Modeling the Effects of Labor Market Reforms in a Developing Economy

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1 Introduction

In many economies, labor market policies such as payroll taxation or mandatory severance payments do not directly affect substantial parts of the labor market. In developing countries and in transition economies, the informal sector – by our definition, the part of the labor market in which these policies are not implemented – is typically quite large. The economic activity that takes place in this sector is simply beyond the ability of governments to monitor and control. The informal sector is also important in many developed economies.[1] Some of these economies have substantial illegal sectors, while in others, some parts of the labor market are intentionally left relatively free of regulation.[2]

There is a substantial literature that analyzes the equilibrium effects of labor market policies using a search and matching framework. In this paper, we add to this literature by constructing a model that allows us to analyze the effects of labor market policies in an economy with a significant informal sector. Our model is a substantial extension of Mortensen and Pissarides (1994), hereafter MP, a standard model for labor market policy analysis in a search and matching framework. This model is particularly attractive because it includes

[1] According to Maloney (2005), the informal sector comprises 30-70% of the labor market in most Latin American countries. Friedman, et.al. (2000) estimate that 14-62% of output comes from the informal sector across a range of transition economies. Table 2 of Schneider and Enste (2000) gives estimates for a wide range of countries, which indicate that the informal sector is quantitatively important in developed, as well as developing and transition, economies.

[2] Much attention has been paid to Spain, where employment on (relatively unregulated) temporary contracts is particularly important. See, e.g., Dolado, et. al. (2005).

endogenous job separations. Specifically, we extend MP by (i) adding an informal sector and (ii) allowing for worker heterogeneity. The second extension is what makes the first one interesting. We allow workers to differ in terms of what they are capable of producing in the formal (regulated) sector. All workers have the option to take up informal sector opportunities as these come along, and all workers are equally productive in that sector, but some workers – those who are most productive in formal-sector employment – will reject informal-sector work in order to wait for a formal-sector job. Similarly, the least productive workers are shut out of the formal sector. Labor market policy, in addition to its direct effects on the formal sector, changes the composition of worker types in the two sectors. A policy change can disqualify some workers from formal-sector employment; similarly, some workers accept informal-sector work who would not have done so earlier. Labor market policy thus affects the mix of worker types in the two sectors. These compositional effects, along with the associated distributional implications, are what our heterogeneous-worker extension of MP buys.

The basic MP model can be summarized as follows. First, there are frictions in the process of matching unemployed workers and vacant jobs. These frictions are modelled using a matching function $m(\theta)$, where $m(\theta)$ is the rate at which unemployed workers find work and $\theta = v/u$, i.e., the ratio of vacancies to unemployment, is interpreted as labor market tightness. Second, when an unemployed worker and a vacancy meet, they match if and only if the joint surplus from matching exceeds the sum of the values they would get were they to continue unmatched. This joint surplus is then split using a Nash bargaining rule. Third, there is free entry of vacancies, so θ is determined by the condition that the value of a maintaining a vacancy equal zero. Fourth – and this is the defining innovation of MP – the rate of job destruction is endogenous. Specifically, when a worker and a firm start their relationship, match productivity is at its maximum level. Shocks then arrive at an exogenous Poisson rate, and with each arrival of a shock, a new productivity value is drawn from an exogenous distribution (and the wage is renegotiated accordingly). The productivity of a match can go up or down over time, but it can never exceed its initial level. When productivity falls below an endogenous reservation value, R, the match ends. R is determined by the condition that the value of continuing the match equal the sum of values to the two parties of remaining unmatched. Labor market tightness, θ, and the reservation productivity, R, are the key endogenous variables in MP. Firms create more vacancies (θ is higher), the longer matches last on average (the lower is R); and matches break up more quickly (the higher is R), the better are workers' outside options (the higher is θ). Equilibrium, a (θ, R) pair, is determined by the intersection of a job-creation schedule (θ as a decreasing function of R) and a job-destruction schedule (R as an increasing function of θ).

Our innovation is to assume that workers differ in their maximum productivities in formal-sector jobs. In particular, we assume that maximum productivity ("potential") is distributed across a continuum of workers of measure one according to a continuous distribution function

$F(y), 0 \leq y \leq 1$. Workers with a high value of y start their formal-sector jobs at a high level of match productivity; workers with lower values of y start at lower levels of match productivity. As in MP, job destruction is endogenous in our model. Productivity in each match varies stochastically over time, and eventually the match is no longer worth maintaining. The twist in our model is that different worker types have different reservation productivities; that is, instead of a single reservation productivity, R, to be determined in equilibrium, there is an equilibrium reservation productivity schedule, $R(y)$.

The connection between our assumption about worker heterogeneity and our interest in the informal sector is as follows. We assume that the unemployed encounter informal-sector opportunities (to work at black-market jobs or casual labor, to engage in home production, etc.) at an exogenous Poisson rate α; correspondingly, informal-sector jobs end at exogenous Poisson rate δ. Any informal-sector opportunity, if taken up, produces output at flow rate y_0, all of which goes to the worker. There is, however, a cost to taking one of these jobs; namely, we assume that informal-sector employment precludes search for a formal-sector job. This cost is, of course, increasing in y. The decision about whether or not to accept an informal-sector job thus depends on a worker's type. Workers with particularly low values of y take informal-sector jobs but not formal-sector jobs. The fact that these workers can spend some of their time in informal-sector employment makes them unqualified for formal-sector jobs. For each of these workers, the value of waiting for an informal-sector opportunity exceeds the expected surplus that he or she would generate by taking a formal-sector job. These "low-productivity" workers are indexed by $0 \leq y < y^*$. Workers with intermediate values of y, "medium-productivity workers," find it worthwhile to take up informal-sector opportunities but are nonetheless still qualified for formal-sector jobs. These workers are indexed by $y^* \leq y < y^{**}$. Finally, workers with high values of y, "high-productivity workers," reject informal-sector opportunities in order to continue searching for formal-sector jobs. These workers are indexed by $y^{**} \leq y < 1$. The cutoff values, y^* and y^{**}, are endogenous and are influenced by labor market policy.

