Directed Search with Multiple Job Applications*

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Abstract

We develop an equilibrium directed search model of the labor market where workers can simultaneously apply for multiple jobs. The main result is that all equilibria exhibit wage dispersion despite the fact that workers and firms are homogeneous. Wage dispersion is driven by the simultaneity of application choice. Risk-neutral workers apply for both 'safe' and 'risky' jobs. The former yield a high probability of a job offer, but for low pay, and act as a fallback option; the latter provide with higher potential payoff, but are harder to get. Furthermore, the density of posted wages is decreasing, consistent with stylized facts. Unlike most directed search models, the equilibria are not constrained efficient.

1 Introduction

"Why are similar workers paid differently" is a classic question in economics. In his recent book on the topic, Mortensen points out that "observable worker

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characteristics that are supposed to account for productivity differences typically explain no more than 30 percent of the variation in compensation" (2003, p.1). Controlling for firm characteristics helps account for part of the other 70 percent, but a large residual remains, suggesting that a model with search frictions might be a useful way to think about this issue.[1]

Prominent examples of random search models that generate equilibrium wage dispersion include Burdett and Judd (1983), Albrecht and Axell (1984), and Burdett and Mortensen (1998). We propose a new model of wage dispersion with homogeneous workers and firms, based not on random but on directed search, and one additional feature that we think is an important characteristic of the search process: job seekers can apply for several jobs at the same time. In random search models, workers looking for employment do not know the wages offered by different firms. In directed search they observe the wages posted by all firms before deciding where to apply. However, they do not know how many other workers apply to the same firm and, since firms have a limited number of vacancies, they may get rationed. Nevertheless, the equilibria of these models are usually constrained efficient.[2]

So far, most research in the area has focused on workers applying for one job at a time, which results in a unique equilibrium with a single wage (at least when agents are homogeneous). In this paper, workers apply for N jobs simultaneously, which yields very different results. Despite the assumption of homogeneity, all equilibria exhibit wage dispersion. Even though workers are risk neutral, they care about the probability of success of each job application because their payoffs only depend on the most attractive offer they receive. The resulting portfolio choice problem is the driving force for dispersion. Furthermore, the density of posted wages is declining, matching well known stylized facts.[3] Last, in our model, equilibria are *not* constrained efficient.

[1] Postel-Vinay and Robin (2002) estimate a model with observed and unobserved worker heterogeneity as well as productivity heterogeneity in firms and conclude that "the contribution of market imperfections to wage dispersion is typically around 50 [percent]." In a similar exercise, van den Berg and Ridder (1998) report that "search frictions explain about 20 [percent] of the variation in observable wage offers."

[2] One of the reasons why directed search has become more popular is that it provides with a more explicit explanation of the matching process and wage determination than random search (Rogerson, Shimer, and Wright (in press) discuss this point in their recent survey of search-theoretic labor models).

[3] In contrast, the Burdett-Mortensen model delivers a wage density that is upward slopping. While this can be fixed by extending the framework, it is often said to be a failing of the basic model (see Mortensen (2003)).

The intuition behind the main result of dispersion is quite straightforward. A worker faces a portfolio choice problem when deciding where to send each of his N applications, since the probability of getting a job is different at different wage levels. This occurs because higher paying firms attract more applicants on average and hence an application to such a firm succeeds with lower probability. Loosely speaking, a worker's optimal strategy is to apply to jobs that offer different levels of risk and payoff. Some applications are sent to 'safe' wages that guarantee a high probability of getting a job, but for low pay. Since this provides insurance, it is optimal to take on more risk with the other applications. As a result, he also applies to firms where the probability of getting the job is lower but the potential payoff is high.

The willingness of workers to send each application to a separate wage level creates an incentive for firms to post different wages. It turns out that in any equilibrium exactly N wages are posted, and every worker applies once to each distinct wage. From the firms' perspective, the lower margins of high wages are balanced with a higher probability of filling a vacancy, leading to the same expected profits. It is important to reiterate, however, that this intuition fails in the single application case. It is only because workers apply multiple times that firms have the incentive to post different wages.

Well-known papers on directed search include Montgomery (1991), Peters (1991), Shimer (1996), Moen (1997), Julien, Kennes, and King (2000), Burdett, Shi, and Wright (2001), Shi (2002), and Shimer (in press). Delacroix and Shi (in press) develop a directed search model with on-the-job search, which shares some features with our model since employed workers can take on more risk when looking for jobs. Albrecht, Gautier, and Vroman (2005) is the only other directed search paper where workers apply multiple times simultaneously. The authors make different assumptions and they reach very different results as will be discussed in detail.[4] Chade and Smith (2004) solve a portfolio choice problem that is similar to ours, but in a very different partial equilibrium context.

The rest of the paper is structured as follows. Section 2 presents the model, states the main theorem, and proves some straightforward initial results. Section 3 discusses the special case of two applications, which provides many of the important insights. The following section extends the results

[4] The basic difference is that, in this paper, firms commit to the wages they post, while Albrecth, Gautier, and Vroman (2005) assume that firms making job offers to the same worker engage in Bertrand competition. See the conclusions for a more detailed comparison.

to an arbitrary (finite) number of applications. Section 5 evaluates the efficiency of the equilibrium and the empirical distribution of wages. Section 6 considers the endogenous choice of the number of applications and section 7 concludes.

2 The Model

In this section we introduce the main features of the model, and define outcomes, payoffs, and equilibrium. At the end we state the main theorem and prove some preliminary results.

2.1 Environment and Strategies

There are continua of measure b workers and measure 1 firms with one vacancy each. All workers and all firms are identical, risk neutral and they produce one unit when matched and zero otherwise. The matching process is as follows. Firms start by posting (and committing to) wages. Workers observe all postings and send out N applications. Firms that receive one or more applicants make a job offer to one of them. Workers that get one or more offers choose which job to accept. Therefore, the game can be separated in four distinct stages. If a firm's chosen applicant rejects the job offer then the firm remains unmatched. Firms therefore compete for workers in two separate stages: they want to attract at least one applicant in the second stage and they try to keep that applicant in the last stage; we label these 'ex ante' and 'ex post' competition, respectively. The utility of an employed worker is equal to his wage and the profits of a firm that employs a worker at wage w are given by $1 - w$.

As is common in the directed search literature, trading frictions are introduced by focusing attention on symmetric mixed strategies for workers. The assumption is that, since the market is large, workers cannot coordinate their search and hence they all use the same strategy. For simplicity, we also assume that their strategies are anonymous, i.e. all firms that post the same wage are treated identically by workers. This assumption, however, is not necessary: it is possible to let workers condition on the firms' names (say, a real number in [0,1]) but this would clutter the exposition without changing the results. Last, the firms also follow anonymous strategies, meaning that they treat all workers the same in the event that they receive multiple applicants. This is the standard environment in the directed search literature, such as Peters (1991) or Burdett, Shi, and Wright (2001), except for the innocuous assumption of the anonymity of workers' strategies, and the key difference that we allow multiple applications.

Before describing the actual strategies, observe that the last two stages of the game can be solved immediately. In the fourth stage, workers with multiple job offers choose the highest wage, and randomize with equal probabilities in the case of a tie. In the third stage, firms with many applicants choose one at random. Therefore we only need to consider the strategies for the first two stages. A strategy for the firm is a wage w that it posts in the beginning of the game. Workers observe all the wages and decide where to apply. Denote the distribution of posted wages by F and note that, due to anonymity, the workers' strategies can be summarized by the wages to which the applications are sent. Therefore, a pure strategy for a worker is an N-tuple of wages to which he applies and a mixed strategy is a randomization over different N-tuples. We denote the workers' strategy by $G(F)$, which is a mapping from the posted wages to the set of all cumulative distribution functions on $[0,1]^N$. Observe that a worker does not need to send each of his N applications independently.

2.2 Outcomes and Equilibrium

We define $q(w)$ to be the probability that a firm posting w receives at least one application and $\psi(w)$ to be the conditional probability that a worker who has applied to such a firm accepts a *different* job offer (i.e. the probability that the firm does *not* get the worker). Let $p(w)$ be the probability that a worker applying to wage w gets an offer and $\mathcal{W}$ be the support of the posted wages (i.e., $\mathcal{W} \equiv suppF$). When a wage is not posted by any firm ($w \notin \mathcal{W}$), we have $p(w) = 0$. Last, we define the *value* of an *individual* application to a wage w to be $p(w)\ w$. Given any N-tuple $\mathbf{w} = (w_1, w_2, ..., w_N)$ chosen by the worker, we assume without loss of generality that $w_N \geq w_{N-1} \geq ... \geq w_1$ for the remainder of the paper.

The expected profits of a firm that posts w and the expected utility of a worker who applies to $\mathbf{w}$ are given by

$$\begin{aligned} \pi(w) &= q(w)\ (1-\psi(w))\ (1-w) && (1) \\ U(\mathbf{w}) &= p(w_N)\ w_N + (1-p(w_N))\ p(w_{N-1})\ w_{N-1} \\ &+ \ ... + \prod_{i=2}^{N}(1-p(w_i))\ p(w_1)\ w_1 && (2) \end{aligned}$$

The expected profits are equal to the probability that at least one applicant appears times the retention probability times $(1-w)$. A worker gets utility w_N from his highest application, which is successful with probability $p(w_N)$. With the complementary probability that application fails and with probability $p(w_{N-1})$ he receives w_{N-1}. And so on.

On $\mathcal{W}$, both $p(w)$ and $q(w)$ depend on the average *queue length* at w, which is denoted by $\lambda(w)$. Intuitively, the queue length is the number of applications divided by the number of firms at a particular wage rate. Formally it is defined by the integral equation

$$\int_0^w \lambda(\tilde{w})\ dF(\tilde{w}) = b\ \hat{G}(w) \tag{3}$$

where $\hat{G}(w)$ is the expected number of applications that a single worker sends to wages no greater than w.[5] The right hand side of equation (3) gives the number of applications that are sent up to wage w by *all* workers, while the left hand side gives the number of firms that post up to that wage multiplied by the average number of applications they receive.

