A Spuriously New Economy

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July 2005

Abstract

We introduce a vintage capital model in which workers are matched with machines of increasing quality. Quality improvements of the machines are the sole source of technological change in this economy. However, the matching of workers with machines implies that there is no well defined capital aggregate in this economy. Hence, investment price indices are a spurious measure of price changes in capital goods. We show that the use of such spurious measures of investment price changes can lead to misleading conclusions about (changes in) the trend properties of the economy. We illustrate how this is a potential problem with recent claims of the emergence of a 'New Economy' in the U.S.

keywords: imperfect competition, price indices, vintage capital.

JEL-code: O310, O470, C190.

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1 Introduction

Many recent empirical studies of technological change have used changes in the relative price of investment goods with respect to consumption goods as a measure of the degree of investment specific or embodied technological change. These studies include, among others, Greenwood, Hercowitz, and Krusell (1997,2000), Violante, Ohanian, Ríos-Rull, and Krusell (2000), Cummins and Violante (2002), Fisher (2002), and Altig, Christiano, Eichenbaum, and Linde (2005).

Other studies, like Jorgenson and Stiroh (2000) and Oliner and Sichel (2000), emphasize the importance of information technology (IT) production and capital for aggregate U.S. productivity growth.

What these studies have in common is that their main conclusions in large part hinge on the way investment prices are measured. In particular, they hinge on the assumption of the existence of an aggregate or IT capital stock, the price of which is properly reflected by the price index used.

A large literature has evolved around the question whether price indices propertly reflect the quality improvements embodied in capital. Among the recent contributions along this strand of the literature, for example, is Pakes (2003).

In this paper we take another angle on this issue. Instead of considering whether price indices properly reflect capital price changes, we consider a case in which the assumed capital aggregate, the price of which is supposed to be approximated by the price index, does not even exist in the first place.

To illustrate our case, we introduce a vintage capital model, in the spirit of Johansen (1959), Arrow (1962), and Jovanovic (1999,2004). In it, workers of different skill levels are matched with machines of different and increasing quality. The quality improvements of machines are the sole source of economic growth in our model. Each worker can only use one machine, such that the capital labor ratio is fixed. The assignment of workers across machines means that capital vintages and labor are intertwined to such a degree that there is no aggregate production function representation in terms of labor and an aggregate capital stock.

The non-existence of an aggregate capital stock is nothing new. Fisher (1969) already showed that, in case of embodied technological change, such a capital stock only exists if the vintage specific production functions are Cobb-Douglas. Because of the fixed capital labor ratio, in our model the vintage specific production functions are Leontief instead. The problem is that the spurious application of a capital price index in the absence of an aggregate capital stock can lead to deceiving conclusions in our model.

We illustrate this problem using a numerical example in which the measured trend properties of the economy change because of shifts in the competitive structure of the capital goods producing sector, rather than a change in the rate of (embodied) technological progress. The changes in the measured trend properties induced by such a competitive shift are an increase in the rate of decline in the relative price of investment goods, and increases in the growth rates of real investment, real GDP, and investment specific technological change.

These are exactly the facts that proponents of the 'New Economy' hypothesis, like Oliner and Sichel (2000), point at when they claim that increases in technological progress in the capital goods sector have driven recent changes in the growth rates of productivity and GDP in the United States. Hence, our example is one in which the 'New Economy' facts are spurious because they are based on the improper application of capital price indices.

The discussion about the existence or non-existence of appropriate capital aggregates has a long history in macroeconomics. It was at the heart of what Harcourt (1969) designated the Cambridge Capital Controversy. This controversy was initiated by Robinson's (1959) criticism of the neoclassical assumption of the existence of an aggregate capital stock. Recent discussions of this controversy generally dismiss it as 'silly' and irrelevant[1].

This paper points out a specific new issue where it actually might be relevant: The measurement of the growth rate of aggregate economic activity.

The assumption of the existence of proper capital aggregates underlies almost all of the existing measures of economic activity. However, we show how spuriously applying price index methods when such aggregates do not exist might lead to misleading results about economic growth and productivity.

We do not provide any direct measurement solutions for what to do when one suspects the identifying assumption of the existence of a proper capital aggregate is invalid. Therefore, we would like to emphasize that this paper merely provides a counterexample as a note of caution. That is, we do not claim that all studies that rely on investment price indices have yielded spurious results. Instead, we would just like to point out that there might be an alternative explanation for the

[1] Prescott (2005, p.523), for example, describes the Cambridge Capital Controversy as such.

'stylized facts' that these studies claim to document.

The structure of this paper is as follows. In the next section we introduce our model economy. Because our argument does not hinge on transitional dynamics, we consider a model that is always on its balanced growth path. In Section 3 we derive the equilibrium balanced growth path of the economy and prove its relevant properties. In Section 4 we consider what we would actually measure in terms of economic aggregates in our model economy. We do so in two stages. First, we derive an aggregate production function representation and show that there is no aggregate capital stock. Secondly, we show how the spurious application of a capital price index in this case can yield misleading statistics on the decline of the relative price of capital, and the growth rates of real investment, real GDP, total factor productivity, and investment specific technological change. Section 5 illustrates the issues presented in the previous section with an example of a 'Spuriously New Economy'. We discuss the empirical relevance of our theoretical results in Section 6. Finally, we conclude in Section 7. Mathematical details are left for Appendix A.

2 Model

The model that we introduce is a model of embodied technological change. In our model, a continuum of workers with heterogenous levels of human capital in each period choose a type of machine that they use to produce a homogenous final good. The machines are supplied by a set of firms that compete monopolistically. The final good is used as a consumption good as well as the input in the production of machines.

The main results of this paper are easiest explained along a balanced growth path. For this reason, we develop a model economy that is always on its balanced growth path. This allows us to make the simplifying assumptions of linear preferences and innovations of equal size at a constant frequency.

The following four subsections introduce the household sector, final goods sector, capital goods sector, as well as the type of exogenous embodied technological progress in our model economy respectively.

2.1 Households

A household in our economy consists of a single infinitely-lived worker. All households have linear preferences in the sense that a household, which, for reasons explained below, we index by h, that consumes $c_{t+s}(h)$ for $s = 0, 1, 2, \ldots$ gets the following level of utility

$$\sum_{s=0}^{\infty} \beta^s c_{t+s}(h) \text{ where } 0 < \beta < 1 \tag{1}$$

The household maximizes this objective subject to the intertemporal budget constraint that

$$a_{t+s+1}(h) = (1 + r_{t+s}) a_{t+s}(h) + w_{t+s}(h) + \pi_{t+s} - c_{t+s}(h) \tag{2}$$

Here $a_{t+s}(h)$ denotes the real assetholdings of the household at the beginning of period $t+s$, r_{t+s} is the real interest rate at time $t+s$, $w_{t+s}(h)$ is the real wage rate the household earns, π_{t+s} are the dividend payments that the household receives over the shares it owns in capital goods producing firms[2].

The intertemporal optimality condition for the households in this economy implies that for consumption to be positive in each period, the real interest rate has to satisfy

$$r_t = \frac{1-\beta}{\beta} \equiv r \text{ for all } t \tag{3}$$

which is what we will assume throughout the rest of this paper.

2.2 Final goods producers

Firms produce a homogenous final (consumption) good by matching workers of different skill-levels with machines of different quality. The capital labor ratio is fixed such that each worker is only matched up with a single machine. The labor is supplied to the competitive firms in the final goods sector.

We will take a certain degree of heterogeneity among workers as given. The relevant dimension of heterogeneity across workers is their human capital levels. We denote the human capital level of a particular worker by h. There is a continuum of workers of measure one whose human capital levels are uniformly distributed on the interval $(\underline{h}, \overline{h}]$, such that $h \backsim unif\left(\underline{h}, \overline{h}\right)$.

[2]We will assume that the shares in these firms are equally distributed among the households, because of which they all get equal dividend payments. However, as Caselli and Ventura (2000) show, the aggregate behavior of our economy will not depend on the distribution of shares.

Just like workers, machines are also heterogenous in this economy. There is a countably finite number of machines supplied in each period. We denote a particular type, or vintage, of machine by τ[3]. Each vintage of machine embodies a different quality, where $A_{t-\tau} > 0$ denotes the number of efficiency units embodied in a machine of vintage age τ. Throughout, we will assume that there is no technological regress such that $A_t - A_{t-1} > 0$ for all t.

The combination of a worker of type h and a machine of vintage age τ yields $hA_{t-\tau}$ units of output[4].

In order to avoid having to consider intractable intertemporal optimization problems and having to make assumptions about possible second hand markets, we will assume that machines fully depreciate in one period. This assumption basically implies that the machines considered here are equivalent to intermediate goods in the sense of Aghion and Howitt (1992) and Romer (1990).

Firms can not use these machines for nothing. The price of a machine of quality $A_{t-\tau}$ at time t is $P_{t,\tau}$. This price is measured in units of the final good, which we will use as the numeraire good throughout.

Given this production technology, vintage profile of prices, and the menu of available vintages of machines, in each period a firm that employs a worker with skill level h chooses, from this menu, the type of machine that maximizes labor service flows. These labor service flows are the difference between the revenue generated by the sale of the final goods produced and the cost of the machine used to produce them.

That is, if $\mathbf{T}_t$ denotes the set of available technology vintage ages and $\mathbf{A}_t$ the set of associated productivity levels of the machines, then firms will assign a worker with human capital level h to a technology from the technology choice set $\Upsilon_t(h)$, which is defined as

$$\Upsilon_t(h) = \left\{ \tau \in \mathbf{T}_t \,\middle|\, \tau \in \arg\max_{s \in \mathbf{T}_t} \{hA_{t-s} - P_{t,s}\} \right\} \tag{4}$$

Let $w_t(h)$ be the wage rate of a worker with human capital level h, then competition and free entry on the demand side of the labor market implies zero profits such that the wage rate of a worker

[3]The notational convention that we will use in this paper follows Chari and Hopenhayn (1991) in the sense that τ represents 'vintage age'. That is, A_t represents the frontier technology level and $A_{t-\tau}$ is the frontier technology level of τ periods ago.

[4]This setup of the production function is similar to the preference setup used by Bresnahan (1981) to estimate marginal cost profiles and markups in the American Automobile Industry.

with skill level h equals revenue minus capital expenditures. Mathematically, this implies

$$w_t(h) = hA_{t-\tau} - P_{t,\tau}, \text{ for all } \tau \in \Upsilon_t(h) \tag{5}$$

When we aggregate over workers of all human capital levels, we obtain the relevant capital demand sets. Let $\mathbf{P}_t$ be the vector of prices charged for the available machines, then the set of buyers of machines of vintage age τ, which we will denote by $D_t(\tau, \mathbf{P}_t, \mathbf{A}_t)$, is given by

$$D_t(\tau, \mathbf{P}_t, \mathbf{A}_t) = \left\{ h \in (\underline{h}, \overline{h}] \middle| \tau \in \arg\max_{s \in \mathbf{T}_t} (hA_{t-s} - P_{t,s}) \right\} \tag{6}$$

These sets determine the demand for each of the available vintages of machines.

2.3 Capital goods producers

Machine designs are assumed to be patented for M periods and each period there is one new machine design patented.

During the first M periods of a machine design's life, the particular machine is supplied by a monopolist firm. After the patent expires the machine design is public domain and there is perfect competition in the supply of these machines.

In order to show the generality of our results we will allow for one monopolist selling all M patented machines, M monopolistic competitors that each sell one particular vintage of machine, or any case in between.

Hence, each supplier may supply more than one patented machine design. We will denote the number of supplies of patented machines by $N \leq M$ and index them by n. The function $\iota_t(\tau)$ identifies the supplier of machines of vintage τ.