We solve the model numerically and perform several policy experiments. We find that a severance tax dramatically reduces the rate at which workers find formal sector jobs, but, at the same time, also greatly increases the average employment duration in the formal sector. There are also compositional effects, viz, fewer workers reject formal sector jobs and more workers reject informal sector jobs. Unemployment among high-productivity workers falls substantially leading to a fall in aggregate unemployment. Net output increases even though average productivity in the formal sector falls. A payroll tax has somewhat different effects. It also reduces the rate at which workers find formal sector jobs, but, unlike a severance tax, it decreases average employment duration in the formal sector. Again, there are compositional effects. As with the severance tax, more workers reject the informal sector, but now more workers also reject the formal sector. Unemployment among high-productivity workers increases as does aggregate unemployment. Average productivity in the formal sector and net output both rise.

In the next two sections, we give the details of our model. In Section 2, we derive the basic equations of the model and prove the existence of a unique equilibrium. Then, in Section 3, we work out the implications of the model for the distributions of productivity and wages across workers in formal-sector jobs and examine how these vary with changes in policy parameters. Section 4 is devoted to our simulations. These simulations give a qualitative sense of the properties of our model as well as a quantitative sense for the impact of labor market policy. Finally, Section 5 concludes.

2 Basic Model

We consider a model in which workers can be in one of three states: (i) unemployed, (ii) employed in the informal sector, or (iii) employed in the formal sector. Unemployment is the residual state in the sense that workers whose employment in either an informal- or a formal-sector job ends flow back into unemployment. Unemployed workers receive b, which is interpreted as the flow income equivalent to the value of leisure. The unemployed look for job opportunities. Formal-sector opportunities arrive at endogenous rate $m(\theta)$, and informal-sector opportunities arrive at exogenous rate α. Not all of these opportunities are taken up. For low-productivity workers – those with $y < y^*$ – it is not worthwhile to take formal-sector jobs, and for high-productivity workers – those with $y > y^{**}$ – informal-sector jobs are not worth taking. These cutoff values, y^* and y^{**}, are endogenous and determine the relative sizes of the informal and formal sectors.

In the informal sector, a worker receives flow income y_0, where $y_0 > b$. As mentioned above, opportunities to work in the informal sector arrive to the unemployed at Poisson rate α. Employment in this sector ends at Poisson rate δ. We assume that employment in the informal sector precludes search for a formal-sector job; i.e., there are no direct transitions from the informal to the formal sector.[3] Since formal-sector output depends on a worker's type, this means that the most productive workers (those with $y > y^{**}$) will not take up opportunities to work in the informal sector. These workers prefer to remain unemployed in hopes of receiving a formal-sector offer.

A worker's output in a formal-sector job depends on his or her type. Formal-sector matches initially produce at the worker's maximum potential productivity level y. Thereafter, as in MP, productivity shocks arrive at Poisson rate λ, which change the match productivity. These shocks are *iid* draws from a continuous distribution $G(x)$, where $0 \leq x \leq 1$. There are three possibilities to consider. First, if the realized value of a draw x is sufficiently low, it is in the mutual interest of the worker and the firm to end the match. Here "sufficiently low" is defined in terms of an endogenous reservation productivity, $R(y)$, which depends on the worker's type. Thus, with probability $G(R(y))$, a shock ends the match.

[3] Alternatively, one could assume that workers in informal-sector jobs can search for formal-sector opportunities but less effectively than if they were unemployed.

Second, if $R(y) \leq x \leq y$, the productivity of the match changes to x. That is, with probability $G(y) - G(R(y))$, the match continues after a shock, but at the new level of productivity. Finally, if the draw is such that $x > y$, we assume that the productivity of the match reverts to y. That is, with probability $1 - G(y)$, the match continues after a shock, but the productivity of the match is reset to its maximum value.[4]

The surplus from a formal-sector match is split between worker and firm using a Nash bargaining rule with an exogenous worker share, β. This surplus depends both on the current productivity of the match, y', and on the worker's type, y. As in MP, we assume the wage is renegotiated whenever match productivity changes.

2.1 Value functions

The worker side of the model is thus summarized by the value functions

$$\begin{aligned}
rU(y) &= b + \alpha \max\left[N_0(y) - U(y), 0\right] + m(\theta) \max\left[N_1(y, y) - U(y), 0\right] \\
rN_0(y) &= y_0 + \delta \left(U(y) - N_0(y)\right) \\
rN_1(y', y) &= w(y', y) + \lambda G(R(y)) \left(U(y) - N_1(y', y)\right) \\
&\quad + \lambda \int_{R(y)}^{y} \left(N_1(x, y) - N_1(y', y)\right) g(x) dx + \lambda \left(1 - G(y)\right) \left(N_1(y, y) - N_1(y', y)\right),
\end{aligned}$$

where $U(y)$ is the value of unemployment, $N_0(y)$ is the value of informal-sector employment, and $N_1(y', y)$ is the value of formal-sector employment in a job with current productivity level y', all of the above for a worker of type y.

Next, consider the vacancy-creation problem faced by a formal-sector firm. Let V be the value of creating a formal-sector vacancy, and let $J(y', y)$ be the value of employing a worker of type y in a match with current productivity y'. The latter value can be written as

$$\begin{aligned}
rJ(y', y) &= y' - w(y', y)(1 + t) + \lambda G(R(y)) \left(V - J(y', y) - s\right) \\
&\quad + \lambda \int_{R(y)}^{y} \left(J(x, y) - J(y', y)\right) g(x) dx + \lambda \left(1 - G(y)\right) \left(J(y, y) - J(y', y)\right).
\end{aligned}$$

This expression can be understood as follows. A firm that employs a worker of type y in a match of productivity y' receives flow output y' and pays a wage (inclusive of a payroll tax) of $w(y', y)(1 + t)$. At rate λ, a productivity shock arrives. With probability $G(R(y))$, the job ends, in which case the firm suffers a capital loss of $V - J(y', y)$ and pays a separation tax of

[4] Two alternative assumptionss that we considered are: (i) if a shock is drawn such that $x > y$, the productivity of the match remains where it is rather than reverting to y and (ii) shocks are drawn randomly over $[0, y]$ rather than $[0, 1]$.

s. If the realized shock x falls in the interval $[R(y), y]$, the job's value changes from $J(y', y)$ to $J(x, y)$. Finally, with probability $1 - G(y)$, the shock resets the job's value of employing a worker of type y to its maximum level, $J(y, y)$.

