Sustainability, Debt Management, and Public Debt Policy in Japan

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Abstract

The purpose of this paper is to analyze sustainability issues of Japan's fiscal policy and then to discuss the debt management policy based on the maturity structure of government bonds using theoretical models and numerical studies. We also investigate the desirable coordination of fiscal and monetary authorities toward fiscal reconstruction. The partial default attempt by the fiscal authority is not effective in the long run because the gross rate of return on debt is adjusted to offset changes in the size of default. Inflationary taxes by the central bank will not have such an offsetting effect. Thus, the emergency reform by the monetary authority may well be better than the emergency reform by the fiscal authority so as to avoid bankruptcy. Given the present maturity structure of bonds, Japan's debt management policy should be based on the smoothing rule over time as follows. As the termination date concentrates within a ten-year period, it is preferable not to raise taxes or the inflation rate or to cut expenditures in this period only but to reserve the fiscal resources to a certain extent even if the reliance on government bonds will rise temporarily. It is also necessary not to cut taxes or increase expenditures much but to reduce reliance on the government bonds gradually when there are relatively a few expiring bonds. Moreover, it is indispensable to restrain the increasing trend of reliance on government bonds.

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I. Introduction

Currently it is crucial for the Japanese government to implement tight public debt policy, because the Japanese government has issued a very huge amount of government debts. Japan's fiscal situation has deteriorated rapidly with the collapse of the 'bubble economy' in the early 1990s and the deep and prolonged period of economic recession which ensued, and from which recovery has been slow and modest despite the implementation of counter-cyclical Keynesian policy. Since national income did not grow much, tax revenue did not increase either. On the contrary, government spending has been gradually raised due to political pressures of interest groups, resulting in large budget deficits.

In 1997, the Japanese government tried to implement the Fiscal Structural Reform so as to reduce budget deficits. However, in 1998, it stopped the reform and reduced taxes and increased public investment based on the traditional Keynesian policy because of the severe economic and financial situation, and the defeat of the governing party (the Liberal Democratic Party) in the Upper House election.

The concern for sustainability of fiscal deficits is a background for the fiscal reconstruction and structural reform movement by the current Koizumi Administration. The "Structural Reform of the Japanese Economy: Basic Policies for Macroeconomic Development" was decided upon after acceptance of the report compiled by the Council on Economic and Fiscal Policy, an advisory council to the Prime Minister. In this report the core of policies for the structural reform of the economic society was made clear. In part of the policies shown, a goal to limit the amount of government bond issues to less than 30 trillion yen in the fiscal 2002 budget, and afterwards to achieve a primary surplus, was set to show that there exists a necessity to take on full-scale measures towards fiscal consolidation or fiscal reconstruction. However, in order to cope with the bad situation of macro-economy, 1.8 trillion yen of the advance tax cuts was employed with a view to strengthening the competitiveness of industry, facilitating a smooth transference of assets to the next generation, promoting a shift from "saving to investment", advancing effective land use, and so on. The goal to limit the amount of government bond issues to less than 30 trillion yen in the fiscal 2002 budget was finally abandoned. In the fiscal 2003, new government bond issues are 36.4 trillion yen and the bond dependency ratio rises to 44.6%.

If creditors fear that the government is going to be in a debt trap, the long-term interest rate begins to rise, reflecting an enlarged credit risk. It is noted that although the Japanese Government Bonds (JGBs) have been issued too much, their yields are the lowest among G7 countries in the bond market. In this regard, despite its weakening credit ratings, the 10-year JGB nominal yield of about 1.5% in 2005 remains lower than the U.S. bond yield of about 1.8% registered during the Great Depression. However, we also have to pay attention to the possibility that the performance in the yield of the JGBs may not accurately reflect its credit risk. The Japanese banking sector continues to purchase the JGBs simply because short-term capital gains from the JGBs have been an easy option to offset the existing stock losses.

The purpose of this paper is to analyze sustainability issues of Japan's fiscal policy and then to discuss the debt management policy based on the given maturity structure of government bonds using theoretical models and numerical studies. We also investigate the desirable coordination of fiscal and monetary authorities toward fiscal reconstruction.

This paper consists of five sections. In section 2 we survey previous studies on sustainability issues. Section 2 qualitatively studies the dynamic effects of emergency policy alternatives. These alternatives are the ones available to the fiscal and monetary authorities confronting a potential debt crisis such as an increase in inflationary taxes and a partial use of default. We investigate the dynamic implications of future emergency reform in a debt-financed economy. In section 3, we evaluate Japan's debt management policy by providing a theoretical model to analyze the maturity structure of government bonds. We then implement a simple numerical analysis based on the smoothing rule derived by the theoretical model. In section 4, we discuss the desirable coordination of monetary and fiscal authorities towards fiscal reconstruction. The problem in Japan is that the fiscal authority is too weak to cope with political pressures from various interest groups. We explain this aspect using a simple fiscal reconstruction model. Finally, concluding remarks follow in Section 5.

2. Sustainability Issues and Emergency Reform

2.1 Concerns about sustainability

The purpose of this section is to study theoretically the dynamic effects of various policy alternatives available to a government confronting a potential debt crisis. In this section we study the dynamic behavior of the model, where public debt and money holding is explicitly incorporated.