The technology used to produce machines is as follows. Units of the final (consumption) good are the only input needed in machine production. We make this assumption to avoid having to deal with the selection of workers across the final goods and capital good producing sectors. Production of a continuum of mass $K_{t,\tau}$ of machines of type $A_{t-\tau}$ requires the use of

$$\underline{h}A_{t-\tau}K_{t,\tau} + \frac{c_\tau}{2}A_{t-\tau}K_{t,\tau}^2 \tag{7}$$

units of the final good. The cost parameter $c_\tau > 0$ depends on vintage age, in order to allow us to take into account potential learning by doing in the production of machines. For example, Irwin and Klenow (1994) show that learning by doing effects are important in the production of random access memory (RAM) chips.

The question that is left is how these machine producers end up choosing the prices of their machines. Suppose supplier n supplies a total of v_n vintages. Let $\tau_{n_1}, \tau_{n_2}, \tau_{n_3}, ...\tau_{n_{v_n}}$ be the vintages supplied by supplier n. Then the vector of prices chosen by supplier n can be denoted

$$\mathbf{P}_{t,n} = \left\{ P_{t,\tau_{n_1}}, \ldots, P_{t,\tau_{n_{v_n}}} \right\}. \tag{8}$$

Throughout this paper, we will focus on Pure Strategy Nash (PSN) equilibria. For the particular problem at hand here this implies that supplier n takes the prices set by the other supplies, which we will denote by the vector $\mathbf{P}'_{t,n}$, and the productivity levels the machines, i.e. the $A_{t-\tau}$ for $\tau \in \mathbf{T}_t$, as given.

Given these variables, producer n chooses the prices of his machines to maximize profits. This implies that $\mathbf{P}_{t,n}$ is an element of the best response set

$$\begin{aligned} BR_t\left(\tau; \mathbf{P}'_{t,n}, \mathbf{A}_t\right) &= \Big\{ \mathbf{P}_{t,n} \in \mathbb{R}_+^{v_n} \\ &\quad \Big| \mathbf{P}_{t,n} \in \arg\max_{\mathbf{P} \in \mathbb{R}_+^{v_n}} \left\{ \sum_{i=1}^{v_n} \left(\mathbf{P}_i K_{t,\tau_{n_i}} - \underline{h} A_{t-\tau_{n_i}} K_{t,\tau_{n_i}} - \frac{c_{\tau_{n_i}}}{2} A_{t-\tau_{n_i}} K_{t,\tau_{n_i}}^2 \right) \right\} \Big\} \end{aligned} \tag{9}$$

Where $K_{t,\tau_{n_i}}$ equals the mass of workers that demand machines of vintage age τ_{n_i} at the prices set[5].

Because patents expire after M periods, these best response sets only apply to $\tau = 0, \ldots, M-1$. For machines that were designed M or more periods ago, perfect competition implies that price must equal average cost and that free entry drives both to $\underline{h} A_{t-\tau}$. Hence, $P_{t,\tau} = \underline{h} A_{t-\tau}$ for $\tau \geq M$.

The corresponding profits are

$$\pi_{t,n} = \sum_{i=1}^{v_n} \left(P_i K_{t,\tau_{n_i}} - \underline{h} A_{t-\tau_{n_i}} K_{t,\tau_{n_i}} - \frac{c_{\tau_{n_i}}}{2} A_{t-\tau_{n_i}} K_{t,\tau_{n_i}}^2 \right) \text{ for all} \tag{10}$$

$$\mathbf{P}_{t,n} \in BR_t\left(\tau; \mathbf{P}'_{t,n}, \mathbf{A}_t\right) \tag{11}$$

for $\tau = 0, \ldots, M-1$ and are zero for $\tau \geq M$.

2.4 Technological progress

What generates the improvements in the quality of machines does not matter for the argument in this paper. Therefore, we will simply assume that the quality of machines grows exogenously. In particular, we will assume that the quality of machines grows at a constant rate g, such that

[5] Formally, $K_{t,\tau_{n_i}}$ is the Lebesque measure of the demand set $D\left(\tau_{n_i}, \left(\mathbf{P}'_{t,n}, \mathbf{P}_{t,n}\right), \mathbf{A}_t\right)$.

$A_{t+1} = (1+g)\, A_t$, where $g > 0$. We will also assume that the patent for this innovation is randomly awarded to one of the households in the economy[6].

Throughout, we will consider two cases. The first is the one in which the household hangs on to this patent and becomes the owner of one of the monopolistically competing machine suppliers. Because there is a continuum of households, the probability that one household holds the patent for two of the M patented machines is a zero probability event. Hence, this is the case in which there is monopolistic competition between M different suppliers. We will refer to this as the 'monopolistic competition' case.

The second is the one in which the market for machines is dominated by a monopoly firm that buys out the patentholders of new machine designs in each period. In this market the same firm will hold a monopoly in the supply of machines along the equilibrium path. We will refer to this as the 'monopoly' case.

3 Equilibrium

In this section we derive the equilibrium outcome and prove the relevant properties of this economy along its balanced growth path. These are the properties that drive our main measurement issues results that are presented in the next section.

We derive the equilibrium in three steps. First of all, we solve for the machine demand decisions made by the workers in the final goods sector. Secondly, we obtain the optimal price setting strategies by the suppliers of the different vintages of machines in response to the demand functions derived in the first step. Finally, we combine the results of the first three steps to derive the balanced growth path of our model economy. We only describe the main results and their intuition here. The details of the derivations are left for Appendix A.

3.1 Demand for machines

Because our setup in the final goods sector is similar to that of the car market in Bresnahan (1981), so are the resulting demand functions. They satisfy the following two main properties, independent

[6] In principle, one could include an R&D sector in the model that is financed using the profits made by the monopolistic competitors. This would significantly complicate the equilibrium conditions in this economy and would not change the main results that are the focus of this paper.

of the set of technologies sold, i.e. $\mathbf{A}_t$, and the prices set for the patented designs, i.e. $\mathbf{P}_t$.

First of all, better workers end up using better machines. That is, there is endogenous assortative matching between workers and machines. Mathematically, this can be written as

$$\text{For } h' > h, \text{ if } h \in D_t(\tau, \mathbf{P}_t, \mathbf{A}_t) \text{ then } h' \notin D_t(\tau', \mathbf{P}_t, \mathbf{A}_t) \text{ forall } \tau' > \tau. \tag{12}$$

Assortative matching between machines and workers is a natural outcome when a technology exhibits capital-skill complementaries, like in the final goods sector in our model. Jovanovic (1999,2004) are examples where this is the case as well.

This assortative matching result also implies that the demand sets are connected. That is, for vintages of machines for which there is positive demand, they are of the form

$$D_t(\tau, \mathbf{P}_t, \mathbf{A}_t) = \left(\underline{h}_{t,\tau}, \overline{h}_{t,\tau}\right] \text{ where } \underline{h} < \underline{h}_{t,\tau} < \overline{h}_{t,\tau} \leq \overline{h} \tag{13}$$

where the upper and lowerbounds of the set are determined by the prices and the productivity levels of the vintages sold. It also follows from this assortative matching result that the set of workers that is indifferent between the choice of two machines is negligible. That is, the size of these demand sets, and thus the demand for each of the different vintages, is uniquely determined by the prices that are set and the productivity levels of the machines.

Secondly, perfect competition for the production of machines of vintage age M and older implies that machines of a design older than M, i.e. a design for which the patent has expired for more than one period, are not demanded anymore. Their demand set is the empty set in equilibrium. That is,

$$D_t(\tau, \mathbf{P}_t, \mathbf{A}_t) = \emptyset \text{ for } \tau > M \tag{14}$$

The derivation of this result is straightforward. The quadratic production technology for machines implies that perfect competition on the machines for which the patent has expired will drive their prices to $\underline{h}A_{t-\tau}$. Considering only the machines in the public domain, the firm's problem becomes

$$\max_{\tau \geq M} \left\{hA_{t-\tau} - \underline{h}A_{t-\tau}\right\} \tag{15}$$

for some $h > \underline{h}$, which is solved by choosing the largest value of $A_{t-\tau}$.

3.2 Price schedule of machines

The properties of the demand sets proven above imply that the amount of machines of vintage age τ equals

$$K_{t,\tau} = \left(\overline{h}_{t,\tau} - \underline{h}_{t,\tau}\right) / \left(\overline{h} - \underline{h}\right) \tag{16}$$

This result can be used to derive the equilibrium price schedule of machines. Before doing so, we first formally define what we mean by the PSN price setting equilibrium in this market.

For a given set of available technologies, $\mathbf{A}_t$, a PSN equilibrium price schedule $\mathbf{P}_t^* = \left\{\mathbf{P}_{t,1}^*, \dots, \mathbf{P}_{t,N}^*\right\}$ in this market satisfies two properties. First of all, for those vintages for which the patent has expired the price is driven to the minimum average cost level. That is, $P_{t,\tau} = \underline{h}A_{t-\tau}$ for all $\tau \geq M$. Secondly, each of the suppliers of one or more patented vintages of machines chooses its prices as part of its best response set with respect to the prices set by the other producers. That is, let $\mathbf{P}_{t,n}^*$ be the prices set by supplier n for the machines it supplies and let $\mathbf{P}_{t,n}^{*'}$ be the prices set by the other producers in the PSN equilibrium, then

$$\mathbf{P}_{t,n}^* \in BR_t\left(\tau; \mathbf{P}_{t,n}^{*'}, \mathbf{A}_t\right) \text{ for all } n = 1, \dots, N \tag{17}$$

It turns out that, for all possible technology menus $\mathbf{A}_t$ and all possible permutations of suppliers over the M patented vintages, there exists a unique equilibrium price schedule. The equilibrium price schedule has several relevant properties that are independent of $\mathbf{A}_t$, of the way suppliers are distributed over the M newest machine designs, and of the cost coefficients $\{c_\tau\}_{\tau=0}^{\infty}$. The existence and uniqueness of the price schedule as well as the details underlying the properties are derived in Appendix A. Here we limit ourselves to the description of the properties that are relevant for the rest of our analysis.

The first property is that, in equilibrium, prices are set such that there is strictly positive demand for all M patented vintages. Mathematically, this boils down to

$$D_t\left(\tau, \mathbf{P}_t^*, \mathbf{A}_t\right) \neq \emptyset \text{ for } \tau = 0, \dots, M \tag{18}$$

in the PSN price setting equilibrium.

The second property is that, in this equilibrium, suppliers make strictly positive profits of the supply of each of the individual patented designs. That is,

$$P_{t,\tau} > \underline{h}A_{t-\tau} + c_\tau A_{t-\tau} K_{t,\tau} > 0 \text{ for all } \tau = 0, \dots, M-1 \tag{19}$$

such that for each patented vintage, all of which are produced with a decreasing returns to scale technology, price exceeds average cost and thus profits are strictly positive.

The final two properties are most easily written in terms of prices per efficiency units. For this purpose, we define the price per efficiency unit of a machine of vintage age τ as $\widehat{P}_{t,\tau} \equiv P_{t,\tau}/A_{t-\tau}$.

In terms of the price schedule per efficiency unit, the third relevant property for what is to come is that prices per efficiency unit are increasing in the quality of the machines. Formally,

$$\widehat{P}_{t,\tau} \text{ is strictly decreasing in } \tau \tag{20}$$

That is, the older the vintage age of the machine, the lower the quality, and the lower the price per efficiency unit.

The final property of the price per efficiency unit schedule is that it only depends on the cost parameters, $\{c_\tau\}_{\tau=0}^{M-1}$, the patent length, M, and the productivity profile of the vintages, $\mathbf{A}_t = \{A_t, \ldots, A_{t-M}\}$. Moreover the price per efficiency unit schedule is homogenous of degree zero in the productivity levels of the vintages.