When a worker applies for a wage w he randomizes over all firms at that wage rate, due to anonymity. As a result, the number of applications received by a firm posting w is random and follows a Poisson distribution with mean $\lambda(w)$.[6] Therefore the probability that a firm posting w receives at least one application is $q(w) = 1-e^{-\lambda(w)}$ and the probability that a worker who applies to such a firm gets an offer is $p(w) = (1-e^{-\lambda(w)})/\lambda(w)$.[7]

[5] If $G_i(w)$ is the marginal distribution of w_i, then $\hat{G}(w) = \sum_{i=1}^N G_i(w)$.

[6] Suppose that n applications are sent at random to m firms. The number of applications received by a firm follows a binomial distribution with sample size n and probability of success $1/m$. As $n, m \to \infty$ keeping $n/m = \lambda$ the distribution converges to a Poisson distribution with mean λ.

[7] Notice that the anonymity of the worker strategies is not a necessary condition for this point. Symmetry and optimality clearly imply that firms with the same wage must have the same queue length. Poisson matching follows.

In order to evaluate $\psi(w)$ for some $w \in \mathcal{W}$ we need to find the probability that, after applying to w, a worker takes a different job. Let $R_j(w_j, w_{-j})$ be the probability that a worker who applies to (w_j, w_{-j}) accepts the job posting w_j if made an offer. This occurs if the worker has no offer that is strictly higher *and* w_j is picked in the case of a tie after randomizing. The indexes of applications can be relabeled so that higher indexes are given preference when tied. This means that $R_j(w_j, w_{-j}) = \prod_{k>j}(1 - p(w_k))$ and we can integrate over all possible wages where workers apply to.[8] Letting $Pr[j|w]$ be the conditional probability that a worker who applied to $w \in \mathcal{W}$ did so with his j^{th} application and $G_j(w_{-j}|w)$ be the conditional distribution over the other applications, given that the j^{th} application was sent to wage w, $\psi(w)$ is given by

$$\psi(w) = 1 - \sum_{j=1}^{N} \Pr[j|w] \int R_j(w, w_{-j})\, dG_j(w_{-j}|w) \tag{4}$$

So far $\lambda(w)$ and $\psi(w)$ have been defined for wages on the support of F, meaning that the workers' optimization problem can be solved for a given distribution of posted wages. However, off the equilibrium path payoffs need to be evaluated in order to solve the firms' problem, and this requires that $\lambda(w)$ and $\psi(w)$ are well defined on the full domain [0,1]. That is, a firm needs to know the queue length it will face at *any* wage. Therefore, although no one is actually applying to wages that are not posted, the queue lengths at such wages could be positive as they represent how many workers *would* apply there if these wage were offered; and similarly for $\psi(w)$. The problem is that when $w \notin \mathcal{W}$, $\lambda(w)$ and $\psi(w)$ are not pinned down by equations (3) and (4), as both F and G have zero density at those wages.

To get around this problem we define λ and ψ as if 'many' firms post every wage in [0,1] so that the reaction of workers can be meaningfully evaluated. We introduce a perturbation by assuming that firms make a mistake with probability ϵ and post a wage at random from a full support distribution, $\tilde{F}$. Equivalently, there is a measure ϵ of noise firms that do not optimize but simply post a wage at random. Given a candidate F, the distribution of posted wages becomes $F_\epsilon(w) = (1-\epsilon)\ F(w) + \epsilon\ \tilde{F}(w)$ and the game can be analyzed from the second stage onwards. Workers observe F_ϵ and their

[8]The relabeling is without loss of generality since the randomization can occur before the applications are actually sent.

best response is $G(F_\epsilon)$. The outcomes λ_ϵ and ψ_ϵ can be calculated in the entire domain [0,1] using F_ϵ, $G(F_\epsilon)$, and equations (3) and (4). As $\epsilon \to 0$ the perturbed distribution converges to F, and we define $\lambda(w) = \lim_{\epsilon\to 0} \lambda_\epsilon(w)$ and $\psi(w) = \lim_{\epsilon\to 0} \psi_\epsilon(w)$ for all $w \in [0,1]$. In the lemma at the end of the section we show that the qualitative properties of λ and ψ do not depend on the choice of $\tilde{F}$.[9]

We can now define an equilibrium.

Definition 2.1 *Given a distribution with full support $\tilde{F}$, an equilibrium is a set of strategies $\{F, G\}$ such that*

1. $\pi(w) \geq \pi(w')$ *for all* $w \in W$ *and* $w' \in [0,1]$.
2. $U(\mathbf{w}) \geq U(\mathbf{w}')$ *for all* $\mathbf{w} \in suppG(F)$ *and* $\mathbf{w}' \in [0,1]^N$.

The first condition captures the profit maximization by firms and the second one ensures that workers best respond.

We now state the main theorem of this paper.

Theorem 2.1 *An equilibrium always exists and it is unique when $N = 2$. N different wages are posted by firms and every worker sends one application to each distinct wage. The number of firms that post a given wage is decreasing with the wage. The equilibria are not constrained efficient.*

2.3 Preliminary Results

The next lemma will be useful in the following sections. Let $\underline{w}$ be the lowest posted wage that receives some applications with positive probability, i.e. $\underline{w} = \inf\{w \in \mathcal{W} | \lambda(w) > 0\}$.

[9] Two different approaches have been taken to solve the same problem in the $N = 1$ case. The market utility approach, used in Shimer (1996, 2004), Moen (1997), Acemoğlu and Shimer (1999), posits that workers respond to deviations by firms so as to be indifferent between applying anywhere. In our framework this approach yields identical result, but it is less appealing due to the complexity of specifying indifferences over sets of wages. Peters (2000) and Burdett, Shi, and Wright (2001), on the other hand, solve for the subgame perfect Nash equilibrium of the finite model and then take the limit of that equilibrium as the number of agents goes to infinity. While arguably the correct (or most reasonable) approach, with multiple applications this is intractable because the probability of success of each application is correlated (see Albrecht, Gautier, Tan, and Vroman (2004)).

Lemma 2.1 *Given any distribution of posted wages, worker optimization implies that* $\lambda(w)$ *is continuous, strictly positive and strictly increasing on* $(\underline{w}, 1] \cap \mathcal{W}$.

Proof: Recall that the probability of getting a job is given by $p(w) = (1 - e^{-\lambda(w)})/\lambda(w)$ for $w \in \mathcal{W}$. If $\lambda(w)$ is not strictly increasing there exist $w, w' \in W$ such that $w > w'$ and $p(w) \geq p(w')$. When $\lambda(w') > 0$, a worker who applies to that firm with positive probability can profitably deviate by switching to w since the wage is higher and the probability of getting an offer is at least as high. Therefore $\lambda(w)$ is strictly increasing above any posted wage that has a positive expected queue length, and hence on $(\underline{w}, 1] \cap \mathcal{W}$. Suppose that $\lambda(w)$ is not continuous on $[\underline{w}, 1] \cap W$. Then there is a $\hat{w} \in \mathcal{W}$ such that for $w \in \mathcal{W}$ arbitrarily close to $\hat{w}$ it holds that $|\lambda(\hat{w}) - \lambda(w)| > k$ for a given $k > 0$. Therefore the probability of getting a job offer is discontinuous at $\hat{w}$ and a worker applying in a neighborhood of $\hat{w}$ has an obvious profitable deviation. *QED*

The properties described in the lemma are very natural. The expected number of applicants increases with the wage that a firm posts, which also implies that the probability of getting an offer for that job is strictly decreasing. $\lambda(w)$ is continuous because the workers' best response to the offered wages 'smooths out' any discontinuities of F: even if a positive measure of firms posts a particular wage, the workers respond by sending a positive measure of their applications to that wage so that the queue length does not jump. It is important to note that the derived results hold for any perturbation and therefore they hold in the full [0,1] range of the unperturbed game. Moreover, any distribution with full support leads to monotonicity and continuity which are the main points of the lemma. As a result, the particular choice of $\tilde{F}$ does not make a difference.

3 A Special Case: $N = 2$

We look at the special case where workers send only two applications which provides many of the main insights. The case of a general N is discussed in the next section. We start by solving for the best response of workers given an arbitrary distribution of posted wages. We then characterize the wages that firms post. Finally, the existence and uniqueness of equilibrium is proved.

3.1 Worker Optimization

We first find the best response of workers for an arbitrary distribution of posted wages. The posted distribution could be the result of a perturbation but in that case the subscript ϵ is omitted to keep notation simple. When a worker decides where to apply he faces a menu of wage and probability pairs from which to choose. The queue length, and hence the probability of success, is determined by the distribution of posted wages, F, and the strategy that other workers use to apply for jobs, $G(F)$. Recalling that $w_2 \geq w_1$ by convention, the worker solves

$$\max_{(w_2,w_1)\in[0,1]^2} \; p(w_2)\; w_2 + (1 - p(w_2))\; p(w_1)\; w_1 \tag{5}$$

Differentiability of $p(w)$ is not guaranteed so the problem cannot be solved by taking the first order conditions. We show that each application can be evaluated separately, even though this is still a simultaneous choice problem. That is, the problem admits a convenient recursive solution.

The low wage application is exercised only if w_2 fails, which means that the optimal choice for w_1 has to solve

$$\max_{w\in[0,1]} \; p(w)\; w \tag{6}$$

Let u_1 denote this maximum value. Given that a worker sends his low wage application to a particular w_1 that solves (6), his optimal choice for the high wage application is a solution to

$$\max_{w\geq w_1} \; p(w)\; w + (1 - p(w))\; u_1 \tag{7}$$

Let u_2 denote the highest utility a worker can receive from two applications. An implication of worker optimization is that all low wage applications offer the same value u_1, and all pairs of wages where workers apply give the same total utility u_2.[10]

The next step is to show that the two problems can actually be solved independently of each other. Let $\bar{w}$ be the highest wage that offers u_1, i.e. $\bar{w} = \max\{w \in \mathcal{W} | p(w)\; w = u_1\}$.[11] The first proposition follows.

[10] It is not hard to see that a pair of wages is a solution to (5) if and only if it solves (6) and (7).

[11] The maximum is well defined since $\lambda(w)$ is continuous and $\mathcal{W}$ is a closed set.

