Can Uncertainty Alleviate the Commons Problem?

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Abstract: Global commons problems, such as climate change, are often affected by severe uncertainty. The paper examines the effect of uncertainty on pollution emissions and welfare in a strategic context. We find that emissions are always lower under uncertainty than under certainty, reflecting risk-reducing considerations. We show that uncertainty can have a net positive impact on the welfare of risk-averse polluters. We extend the analysis to increases in risk, increases in risk-aversion, and risk heterogeneity. A policy implication is that the establishment of risk-sharing institutions should generally not predate arrangements addressing the commons problem.

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I. Introduction

Uncertainty is generally viewed as a "bad". For instance, severe uncertainty affecting future environmental damages often goes along with the possibility of an environmental catastrophe. In this paper, we show that uncertainty may have a positive impact in a strategic context. Uncertainty can lower the incentives to pollute and make risk-averse polluters better-off.[1]

This result may have interesting implications for the way economists traditionally view insurance and risk-sharing institutions. Take the climate change problem. In the summary for policy-makers, the Intergovernmental Panel of experts on Climate Change argues that *"one of the important potential gains from cooperating in a collective framework (...) is that of risk-sharing"* (IPCC, 1995, p.7). It further promotes the creation of insurance institutions capable of sharing climate change risks on a global basis. Our analysis suggests that the absence of such a global insurance system may actually be a good thing. Indeed, the common risk-reduction that this insurance system would induce may lead countries to increase their emissions. This is a simple moral hazard effect arising in the absence of a binding international agreement to abate emissions. The net impact of an insurance mechanism may then be negative. This suggests that the establishment of risk-sharing institutions should generally not predate international arrangements addressing the pollution problem.

Most global commons problems such as climate change share two main features: strategic interactions between polluters and uncertainty about the effects of pollution. Either feature has been extensively studied by economists, but they are usually considered separately. In this paper, we look at their combined effect. We study a simple model where agents' actions impose a negative externality on others and the damages from this externality are subject to uncertainty. In our analysis, we contrast two polar situations: when polluters cooperate and when they free-ride. We look at the effect of uncertainty on pollution emissions and on welfare in these two cases.[2]

[1] We do not distinguish risk and uncertainty in our framework.

[2] Some papers have examined through numerical simulations the effect of learning on the optimal emissions path both in the cooperative and non cooperative case. See, e.g., Hammitt and Adams (1996), Ulph and Maddison (1997) and Ulph and Ulph (1996). Our paper is aalso related to the growing literature on uncertainty and international environmental agreements (see, e.g., Kolstad (2003) for a recent reference). This literature usually considers a stock pollution model where polluters can sign agreements or form coalitions between each other. It typically examines how the size of the coalitions changes as a result of learning. Here, we consider the simpler question of the effect of uncertainty on strategic pollution in a static framework, and without coalition formation. See Baker (2004) for a link between the effect of uncertainty and learning in this context.

In a first stage, we compare uncertainty to certainty. We find that cooperative pollution emissions and welfare are always lower under uncertainty. Essentially, reducing emissions provides a way to lower the variability of everybody's damage, which is socially desirable under risk-aversion. This motive also emerges when polluters act non-cooperatively. By lowering his emissions, a polluter may reduce the variability of his own damage. We show that uncertainty leads to a decrease of total pollution at the Nash equilibrium. Because of the negative externality, this has an indirect positive effect on welfare which works against the direct negative effect of uncertainty. We provide an example where the net effect is positive and welfare in equilibrium is higher under uncertainty.

In a second stage, we extend the analysis in three different directions. We first look at the effect of an increase in uncertainty in the sense of Rothschild and Stiglitz (1970). We then study an Arrow-Pratt increase in risk-aversion. We finally relax the homogeneity assumption and look at the effect of different risks. Our analysis confirms but somehow qualifies the previous findings. First, we provide conditions under which more uncertainty leads to lower emissions. Yet, we also show that an opposite effect comes into play, related to prudence, and which tends to increase emissions. Second, we show that more risk-aversion always leads to less pollution. Third, we find that facing uncertainty alone may be a benefit under cooperation, but is a large disadvantage at the Nash equilibrium.

Few other papers have examined the effect of uncertainty in a strategic context. Gradstein et al. (1992) argue that the comparative statics of uncertainty is generally ambiguous. Sandler and Sterbenz (1990) look at the exploitation of a renewable resource. They show that uncertainty on the resource stock leads risk-averse firms to reduce their exploitation effort compared to certainty, see also Sandler et al. (1987). More recently, Eso and White (2004) set out a general theory of uncertainty in auctions and White (2004) introduces uncertainty in a Rubinstein bargaining model. They study the effect of uncertainty on decisions and welfare and show that uncertainty may be always beneficial for risk-averse players. Our paper complements these studies. We provide the first analysis of the effect of uncertainty and risk-aversion in a context of global pollution.

The paper's contribution can be viewed from two perspectives. With respect to the externality literature, we show that uncertainty strongly affects agents' behaviors and welfare. Especially, uncertainty can help to alleviate commons problems, both by lowering emissions and increasing welfare. With respect to the uncertainty literature, we find that strategic interactions may substantively modify the analysis and results compared to situations with a single decision maker.

The paper is organized as follows. In the next section, we introduce the global pollution model. In section 3, we present the benchmark comparison between uncertainty and certainty. In section 4, we examine the effect of an increase in uncertainty, while in section 5 we examine the effect on an increase in risk-aversion. In section 6, we look at risk heterogeneity. The last section concludes.

II. The Model

Consider the following n-player game. Each agent i emits a level of pollution e_i. Agents benefit from emitting pollution, but suffer from the global level of pollution $\sum_{j=1}^{n} e_j$. Damages from pollution are uncertain, subject to a risk $\tilde{\theta}_i > 0$. We make the following assumptions regarding the benefits and costs of pollution. First, the benefits are simply equal to e_i in any state of nature. That is, benefits from polluting are not affected by uncertainty, and the individual level of pollution emission is identified with its monetary value. Second, there exists a damage function d such that the individual damage from pollution is equal to $\tilde{\theta}_i d(\sum_{j=1}^{n} e_j)$. The damage function d is strictly increasing $d' > 0$ and convex $d'' \geq 0$ with $d(0) = 0$. Finally, we assume that the preferences of the agents can be represented by the same von Neumann-Morgenstern utility function u, which is strictly increasing $u' > 0$ and concave $u'' \leq 0$. Agent i's expected utility is thus equal to

$$E_{\tilde{\theta}_i} u(e_i - \tilde{\theta}_i d(\sum_{j=1}^{n} e_j)) \tag{II.1}$$

In this paper, we focus on situations where individual risks are *ex-ante* identical.[3] For instance, damage may be the same for everybody, $\widetilde{\theta}_i = \widetilde{\theta}$ or individual damages $\widetilde{\theta}_i$ may be identically and independently distributed.[4] This assumption allows us to write $\tilde{\theta}_i = \tilde{\theta}$ in the former expression.

The remainder of the paper is devoted to the analysis of this game. We will especially contrast properties of cooperative and non-cooperative solutions. In the cooperative case, we suppose that

[3] This assumption is relaxed in section VI.

