the $t-1$ and $t+1$ (log) input index estimates, where the weights in the index are the estimated production function parameters. All of our findings are robust to dropping these observations.

TABLE 1

Comparison of Social PPF Productivity Index
to the BHC Productivity Index

Ordinary Least Squares Estimates, ISIC 311, Chile

		Rate of Growth in:		
Year	Value Added	Social PPF Index	BHC Index	Difference (BHC "Reallocation" Term)
		$\sum_i \frac{(s_{it}+s_{i,t-1})}{2} ln(\frac{\omega_{it}}{\omega_{i,t-1}})$	$\sum_i s_{it} ln\omega_{it} - \sum s_{i,t-1} ln\omega_{i,t-1}$	$\sum_i \frac{(ln\omega_{it}+ln\omega_{i,t-1})}{2} * \Delta s_i$
1988	11.75	-3.12	-14.96	-11.84
1989	7.36	3.64	-7.45	-11.10
1990	4.59	-1.10	-10.52	-9.43
1991	13.82	7.36	-13.28	-20.63
1992	14.67	6.09	-8.11	-14.20
1993	8.98	5.09	-0.19	-5.28
1994	8.20	2.95	7.09	4.13
1995	7.06	-0.74	-7.24	-6.50
1996	-1.30	12.56	28.25	15.69
Average	8.35	3.64	-2.93	-6.57
Std. Dev.	4.89	4.72	13.48	10.74

The last column is the discrepancy between the BHC index and the social PPF index, which is equal to a reallocation-like term given by $\sum_i \overline{ln\omega_i} * \Delta s_i$. The comparison is done on firms that exist in period t and $t-1$, which account for 94.4% of the plant-year observations and 96.4% of industry value added. See text for details.

TABLE 2

Comparison of Productivity Indexes Across

OLS, Levinsohn-Petrin, and Revenue Share Productivity Estimates

ISIC 311, Chile

Year	Value Added	OLS	Levinsohn-Petrin	Revenue Shares
		Social PPF Index		
1988	11.75	-3.12	-0.04	0.77
1989	7.36	3.64	4.39	4.86
1990	4.59	-1.10	-1.26	-1.91
1991	13.82	7.36	7.68	8.98
1992	14.67	6.09	6.82	9.69
1993	8.98	5.09	5.87	4.41
1994	8.20	2.95	4.45	2.84
1995	7.06	0.74	1.41	-5.22
1996	-1.30	12.56	11.07	6.80
		BHC Index		
1988	11.75	-14.96	-19.16	-5.04
1989	7.36	-7.45	-11.94	-4.40
1990	4.59	-10.52	-20.43	-2.21
1991	13.82	-13.28	-24.42	-4.09
1992	14.67	-8.11	-15.54	-0.01
1993	8.98	-0.19	5.16	0.09
1994	8.20	7.09	4.81	4.33
1995	7.06	-7.24	-9.09	-10.40
1996	-1.30	28.25	39.66	1.07

Growth rates in productivity compared across methods used to estimate production function parameters. Comparison is done on firms that exist in period t and $t-1$.

TABLE 3

Regression of Social PPF Productivity Index on BHC Productivity Index
Chilean Manufacturing, Industry by Industry, 1988-1996 (9 observations)

Industry Code	Coefficient Estimator	Intercept (Std. Err.)	Slope (Std. Err.)	R-squared
311	OLS	.043	.247	0.47
		(.012)	(.099)	
	LP	.051	.122	0.38
		(.011)	(.058)	
	Rev.	.036	.314	0.22
	Shares	(.015)	(.223)	
381	OLS	.031	.228	0.47
		(.023)	(.091)	
	LP	.028	.230	0.44
		(.024)	(.096)	
	Rev.	.008	.373	0.62
	Shares	(.021)	(.109)	
321	OLS	.010	.177	0.21
		(.023)	(.129)	
	LP	.017	.092	0.14
		(.020)	(.085)	
	Rev.	-.009	.503	0.38
	Shares	(.026)	(.026)	
331	OLS	.017	.207	0.25
		(.046)	(.132)	
	LP	.018	.146	0.20
		(.047)	(.108)	
	Rev.	.043	.501	0.57
	Shares	(.045)	(.163)	

Row one for ISIC 311 is the regression of column 2 on column 3 from table 1 (for example). An r-squared of 1 and an intercept of 0 and slope of 1 would indicate that the measures are identical. An r-squared of 1 would indicate that the BHC measure is a linear transformation of the Social PPF Index.