Proposition 3.1 *Given any distribution of posted wages, workers optimize only if $w_1 \le \bar{w} \le w_2$ holds for every pair (w_1, w_2) where they apply.*

Proof: Suppose this is not true. Since $w_1 \le w_2$ the only other possibilities are $\bar{w} < w_1$ or $w_2 < \bar{w}$. By construction $w_1 > \bar{w}$ implies that $p(w_1)\ w_1 < u_1$ which cannot be optimal. If $w_2 < \bar{w}$ then a worker can deviate and send his high wage application to $\bar{w}$ instead of w_2. This deviation is profitable because

$$\begin{aligned} p(\bar{w})\ \bar{w} + (1 - p(\bar{w}))\ p(w_1)\ w_1 - [p(w_2)\ w_2 + (1 - p(w_2))\ p(w_1)\ w_1] &= \\ (p(\bar{w})\ \bar{w} - p(w_2)\ w_2) + [p(w_2) - p(\bar{w})]\ p(w_1)\ w_1 &> 0 \end{aligned}$$

The first term is non-negative since $\bar{w}$ provides the highest possible value by definition. The second term is strictly positive because $\bar{w} > w_2 \Rightarrow p(\bar{w}) < p(w_2)$. *QED*

This result has several implications. The workers are indifferent about which combination of wages they apply to so long as they are on opposite sides of $\bar{w}$. All wages below $\bar{w}$ offer the same value, u_1, since every worker sends his low application there; similarly, all wages above $\bar{w}$ offer u_2 when paired with a low wage. These results hold for any perturbed distribution of wages and hence they hold in the limit as $\epsilon \to 0$. Recalling that $\lambda(w)$ is strictly increasing in w and that $p(w) = (1 - e^{-\lambda(w)})/\lambda(w)$, the following conditions uniquely define the queue length:

$$\begin{aligned} p(w)\ w &= u_1,\ \forall\ w \in [u_1, \bar{w}] &(8)\\ p(w)\ w &+ (1 - p(w))\ u_1 = u_2,\ \forall\ w \in [\bar{w}, 1] &(9) \end{aligned}$$

Wages below u_1 are not relevant because the value of these openings is too low. Even if applicants receive an offer with probability one they would not apply to such a wage.

These observations are illustrated in figure (1). The high indifference curve (IC-H) traces the wage and queue length pairs where workers are willing to send a high wage application, while IC-L is the indifference curve for the low wage applications. The two curves intersect at $\bar{w}$ where workers are indifferent about whether they apply with a 'high' or a 'low' application. Finally, a wage above $\bar{w}$ attracts a high wage application. This means that the queue length is 'bid up' to IC-H and similarly for wages below the cutoff. Hence the dashed line is the indifference curve that firms anticipate.

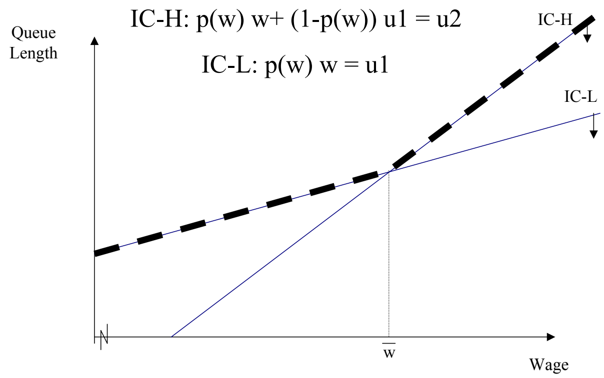

Figure 1: Workers' application behavior.

It is interesting to note that while the *total* utility of any pair of wages is always equal to u_2, wages that are strictly above $\bar{w}$ give value that is *stricly lower* than u_1 and workers nevertheless apply there. This is illustrated in figure (1) by the fact that IC-H provides with greater utility than IC-L in the high wages. This point appears to be counterintuitive at first sight: if workers can apply to wages that offer value u_1, why would they choose some wage with a strictly lower individual value? The answer is that the return to failure in the high wage application is not zero: it is equal to the value that the next application brings in. As a result, when the worker chooses where to send his high wage application he faces a tradeoff between the value that he can get from that particular application and the probability of exercising his fallback option, i.e. the low wage application. Since the low wage provides with insurance against the possible failure of w_2, it is profitable for the worker to try a risky application that has high returns conditional on success (i.e., the wage is high) and also offers a high chance of continuing to the next application. Therefore, the low wage application goes to a relatively 'safe' region and the high application is sent to a 'risky' part of the wage distribution.[12]

The next result proves that any equilibrium exhibits wage dispersion.

Proposition 3.2 *There does not exist an equilibrium in which only one wage is posted.*

Proof: See the appendix. QED

The main intuition of the proof is straightforward. When a single wage is posted, workers are indifferent about which firm to work for and hence they randomize when they get multiple job offers. However, if a firm deviates and posts a slightly higher wage, it wins the worker for sure even when he receives other offers. Since the increase in the hiring probability is discrete, while the increase in the wage can be arbitrarily small, this deviation raises profits. Note that it is the ex post competition among firms that precludes the possibility of a single wage equilibrium.

3.2 Characterization of Firm Optimization

We now turn to the analysis of the first stage of the model. We prove that exactly two wages are posted in equilibrium and we characterize them.

When posting a wage, firms solve

$$\max_{w\in[0,1]} \; q(w)\; (1-\psi(w))\; (1-w) \tag{10}$$

taking as given the equilibrium objects $\{\bar{w}, u_1, u_2\}$. The probability that a firm receives at least one applicant, $q(w)$, depends on the average queue length according to $q(w) = 1 - e^{-\lambda(w)}$. Whether a wage is above or below the cutoff $\bar{w}$ determines the type of application it receives (high or low). This helps evaluate the probability of losing a worker after making an offer, $\psi(w)$. We label the firms that attract high (low) wage applications as high (low) wage firms. The next proposition states the result of the maximization which is proved in the appendix. A discussion follows to provide intuition about the main points.

[12]As noted in the introduction, this is an important difference between our paper and other papers on directed search with wage dispersion in which the value of sending an application is always the same for identical workers. This is solely due to the fact that the same worker applies multiple times and hence he faces a portfolio choice problem.

Proposition 3.3 *In equilibrium, all high wage firms post $\bar{w}$ and all low wage firms post $\hat{w}_1 \in (u_1, \bar{w})$ which is derived by the first order conditions.*

Proof: See the appendix. *QED*

The reason why one wage is posted by each type of firms is not surprising: conditional on attracting a particular type of applications, firms compete with each other as in the one application case (e.g. Burdett, Shi, and Wright (2001)), subject to some additional boundary conditions. As a result there is a unique solution to each of their profit maximization problems and two distinct wages are posted, (w_1^*, w_2^*).

To examine this in some more detail note that high wage firms are the applicants' best alternative and hence workers never reject an offer by such a firm. Therefore $\psi(w) = 0$ and the maximization problem of high wage firms is given by

$$\max_{w \in [\bar{w}, 1]} [1 \quad - \quad e^{-\lambda(w)}] \; (1 - w) \tag{11}$$

$$\text{s.t. } \; p(w) \; w \quad + \quad (1 - p(w)) \; u_1 = u_2 \tag{12}$$

When profits are equalized across firms, the point of tangency between the isoprofit curve of the high wage firms and the high indifference curve of workers, $\hat{w}_2$, always occurs at a wage which is below $\bar{w}$, as illustrated in figure (2). This means that in any equilibrium the high wage firms have to post at their lower boundary and $w_2^* = \bar{w}$.

The retention probability of low wage firms can now be calculated. When a low wage firm makes a job offer, it loses its applicant only if he is successful in his high wage application which occurs with probability $p(\bar{w})$. As a result, low wage firms solve

$$\max_{w \in [0, \bar{w}]} (1 - e^{-\lambda(w)}) \; (1 \quad - \quad p(\bar{w})) \; (1 - w) \tag{13}$$

$$\text{s.t. } p(w) \; w \quad = \quad u_1 \tag{14}$$

Since the retention probability enters the maximization problem as a constant, it has no marginal effect on the choice of low wage firms. Proposition (3.2) ensures that in equilibrium low wage firms cannot be posting at the boundary. As a result their profit maximizing wage occurs at the point of tangency between their isoprofit curve and the low indifference curve of workers, i.e. $w_1^* = \hat{w}_1$. Last, note that the profit functions are different for the two types of firms which is why profits are equal even though the two isoprofit curves do not intersect in figure (2).

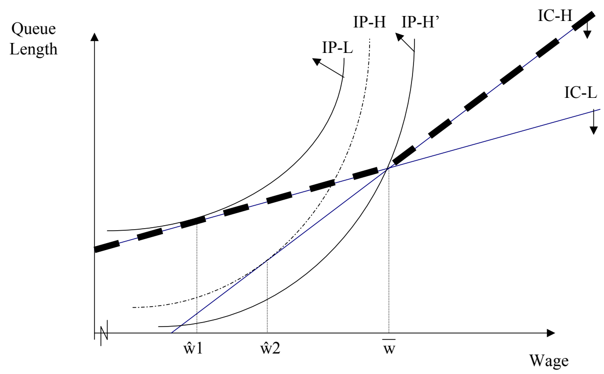

Figure 2: Firms' equilibrium behavior.

It is now easy to see that the density of posted wages is falling. Each wage receives one application per worker so $\lambda(w_i^*) = b/d_i$ where d_i is the fraction of firms posting w_i^*. $d_1 > d_2$ follows from the fact that the queue length is increasing with the wage rate. This result is driven by the fact that workers 'want' their high wage application to be risky (or, the queue length to be high). If this is not the case, a worker would be better off by not applying to the low wage and instead sending both applications to the high wage.

3.3 Existence and Uniqueness of Equilibrium

Turning to the existence of equilibrium, we need to find the 'correct' fraction of firms to post each wage so that profits are equalized across types of firms. More formally, an equilibrium exists if there is $\{d_1, d_2\}$ such that $d_1 + d_2 = 1$ and there is no profitable deviation when w_i^* is posted by d_i firms. Furthermore, the equilibrium is unique when there is a single pair of d_i's that satisfies the two conditions above.

