On-the-Job Search and Strategic Bargaining

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Abstract

This paper studies wage bargaining in a simple economy in which both employed and unemployed workers search for better jobs. The axiomatic Nash bargaining solution and standard strategic bargaining solutions are inapplicable because the set of feasible payoffs is non-convex. I instead develop a strategic model of wage bargaining between a single worker and firm that is applicable to such an environment. I show that if workers and firms are homogeneous, there are market equilibria with a continuous wage distribution in which identical firms bargain to different wages, each of which is a subgame perfect equilibrium of the bargaining game. If firms are heterogeneous, I characterize market equilibria in which more productive firms pay higher wages. I compare the quantitative predictions of this model with Burdett and Mortensen's (1998) wage posting model and argue that the bargaining model is theoretically more appealing along important dimensions.

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1 Introduction

In recent years, search theorists have grown increasingly aware of the need to incorporate on-the-job search into their models. In part this is because job-to-job transitions are pervasive in the United States economy. According to conservative estimates, job-to-job transitions are about half as common as unemployment-to-employment transitions (Blanchard and Diamond 1989). Using evidence from a newer data set, Fallick and Fleischman (2004) argue that half of all new employment relationships result from a job-to-job transition rather than a movement from unemployment or out of the labor force into employment.

But the interest in on-the-job search models is also a consequence of the novel theoretical results that they generate. Burdett and Mortensen (1998) develop a wage-posting model in which firms offer high wages to attract workers from other firms and to reduce worker turnover. They show that the unique equilibrium of the labor market is characterized by a continuous wage distribution, even if all workers and firms are identical. If firms are heterogeneous, higher productivity firms pay higher wages. This paper has spawned a number of extensions. Stevens (2004) and Burdett and Coles (2003) allow firms to post wage contracts rather than just a single wage. The latter paper shows that if workers are risk averse, equilibrium involves a distribution of contracts, each with an upward-sloping wage profile. Postel-Vinay and Robin (2002) allow firms to match outside offers and show that workers may voluntarily take a wage cut in order to move to a firm that is likely to be more aggressive in matching outside offers in the future. Cahuc, Postel-Vinay, and Robin (2003) explicitly model the bargaining game between a worker and one or more potential employers. Moreover, many of these models have been tested using matched worker-firm data sets; Mortensen (2003) is a prominent example.

At the same time, there is a substantial gap between this model and the 'standard' labor market model of search, summarized in Pissarides's (2000) textbook. In the simplest version of that model, only unemployed workers search for jobs. When a worker and firm meet, the wage is set in accordance with the axiomatic Nash (1953) bargaining solution. Pissarides shows that this results in the worker and firm splitting the gains from trade, with the worker's share determined by her (exogenous) bargaining power. There have been some attempts to introduce on-the-job search into the bargaining model. Pissarides (1994) assumes that a worker and firm split the surplus from matching. The equilibrium of the resulting model is qualitatively different from the equilibrium of the Burdett and Mortensen (1998) model: if workers and firms are homogeneous, then all workers earn the same wage at all jobs, so there is no wage dispersion. The natural conclusion is that whether there is wage dispersion in a homogeneous agent economy with on-the-job search depends critically on whether firms post wages or wages are bargained.

This paper revisits this conclusion. The first finding is that the axiomatic Nash bargaining solution is inapplicable in this environment. Nash (1953, p. 129) writes "The only important thing is the set of those pairs (u_1, u_2) of utilities which can be realized by the players if they cooperate.... It should be a compact convex set in the (u_1, u_2) plane." I find that in the model with on-the-job search, the set of feasible payoffs is typically non-convex because an increase in the wage raises the duration of an employment relationship. This possibility is absent from models without on-the-job search, but is central to wage setting in the environment of interest to this paper.

This leads me to focus on strategic bargaining games. I assume that when a worker and firm first meet, they bargain over the wage for the duration of the employment relationship, taking as given the wage bargained by other workers and firms, the "wage distribution." I model bargaining as an infinite horizon alternating offers game with a small risk that bargaining breaks down between offers. I require that any wage w that is paid in a market equilibrium be a subgame perfect equilibrium of the strategic bargaining game when the risk of breakdown is sufficiently small. The existing literature on such games, including Rubinstein (1982), Shaked and Sutton (1984), and Binmore, Rubinstein, and Wolinsky (1986) shows that under some conditions there is a unique subgame perfect equilibrium in this strategic bargaining game. Unfortunately, these results are also inapplicable to my environment because all of these papers also assume that the set of feasible payoffs is convex. When I extend their approach to handle models with non-convex payoffs, I find that the subgame perfect equilibrium of the bargaining game with a given wage distribution is no longer unique. Instead I get a precise characterization of the set of subgame perfect equilibria. In a market equilibrium, each wage in the support of the wage distribution corresponds to one of these subgame perfect equilibria.

In an environment with homogeneous firms and on-the-job search, I find there are many market equilibria. There is a continuum of market equilibria each characterized by a different continuous wage distribution. In each market equilibrium every wage in the support of the distribution is a subgame perfect equilibrium of the bargaining game. Depending on how employed workers behave when they encounter a firm paying their current wage, there may also be a continuum of market equilibria with a degenerate wage distribution and more generally a continuum of market equilibria with an n-point wage distribution for arbitrary n.

I then extend the model to have heterogeneous firms, with a continuous distribution of productivity x across firms. I provide a simple characterization of market equilibria in which more productive firms pay strictly higher wages: there is a function $\phi_x(y)$ such that for each firm type x, $\phi_x(x) > \phi_x(y)$ for all y in a neighborhood of x. This is a generalization of a naïve application of the Nash bargaining solution to this model (see Mortensen 2003, Section 4.3.4), which imposes the stronger condition that $\phi_x(x) > \phi_x(y)$ for all firm types x and y.

This paper proceeds as follows. Section 2 lays out the basic model with homogeneous workers and firms and discusses convexity of the set of feasible payoffs. Section 3 characterizes the set of market equilibria with a continuous wage distribution, while Section 4 shows that, if workers never switch employers when they are indifferent, the model has many market equilibria characterized by a mass of firms paying the same wage. I argue that such market equilibria seem contrived compared to the ones with a continuous wage distribution, since they are broken if firms are concerned that workers might sometimes accept equal outside offers. Section 5 explores the model with heterogeneous firms. I provide a concise definition of a market equilibrium when more productive firms pay higher wages. I then show that, like the Burdett and Mortensen (1998) model, the strategic bargaining model of on-the-job search predicts the productivity of each worker conditional on her wage and the entire wage distribution. Moreover, the model implies that some wage distributions cannot be produced by this model regardless of the distribution of productivity. Section 6 discusses the connection between this paper and existing attempts to use the Nash (1953) bargaining solution to set wages in models with on-the-job search. Finally, the paper concludes in Section 7 by evaluating the advantages and disadvantages of bargaining and wage posting models of on-the-job search.