[4] Another example is when one and only one individual can be affected by the total damage, but this individual is unknown *ex-ante*.

agents jointly maximize welfare, defined as the sum of the expected utilities. Since agents are ex-ante identical and utilities are concave, welfare is always greater when emissions are identical across agents.[5] We therefore define $W(e)$ as the expected utility obtained when all agents emit e:

$$W(e) = E_{\tilde{\theta}} u(e - \tilde{\theta} d(ne)).$$

It is easy to check that our assumptions on u and d imply concavity of W.[6] We define the cooperative level of pollution emissions, e^C, as the solution of the maximization of $W(e)$. It is usually the unique solution to the following first-order condition:[7]

$$E_{\tilde{\theta}}(1 - n\tilde{\theta} d'(ne))u'(e - \tilde{\theta} d(ne)) = 0 \tag{II.2}$$

This is essentially a marginal benefit equals marginal cost condition. At the optimum, the *private* expected marginal benefits from emitting pollution $E_{\tilde{\theta}} u'(e - \tilde{\theta} d(ne))$ are equal to the *social* expected marginal damages generated by pollution $nE_{\tilde{\theta}} \tilde{\theta} d'(ne)u'(e - \tilde{\theta} d(ne))$.

Next, we characterize the symmetric Nash equilibrium of the game. Consider an agent i and suppose that others' emission levels e_{-i} are given. Agent i's expected utility II.1 is a concave function of e_i, which means that best-response is unique.[8] We define the non-cooperative level of pollution emissions, e^N, as the level of emissions such that the symmetric profile $\forall i, e_i = e^N$ is a Nash equilibrium and solves the following equation:

$$E_{\tilde{\theta}}(1 - \tilde{\theta} d'(ne))u'(e - \tilde{\theta} d(ne)) = 0 \tag{II.3}$$

We assume that equation II.3 has a unique solution.[9] At the equilibrium, agents set their emissions by equalizing their private marginal benefits to their *private* marginal damages generated by pollution $E_{\tilde{\theta}} \tilde{\theta} d'(ne)u'(e - \tilde{\theta} d(ne))$. They do not account for the effect of their emissions on others,

[5] Applying Jensen's inequality, we see that $\sum_i E_{\tilde{\theta}} u(e_i - \tilde{\theta} d(\sum_{j=1}^n e_j)) \leq nE_{\tilde{\theta}} u(\frac{1}{n} \sum_{j=1}^n e_j - \tilde{\theta} d(\sum_{j=1}^n e_j))$

[6] Omitting the functions' arguments for clarity, we see that $W''(e) = E_{\tilde{\theta}}[-n^2 \tilde{\theta} d'' u' + (1 - n\tilde{\theta} d')^2 u''] \leq 0$.

[7] To insure existence and interiority, we also need that $W'(0) > 0$ and $W'(e) < 0$ if e is large enough. This holds for instance if $\overline{\theta} d'(0) < 1/n$ and $d'(\infty) = \infty$.

[8] Differentiating the individual payoff II.1 twice with respect to e_i yields $E_{\tilde{\theta}}[-\tilde{\theta} d'' u' + (1 - \tilde{\theta} d')^2 u''] \leq 0$.

[9] The conditions of footnote 7 also ensure the existence of a symmetric interior equilibrium. Our results can be easily extended to the case of multiple symmetric equilibria. Comparative statics with multiple equilibria usually specifies how the minimum and the maximum non-cooperative emission levels vary (see, e.g., Milgrom and Roberts, 1994). This issue is discussed in more detail in the Appendix.

which explains that the private marginal cost is n times lower than the social marginal cost. Clearly, this yields overpollution at the equilibrium. (All proofs are given in Appendix).

Proposition 1. $e^N \geq e^C$

III. Certainty and Risk Neutrality

A. Emissions

We begin the analysis of the game by looking at the natural benchmarks of certainty and risk-neutrality. In our model, the curvature of u only captures risk aversion motives. Uncertainty does not affect risk-neutral agents' emissions and the emissions of risk-averse agents under certainty are identical to the emissions of risk-neutral agents. This is arguably a nice feature that makes the benchmarks of certainty and risk-neutrality equivalent. Formally, denote by $\bar{\theta}$ the expected value of $\tilde{\theta}$. When damages are certain or when agents are risk neutral, condition II.2 becomes

$$1 - n\bar{\theta}d'(ne) = 0$$

while condition II.3 becomes

$$1 - \bar{\theta}d'(ne) = 0$$

Denote by $\bar{e}^C$ and $\bar{e}^N$ the corresponding cooperative and non-cooperative emission levels. Comparing these benchmarks with the general case of uncertainty and risk aversion, we obtain the following result.

Proposition 2. $e^C \leq \bar{e}^C$ *and* $e^N \leq \bar{e}^N$

Pollution emissions are thus lower under uncertainty (resp. risk-aversion) than under certainty (risk-neutrality).[10] The intuition behind the result is quite simple. Consider first the cooperative case. Agents' uncertain payoffs are equal to

$$\pi = e - \tilde{\theta}d(ne).$$

[10] A similar result is obtained in Sandler and Sterbenz (1990).

The benchmark emissions $\bar{e}^C$ maximize the expected value of the payoff $E_{\tilde{\theta}}\pi = e - \bar{\theta}d(ne)$. Hence decreasing e leads to a reduction in expected payoffs. At the same time, it also reduces the variance of the payoff $var(\pi) = var(\tilde{\theta})d^2(ne)$. Lowering emissions may then be viewed as a form of insurance for cooperative risk-averse agents. By reducing their emissions, they trade-off some loss in expected payoff for a reduction in risk.

Next, consider the strategic setting. By the same logic, uncertainty leads a risk-averse agent to decrease his level of emissions, *assuming others' emissions are fixed.* Situation is now complicated by the fact that others will adjust and may increase their emissions in response. However, since the whole best-response functions are lower under uncertainty, the intersection of the best-responses with the 45 degree line is also lower. At the symmetric equilibrium, the uncertainty effect is thus always greater than possible strategic effects and the non-cooperative level of emissions is lower under uncertainty.

B. Welfare

The previous result showed that pollution is lower under uncertainty. How does this affect social welfare? We see now that the cooperative level of welfare is always lower, while the effect of uncertainty on the non-cooperative level of welfare is ambiguous.

Formally, let $\bar{W}$ denote welfare under certainty. Since agents are risk-averse, by Jensen's inequality for any level of emission e, $W(e) \leq \bar{W}(e)$. Therefore the maximum of W is always lower than the maximum of $\bar{W}$, which means that $W(e^C) \leq \bar{W}(\bar{e}^C)$. Next, compare the equilibrium levels of welfare $W(e^N)$ and $\bar{W}(\bar{e}^N)$. It is useful to decompose their difference as the sum of two terms:

$$W(e^N) - \bar{W}(\bar{e}^N) = [W(e^N) - W(\bar{e}^N)] + [W(\bar{e}^N) - \bar{W}(\bar{e}^N)]$$

Uncertainty has two effects, as captured by the two terms of the right hand side of this equality. It leads to a decrease in emissions, keeping the risk fixed, (first term) and it changes the risk faced, keeping pollution emissions constant (second term). The first term is positive by concavity of W and by Proposition 2, while the second term is negative by risk-aversion. The overall effect is thus ambiguous. *Uncertainty may be socially beneficial*, if the indirect positive effect of reduced emissions is greater than the direct negative effect of uncertainty. This discussion is illustrated graphically in Figure 1 and by the following numerical example.