Proposition 3.4 *An equilibrium exists and it is unique.*

Proof: See the appendix. QED

At this point it should be remarked that the full support of $\tilde{F}$ is the only property of the trembling distribution that is used in solving the model. As a result, the unique equilibrium that was constructed survives any choice of $\tilde{F}$.

4 The General Case: $N \geq 2$

We turn to the model with a general N. The analysis mirrors the one of section 3 and we prove that all results generalize, except for uniqueness. We provide computational evidence for uniqueness at the end of the section. Figure 3 illustrates the distribution of posted and received wages for an economy with equal number of workers and firms and $N = 15$. Properties of the distribution of received wages are discussed in the next section.

4.1 Worker Optimization

Let $\mathcal{W}_i$ be the support of w_i for all i, i.e. $\mathcal{W}_i$ is the set of wages that receive the i^{th} application of workers. As before, the utility of the lowest i applications has to be the same in any N-tuple of wages which defines the following recursive relationship

$$u_i = p(w_i)\ w_i + (1 - p(w_i))\ u_{i-1},\ \forall\ w_i \in \mathcal{W}_i,\ i \in \{1, 2, ...N\} \tag{15}$$

where $u_0 \equiv 0$. Note that $u_i > u_{i-1}$ since $w_i \geq w_{i-1}$. Moreover, u_i is the highest possible utility a worker can get from i applications when his fallback option is u_{i-1}. Let $\bar{w}_i$ be the highest wage that provides with total utility equal to u_i when the fallback option is u_{i-1}, i.e $\bar{w}_i = \sup\{w | p(w)\ w + (1 - p(w))\ u_{i-1} = u_i\}$. Also, let $\bar{w}_0$ be the lowest wage that receives applications with positive probability. We now generalize proposition (3.1).

Proposition 4.1 *When workers optimally send N applications, $w \in \mathcal{W}_i \Rightarrow w \in [\bar{w}_{i-1}, \bar{w}_i]$ for $i \in \{1, 2, ...N\}$.*

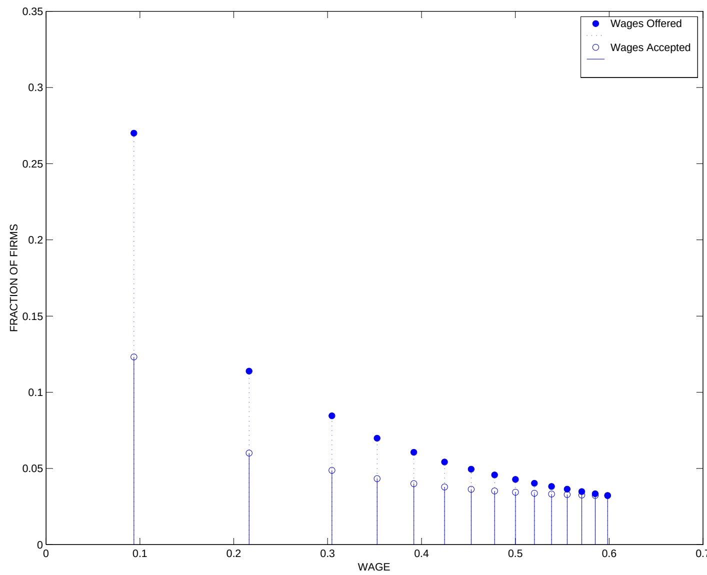

Figure 3: Equilibrium wage distribution for $N = 15$ and $b = 1$.

Proof: The proof is by induction. It is sufficient to show that the following property holds for all i: $w < \bar{w}_i \Rightarrow w \notin \mathcal{W}_k$ for $k \geq i+1$. The initial step for $i = 1$ was proved in the previous section, where $\bar{w}_1 = \bar{w}$. We assume that the property holds for $i-1$ and show that a contradiction is reached if it does not hold for i. In other words, if $w < \bar{w}_{i-1} \Rightarrow w \in \mathcal{W}_{i-1}$ holds, then there is no $\tilde{w} \in \mathcal{W}_{i+1}$ such that $\tilde{w} < \bar{w}_i$ (if $\tilde{w} \in W_k$ for $k > i+1$ the same argument goes through). Define $v(w, u_{i-1}) = p(w)\ w + (1-p(w))\ u_{i-1}$ to be the utility of applying to a particular wage w when the fallback option is u_{i-1}. We want to show that $v(\bar{w}_i, u_i) > v(\tilde{w}, u_i)$ for all $\tilde{w} < \bar{w}_i$. Note that

$$\begin{aligned} v(\tilde{w}, u_{i-1}) &= p(\tilde{w})\ \tilde{w} + (1-p(\tilde{w}))\ u_{i-1} \leq \\ v(\bar{w}_i, u_{i-1}) &= p(\bar{w}_i)\ \bar{w}_i + (1-p(\bar{w}_i))\ u_{i-1} \end{aligned}$$

since the second line is the optimal choice when u_{i-1} is the fallback option and hence it provides with the maximum level of utility. Replacing u_{i-1} with u_i in both lines above we get the terms to be compared. Since $\bar{w}_i > \tilde{w} \Rightarrow (1-p(\bar{w}_i)) > (1-p(\tilde{w}))$ the second term increases by more and the inequality becomes strict which proves the result. QED

An implication of the proposition is that when a worker applies to a firm of type i he receives the posted wage w if he is successful in his application or u_{i-1} if he is unsuccessful. Therefore the queue lengths facing the firms attracting the i^{th} application are given by the following equation:

$$p(w)\ w + (1 - p(w))\ u_{i-1} = u_i, \quad \forall\ w \in [\bar{w}_{i-1}, \bar{w}_i] \tag{16}$$

which is a straight generalization of equations (8) and (9).

4.2 Firm Optimization

We now turn to the first stage of the model. For the remainder of the paper firms that receive the i^{th} lowest application of workers are referred to as *type i firms*. The profit maximization problem of each type of firm is solved and profits are then equalized across types.

When posting a wage, firms take as given the cutoffs $\{\bar{w}_k\}_{k=0}^{N}$ and the equilibrium utility levels $\{u_k\}_{k=1}^{N}$, which determine the utility provided to workers for their lowest k applications. A firm of type i solves the following profit maximization problem:

$$\max_{w \in [\bar{w}_{i-1}, \bar{w}_i]} q(w)\ (1 - \psi(w))\ (1 - w) \tag{17}$$

where the queue lengths are determined by equations (16).

Proposition 4.2 *In equilibrium, all type i firms post $w_i^* = \bar{w}_{i-1}$ for $i \geq 2$. The wage that the lowest type of firms is $\hat{w}_1$ and it is given by the first order conditions.*

Proof: See the appendix. QED

The logic of the proof is similar to the one of proposition (3.3). The solution to the problem of type N firms is shown to be $\bar{w}_{N-1}$. This means that $\psi(w) = p(\bar{w}_{N-1})$ for type $N-1$ firms and the solution to their profit maximizing problem is $\bar{w}_{N-2}$. This implies that $\psi(w) = (1 - p(\bar{w}_{N-1}))\ (1 - p(\bar{w}_{N-2}))$ for type $N-3$ firms and so on. In general, the retention probability of a type i firms is $1 - \psi(w) = \prod_{n=i+1}^{N}(1 - p(w_n^*)) \equiv 1 - \psi_i$. Given ψ_i, the maximization problem for a type i firm becomes

$$\max_{w \in [\bar{w}_{i-1}, \bar{w}_i]} q(w)\ (1 - w)\ (1 - \psi_i) \tag{18}$$

$$\text{s.t.} \quad p(w)\ w + (1 - p(w))\ u_{i-1} = u_i \tag{19}$$

and the solution lies at the lower boundary for all $i \geq 2$. Finally, it should be noted that the density of posted wages is falling for the same reasons as in section 3.

4.3 Existence and Uniqueness of Equilibrium

The next proposition proves the existence of an equilibrium. We then provide some sufficient conditions for uniqueness and show computationally that they are plausible.

Proposition 4.3 *An equilibrium exists for any N.*

Proof: See the appendix. *QED*

In the appendix we show that given an arbitrary number of type one firms, d_1, we can find a unique sequence $d_2(d_1), d_3(d_1)...d_N(d_1)$ such that all types of firms make the same profits when w_i^* is posted by d_i firms. Furthermore, there is some d_1' such that $S(d_1') = 1$ where $S(d_1) \equiv d_1 + \sum_{i=2}^{N} d_i(d_1)$. The uniqueness of an equilibrium has not been proved analytically for a general N which means that there may be $d_1'' \neq d_1'$ with $S(d_1'') = S(d_1') = 1$.

We now fix N and derive some sufficient conditions for uniqueness. Let $S_b(d_1)$ denote the sum of the d_is when the number of firms posting w_1^* is d_1, the worker-firm ratio is b, and all firms make the same profits. Also, let $D(b)$ be the set of all N-tuples where $S_b(d_1) = 1$. The following lemma describes the result.

Lemma 4.1 *Given N, the equilibrium is unique for any $b > 0$ if $S_{b^*}(d_1)$ is strictly increasing for some b^*.*

Proof: Given any b and b' let $d_i' = (d_i\ b')/b$, $\lambda_i = b/d_i$, and $\lambda_i' = b'/d_i'$. Then $\lambda_i = \lambda_i'$ and therefore $\{d_1, d_2, ..., d_N\} \in D(b)$ if and only if $\{d_1', d_2', ..., d_N'\} \in D(b')$. Furthermore, the equilibrium conditions are fulfilled in one case if and only if they are fulfilled in the other. If $S_{b^*}(d_1)$ is strictly increasing for some b^*, then there is a unique d_1^* such that $S_{b^*}(d_1^*) = 1$ and hence there is a unique equilibrium. This means that the equilibrium is unique for any b. *QED*

Figure 4 graphs $S_b(d_1)$ when $b = 1$ for various N. Graphs for other N look similar, which suggests that the equilibrium is unique.