2 Model

2.1 Preferences and Technology

I consider a continuous time, infinite horizon economy. There are two types of economic agents, firms and workers. All agents are risk-neutral, infinitely-lived, and discount future income at rate $r > 0$.

Let v denote the measure of firms in the economy, indexed by $j \in [0, v]$. Firms are ex ante identical but may pay different wages. Let $W(j)$ denote the wage that firm j pays its workers, $[\underline{w}, \bar{w}]$ denote the support of the wage distribution, and $F(w)$ denote the fraction of firms paying a wage *strictly* less than w. This is a critical object determined in the market equilibrium of this economy but taken as given by each individual agent. Each firm is endowed with a constant returns to scale production technology using only labor. More precisely, each employee produces output x and hence yields a flow profit $x - W(j)$ to firm j. Each firm contacts a worker at a constant rate, regardless of the firm's bargained wage or how many filled jobs it has. These means that there is no opportunity cost of hiring a worker.[1] Employment relationships end exogenously at rate $s > 0$, leaving the worker unemployed and the firm with nothing.

Normalize the measure of workers to 1. Each worker may be employed or unemployed. An unemployed worker gets flow utility $z < x$ from leisure and unemployment income, while a worker employed by firm j earns the wage $W(j)$. All workers search for jobs, contacting a randomly selected firm at rate $\lambda > 0$. A worker's optimal search behavior is simple: she takes any job that raises the present value of her income. There is one subtle but important tie-breaking assumption: I look at market equilibria in which a worker switches jobs when indifferent. Relaxing this behavioral restriction enlarges the set of market equilibria, a possibility I explore in Section 4.

For a given wage distribution F, I can characterize the equilibrium through a series of Bellman equations. Let $E(w)$ denote the expected value of income for a worker currently employed at wage w and U denote the expected value of income for an unemployed worker. These satisfy

$$rE(w) = w + s\big(U - E(w)\big) + \lambda \int_{\underline{w}}^{\bar{w}} \max\{E(w') - E(w), 0\} dF(w') \tag{1}$$

$$rU = z + \lambda \int_{\underline{w}}^{\bar{w}} \max\{E(w') - U, 0\} dF(w'). \tag{2}$$

An employed worker earns a wage w; the match ends at rate s, leaving the worker unemployed; and the worker gets another wage offer at rate λ, leading to a capital gain $E(w') - E(w)$ if $E(w') \geq E(w)$ and zero otherwise. An unemployed worker earns income z and finds a firm at rate λ as well.

It is useful to simplify these expressions to obtain an expression for the surplus a worker gets from a match. Observe from (1) and (2) that $E(z) = U$, so a worker is indifferent between unemployment and working at a wage equal to her unemployment income. Then

[1] In a market equilibrium, a firm's profit depends on its bargained wage, unlike in Burdett and Mortensen (1998).

differentiate (1) to prove $E'(w') = \frac{1}{r+s+\lambda(1-F(w'))} > 0$. Integrate this using the terminal condition $E(z) = U$ to get

$$E(w) - U = \int_z^w \frac{1}{r+s+\lambda(1-F(w'))} dw'. \tag{3}$$

In particular, workers prefer higher wages and move whenever they find a job that pays a higher wage, at rate $\lambda(1 - F(w))$.

Next consider a firm paying a wage w. Since it has a constant returns to scale production technology, one can evaluate each of its filled jobs in isolation. Their value is defined recursively by

$$rJ(w) = x - w - \big(s + \lambda(1 - F(w))\big)J(w). \tag{4}$$

The job produces flow profit $x - w$, but ends either exogenously at rate s or endogenously when the worker finds at least as good a wage offer, at rate $\lambda(1 - F(w))$. When a job ends, the firm loses the full value of the job $J(w)$ since its opportunity cost is zero.

2.2 Wage Bargaining and Equilibrium Concept

A critical issue in this environment is how wages are set. A worker will take any job paying at least her value of unemployment z, $E(w) \geq U$ if $w \geq z$. Similarly, a firm will hire any worker if the wage is no more than x, $J(w) \geq 0$ if $w \leq x$. This introduces a bilateral monopoly problem in wage setting. Following Diamond (1982) and Mortensen (1982), I assume that the worker and firm settle on a wage by bargaining.

Wage bargaining is complicated in this environment. Standard axiomatic bargaining solutions (Nash 1953) and strategic bargaining games (Rubinstein 1982, Shaked and Sutton 1984, Binmore, Rubinstein, and Wolinsky 1986) assume that the set of feasible allocations is convex, a restriction that I show below may be violated in this model. So rather than apply an out-of-the-box bargaining solution, I am forced to return to the foundations of two person bargaining theory and analyze the subgame perfect equilibria of a particular extensive form game.

Before discussing a particular bargaining game, it is important to mention some important features of wage setting in this environment. I assume that when an unemployed worker meets a firm, the pair bargains over a wage. The wage subsequently remains fixed for the duration of the match.[2] At some later date, the worker may meet another firm. At this

[2]This rules out the possibility that the pair bargains over a wage contract. If they could, the optimal

juncture, the worker must choose an employer and then, if she switches employers, bargain with the new employer with no possibility of recalling her old job. If she stays at her old job, her wage is unchanged. I make this assumption to parallel Pissarides (1994) as closely as possible. This rules out the possibility that a worker can exploit multiple job opportunities to raise her wage. Postel-Vinay and Robin (2002) examine what happens if a worker who has multiple job opportunities can get her employers to bid for her labor and Moscarini (2004) looks at firms' incentive not to match outside job offers.

Another important assumption is that a worker who contacts a firm can observe the firm's index j and rationally anticipate the bargained wage $W(j)$. Although this assumption might seem extreme in an environment with homogeneous firms, it is more plausible when firms are heterogeneous. I show in Section 5 that with heterogeneous firms, there is a market equilibrium in which more productive firms pay higher wages and workers move whenever they contact a more productive firm. Finally, I do not permit bargaining over wage *lotteries* rather than simply over wages. I show in Section 3.4 that, because the set of feasible allocations may be nonconvex, wage lotteries could play a nontrivial role.

I now describe how a worker and a firm set the wage. I consider an alternating offers game, based closely on Binmore, Rubinstein, and Wolinsky's (1986) "strategic model with exogenous risk of breakdown." For simplicity, assume that a worker and firm bargain in artificial time, so wage bargaining does not delay production. Denote time in the bargaining game by $t = 1, 2, 3, \ldots$, with an infinite horizon. In each 'odd' stage $t = 1, 3, 5, \ldots$, the firm makes a wage offer w, which the worker can accept or reject.[3] If the worker accepts the offer, bargaining ends, production starts, and the worker is paid the negotiated wage for the remainder of the match, giving the worker and firm expected values $E(w)$ and $J(w)$, respectively. If the worker rejects the offer, negotiations break down with probability δ, leaving the worker unemployed with expected income U and the firm with nothing. Otherwise, the game proceeds to stage $t + 1 = 2, 4, 6, \ldots$, an 'even' stage. Now the worker makes a wage demand to the firm, which again may be accepted or rejected. Rejection leads to a probability δ that negotiations break down; otherwise the game proceeds to stage $t + 2$, another 'odd' stage

contract would be simple to describe: the firm would pay the worker her full marginal product x for the duration of the match and the pair would bargain over an initial transfer from the worker to the firm. Since the worker receives her entire marginal product, she has no incentive to switch employers at a later date. Thus this contract eliminates all job-to-job transitions, which are inefficient from the perspective of a particular worker and firm. This is reminiscent of Stevens's (2004) findings in the Burdett and Mortensen (1998) wage posting model of on-the-job search.