Insert Figure 1 about here

Consider simple quadratic forms

$$d(e) = \frac{e^2}{2}; u(\pi) = 1 - \frac{(1-\pi)^2}{2}$$

with $\pi \leq 1$. Assume that the damage is binary, $\widetilde{\theta} = 1.5$ or 0.5 with equal probability (hence $\overline{\theta} = 1$). Consider $n = 2$ agents. Solving numerically for cooperative emissions yields $\bar{e}^C = 0.25$ and $e^C = 0.2419$. In the non-cooperative case, there is a unique symmetric equilibrium. We find $\bar{e}^N = 0.5$ and $e^N = 0.4517$. This illustrates Proposition 2: emissions are lower under uncertainty than under certainty. Observe that here the difference is more important for non-cooperative emissions. Next look at the impact on welfare. We find $W(e^C) = 0.6153 < \bar{W}(\bar{e}^C) = 0.6171$. In contrast, $W(e^N) = 0.5217 > \bar{W}(\bar{e}^N) = 0.5$. The indirect positive effect due to emissions reduction, i.e., $W(e^N) - W(\bar{e}^N) = 0.053$, is larger than the negative effect of uncertainty on welfare, keeping emissions constant, i.e. $W(\bar{e}^N) - \bar{W}(\bar{e}^N) = -0.0313$.

In this section, we have shown that uncertainty reduces pollution, since cooperative and non-cooperative emission levels are lower under uncertainty. In equilibrium, social welfare may actually be higher under uncertainty.[11] In the following sections, we question the robustness of these findings and extend our analysis in different directions.

IV. Increase in Uncertainty

We first look at an increase in uncertainty. That is, instead of comparing certainty to uncertainty, we compare an uncertain risk $\tilde{\theta}$ to a risk $\tilde{z}$ that is *more* uncertain, in the sense of Rothschild and Stiglitz (1970). Throughout this section, we make explicit the dependence of e^C and e^N on the risk faced $\tilde{\theta}$ by denoting them $e^C(\widetilde{\theta})$ and $e^N(\widetilde{\theta})$.

[11] Hammitt and Adams (1996) found that, under uncertainty, welfare at the non-cooperative solution is very close (about 98%) to the welfare at the cooperative solution. Our result may potentially explain their numerical findings.

A. Cooperative Emissions

First, how does an increase in risk affect cooperative emissions? We know from the literature on optimal investments in risky assets that risk-aversion is in general not sufficient to insure that an increase in uncertainty has a non-ambiguous effect on the agent's decision. Additional conditions on the utility function must usually be satisfied (Gollier, 2001). We will see that a similar conclusion holds in our setting. Especially, considerations of *prudence* have to be introduced. Risk-aversion might be sufficient, however, to guarantee a non-ambiguous effect for specific increases in risk. We will show how this applies to a class of *catastrophic* risks.

Proposition 3 below gives a sufficient condition on the utility function for an increase in risk to yield a decrease in emissions. To understand this result, we find useful to present first an informal analysis of the situation where damages are small. This analysis illustrates the trade-offs involved in the general case. Specifically, assume that $\tilde{\theta}d(ne)$ is small compared to e. A second-order Taylor approximation of the expected utility function yields:

$$W(e,\tilde{\theta}) = E_{\tilde{\theta}}u(e-\tilde{\theta}d(ne)) \approx u(e) - \overline{\theta}d(ne)u'(e) + \frac{1}{2}E_{\tilde{\theta}}\tilde{\theta}^2 d(ne)^2u''(e) \tag{IV.1}$$

The cooperative level of emission is characterized by the condition $W_e(e^C,\tilde{\theta}) = 0$. Consider $\tilde{z}$ a mean-preserving spread of $\tilde{\theta}$ and replace W by its approximate expression. This yields:

$$W_e(e,\tilde{z}) - W_e(e,\tilde{\theta}) = \frac{1}{2}(E_{\tilde{z}}\tilde{z}^2 - E_{\tilde{\theta}}\tilde{\theta}^2)\frac{\partial}{\partial e}[d(ne)^2u''(e)]$$

Since W_e is decreasing in e and $E_{\tilde{z}}\tilde{z}^2 > E_{\tilde{\theta}}\tilde{\theta}^2$, the effect of uncertainty on emissions is controlled by the sign of $\frac{\partial}{\partial e}[d(ne)^2u''(e)]$. Effectively, if this sign is negative, $W_e(e,\tilde{z}) < W_e(e,\tilde{\theta})$ for any e, which means that $e^C(\tilde{z}) < e^C(\tilde{\theta})$. In contrast, a positive sign implies that $e^C(\tilde{z}) > e^C(\tilde{\theta})$. The derivative is equal to the sum of two terms

$$\frac{\partial}{\partial e}[d(ne)^2u''(e)] = nd(ne)d'(ne)u''(e) + \frac{1}{2}d(ne)^2u'''(e) \tag{IV.2}$$

The first term captures the effect of uncertainty on pollution damage $\tilde{\theta}d(ne)$ and is always negative under risk aversion. Greater uncertainty increases the damage's variance. In compensation, risk-averse agents have an incentive to reduce their emissions. The second term captures the effect of uncertainty on pollution benefits e. Following Kimball (1990), agents are said to be prudent when $u''' \geq 0$. Under prudence, the marginal value of a extra unit of emissions e is higher when uncertainty is greater.[12] Prudent agents have thus an incentive to *increase* their emissions when they face more uncertainty. This effect operates as a precautionary savings motive. By polluting more, the agents increase the portion of their payoff which is not subject to uncertainty.

When is the global effect negative? Observe that since d is convex and $d(0) = 0$, $d'(x) > d(x)/x$ for any x, so that expression IV.2 is negative if

$$2 + e\frac{u'''(e)}{u''(e)}$$

is positive. Introduce relative prudence as $P_r(e) = -e\frac{u'''(e)}{u''(e)}$. When damages are relatively small, a greater risk thus leads to lower cooperative emissions if relative prudence is lower than 2.[13] Our next result shows that this condition in fact plays a role for arbitrary risks and arbitrary damage functions.

Proposition 3. *Let $\tilde{z}$ be a mean-preserving spread of $\tilde{\theta}$. Suppose that $u''' \geq 0$ and $P_r \leq 2$, then for any damage function d, $e^C(\tilde{z}) \leq e^C(\tilde{\theta})$.*

This result says that if agents are prudent, but not too prudent, an increase in risk always lowers cooperative emissions. The condition $P_r \leq 2$ puts a limit on the magnitude of prudence effects. It is satisfied, for example, for quadratic utility functions ($u''' = 0$) and for CRRA utility function $u(\pi) = \frac{1}{1-\gamma}\pi^{1-\gamma}$ where $0 \leq \gamma \leq 1$.

By imposing certain conditions on the preferences, we could thus obtain a comparative statics result valid for any increase in risk. An alternative approach consists in restricting the type of risk in order to obtain results valid for any risk-averse agents. We next illustrate this approach for a class of catastrophic risks.