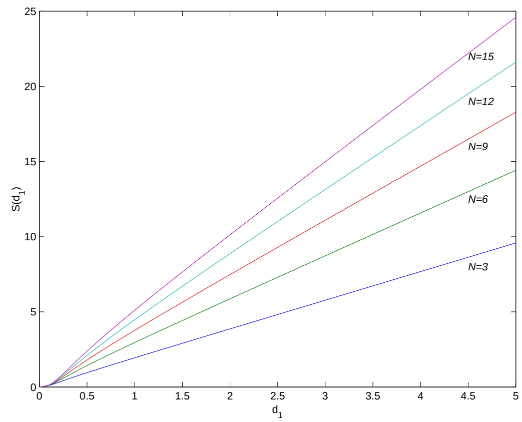

Figure 4: Sum of firms for $b = 1$ and various N.

5 Further Equilibrium Properties

In this section we investigate the efficiency properties of the matching process and the empirical distribution of the model.

5.1 Efficiency

The criterion for constrained efficiency is whether the output (or, the number of matches) is maximized conditional on the matching frictions, given the worker-firm ration b. The main result is that efficiency does not obtain, since workers send too many applications to high wage firms.

It was shown in the earlier sections that in equilibrium workers send each of their N applications to a different group of firms, which was identified by its distinct wage. Since wages are irrelevant for efficiency purposes we label the firms posting w_i as group i. As before, $p_i = p(w_i)$ and $\lambda_i = \lambda(w_i) = b/d_i$. Letting $\mathbf{d} = (d_1, d_2, ..., d_N)$ be the vector of the fraction of firms within each group, the total number of matches is given by $b\ m(\mathbf{d})$ where $m(\mathbf{d}) \equiv 1 - \prod_{i=1}^{N} (1 - p_i)$ is the probability that a particular worker receives a job offer. The planner has to decide how many firms to allocate to each group in order to maximize output or, equivalently, to maximize $m(\mathbf{d})$.

An immediate necessary condition for optimality (which fails) is that the probability of a match cannot be increased by reallocating firms between any two groups. This condition follows directly from observing that $m(\mathbf{d}) = 1 - (1-p_k)\ (1-p_l)\ \prod_{i \neq k,l} (1-p_i)$, given any two groups of firms, k and l. Therefore, an equilibrium is efficient only if d_k and d_l minimize $(1-p_k)\ (1-p_l)$, which is the same as solving

$$\begin{aligned} \max_{d_k, d_l \geq 0} (p_k \quad &+ \quad p_l \ - \ p_k\ p_l) \\ \text{s.t.}\ \ d_k + d_l \quad &= \quad 1 - \sum_{i \neq k,l} d_i \end{aligned} \tag{20}$$

This problem is identical to the case of two applications where the worker-firm ratio is given by $b/(1 - \sum_{i \neq k,l} d_i)$. Therefore, we consider the $N = 2$ case for an arbitrary b, letting d be the fraction of firms in the first group and $1 - d$ the fraction in the second group. The planner has to decide the optimal value of d.

Proposition 5.1 *When $N = 2$ the number of matches is maximized only if $d = 1/2$ or $d \in \{0, 1\}$.*

Proof: See the appendix. *QED*

The proposition shows that the number of firms should be equal in both groups when it is optimal to send two applications. Note, however, that it may be optimal to send only one application due to congestion. As a result, all non-degenerate groups should also have equal size when N applications are sent. However, we know that in equilibrium the number of firms posting the lower wages is larger and hence this efficiency condition is never met. This fundamental lack of efficiency arises because workers apply to a portfolio of wages and hence they are willing to accept higher queue lengths at higher wages. As this effect is not driven by any productivity differentials, it leads to inefficiencies. Moreover, since the lack of efficiency arises from the matching process it carries over even if the number of applications is endogenized or if the ratio of workers to firms is determined according to a zero profit condition. It is worthwhile to mention that in the usual directed search environment with one application efficiency does obtain: first, the matching process is constrained efficient by default; furthermore, entry is efficient and, in the case of agent heterogeneity, there is wage dispersion which leads to an efficient allocation of labor across firms (see Mortensen and Wright (2002) for a discussion).

5.2 The Empirical Distribution

As already noted, the density of posted wages is decreasing. Since higher wages are accepted more often, however, the density of *received* wages need not be decreasing. We find that a sufficient condition for the empirical density to be declining is that the worker-firm ratio, b, is large enough. The density may be non-monotonic for intermediate values of b and is increasing for very small b.

The measure of workers who are employed at wage w_i^* is given by $b\ (1 - \psi_{i+1})\ p_i \equiv E_i$. Moreover, $E_{i-1} = b\ (1 - \psi_i)\ p_{i-1} = b\ (1 - \psi_{i+1})\ (1 - p_i)\ p_{i-1}$. For the density to be declining, $E_i < E_{i-1}$ has to hold for all i which happens if and only if $p_i < (1 - p_i)\ p_{i-1}$. Equal profits imply that $q(w_i^*)\ (1 - w_i^*) = q(w_{i-1}^*)\ (1 - p_i)\ (1 - w_{i-1}^*)$ or $p_i\ \lambda_i\ (1 - w_i^*) = p_{i-1}\ \lambda_{i-1}\ (1 - p_i)\ (1 - w_{i-1}^*)$ yielding the condition $\lambda_i\ (1 - w_i^*) > \lambda_{i-1}\ (1 - w_{i-1}^*)$. Using the equilibrium conditions $w_i^* = (u_{i-1} - u_{i-2})/p_i + u_{i-1}$ and $w_{i-1}^* = (u_{i-1} - u_{i-2})/p_{i-1} + u_{i-1}$ the inequality becomes $\lambda_i(1 - x/p_i) > \lambda_{i-1}(1 - x/p_{i-1})$ where $x \equiv (u_{i-1} - u_{i-2})/(1 - u_{i-2})$. Therefore, the empirical distribution is decreasing if $g(\lambda) \equiv \lambda\ (1 - \lambda\ x/(1 - e^{-\lambda}))$ is increasing with respect to the queue length. The first derivative yields $\partial g/\partial \lambda = (1 - e^{-\lambda})\ (1 - e^{-\lambda} - 2\ \lambda\ x) + \lambda^2\ x\ e^{-\lambda}$ which is positive if $\lambda\ x$ is small. Noting that $\lambda_i\ x = (1 - e^{-\lambda_i})\ (w_i^* - u_{i-2})/(1 - u_{i-2})$ and that the right hand side goes to zero for b large enough the result is established.

6 Endogenizing N

We introduce a cost per application c and endogenize the number of applications that a worker sends. As earlier, attention is restricted to symmetric equilibria where every worker sends the same number of applications in expectation. Two separate issues are investigated. It is shown that there is a non-trivial range of the cost parameter that supports the equilibria described in the previous sections. We then discuss the equilibria that can arise for an arbitrary value of c.

To analyze the first issue, recall that u_i is the maximum payoff a worker receives when applying i times. To determine the marginal benefit of the i^{th} application note that in equilibrium for $i \geq 2$

$$
\begin{aligned}
u_i &= p_i\ w_i^* + (1 - p_i)\ u_{i-1} && (21)\\
u_{i-1} &= p_i\ w_i^* + (1 - p_i)\ u_{i-2} && (22)
\end{aligned}
$$

where the first expression holds by the definition of u_i and the second holds because $w_i^* = \bar{w}_{i-1}$. Subtracting (22) from (21), the marginal benefit of the i^{th} application is given by $u_i - u_{i-1} = (1-p_i)\ (u_{i-1} - u_{i-2}) = \prod_{j=2}^{i}(1-p_j)\ u_1$. Clearly, the marginal benefit of an additional application is decreasing in i and as a result $u_N - u_{N-1} > c$ is a sufficient condition for workers to send at least N applications. Moreover, since the left hand side is strictly positive, the equilibrium is robust to the introduction of small costs of search.[13]

The next step is to ensure that no worker applies more than N times. It is easy to see that a worker who contemplates sending $N+1$ applications will send his additional application to the highest wage, w_N^*. His utility from applying $N+1$ times is therefore given by $u_{N+1} = p_N\ w_N^* + (1-p_N)\ u_N$ which means that the marginal benefit of the 'extra' application is $u_{N+1} - u_N = (1-p_N)\ (u_N - u_{N-1})$. As a result, an equilibrium where workers apply exactly N times can be supported when the cost parameter lies in the open set $((1-p_N)\ (u_N - u_{N-1}), u_N - u_{N-1})$.

Turning to the case of determining N for an arbitrary c, let the superscript n denote the equilibrium outcomes that arise when workers send n applications. It is possible that $c < (1-p_N^N)\ (u_N^N - u_{N-1}^N)$ and $c > u_{N+1}^{N+1} - u_N^{N+1}$ hold simultaneously. The first inequality means that a worker has an incentive to apply $N+1$ times, when everyone else sends N applications, while the second inequality implies that applying N times is preferable when everyone applies $N+1$ times. As a result, an equilibrium in the (now endogenous) number of applications has to involve some randomization in the number of applications: proportion $\alpha \in (0,1)$ of workers apply $N+1$ times while $1-\alpha$ apply N times, where α is chosen so that both types of workers receive the same ex ante utility.[14] It is worth noting that an equilibrium where workers randomize over how many times to apply looks very much like the one we have already developed. Some firms will post a wage, w_{N+1}^*, which is visited only by workers who send $N+1$ applications. The maximization problem of that group can be solved in a similar way to the previous sections, though characterization may be slightly different as it is possible that w_{N+1}^* is not constrained. Numerical simulations suggest that any possible possible cost per application can be supported by such an equilibrium. However, there

[13]This is not the case in other models, e.g. Albrecht and Axell (1984); see Gaumont, Schindler, and Wright (2005) for a discussion.

[14]It is relatively straightforward to show that the number of applications that workers send in equilibrium can only be one apart. This is due to the decreasing returns of additional applications.

may be multiplicity of equilibria.

7 Conclusions

We develop a directed search model where workers apply simultaneously for N jobs. We find that an equilibrium always exists. We prove analytically that the equilibrium is unique for $N = 2$ and show computational evidence suggesting that it is also unique for any N. All equilibria exhibit wage dispersion, with firms posting N different wages and workers sending one application to each distinct wage. In line with stylized facts, the density of posted wages is decreasing. The main distinguishing feature of our model is that dispersion is driven by the portfolio choice that workers face. As a result, the equilibrium is inefficient because the higher paying firms enjoy higher probability of hiring a worker without underlying productivity differentials. A social planner can increase the total number of matches by having the same probability of success across firms.