[3] This notation suggests that the firm makes the first wage offer. The results are unchanged if the worker makes the first wage offer or if the first mover is determined by a coin flip.

that is identical to stage t.

The worker and firm treat the wage distribution F, and hence the Bellman values J, E, and U, as fixed when bargaining. In a market equilibrium, some firm must be willing to offer each wage in the support of the wage distribution F. To be precise, market equilibrium imposes that for all $\varepsilon > 0$ and all firms j, there is a sufficiently small $\delta > 0$ such that there is a subgame perfect equilibrium of the strategic bargaining model in which the bargained wage lies within ε of $W(j)$. The wages $W(j)$ must integrate up to the distribution F.

2.3 Nonconvexity of the Set of Feasible Payoffs

Nash (1953) examined two person bargaining problems in which the feasible set of payoffs is convex. He argued that there are four reasonable restrictions on the outcome of a bargaining game: it should be invariant to equivalent representations of players' von Neumann-Morgenstern utility functions; it should be independent of irrelevant alternatives;[4] it should be Pareto efficient; and it should be symmetric if the underlying problem is symmetric. Nash proved that if these four axioms hold, there is a unique solution to the bargaining problem and it maximizes $(E(w)-U)J(w)$. Similarly, uniqueness theorems in the literature on alternating offers bargaining games (Rubinstein 1982, Shaked and Sutton 1984, Binmore, Rubinstein, and Wolinsky 1986) assume that the set of feasible payoffs is convex.

Unfortunately, the set of feasible payoffs is typically nonconvex in this environment and so these results are inapplicable. Consider the simplest wage distribution, $F(w)$ degenerate at some wage $\bar{w} \in (z, x)$. Then the value functions (3) and (4) reduce to

$$E(w) - U = \begin{cases} \dfrac{w-z}{r+s+\lambda} & \text{if} \quad w \leq \bar{w} \\ \dfrac{\bar{w}-z}{r+s+\lambda} + \dfrac{w-\bar{w}}{r+s} & \text{if} \quad w > \bar{w} \end{cases} \tag{5}$$

and

$$J(w) = \begin{cases} \dfrac{x-w}{r+s+\lambda} & \text{if} \quad w \leq \bar{w} \\ \dfrac{x-w}{r+s} & \text{if} \quad w > \bar{w}. \end{cases} \tag{6}$$

Notably $E(w)-U$ is continuous but not differentiable at $\bar{w}$, while $J(w)$ is discontinuous at $\bar{w}$. For sufficiently small ε, both the worker and firm prefer a fair lottery between $\bar{w}-\varepsilon$ and

[4] More precisely, suppose some outcome x is the bargaining solution in one problem. We now eliminate some feasible payoffs but x remains feasible. Then it should still be the outcome of the restricted problem.

$\bar{w} + \varepsilon$ to a wage of $\bar{w}$ for sure. The worker prefers the lottery because $E(w)$ has a convex kink at $\bar{w}$ while the firm prefers it because $J(w)$ jumps up discontinuously at $\bar{w}$. In the next section I show that this nonconvexity carries over to many other wage distributions, including any wage distribution associated with a market equilibrium.

3 Market Equilibria with Wage Dispersion

This section considers market equilibria in which the wage distribution is continuous. I prove that there is a family of such market equilibria, parameterized by the lower bound of the wage distribution $\underline{w} \in \left[\frac{1}{2}(x+z), x\right)$:

$$F_{\underline{w}}(w) = \frac{r+s+\lambda}{\lambda}\left(1 - \left(\frac{x-w}{x-\underline{w}}\right)\sqrt{1 - 2\left(\frac{x-\underline{w}}{\underline{w}-z}\right)\log\left(\frac{x-w}{x-\underline{w}}\right)}\right) \tag{7}$$

with support $(\underline{w}, \bar{w})$. Clearly $F_{\underline{w}}$ is continuous and differentiable on its support. The restriction that $\underline{w} \in \left[\frac{1}{2}(x+z), x\right)$ ensures that $F_{\underline{w}}$ is increasing. Moreover, $F_{\underline{w}}(\underline{w}) = 0$ while $F_{\underline{w}}(\bar{w}) = 1$ pins down the upper bound of the support of $F_{\underline{w}}$; since $F_{\underline{w}}(x) = 1$, $\bar{w} \in (\underline{w}, x)$.

I start in Sections 3.1 and 3.2 by proving that each of these wage distributions is consistent with market equilibrium. Section 3.3 proves that there are no other market equilibria with a continuous wage distribution while Section 3.4 considers wage lotteries.

3.1 Bellman Values

The first step is to characterize the Bellman values when wages satisfy (7). Substitute (7) into equation (3) to show that for $w \in [\underline{w}, \bar{w}]$,

$$E(w) - U = \frac{(\underline{w} - z)\sqrt{1 - 2\left(\frac{x-\underline{w}}{\underline{w}-z}\right)\log\left(\frac{x-w}{x-\underline{w}}\right)}}{r+s+\lambda} \tag{8}$$

This is strictly increasing in w. One can also solve for $E(w) - U$ when $w \notin [\underline{w}, \bar{w}]$ and confirm that E is globally increasing. Similarly, substituting (7) into equation (4) shows that a firm paying a wage $w \in [\underline{w}, \bar{w}]$ earns expected profit

$$J(w) = \frac{x - \underline{w}}{(r+s+\lambda)\sqrt{1 - 2\left(\frac{x-\underline{w}}{\underline{w}-z}\right)\log\left(\frac{x-w}{x-\underline{w}}\right)}}. \tag{9}$$

This is strictly decreasing in w. Again, one can solve for $J(w)$ when $w \notin [\underline{w}, \bar{w}]$ and confirm that J is globally decreasing.

The set of feasible payoffs is nonconvex when the wage distribution F satisfies equation (7). In particular, equations (8) and (9) imply

$$(E(w) - U)J(w) = \frac{(x - \underline{w})(\underline{w} - z)}{(r + s + \lambda)^2} \tag{10}$$

is constant for $w \in [\underline{w}, \bar{w}]$, so this region of the Pareto frontier of the bargaining set is convex. That the Pareto frontier is convex under the wage distribution $F_{\underline{w}}$ is not an accident. Section 3.3 shows that in any market equilibrium with a continuous wage distribution, $(E(w) - U)J(w)$ is constant on the support of the wage distribution.