Formally, let $\widehat{\mathbf{P}}_t^*$ be the equilibrium schedule of prices per efficiency unit, then this last property implies

$$\widehat{\mathbf{P}}_t^* = \widehat{\mathbf{P}}\left(\mathbf{A}_t, \{c_\tau\}_{\tau=0}^{M-1}\right) \tag{21}$$

such that $\widehat{\mathbf{P}}_t^*$ is solely a function of the cost parameters for the vintages sold in the market, i.e. $\{c_\tau\}_{\tau=0}^{M-1}$, and the productivity profile, i.e. $\mathbf{A}_t$. Furthermore, the function $\widehat{\mathbf{P}}$ is homogenous of degree zero in $\mathbf{A}_t$. So are the demand sets and the equilibrium demand levels, $K_{t,\tau}$.

3.3 Balanced growth path

What is important for the behavior of this economy on its balanced growth path is the last property. That is, that the equilibrium price per efficiency unit profile, $\widehat{\mathbf{P}}_t^*$, as well as the demand levels, $K_{t,\tau}$, are homogenous of degree zero in $\mathbf{A}_t$. Combined with our assumption that the productivity profile grows at a constant rate over time, this implies that the balanced growth path has several important properties. We will describe them here. They are proved in Appendix A.

First of all, the vintage age distributions of machines and investment and the prices per efficiency units for particular vintage ages are constant over time. That is

$$K_{t,\tau} = \overline{K}_\tau \text{ and } \widehat{P}_{t,\tau} = \overline{\widehat{P}}_\tau \tag{22}$$

Here, $-$ denotes equilibrium values along the balanced growth path and $\hat{}$ denotes detrended equilibrium values in terms of efficiency units.

Secondly, equilibrium consumption is defined as final goods output minus intermediate goods demand. That is,

$$C_t = Y_t - X_t \tag{23}$$

Equilibrium investment, expressed in terms of the (numeraire) final good, is defined as

$$I_t = \sum_{\tau=0}^{M} P_{t,\tau} K_{t,\tau} \tag{24}$$

Gross output of the final goods sector equals

$$Y_t = \int_{\underline{h}}^{\overline{h}} A_{t-\Upsilon_t(h)} h dh \tag{25}$$

While the intermediate inputs demand for final goods is

$$X_t = \frac{1}{2} \sum_{\tau=0}^{M-1} c_\tau K_{t,\tau}^2 \tag{26}$$

and reflects the amount of final goods output needed to produce the capital goods.

Along the balanced growth path, consumption, investment expressed in terms of finals goods, gross output of the final goods sector, and intermediate input demand of the capital goods sector all grow at the constant rate g. Mathematically, this means that

$$C_t = A_t \overline{\widehat{C}}\ , \ I_t = A_t \overline{\widehat{I}},\ Y_t = A_t \overline{\widehat{Y}}_t,\ X_t = A_t \overline{\widehat{X}} \tag{27}$$

In terms of the theoretical equivalents of things that are actually measured in the national accounts, the balanced growth path implies the following. Gross Domestic Product (GDP), expressed in units of the consumption (final) good grows at the constant rate, $g > 0$, over time. GDP here equals the sum of the value added of the final goods sector, which is Y_t, and that of the capital goods sector, which equals $I_t - X_t$. Hence, GDP follows

$$GDP_t = Y_t + (I_t - X_t) = A_t \left(\overline{\widehat{Y}} + \overline{\widehat{I}} - \overline{\widehat{X}} \right) \tag{28}$$

Note that GDP does not take into account capital depreciation. GDP corrected for capital depreciation is known as Net Domestic Product (NDP). Since capital fully depreciates in every period here, NDP in this economy equals net value added for the final goods sector plus net value added for the capital goods sector. That is,

$$NDP_t = (Y_t - I_t) + (I_t - X_t) = GDP_t - I_t = A_t\left(\overline{\overline{\widehat{Y}}} - \overline{\overline{\widehat{X}}}\right) = A_t\overline{\overline{\widehat{C}}} \tag{29}$$

This is again denoted in terms of the consumption good, which we use as the numeraire good.

Finally, for growth accounting purposes, it is useful to consider factor shares, most notably labor shares. The labor shares in both the overall economy as well as in the final goods producing sectors are constant along the balanced growth path. In particular, the aggregate labor share equals

$$s_L = \frac{\text{wages}}{GDP_t} = \frac{Y_t - I_t}{GDP_t} = \frac{\left(\overline{\overline{\widehat{Y}}} - \overline{\overline{\widehat{I}}}\right)}{\left(\overline{\overline{\widehat{Y}}} + \overline{\overline{\widehat{I}}} - \overline{\overline{\widehat{X}}}\right)} = 1 - \frac{2\overline{\overline{\widehat{I}}} - \overline{\overline{\widehat{X}}}}{\left(\overline{\overline{\widehat{Y}}} + \overline{\overline{\widehat{I}}} - \overline{\overline{\widehat{X}}}\right)} \tag{30}$$

while the share of labor in the final goods sector equals

$$s_L^f = \frac{\text{wages}}{\text{gross value added in final goods sector}} = \frac{\left(\overline{\overline{\widehat{Y}}} - \overline{\overline{\widehat{I}}}\right)}{\overline{\overline{\widehat{Y}}}} = 1 - \frac{\overline{\overline{\widehat{I}}}}{\overline{\overline{\widehat{Y}}}} \tag{31}$$

Together, the equilibrium properties along the balanced growth path described above imply that the trend properties of this economy are fully defined by the exogenous growth rate $g > 0$. All other parameters only influence the detrended equilibrium levels, $\overline{\overline{\widehat{Y}}}$, $\overline{\overline{\widehat{C}}}$, $\overline{\overline{\widehat{I}}}$, and $\overline{\overline{\widehat{X}}}$, as well as the equilibrium factor shares.

4 Measurement

The main point of this paper is that standard measures of the trend properties of our model economy will paint a misleading picture of the actual economic developments. In this section we will show that this is the case because the measured trend properties of this economy turn out to depend on much more than only g. This reveals a potential source for persistent measurement error in the growth rates of several important economic aggregates.

Before we consider the measurement of these aggregate variables, we first consider whether they acually exist. This turns out not to be the case. The crux for our results is that an aggegregate capital stock does not exist in our model economy. In the first part of this section we derive this non-existence result and discuss how it is closely related to the Cambridge Capital Controversy in macroeconomics.

In the subsequent part of this section we consider the measured growth rates of several commonly studied economic aggregates. In particular, we focus on the growth rates of the relative price of capital goods relative to consumption goods, of real investment and the capital stock, of real GDP, and those of both TFP and investment-specific technological change. We will deal with them in the order mentioned.

4.1 The absence of an aggregate capital stock

At the heart of the potential measurement errors in the growth rates of economic aggregates in this economy is the fact that, in this economy, there is not a theoretically well-defined aggregate capital stock. Because the final goods sector is the only sector that uses capital goods in our economy, we will focus on the non-existence of an appropriate capital aggregate for that sector.

The argument that a proper capital aggregate only exists under very restrictive assumptions goes back to Robinson (1959). Her article was at the heart of the Cambridge capital controversy that was a prominent topic in macroeconomics in the 1960's and 1970's[7]. After Robinson's criticism of the neoclassical production function, a large literature evolved in which the conditions under which an aggregate capital stock exists are derived.

Fisher (1969) showed that this was only the case when the vintage specific production functions are Cobb-Douglas with equal capital elasticities. The final goods sector in our model does not satisfy this assumption, i.e. the vintage specific production functions are Leontief. This means that no aggregate capital stock exists for this sector. In order to show this, we derive an aggregate production function for the final goods sector in the way Fisher (1969) proposed. The basic issue is the following.

The common assumption in both neoclassical macroeconomic theory as well as in most statistical measurements of aggregate economic activity is that value added is generated using capital and labor and that the production function can be represented as

$$Y_t = Z_t F\left(K_t, L_t\right) \tag{32}$$

Here Z_t is factor neutral technological progress, also known as Total Factor Productivity, L_t is an appropriately defined aggregate of labor inputs, and K_t is an appropriately defined capital aggregate

[7]Harcourt (1969, 1976), and Cohen and Harcourt (2003) are three surveys of the capital controversy.

that is a composite of all the underlying different capital inputs[8]. The composite K_t is assumed to be homogenous of degree one in the underlying capital stocks. These capital stocks would in our case be the stocks of different machines used in production. Hence, in our model this capital stock would be a composite of all machine vintages, such that

$$K_t = J\left(\{K_{t,\tau}\}_{\tau=-\infty}^{\infty}\right) \tag{33}$$

What we will show in the following is that such a representation of the production function in the final goods sector does not exist in our model. Instead, the best we can do is write the production function in that sector as

$$Y_t = Z_t F\left(\{K_{t,\tau}\}_{\tau=-\infty}^{\infty}, L_t\right) \tag{34}$$

Because all increases in output in this sector are due to a shift in the distribution of machines used in production towards better vintages, factor neutral technological progress is zero. That is TFP, Z_t, is constant over time.

For the derivation of the aggregate production function, (34), for the final goods sector, we follow Fisher (1969). We consider the decision of a planner that is endowed with a continuum for workers of measure L_t that is uniformly distributed over the interval $(\underline{h}, \overline{h}]$ as well as with a sequence of capital stocks of different vintages $\{K_{t,\tau}\}_{\tau=0}^{M}$. Given these endowments of production factors, the planner chooses an allocation of labor over the capital stocks to maximize output.

Let $K_\tau(h) \geq 0$ be the amount of capital of vintage age τ that is assigned to workers of type h and,equivalently, let $L_h(\tau) \geq 0$ be the amount of workers of human capital level h that is assigned to machines of vintage age τ.

The planner chooses these allocations to maximize output, which is given by the production function

$$Y_t = \sum_{\tau=0}^{M} A_{t-\tau} \int_{\underline{h}}^{\overline{h}} h \min\left\{K_\tau(h), L_h(h)\right\} dh \tag{35}$$

The allocations are chosen subject to the resource constraints that the capital assigned does not exceed the capital available

$$\int_{\underline{h}}^{\overline{h}} K_\tau(h)\, dh \leq K_{t,\tau} \tag{36}$$

[8]We make this argument here for the maximum level of aggregation. Our argument equally applies to the problem of the existence of capital stocks of subaggregates, like computers for example.

and that the amount of labor assigned does not exceed the amount of labor available

$$\sum_{\tau=0}^{M} L_h(\tau) \leq \frac{L_t}{\left[\overline{h} - \underline{h}\right]} \tag{37}$$

The solution to this optimization problem coincides with the decentralized equilibrium outcome in our model economy. It entails the assortative matching between workers and machines.

Denote the human capital level of the least skilled worker that is still assigned a machine as

$$\underline{h}^* = \overline{h} - \left(\overline{h} - \underline{h}\right) \min\left\{\frac{1}{L_t}\sum_{\tau=0}^{M} K_{t,\tau}, 1\right\} \tag{38}$$

and let the oldest vintage of machines assigned to workers be

$$\tau^* = \max_{\tau=0,\dots,M}\left\{\sum_{s=0}^{\tau-1} K_{t,s} < L_t\right\} \tag{39}$$

These definitions allow us to write the optimal assignment as follows.

$$K_\tau(h) = L_h(\tau) = \begin{cases} L_t & \text{for } \tau \leq \tau^* \text{ and } h \in \left(h^*_{\tau-1}, h^*_\tau\right] \\ 0 & \text{otherwise} \end{cases} \tag{40}$$

Where the boundaries of the matching sets are given by

$$h^*_\tau = \begin{cases} \overline{h} & \text{for } \tau = 0 \\ \max\left\{\underline{h}, \overline{h} - \frac{\left(\overline{h}-\underline{h}\right)}{L}\sum_{s=0}^{\tau-1} K_{t,s}\right\} & \text{otherwise} \end{cases} \tag{41}$$

The level of output that results from this assignment equals

$$\begin{aligned} Y\left(L_t, \{K_{t,\tau}\}_{\tau=0}^{M}\right) &= \frac{L_t}{\overline{h}-\underline{h}}\sum_{\tau=1}^{\tau^*}\int_{h^*_{\tau-1}}^{h^*_\tau} A_{t-\tau} h \\ &= \frac{1}{2}\frac{L_t}{\overline{h}-\underline{h}}\left[A_t\overline{h}^2 - \sum_{\tau=1}^{\tau^*}\left(A_{t-\tau-1} - A_{t-\tau}\right)h^{*2}_\tau - A_{t-\tau^*}\underline{h}^{*2}\right] \end{aligned}$$

This production function exhibits constant returns to scale. In fact, it is a perfectly Neoclassical production function in labor, L_t, and the heterogenous capital inputs $\{K_{t,\tau}\}_{\tau=0}^{M}$.