The events of the 1980s and 1990s in Japan suggest that when a government becomes strapped for funds, it will tend to borrow from the world credit market rather than raise taxes to finance additional public spending. Indeed, many governments did either not raise broadly based taxes, e.g., the Thatcher government in Great Britain or the Reagan and Bush Administrations in the United States, or simply could not raise taxes to prevent causing riots, e.g., countries in Latin American and Eastern Europe, and, arguably, France in the reign of Louis XVI. There are long-term concerns about the accumulated fiscal deficit. Important one is whether such a large deficit can be sustained. The system will be paralyzed if public finance collapses under the weight of massive deficit. As a result, the financial system and the economy as a whole will be seriously affected. An extreme case of hyperinflation or default could develop.

The so-called chain-letter mechanism (or a Ponzi debt game) involves a situation where the future time path of taxes is fixed and debt finance is used to pay for any additional public spending; debt issuance is thus endogenously determined by the government's budget constraint. If the mechanism is sustainable, increased taxation need not necessarily be required in order to finance increased government spending as the economy converges to the steady state equilibrium. If the mechanism is unsustainable, the government will eventually go bankrupt in the sense that it will be unable to raise enough revenue to finance public spending and debt repayment. As debt crowds out private capital formation, the economy will also eventually go bankrupt if the mechanism fails. This suggests that studying the chain-letter mechanism and associated sustainability issues is quite important in terms of understanding the effects of government austerity (fiscal reconstruction) measures on the macroeconomy.

A simple way to evaluate the fiscal sustainability problem is to focus on the government bond market. In this regard for Japan, despite its weakening credit ratings, the 10-year Japanese Government Bond (JGB) nominal yield of about 1.5% in 2005 remains. So far the myth that the JGBs are risk-free has been somehow propagated. This episode may imply that Japan's government solvency is not a serious issue right now. However, Japan has experienced deep deflation, so the real rate of interest is about 2%, which is not so low. We also have to pay attention to the possibility that the performance in the yield of the JGB may not accurately reflect its credit risk.

Ihori, Nakazato, and Kawade (2002) attempted a standard approach to test the fiscal sustainability condition, using the methodology of Hamilton and Flavin (1986). They conducted the empirical analysis for the Japanese fiscal data from 1957 to 1999. To conduct the test, the values for the nominal growth rate, n, and the nominal interest rate, r, must be specified. Their strategy was to set various values for $r - n$ and to check whether the results are sensitive to the values chosen. The estimated results imply that the null hypothesis cannot be rejected at a 5% significance level, suggesting that government solvency was not a serious problem until FY 1996. On the contrary, the result for the period 1957-1997 rejects the null hypothesis when $r - n$ is above 0.05, and the results for the period 1957-1998 and the period 1957-1999 also reject the null hypothesis when $r - n$ is above 0.04.

Bohn (1998) proposed a new method different from existing tests for sustainability of government debt. According to Bohn (1998), the test has better properties than the tests based on estimating a transversality condition and on cointegration tests. The condition that fiscal policy satisfies the intertemporal budget constraint, i.e. the condition on sustainability of government debt, is that the primary surplus to GDP (s_t) increases with the ratio of (start-of-period) debt to GDP (d_t). Strictly speaking, when we can express a relation between the two as

$$s_t = f(d_t) + \mu_t$$

Suppose other determinants, μ_t, is bounded and the present value of future GDP is finite. Then, government debt satisfies a transversality condition if there is a debt-GDP ratio d^* such that $f'(d_t) \geq \beta > 0$ for all $d_t \geq d^*$, where β is a positive constant. We draw a scatter plot of s_t against d_t in Figure 1 (only the general account of the central government) and Figure 2 (the consolidated account of the central and local governments). Until the early 1990s, the Japanese fiscal policy held the quadratic relation between the two. Recently, the Japanese fiscal policy deviates from the relation excessively. Doi and Ihori (2003) showed that Japanese government debt does not satisfy a transversality condition for FY 1965-2000 by estimating β.

These observations indicate that fiscal sustainability may become a serious issue. The longer the sample period, the more likely we face the fiscal crisis. It follows that the chain letter mechanism will cause the public debt crisis to occur in the near future. Japan has two serious difficulties in terms of sustainability. First, the Japanese primary surplus is apparently a decreasing function of the debt-GDP ratio since 1990 and hence it does not satisfy Bohn's test. Second, the rate of interest is greater than the growth rate in Japan in the 1990s. Hence, it is important to reduce the government deficit in the near future.

2.2. Emergency Reform

Many governments prefer to rely on the issuance of debt rather than explicit taxation in financing expenditures. Recent experience suggests that a number of countries are facing potential bankruptcy as a result of issuing too much debt. As shown in Ihori (1988), the chain-letter mechanism would most likely be sustainable when the initial interest rate and stock of government debt are smaller or when the propensity to save and the growth rate are higher.

When the government goes eventually bankrupt, austerity measures as fiscal reconstruction will be required. This will depend critically on the response of the private sector to the specific austerity policy and more specifically the response of capital accumulation. Serious mistakes, which will possibly exacerbate the bankruptcy problem, may occur if the wrong action is taken. The conventional wisdom suggests that either the government must raise taxes or dramatically reduce spending. This is contingent on an increase in capital accumulation taking place in response to the change in policy. However, whether these contractions will be effected through cuts in spending or increases in explicit tax collections, and when these actions will be taken is in general unknown. Expectations of future policy changes are crucial in understanding seemingly counterintuitive macroeconomic dynamics. Bertola and Drazen (1993) argued that expectations about the discrete character of future fiscal adjustments can help explain the effects of current fiscal policy. They showed that if government spending follows an upward-trending stochastic process which the public believes may fall sharply when it reaches specific 'trigger' points, then optimizing consumption behavior and simple budget-constraint arithmetic imply a nonlinear relationship between private consumption and government spending. This theoretical relation is consistent with the experience of several countries.