3.2 Subgame Perfect Equilibria of the Bargaining Game

To prove that the wage distribution (7) is consistent with a market equilibrium, I must show that for any $\varepsilon > 0$ and all $w \in (\underline{w}, \bar{w})$, there is a $\delta > 0$ such that there is a subgame perfect equilibrium of the strategic bargaining model in which the bargained wage lies within ε of w.

Consider the following strategies: In each odd period, the firm proposes a low wage $w^f \in [\underline{w}, \bar{w})$. The worker accepts any offer $w \geq w^f$ and refuses lower wage offers. In each even period, the worker proposes a high wage $w^w \in (w^f, \bar{w}]$. The firm accepts $w \leq w^w$ and rejects higher wage demands. To prove that this is a subgame perfect equilibrium, I must show that no one-stage deviation is profitable, which puts strong restrictions on the relationship between w^f and w^w.

First, suppose the firm considers offering a wage different than w^f. Any higher wage is accepted, but so is w^f, and since J is decreasing (equation 9), a wage increase reduces the firm's payoff. Any lower wage is rejected, in which event the firm accepts the higher wage w^w in the next period if negotiations do not break down first. Again, this reduces firm's payoff from $J(w^f)$ to $(1 - \delta)J(w^w)$. Similarly, offering w^w is the worker's best response to the firm's strategy since E is increasing (equation 8).

Next turn to the acceptance thresholds. Monotonicity of $J(w)$ and $E(w) - U$ in w ensure that threshold rules are optimal. The threshold must be at the point where the respondent is indifferent. A firm is indifferent between accepting w^w now or facing the risk that negotiations break down but otherwise having w^f accepted next period if

$$J(w^w) = (1 - \delta)J(w^f). \tag{11}$$

Worker's analogous indifference condition is

$$E(w^f) = (1-\delta)E(w^w) + \delta U. \tag{12}$$

This is a pair of equations in w^f and w^w. But substituting from (8) and (9) indicates that for any $\underline{w} \le w^f < w^w \le \bar{w}$, *both* equation (11) *and* equation (12) imply

$$\sqrt{1 - 2\left(\frac{x-\underline{w}}{\underline{w}-z}\right)\log\left(\frac{x-w^f}{x-\underline{w}}\right)} = (1-\delta)\sqrt{1 - 2\left(\frac{x-\underline{w}}{\underline{w}-z}\right)\log\left(\frac{x-w^w}{x-\underline{w}}\right)}. \tag{13}$$

That the two equations imply the same relationship between w^f and w^w is due to the constancy of $(E(w) - U)J(w)$; this would not be true for an arbitrary distribution F.

Any pair $\{w^f, w^w\} \in [\underline{w}, \bar{w}]^2$ satisfying equation (13) is a subgame perfect equilibrium of the bargaining game with fixed δ. In particular, take an arbitrary $w \in (\underline{w}, \bar{w})$. Let $w^f = w$ and select w^w using (13). For sufficiently small δ, this defines $w^w \in (w^f, \bar{w}) \cap (w - \varepsilon, w + \varepsilon)$. Since $w^w \in (w^f, \bar{w})$, this is a subgame perfect equilibrium of the bargaining game. And since $w \in (w - \varepsilon, w + \varepsilon)$, the bargaining outcome lies with ε of w.

All that remains is to assign wages to firms. One possibility is to let firm $j \in (0, v)$ pay a wage $W(j)$ solving $F_{\underline{w}}(W(j)) = j/v$, which gives rise to the desired wage distribution. But any reshuffling of the wages paid is also consistent with market equilibrium, as long as the correct density of firms pay the correct wage and workers know which firm pays which wage.

In summary, when a worker encounters firm j, she rationally anticipates that the bargaining game will conclude at some wage $W(j)$. The worker prefers to meet a firm j' with $W(j') > W(j)$, but if firm j always offers the wage $W(j)$ and refuses any higher offer when playing the bargaining game, the worker's best response is to accept the low wage offer. Conversely, firms that bargain to lower wages earn more profits per worker, but it is impossible for a firm that is expected to offer a high wage $W(j')$ to get away with paying its worker a low wage $W(j)$. To readers accustomed to the logic of wage posting models like Burdett and Mortensen (1998), this might seem perverse: why can't firms unilaterally lower the wage they pay if this is in their interest? Here the possibility of doing so is limited by workers' expectations and their associated strategies in the bargaining game.

3.3 Other Market Equilibria

There is no market equilibrium in which a positive measure of firms pay the same wage. To prove this, suppose to the contrary that a positive of measure of firms pay $w < x$.[5] Suppose one of those firms considers offering its worker a slightly higher wage $w + \varepsilon$ in the odd stage of the bargaining game. Workers prefer higher wages and so the worker will naturally accept the offer. For sufficiently small ε, the firm also benefits from the higher wage offer: by assumption, workers switch employers whenever they are indifferent, so by raising its wage offer slightly above the mass point, the firm discretely reduces its turnover, increasing $J(w)$. Thus this is not a market equilibrium.

Now consider an arbitrary market equilibrium with a continuous wage distribution F with support $(\underline{w}, \bar{w})$, so each wage in the support is a subgame perfect equilibrium of the bargaining game. First note that the worker's value E must be increasing and firm's value J must be decreasing on the support of the wage distribution. That E is increasing follows immediately from equation (3). Equation (4) allows for the possibility that J is increasing in w, but this cannot happen in a market equilibrium: If J is increasing at w, a worker and firm would not agree on a wage of w since both would prefer a higher wage, i.e. w is not in the support of the wage distribution; but if w is not on the support of the wage distribution, equation (4) indicates that J is decreasing at w.

It follows that equations (11) and (12) carry over to this environment and jointly imply

$$\big(E(w^w) - U\big) J(w^w) = \big(E(w^f) - U\big) J(w^f)$$

for all w^w, $w^f \in (\underline{w}, \bar{w})$. In other words, the product of the surplus that the worker gets from matching and the surplus that the firm gets from matching must be constant on the support of the wage distribution.

This is a strong restriction on the wage distribution. To see how strong, note from equations (3) and (4) that for $w \in (\underline{w}, \bar{w})$,

$$\big(E(w) - U\big) J(w) = \left(\int_z^w \frac{1}{r + s + \lambda(1 - F(w'))} dw' \right) \left(\frac{x - w}{r + s + \lambda(1 - F(w))} \right). \tag{14}$$

[5] It is easy to rule out market equilibria in which the wage is $w \geq x$.