However, because of the assignment of capital over workers, capital and labor are not separable in this production function. On the contrary, the amounts of capital and labor interact in a complex manner through the assignment of machines to workers, which determines the h^*_τ's.

Hence, there is no aggregate production function representation for the final goods sector in terms of a single capital aggregate $J\left(\{K_{t,\tau}\}_{\tau=-\infty}^{\infty}\right)$ that is homogenous of degree one in the capital inputs $\{K_{t,\tau}\}_{\tau=-\infty}^{\infty}$ and the aggregate labor input L_t. Therefore, the concept of a capital price index is ill-defined in this model. A capital price index does not exist, because there is no properly defined theoretical aggregate capital stock.

4.2 Measured relative price of investment

The theoretical non-existence of an aggregate capital stock does not mean that one cannot apply price index methods to obtain a spurious estimate of it.

Such an estimate would be spurious in the same sense as the regressions in Granger and Newbold (1974). That is, the spurious regressions in Granger and Newbold (1974) involve the estimation of a non-existent stationary linear relationship between two random walks. Here, the spurious capital measure involves the estimation of a non-existent aggregate capital stock.

There are, in principle, many different ways to construct such a price index $P_{K,t}$, each of which essentially employs a different price index formula. Furthermore, since in every period some machines exit the market while others enter, one also has to decide on how to deal with the inclusion of new goods.

The aim of this paper is not to be an exposition on the many price index methods. Instead, it is meant to illustrate a conceptual problem with the application of them in the model economy introduced. Therefore, we will limit our analysis to one of the most common price index formulas. Furthermore, we will consider only two ways of dealing with the inclusion of new goods. The qualitative results derived from the resulting price indices also hold for the application of other common price index methods. That is, we will emphasize the conceptual issues with constructing a capital price index in this model and these issues are robust to what type of capital price index is constructed.

The price index formula we use is the Laspeyres formula. It is a useful benchmark, because as Frisch (1936) and Konüs (1939) already showed, it yields an upperbound on inflation in the standard case in which there are no new capital goods and there exists a proper capital aggregate.

The first way we deal with new goods is to ignore them and simply apply the price index formulas to models of machines that are sold in the two periods between which we calculate capital price inflation. This yields the matched model indices used in, for example, Aizcorbe et al. (2000) and that are commonly applied to capital price indices by the Bureau of Labor Statistics.

The Laspeyres matched model index that aims to measure capital price inflation between $t-1$ and t in our model would yield

(42) $$\pi_t^{(M)} = \frac{\sum_{\tau=1}^{M} P_{t,\tau} K_{t-1,\tau-1}}{\sum_{\tau=0}^{M-1} P_{t-1,\tau} K_{t-1,\tau}} - 1$$

It measures the percentage change in the cost from $t-1$ to t of buying the period $t-1$ machines that are available in period t.

For this matched model Laspeyres index we find that, on the balanced growth path of our economy, it yields a constant percentage decline in the relative price of capital goods relative to consumption goods. That is,

(43) $$\pi_t^{(M)} = \pi^{(M)} < 0 \text{ for all } t$$

The magnitude of the measured price declines depends on cross-vintage profile of the price declines[9]

(44) $$\frac{P_{t,\tau} - P_{t-1,\tau-1}}{P_{t-1,\tau-1}} = \frac{\widehat{P}_{t,\tau} - \widehat{P}_{t-1,\tau-1}}{\widehat{P}_{t-1,\tau-1}}$$

which in its turn depends on the length of the patent M, the cost parameters $\{c_\tau\}_{\tau=0}^{M-1}$ and the growth rate g.

The second way we deal with new goods is to include them by using a hedonic regression model to impute the price of the models that enter and exit for the periods that their prices are not observed. This would result in a hedonic price index.

The Laspeyres hedonic price index that aims to measure capital price inflation between $t-1$ and t in our model would yield

(45) $$\pi_t^{(H)} = \frac{\sum_{\tau=1}^{M} P_{t,\tau} K_{t-1,\tau-1} + P'_{t,M+1} K_{t-1,M}}{\sum_{\tau=0}^{M} P_{t-1,\tau} K_{t-1,\tau}} - 1$$

where $P'_{t,M+1}$ is the imputed price of the machines of vintage age $M+1$ at time t that is imputed using a hedonic regression. In general $P'_{t,M+1}$depends on the specific hedonic regression applied. However, for simplicity we will assume that the price of the obsolete vintage is imputed as $P'_{t,M+1} = P_{t-1,M} = A_{t-M-1}\underline{h}$, then

(46) $$\pi_t^{(H)} = (1 - s_{t-1,M})\, \pi_t^{(M)}$$

[9] Aizcorbe and Kortum (2004) call the price path of a specific vintage over its lifecycle a 'price contour'.

where $s_{t-1,M}$ is the share of the vintage of age M at time $t-1$ and $\pi_t^{(M)}$ is the inflation rate measured using the Laspeyres matched model index defined above. Because $s_{t-1,M} > 0$ and $\pi_t^{(M)} < 0$ are both constant over time on the balanced growth path, we obtain that

$$\pi_t^{(H)} = \pi^{(H)} < 0 \quad \text{for all } t \tag{47}$$

Thus, just like the matched model index, the hedonic Laspeyres capital price index would yield a constant rate of decline in the relative price of capital compared to the consumption good along the balanced growth path. Note, however, that in this case the hedonic price index measures smaller price declines than the matched model one.

Hence, both price indices that we consider here would find a constant rate of decline in the relative price of investment goods, consistent with the observation that drives the results in Greenwood, Hercowitz, and Krusell (1997) for example.

This is because older vintages of machines are assigned to workers with lower skill levels, their prices decline over their product life-cycle. These price declines are aggregated into a decline in the aggregate capital price index. This decline in the aggregate price index consequently reflects four things in this model.

First of all, it reflects the matching of machines with workers and the price declines thus depend on shape of the skill distribution, which we assumed to be uniform here. Secondly, it captures changes in the cross-vintage profile of production costs, $\{c_\tau\}_{\tau=0}^{M-1}$. Learning by doing allows suppliers of older vintages to charge a lower price than their competitors that sell newer vintages. Thirdly, it depends on the competitive structure on the supply side of machines. That is, which suppliers supply which vintages and how many vintages are sold in the market. Finally, it depends on the growth rate of embodied technological change g.

4.3 Real investment, capital stock, and output

Since there is full depreciation of machines in every period, the capital stock is equal to gross investment in each period. That is, both the capital aggregate K_t and real investment I_t in our model economy would be measured as the capital expenditures in period t deflated by a capital price index. Since firms in the final goods sector make zero profits in equilibrium, capital expenditures equal total revenue minus the wage bill. That is, capital expenditures equal $(1 - s_L)\, Y_t$. Consequently, the capital aggregate and real investment are constructed as

$$K_t = I_t = (1 - s_L) \frac{Y_t}{P_{K,t}} \tag{48}$$

where $P_{K,t}$ is the capital price index.

This implies that the growth rates of real investment and the capital stock equal

$$g - \pi \tag{49}$$

Where π is the percentage change in the capital price index derived in the subsection above.

Real GDP in this economy would equal the value added of the final goods sector deflated by the price of the final good, which we normalized to 1 here because it is the numeraire good, plus the value added of the investment goods sector deflated by the investment price index, $P_{K,t}$.

That is real GDP equals

$$\text{real}\, GDP_t = Y_t + \frac{(I_t - X_t)}{P_{K,t}} \tag{50}$$

On the balanced growth path, this implies that the measured growth rate of real GDP equals

$$\frac{\text{real}\, GDP_t - \text{real}\, GDP_{t-1}}{\text{real}\, GDP_{t-1}} = g - \frac{\left(\overline{\widehat{I}} - \overline{\widehat{X}}\right)}{\overline{\widehat{Y}} + \left(\overline{\widehat{I}} - \overline{\widehat{X}}\right)} \pi \tag{51}$$

Hence, beyond the trend in technological progress, g, measured economic growth in this economy depends on the other three factors that affect the spurious investment price inflation rate π. That is, if any of these factors change, measured economic growth will change, even though there is no shift in the rate of technological change in the economy.

4.4 Productivity

We already showed that there is no growth in total factor productivity in the final goods sector of our model economy. This follows from the construction of the aggregate production function above. That is, if the final goods sector uses the same amounts of labor and the same number of machines for each particular vintage at two different points in time, then it would produce the same amount of output at both points in time. There is no technological progress in this model that shifts the productivity of all factors of production in the same way, where each vintage of machine is considered a separate production factor because there is no aggregate capital stock, and thus TFP growth is zero.

All productivity growth in this model is embodied in the new machines that become available over time. Without the adoption of the new machines productivity levels in the final goods sector would not be increasing over time.

Hence, what we would like to get out of an accounting exercise that distinguishes between total factor productivity and embodied technological change is that TFP growth is zero in the final goods sector and that all growth is due to the quality improvements of machines.

Would our current methods of measuring investment specific technological change (and of growth accounting) yield this result in the model economy here? What would happen if we would apply growth accounting techniques in our model economy to assess the contributions of total factor productivity growth and of investment specific technological change?

Using growth accounting for the final goods sector involves dividing the growth of output in this sector into its three contributing factors. The first is the growth of the labor input. The second is capital deepening, i.e. the growth of capital inputs as measured by the "quality adjusted" real capital stock we discussed in the previous subsection.The final part is TFP growth, i.e. the Solow residual, it is simply the part of output growth that is not attributed to growth of the capital and labor inputs.

In practice, this boils down to applying a log-linear approximation of the neoclassical production function and assuming that marginal products equal factor prices, (32), to obtain the decomposition

$$\left(\frac{Y_t - Y_{t-1}}{Y_{t-1}}\right) \approx \left(\frac{Z_t - Z_{t-1}}{Z_{t-1}}\right) + s^f_{L,t}\left(\frac{L_t - L_{t-1}}{L_{t-1}}\right) + \left(1 - s^f_{L,t}\right)\left(\frac{K_t - K_{t-1}}{K_{t-1}}\right) \tag{52}$$

where Z_t represents the measured level of TFP, $s^f_{L,t}$ is the share of labor in the final goods sector, and K_t is the measured capital aggregate.

As derived above, on the balanced growth path, output of the final goods sector grows at a constant rate g, the labor share in the final goods sector is constant, i.e. $s_{L,t} = s^f_L$, and the labor inputs are constant and equal one, i.e. $L_t = 1$ for all t. This implies that, along the balanced growth path in our model economy, this decomposition simplifies to

$$g = \left(\frac{Z_t - Z_{t-1}}{Z_{t-1}}\right) + \left(1 - s^f_L\right)\left(\frac{K_t - K_{t-1}}{K_{t-1}}\right) \tag{53}$$

Thus, on the balanced growth path our growth accounting exercise will attribute output growth either to TFP growth, i.e. to the growth of Z_t, or to capital deepening, i.e. the growth of K_t. The growth rate of TFP is the residual, after the subtraction of the capital deepening contribution from g.