Such a situation might be relevant for the recent Japanese economy. A recent line of economic research suggests that private agents realize that current bond-financed deficits carry with them future tax obligations. Anticipating higher future taxes, private agents change current spending behavior to smooth consumption intertemporally. Although the econometric study of this issue is still in its infancy, some recent research indicates that private Japanese behavior has partially offset recent changes in fiscal policy (see Ihori and Sato (2002) among others).

In reality, however, it may be difficult to employ the standard austerity measures in a proper time. For example, Japan's fiscal policy in 1990s created a problem of a tendency to postpone fiscal reconstruction reforms. The consensus at the time was that there was no immediate need for such painful measures as long as government policy prevented the economy from slipping into recession. There was, indeed, a widespread feeling in the private sector that the government would come to its aid if the economic situation worsened. That feeling fostered a certain complacency in the business world, making many corporate managers liable to "moral hazards" – risks stemming from lack of self-discipline. The continuation of the short-term stimulus policy, at a time when the economy needed long-term structural changes, discouraged self-help efforts in the private sector. Lobbying activities of local interest groups was exaggerated in the 1990s, as showed in Ihori, Doi and Kondo (2001) and Doi and Ihori (2002)'s empirical evidence. This is also one of the main reasons why Japan's fiscal reconstruction did not perform very well in the 1990s.

It is thus argued that if the current deficits seem not sustainable, governments in such countries will be forced to in effect repudiate their debt, either explicitly through an introduction of partial default or through inflation depreciation (inflationary taxes). We may call such a policy change the emergency reform for debt repudiation. The consequent fiscal reconstruction postponement is not free from credibility problems: Will the additional debt be paid off in full, or will the government find it optimal to resort to higher inflation or partial default to diminish the burden of the debt, etc? It should be stressed that if the private sector recognizes such possibilities of future emergency reforms for debt repudiation, government bonds and real capital may no longer be regarded as perfect substitutes. The more likely the current deficits seem not sustainable, the higher the subjective probability of the future emergency reform.

Several important papers investigated debt Ponzi games under uncertainty. The average riskless rate may be a poor guide as to whether permanent rollover of debt is feasible when economies are stochastic. Blanchard and Weil (1992) showed that whether or not governments can rollover debt in dynamically efficient economies depends on whether the issuance of public debt can partially substitute missing markets. Bohn (1991) showed that the sustainability even of simple policy rules like balanced budgets or tax rate smoothing should not be taken for granted in a stochastic economy and that sustainability is often sensitive to assumptions about debt management. The sustainability question in stochastic models is an aspect of fiscal policy that deserves more attention in future research and in policy-making.

Bearing these aspects in mind, we formalize one such psychological phenomenon: confidence. Holding government debt to provide for old-age consumption requires confidence and trust since no one can guarantee the young that the rate of return on debt is the same as that of real capital. Put another way, it is not sure to what extent the debt burden will be transferred to the next generation. This fact depends on the possibility of future emergency reform.

2.3 Remarks

Tirole (1985) and Weil (1987a) examined in the overlapping generations framework deterministic and speculative bubbles which are, like government debt, intergenerational schemes based on trust. Weil considered a two-state model with real capital and a bubble. The bubble has probability θ of bursting every period. The main result in Weil is that the highest sustainable bubble (the equivalent of the highest sustainable debt in the present paper) decreases with the probability of bursting (debt repudiation).

Economic theory has begun to catch up with political reality. It has done this by not only studying the optimality of fiscal policy in a context where explicit account is taken of the government's budget constraint but it has gone a step further by examining the time consistency of optimal policy. Here, it is the issue of whether it is optimal to keep promises that were optimal to make in the past. The latter lies at the heart of the credibility dilemma faced by any serious politician. Calvo (1988) studied models in which debt repudiation is possible and showed that expectations may play a crucial role in the determination of equilibrium. See also Chari and Kehoe (1993), and Bulow and Rogoff (1989).

The fiscal regime prevailing in an economy, as well as the type of fiscal relationships expected to arise from such a regime, is an important factor in determining the response of private agents to fiscal signals. Fiscal regimes differ across countries and change over time. At each point in time there is uncertainty about the regime that will prevail from then on. A high government deficit financed by debt can be regarded as unsustainable and therefore may be taken to signal future contractions in the deficits. Alesina, Prati and Tabellini (1989) showed that the maturity structure of public debt may influence the likelihood of a confidence crisis on the debt. The shorter and more concentrated is the maturity, the more likely is a confidence crisis. See also Giavazzi and Pagano (1989).

3. Debt Management Policy of the Japanese Government

3.1 Japan's Government Bonds

The Japanese government currently issues government bonds, which can be classified into six categories: short-term (6-month and 1-year Treasury bills); medium-term (2-year and 5-year bonds); long-term (10-year bonds); super-long-term (15-year, 20-year and 30-year bonds); government bonds for individual investors; and inflation-indexed bonds. The short-term government bonds are all discount bonds. On the other hand, all medium-, long-, and super-long-term government bonds, except for the 15-year floating-rate bonds, are the bonds with fixed-rate coupons. The 15-year floating-rate bonds and the government bonds for individual investors feature a coupon rate that varies according to certain rules. The inflation-indexed bonds are issued as the 10-years bonds to finance funds for the Fiscal Investment and Loan Program.[1]

The planned issue amount of each JGB for fiscal 2004 is shown in Table 1. In the past, there used to be some other types of government bonds. But after the August

[1] The Fiscal Investment and Loan Program (FILP) has been called "the second budget" because the government initially used FILP to undertake projects it was unable to include in the general account budget. Doi and Hoshi (2003) has a good summary of the structure, components, and history of FILP and PSS, and provides estimates of the costs FILP has and might impose on Japanese taxpayers; its appendix provides a further review of the literature. Also see Cargill and Yoshino (2000, 2003).