In particular, $\big(E(\underline{w})-U\big)J(\underline{w})=\big(E(w)-U\big)J(w)$ for all w or

$$\left(\frac{\underline{w}-z}{r+s+\lambda}\right)\left(\frac{x-\underline{w}}{r+s+\lambda}\right)=\left(\int_z^w \frac{1}{r+s+\lambda(1-F(w'))}dw'\right)\left(\frac{x-w}{r+s+\lambda(1-F(w))}\right).$$

Differentiating this yields a first order nonlinear differential equation for F:

$$F'(w)=\frac{r+s+\lambda(1-F(w))}{\lambda(x-w)}-\frac{(x-w)\big(r+s+\lambda\big)^2}{(\underline{w}-z)(x-\underline{w})\lambda\big(r+s+\lambda(1-F(w))\big)}. \tag{15}$$

Any continuous wage distribution must satisfy this condition for $w\in(\underline{w},\bar{w})$. Integrating (15) with the terminal condition $F(\underline{w})=0$ gives equation (7) for $F_{\underline{w}}$. Finally, if $\underline{w}<\frac{x+z}{2}$, $F'(0)<0$, which is inconsistent with F being a cumulative distribution function. Thus the only possible market equilibria are those already analyzed.

3.4 Wage Lotteries

I have so far assumed that a worker or firm can offer its counterpart a wage in the bargaining game, but it cannot offer a wage lottery. To understand why this restriction may be important, consider a firm j that is supposed to offer a worker a wage $W(j)\in(\underline{w},\bar{w})$ when the wage distribution is $F_{\underline{w}}$. Since workers' value function E is increasing, this gives the worker a weighted average of her utility from the lowest possible and highest possible wages:

$$E(W(j))\equiv\alpha E(\underline{w})+(1-\alpha)E(\bar{w}) \tag{16}$$

for some $\alpha\in(0,1)$.

Suppose instead the firm offers the worker a lottery. If the worker accepts the lottery, the wage is $\underline{w}$ with probability α and $\bar{w}$ with probability $1-\alpha$. Since the worker is indifferent about accepting $W(j)$, she is also indifferent about accepting this lottery and strictly prefers any more generous lottery. But the firm's payoff is higher under this lottery. To prove this, recall that $(E(W(j))-U)J(W(j))=(E(\underline{w})-U)J(\underline{w})=(E(\bar{w})-U)J(\bar{w})$. Substitute this into equation (16) to get

$$E(W(j))-U=\alpha\frac{\big(E(W(j))-U\big)J(W(j))}{J(\underline{w})}+(1-\alpha)\frac{\big(E(W(j))-U\big)J(W(j))}{J(\bar{w})}$$

or

$$\frac{1}{J(W(j))} = \alpha \frac{1}{J(\underline{w})} + (1-\alpha) \frac{1}{J(\bar{w})},$$

so Jensen's inequality implies $J(W(j)) < \alpha J(\underline{w}) + (1-\alpha) J(\bar{w})$. Lotteries enable the worker and firm to convexify the feasible set of payoffs, raising the possibilities for both. I have ruled out lotteries by fiat, but one reason that lotteries might not be possible is if there is no third party who can verify their outcome. Future research should explore how allowing for lotteries affects the conclusions of this paper.

4 Degenerate Market Equilibria

This section modifies the restriction that workers switch from firm j to j' even if they are indifferent. Instead, I consider the opposite tie-breaking assumption: a worker moves only when she encounters a firm paying a strictly higher wage. It is straightforward to see that the continuous wage distributions $F_{\underline{w}}$ found in the previous section remain market equilibria under this alternative restriction, since workers never encounter a firm paying their current wage, but I show that this change in behavior introduces many additional market equilibria, each with a discrete wage distribution. I start with the simplest type of market equilibrium.

4.1 Single Wage Market Equilibrium

Suppose all firms offer a common wage $\bar{w} \in (z, x)$. Equation (3) is unaffected by the change in workers' behavior since they are indifferent about whether they move when they encounter a firm offering the same wage. Specializing it to this case gives

$$E(w) - U = \begin{cases} \dfrac{w-z}{r+s+\lambda} & \text{if} \quad w < \bar{w} \\ \dfrac{\bar{w}-z}{r+s+\lambda} + \dfrac{w-\bar{w}}{r+s} & \text{if} \quad w \geq \bar{w}. \end{cases} \tag{17}$$

This is continuous in w even at $\bar{w}$.

Since workers do not switch when they are indifferent, a firm paying $\bar{w}$ or higher does not suffer any turnover, while a lower paying firm loses a worker whenever she encounters another firm. Adapting equation (4) to this environment gives

$$J(w) = \begin{cases} \dfrac{x-w}{r+s+\lambda} & \text{if} \quad w < \bar{w} \\ \dfrac{x-w}{r+s} & \text{if} \quad w \geq \bar{w}. \end{cases} \tag{18}$$

Notably this jumps up discontinuously at $\bar{w}$

To see whether this is a market equilibrium, I again examine the bargaining game with a small risk δ of a breakdown in negotiations. I look for a subgame perfect equilibrium in which the firm always offers $\bar{w}$ and the worker always demands $w^w > \bar{w}$. These wages have the property that the firm is strictly indifferent between accepting w^w this period or taking its chances that negotiations break down and offering $\bar{w}$ next period. On the other hand, subgame perfect equilibrium requires only that the worker weakly prefer accepting $\bar{w}$ this period rather than waiting until next period to offer w^w. To understand why, note that even if the worker strictly prefers to accept the firm's offer $\bar{w}$, the firm might choose not to cut its wage, knowing that if it did so its profits would fall discretely from the increase in turnover.[6]

The first step is to solve for the worker's offer w^w as a function of the firm's offer $\bar{w}$. The worker's offer must be acceptable but must leave the firm indifferent, for otherwise the worker would benefit from demanding a higher wage: $J(w^w) = (1-\delta)J(\bar{w})$ or from (18),

$$w^w = (1-\delta)\bar{w} + \delta x. \tag{19}$$

Next, the worker must be willing to accept the firm's offer: $E(\bar{w}) \geq (1-\delta)E(w^w) + \delta U$. Using (17) to solve for $E(w) - U$ and (19) to eliminate w^w gives

$$\bar{w} \geq \frac{(1-\delta)(r+s+\lambda)x + (r+s)z}{(2-\delta)(r+s) + (1-\delta)\lambda}.$$

Note that the right hand side is continuously decreasing in δ. In particular, for any

$$\bar{w} > \frac{(r+s+\lambda)x + (r+s)z}{2(r+s) + \lambda} \equiv w^*, \tag{20}$$

[6] Since the firm's value function is nonmonotone in the wage, one also has to verify that a large reduction in the wage is unacceptable to the worker. In particular, the firm earns the same profit from the low wage $w = \bar{w} - \lambda(x - \bar{w})/(r+s)$ as from the high wage $\bar{w}$, $J(w) = J(\bar{w})$. If the probability of breakdown is sufficiently small, the worker will refuse this or any lower wage offer, preferring to wait one period and receive w^w.

there is a $\delta > 0$ such that if all firms pay $\bar{w}$, it is a subgame perfect equilibrium of the bargaining game for a firm to offer $\bar{w}$ and a worker to offer $w^w = (1-\delta)\bar{w} + \delta x$. In summary, any single wage $\bar{w} \in (w^*, x)$ is associated with a market equilibrium of this model when workers stay at their employer if indifferent.