Substitution of (48) into the above equation yields that TFP will be measured as a weighted average of output growth and the capital price declines. That is,

$$\left(\frac{Z_t - Z_{t-1}}{Z_{t-1}}\right) = s_L^f g + \left(1 - s_L^f\right)\left(\frac{P_{K,t} - P_{K,t-1}}{P_{K,t-1}}\right) \tag{54}$$

Hence, what is crucial for the growth accounting results in our model is the capital price index $P_{K,t}$ used for it.

Since we already argued that all growth in the final goods sector of this economy is due to quality increases in capital and that there is no TFP growth, i.e.

$$\left(\frac{Z_t - Z_{t-1}}{Z_{t-1}}\right) = 0 \tag{55}$$

in the sector, we would like our capital price index to satisfy that its measured growth rate, π, satisfies

$$\pi = \left(\frac{P_{K,t} - P_{K,t-1}}{P_{K,t-1}}\right) = -\frac{s_L^f g}{\left(1 - s_L^f\right)} \tag{56}$$

However, there is nothing in our model that assures us that this is the actual percentage change in the relative price of capital, $P_{K,t}$, measured using common price index methods.

5 Numerical example: A New Economy?

In order to illustrate the implications of our theoretical results, we provide a numerical example. Our approach will be to calibrate a benchmark set of parameter values. We then proceed by changing the competitive structure in the capital goods market, by changing the number of models sold, M, the cost parameters $\{c_\tau\}_{\tau=0}^{\infty}$, the distribution of suppliers over machine vintages, and show how these changes affect the measured growth rates of economic aggregates. We also show how a shift in the skill distribution of workers affects the measured growth rates of aggregates in our model economy. Finally, we show that an increase in the growth rate of embodied technological change g, leads to an even bigger measured increase in the growth rate of real GDP.

For our benchmark case we use a year as a period and calibrate our parameters based on the machines in our model representing equipment in the U.S. economy. This means that the theoretical labor input in the model should be interpreted as a composite of labor and structures. Because of this interpretation, we would like to emphasize the illustrative nature of the numerical results here.

They are by no way meant to quantify any biases in existing empirical analyses. Instead, they are meant to illustrate conceptual measurement issues.

We choose the growth rate, g, to equal the average growth rate of output, which in our case equals the sum of personal consumption expenditures and fixed private non-residential investment, divided by the PCE deflator for 1960-2005. It turns out to equal 3.7% annually. The number of models sold is calibrated to equal the length of a U.S. patent in years, i.e. 20. For illustrative purposes, we chose the monopolist case a our benchmark. The cost parameters are constant across vintages, such that $c_\tau = c$ for all τ. We choose c, and the skill distribution parameters, $\underline{h}$ and $\overline{h}$, such that our model approximately satisfies the following three conditions.

The aggregate labor share, i.e. the share of labor and structures, equals 83%, such that the share of equipment in value added in this economy is 17%. This is consistent with the output elasticity of 0.17 that Greenwood, Hercowitz, and Krusell (1997) for their Cobb-Douglas production function. The second condition is that equipment investment as a share of output equals the average of 9% observed over 1960-2005 in the data. The final condition is that measured investment price deflation is about 6% annually, which is in line with the estimates reported in Cummins and Violante (2002).

The column labeled 'benchmark' of Table 1 lists the results for our benchmark case. In this case, the spuriously measured growth rate of real investment equals 10.6%, while that of real GDP is 4.2%. Contrary to the theoretical results derived above, growth accounting does find positive TGP growth, both for the overall economy, by applying the methodology of Greenwood, Hercowitz, and Krusell (1997), as well as in the final goods sector. Even though production in the latter does not exhibit and factor neutral technological change, a standard growth accounting exercise in this case would suggest TFP growth of 2.6% annually. The corresponding price per efficiency contour, $\widehat{P}_\tau$, that drives these results is depicted as the solid line in Figure 1.

We will consider the effect of five particular changes in the benchmark parameters. The first four are changes to the competitive and cost structure underlying the supply of capital goods. The fifth, and final, change is one in which the growth rate of embodied technological change doubles. In theory, the first four changes should not affect the measured growth rates of economic aggregates, since they do not affect theoretical trend growth in the economy. We will show, however, that it does affect measured growth.

Case (I) is one in which machines are supplied by monopolistic competitors rather than by one monopolist. As one can see from Figure 1, in this case the increased degree of competition lowers all prices. Furthermore, the competition between suppliers of adjacent vintages also leads to more rapid price declines for the most advanced vintages with the highest market shares in this case. As a result, measured investment prices decline 4.4% to 4.7% faster in this case than under the monopoly. This also means an increase in the rate of growth of real investment. The effect of this change in the competitive structure on other measured growth rates is subdued because the increased competition implies that the suppliers of capital goods extract less of the value added produced in the final goods sector. This results in a decrease in the nominal investment ratio as well as increases in the labor shares in both the overall economy and the final goods sector. These changes in the composition of value added almost counterbalance the change in the growth rate of real investment. Consequently,.the measured growth rates of real GDP as well as aggregate TFP and TFP in the final goods sector are almost the same in this case as in our benchmark.

In case (II) we illustrate what happens when the product lifecycle is shortened. In this case we halved it from 20 models to 10. This shortening reduces the opportunity for the monopolist to price discriminate between the different workers and thus the capital goods supplier extracts less of the producer surplus in the final goods sector. This leads to a lower investment ratio as well as a steeper decline in prices over the lifecycle. The outcome is that measured investment price declines are almost double that of the benchmark case. Furthermore, the growth rate of real investment increases from 10.6% to 18.4%. Just like in the previous case, this change in the growth rate of real investment is in large part offset by the change in the nominal investment share, leading to little change in the measured growth rates of the other aggregates.

Case (III) is one in which there is an increasing disparity in skills. In particular, we will assume that the best workers become twice as good. In that case the monopolists price differentation is more effective, because of the increased heterogeneity on its demand side. Consequently, the nominal investment share increases. All of a sudden, the measured investment price declines, which themselves are almost the same as in the benchmark case, become more important in the measurement of the economic aggregates. This leads to an increase in the growth rate of real GDP and to substantial decreases in the growth rates of aggregate and final goods sector TFP.

In case (IV) an decrease in the returns to scale in the machine producing sector leads to a reduction in the nominal investment ratio. This has the opposite effect on the measured growth rates of economic aggregates as case (III).

In case (V) we double the growth rate of embodied technological change. The most remarkable result for this case is that this increase in embodied technological change is actually mostly captured through a doubling of the growth rates of TFP in both the aggregate economy as well as the final goods sector. That is, even though the observed acceleration in output growth in this case would be completely embodied in machines, existing growth accounting techniques would attribute the majority of it to an acceleration in factor neutral technological progress.

The cases that we considered here are definitely not exhaustive. What they do show, however, is how deviations from the neoclassical assumptions used to measure economic activity and productivity growth, can result in these measures being distorted by factors that should, in principle, not affect them.

6 Implications

So far, we have presented a theoretical example to illustrate how the construction of an aggregate investment price index for a non-existent capital aggregate can lead to misleading inference about the trend properties of the macroeconomy. That is, we revisited the Cambridge Capital Controversy and showed how it might apply to our measures of economic growth and productivity.

The Capital Controversy, however, is often disregarded as 'silly' and irrelevant. In this section we set out to do two things. First of all, we aim to show that it might not be so silly at all. We do so by discussing a set of facts that are inconsistent with the assumption of the existence of a proper capital aggregate. Secondly, we discuss a set of recent empirical results for which the existence of such aggregates is very relevant, because they are especially driven by the capital price indices that are applied.

6.1 Does an aggregate capital stock exist?

We have shown, in our analysis here, that the application of a capital price index in the absence of a capital aggregate can potentially lead to misleading inference about growth in economic aggregates.

Contrary to the the spurious regressions in Granger and Newbold (1974), for which their identifying assumption can be tested, there is no statistical method that allows us to test for the existence of an aggregate capital stock. However, from Fisher (1969) we know that the only case in which an aggregate capital stock representation exists is when all vintage production functions are Cobb-Douglas. In that case, the aggregate production function representation is the Cobb-Douglas representation used in, for example, Greenwood, Hercowitz, and Krusell (1997). This specification implies several testable empirical implications that can potentially be falsified.

On the aggregate level, the transitional dynamics in the model in Greenwood, Hercowitz, and Krusell (1997) are basically the same as those in the Solow growth model with a depreciation rate that is adjusted for the relative price declines of capital goods. Gilchrist and Williams (2000,2001) argue, however, that the actual transitional dynamics of the U.S. economy, and that of Germany and Japan, are probably better described by a putty-clay vintage capital model in which an aggregate capital stock does not exist than by the conventional Solow model.

At the disaggregate level, there is actually some relevant information in the cross sectional behavior of prices across models sold. The Cobb-Douglas model has very stark predictions about the prices of different capital vintages. As we show in Appendix A, it implies that relative prices reflect relative quality differences across machines, no matter what the human capital level of the worker is that they are matched with.

Formally, it implies that for all vintages τ for which there is non-zero investment

$$\widehat{P}_{t,\tau} = \frac{P_{t,\tau}}{A_{t-\tau}} = \widetilde{\widehat{P}}_t \text{ and does not depend on } \tau \tag{57}$$

Hence, the price per efficiency unit is the same for all vintages for which there is strictly positive demand.

In such a model, the introduction of a new machine does not necessarily imply that the prices of the other models decline. Furthermore, assuming that the quality of each particular vintage is constant over time, such a model implies that the prices of all machines decline at the same rate to maintain their relative quality ratios.

In our model, the relative prices of machines depend both on their relative quality levels as well as their production costs and the workers that they are matched up with. Consequently, our model implies that the prices of older models decline when a new and better model is introduced. This is because the assortative matching between machines and workers implies that the older models are now sold to workers with lower human capital levels that are less productive using them.

This implication of our model is consistent with the behavior of prices of high-tech goods, like computers, printers, and microprocessors. For example, in May 2004 Intel introduced three faster Pentium M chips and reduced the price of older Pentium M chips by as much as 30 percent[10].

[10] Source: "Daily Briefing." *The Atlanta Journal Constitution.* May 11, 2004. Business Section. Page 2D.

Similarly, in April 2003 Hewlitt Packard introduced new models of its LaserJet Printers and reduced the price of older models by as much as 20 percent[11].

The price contours of semiconductors plotted in Aizcorbe and Kortum (2004) also suggest that not all models exhibit the same percentage price decline at each point in time. This is not consistent with the Cobb-Douglas assumption needed for the existence of an aggregate capital stock and suggests that relative prices reflect more than only relative quality differences.

Hence, both at the aggregate and the disaggregate level, there is evidence that the assumption of the existence of an aggregate capital stock might not be valid and that, thus, the application of a capital price index might be misleading.

6.2 Investment price index driven results

In recent years, there have been many studies that have heavily relied on investment price indices. The reason for this is the increased importance of information technology equipment and the inherent problem of accounting for the quality improvements embodied in it. Two strands of the literature stand out in particular in this respect.

The first is that on investment specific technological change, i.e. higher productivity growth in the capital goods producing sector than in the consumption goods producing sector, as initiated by Greenwood, Hercowitz, and Krusell (1997).

Greenwood, Hercowitz, and Krusell (1997) were the first to use the changes in a quality adjusted capital price index, in particular one based on Gordon (1990), relative to the changes in the consumption price index as a measure of investment specific technological change in a general equilibrium framework. They use the capital price index to decompose productivity growth into disembodied TFP growth and growth induced by the decline of the quality adjusted relative price of capital goods, known as investment specific technological change.