The value of a vacancy is defined by

$$rV = -c + \frac{m(\theta)}{\theta} E\left[\max[J(y, y) - V, 0]\right].$$

This expression reflects the assumption that match productivity initially equals the worker's type. A vacancy, however, does not know in advance what type of worker it will meet. It may, for example, meet a worker of type $y < y^*$, in which case it is not worth forming the match. If the worker is of type $y \geq y^*$, the match forms, but the job's value depends, of course, on the worker's type. Finally, note that in computing the expectation, we need to account for contamination in the unemployment pool. That is, the distribution of y among the unemployed will, in general, differ from the corresponding population distribution. We deal with this complication below in the subsection on steady-state conditions.

As usual in this type of model, the fundamental equilibrium condition is the one given by free entry of vacancies, i.e., $V = 0$. Equation (5), with $V = 0$, determines the equilibrium value of labor market tightness. The other endogenous objects of the model, namely, the wage schedule, $w(y', y)$, the reservation productivity schedule, $R(y)$, and the cutoff values, y^* and y^{**}, can all be expressed in terms of θ.

2.2 Wage Determination

We use the Nash bargaining assumption with an exogenous share parameter β to derive the wage function. Given $V = 0$, the wage for a worker of type y on a job producing at level y' solves

$$\max_{w(y',y)} \left[N_1(y', y) - U(y)\right]^{\beta} J(y', y)^{1-\beta}.$$

It is relatively straightforward to verify that

$$w(y', y) = \frac{\beta(y' - \lambda G(R(y))s) + (1-\beta)(1+t)rU(y)}{1+t}.$$

Note that the wage is a weighted sum of the current output (less the expected instantaneous capital loss) and the worker's continuation value. Were workers homogeneous, this would reduce to the expression in MP amended to include a severance payment and a payroll tax.

2.3 Reservation Productivity

Filled jobs are destroyed when a sufficiently unfavorable productivity shock is realized. The reservation productivity $R(y)$ is defined by

$$N_1(R(y), y) - U(y) + J(R(y), y) = -s.$$

Given the surplus sharing rule, this is equivalent to

$$J(R(y), y) = -s.$$

Substitution gives

$$R(y) = (1+t)rU(y) - \frac{s[r + \lambda\beta G(R(y))]}{1-\beta} - \frac{\lambda}{r+\lambda}\int_{R(y)}^{y}[1-G(x)]dx. \tag{1}$$

For any fixed value of y, this is analogous to the reservation productivity in MP. Since $U(y)$ is increasing in θ for all workers who take formal-sector jobs, equation (1) defines an upward-sloping "job-destruction" locus in the $(\theta, R(y))$ plane.

An interesting question is how the reservation productivity varies with y. On the one hand, the higher is a worker's maximum potential productivity, the better are her outside options. That is, $U(y)$ is increasing in y. This suggests that $R(y)$ should be increasing in its argument. On the other hand, a "good match gone bad" retains its upside potential. The final term in equation (1), which can be interpreted as a labor-hoarding effect, is decreasing in y. This suggests that $R(y)$ should be decreasing in y. As will be seen below once we solve for $U(y)$, which of these terms dominates depends on parameters.

2.4 Unemployment values and cutoff productivities

Workers with $y < y^*$ only work in the informal sector, workers with $y^* \leq y \leq y^{**}$ accept both informal-sector and formal-sector jobs, and workers with $y > y^{**}$ accept only formal-sector jobs. Thus y^* is defined by the condition that a worker with productivity $y = y^*$ be indifferent between unemployment and a formal-sector offer, and y^{**} is defined by the condition that a worker with productivity $y = y^{**}$ be indifferent between unemployment and an informal-sector offer.

Consider a worker with $y^* \leq y \leq y^{**}$. The value of unemployment for this worker is given by

$$rU(y) = b + \alpha[N_0(y) - U(y)] + m(\theta)[N_1(y, y) - U(y)].$$

The condition that $N_1(y^*, y^*) = U(y^*)$ then implies

$$rU(y^*) = b + \alpha[N_0(y^*) - U(y^*)]$$

and substitution gives

$$rU(y^*) = \frac{b(r+\delta) + \alpha y_0}{r+\alpha+\delta}.$$

(Note $rU(y^*)$ takes this value for all $y \leq y^*$.) Setting this equal to $N_1(y^*, y^*)$ gives

$$y^* = (1+t)\frac{b(r+\delta) + \alpha y_0}{r+\alpha+\delta} + \lambda G(R(y^*))\, s - \frac{\lambda}{r+\lambda}\int_{R(y^*)}^{y^*} (1 - G(x))\, dx. \tag{2}$$

Note that since $R(y)$ is increasing in θ, y^* is also increasing in θ.