One interesting feature that all these market equilibria share is that the worker's surplus $E(\bar{w}) - U = \frac{\bar{w}-z}{r+s+\lambda}$ exceeds the firms' surplus $J(\bar{w}) = \frac{x-\bar{w}}{r+s}$, although in the limiting case of $\bar{w} = w^*$, the two terms are equal. It is straightforward to show that without on-the-job search, the worker and firm would divide the match surplus equally, giving rise to a unique equilibrium wage w^*. The possibility of on-the-job search therefore raises the wage if there is a degenerate wage in the market equilibrium.

4.2 Many-Wage Market Equilibria

Using the same logic, one can construct other market equilibrium wage distributions. For example, there can be a market equilibrium with N wages, $w_1 < w_2 < \cdots < w_N < x \equiv w_{N+1}$, in which a fraction π_i of firms pay a wage w_i, $\sum_{i=1}^{N} \pi_i = 1$. Workers move to higher wage firms whenever presented with the possibility. In the bargaining game, a 'type i' firm offers a wage w_i and a worker bargaining with such a firm responds with a slightly higher wage $w_i^w = (1-\delta)w_i + \delta x$. If δ is sufficiently small, $w_i^w < w_{i+1}$. The firm is indifferent about accepting an offer, while the worker weakly prefers to accept w_i.

This is a market equilibrium for sufficiently small δ. Firms are indifferent about accepting w_i^w or waiting one period to have w_i accepted. Workers are willing to accept w_i when $E(w_i) - U \geq (1-\delta)(E(w_i^w) - U)$, which holds for small δ if

$$w_i - z + \lambda \sum_{j=1}^{i-1} \frac{(w_{j+1} - w_j) \sum_{k=1}^{j} \pi_k}{r + s + \lambda \sum_{k=j+1}^{N} \pi_k} > \frac{(r+s+\lambda)(x - w_i)}{r + s + \lambda \sum_{k=i+1}^{N} \pi_k}$$

for all i. In the special case $N = 1$, this reduces to condition (20), while for $N = 2$, two wages are a market equilibrium if

$$w_1 - z > \frac{(r+s+\lambda)(x-w_1)}{r+s+\lambda(1-\pi_1)} \quad \text{and} \quad w_2 - z + \frac{\lambda(w_2 - w_1)\pi_1}{r+s+\lambda(1-\pi_1)} > \frac{(r+s+\lambda)(x-w_2)}{r+s}.$$

For a given π, the first condition places a lower bound on w_1 while the second condition places a lower bound on w_2 that is increasing in w_1. Both conditions hold when w_1 and w_2 are sufficiently close to productivity x.

To summarize, this simple model of on-the-job search admits a plethora of market equilibria with mass points in the wage distribution. These equilibria hinge on workers' willingness not to switch employers when they are indifferent. It is unclear whether that assumption is more reasonable than the extreme alternative that workers always switch when indifferent. What happens if one looks for a middle ground? I can think of at least two reasonable 'refinements':

1. When a worker at firm j contacts firm j', she moves if $W(j') > W(j)$ or $W(j') = W(j)$ and $j' > j$. Otherwise she remains at firm j.

2. When a worker at firm j contacts firm j' she moves with probability 1 if $W(j') > W(j)$ and with probability $p > 0$ if $W(j') = W(j)$.

Either refinement eliminates the possibility of a mass in the wage distribution, since, at least for some firms, an arbitrarily small increase in the wage above the mass point leads to a discrete increase in the duration of the match and hence in the firm's value. The only market equilibria that are robust to this refinement are the ones with a continuous wage distributions stressed in Section 3.

5 Heterogeneous Firms

I now extend the basic model to introduce firm heterogeneity. I assume that productivity x is distributed across firms according to a cumulative distribution function $H(x)$, continuously differentiable with convex support $[\underline{x}, \bar{x}]$. Each firm contacts a worker at the same constant rate, regardless of the firm's bargained wage or how many filled jobs it has. Put differently, I treat the distribution $H(x)$ as a primitive of the model and do not ask why both high and low productivity firms recruit workers. I also maintain the assumption that the opportunity cost of hiring a worker is zero, independent of x. This follows if all firms costlessly contact workers at a constant rate.

I abuse notation slightly to allow for firm heterogeneity. I refer to a firm by its productivity x rather than its index j and let $W(x)$ denote the wage paid by firm x. I also let $J_x(w)$ denote the expected present value of a match for firm x if the worker receives a wage w.

To simplify the exposition, I assume that the lower bound of the productivity distribution is workers' value of leisure, $\underline{x} = z$. This ensures that $W(z) = z$, since that is the only wage that both the worker and firm are willing to accept. Finally, I look only at market equilibria in which the wage function is increasing and continuously differentiable, $W'(x) > 0$ for all x.

This implies that the fraction of firms with productivity less than x is equal to the fraction of firms that pay a wage less than $W(x)$, $H(x) = F(W(x))$, and that F inherits the continuous differentiability of H and W. It seems likely that for some parameterizations of the model, other market equilibria exist, but I do not characterize them here.

5.1 Definition of Equilibrium

To characterize a market equilibrium, start again with the Bellman values. The worker's surplus from a match is unchanged from equation (3) and is continuously differentiable. The value of a match to a firm is a trivial generalization of equation (4),

$$J_x(w) = \frac{x - w}{r + s + \lambda(1 - F(w))}. \tag{21}$$

Since F is assumed continuously differentiable, $J_x(w)$ is also a continuously differentiable function of w. Assume that $F(w)$ is such that $J_x(w)$ is a decreasing function of w, at least for $w \leq x$.