Their analysis yields the result that, since the middle of the 1970's, the quality adjusted relative price declines of investment goods have accelerated, increasing the contribution of investment specific technological change to U.S. output growth. This, in principle, is not inconsistent with the observation that quality improvements in computers and other IT capital goods have accelerated since the middle of the 1970's. There is, however, one catch.

[11] Source: HP News Release. "HP Announces Innovative New Products and Services for Small- and Medium-sized Business." April 2, 2003.

The results in Greenwood, Hercowitz, and Krusell (1997) also yield that the rate of investment specific technological change measured using a quality adjusted investment price index implies that TFP growth in the U.S. has been persistently negative between 1973 and 1990. The average annual decline in TFP for the period between 1973 and 1990 reported in their analysis is 0.9%.

In principle, it is not hard to come up with an explanation why TFP could temporarily decline. It is much harder, however, to come up with a story why TFP would decline persistently over a 17 year long period and why this decline would exactly coincide with the time that investment specific technological change accelerates. This raises the issue whether the price index representing the relative price of capital might not be the appropriate measure of investment specific technological change and whether it might overstate the contribution of quality improvements of capital goods to economic growth. The latter is exactly the case in the model economy here.

Subsequently, many other empirical studies of technological change have used changes in the relative price of investment goods with respect to consumption goods as a measure of the degree of investment specific or embodied technological change. These studies include, among others, Greenwood, Hercowitz, and Krusell (1997,2000), Violante, Ohanian, Ríos-Rull, and Krusell (2000), Cummins and Violante (2002), Fisher (2002), and Altig, Christiano, Eichenbaum, and Linde (2005).

The second strand of the literature is made up of growth accounting studies that emphasize the importance of IT capital deepening for U.S. output growth. These studies also have the potential to suffer from the same measurement problem introduced in this paper. That is, just like Greenwood, Hercowitz, and Krusell (1997), these growth accounting studies, like Jorgenson and Stiroh (2000) and Oliner and Sichel (2000), also assume the existence of an aggregate IT capital stock. Therefore, the same issue raised by us in this paper might lead them to overestimate the contribution of IT goods to U.S. output growth.

IT capital seems to be especially subject to the issue raised in this paper, because, as documented by Aizcorbe and Kortum (2004), the price contours that we observe for these capital goods are much more similar to those implied by our model than those implied by the Cobb-Douglas case in which an IT capital aggregate would be well-defined.

7 Conclusion

This paper is a note of caution on the use of capital price indices. We use a vintage capital model in which workers are matched with machines of increasing quality to illustrate a potential problem with the application of such price indices. The most important feature of our model economy, for the results in this paper, is that there does not exist an aggregate production function representation in terms of a single capital aggregate. Hence, a capital price index in this economy is a spurious measure of something that is not defined.

However, the nonexistence of a capital stock does not mean that one cannot apply a capital price index to obtain measures of the trend properties of aggregate economic activity. We show that when one does so in our model economy, these measures yield misleading results on productivity and economic growth. Besides technological progress, measured growth rates in our model economy also depend on the competitive structure of the capital goods producing sector, which is irrelevant for the economy's long-term growth rate.

We use a numerical example to show that a shift in this competitive structure might lead to spurious increases in the measured rate of decline in the relative price of investment, and the perceived growth rates of real investment, real GDP, and investment specific technological change.

Since these are exactly the facts emphasized by proponents of the hypothesis that there is a 'New Economy' in the U.S. driven by information technology and since the predictions of our model about the path of prices of machines over their life-cycle seems to be broadly consistent with that observed for IT capital goods, our note of caution seems especially relevant for this 'New Economy Hypothesis'.

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A Proofs

Proof of equations (12) and (13):

To see why (12) is true, consider $h' > h$ and $\tau' > \tau$, then $h \in D(\tau, \mathbf{P}_t, \mathbf{A}_t)$ implies that

$$\forall\ s \in \mathbf{T}_t: \ A_{t-\tau}h - P_{t,\tau} \geq A_{t-s}h - P_{t,s} \tag{58}$$

or, equivalently, in terms of marginal benefits and costs

$$\forall\ s \in \mathbf{T}_t: \ \left(A_{t-\tau} - A_{t-s}\right) h \geq P_{t,\tau} - P_{t,s} \tag{59}$$

Consequently, because for all $\tau' > \tau$ strictly positive technological progress implies $A_{t-\tau'} > A_{t-\tau}$, the marginal benefits from updating for the worker of type h' exceed those of the worker of type h. That is,

$$\forall \tau' > \tau: \ \left(A_{t-\tau} - A_{t-\tau'}\right) h' > \left(A_{t-\tau} - A_{t-\tau'}\right) h \geq P_{t,\tau} - P_{t,\tau'} \tag{60}$$

This implies that it must thus be true that $h' \notin D_t(\tau', \mathbf{P}_t, \mathbf{A}_t)$ for all $\tau' > \tau$.

The result of equation (12) implies that the demand sets are connected for the following reason. Suppose there would be a demand set that was not connected, then there exist $h'' > h' > h$ such that $h'' \in D_t(\tau, \mathbf{P}_t, \mathbf{A}_t)$, $h' \in D_t(\tau', \mathbf{P}_t, \mathbf{A}_t)$, and $h \in D_t(\tau, \mathbf{P}_t, \mathbf{A}_t)$ where $\tau \neq \tau'$. However, if $\tau > \tau'$, then the choices of h'' and h' do not satisfy assortative matching. On the other hand, if $\tau' > \tau$, then the choices of h' and h do not satisfy assortative matching. Hence, the demand sets need to be connected.

If the demand sets are connected and subsets of the interval $(\underline{h}, \overline{h}]$, then they have to be of the form given in equation (13).

The proof that the set of all workers that is indifferent between two machines is negligible is a bit more involved. Let $\mathcal{H}_t$ denote the set of all human capital levels for which the workers are indifferent between two vintages of machines at time t. Since the human capital levels are uniformly distributed, it suffices to prove that $\mathcal{H}_t$ contains a finite number of elements. Since we have already derived that workers will only use technologies $\{0, \dots, M\}$ there are only a finite number of combinations between which workers can be indifferent.

We will show that, if a worker of type h is indifferent between two intermediate goods, then no other worker will be. That is, define the set

$$\mathcal{H}_t^*\left(\tau, \tau'\right) = \left\{h \in (\underline{h}, \overline{h} \,\middle|\, h \in D_t(\tau) \wedge h \in D_t\left(\tau'\right)\right\} \tag{61}$$

such that

$$\mathcal{H}_t = \bigcup_{\tau=0}^{M-1} \bigcup_{\tau'=\tau+1}^{M} \mathcal{H}_t^*\left(\tau, \tau'\right) \tag{62}$$

and, denoting the Lebesque measure as $\mu(.)$, we obtain

$$\mu\left(\mathcal{H}_t\right) \leq \frac{1}{\overline{h}} \sum_{\tau=0}^{M-1} \sum_{\tau'=\tau+1}^{M} \mu\left(\mathcal{H}_t^*\left(\tau, \tau'\right)\right) \tag{63}$$

We will simply show that $\forall \tau' > \tau : \mu\left(\mathcal{H}_t^*\left(\tau, \tau'\right)\right) = 0$. Let $h \in (\underline{h}, \overline{h}]$ be such that $h \in D_t(\tau)$ as well as $h \in D_t(\tau')$ for $\tau' > \tau$. In that case

$$A_{t-\tau}h - P_{t,\tau} = A_{t-\tau'}h - P_{t,\tau'} \tag{64}$$

or equivalently

$$(65) \qquad \left(A_{t-\tau} - A_{t-\tau'}\right) h = P_{t,\tau} - P_{t,\tau'}$$

This, however implies that for all $h' > h > h''$

$$(66) \qquad \left(A_{t-\tau} - A_{t-\tau'}\right) h' > P_{t,\tau} - P_{t,\tau'} > \left(A_{t-\tau} - A_{t-\tau'}\right) h''$$

such that the workers of type $h' > h$ will prefer τ over τ', while workers of type $h'' < h$ will do the opposite. Hence, $\mathcal{H}_t^* \left(\tau, \tau'\right) = \{h\}$ and is of measure zero.

Proof of equations (18) through (21):

We will prove these equations in three steps. In the first step, we prove equation (18) and show that, irrespective of $\mathbf{A}_t$, M, and $\{c_\tau\}_{\tau=0}^M$, the suppliers will set their prices such that there is demand for each of the vintages. In the second step, we derive the first order conditions that, given that it is interior, determine the optimal price schedule and show that the suppliers make strictly positive profits of the supply of each of the patented vintages. That is, we prove equation (19). In the final step, we prove the properties of the price schedule per efficiency unit that are formalized in equations (20) and (21).

Strictly positive demand for all M newest vintages: In order to prove equation (18), it turns out to be useful to introduce the function that relates a vintage back to its supplier. We denote this function by $\iota(\tau)$. It is equal to the index number of the supplier that supplies machines of vintage τ.

Furthermore, to keep track of which vintages are supplied by the same supplier and which are not, we define the indicator function

$$(67) \qquad I\left[a = b\right] = \left\{ \begin{array}{l} 1 \text{ if } a = b \\ 0 \text{ if } a \neq b \end{array} \right\}$$

so that $I\left[\iota(\tau) = \iota(\tau')\right]$ is equal to one if vintages τ and τ' are supplied by the same supplier and zero otherwise.

For this proof we will consider the supplier of vintage τ and consider the effect of its price setting on the profits made from the supply of vintage τ, as well as that of vintage ages $\tau - 1$ and $\tau + 1$. Here we assume, without loss of generality that these adjacent vintages have prices set such that $K_{t,\tau-1}, K_{t,\tau+1} > 0$ in case vintage τ would not be supplied. We will distinguish the cases $\tau = 0$, for which $K_{t,\tau-1}$ is irrelevant, and $\tau = M - 1$, for which we know that there are no profits made of vintage $\tau + 1$.

For this vintage τ, we will show that there exists a price $P_{t,\tau} > 0$ such that the supplier makes strictly positive profits of the supply of vintage τ as well as that this price increases the sum of the profits over all three vintages $(\tau - 1, \tau, \tau + 1)$, or any two of these vintages that include τ. That is, independent of the prices of the adjacent vintages for which there is demand, the supplier of vintage τ can increase its profits, no matter whether it only owns the patent for vintage τ or any of the patents for the adjacent vintages. The assortative matching result implies that looking at three adjacent vintages is enough for this argument, because the price set for vintage τ at the margin only affects the demand for the adjacent vintages.

Let us first determine the reservation price level, above which vintage τ will not be demanded at all. This price level is determined by the type of worker, that, without the availability of vintage τ, is indifferent between vintage

$\tau-1$ and $\tau+1$. We denote the human capital level of this worker by $\widetilde{h}$. It has to satisfy

$$A_{t-\tau+1}\widetilde{h}-P_{t,\tau-1}=A_{t-\tau-1}\widetilde{h}-P_{t,\tau+1} \tag{68}$$

such that

$$\widetilde{h}=\begin{cases}\overline{h} & \text{for } \tau=0\\ \frac{P_{t,\tau-1}-P_{t,\tau+1}}{A_{t-\tau+1}-A_{t-\tau-1}} & \text{for } \tau>0\end{cases} \tag{69}$$

Hence, demand for vintage τ implies that its price level much be such that

$$A_{t-\tau}\widetilde{h}-P_{t,\tau}\geq A_{t-\tau+1}\widetilde{h}-P_{t,\tau-1}=A_{t-\tau-1}\widetilde{h}-P_{t,\tau+1} \tag{70}$$

In terms of the price per efficiency unit, this implies that

$$\begin{aligned}\hat{P}_{t,\tau} &\leq \begin{cases}\frac{A_t-A_{t-1}}{A_t}\overline{h}+\frac{A_{t-1}}{A_t}\widehat{P}_{t,1} & \text{for } \tau=0\\ \left[\begin{array}{c}\frac{A_{t-\tau+1}}{A_{t-\tau}}\left(\frac{A_{t-\tau}-A_{t-\tau-1}}{A_{t-\tau+1}-A_{t-\tau-1}}\right)\widehat{P}_{t,\tau-1}\\ +\frac{A_{t-\tau-1}}{A_{t-\tau}}\left(\frac{A_{t-\tau+1}-A_{t-\tau}}{A_{t-\tau+1}-A_{t-\tau-1}}\right)\widehat{P}_{t,\tau+1}\end{array}\right] & \text{for } \tau>0\end{cases}\\ &\equiv \widetilde{P}_{t,\tau}\end{aligned} \tag{71}$$

Hence, $\widetilde{P}_{t,\tau}$ is the maximum price per efficieny unit at which the supplier of vintage τ faces positive demand.