1988 3-year fixed-rate bonds, the September 2000 5-year discount bonds, the February 2001 4-year fixed-rate bonds, the March 2001 6-year fixed-rate bonds, and the November 2002 3-year discount bonds, these bonds have never been issued. The current maturity structure of the government bonds (outstanding basis) is shown in Figure 3.

3.2 Theoretical Analysis of Debt Management Policy

We construct a theoretical model based on Beetsma and Bovenberg (1997a, 1997b). We include the maturity structure of the government bonds in the model. There are households, firms, the fiscal authority (government) and the monetary authority (central bank). The households live for three periods. The firms produce a private good by using labor, as given price level, P_t (t = 1, 2, 3). Their production functions are $Y_t = L_t^{\eta}$ $(0 < \eta < 1)$, where Y_t denotes output, L_t denotes input of labor. Their profits are described as $(1-\tau_t)P_tL_t^{\eta}-W_tL_t$, where W_t denotes nominal wage rate, The firms' output is taxed at a rate τ_t, as will be described later.

The households organize labor unions, the objective of which is to obtain a target real wage rate. They are assumed to make an expectation to inflation rationally. We also assume that the unions have monopoly power in the labor market. We can normalize the logarithm of real wage rate to zero. Therefore, the (log) of the nominal wage rate is set equal to the (rationally) expected price level.

Under such a situation, the logarithm of output $y_t \equiv \ln Y_t$ is written as

$$y_t = \frac{\eta}{1-\eta}(\pi_t - \pi^e{}_t - \tau_t + \ln\eta).$$

where $\pi_t \equiv \dfrac{P_t-P_{t-1}}{P_t}$, $\pi^e{}_t$ denotes the inflation rate expected by the private sector. Since $\dfrac{\eta}{1-\eta}\ln\eta$ is a constant, we set $v\equiv\dfrac{\eta}{1-\eta}$, and normalize y_t as follows

$$x_t \equiv y_t - v\ln\eta = v(\pi_t - \pi^e{}_t - \tau_t) \tag{1}$$

Eq. (1) is the Lucas supply function.

In a rational expectations equilibrium ($\pi_t = \pi^e{}_t$), if there exist no tax distortion (τ_t = 0), the normalized output is given as x_t = 0. This normalized output level corresponds to the natural rate of employment, as mentioned in Fujiki et al. (1998). Moreover, the socially desirable output, $\tilde{x}_t$, without any distortion of resource allocation is positive, because the socially desirable employment is allowed to exceed the natural rate of employment, as pointed out in Beetsma and Bovenberg (1997a, 1997b). Hereafter $\tilde{x}_t$ is assumed to be given as a positive constant exogenously.

Next, we describe behavior of the monetary authority. The monetary authority decides level of money supply in each period. We presume that the quantity theory of money is held;

$$\frac{M_t}{P_t} = \kappa\tilde{X}_t$$

where κ is a constant, M_t denotes nominal money supply, and $\tilde{X}_t \equiv \exp(\tilde{x}_t)$. Since $\tilde{X}_t$ is given exogenously, the monetary authority determines the inflation rate directly through controlling money supply. Therefore, $\dfrac{M_t-M_{t-1}}{M_t} = \pi_t$ in this model.

Finally, we consider the government's behavior. The government (or fiscal authority) collects revenues from taxes, bond issuing and seigniorage. Its revenues are used for fiscal expenditures and repayment of government bond. The government can issue (inflation-indexed) bonds, and choose their maturity. We assume that the pure expectation hypothesis is held. In such a situation, the fiscal authority faces the following budget constraint in each period;

$$P_1G_1 + (1+r_{B1})P_1B_{01} = \tau_1P_1X_1 + (M_1-M_0) + P_1(B_{12} + B_{13})$$

$P_2G_2 + (1+r_{B1})(1+r_{B2})P_2B_{02} + (1+r_{B2})P_2B_{12} = \tau_2P_2X_2 + (M_2-M_1) + P_2B_{23}$

$P_3G_3 + (1+r_{B1})(1+r_{B2})(1+r_{B3})P_3B_{03}$

$+ (1+r_{B2})(1+r_{B3})P_3B_{13} + (1+r_{B3})P_3B_{23} = \tau_3P_3X_3 + (M_3-M_2)$

where G_t denotes real government expenditures, r_{Bt} denotes interest rate of bonds in period t, and B_{st} denotes the amount of bonds issued in period s with a prescribed payout in period t. Since B_{01}, B_{02}, and B_{03} are issued in period 0, they are exogenously given for the government. The government chooses G_t, τ_t, B_{ts} ($t+1\leq s\leq 3$).