Similarly, the condition that $N_0(y^{**}) = U(y^{**})$ implies that at $y = y^{**}$

$$rU(y^{**}) = b + m(\theta)\left[N_1(y^{**}, y^{**}) - U(y^{**})\right].$$

Setting $U(y^{**}) = N_0(y^{**})$ implies that $rU(y^{**}) = y_0$ and substitution gives

$$N_1(y^{**}, y^{**}) = \frac{(r + m(\theta))y_0 - rb}{rm(\theta)}.$$

Substituting the value function for $N_1(y^{**}, y^{**})$ and solving gives

$$\begin{aligned} y^{**} &= (1+t)\frac{(y_0 - b)(r+\lambda) + m(\theta)\beta y_0}{m(\theta)\beta} \\ &\quad + \lambda G(R(y^{**}))s - \frac{\lambda}{r+\lambda}\int_{R(y^{**})}^{y^{**}} [1 - G(x)]dx \end{aligned} \tag{3}$$

While $rU(y)$ was shown to have simple forms at $y \leq y^*$ and at y^{**}, it is more complicated at other values of y. For $y^* \leq y < y^{**}$, we have

$$rU(y) = \frac{\left[b(r+\delta) + \alpha y_0\right](r+\lambda) + \frac{(r+\delta)m(\theta)\beta}{1+t}\left\{y - \lambda G(R(y))s + \frac{\lambda}{r+\lambda}\int_{R(y)}^{y}[1 - G(x)]dx\right\}}{(r+\alpha+\delta)(r+\lambda) + (r+\delta)m(\theta)\beta}$$

and for $y \geq y^{**}$, we have

$$rU(y) = \frac{b(r+\lambda) + \frac{m(\theta)\beta}{1+t}\left\{y - \lambda G(R(y))s + \frac{\lambda}{r+\lambda}\int_{R(y)}^{y}[1 - G(x)]dx\right\}}{r + \lambda + m(\theta)\beta}.$$

Note that the two expressions would be identical if the informal sector did not exist, that is, were $\alpha = \delta = 0$.

Given the expression for $R(y)$ (equation 1), the differing forms for $rU(y)$ mean that the form of $R(y)$ differs for high and intermediate productivity workers. For any fixed value of θ, equation (1) has a unique solution for $R(y)$. One can also check, given a unique schedule $R(y)$, that equations (2) and (3) imply unique solutions for the cutoff values, y^* and y^{**}, respectively.

2.5 Steady-State Conditions

The model's steady-state conditions allow us to solve for the unemployment rates, $u(y)$, for the various worker types. Let $u(y)$ be the fraction of time a worker of type y spends in unemployment, let $n_0(y)$ be the fraction of time that a worker of type y spends in informal-sector employment, and let $n_1(y)$ be the fraction of time a worker of type y spends in formal-sector employment. Of course, $u(y) + n_0(y) + n_1(y) = 1$.

Workers of type $y < y^*$ flow back and forth between unemployment and informal-sector employment. There is thus only one steady-state condition for these workers, namely, the flows out of and into unemployment must be equal,

$$\alpha u(y) = \delta(1 - u(y)).$$

For $y < y^*$ we thus have

$$\begin{aligned} u(y) &= \frac{\delta}{\delta + \alpha} \\ n_0(y) &= \frac{\alpha}{\delta + \alpha} \\ n_1(y) &= 0. \end{aligned} \tag{4}$$

There are two steady-state conditions for workers with $y^* \leq y \leq y^{**}$, (i) the flow out of unemployment to the informal sector equals the reverse flow and (ii) the flow out of unemployment into the formal sector equals the reverse flow,

$$\begin{aligned} \alpha u(y) &= \delta n_0(y) \\ m(\theta) u(y) &= \lambda G(R(y)) (1 - u(y) - n_0(y)). \end{aligned}$$

Combining these conditions gives

$$\begin{aligned} u(y) &= \frac{\delta \lambda G(R(y))}{\lambda (\delta + \alpha) G(R(y)) + \delta m(\theta)} \\ n_0(y) &= \frac{\alpha \lambda G(R(y))}{\lambda (\delta + \alpha) G(R(y)) + \delta m(\theta)} \\ n_1(y) &= \frac{\delta m(\theta)}{\lambda (\delta + \alpha) G(R(y)) + \delta m(\theta)} \end{aligned} \tag{5}$$

for $y^* \leq y \leq y^{**}$.

Finally for workers with $y > y^{**}$ there is again only one steady-state condition, namely, that the flow from unemployment to the formal sector equals the flow back into unemployment, i.e.,

$$m(\theta) u(y) = (1 - u(y)) \lambda G(R(y)).$$

This implies

$$\begin{aligned} u(y) &= \frac{\lambda G(R(y))}{\lambda G(R(y)) + m(\theta)} \\ n_0(y) &= 0 \\ n_1(y) &= \frac{m(\theta)}{\lambda G(R(y)) + m(\theta)} \end{aligned} \tag{6}$$

for $y > y^{**}$.

Total unemployment is obtained by aggregating across the population.

$$u = \int_0^{y^*} u(y) f(y)dy + \int_{y^*}^{y^{**}} u(y) f(y)dy + \int_{y^{**}}^{1} u(y) f(y)dy.$$

2.6 Equilibrium

Finally, we use the free-entry condition to close the model and determine equilibrium labor market tightness. Setting $V = 0$ we have

$$c = \frac{m(\theta)}{\theta} E\left[\max[J(y,y),0]\right].$$

To determine the expected value of meeting a worker, we need to account for the fact that the density of types among unemployed workers is contaminated. Let $f_u(y)$ denote the density of types among the unemployed. Using Bayes Law,

$$f_u(y) = \frac{u(y)f(y)}{u}.$$

The free-entry condition can thus be rewritten as

$$c = \frac{m(\theta)}{\theta} \int_{y^*}^{1} J(y,y) \frac{u(y)}{u} f(y)dy.$$

After substitution, the free-entry condition becomes

$$c = \frac{m(\theta)}{\theta} (1-\beta) \int_{y^*}^{1} \left(\frac{y - R(y)}{r + \lambda} - \frac{s}{1-\beta} \right) \frac{u(y)}{u} f(y)dy. \tag{7}$$

This expression takes into account that $J(y,y) < 0$ for $y < y^*$, i.e., some contacts do not lead to a match. Note also that the forms of $R(y)$ and of $u(y)$ differ for intermediate-productivity and high-productivity workers.