The supplier of vintage τ has three options. First of all, it can choose to make vintage τ available at minimum cost, in which case $P_{t,\tau}=\underline{h}A_{t-\tau}$ and the firm would make non-positive profits. Secondly, it could choose $P_{t,\tau}\geq A_{t-\tau}\widetilde{P}_{t,\tau}$ at which it faces no demand and profits are zero. Finally, it can choose a price $P_{t,\tau}\geq A_{t-\tau}\left(\widetilde{P}_{t,\tau}-\varepsilon\right)$ where $0<\varepsilon<\widetilde{P}_{t,\tau}$.

The firm will choose the third option, whenever that option increases the profits its makes over all the vintages its supplies. In the following we will show that, independent of the prices $P_{t,\tau-1}$ and $P_{t,\tau+1}$, there exists an $\varepsilon>0$ for which this is the case.

We will consider the profits that the supplier of vintage τ makes when it chooses a price equal to

$$\hat{P}_{t,\tau}=\widetilde{P}_{t,\tau}-\varepsilon \text{ for } \varepsilon>0 \tag{72}$$

For a small enough $\varepsilon>0$ when $K_{t,\tau-1},K_{t,\tau+1}>0$ the choice of $\hat{P}_{t,\tau}$will not affect the demand of vintages other than those of vintage ages $\tau-1$, τ and $\tau+1$. Hence, for small $\varepsilon>0$, which turns out to be the relevant case in this proof, what matters for the supplier of vintage τ and what determines the price it chooses is whether it also supplies vintage $\tau-1$, and/or $\tau+1$, or neither of them.

At the price $\hat{P}_{t,\tau}=\widetilde{P}_{t,\tau}-\varepsilon$ the demand for vintage τ can be shown to equal

$$K_{t,\tau}=\left(I\left[\tau\neq 0\right]\frac{A_{t-\tau}}{A_{t-\tau+1}-A_{t-\tau}}+\frac{A_{t-\tau}}{A_{t-\tau}-A_{t-\tau-1}}\right)\frac{\varepsilon}{\overline{h}-\underline{h}} \tag{73}$$

The new profits over the three adjacent vintages for the supplier of vintage τ are given by

$$I\left[\iota(\tau-1)=\iota(\tau)\right]A_{t-\tau+1}\left(\widehat{P}_{t,\tau-1}-\underline{h}-\frac{c_{\tau-1}}{2}\left(K_{t,\tau-1}-\frac{A_{t-\tau}}{A_{t-\tau+1}-A_{t-\tau}}\frac{\varepsilon}{\overline{h}-\underline{h}}\right)\right)\times$$
$$\left(K_{t,\tau-1}-\frac{A_{t-\tau}}{A_{t-\tau+1}-A_{t-\tau}}\frac{\varepsilon}{\overline{h}-\underline{h}}\right)+$$
$$A_{t-\tau}\left(\widetilde{P}_{t,\tau}-\underline{h}-\left[1+\frac{c_{\tau}}{2}\left(I\left[\tau\neq 0\right]\frac{A_{t-\tau}}{A_{t-\tau+1}-A_{t-\tau}}+\frac{A_{t-\tau}}{A_{t-\tau}-A_{t-\tau-1}}\right)\right]\frac{\varepsilon}{\overline{h}-\underline{h}}\right)\times$$
$$\left(I\left[\tau\neq 0\right]\frac{A_{t-\tau}}{A_{t-\tau+1}-A_{t-\tau}}+\frac{A_{t-\tau}}{A_{t-\tau}-A_{t-\tau-1}}\right)\frac{\varepsilon}{\overline{h}-\underline{h}}+$$
$$I\left[\iota(\tau+1)=\iota(\tau)\right]A_{t-\tau-1}\left(\widehat{P}_{t,\tau+1}-\underline{h}-\frac{c_{\tau+1}}{2}\left(K_{t,\tau+1}-\frac{A_{t-\tau}}{A_{t-\tau}-A_{t-\tau-1}}\frac{\varepsilon}{\overline{h}-\underline{h}}\right)\right)\times$$
$$\left(K_{t,\tau+1}-\frac{A_{t-\tau}}{A_{t-\tau}-A_{t-\tau-1}}\frac{\varepsilon}{\overline{h}-\underline{h}}\right)$$

Which simplifies to

(74)
$$\begin{aligned}&I\left[\iota(\tau-1)=\iota(\tau)\right]A_{t-\tau+1}\left(\widehat{P}_{t,\tau-1}-\underline{h}-\frac{c_{\tau-1}}{2}K_{t,\tau-1}\right)K_{t,\tau-1}+\\&I\left[\iota(\tau+1)=\iota(\tau)\right]A_{t-\tau-1}\left(\widehat{P}_{t,\tau+1}-\underline{h}-\frac{c_{\tau+1}}{2}K_{t,\tau+1}\right)K_{t,\tau+1}+\\&a\left(\frac{\varepsilon}{\overline{h}-\underline{h}}\right)-b\left(\frac{\varepsilon}{\overline{h}-\underline{h}}\right)^2\end{aligned}$$

where $a>0$ and $b>0$. In particular, they equal

(75)
$$a = I\left[\tau=0\right]A_t\left(\overline{h}-\underline{h}\right)$$

(76)
$$+I\left[\tau\neq 0\right]\left(1-I\left[\iota(\tau)=\iota(\tau-1)\right]\right)\frac{A_{t-\tau+1}A_{t-\tau}}{A_{t-\tau+1}-A_{t-\tau}}\left(\widehat{P}_{t,\tau-1}-\underline{h}\right)$$

(77)
$$+\left(1-I\left[\iota(\tau+1)=\iota(\tau)\right]\right)\frac{A_{t-\tau}A_{t-\tau-1}}{A_{t-\tau}-A_{t-\tau-1}}\left(\widehat{P}_{t,\tau+1}-\underline{h}\right)$$
$$+I\left[\iota(\tau-1)=\iota(\tau)\right]\frac{A_{t-\tau+1}A_{t-\tau}}{A_{t-\tau+1}-A_{t-\tau}}c_{\tau-1}K_{t,\tau-1}$$

(78)
$$+I\left[\iota(\tau+1)=\iota(\tau)\right]\frac{A_{t-\tau-1}A_{t-\tau}}{A_{t-\tau}-A_{t-\tau-1}}c_{\tau+1}K_{t,\tau+1}$$

and

(79)
$$b = \frac{c_{\tau-1}}{2}I\left[\iota(\tau-1)=\iota(\tau)\right]A_{t-\tau+1}\left(\frac{A_{t-\tau}}{A_{t-\tau+1}-A_{t-\tau}}\right)^2$$
$$+\left[1+\frac{c_{\tau}}{2}\left(I\left[\tau\neq 0\right]\frac{A_{t-\tau}}{A_{t-\tau+1}-A_{t-\tau}}+\frac{A_{t-\tau}}{A_{t-\tau}-A_{t-\tau-1}}\right)\right]\times$$

(80)
$$\left(I\left[\tau\neq 0\right]\frac{A_{t-\tau}}{A_{t-\tau+1}-A_{t-\tau}}+\frac{A_{t-\tau}}{A_{t-\tau}-A_{t-\tau-1}}\right)+$$
$$+I\left[\iota(\tau+1)=\iota(\tau)\right]A_{t-\tau-1}\frac{c_{\tau+1}}{2}\left(\frac{A_{t-\tau}}{A_{t-\tau}-A_{t-\tau-1}}\right)^2$$

Note that the first two terms of equation (74) equal the profits that the supplier of vintage τ would have made of the two adjacent vintages, if it would have owned any of them. The term $a\varepsilon-b\varepsilon^2$ represents the additional profits earned due to the supply of vintage τ at price $\widetilde{P}_{t,\tau}-\varepsilon$. Hence, the supplier of vintage τ would always set a price that generates strictly positive demand for that vintage if there exists an $\varepsilon>0$ for which this additional profit is strictly positive. Since there always is an $\varepsilon > 0$ for which $a\varepsilon - b\varepsilon^2 > 0$, it always the case that the supplier of vintage τ will supply that vintage at a price that generates strictly positive demand.

Strictly positive profits: This follows as a corollary from the proof above. The supplier of the vintage τ can always choose its price to strictly increase its profits relative to zero.

$\widehat{P}_{t,\tau}$ **is strictly decreasing in** τ**:** This follows from an induction argument. We have proven above that in the equilibrium there must be strictly positive demand for each of the vintages of age $\tau = 0, \ldots, M-1$, i.e. $K_{t,\tau} > 0$ in equilibrium. In terms of the prices per efficiency unit, the demand sets are

$$K_{t,\tau} = \begin{cases} \frac{1}{\overline{h}-\underline{h}} \left[\overline{h} - \frac{A_t}{A_t - A_{t-1}} \widehat{P}_{t,0} + \frac{A_{t-1}}{A_t - A_{t-1}} \widehat{P}_{t,1} \right] & \text{for } \tau = 0 \\ \frac{1}{\overline{h}-\underline{h}} \left[\frac{A_{t-\tau+1}}{A_{t-\tau+1} - A_{t-\tau}} \left(\widehat{P}_{t,\tau-1} - \widehat{P}_{t,\tau} \right) \right. \\ \quad \left. - \frac{A_{t-\tau-1}}{A_{t-\tau} - A_{t-\tau-1}} \left(\widehat{P}_{t,\tau} - \widehat{P}_{t,\tau+1} \right) \right] & \text{for } \tau = 1, \ldots, M-1 \end{cases} \tag{81}$$

This implies that for $\tau = 1, \ldots, M-1$

$$\frac{A_{t-\tau+1}}{A_{t-\tau+1} - A_{t-\tau}} \left(\widehat{P}_{t,\tau-1} - \widehat{P}_{t,\tau} \right) > \frac{A_{t-\tau-1}}{A_{t-\tau} - A_{t-\tau-1}} \left(\widehat{P}_{t,\tau} - \widehat{P}_{t,\tau+1} \right) \tag{82}$$

Hence, if the price per efficiency unit for vintage age τ is larger than that for $\tau+1$, then it must be the case that the price per efficiency unit for vintage age $\tau - 1$ is higher than that of vintage τ. The only thing we need to proof our claim is a initial result and then we can apply an induction argument.

We do know that in equilibrium the supplier of vintage age $M-1$ will choose a price that yields strictly positive profit, which implies $\widehat{P}_{t,M-1} > \underline{h}$. Furthermore, we know that perfect competition in the supply of vintage M will drive its price to minimum cost, such that $\widehat{P}_{t,M} = \underline{h}$. Hence, we know that $\left(\widehat{P}_{t,M-1} - \widehat{P}_{t,M} \right) > 0$. Applying our induction argument thus yields that this implies that $\left(\widehat{P}_{t,\tau} - \widehat{P}_{t,\tau+1} \right) > 0$ for $\tau = 0, \ldots, M-1$. Hence $\widehat{P}_{t,\tau}$ is strictly decreasing in τ.