We introduce and define catastrophic risks as follows. Damages may only be low or high (a catastrophe occurs). We denote by θ_L the risk associated with low damages, by $\bar{\theta}$ the expected value of the risk, and by $1/k$ (with $k > 1$) the catastrophe's probability. Thus, $\tilde{\theta}$ is equal to

[12] Formally, prudence implies that $E_{\tilde{z}}u'(e - \tilde{z}d(ne)) \geq E_{\tilde{\theta}}u'(e - \tilde{\theta}d(ne))$.

[13] The same condition appears in the comparative statics of an increase in uncertainty on the optimal investment in risky assets (see, Hadar and Seo, 1990).

θ_L with probability $1-\frac{1}{k}$ and to $\theta_L + k(\bar{\theta}-\theta_L)$ with probability $1/k$. We define *an increase in catastrophic risk* as an increase in k, keeping θ_L and $\bar{\theta}$ constant. An increase in catastrophic risk is a particular case of Rothschild and Stiglitz's increase in uncertainty. As k increases, the catastrophe becomes less probable but more damaging. We can then show the following result.

Proposition 4. *For any utility function u and any damage function d, an increase in catastrophic risk always leads to lower cooperative emissions.*

In this case, risk-aversion is sufficient to obtain non-ambiguous comparative statics. Overall, we draw two lessons from this section. First, we find that an increase in uncertainty often leads to lower cooperative emissions. Second, we find that effects related to prudence might push towards increasing emissions; this makes the comparative statics analysis quite complicated. We show that these prudence effects may be dominated, however, for specific classes of risks. Finally, observe that no matter how cooperative emissions vary, welfare is always lower when uncertainty is greater.

B. Non-cooperative Emissions

Next, turn to the equilibrium analysis. How does an increase in uncertainty affect the non-cooperative level of pollution? To answer this question, we find useful to look at the small damages heuristics again. Consider the expected utility's approximation:

$$E_{\tilde{\theta}}u(e_i - \tilde{\theta}d(e_i+e_{-i})) \approx u(e_i) - \overline{\theta}d(e_i+e_{-i})u'(e_i) + \frac{1}{2}E_{\tilde{\theta}}\tilde{\theta}^2 d(e_i+e_{-i})^2 u''(e_i)$$

Through a similar reasoning as above, we find that non-cooperative emissions are lower under $\tilde{z}$ if and only if the following expression is negative

$$d(ne)d'(ne)u''(e) + \frac{1}{2}d^2(ne)u'''(e)$$

This expression is still the sum of a damage effect (first term) and a benefit effect (second term). While the benefit effect is unchanged, the damage effect is now n times lower. This arises from the negative externality. Agents do not take into account the effect of uncertainty on others' damages. We thus expect conditions leading to a decrease in non-cooperative emissions to be more demanding than those leading to a decrease in cooperative emissions. For instance, when damages are relatively small, a greater risk leads to a fall in non-cooperative emissions as soon as $P_r \leq \frac{2}{n}$.

This shows that the analysis of an increase in uncertainty is strongly affected by strategic interactions. Especially, we could not find a counterpart of Proposition 3. Computations presented in the Appendix show that the effect of an increase in uncertainty on non-cooperative emissions depend in a complex way on the risk and on properties of the utility function and the damage function. An exception is provided by quadratic utility functions, however.

Proposition 5. *Let $\tilde{z}$ be a mean-preserving spread of $\tilde{\theta}$. If $u''' = 0$, then for any damage function d, $e^N(\tilde{z}) \leq e^N(\tilde{\theta})$.*

For quadratic utility functions, an increase in risk always lowers non-cooperative emissions. In contrast when the utility function is not quadratic, effects due to prudence might come into play and the comparative statics analysis is in general ambiguous. In the non-cooperative setting, restricting the type of risk turns out to be a more fruitful approach. The following result shows that an increase in catastrophic risks has unambiguous effects on non-cooperative emissions.

Proposition 6. *For any utility function u and damage function d, an increase in catastrophic risk always leads to lower non-cooperative emissions*

Thus, an increase in catastrophic risks also leads to lower emissions when polluters act strategically. Observe that our previous welfare analysis carries on. When uncertainty increases and non-cooperative emissions decrease, welfare might increase if the indirect positive effect of less pollution is greater than the direct negative effect of more uncertainty. We illustrate this possibility with the following simple example.

Suppose that d is linear and u is quadratic, i.e. $d(e) = e; u(\pi) = -\frac{(1-\pi)^2}{2}$ with $\pi \leq 1$. Also, suppose that the risk is catastrophic with $\theta_L = 0$, $\bar{\theta} = \frac{1}{3}$, and $k > 3$. Thus, $\tilde{\theta}$ is equal to 0 with probability $1 - \frac{1}{k}$ and $\frac{1}{3}k$ with probability $\frac{1}{k}$. Finally, consider $n = 2$ agents. Direct computations yield:

$$e^C = \frac{3}{4k-3}; e^N = \frac{3}{k}$$

for cooperative and non-cooperative emissions. An increase in k lowers both levels of emissions, consistently with Propositions 4 and 6. Next, examine welfare. We obtain:

$$W(e^C) = \frac{2-2k}{-3+4k}; W(e^N) \equiv \frac{3-2k-k^2}{2k^2}$$

the cooperative level of welfare decrease in k while the non-cooperative level of welfare always increase in k. In this example, *more uncertainty is socially beneficial* in the non-cooperative setting. Moreover, $W(e^C)$ and $W(e^N)$ both tend asymptotically towards -0.5 when k is large.

V. Increase in Risk Aversion

In this section, we extend our analysis in another direction. Keeping the risk constant, we now look at the effect of *an Arrow-Pratt increase in risk aversion.* More precisely, individuals with utility function v are more risk-averse than those with utility u if there exists a function Φ satisfying $\Phi' > 0$ and $\Phi'' \leq 0$ and such that $v = \Phi(u)$ (Pratt, 1964). We question whether more risk-averse agents emit more or less pollution. Throughout this section we denote by $e^C(u)$, $e^N(u)$ the cooperative and non-cooperative emission levels for the utility function u.

A. Cooperative Emissions

Proposition 7. *Suppose that individuals with utility v are more risk-averse than those with utility u. Then $e^C(v) \leq e^C(u)$.*

When agents are more risk averse, the socially optimal level of emissions is always lower. This result is quite intuitive. Less emissions leads to a reduction in the variability of the future payoffs. Such a reduction is more and more desirable as risk-aversion increases. This result extends our benchmark comparison with risk-neutrality, Proposition 2. Comparing Propositions 7 and 3, we see that the comparative statics of an increase in risk-aversion is less ambiguous, and easier to work out, than for an increase in uncertainty.[14] This is similar to the results obtained in other

[14] As agents become more risk-averse, they might also become more prudent. The analysis of the previous section suggests that it could lead to an increase in emissions. Our result shows that this never happens. Here, the negative effect on emissions due to more risk-aversion is always greater than potentially positive effects due to more prudence.

contexts with single decision makers, such as optimal investments in risky assets, see Gollier (2001). We next question whether these findings hold for non-cooperative emissions.

B. Non-cooperative Emissions

Proposition 8. *Suppose that individuals with utility v are more risk-averse than those with utility u. Then $e^N(v) \leq e^N(u)$.*

Thus, the non-cooperative level of emissions also decreases when agents become more risk-averse. As in the cooperative case, a more risk-averse agent has an additional incentive to reduce emissions, keeping others' emissions constant. However, others will adjust and might well increase their emissions in response to the initial decrease. This would happen when emissions are strategic substitutes. Proposition 8 shows that this strategic effect is never strong enough to compensate for the risk-aversion effect.