A steady-state equilibrium is a labor market tightness θ, together with a reservation productivity function $R(y)$, unemployment rates $u(y)$, and cutoff values y^* and y^{**} such that

(i) the value of maintaining a vacancy is zero

(ii) matches are consummated and dissolved if and only if it is in the mutual interest of the worker and firm to do so

(iii) the steady-state conditions hold

(iv) formal-sector matches are not worthwhile for workers of type $y < y^*$

(v) informal-sector matches are not worthwhile for workers of type $y > y^{**}$.

A unique equilibrium exists if there is a unique value of θ that solves equation (7), taking into account that $R(y)$, $u(y)$, and y^* all depend on θ. Note that

1. $R(y)$ is increasing in θ for each y;
2. $\dfrac{u\left(y\right)}{u}$ is decreasing in θ for each $y \geq y^*$;
3. y^* is increasing in θ.

These three facts imply that the right-hand side of the free-entry condition is decreasing in θ. This result, together with the facts that the limit of the right-hand side of (7) as $\theta \to 0$ equals ∞ and equals 0 as $\theta \to \infty$, implies the existence of a unique θ satisfying the free-entry condition.

3 Distributional Characteristics of Equilibrium

Given assumed functional forms for the distribution functions, $F(\cdot)$ and $G(\cdot)$, and for the matching function, $m(\theta)$, and given assumed values for the exogenous parameters of the model, equation (7) can be solved numerically for θ. Given θ, we can then recover the other equilibrium objects of the model, namely, the cutoff values y^* and y^{**}, the reservation productivity schedule, $R(y)$ (defined for all $y \geq y^*$), the wage as a function of current productivity and type, $w(y', y)$ (defined for all $y \geq y^*$ and $R(y) \leq y' \leq y$), the type-specific unemployment rates, $u(y)$, etc. In fact, we can do more than this. Once we solve for equilibrium, we can simulate the distributions of wages and of match productivities and then use these simulated distributions to compute both the aggregate and distributional effects of labor market policy.

To start, we discuss the computation of the joint distribution of (y', y) across workers employed in the formal sector. Once we compute this joint distribution (and the corresponding marginals), we can find (i) the distribution of wages (a worker's wage is a function of both his current productivity and his type, i.e., of (y', y)) and (ii) the distribution of productivity, y'.

To find the distribution of (y', y) across workers employed in the formal sector, we use

$$h(y', y) = h(y'|y)h(y).$$

Here $h(y', y)$ is the joint density, $h(y'|y)$ is the conditional density, and $h(y)$ is the marginal density across workers employed in the formal sector. It is relatively easy to compute $h(y)$. Let E denote "employed in the formal sector." Then by Bayes Law,

$$h(y) = \frac{P[E|y]f(y)}{P[E]} = \frac{n_1(y)f(y)}{\int n_1(y)f(y)dy},$$

where from equations (4) to (6),

$$\begin{aligned} n_1(y) &= 0 && \text{for } y < y^* \\ &= \frac{\delta m(\theta)}{\delta m(\theta) + (\alpha+\delta)\lambda G(R(y))} && \text{for } y^* \leq y < y^{**} \\ &= \frac{m(\theta)}{m(\theta) + \lambda G[R(y)]} && \text{for } y \geq y^{**}. \end{aligned}$$

Next, we need to find $h(y'|y)$. Consider a worker of type y who is employed in the formal sector. Her match starts at productivity y; later, a shock (or shocks) may change her match productivity. Let N denote the number of shocks this worker has experienced to date (in her current spell of employment in a formal-sector job). Since we are considering a worker who is employed in the formal sector, we know that none of these shocks has resulted in a productivity realization less than $R(y)$.

If $N = 0$, then $y' = y$ with probability 1. If $N > 0$, then $y' = y$ with probability $\frac{1-G(y)}{1-G(R(y))}$, i.e., the probability that the productivity shock is greater than or equal to y (conditional on the worker being employed) in which case the productivity reverts to y. Combining these terms, we have that conditional on y, $y' = y$ with probability $P[N = 0] + \frac{1-G(y)}{1-G(R(y))}P[N > 0]$. Similarly, the conditional density of y' for $R(y) \leq y' < y$ is

$$h(y'|y) = \frac{g(y')}{1-G(R(y))}P[N > 0] \text{ for } R(y) \leq y' < y.$$

Thus, for a worker of type y, we need to find $P[N = 0]$.

To do this, we first condition on elapsed duration. Consider a worker of type y whose elapsed duration of employment in her current formal sector job is t. This worker type exits formal sector employment at Poisson rate $\lambda G(R(y))$; equivalently, the distribution of completed durations for a worker of type y is exponential with parameter $\lambda G(R(y))$. The exponential has the convenient property that the distributions of completed and elapsed durations are the same.

Let N_t be the number of shocks this worker has realized given elapsed duration t. Shocks arrive at rate λ. However, as the worker is still employed, we know that none of the realizations of these shocks was below $R(y)$. Thus, N_t is Poisson with parameter $\lambda(1 - G(R(y))t$, and $P[N_t = 0] = \exp\{-\lambda(1 - G(R(y))t\}$. Integrating $P[N_t = 0]$ against the distribution of elapsed duration gives

$$P[N = 0] = \int_0^\infty \exp\{-\lambda(1 - G(R(y))t\}\lambda G(R(y)) \exp\{-\lambda G(R(y))t\}dt = G(R(y))$$

for a worker of type y.

Thus, the probability that a worker of type y is working to her potential when she is employed in a formal sector job (i.e., that $y' = y$) is

$$P[y' = y|y] = G(R(y)) + \frac{1 - G(y)}{1 - G(R(y))}(1 - G(R(y))) = 1 - (G(y) - G(R(y))).$$

The density of y' across all other values that are consistent with continued formal-sector employment for a type y worker is

$$h(y'|y) = \frac{g(y')}{1 - G(R(y))}(1 - G(R(y))) = g(y') \text{ for } R(y) \leq y' < y.$$

In principle, we can compute the joint distribution of (y', y) analytically, but in practice the algebraically complicated form of $R(y)$ makes this difficult. We use the following simulation procedure to solve this problem. First, make a random draw from $H(y)$[5], the distribution function corresponding to $h(y)$. Then, given y, make a draw from $h(y'|y)$. (With probability $1 - (G(y) - G(R(y)))$, $y' = y$; with probability $G(y) - G(R(y))$, y' is a random draw from $G(\cdot)$ over the interval $[R(y), y)$ To sample from the latter, make a random draw from $[R(y), y)$, etc.) The (y', y) pair generated in this way constitutes a random draw from $h(y', y)$. Repeat this procedure many (e.g., 1000) times. We then have a pseudo-random sample from the joint distribution.