Dividing both sides of the above budget constraints by $P_t\tilde{X}_t$ gives the following budget constraints in share of non-distortionary (normalized) output:

$$g_1 + (1+r_{B1})b_{01} = \tau_1 + \kappa\pi_1 + b_{12} + b_{13} \tag{2-1}$$

$$g_2 + (1+r_{B1})(1+r_{B2})b_{02} + (1+r_{B2})b_{12} = \tau_2 + \kappa\pi_2 + b_{23} \tag{2-2}$$

$$g_3 + (1+r_{B1})(1+r_{B2})(1+r_{B3})b_{03}+(1+r_{B2})(1+r_{B3})b_{13} + (1+r_{B3})b_{23} = \tau_3 + \kappa\pi_3 \tag{2-3}$$

where $g_t\equiv\frac{G_t}{\tilde{X}}$, $b_{st}\equiv\frac{B_{st}}{\tilde{X}}$,. We presume that $X_t \approx \tilde{X}_t \approx \tilde{X}$ (a constant).

For simplicity, the real interest rate is assumed to be equal to the world interest rate ρ, which is constant over time. Hence $r_{Bt} = \rho$. From (2-1,2,3) we can obtain the integrated government budget constraint as follows,

$$\begin{aligned} & g_1+\frac{g_2}{1+\rho}+\frac{g_3}{(1+\rho)^2}+(1+\rho)(b_{01}+b_{02}+b_{03}) = \\ & =\tau_1+\kappa\pi_1+\frac{\tau_2+\kappa\pi_2}{1+\rho}+\frac{\tau_3+\kappa\pi_3}{(1+\rho)^2} \end{aligned} \tag{3}$$

3.3 Second Best Solution

In this subsection, we analyze the most desirable case with distortionary taxes, where the two policy makers are integrated and are committed to their policy announcements. We deal with the situation in which the government and the central bank are integrated and are credibly committed to their policy announcements. The credible commitment particularly implies that the policy makers announce an inflation rate and commit themselves to the announced rate at the beginning of each period before nominal wages are concluded.

The society has the social loss function V, which is represented by

$$\mathrm{V} = \frac{1}{2}\sum_{t=1}^{3}\beta_S^{t-1}[\alpha_{\pi S}\pi_t^2 + (x_t-\tilde{x})^2 + \alpha_{gS}(g_t-\tilde{g}_t)^2] \tag{4}$$

where $\alpha_{\pi S} > 0$, $\alpha_{gS} > 0$, and β_S denotes the discount factor, $0< \beta_S\leq 1$. We define $\tilde{g}_t$ as the government spending target as the optimal share of the output realized without tax distortions or inflation surprises in period t. Now, for simplicity of the analysis, $\tilde{g}_t$ is assumed to be constant over time: $\tilde{g}_t = \tilde{g}$.

The policy makers minimize the above loss function. The constraints of each period consist of the Lucas supply function (1), the government budget constraint (3), and the restriction generated by the rational expectations formation of the private sector ($\pi^e_t = \pi_t$). The optimality conditions are given as follows:

$$v^2(\tau_t+\frac{\tilde{x}}{v}) = \alpha_{gS}(\tilde{g} - g_t) = \frac{\alpha_{\pi S}}{\kappa}\pi_t \qquad \text{(t=1, 2, 3)} \tag{5-1}$$

$$\pi_1 = \beta_S(1+\rho)\pi_2 = \beta_S^2(1+\rho)^2\pi_3 \tag{5-2}$$

$$\tau_1+\frac{\tilde{x}}{v} = \beta_S(1+\rho)(\tau_2+\frac{\tilde{x}}{v}) = \beta_S^2(1+\rho)^2(\tau_3+\frac{\tilde{x}}{v}) \tag{5-3}$$

$$\tilde{g} - g_1 = \beta_S(1+\rho)(\tilde{g} - g_2) = \beta_S^2(1+\rho)^2(\tilde{g} - g_3) \tag{5-4}$$

Eq. (5-1) is the static optimization condition in each period. (5-2)(5-3) and (5-4) are the intertemporal optimization conditions for inflation, tax rate, and government spending, respectively. For example, if $\beta_S(1+\rho) = 1$ (the discount rate is equal to the rate of interest), it is desirable to have the same levels of inflation, tax rate, and government spending over time, respectively. This is a well known smoothing condition over time a la Barro (1979). See also Barro (1995, 1997).

Several remarks are useful. Firstly, as Beetsma and Bovenberg (1997a, 1997b) mention, the social loss is affected by the initial level of government debt outstanding. The larger the initial debt $b_{01}+b_{02}+b_{03}$, the larger is the social loss.

Secondly, the income tax and individual preferences of leisure and labor affect the production level of the nation.

Thirdly, the term of expiration affects neither the inflation rate, the tax rate, nor fiscal expenditures, but affects the desirable maturity level of issued bonds. Namely, so long as the total of the initial debt $b_{01}+b_{02}+b_{03}$ is fixed, the maturity structure of the initial debt b_{01}, b_{02}, b_{03} does not affect the optimal values of inflation, tax rate and government spending. However, the maturity structure of the initial debt b_{01}, b_{02}, b_{03} does affect the optimal values of b_{12}, b_{13}, b_{23}. The budget constraints (2-1, 2, 3) determine the optimal values of b_{12}, b_{13}, b_{23} as a function of b_{01}, b_{02}, b_{03} once the optimal values of inflation, tax rate and government spending are determined by the smoothing conditions (5-1, 2, 3, 4). We may derive bond issuance equations from (2-1, 2, 3) and (5-1, 2, 3, 4). Namely, bond issuance in each period is respectively given as