Overall, this section has shown that more risk-averse agents always emit less pollution.

VI. Risk Heterogeneity

In this section, we relax the assumption that the agents are homogeneous. In order to focus on the effect of uncertainty, we consider two agents who only differ in the risk they face: One faces uncertain damage $\widetilde{\theta}$ while the other faces certain damage $\bar{\theta} = E\widetilde{\theta}$.[15] We study how this asymmetry affects cooperative and non-cooperative emissions. We especially ask: Which agent is going to emit the most? Is uncertainty detrimental or an advantage? As before we examine cooperative and non-cooperative levels of emissions.

A. Cooperative Case

Let agent 1 be the agent who faces no uncertainty, and agent 2 the one who faces uncertainty. Recall, social welfare is defined as the sum of both agents' payoffs. Thus, the cooperative levels of emission solves the following maximization program

$$\max_{e_1, e_2} u(e_1 - \overline{\theta} d(e_1 + e_2)) + E_{\widetilde{\theta}} u(e_2 - \widetilde{\theta} d(e_1 + e_2))$$

[15] This analysis also covers the case of heterogeneity in the preferences, i.e., when both agents face the same risk, one agent is risk-averse and the other is risk neutral.

After simple manipulations, the condition for an interior cooperative solution (e_1^C, e_2^C) may be written

$$u'(e_1^C - \overline{\theta}d(e_1^C + e_2^C)) = E_{\widetilde{\theta}}u'(e_2^C - \widetilde{\theta}d(e_1^C + e_2^C)), \tag{VI.1}$$

which is the standard equalization of expected marginal utilities. Observe that if the marginal utility is convex, $E_{\widetilde{\theta}}u'(e_2 - \widetilde{\theta}d(e_1 + e_2)) \geq u'(e_2 - \overline{\theta}d(e_1 + e_2))$ for any e_1, e_2. Hence under prudence, we obtain that agent 2 emits more than agent 1 at the optimum.[16] The intuition for this result is simple. Under prudence, agent 2 values more an extra unit of emissions than agent 1. Allowing him to emit more is therefore socially efficient.

Proposition 9. *Consider two identical agents, except that agent 1 faces no uncertainty $\overline{\theta}$ while agent 2 faces uncertainty $\widetilde{\theta}$. If $u''' \geq 0$, then $e_2^C \geq e_1^C$.*

How does uncertainty affect welfare? Since the objective is concave in the risk variable $\widetilde{\theta}$, uncertainty clearly reduces aggregate expected utilities compared to the certainty case. We further question the distribution of this welfare reduction. Does agent 1 end up with more or less utility than agent 2? The answer depends again on the shape of the utility function. We show in the Appendix that when u displays decreasing absolute risk aversion (DARA), agent 2's expected utility is higher at the social optimum. This is because under DARA $-u'$ is more concave than u. As a result, the increase in emissions that make agent 2's marginal utility equal to agent 1's marginal utility is larger than the increase in emissions needed to equate both expected utilities. All in all, these results thus suggest that, under the usual DARA assumption (which implies prudence), uncertainty may be viewed as an advantage at the optimum.

B. Non-cooperative Case

We now examine the Nash equilibria in this economy. We first show that equilibria cannot be interior. Consider the first-order conditions

$$\begin{aligned} 1 - \overline{\theta}d'(e_1^N + e_2^N) &= 0 && \text{(VI.2)} \\ E(1 - \widetilde{\theta}d'(e_1^N + e_2^N))u'(e_2^N - \widetilde{\theta}d(e_1^N + e_2^N)) &= 0, && \text{(VI.3)} \end{aligned}$$

[16] It can be shown that this result holds as well for corner solutions. Namely suppose that one of the two agents emits zero emissions, $e_i^C = 0$. Then, when $u''' > 0$, it must be agent $i = 1$.

where e_1^N is the equilibrium level of emissions of the agent 1 while e_2^N is the equilibrium level of emissions of the agent 2. Through (VI.2), we obtain

$$E(1-\widetilde{\theta}d'(e_1^N+e_2^N))u'(e_2^N-\widetilde{\theta}d(e_1^N+e_2^N)) = cov[1-\widetilde{\theta}d'(e_1^N+e_2^N), u'(e_2^N-\widetilde{\theta}d(e_1^N+e_2^N))]$$

This covariance is strictly negative when u is strictly concave, which contradicts (VI.3). This means that, in equilibrium, one agent must emit no emissions at all. We then show that it can only be agent 2, and that this indeed constitutes an equilibrium.

Proposition 10. *Consider two identical agents, except that agent 1 faces no uncertainty $\overline{\theta}$ while agent 2 faces uncertainty $\widetilde{\theta}$. There is a unique equilibrium where $e_2^N = 0$ and e_1^N solves $1 - \overline{\theta}d'(e_1^N) = 0$.*

The intuition for this result may be presented as follows. Start with a situation where both agents face no uncertainty. Suppose then that agent 2 faces some degree of uncertainty about his own damage. This leads agent 2 to reduce his emissions, for a given level of emissions of agent 1. Since emissions are perfect strategic substitutes under certainty, this leads agent 1 to increase his own emissions. This increase in emissions of agent 1 gives a further incentive to agent 2 to reduce his emissions, and so on. The process continues until agent 2 emits no more and agent 1 emits as much as if he were alone in the economy.

Note also that agent 1's level of emissions is actually equal to the total emissions obtained in equilibrium under certainty, and that total pollution is the same in the economy. Therefore, it is immediate that agent 1 is better-off compared to the full certainty case and agent 2 is worse-off. This raises the question of whether the increase in agent 1's expected utility may compensate agent 2's decrease. That is, how does heterogeneous uncertainty affects welfare in equilibrium compared to certainty? It is actually easy to see that it always reduces welfare.[17]

In this section, we have shown that when agents are heterogeneous in the risk they face, the effect of uncertainty goes in opposite directions in the cooperative and the non-cooperative case. Facing uncertainty is generally beneficial at the social optimum. In contrast, facing uncertainty is

[17] Let e be the level of emissions of agent 1 in equilibrium, which satisfies $1-\overline{\theta}d'(e) = 0$. Welfare in equilibrium equals $u(e-\overline{\theta}d(e)) + Eu(0-\widetilde{\theta}d(e))$. Under risk aversion, this is lower than $u(e-\overline{\theta}d(e)) + u(0-\overline{\theta}d(e))$ which is itself lower than $2u(\frac{1}{2}e-\overline{\theta}d(e))$ by Jensen's inequality. This last quantity is simply the aggregate expected utility that is reached at the symmetric equilibrum under certainty .

always detrimental in equilibrium. In both cases, uncertainty reduces aggregate expected utility.

VII. Conclusion

Uncertainty and free-riding are both primary concerns for environmental issues. Most of the existing literature has examined one or the other of these concerns, but not the two combined. It is, however, natural to ask whether uncertainty can alleviate the commons problem or whether both problems are mutually aggravating.