To our knowledge, the only other directed search model where workers can simultaneously apply for multiple jobs is Albrecht, Gautier, and Vroman (2005). Their set-up is similar to ours, except for a crucial assumption: in their model, when two or more firms make an offer to the same applicant the potential employers engage in Bertrand competition for the worker and hence he ends up receiving the full surplus of the match. It is not hard to see that such an assumption negates our proof for the necessity of wage dispersion. Indeed, in their model the unique equilibrium has all firms posting the reservation value of workers, with some workers receiving their marginal product due to Bertrand competition, regardless of the number of applications. While we think that ex post bidding is not an unreasonable assumption, we believe that it is useful to explore alternative formulations. In our setting, commitment to posted wages results in dispersion in posted, as well as received, wages. Moreover, the number of times that workers apply affects both the number of posted wages and the variation of dispersion in the market.

We should mention that our model can be easily extended in a number of ways. First, it is very straightforward to show that all the static characterization results can be replicated every period in a discrete time infinite horizon setting. Other potentially interesting extensions include firm and worker heterogeneity. It is worth noting that the homogeneity of firms was not used when analyzing workers' optimization and, therefore, the results carry over in the case of productivity differentials among firms. Their optimization problems will be different, of course, and we conjecture that more productive firms will attract higher applications since they place a premium on hiring. Similarly, in the case of observable worker heterogeneity each firm posts a menu of type-specific wages and each type of workers has its own set of utility levels and cutoffs. In conclusion, we believe that this paper provides some basic structure for further analysis.

8 Appendix

Proof of Proposition (3.2).
We show that when a single wage is posted, firms have a profitable deviation. In order to evaluate off the equilibrium path payoffs we perturb the game and find the limits of the outcomes as the perturbation vanishes. Assume an equilibrium exists such that all firms post the same wage w^*. The expected profits are given by $\pi(w^*) = q(w^*)\ (1-w^*)\ (1-\psi(w^*))$, where $\psi(w^*) > 0$ since a worker turns down a firm with positive probability in the case of multiple offers. Suppose $w^* \in (0,1)$ and note that $w^* = \bar{w}$ when trembles are sufficiently small, since each worker sends at most one application to trembling firms. This immediately implies that in the limit $\psi(w) = 0$ for all $w > w^*$. Since the queue length (and $q(w)$) is increasing in w, the profits of a firm that posts a wage just above w^* are equal to $\lim_{w\searrow w^*} \pi(w) = q(w^*)\ (1-w^*) > q(w^*)\ (1-w^*)\ (1-\psi(w^*)) = \pi(w^*)$. Therefore offering a wage just above w^* is a profitable deviation.

If all firms post $w^* = 1$ they make zero expected profits. It is easy to see that there is some $\tilde{w}$ close enough to one which receives applications with probability that is bounded away from zero for all trembles and hence there is a profitable deviation at the unperturbed game. Last, if $w^* = 0$ workers receive zero expected utility and so for any positive trembles they send both applications to positive wages. As the trembles become smaller the hiring probability of a firm with a positive wage converges to one and since $q(0)\ (1-\psi(0)) < 1$ posting a wage slightly above zero increases the firm's profits. *QED*

Proof of Proposition (3.3).
The proposition is proved in two stages. The problem of the high wage firms is solved first and that of the low wage firms follows. As shown in section 3,

the maximization problem of the high wage firms is given by

$$\max_{w \in [\bar{w},1]} \ (1 \ - \ e^{-\lambda(w)}) \ (1-w) \tag{23}$$

$$\text{s.t.} \ \ p(w) \ w \ + \ (1-p(w)) \ u_1 = u_2 \tag{24}$$

Using the constraint we can solve for $w = (u_2 - u_1) \ \lambda/(1-e^{-\lambda}) + u_1$ and substitute that expression into the objective function. The maximization problem can be rewritten with respect to λ as $\max_{\lambda \geq \bar{\lambda}} \ 1 - u_1 - \lambda \ (u_2 - u_1) - e^{-\lambda} \ (1-u_1)$ where $\bar{\lambda} = \lambda(\bar{w})$. This problem is strictly concave in λ since $u_1 < 1$ and hence it has a unique solution λ_2^*, which corresponds to some w_2^*. Note that we proceeded as if $\psi(\bar{w}) = 0$ which is not necessarily the case. However, if $w_2^* > \bar{w}$ then the value of $\psi(\bar{w})$ is irrelevant; if $w_2^* = \bar{w}$ then proposition (3.2) shows that low wage firms cannot post $\bar{w}$ in equilibrium and hence $\psi(\bar{w}) = 0$. Therefore the maximization problem is specified correctly.

There are two candidate solutions for w_2^*. If the constraint does not bind, the wage is determined by the first order conditions of the problem, $\hat{w}_2$. If the constraint does bind then $w_2^* = \bar{w}$. We show that high wage firms enjoy strictly higher profits than low wage firms when $w_2^* = \hat{w}_2$. Setting the derivative of the problem to zero yields $u_2 - u_1 = e^{-\lambda_2^*} \ (1-u_1)$. Substituting this expression back into the profit function and rearranging gives the following:

$$\pi(\hat{w}_2) = (1-e^{-\lambda_2^*}) \ (1-u_1) \ (1 - \frac{\lambda_2^* \ e^{-\lambda_2^*}}{1-e^{-\lambda_2^*}}). \tag{25}$$

The profits of a low wage firm that posts w_1 and has expected queue length $\lambda_1 = \lambda(w_1)$ are given by

$$\pi(w_1) = (1-e^{-\lambda_1}) \ (1-w_1) \ (1 - \frac{1-e^{-\lambda_2^*}}{\lambda_2^*}), \tag{26}$$

where the first term is the probability of getting at least one applicant, the second term is the margin of the firm, and the last term is the probability that the chosen applicant does not have an offer from a high wage firm.

Comparing the two equations term by term it is easy to see that the profits of high wage firms are strictly higher: firms offering a higher wage have longer queues, so $\lambda_2^* > \lambda_1$ which means that $1 - e^{-\lambda_2^*} > 1 - e^{-\lambda_1}$; $u_1 = p(w_1) \ w_1$ which implies that $u_1 \leq w_1$; to prove the the third term we need to show that $\lambda \ e^{-\lambda}/(1-e^{-\lambda}) < (1-e^{-\lambda})/\lambda$ for any $\lambda > 0$. This expression can be rearranged as $\lambda^2 \ e^{-\lambda} < (1-e^{-\lambda})^2$ or $\lambda^2 \ e^{\lambda} - e^{2 \ \lambda} + 2 \ e^{\lambda} - 1 < 0$. Denote the left hand side by $H_1(\lambda)$ and note that $H_1(0) = 0$. If $H_1'(\lambda) < 0$ for all $\lambda > 0$ we have our result. But, $H_1'(\lambda) = (2\ \lambda + \lambda^2 + 2 - 2\ e^{\lambda})\ e^{\lambda}$ and $H_1'(0) = 0$. Call the term in the bracket $H_2(\lambda)$ and note that $H_2(0) = 0$. Then $H_2'(\lambda) = 2\ (1 + \lambda - e^{\lambda})$ which is negative for $\lambda > 0$. Therefore, $w_2^* = \bar{w}$ is a necessary condition for any equilibrium.

Turning to low wage firms, they solve

$$\max_{w}\ (1 - e^{-\lambda(w)})\ (1 \quad - \quad \psi(w))\ (1 - w) \tag{27}$$

$$\text{s.t. } p(w)\ w \quad = \quad u_1 \tag{28}$$

As argued above, $\psi(w) = p(w_2^*)$ for $w \in [0, \bar{w}) \cap \mathcal{W}$, i.e. for all wage levels that are actually posted. We solve the maximization problem as if $\psi(w)$ is the same for all w, whether posted or not, which is the case when, for instance, workers randomize independently inside each of the two intervals. We then show that this simplification is innocuous. Using equation (28), we can solve for $w = u_1\ \lambda/(1 - e^{-\lambda})$ and substitute it into the profit function to get $\max_{\lambda}\ (1 - e^{-\lambda} - \lambda\ u_1)\ (1 - p(w_2^*))$. The term in the second bracket has no marginal effect so the problem is strictly concave and therefore it has a unique solution λ_1^*. The first order conditions imply $u_1 = e^{-\lambda_1^*}$ and hence $w_1^* = \lambda_1^*\ e^{-\lambda_1^*}/(1 - e^{-\lambda_1^*})$.

We now consider the case where the worker strategies are such that $\psi(w)$ takes different values in $[0, \bar{w})$. An example of why this could happen is the following. Suppose that one of the pairs of wages that the workers randomize over in response to every perturbed distribution is $(\tilde{w}_1, \tilde{w}_2)$ where $\tilde{w}_2 \approx 1$. If workers applying to $\tilde{w}_1$ send their high wage application to $\tilde{w}_2$ only, then the retention probability at $\tilde{w}_1$ is very high since $\tilde{w}_2$ being close to one implies that $p(\tilde{w}_2)$ has to be very low. As the trembles become smaller, the probability that this particular pair is chosen converges to zero, however $\psi_\epsilon(\tilde{w}_1)$ remains equal to $p(\tilde{w}_2)$ and so it converges to a relatively high value. This would be troublesome if a different equilibrium could be supported in the way described. Suppose that there is such an equilibrium in which low wage firms post some $\tilde{w} \neq \hat{w}_1$. For $\tilde{w}$ to be posted it needs to provide the highest possible profits, implying in particular that $\pi(\tilde{w}) \geq \pi(\hat{w}_1)$. The last inequality can only hold if $\psi(\hat{w}_1) > \psi(\tilde{w})$ since $\{\hat{w}_1\} = \text{argmax}(1 - e^{-\lambda(w)})\ (1 - w)$. However, the fact that $\tilde{w}$ is actually posted means that $\psi(\tilde{w}) = p(w_2^*)$. Moreover, $w_2^* = \bar{w}$ implies that $p(w) \leq p(w_2^*)$ for all wages w that high firms can post and hence $\psi(\tilde{w}) = p(w_2^*) \geq \psi(\hat{w}_1)$, yielding a contradiction. Therefore no other equilibrium can be supported.