$\widehat{\mathbf{P}}_t^* = \widehat{\mathbf{P}} \left(\mathbf{A}_t, \{c_\tau\}_{\tau=0}^{M} \right)$ **where** $\widehat{\mathbf{P}}(.)$ **is homogenous of degree zero in** A_t**:** Supplier i sets the prices of the vintages of machines its supplies to maximize the profits

$$\pi_{t,i} = \sum_{\tau=0}^{M-1} I \left[\iota(\tau) = i \right] A_{t-\tau} \left(\widehat{P}_{t,\tau} - \underline{h} - \frac{c_\tau}{2} K_{t,\tau} \right) K_{t,\tau} \tag{83}$$

The necessary and sufficient conditions for profit maximization in equilibrium imply that this supplier will set the price of each vintage τ which it supplies, i.e. $\iota(\tau) = i$, to satisfy the condition

$$0 = K_{t-\tau} + \sum_{s=0}^{M-1} I \left[\iota(s) = i \right] \frac{A_{t-s}}{A_{t-\tau}} \left(\widehat{P}_{t,s} - \underline{h} - c_s K_{t,s} \right) \frac{\partial K_{t,s}}{\partial \widehat{P}_{t,\tau}} \tag{84}$$

However, note that these optimality conditions are homogenous of degree zero in $\mathbf{A}_t = \{A_t, \ldots, A_{t-M}\}$. This is because the demand functions that determine $K_{t,\tau}$ are homogenous of degree zero in $\mathbf{A}_t = \{A_t, \ldots, A_{t-M}\}$ and so are $\partial K_{t,\tau} / \partial \widehat{P}_{t,\tau}$. Furthermore, besides the productivity levels in $\mathbf{A}_t$ the only other parameters that show up in these equilibrium conditions are the cost parameters $\{c_\tau\}_{\tau=0}^{M-1}$ and the bounds of the skill distribution, i.e. $\overline{h}$ and $\underline{h}$. Thus the equilibrium price per efficiency unit profile is only a function of the productivity levels, the cost parameters, and skill distribution and it is homogenous of degree zero in the productivity levels.

Furthermore, the system of equilibrium conditions, implied by the optimality conditions above, is linear in the prices per efficiency unit and it turns out to be straightforward to show that it has one unique solution. That is, the PSN equilibrium exists and it is unique.

Proof of equation (43) :

The following is the proof of equation (43). The matched model Laspeyres index yields a capital price inflation estimate of

$$\begin{aligned} \pi_t^{(M)} &= \frac{\sum_{\tau=1}^{M} P_{t,\tau} K_{t-1,\tau-1}}{\sum_{\tau=0}^{M-1} P_{t-1,\tau} K_{t-1,\tau}} - 1 = \frac{\sum_{\tau=0}^{M-1} P_{t,\tau+1} K_{t-1,\tau}}{\sum_{\tau=0}^{M-1} P_{t-1,\tau} K_{t-1,\tau}} - 1 \\ &= \sum_{\tau=0}^{M-1} s_{t-1,\tau}^{*} \widehat{\pi}_{t,\tau} \end{aligned} \tag{85}$$

where the shares $s_{t,\tau}^{*}$ are given by

$$s_{t-1,\tau}^{*} = \frac{P_{t-1,\tau} K_{t-1,\tau}}{\sum_{s=0}^{M-1} P_{t-1,s} K_{t-1,s}} = \frac{A_{t-1-\tau} \widehat{P}_{t-1,\tau} K_{t-1,\tau}}{\sum_{s=0}^{M-1} A_{t-1-s} \widehat{P}_{t-1,s} K_{t-1,s}} \tag{86}$$

and represent the expenditure share in period $t-1$ of vintage age τ in the expenditures on machines that are also available at time t. The inflation rates $\widehat{\pi}_{t,\tau}$ equal

$$\widehat{\pi}_{t,\tau} = \left(\widehat{P}_{t,\tau+1} - \widehat{P}_{t-1,\tau}\right) \Big/ \widehat{P}_{t-1,\tau} \tag{87}$$

On the balanced growth path both $s_{t-1,\tau}^{*}$ and $\widehat{\pi}_{t,\tau}$ will be constant over time. Furthermore, because the price per efficiency unit is declining in the vintage age, $\widehat{\pi}_{t,\tau} < 0$ for all τ. And thus $\pi_t^{(M)}$ is constant over time and negative.

Proof of equation (57) :

Instead of the Leontief technology that we consider in our model consider a firm that has hired a worker of skill-level h which it matches with $K_{t,\tau}$ units of the capital good of vintage age τ. Here $K_{t,\tau}$ is not fixed at one but the firm can choose it. The production technology in this case is Cobb-Douglas in the sense that output produced equals

$$Y_t = h \left(A_{t-\tau} K_{t,\tau}\right)^{\alpha} \text{ where } 0 < \alpha < 1 \tag{88}$$

The firm thus demands the amount of capital inputs that maximizes

$$Y_t - P_{t,\tau} K_{t,\tau} \tag{89}$$

The optimal capital input choice for the firm is

$$K_{t,\tau} = \left(\frac{P_{t,\tau}}{\alpha h A_{t-\tau}^{\alpha}}\right) \tag{90}$$

and the resulting level of profits equals

$$Y_t - P_{t,\tau} K_{t,\tau} = \left[\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right] h^{\frac{1}{1-\alpha}} \left(\frac{A_{t-\tau}}{P_{t,\tau}}\right) \tag{91}$$

The firm will choose the technology vintage τ that maximizes these profits. Independently of the skill level, h, of the worker, this turns out to be the technology with the lowest price per efficiency unit $P_{t,\tau}/A_{t-\tau}$. Thus, for this case, i.e. the case in which a proper capital aggregate exists, all technology vintages in which there is positive investment must have the same price per efficiency unit.

Table 1: Numerical results

Parameters	benchmark	I	II	III	IV	V
Growth rate of embodied technological change (g)	3.7%	3.7%	3.7%	3.7%	3.7%	7.3%
Number of models sold (M)	20	20	10	20	20	20
Market structure*	M	MC	M	M	M	M
Cost level (c)	0.5	0.5	0.5	0.5	5	0.5
Upperbound skill distribution ($\overline{h}$)	0.325	0.325	0.325	0.650	0.325	0.325
Lowerbound skill distribution ($\underline{h}$)	0.008	0.008	0.008	0.008	0.008	0.008
Measured variables						
Aggregate labor share	82.7%	88.2%	90.3%	68.5%	88.0%	73.5%
Final goods sector labor share (s_L^f)	89.9%	92.2%	94.4%	80.4%	92.5%	83.8%
Nominal investment ratio (I/Y)	9.3%	7.5%	5.4%	16.6%	7.2%	14.2%
Investment price inflation (Matched model)	-6.3%	-10.7%	-12.5%	-6.3%	-6.2%	-5.7%
Investment price inflation (Hedonic)	-5.8%	-10.5%	-10.3%	-6.1%	-5.0%	-5.4%
Growth rate of real investment	10.6%	16.1%	18.4%	10.7%	10.5%	13.8%
Growth rate of real GDP	4.2%	4.2%	4.3%	4.7%	4.0%	8.1%
Aggregate TFP growth (GHK-methodology)	1.9%	1.9%	2.0%	0.4%	2.5%	3.8%
Final goods sector TFP growth	2.6%	2.5%	2.7%	1.7%	2.9%	5.2%

Note: *M means 'monopolist' case, MC means 'monopolistic competition'.

Cases: (I) Monopolistic competition, (II) Shorter product life cycle, (III) Skill accumulation, (IV) Decreasing returns to scale, (V) Increase in growth rate of embodied technological change.

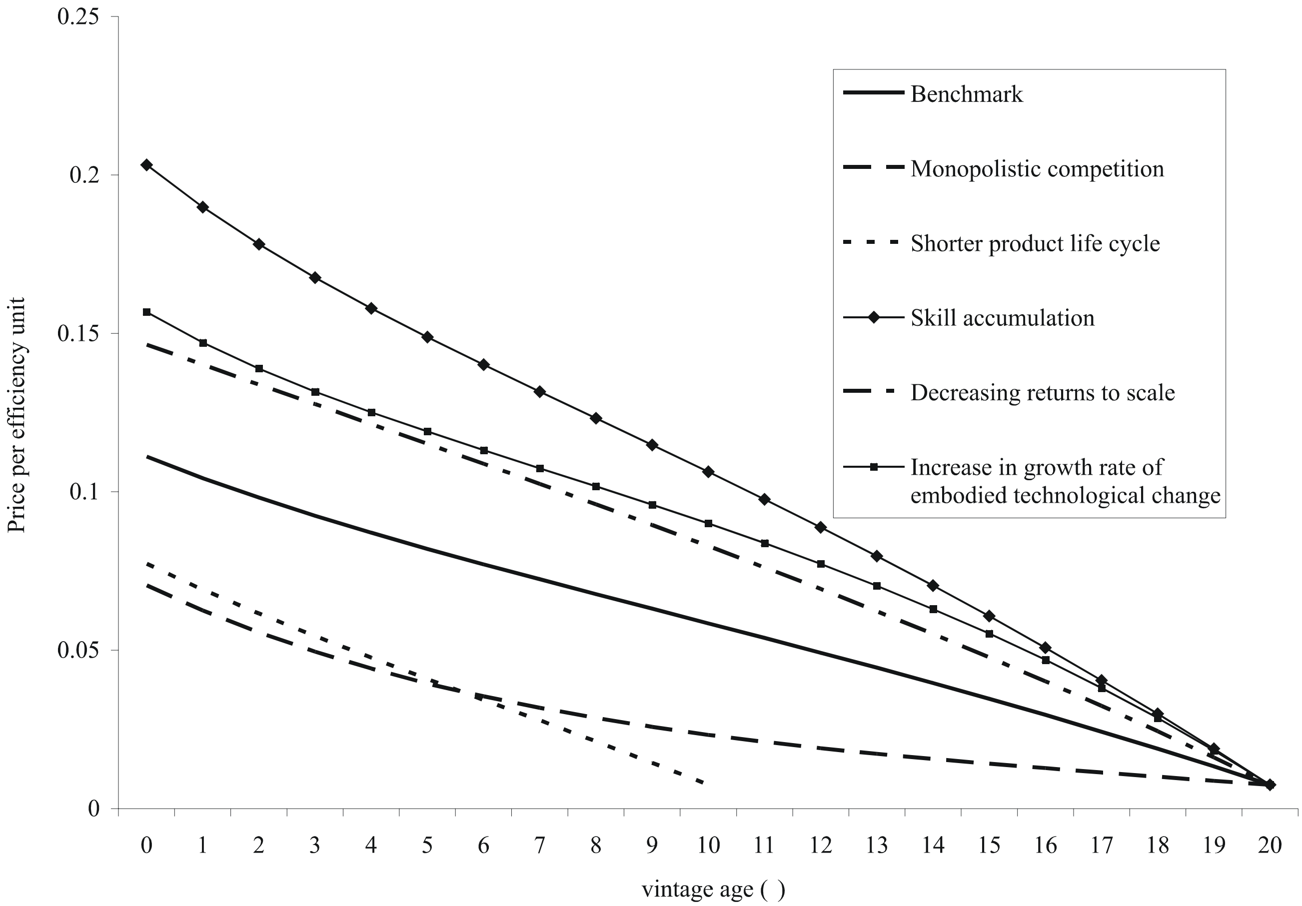

Figure 1: Equilibrium price contours