To think about this question, we have introduced a simple model with both uncertainty and strategic interactions between polluters. Within this model, we have shown that uncertainty always leads to reduce emissions compared to certainty. Similarly, we have shown that an increase in risk-aversion always leads to reduce emissions. The intuition for these results is quite simple. In our model, polluters face more risk when they emit more. The introduction of uncertainty gives an incentive for every risk-averse polluter to reduce emissions. Importantly, this effect may be strong enough so that uncertainty actually increases welfare at the non-cooperative equilibrium, even though uncertainty always lowers welfare under cooperation. By reducing the incentives to free-ride, uncertainty may increase the welfare of risk-averse polluters.

The previous set of results arguably supports the idea that uncertainty alleviates the commons problem. This idea needs to be qualified, however. In particular, we have shown that another effect takes place in face of an increase in uncertainty. Prudence may lead polluters to increase their emissions, in order to increase the risk-free portion of their payoffs. This would tend to aggravate the commons problem. Also, an heterogeneous increase in uncertainty has different implications. Being the only agent to face uncertainty is an important disadvantage in equilibrium in our setting.

Our analysis sheds new light on the effect of uncertainty on the incentives to reach an agreement. A classical argument is that reaching an agreement may be easier under a "veil of uncertainty", see Young (1994) and Na and Shin (1998) for a formal analysis. In this view, cooperation is compared ex ante, before uncertainty is resolved, and ex post, once uncertainty is resolved. Cooperation is more likely to emerge ex-ante than ex-post, because more agents potentially gain from the agreement before the uncertainty is resolved. In contrast our results show that, *from an ex ante perspective*, cooperation may be less likely under uncertainty. The reason is that the difference in social welfare between cooperation and non-cooperation may be lower under uncertainty and that this difference exactly measures the collective gain to reach an agreement. By partly alleviating the commons problem, uncertainty also reduces the incentives to fully solve it.

More work is certainly needed to better understand the effect of uncertainty in a strategic context. Our results are likely sensitive to the specificities of the model. For instance, the fact that uncertainty affects damages but not benefits clearly plays an important role in our analysis. Uncertain benefits would probably limit the risk-reducing incentives to lower emissions. Also, we have studied a static model. A number of dynamic features could be fruitfully added to the model, such as stock pollution, irreversibilities, learning, or sequentiality in decision-making. Still, we believe that the basic insights presented here are robust and will likely carry on to more complicated settings. In addition, they should apply to similar strategic contexts, typically models of voluntary contribution to a public good (Bergstrom et al., 1986). Our model offers simple predictions as to the effect of uncertainty on individual decisions. Testing these theoretical predictions experimentally provides another direction for future research.

Appendix.

Two functions play an important role in our analysis of the game. The first one is $W'(e, u, (\tilde{\theta})) = E_{\tilde{\theta}}(1 - n\tilde{\theta}d'(ne))u'(e - \tilde{\theta}d(ne))$ and the second one is $F(e, u, (\tilde{\theta})) = E_{\tilde{\theta}}(1 - \tilde{\theta}d'(ne))u'(e - \tilde{\theta}d(ne))$. Dependence of $W'$and F with respect to u or $\tilde{\theta}$ will be explicitly stated only when needed. The zeros of these functions characterize the cooperative and non-cooperative emissions profiles: $W'(e^C) = 0$ and $F(e^N) = 0$. Note that W' is non-increasing in e since $W'' \leq 0$. Unicity of the symmetric equilibrium would be insured by the property that F is decreasing in e, or by the weaker property that $F(e^N) = 0 \Longrightarrow F'(e^N) < 0$. The derivative of F has a non-trivial expression of possibly ambiguous sign:

$$F'(e) = E_{\tilde{\theta}}[(-n\tilde{\theta}d''(ne))u'(e - \tilde{\theta}d(ne)) + (1 - \tilde{\theta}d'(ne))((1 - n\tilde{\theta}d'(ne))u''(e - \tilde{\theta}d(ne))]$$

One can see that $\partial F/\partial e = \partial^2\Pi/\partial e_i^2 + (n-1)\partial^2\Pi/\partial e_i \partial e_j$ where Π is the objective function $E_{\tilde{\theta}}u(e_i - \tilde{\theta}d(\sum_{j=1}^n e_j))$ and the derivatives are computed at $e_i = e$. Since $\partial^2\Pi/\partial e_i^2 \leq 0$, F is decreasing as soon as emissions are strategic substitutes $\partial^2\Pi/\partial e_i \partial e_j \leq 0$.

Observe that our results directly extend to situations with multiple symmetric equilibria. Consider, for example, Proposition 8: If utility function v is more risk-averse than utility function u, $e^N(v) \leq e^N(u)$. In order to show this result, we show in the proof below that for any level of emissions e, $F(e, v) \leq F(e, u)$. Thus, if $F(., v)$ and $F(., u)$ both have a unique zero, our result follows. If they have multiple zeros, this property ensures that the lowest and the highest zeros of $F(., v)$ are lower, respectively, than the lowest and highest zero of $F(., u)$. Denoting by $e^N_{\min}(u)$ and $e^N_{\max}(u)$ the lowest and highest symmetric equilibrium, it means that $e^N_{\min}(v) \leq e^N_{\min}(u)$ and $e^N_{\max}(v) \leq e^N_{\max}(u)$. The other results extend in a similar fashion.

Proof of Proposition 1: For any θ, we have $1 - \theta d'(ne) \geq 1 - n\theta d'(ne)$, hence $F(e) \geq W'(e)$. Since W' is decreasing, we know that the zero of F must be greater than the zero of W'. Hence $e^N \geq e^C$.

Proof of Proposition 2: We make use of the following result: If $X(\theta)$ is non-increasing in θ and $Y(\theta)$ is non-decreasing in θ, then $cov_{\tilde{\theta}}(X(\tilde{\theta}), Y(\tilde{\theta})) \leq 0$.

First, consider the first-best. Introduce $X(\theta) = 1 - n\theta d'(ne^C)$ and $Y(\theta) = u'(e^C - \theta d(ne^C))$. It is easy to check that X is non increasing and Y is non decreasing in θ (because u is concave).

This implies that $cov_{\tilde{\theta}}(X(\tilde{\theta}), Y(\tilde{\theta})) \leq 0$. We have $cov_{\tilde{\theta}}(X(\tilde{\theta}), Y(\tilde{\theta})) = EX(\tilde{\theta})Y(\tilde{\theta}) - EX(\tilde{\theta})EY(\tilde{\theta})$. Then, $EX(\tilde{\theta})Y(\tilde{\theta}) = W'(e^C) = 0$ and $EY(\tilde{\theta}) > 0$. Therefore, $EX(\tilde{\theta}) \geq 0$, which is equivalent to $1 - n\bar{\theta}d'(ne^C) \geq 0$. Since the function $1 - n\bar{\theta}d'(ne)$ is non increasing and $1 - n\bar{\theta}d'(n\overline{e}^C) = 0$, we obtain $\overline{e}^C \leq e^C$.

Second, consider the symmetric Nash equilibrium. Introduce now $X(\theta) = 1 - \theta d'(ne^N)$ and $Y(\theta) = u'(e^N - \theta d(ne^N))$. Again, we have $cov_{\tilde{\theta}}(X(\tilde{\theta}), Y(\tilde{\theta})) \leq 0$, $EX(\tilde{\theta})Y(\tilde{\theta}) = F(e^N) = 0$ and $EY(\tilde{\theta}) > 0$. Therefore $EX(\tilde{\theta}) = 1 - \bar{\theta}d'(ne^N) \geq 0$. Since the function $1 - \bar{\theta}d'(ne)$ is non increasing and $1 - \bar{\theta}d'(n\overline{e}^N) = 0$ we obtain $\overline{e}^N \leq e^N$.