This completes the proof of proposition (3.3). *QED*

Proof of Proposition (4.2).
To generalize (3.3) to any N it is sufficient to show that unless type $i \geq 2$ firms post $\bar{w}_{i-1}$ they make strictly higher profits than firms of type $i-1$. After using the constraint to solve for the wage, and taking the first order conditions, the profits of a type i firm are

$$\pi(\hat{w}_i) = (1 - e^{-\lambda_i^*})\ (1 - u_{i-1})\ (1 - \frac{\lambda_i^*\ e^{-\lambda_i^*}}{1 - e^{-\lambda_i^*}})\ (1 - \psi_i) \tag{29}$$

The profits of a type $i-1$ firm are given by

$$\pi(w_{i-1}) = (1 - e^{-\lambda_{i-1}})\ (1 - w_{i-1})\ (1 - \frac{1 - e^{-\lambda_i^*}}{\lambda_i^*})\ (1 - \psi_i) \tag{30}$$

and they are lower for the same reasons as before. *QED*

Proof of Propositions (3.4) *and* (4.3).
We show that there is a sequence $\{d_i\}_{i=1}^{N}$ such that when w_i^* is posted by d_i firms (call these type i firms), there is no profitable deviation for any type of firm. Afterwards, uniqueness is proven for the $N = 2$ case.

First, consider deviations within the same type. Since $w_1^* = \hat{w}_1$ it is immediate that type 1 firms cannot profitably deviate within their type. For type $i \geq 2$ firms, $w_i^* = \bar{w}_{i-1}$ is a necessary condition for equilibrium. $\bar{w}_{i-1}$ is the profit maximizing wage within type i only if $\bar{w}_{i-1} > \hat{w}_i$, i.e. when then wage derived from the first order condition is not feasible. We show that profits can be equalized across types only if the above condition holds. The previous proposition proved that if type i firms post $\hat{w}_i$ then they necessarily make higher profits than type $i-1$ firms, or $\pi(\hat{w}_i) > \pi(w_{i-1}^*)$. If $\bar{w}_{i-1} < \hat{w}_i$, and if all type i firms post $\bar{w}_{i-1}$ they make higher profits than if they all posted $\hat{w}_i$. This happens because they receive the same number of applications (one per worker) but pay them less (however, each firm could individually increase its profits even more by posting $\hat{w}_i$). As a result, $\pi(\bar{w}_{i-1}) > \pi(\hat{w}_i)$ and profits cannot be equalized across types i and $i-1$. If, on the other hand, $\bar{w}_{i-1} > \hat{w}_i$, then $\pi(\bar{w}_{i-1}) < \pi(\hat{w}_i)$. Therefore, if profits can be equalized across types, then $\bar{w}_{i-1}$ is the profit maximizing wage of type i firms.

The next step is to prove that profits can be equalized across types of firms. To simplify notation let $\pi_i = \pi(w_i^*)$, $p_i = p(w_i^*)$, $\tilde{\pi}_i = \pi_i/(1 - \psi_i)$, and $\lambda_i^* = b/d_i$. For equal profits across types it is sufficient to show that $\pi_i = \pi_{i-1}$ for all i, which is the same as $\tilde{\pi}_i = (1-p_i)\ \tilde{\pi}_{i-1}$ since the term $(1-\psi_i)$ is common to both sides. We show that given a d_{i-1} we can find a d_i in $(0, d_{i-1})$ such that $\Delta\pi_i(d_i|d_{i-1}) \equiv \tilde{\pi}_{i-1} - \tilde{\pi}_i/(1-p_i) = 0$. This allows us to construct a sequence of d_is such that all firms make the same profits for an arbitrary initial d_1. We then show that the d_i's sum up to one.

It is useful to recall the following two equations (for $i \geq 2$).

$$
\begin{aligned}
u_{i-1} &= p_{i-1}\ w_{i-1}^* + (1-p_{i-1})\ u_{i-2} && (31)\\
u_{i-1} &= p_i\ w_i^* + (1-p_i)\ u_{i-2} && (32)
\end{aligned}
$$

Equation (31) holds by the definition of u_{i-1}. Equation (32) holds because $w_i^* = \bar{w}_{i-1}$ and hence the i firm has to provide the same utility as w_{i-1}^* if it is used for the $i-1$ lowest application.

Note that the queue lengths are the same when $d_i = d_{i-1}$, which means that $p_{i-1} = p_i$, $w_{i-1}^* = w_i^*$, and $\tilde{\pi}_{i-1} = \tilde{\pi}_i$ leading to $\Delta\pi_i(d_{i-1}; d_{i-1}) < 0$. On the other hand, $\lambda_i \approx \infty$ when $d_i \approx 0$ which means that $p_i \approx 0$ and therefore equation (32) requires a very large w_i^* leading to $\tilde{\pi}_i < 0$ (this occurs because the firm is assumed to post $\bar{w}_{i-1}$). $\Delta\pi_i(d_i|d_{i-1}) > 0$ when $d_i \approx 0$, and there is a $d_i(d_{i-1})$ such that type i and $i-1$ firms make the same profits. Moreover, the solution $d_i(d_{i-1})$ is unique because

$$
\frac{\partial \Delta\pi_i}{\partial d_i} = -\tilde{\pi}_i\ \frac{\partial(1/(1-p_i))}{\partial d_i} - \frac{1}{1-p_i}\ \frac{\partial \tilde{\pi}_i}{\partial d_i} < 0 \tag{33}
$$

When d_i increases the queue length decreases and hence the probability of getting a job increases. Therefore the first partial is positive and the first term as a whole is strictly negative. The second partial is non-positive since $\partial\tilde{\pi}_i/\partial\lambda_i \leq 0$. Recall that when $i = 1$ the first order conditions are equal to zero because $w_1^* = \hat{w}_1$. Furthermore, when $i \geq 2$ the firm would like to post a lower wage when profits are equalized (i.e., $w_i^* = \bar{w}_{i-1} > \hat{w}_i$) which implies that the first order conditions with respect to λ are strictly negative. This proves that equation (33) is strictly negative.

Therefore, for a given d_1 the rest of the sequence $d_2(d_1), d_3(d_1)...d_N(d_1)$ can be uniquely constructed such that all types of firms make the same profits. To find the sequence whose elements sum up to one define $S(d_1) \equiv \sum_{i=1}^N d_i(d_1)$ and note that it is continuous since all of its components vary continuously with d_1. Moreover, $S(1/N) < 1$ since $d_i(d_{i-1}) < d_{i-1}$ and

$S(1) > 1$ so there is some d_1^* such that $S(d_1^*) = 1$ and an equilibrium exists for any N.

To prove the uniqueness of equilibrium when $N = 2$ we show that d_1 and $d_2(d_1)$ are positively related along the isoprofit curve, and hence there is a unique pair that sums up to one. Implicit differentiation of d_2 with respect to d_1 while keeping profits equal yields $\partial d_2/\partial d_1 = -(\partial\Delta\pi_2/\partial d_1)/(\partial\Delta\pi_2/\partial d_2)$. The denominator is positive by (33). A little algebra shows that the numerator is given by $\partial\Delta\pi_2/\partial d_1 = (\partial\lambda_1^*/\partial d_1)\ e^{-\lambda_1^*}\ (\lambda_1^* - \lambda_2^*/(1-p_2))$, which is positive since the queue length is inversely related to the number of firms and $\lambda_1^* < \lambda_2^*$. This proves that the equilibrium is unique when $N = 2$. *QED*

Proof of Proposition (5.1).
The planner solves the following problem: $\max_{d\in[0,1]}\ m(d) = p_1 + p_2 - p_1\ p_2$. If the problem has an interior solution, the first order conditions yield

$$\frac{\partial p_2}{\partial d_1}(1-p_1) + \frac{\partial p_1}{\partial d_1}(1-p_2) = 0 \tag{34}$$

Recalling that $\lambda_1 = b/(1-d)$ and $\lambda_2 = b/d$ it is easy to see that $\partial p_i/\partial d = -\partial\lambda_i/\partial d\ (1 - e^{\lambda_i} - \lambda_i\ e^{-\lambda_i})/\lambda_i^2$, $\partial\lambda_1/\partial d = b/(1-d)^2 = \lambda_1^2/b$, and $\partial\lambda_2/\partial d = -b/d^2 = -\lambda_2^2/b$, so equation (34) can be rewritten as

$$(1 - e^{-\lambda_2} - \lambda_2 e^{-\lambda_2})(1 - \frac{1-e^{-\lambda_1}}{\lambda_1}) = (1 - e^{-\lambda_1} - \lambda_1 e^{-\lambda_1})(1 - \frac{1-e^{-\lambda_2}}{\lambda_2}) \tag{35}$$

It is immediate that one extremum occurs when $\lambda_1 = \lambda_2$, or $d = 1/2$. The second derivative is given by

$$\begin{aligned}\frac{\partial^2 m}{\partial d^2} = \quad & \tfrac{1}{b^2}(1 - e^{-\lambda_2} - \lambda_2 e^{-\lambda_2})(1 - e^{-\lambda_1} - \lambda_1 e^{-\lambda_1}) - \tfrac{1}{b^2}\lambda_2^3 e^{-\lambda_2}(1-p_1) \\ & \tfrac{1}{b^2}(1 - e^{-\lambda_2} - \lambda_2 e^{-\lambda_2})(1 - e^{-\lambda_1} - \lambda_1 e^{-\lambda_1}) - \tfrac{1}{b^2}\lambda_1^3 e^{-\lambda_1}(1-p_2).\end{aligned} \tag{36}$$

Substitution of (35) and dividing by $(1-p_1)(1-p_2)/b^2$ establishes that at any candidate extreme point the sign of the second derivative is given by $sign(\partial^2 m/\partial d^2) = sign(f(\lambda_2) + f(\lambda_1))$, where

$$f(\lambda) = \frac{(1 - e^{-\lambda} - \lambda e^{-\lambda})^2}{(1 - (1-e^{\lambda})/\lambda)^2} - \frac{\lambda^3 e^{-\lambda}}{1 - (1-e^{\lambda})/\lambda}. \tag{37}$$

Therefore, we want to show that there is no $b > 0$ such that there exists $d \in (1/2, 1)$ where (35) holds and

$$f(\frac{b}{d}) + f(\frac{b}{1-d}) \leq 0. \tag{38}$$

Figure 5 shows $f(\lambda)$ for $\lambda \geq 0$. The function is strictly decreasing on $(0, a_1)$, strictly increasing on (a_1, a_4), again strictly decreasing on (a_4, ∞) and converges to 1 for $\lambda \to \infty$. The only roots of the function are 0 and a_2. We will discuss this function in order to establish the result. Note that for any b, the specific value of d defines $\lambda_1 = b/d$ and $\lambda_2 = b/(1-d)$. Note that for $\lambda_2 > a_3$ it is not possible to fulfill (38), where a_3 is such that $f(a_3) = -f(a_1)$. Therefore we will restrict the discussion to $\lambda_2 < a_3$. This also implies that we do not have to discuss any b where $2b > a_3$. For $d = 1/2$ we know that $\lambda_1 = \lambda_2$, and therefore the first order condition holds and $sign(\partial^2 m/\partial d^2) = sign f(2b)$.