$$b_{12}+b_{13} = \frac{1}{1+\beta_S(1+\rho)^2+\beta_S^2(1+\rho)^4}[\{1+\beta_S(1+\rho)^2\}\{\tilde{K}+(1+\rho)b_{01}\} - \beta_S^2(1+\rho)^2\{\tilde{K}+(1+\rho)\tilde{K}+(1+\rho)^3(b_{02}+b_{03})\}] \tag{5-5}$$

$$b_{23} = \frac{1}{1+\beta_S(1+\rho)^2}[\tilde{K} + (1+\rho)^2 b_{02} + (1+\rho)b_{12} - \beta_S(1+\rho)\{\tilde{K}+(1+\rho)^3 b_{03}+(1+\rho)^2 b_{13}\}] \tag{5-6}$$

where $\tilde{K} \equiv \tilde{g} + \tilde{x}/v$.

Eq. (5-5) implies the maturity structure at the beginning of the first period $\{b_{01}, b_{02}, b_{03}\}$ cannot determine the maturity structure on issuing basis in the first period, or the optimal combination of $\{b_{12}, b_{13}\}$, but can determine the total amount of bond issuance in the first period $\{b_{12}+b_{13}\}$. According to (5-6), however, the maturity structure on issuing basis in the first period affects bond issuance in the second period. For example, an increase in b_{01} would raise relatively the optimal values of b_{12}, b_{13}. An increase in b_{02} would raise the optimal value of b_{23}. An increase in b_{03} would reduce the optimal values of b_{13}, b_{23}.

Intuition is as follows. To maintain the neutrality of bonds toward social welfare (social loss), it is necessary to issue bonds to cover the part of fiscal expenditures and redemption that cannot be covered from tax revenues and re-coinage profits while maintaining budget constraints and not distorting the inflation rate, tax rate, and fiscal expenditures. Given the maturity structure, issuing bonds should act as a buffer in the budget.

3.4. Numerical Analysis

In this subsection, we numerically examine the second best debt management policy under commitment, which is theoretically analyzed in the previous subsection. We can easily extend the analytical framework to a more general multi-period model.

For the present numerical analysis, we use a 200-period model and incorporate nominal bonds as well.[2]

In doing the numerical analysis, it is necessary to specify values of some exogenous parameters in the theoretical model. Based on the data of Japanese economy, we set $\eta = 0.7$, $\rho = 0.02$, $\beta_S = 0.985$, $\alpha_{\pi S} = 10$, $\alpha_{gS} = 10$, $\tilde{x} = 0.01$, and $\tilde{g}_1 = 0.045$. We also adapt $\kappa = 0.36$, based on Bank of Japan (1996), as mentioned in Fujiki et al. (1998).

With respect to the initial maturity structure of government bonds, values at the end of fiscal 1998, are used for calculation. Values of the maturity structure on issuing basis in fiscal 1998 are used, as shown in Table 2. The initial level of outstanding debt is also an important figure in the analysis. We set the outstanding debt to (normalized) output ratio as 0.27, based on the value in 1998.

Under such values of parameters, we derive numerical results by expanding the model to 200 periods. Figure 4 shows transitions of debt dependence ratio (in the upper figure), and inflation rate, government expenditure (to the desirable output ratio), and tax rate (in the lower figure). The lower figure indicates the smoothing effects of these flow variables a la Barro (1979). The upper figure also reflects the smoothing rule. Following the maturity structure on issuing basis in fiscal 1998, shown in Table 2, a 10-year-bond-centered maturity structure is used for calculation. Hence a rise of the bond dependence ratio is on a 10-year cycle. It suggests that it is preferable to have an increase in bond issue when many (10-year) bonds are matured, rather than to have a sudden increase in tax revenue. Then the bond dependence ratio rises temporarily at this period.

Figure 5 also indicates transitions of government debt outstanding (to the desirable output ratio), and the desired maturity structure of government debt. This figure suggests that it is desirable to reduce the bond dependence ratio gradually to redeem fully in the 200th period.

Given the present structure of bonds by maturity (structure centering on 10-year bonds), the calculated desired schedule of issuing bonds implies the following debt management policy based on the smoothing rule. Namely, as the termination date concentrates within a ten-year period, it is preferable not to raise taxes or the inflation rate or to cut expenditures in this period only but to reserve the fiscal resources to a certain extent even if the reliance on government bonds will rise temporarily. At the same time, it is necessary not to cut taxes or increase expenditures much but to reduce reliance on the government bonds gradually when there are relatively a few expiring bonds. Moreover, it is indispensable to restrain the increasing trend of reliance on government bonds.

4. Debt Management and Fiscal Sustainability

4.1 Default of the government bonds

As analyzed in Beetsma and Bovenberg (1997a, b) among others, when monetary and fiscal authorities are not cooperative and not able to commit their policy announcements, an optimally designed conservative, independent central bank is necessary to establish the second best. The central bank must be made more conservative than society. They showed that correcting monetary policy preferences is a direct way to eliminate the distortions due to the inability to commit. Drudi and Giordano (2000) showed that since default risk increases as the maturity structure of the debt shortens, optimal maturity under bankruptcy risk is in general longer than in

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[2] The reason why we set a 200-period model is to weaken effects of the terminal conditions which all stock variables are zero, on this numerical analysis.

the case in which debt repudiation policies can be precommitted or are very much unlikely. See also Persson, Persson and Svensson (1987, 2005).