Proof of Proposition 3: Recall, when $\tilde{z}$ is a mean preserving spread of $\tilde{\theta}$, for any concave function φ, $E\varphi(\tilde{z}) \leq E\varphi(\tilde{\theta})$. Introduce $\varphi(e, \theta) = (1 - n\theta d'(ne))u'(e - \theta d(ne))$ such that $W'(e, (\tilde{\theta})) = E_{\tilde{\theta}}\varphi(e, \tilde{\theta})$. If φ is concave in θ, we have $W'(e, (\tilde{z})) \leq W'(e, (\tilde{\theta}))$ and since W' is decreasing, $e^C(z) \leq e^C(\theta)$. Thus, it is sufficient to show that φ is concave in θ. Compute the second-order derivative of φ with respect to θ :

$$\partial^2\varphi/\partial\theta^2 = 2nd(ne)d'(ne)u''(e - \theta d(ne)) + d^2(ne)(1 - \theta nd'(ne))u'''(e - \theta d(ne))$$

Next, suppose that $u''' \geq 0$ and $P_r \leq 2$. It means that $\forall x, -x\frac{u'''(x)}{u''(x)} \leq 2$ hence $2u''(x) \leq -xu'''(x)$. Applying in $x = e - \theta d(ne)$, we obtain

$$\begin{aligned} \partial^2\varphi/\partial\theta^2 &\leq d(ne)d'(ne)u'''(e - \theta d(ne))[-n(e - \theta d(ne)) + (1 - \theta nd'(ne))\frac{d(ne)}{d'(ne)}] \\ \partial^2\varphi/\partial\theta^2 &\leq d(ne)d'(ne)u'''(e - \theta d(ne))[\frac{d(ne)}{d'(ne)} - ne] \end{aligned}$$

The right hand side is negative since $u''' \geq 0$ and $d'(x) \geq \frac{d(x)}{x}$ for any x when d is convex and $d(0) = 0$.

Proof of Proposition 4: We have here

$$W(e) = (1 - \frac{1}{k})u(e - \theta_L d(ne)) + \frac{1}{k}u(e - (\theta_L + k(\overline{\theta} - \theta_L))d(ne))$$

Define $\pi_L = e - \theta_L d(ne)$ and $\pi_H = e - (\theta_L + k(\overline{\theta} - \theta_L))d(ne)$. The first-order condition becomes:

$$(1 - \frac{1}{k})(1 - \theta_L nd'(ne))u'(\pi_L) + \frac{1}{k}(1 - (\theta_L + k(\overline{\theta} - \theta_L)nd'(ne))u'(\pi_H) = 0$$

It is sufficient to show that

$$H^C(e,k) = (k-1)(1 - \theta_L nd'(ne))u'(\pi_L) + (1 - (\theta_L + k(\overline{\theta} - \theta_L)nd'(ne))u'(\pi_H)$$

decreases in k. $\frac{\partial H^C(e,k)}{\partial k}$ equals

$$(1 - \theta_L nd'(ne))u'(\pi_L) - (\overline{\theta} - \theta_L)nd'(ne)u'(\pi_H) - (1 - (\theta_L + k(\overline{\theta} - \theta_L)nd'(ne))(\overline{\theta} - \theta_L)d(ne)u''(\pi_H)$$

By the first-order condition, observe that $(1 - \theta_L nd'(ne))u'(\pi_L) = \frac{u'(\pi_H)}{1-k}(1 - (\theta_L + k(\overline{\theta} - \theta_L))nd'(ne))$. After some simplifications, $\frac{\partial H^C(e,k)}{\partial k}$ reduces to

$$\frac{u'(\pi_H)}{1-k}(1 - \overline{\theta}nd'(ne)) - (1 - (\theta_L + k(\overline{\theta} - \theta_L)nd'(ne))(\overline{\theta} - \theta_L)d(ne)u''(\pi_H)$$

From Proposition 2, $1 - \overline{\theta}nd'(ne) > 0$. Hence the left member of the previous expression is negative since $k > 1$. Observe also that, since $\overline{\theta} > \theta_L$, then $1 - \theta_L nd'(ne) > 0$. From the first-order condition, this implies $(1 - (\theta_L + k(\overline{\theta} - \theta_L)nd'(ne)) < 0$. Therefore, the right member of the previous expression is negative as well under risk aversion, hence $\frac{\partial H^C(e,k)}{\partial k} < 0$.

Proof of Proposition 5: Introduce the auxiliary function $\psi(e,\theta) = (1 - \theta d'(ne))u'(e - \theta d(ne))$ such that $F(e,(\tilde{\theta})) = E_{\tilde{\theta}}\psi(e,\tilde{\theta})$. Let $\tilde{z}$ be a mean preserving spread of $\tilde{\theta}$. We know that if ψ is concave in θ, $F(e,(\tilde{z})) \leq F(e,(\tilde{\theta}))$, while if ψ is convex in θ, $F(e,(\tilde{z})) \geq F(e,(\tilde{\theta}))$. The second-order derivative of ψ with respect to θ is as follows:

$$\partial^2\psi/\partial\theta^2 = 2d(ne)d'(ne)u''(e - \theta d(ne)) + d^2(ne)(1 - \theta d'(ne))u'''(e - \theta d(ne))$$

The first term of the right hand side is negative, while the second term is ambiguous. However, $\partial^2\psi/\partial\theta^2$ is negative if $u''' = 0$.

Proof of Proposition 6: The proof is similar to the proof for the cooperative case. Define

$$H^N(e,k) = (k-1)(1-\theta_L d'(ne))u'(\pi_L) + (1-(\theta_L + k(\overline{\theta}-\theta_L)d'(ne))u'(\pi_H)$$

It is sufficient to show that $\frac{\partial H^N(e,k)}{\partial k}$ is negative. After some simplifications, $\frac{\partial H^N(e,k)}{\partial k}$ is equal to

$$\frac{u'(\pi_H)}{1-k}(1-\overline{\theta}d'(ne)) - (1-(\theta_L + k(\overline{\theta}-\theta_L)d'(ne))(\overline{\theta}-\theta_L)d(ne)u''(\pi_H)$$

The result then easily follows from $1-\overline{\theta}d'(ne) > 0$.

Proof of Proposition 7: Denote $W'(e,u) = E_{\tilde{\theta}}(1-n\tilde{\theta}d'(ne))u'(e-\tilde{\theta}d(ne))$. We will show that $W'(e^C(u),v) \leq 0$. Since W' is non increasing in e and $W'(e^C(v),v) \leq 0$, we will then conclude that $e^C(v) \geq e^C(u)$. Take θ and consider two cases. Either $1-\theta nd'(ne) \leq 0$, in which case $e-\theta d(ne) \leq e - \frac{d(ne)}{nd'(ne)}$ and $\Phi'[u(e-\theta d(ne))] \geq \Phi'[u(e-\frac{d(ne)}{nd'(ne)})]$ since u is increasing and Φ' is decreasing. Finally

$$[1-\theta nd'(ne)]\Phi'[u(e-\theta d(ne))] \leq [1-\theta nd'(ne)]\Phi'[u(e-\frac{d(ne)}{nd'(ne)})]$$

Or $1-\theta nd'(ne) \leq 0$, and $\Phi'[u(e-\theta d(ne))] \leq \Phi'[u(e-\frac{d(ne)}{nd'(ne)})]$ which yields the same inequality as above. This shows that this inequality is actually valid for any θ. Multiplying by $u'(e-\theta d(ne))$ and taking the expectation over θ leads to

$$W'(e,v) \leq \Phi'[u(e-\frac{d(ne)}{nd'(ne)})]W'(e,u)$$

At $e = e^C(u)$, $W'(e,u) = 0$ and $W'(e,v) \leq 0$.