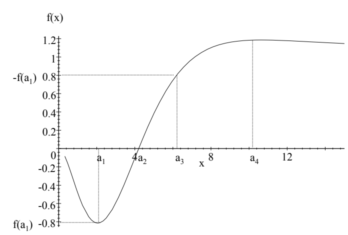

Figure 5: f(x) for $x \geq 0$.

CASE 1: $b \geq a_2/2$. Then at $d = 1/2$ we have $2f(2b) \geq 0$. Starting from $d = 1/2$, i.e. $\lambda_1 = \lambda_2$, we will increase d and thus spread λ_1 and λ_2 apart. We will show that there does not exist $d > 1/2$ such that (38) holds. Assume that (38) holds for the given b at some $d > 1/2$. Then for any $b' \in [a_2/2, b)$ there exists a $d' > 1/2$ such that (38) holds. This is easy to see if there exists $d' > 1/2$ such that $\lambda_1 = b/d = b'/d' = \lambda_1'$. Then $f(\lambda_1) = f(\lambda_1')$. Since $\lambda_2 = b/1-d) > b'/(1-d') = \lambda_2'$, $f(\lambda_2) > f(\lambda_2')$. But then $f(\lambda_1) + f(\lambda_2) \leq 0$ implies $f(\lambda_1') + f(\lambda_2') < 0$. If for some $b' \in [a_2/2, b)$ no such $d' > 1/2$ exists, we reach a contradiction: There is some $b'' \in [b', b)$ such that at $d'' = 1/2$ it holds that $\lambda_1 = b/d = b''/d'' = \lambda_1'$. By the prior argument $f(\lambda_1'') + f(\lambda_2'') < 0$, but this violates $2f(2b) = f(\lambda_1'') + f(\lambda_2'') \geq 0$. Therefore, if we know that (38) does not hold at $\tilde{b} = a_2/2$, then we know that (38) does not hold for any $b > a_2/2$. Figure 6 shows $f(a_2/2d) + f(a_2/(2(1-d)))$ for all $d \geq 1/2$, which is strictly positive for all $d > 1/2$. Therefore, (38) does not hold for any $b \geq a_2/2$.

CASE 2: $b < a_2/2$. In this case we have at $d = 1/2$ that $2f(2b) < 0$,

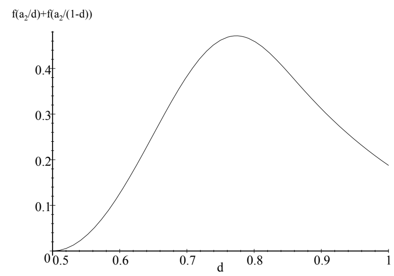

Figure 6: $f(\frac{a_2}{2d}) + f(\frac{a_2}{2(1-d)})$ for $d \in [0, 1]$.

i.e. we are in a local maximum. If there exist any other local maxima at $d > 1/2$, there has to be some $d' \in (1/2, d)$ that constitutes a local minimum. Therefore, if for some d conditions (38) and (35) hold simultaneously, then there exists $1/2 < d' < d$ such that $f(b/d') + f(b/1 - d') > 0$. At d' it has to hold $\lambda_2' = b/(1-d') > a_2$, otherwise $f(\lambda_1') + f(\lambda_2') > 0$ would not be possible. We also know that $\lambda_1' < b/2 < a_2$. Since $d' < d$, we know that $\lambda_1 < \lambda_1'$ and $\lambda_2' < \lambda_2$. Now consider a d' at which $f(\lambda_1') + f(\lambda_2') > 0$. If we increase d to values above d', the derivative of $f(\lambda_1) + f(\lambda_2)$ is

$$
\begin{aligned}
\frac{\partial(f(\lambda_1) + f(\lambda_2))}{\partial d} &= f'(\lambda_1)\frac{\partial \lambda_1}{\partial d} + f'(\lambda_2)\frac{\partial \lambda_2}{\partial d} && (39) \\
&= \frac{1}{b}[-f'(\lambda_1)\lambda_1^2 + f'(\lambda_2)\lambda_2^2]. && (40)
\end{aligned}
$$

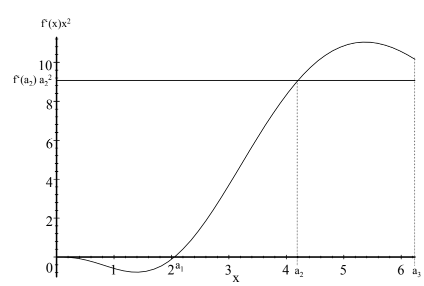

Figure 7: $f'(x)x^2$ for $x \in [0, a_2]$.

If the term in square brackets is positive, then $f(\lambda_1)+f(\lambda_2)$ is increasing as we increase d further. So if we can show that the part in the square brackets is positive for all $(\lambda_1,\lambda_2)\in[0,a_2]\times[a_2,a_3]$, then it is not possible to increase d starting from any d' and achieve a negative value of $f(\lambda_1)+f(\lambda_2)$ (which we would need to arrive at another maximum). Since $max_{[0,a_2]}f'(\lambda)\lambda^2 \leq min_{[a_2,a_3]}f'(\lambda)\lambda^2$, as can be seen in figure 7, it is not possible to have another local maximum in the interior apart from $d=1/2$. *QED*

References

[1] Acemoğlu, Daron, and Robert Shimer. "Efficient Unemployment Insurance." *Journal of Political Economy*, 107 (1999), 893-928.

[2] Albrecth, James, and Bo Axell. "An Equilibrium Model of Search Unemployment." *Journal of Political Economy*, 92 (1984), 824-40.

[3] Albrecht James, Pieter A. Gautier, and Susan Vroman. "Equilibrium Directed Search with Multiple Applications," mimeo (2005).

[4] Albrecht James, Pieter A. Gautier, Serene Tan, and Susan Vroman. "Matching with Multiple Applications Revisited." *Economic Letters*, 84 (2002), 311-314.

[5] van den Berg, Gerard J., and Geert Ridder. "An Empirical Equilibrium Search Model of the Labor Market." *Econometrica*, 66 (1998), 1183-1221.

[6] Burdett, Kenneth, and Kenneth Judd. "Equilibrium Price Dispersion." *Econometrica*, 51 (1983), 955-969.

[7] Burdett, Kenneth, and Dale T. Mortensen. "Wage Differentials, Employer Size, and Unemployment." *International Economic Review*, (1998), 257-273.

[8] Burdett, Kenneth, Shouyong Shi, and Randall Wright. "Pricing and Matching with Frictions." *Journal of Political Economy*, 109 (2001), 1060-1085.

[9] Chade, Hector, and Lones Smith. "Simultaneous Search", mimeo (2004).

[10] Delacroix, Alain, and Shouyong Shi. "Directed Search On the Job and the Wage Ladder." *International Economic Review*, forthcoming.

[11] Gaumont, Damien, Martin Schindler, and Randall Wright. "Alternative Theories of Wage Dispersion." In *Structural Labor Market Models. Essays in Honor of Dale T. Mortensen*, edited by Henning Bunzel, Elsevier North-Holland, forthcoming.

[12] Julien, Benoit, John Kennes, and Ian King. "Bidding for Labor." *Review of Economic Dynamics*, 3 (2000), 619-629.

[13] Moen, Espen R. "Competitive Search Equilibrium. *Journal of Political Economy*, 105 (1997), 385-411.

[14] Montgomery, James D. "Equilibrium Wage Dispersion and Interindustry Wage Differentials." *Quarterly Journal of Economics*, (1991), 163-179.

[15] Mortensen, Dale T. "Wage Dispersion: Why Are Similar Workers Paid Differently?" Cambridge: Zeuthen Lecture Book Series (2003).

[16] Mortensen, Dale T., and Randall Wright. "Competitive Pricing and Efficiency in Search Equilibrium." *International Economic Review*, 43 (2002), 1-20.

[17] Peters, Michael. "Ex Ante Price Offers in Matching Games: Non-Steady States." *Econometrica*, 59 (1991), 1425-1454.

[18] Peters, Michael. "Limits of Exact Equilibria for Capacity Constrained Sellers with Costly Search." *Journal of Economic Theory*, 95 (2000), 139-168.

[19] Postel-Vinay, Fabien, and Jean-Marc Robin. "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity." *Econometrica*, 70 (2002), 2295-2350.

[20] Rogerson, Richard, Robert Shimer, and Randall Wright. "Search-Theoretic Models of the Labor Market: A Survey." *Journal of Economic Literature*, forthcoming.

[21] Shi, Shouyong. “A Directed Search Model of Inequality with Heterogeneous Skills and Skill-Biased Technology.” *Review of Economic Studies*, 69 (2002), 467-491.

[22] Shimer, Robert. “Essays in Search Theory.” PhD dissertation, Massachusetts Institute of Technology, Cambridge, Massachusetts (1996).

[23] Shimer, Robert. “The Assignment of Workers to Jobs in an Economy with Coordination Frictions.” *Journal of Political Economy*, forthcoming.