Now consider an alternating offers wage bargaining game between a worker and a type x firm, taking the wage distribution $F(w)$ as given. Let δ denote the probability that negotiations break down following each rejected offer and let w^w and w^f denote the worker's and firm's wage offers, respectively. The bargaining problem is analogous to the one in Section 3 since both value functions are monotone. In particular, these offers are part of a subgame perfect equilibrium if the firm is indifferent about accepting w^w and the worker is indifferent about accepting w^f:

$$J_x(w^w) = (1 - \delta)J_x(w^f) \quad \text{and} \quad E(w^f) = (1 - \delta)E(w^w) + \delta U.$$

We are interested in characterizing the solution when δ is small, so w^w and w^f converge to $W(x)$. To do so, first differentiate the preceding expressions with respect to δ:

$$\begin{aligned} J_x'(w^w)\frac{dw^w}{d\delta} &= -J_x(w^f) + (1 - \delta)J_x'(w^f)\frac{dw^f}{d\delta} \quad \text{and} \\ E'(w^f)\frac{dw^f}{d\delta} &= -E(w^w) + U + (1 - \delta)E'(w^w)\frac{dw^w}{d\delta}. \end{aligned}$$

In the limit as δ converges to zero, $w^w = w^f = W(x)$. Since J_x and E are continuously differentiable, these expressions reduce to

$$\left(\frac{dw^w}{d\delta} - \frac{dw^f}{d\delta}\right)\Big|_{\delta\to 0} = \frac{-J_x(W(x))}{J_x'(W(x))} = \frac{E(W(x))-U}{E'(W(x))}.$$

The last equation delivers the critical result:

$$\frac{E'(W(x))}{E(W(x))-U} + \frac{J_x'(W(x))}{J_x(W(x))} = 0. \tag{22}$$

Equation (22) generalizes the results from the model with homogeneous firms in Section 3, where I proved that $(E(w)-U)J(w)$ is constant along the support of the wage distribution. With heterogeneous firms, firm x bargains to a wage $W(x)$ only if $W(x)$ is a local extremum of $(E(w)-U)J_x(w)$, so the wage elasticity of a type x firm's value of the match $J_x(w)$ plus the wage elasticity of the worker's value of the match $E(w)-U$ must sum to zero.

To further refine this characterization of a subgame perfect equilibrium wage, differentiate (22) with respect to x:

$$\frac{d}{dW(x)}\left(\frac{E'(W(x))}{E(W(x))-U} + \frac{J_x'(W(x))}{J_x(W(x))}\right)W'(x) + \frac{d}{dx}\left(\frac{J_x'(W(x))}{J_x(W(x))}\right) = 0.$$

One can verify directly from (21) that the second term is $1/(x-W(x))^2 > 0$ and so the first term must be negative. Since $W'(x) > 0$, this implies

$$\frac{d^2 \log\big((E(w)-U)J_x(w)\big)}{dw^2}\Bigg|_{w=W(x)} < 0.$$

That is, $W(x)$ is a local maximum of $\log\big((E(w)-U)J_x(w)\big)$ and hence is a local maximum of $(E(w)-U)J_x(w)$ as well. Since $W(x)$ is continuous and increasing, this is equivalent to requiring that x is a local maximum of

$$\phi_x(y) \equiv \big(E(W(y))-U\big)J_x(W(y)).^7$$

Formally, let $B_\varepsilon(x) \equiv (x-\varepsilon, x+\varepsilon)$ be a ball of radius ε around a point x. Then in a market

[7] By the same logic, if there is a market equilibrium with a decreasing wage function, x must be a local minimum of $\phi_x(y)$.

equilibrium, for every x there is an $\varepsilon > 0$ such that

$$\{x\} = \arg\max_{y \in B_\varepsilon(x)} \phi_x(y). \tag{23}$$

Substituting from (3) and (21), this is equivalent to

$$\{x\} = \arg\max_{y \in B_\varepsilon(x)} \left(\int_z^y \frac{W'(y')}{r+s+\lambda(1-H(y'))} dy'\right) \left(\frac{x - W(y)}{r+s+\lambda(1-H(y))}\right). \tag{24}$$

A market equilibrium is a continuously differentiable and increasing wage function $W(\cdot)$ such that (24) holds.

5.2 Testable Implications

Mortensen (2003) discusses the empirical content of the Burdett and Mortensen (1998) model. If one has data on the wage offer distribution $F(w)$, the model allows us to infer the productivity of each firm. The same is true in this model. Let $X(w)$ be the inverse of $W(x)$, the productivity of a firm that pays a wage of w. Use (3) and (21) to substitute for the worker's and firm's match value in equation (22) and simplify:

$$X(w) = w + \frac{\big(r+s+\lambda(1-F(w))\big)\int_z^w \frac{1}{r+s+\lambda(1-F(w'))} dw'}{1 + \lambda F'(w)\int_z^w \frac{1}{r+s+\lambda(1-F(w'))} dw'}. \tag{25}$$

Given any wage distribution F, one can back out the implied productivity of each firm.

Even if one does not have data on each worker's productivity, the model is still testable. In the proposed market equilibrium, more productive firms pay higher wages, so $X(w)$ should be an increasing function. Differentiating (25) gives $X'(w) > 0$ if and only if

$$\frac{2}{\lambda} + \left(\int_z^w \frac{1}{r+s+\lambda(1-F(w'))} dw'\right) F'(w) > \\ \big(r+s+\lambda(1-F(w))\big)\left(\int_z^w \frac{1}{r+s+\lambda(1-F(w'))} dw'\right)^2 F''(w)$$

This condition holds if the cumulative distribution function F is concave, or equivalently if the wage density F' is decreasing, but otherwise it may be violated. For example, suppose $\lambda = 20(r+s)$ and $z = 0$. Then this model implies the wage distribution $F(w) = w^5$ with support $[0,1]$ is inconsistent with any market equilibrium in which the wage function $W(x)$

is increasing.

5.3 Comparison with Burdett and Mortensen (1998)

It is useful to compare the equilibrium wage function from the bargaining model with a similar function obtained in the Burdett and Mortensen (1998) wage posting model. My treatment of this model follows Mortensen (2003). When a worker meets a firm, the firm unilaterally offers the worker a wage without knowing the worker's employment status. If the firm offers the worker a wage $w \geq z$, the worker accepts the job if she is unemployed, with probability $u = \frac{s}{s+\lambda}$, or employed at a lower wage, with probability $(1-u)G(w)$, where $G(w) = \frac{sF(w)}{s+\lambda(1-F(w))}$ is the steady state distribution of wages paid by firms.[8] In this event, the firm's expected discounted profit is $J_x(w) = \frac{x-w}{r+s+\lambda(1-F(w))}$. Putting this together, a type x firm chooses its wage to maximize

$$\left(\frac{s}{s+\lambda} + \frac{\lambda sF(w)}{(s+\lambda)(s+\lambda(1-F(w)))}\right)\left(\frac{x-w}{r+s+\lambda(1-F(w))}\right). \tag{26}$$

The necessary first order condition is that a type x firm posts a wage w if $x = X_{BM}(w)$ defined by

$$X_{BM}(w) = w + \frac{\big(s+\lambda(1-F(w))\big)\big(r+s+\lambda(1-F(w))\big)}{\big(r+2s+2\lambda(1-F(w))\big)\lambda F'(w)}. \tag{27}$$

This generalizes equation (3.16) in Mortensen (2003) to the case of $r > 0$.