If we allow for political distortions, the preferences of the fiscal authority may depart from the preferences of society. In the presence of political distortions a debt target is also needed. For example, if the government discounts the future too heavily, the optimal debt target would de facto act as a ceiling on public debt.

In Japan, the central bank now acts as an independent policy maker and its concern on inflationary targeting is more conservative than the government. In this sense, we could say that the central bank behaves in a good manner to attain the second best.

Let us explain this aspect using a model that includes confidence crisis of government debt and spontaneous default of fiscal authority in the model introduced in Section 3. We employ the same three-period model in Section 3. Investors of government bonds decide whether they buy bonds or not in prospect of behaviors of the government. If they can perfectly expect the government's default, they do not purchase bonds at all. Hence we should investigate such a situation using backward induction. It means that a subgame perfect Nash equilibrium is adopted as a solution concept in this section. The structure of this policy game is shown in Figure 6.

The fiscal authority and the monetary authority have individual loss functions. Loss function of the fiscal authority is written as

$$V^F = \frac{1}{2}\sum_{t=1}^{3}\beta_S^{t-1}[\alpha_{\pi F}\pi_t^2 + (x_t - \tilde{x})^2 + \alpha_{gS}(g_t - \tilde{g})^2] \quad (6)$$

where $\alpha_{\pi F} > 0$, $\alpha_{gS} > 0$, and β_S denotes the discount factor, $0 < \beta_S \leq 1$. Also Loss function of the monetary authority is written as

$$V^M = \frac{1}{2}\sum_{t=1}^{3}\beta_S^{t-1}[\alpha_{\pi M}\pi_t^2 + (x_t - \tilde{x})^2 + \alpha_{gS}(g_t - \tilde{g})^2] \quad (7)$$

where $\alpha_{\pi M} > \alpha_{\pi F} > 0$. It implies that the monetary authority is more conservative in inflation than the fiscal authority. Each policy maker minimizes the above loss function, taking policies selected by the other authority as given. In this section, we set that both policymakers decide policies simultaneously in each period. Investors of the government bonds have the loss function (4).

Now, we describe a situation that the government triggers a debt default. The government can declare the default before policies are chosen in this period. When the default occurs, the government does not pay at all. However, the production in this economy is deteriorated due to the default. In this situation, the Lucas supply function is assumed to become

$$x_t = zv(\pi_t - \pi^e_t - \tau_t) \qquad 0 < z < 1 \quad (1')$$

where z is constant over time.

It means that the production in default on the government bond is z times as large as that in the normal situation, regardless of the amount of the debt.

The constraints of each period consist of the Lucas supply function (1) or (1'), the government budget constraints (2). We also rewrite the government budget constraints as follows,

$$\tilde{K} + \frac{(1-z)\tilde{x}}{vz} + (1+\rho)b_{01} = (\tau_1 + \frac{\tilde{x}}{vz}) + \kappa\pi_1 + (\tilde{g} - g_1) + b_{12} + b_{13} \quad (8\text{-}1)$$

$$\tilde{K} + \frac{(1-z)\tilde{x}}{vz} + (1+\rho)^2 b_{02} + (1+\rho)b_{12} = (\tau_2 + \frac{\tilde{x}}{vz}) + \kappa\pi_2 + (\tilde{g} - g_2) + b_{23} \quad (8\text{-}2)$$

$$\tilde{K} + \frac{(1-z)\tilde{x}}{vz} + (1+\rho)^3 b_{03} + (1+\rho)^2 b_{13} + (1+\rho)b_{23} = (\tau_3 + \frac{\tilde{x}}{vz}) + \kappa\pi_3 + (\tilde{g} - g_3) \quad (8\text{-}3)$$

Note $(x_t - \widetilde{x})^2 = z^2v^2(\pi^e_t - \pi_t + \tau_t + \frac{\widetilde{x}}{vz})^2$ from equation (1'), z =1 in the normal situation, and $0 < z < 1$ in default of payment.

We assume that, in each period, the government cannot commit inflation rate announced at the beginning of each period before nominal wages are set. Under this situation, the policy authorities take inflation expectations as predetermined.

4.2 Policy choice in the third period

To solve for the three-period decision problem, we use the backward induction method. Thus, we begin with solving for the solution in the third period and then proceed to solve for the solution in the second and first period. It implies that such a policy is a time-consistent policy, which is analyzed in Lucas and Stokey (1983), Persson, Persson and Svensson (1987, 2005), Calvo and Guidotti (1990a, 1990b) and so on. In the third period, the fiscal authority chooses $\{\tau_3, g_3\}$ to minimize its loss function, subject to the budget constraint (8-3). Also the monetary authority chooses $\{\pi_3\}$ to minimize its loss function, taking as given the expected inflation rate (π^e_3), without any regard for the budget constraint (8-3).