Proof of Proposition 8: Denote $F(e,u) = E_{\tilde{\theta}}(1-\tilde{\theta}d'(ne))u'(e-\tilde{\theta}d(ne))$. By a technique similar to the one applied in the previous proof, we can show that $F(e^N(u),v) \leq 0$. Actually, we can also show that $F(e^N(v),u) \geq 0$. To see this, note that $\Psi = \Phi^{-1}$ is increasing and convex and that $u = \Psi v$. The previous technique can then be applied and eventually yields $F(e^N(v),u) \geq 0$. If $e^N(u)$ is the unique zero of $F(e,u)$ and $e^N(v)$ is the unique zero of $F(e,v)$, then the previous inequalities imply that $e^N(v) \leq e^N(u)$.

Proof of the statement below Proposition 9: u is DARA implies that there exists T

increasing and convex such that is $u = T(-u')$. This yields

$$\begin{aligned} u(e_1^C - \overline{\theta} d(e_1^C + e_2^C)) &= T(-u'(e_1^C - \overline{\theta} d(e_1^C + e_2^C))) \\ &= T(-Eu'(e_2^C - \widetilde{\theta} d(e_1^C + e_2^C))) \text{ by VI.1} \\ &\leq E(T(-u'(e_2^C - \widetilde{\theta} d(e_1^C + e_2^C))) \text{ by } T \text{ convex} \\ &= Eu(e_2^C - \widetilde{\theta} d(e_1^C + e_2^C)). \end{aligned}$$

Hence, agent 2's expected utility is higher.

Proof of Proposition 10: Assume first that agent 1 does not emit, $e_1^N = 0$. Then agent 2 chooses e_2^N such that

$$E(1 - \widetilde{\theta} d'(e_2^N))u'(e_2^N - \widetilde{\theta} d(e_2^N)) = 0 \tag{VII.1}$$

To examine whether this can be an equilibrium let us examine agent 1's best response. Agent 1 then simply chooses e_1 to maximize $u(e_1 - \overline{\theta} d(e_1 + e_2^N))$ where e_2^N is characterized by VII.1. So $e_1^N = 0$ would be a best-response if and only if

$$1 - \overline{\theta} d'(e_2^N) \leq 0. \tag{VII.2}$$

Yet $E(1 - \widetilde{\theta} d'(e_2^N))u'(e_2^N - \widetilde{\theta} d(e_2^N))$ is equal to

$$cov_{\widetilde{\theta}}[1 - \widetilde{\theta} d'(e_2^N), u'(e_2^N - \widetilde{\theta} d(e_2^N))] + E(1 - \widetilde{\theta} d'(e_2^N))Eu'(e_2^N - \widetilde{\theta} d(e_2^N))$$

which is strictly negative under u' strictly decreasing and VII.2. This contradicts VII.1. As a result, there is no equilibrium with $e_1^N = 0$. Let us finally assume that the agent facing uncertainty does not emit at all, $e_2^N = 0$. In that case, agent 1 chooses e_1^N such that

$$1 - \overline{\theta} d'(e_1^N) = 0 \tag{VII.3}$$

Agent 2 then chooses e_2 to maximize $W_N(e_2) \equiv Eu(e_2 - \widetilde{\theta} d(e_1^N + e_2))$ where e_1^N is characterized by VII.3. Since W_N is concave, a necessary and sufficient condition for $e_2^N = 0$ to be a best response consists in showing that $W_N'(0) \leq 0$. We get

$$W_N'(0) = E(1 - \widetilde{\theta} d'(e_1^N)) u'(0 - \widetilde{\theta} d(e_1^N)) = cov_{\widetilde{\theta}}[1 - \widetilde{\theta} d'(e_1^N), u'(0 - \widetilde{\theta} d(e_1^N)]$$

which is indeed negative under u' strictly decreasing.

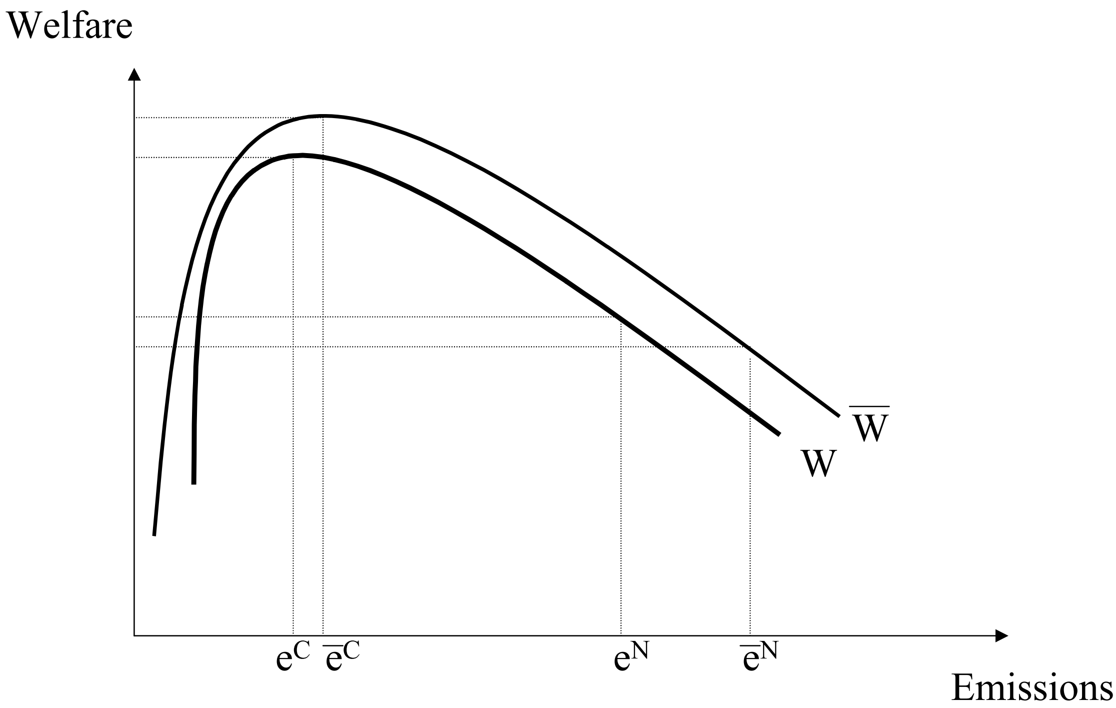

Figure 1:

Welfare as a function of emissions under certainty ($\overline{W}$) and uncertainty (W). Uncertainty always reduces welfare under cooperation, but may increase welfare in the non-cooperative case, as shown on the figure.

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