Given the pseudo-random sample from $H(y', y)$, it is straighforward to simulate the distribution of wages. (Just plug each "sampled " (y', y) pair into the formula for $w(y', y)$.) We extend this to include all employed workers by noting that a fraction $\frac{n_0}{n_0 + n_1}$ of the workers who are employed at any time are employed in the informal sector. Thus, for example, to compute the average "wage" (i.e., averaged across all the employed, including informal sector workers), we can take a weighted average of y_0 and the average formal-sector wage. We can also (trivially) simulate the distribution of productivity as the marginal distribution of y'. (And we can extend this by adding in the mass of workers who are producing at level y_0 in the informal sector.) We present simulations of the model and these distributions in the next section.

[5]That is, we make a random draw from $[0, 1]$. Call this draw z. Then find $H^{-1}(z)$, i.e., the type y with the property that a fraction z of workers employed in the formal sector are of that type or lower.

3.1 Simulation Results

We now present our numerical analysis of the effects of labor market policy. For all our simulations, we assume the following functional forms and parameter values. First, we assume that the distribution of worker types, i.e., y, is uniform over $[0, 1]$ and that the productivity shock, i.e., x, is likewise drawn from a standard uniform distribution. We assume the standard uniform for computational convenience, but it would not be appreciably more difficult to solve the model using flexible parametric distributions, e.g., betas, for $F(\cdot)$ and $G(\cdot)$. Second, we assume a Cobb-Douglas matching function, namely, $m(\theta) = 4\theta^{1/2}$. The Cobb-Douglas form is a standard assumption; we chose the scale parameter to give reasonable results for labor market tightness. Third, we chose our parameter values with a year as the implicit unit of time. We set $r = 0.05$ as the discount rate. We normalize the flow income equivalent of leisure to $b = 0$. The parameters for the informal sector are $y_0 = 0.35$, $\alpha = 5$ and $\delta = 0.5$, and the formal-sector parameters are $c = 0.3$, $\beta = 0.5$, and $\lambda = 1$. Note that the share parameter, β, equals the elasticity of the matching function with respect to labor market tightness, i.e., the Hosios value. Our parameter values were chosen to produce plausible results for our baseline case of $s = t = 0$.

Consider first the baseline case given in row 1 of Table 1. With no severance payment or payroll taxation, our baseline generates labor market tightness of 1.21. More than 30 percent of the labor force is "low productivity" and works only in the informal sector, while about 60 percent of the labor force works only in the formal sector. The remaining 10 percent would work in either sector. The reservation productivity for the worker who is just on the margin for working in the formal sector ($y = y^*$) is the same as that worker's type. With no severance payment, it is worthwhile employing this worker even though his match would end were its productivity to go even a bit below his maximum level. Next, note that $R(y^{**}) < R(y^*)$. As we noted earlier, there are two effects of y on the reservation productivity. First, more productive workers have greater "upside potential"; on the other hand, more productive workers have better alternative options. The first effect dominates among medium productivty workers for this parameterization, however, the first panel of Figure 1 shows that $R(y)$ is rising above y^{**}, i.e., for all high productivity workers. The next four columns in Table 1 give average unemployment rates. The average unemployment rate for the baseline case is 8.6 percent. Among low-productivity workers, the unemployment rate is $\dfrac{\delta}{\alpha + \delta} = 9.1$ percent. The average unemployment rate for medium-productivity workers is much lower, reflecting the fact that these workers take both informal- and formal-sector jobs. Finally, the average unemployment rate for high-productivity workers is 9.1 percent. This reflects the fact that these workers do not take up informal-sector opportunities. Next, we present two summary statistics for the formal sector. Average worker productivity for workers in this sector is 0.629 and the corresponding average wage for these workers is 0.574. The final column gives gross total output (Y), i.e., the sum of outputs from the informal and formal sectors. We could compute output net of vacancy creation costs as $Y - c\theta = 0.12$.

The next four rows of Table 1 show the effect of raising the severance tax. Note that a severance tax of .2 is equivalent to about 40% of the average annual wage in the formal sector. Since the severance tax makes vacancy creation less attractive, we find that θ, labor market tightness, decreases. The severance tax shifts the reservation productivity schedule down. This is a consequence of the fact that the severance tax makes it more costly to end matches. In addition, the severance tax affects the composition of the sectors. The first cutoff value, y^*, falls with s. The reason is that formal sector employment is more attractive to the previously marginal worker because jobs last longer when the severance tax is higher. The effect on the second cutoff value, y^{**}, is more complicated. On the one hand, formal sector jobs are again more attractive because of their longer duration, but, on the other hand, a worker who receives an informal sector opportunity takes into account the fact that θ decreases with the severance tax implying that the arrival rate of formal sector jobs falls. The unemployment for low productivity workers is unaffected by the severance tax. The unemployment rate for high productivity workers falls. The effect of increasing job duration outweighs the reduction in the job arrival rate. For the medium productivity workers, the unemployment rate falls slightly as it is a mixture of the effects on the other two worker types. The overall unemployment falls for two reasons. First, it reflects the reduction in unemployment associated with formal sector jobs. Second, the compositional changes, i.e., the changes in y^* and y^{**}, mean that there are more workers who accept both types of jobs. Given the effect on the reservation productivities, it is not surprising that we find a reduction in average productivity. Correspondingly, we find a reduction in the average formal sector wage. Gross output falls considerably, however, output net of vacancy costs rises.