TABLE 3 (continued)

Regression of Social PPF Productivity Index on BHC Productivity Index

Chilean Manufacturing, Industry by Industry, 1988-1996 (9 observations)

Industry Code	Coefficient Estimator	Intercept (Std. Err.)	Slope (Std. Err.)	R-squared
322	OLS	-.008	.223	0.22
		(.031)	(.154)	
	LP	-.008	.214	0.21
		(.033)	(.156)	
	Rev.	-.008	.482	0.44
	Shares	(.033)	(.201)	
356	OLS	-.047	.024	0.02
		(.029)	(.061)	
	LP	-.043	.028	0.02
		(.031)	(.066)	
	Rev.	-022	.165	0.15
	Shares	(.042)	(.143)	
382	OLS	.093	.068	0.14
		(.036)	(.063)	
	LP	.093	.058	0.11
		(.037)	(.062)	
	Rev.	.072	.180	0.19
	Shares	(.047)	(.137)	
352	OLS	.019	.274	0.27
		(.031)	(.168)	
	LP	.039	.156	0.17
		(.031)	(.128)	
	Rev.	.003	.416	0.50
	Shares	(.024)	(.155)	

Row one for ISIC 311 is the regression of column 2 on column 3 from table 1 (for example). An r-squared of 1 and an intercept of 0 and slope of 1 would indicate that the measures are identical. An r-squared of 1 would indicate that the BHC measure is a linear transformation of the Social PPF Index.

TABLE 4

Comparison of Base-Period Share to Average Share

In Approximating the Shift in the Social PPF

Ordinary Least Squares Estimates, ISIC 311, Chile

Year	Rate of Growth in: Value Added	Average Share (Tornquist): $\sum_i \frac{(s_{it}+s_{i,t-1})}{2} ln(\frac{\omega_{it}}{\omega_{i,t-1}})$	Base Period Share (BHC) (Laspeyres): $\sum_i s_{i,t-1} ln(\frac{\omega_{it}}{\omega_{i,t-1}})$
1988	11.75	-3.12	-21.93
1989	7.36	3.64	-11.17
1990	4.59	-1.10	-16.12
1991	13.82	7.36	-4.03
1992	14.67	6.09	-7.19
1993	8.98	5.09	-8.61
1994	8.20	2.95	-10.12
1995	7.06	-0.74	-8.48
1996	-1.30	12.56	0.02
Average	8.35	3.64	-9.73
Std. Dev.	4.89	4.72	6.41

The comparison is done on firms that exist in period t and $t-1$, which account for 94.4% of the plant-year observations and 96.4% of industry value added. See text for details.

TABLE 5

Petrin and Levinsohn Decomposition of
the Change in Rate of Productivity Growth
"Real Productivity" vs. "Reallocation"

Ordinary Least Squares Estimates, ISIC 311, Chile

Year	Productivity Growth Rate	Change in Growth Rate	Real Productivity Component	Reallocation Component
1988	-3.12	—	—	—
1989	3.64	7.40	-1.79	9.18
1990	-1.10	-5.25	-14.83	9.59
1991	7.36	10.86	2.35	8.51
1992	6.09	-2.30	-10.64	8.34
1993	5.09	-0.66	-9.17	8.51
1994	2.95	-1.12	-9.72	8.60
1995	-0.74	-2.44	-10.08	7.64
1996	12.56	13.61	6.23	7.38
Average	3.64	2.51	-5.95	8.46
Std. Dev.	4.72	7.05	7.33	0.72

The first column is the rate of productivity growth from $t-1$ to t estimated using the social PPF index. The second column is the change in this growth rate for firms that exist in $t-2$, $t-1$, and t The discrepancy between the change in column 1 and the level in column 2 is due to firms that do not exist in all three periods (the net entry term). The third and fourth column decompose column 2. See text for details.