Of course, (27) might represent a minimum of (26). It is in fact a maximum if and only if X_{BM} is increasing or equivalently

$$2\lambda F'(w)^2 > \big(r+2s+2\lambda(1-F(w))\big)F''(w) \tag{28}$$

As in the bargaining model, any concave cumulative wage distribution function F can be rationalized by some underlying productivity distribution. For non-concave distributions, including the example in the previous section, the condition may be violated. Other functions are consistent with one model but not the other; for example, a log-normal wage distribution cannot be justified using the Burdett-Mortensen model but is consistent with the

[8] In steady state, the flow of workers into employment is λu and the flow of workers out of employment is $s(1-u)$. Equating these gives $u = s/(s+\lambda)$. The flow of workers into jobs paying less than w is $\lambda uF(w)$, the rate at which unemployed workers find such jobs. The flow of workers out of such jobs is $\big(s+\lambda(1-F(w))\big)(1-u)G(w)$, the rate at which workers in these jobs either become unemployed or find a better job. Equating these and using $u = s/(s+\lambda)$ delivers the equation for G in the text.

bargaining model.

Despite their apparent similarities, the quantitative predictions of the two models differ substantially. Suppose, for example, that $F(w) = 1 - \exp(-w)$ with support $[0, \infty]$. Also set $r = 0.05$, $s = 0.5$, $\lambda = 10$, and $z = 0$. Both models can explain this data using some underlying productivity distribution, but the distributions are distinct, particularly in the right tail. For example, according to the Burdett-Mortensen model, the productivity of a firm paying a wage of 10 in this example must be $x = 587.4$. In the bargaining model, the implied productivity is a much more reasonable $x = 16.9$. This is not just a theoretical curiosity. Mortensen (2003) makes the same point in his empirical analysis of Danish wage distributions; compare Figures 4.3 and 4.5 in his book.

6 Discussion

Some previous authors have attempted to use the Nash (1953) bargaining solution to set wages in models with on-the-job search. For example, after arguing that Danish wage data are inconsistent with the predictions of the Burdett and Mortensen (1998) model, Mortensen (2003, p. 87) examines "whether the Nash bilateral bargaining model is consistent with the Danish data on the distribution of average wages paid." To implement this, he imposes in section 4.3.4 that the wage function satisfy

$$\{x\} = \arg\max_y \phi_x(y). \tag{29}$$

This paper shows that while (29) is a sufficient condition for equilibrium, equilibrium only imposes the weaker restriction (23). It is unclear whether other market equilibria exist in Mortensen's model.

Other authors, notably Pissarides (1994) and (2000), have examined models like this and assumed that the worker and firm simply split the output from a match,

$$E(W(x)) - U = J_x(W(x)) \tag{30}$$

for all x. From equation (22), this is consistent with equilibrium if and only if $E'(W(x)) + J'_x(W(x)) = 0$. But one can verify directly from (3) and (21) that

$$E'(W(x)) + J'_x(W(x)) = \frac{\lambda F'(W(x))}{\big(r + s + \lambda\big(1 - F(W(x))\big)\big)^2},$$

which is never zero if some firm x is supposed to pay $W(x)$. The 'surplus splitting' rule (30) ignores the fact that by raising the wage, the worker and firm increase the duration of the match, a critical feature for wage bargaining in environments with on-the-job search.

In fact, there are situations in which surplus splitting is Pareto inefficient. Consider a firm that is slightly less productive than most of the other firms in the economy. If all firms split the surplus from matching, this firm will pay a slightly lower wage than most others and suffer high turnover. By raising the wage, the firm increases the worker's utility and may increase its profit by reducing turnover. One does not need a very extreme parameterization of the model to illustrate this possibility. Let $H(x)$ be uniform on $(z, z+1)$. Then if $\lambda > 3(r+s)$ and all firms split the surplus according to (30), one can show that some firms—more precisely, the most productive firms—would gain by unilaterally raising their workers' wages.

7 Conclusion

The Burdett and Mortensen (1998) model has become an important workhorse of theoretically motivated empirical labor economics. This paper introduces a related model of bargaining and on-the-job search that delivers results that are qualitatively, if not quantitatively, similar to the wage posting model. Why might an economist prefer one model to the other?

The wage posting model has one undeniable appeal: it has a unique market equilibrium. Even in the simplest model with homogeneous workers and homogeneous firms, and even if one is willing to ignore the less robust market equilibria with mass points in the wage distribution, the bargaining model admits a multiplicity of market equilibria, each characterized by a continuous wage distribution. Future research should explore which of these market equilibria is most plausible. For example, one can prove that there is only one wage distribution, $F_{(x+z)/2}$, such that all $w \in [\underline{w}, \bar{w}]$ are local maxima of $(E(w)-U)J(w)$. With any other wage distribution $F_{\underline{w}}$ and $\underline{w} > \frac{x+z}{2}$, it is easy to show that $(E(\underline{w})-U)J(\underline{w})$ is a local minimum. The characterization of market equilibrium with heterogeneous firms, condition (23), therefore suggests that only the wage distribution $F_{(x+z)/2}$ is the limit of market equilibria of heterogeneous agent economies, with wages monotonic in productivity, as heterogeneity grows less important.

Along other dimensions, the bargaining model seems more attractive than the posting model. Consider the out-of-steady state dynamics of the two models. In the wage posting model, the payoff-relevant state of the economy is described by the unemployment rate u and the distribution of wages paid to employed workers G. Burdett and Mortensen prove that if these are at their steady state values, then there is a market equilibrium in which the wage offer distribution F is constant over time. But suppose instead the economy starts off out of steady state. Does it converge to steady state? What do the non-stationary dynamics look like? Although it is possible to answer these questions under special conditions, a general characterization of the non-stationary dynamics remains elusive (Shimer 2003).

In the bargaining model, the characterization of market equilibrium when the economy is away from steady state is trivial—in fact, it was not necessary to mention the unemployment rate u or the distribution of wages paid G anywhere in the paper. Whether a wage distribution F is a market equilibrium is independent of whether u and G are in steady state.

Allowing for aggregate shocks, e.g. changes in the arrival rate of offers λ, further complicates the posting model. First is the question of whether firms should be able to post offers that are contingent on the aggregate shock. If they can, one can show that firms will use the shock in order to artificially create an upward-sloping wage profile, much as in Stevens's (2004) and Burdett and Coles's (2003) deterministic wage contracting models. This conclusion seems unappealing, and so one is led to assume that the firms cannot make wage offers contingent on the aggregate state. But in such a model, the payoff relevant state of the economy is the aggregate shock, the unemployment rate, and the wage distribution across workers. Solving for a market equilibrium is complex at best. In this environment, the bargaining model is appealing along two dimensions. First, it is natural to assume that workers and firms continually re-bargain in the face of shocks. Second, the payoff relevant state is again only the aggregate shock, and so it is possible, at least in principle, to find a solution to the model in which the wage offer distribution depends on current and expected future values of the shock.

Finally, the bargaining model addresses an important theoretical concern with the wage posting model. In the latter model, wages are time-inconsistent, since a firm would like to cut the wage as soon as the worker agrees to take a job. Although reputation concerns might keep firms paying high wages, reputations are complicated to model and usually ignored; a notable exception is Coles (2001). In the wage bargaining model, a worker and firm can re-bargain at any time and the old wage would remain a subgame perfect equilibrium.

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