Table 2 presents the effects of varying the payroll tax holding the severance tax at zero. Note that we are considering payroll taxes up to 40 percent. A payroll tax of 40 percent on average is more costly than a severance tax of .2 since the average duration of a formal sector job with $s = .2$ is greater than one year. As indicated in Table 2, increasing the payroll tax reduces θ since it makes formal sector vacancy creation less attractive. In contrast to the effect of the severance tax, the payroll tax decreases job duration by shifting up the reservation productivity schedule. (As can be seen in Figure 1, the payroll tax of .4 shifts the schedule less than the severance tax of .2.) The payroll tax also has compositional effects. The fraction of workers who never take formal sector jobs ($y < y^*$) increases substantially with t, and the fraction that only take formal sector jobs ($y > y^{**}$) decreases substantially with t. Finally, the fraction of workers who will take any job increases with t. The fact that both labor market tightness and expected formal sector job duration decrease implies that unemployment increases among high productivity workers. The effect on overall unemployment, however, is mitigated to some extent by the compositional changes. Consistent with the compositional change and the shift in the reservation productivity schedule, average formal sector productivity rises. Wages received by workers fall, but the wage paid, i.e., the gross wages, increase on average. Gross output falls, but net output rises slightly.

The final table examines the effects of increasing s and t simultaneously, namely, to $s = t = 0.1$. Since both of these taxes make vacancy creation less attractive, labor market tightness falls. Employment duration in the formal sector increases, i.e., the severance tax effect dominates,as can be seen in the final panel of Figure 1. On net, there is a slight decrease in the average unemployment rate among high-productivity workers, which leads to a corresponding fall in the aggregate rate. Average productivity in the formal sector falls as a result of the decrease in the reservation productivity. This leads to a reduction in the wage. While gross output falls, net output rises.

Figures 2 and 3 illustrate the effects of s and t on the distributions of current productivities and wages across workers employed in the formal sector. Figure 2 shows the densities of types (y) and current productivities (y') for various (s, t) combinations. The density of y is the contaminated one; i.e., it incorporates the different job-finding and job-losing experiences of the various worker types. The distribution of y necessarily first-order stochastically dominates that of y' since no worker's current productivity can exceed his or her type. This is illustrated by the two densities in the baseline case in the first panel of Figure 2. The severance tax shifts the density of current productivity to the left, reflecting the decrease in reservation productivities. The payroll tax shifts the densities of y and y' to the right, reflecting the compositional effect as well as the fact that the reservation productivity schedule increases. In addition, the variance of both y and y' decreases. The final panel of Figure 2 illustrates the effects of increasing s and t simultaneously. The severance tax effects dominate. Figure 3 shows the effects of these policies on wages. In all cases, the policies decrease the mean wage and reduce the variance. As in the previous figure, the effect of the payroll tax is more dramatic, but when the two taxes are used together, the severance tax effect dominates.

4 Conclusions

References

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Table 1: Effects of Varying s with $t = 0$

						u rates by type				formal		total
s	θ	y^*	y^{**}	$R(y^*)$	$R(y^{**})$	low	med	high	total	$\overline{y}'$	$\overline{w}$	Y
0	1.21	0.315	0.410	0.315	0.243	0.091	0.037	0.091	0.086	0.629	0.574	0.485
.05	0.93	0.307	0.407	0.275	0.192	0.091	0.036	0.087	0.083	0.604	0.548	0.474
.1	0.68	0.299	0.407	0.241	0.145	0.091	0.035	0.083	0.080	0.576	0.523	0.460
.15	0.46	0.291	0.411	0.213	0.097	0.091	0.035	0.077	0.076	0.546	0.499	0.445
.2	0.30	0.284	0.423	0.188	0.046	0.091	0.033	0.065	0.068	0.508	0.474	0.426

Table 2: Effects of Varying t with $s = 0$

						u rates by type				formal		total
t	θ	y^*	y^{**}	$R(y^*)$	$R(y^{**})$	low	med	high	total	$\overline{y}'$	$\overline{w}$	Y
0	1.21	0.315	0.410	0.315	0.243	0.091	0.037	0.091	0.086	0.629	0.574	0.485
0.1	1.17	0.347	0.459	0.347	0.272	0.091	0.040	0.095	0.088	0.646	0.535	0.483
0.2	1.12	0.378	0.511	0.378	0.302	0.091	0.043	0.100	0.089	0.665	0.504	0.480
0.3	1.07	0.410	0.565	0.410	0.334	0.091	0.045	0.105	0.090	0.683	0.477	0.475
0.4	1.01	0.441	0.623	0.441	0.367	0.091	0.048	0.110	0.090	0.703	0.454	0.467

Table 3: Results with $s = t = 0$ and $s = t = 0.1$

					u rates by type				formal		total
θ	y^*	y^{**}	$R(y^*)$	$R(y^{**})$	low	med	high	total	$\overline{y}'$	$\overline{w}$	Y
1.21	0.315	0.410	0.315	0.243	0.091	0.037	0.091	0.086	0.629	0.574	0.485
0.66	0.331	0.460	0.268	0.167	0.091	0.038	0.087	0.082	0.593	0.488	0.459

Figure 1: $R(y)$ for Various (s, t) Combinations

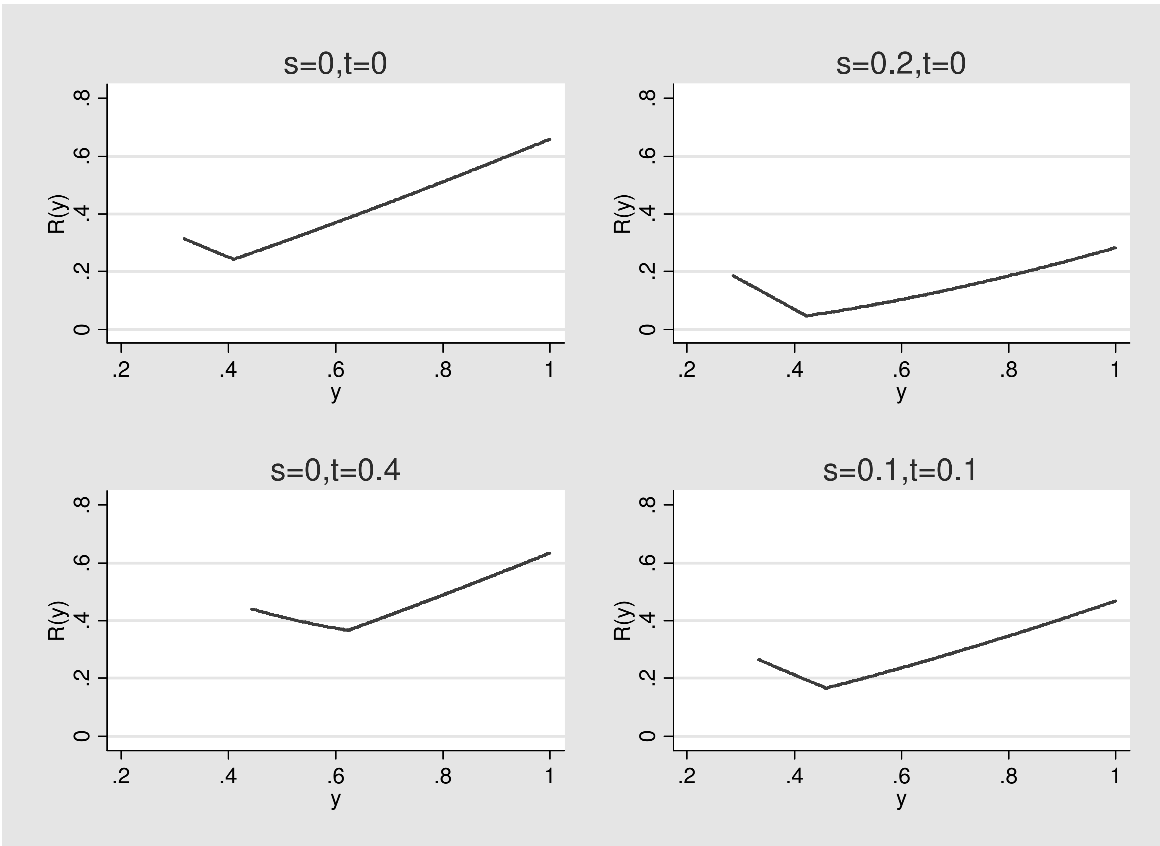

Figure 2: Densities of Types and Current Productivities

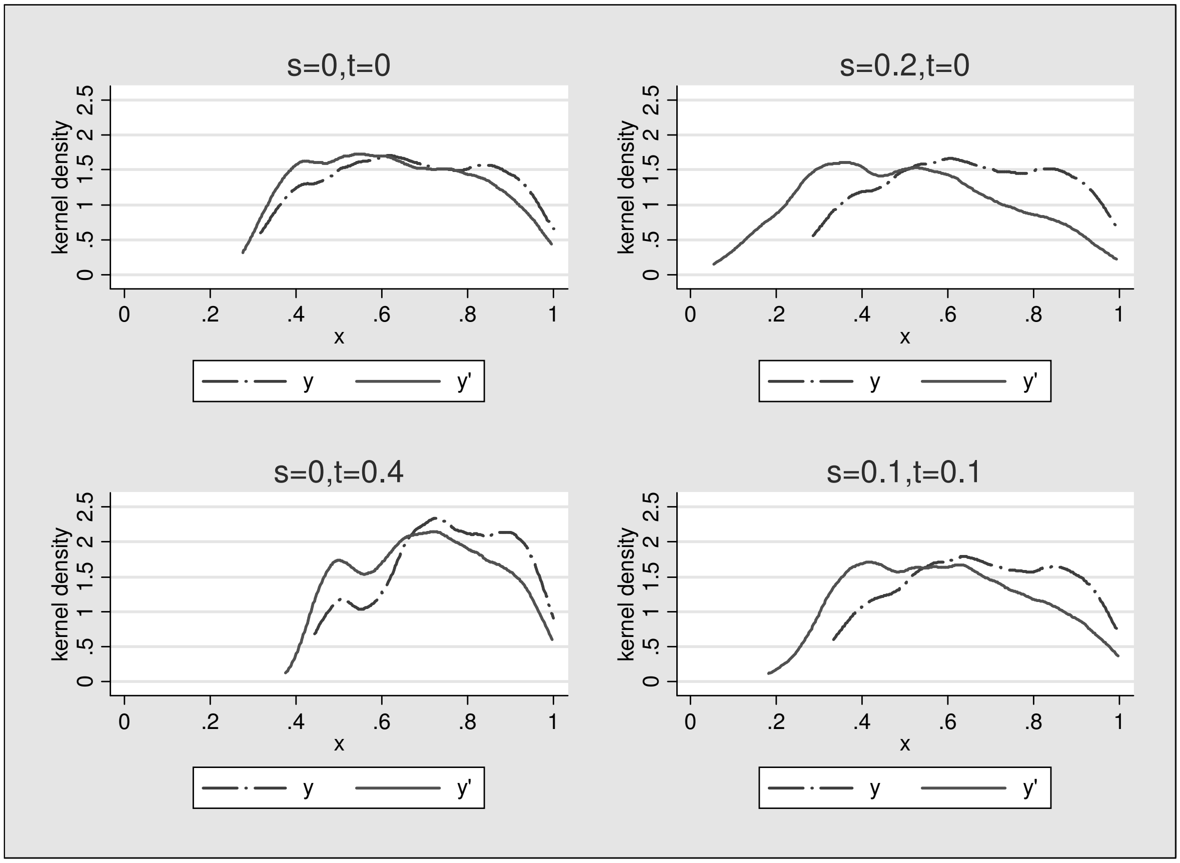

Figure 3: Densities of Current Productivities and Wages