4.2.1 The normal case (Case I)

If the government does not trigger a debt default in the third period, we obtain the following conditions from the first order conditions for the choice of $\{\pi_3, \tau_3, g_3\}$, taking policies decided by the other authority and inflation expectation as given,

$$v(\widetilde{x} - x_3) = \alpha_{gS}(\widetilde{g} - g_3) = \alpha_{\pi M}\pi_3 \quad (9)$$

Moreover, from the above conditions and the government budget constraint and the restriction generated by the rational expectations formation of the private sector ($\pi^e_3 = \pi_3$), the following relations are held

$$\pi_3 = \frac{1}{\mathrm{N}\alpha_{\pi S}}[\widetilde{K} + b_3]$$

$$\tau_3 + \frac{\widetilde{x}}{v} = \frac{1}{\mathrm{N}v^2}[\widetilde{K} + b_3] \quad (10)$$

$$\widetilde{g} - g_3 = \frac{1}{\mathrm{N}\alpha_{gS}}[\widetilde{K} + b_3]$$

where $\mathrm{N} \equiv \frac{\kappa}{\alpha_{\pi M}} + \frac{1}{v^2} + \frac{1}{\alpha_{gS}}$, $\quad b_3 \equiv (1+\rho)^3 b_{03} + (1+\rho)^2 b_{13} + (1+\rho) b_{23}$

Hence, the value of the loss function of the fiscal authority is

$$\mathrm{V}_3^{\mathrm{FI}} \equiv \frac{1}{2}\frac{\mathrm{N_F}^*}{\mathrm{N}^2}[\widetilde{K} + b_3]^2 \quad (11)$$

where $\mathrm{N_F}^* \equiv \frac{\alpha_{\pi F}}{\alpha_{\pi M}^2} + \frac{1}{v^2} + \frac{1}{\alpha_{gS}}$

4.2.2 The case of default (Case II)

If the government *does* declare a debt default in the third period, the government budget constraint becomes as follows,

$$\widetilde{K} + \frac{(1-z)\widetilde{x}}{vz} = (\tau_3 + \frac{\widetilde{x}}{vz}) + \kappa\pi_3 + (\widetilde{g} - g_3) \qquad 0 < z < 1 \quad (8\text{-}3')$$

Under this situation, we obtain the following conditions from the first order conditions for the choice of $\{\pi_3, \tau_3, g_3\}$, taking policies decided by the other authority and inflation expectation as given,

$$vz(\widetilde{x} - x_3) = \alpha_{gS}(\widetilde{g} - g_3) = \alpha_{\pi M}\pi_3 \quad (12)$$

Moreover, from the above conditions (1') and (12), and the government budget constraint (8-3') and the restriction generated by the rational expectations formation of the private sector ($\pi^e{}_3 = \pi_3$), the following relations are held

$$\pi_3 = \frac{1}{\mathrm{H}\alpha_{\pi S}}[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}]$$

$$\tau_3 + \frac{\tilde{x}}{vz} = \frac{1}{\mathrm{H}v^2z^2}[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}]$$

$$\tilde{g} - g_3 = \frac{1}{\mathrm{H}\alpha_{gS}}[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}] \tag{13}$$

where $\mathrm{H} \equiv \frac{\kappa}{\alpha_{\pi M}} + \frac{1}{v^2z^2} + \frac{1}{\alpha_{gS}}$

Hence, the value of the loss function of the fiscal authority is

$$V_3^{FII} \equiv \frac{1}{2}\frac{\mathrm{H_F}^*}{\mathrm{H}^2}[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}]^2 \tag{14}$$

where $\mathrm{H_F}^* \equiv \frac{\alpha_{\pi F}}{\alpha_{\pi M}^2} + \frac{1}{v^2z^2} + \frac{1}{\alpha_{gS}}$

4.2.3 Welfare comparison of both cases

We analyze whether the government has an incentive of a debt default. If $V_3^{FI} > V_3^{FII}$, the government minimize the loss by making default. From (11) and (14), $V_3^{FI} > V_3^{FII}$ is satisfied under the following conditions

$$b_3 > \sqrt{D} - \tilde{K}, \qquad \text{if } \tilde{K} \le \frac{N\tilde{x}vz}{1+z}$$

$$b_3 > \sqrt{D} - \tilde{K}, \qquad \text{if } \tilde{K} > \frac{N\tilde{x}vz}{1+z} \text{ and } D > \tilde{K}^2 \tag{15}$$

$$b_3 > 0, \qquad \text{if } \tilde{K} > \frac{N\tilde{x}vz}{1+z} \text{ and } D \le \tilde{K}^2$$

where $D \equiv \frac{\mathrm{N}^2\mathrm{H_F}^*}{\mathrm{N_F}^*\mathrm{H}^2}[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}]^2$

These conditions suggest that the government has an incentive to default when the amount of debt outstanding is more than a certain level.

Since the condition (15) is very complicated, we focus on a case that is consistent with the numerical analysis in Section 3. We set $\eta = 0.7$, $\rho = 0.02$, $\beta_S = 0.985$, $\alpha_{\pi S} = 10$, $\alpha_{gS} = 10$, $\kappa = 0.36$, and $\tilde{g}_1 = 0.045$. In the case where we adopt $\tilde{x} = 0.01$, as set in Section 3, trivially $V_3^{FI} > V_3^{FII}$, that is, the government always triggers a debt default in the third period. Hence, we rearrange $\tilde{x} = 0.2$. Under this situation, $\tilde{K} > \frac{N\tilde{x}vz}{1+z}$ and $D > \tilde{K}^2$ are held. We investigate theoretical properties on these numerical assumptions in the following sections.

Incidentally, confidence crisis does not occur in the third period, since the government does not newly issue bonds in this period.

4.3 Policy choice in the second period

In the second period, investors of the government bonds firstly expect whether the government trigger a debt default (in the second period or the third period). If they believe the default occurs, they do not buy the bonds at all. This situation is confidence crisis. Under this situation, the government cannot newly issue bonds (b_{23}). If investors expect the default does not occur, the government bonds are freely treaded.

After that, the fiscal authority chooses $\{\tau_2, g_2, b_{23}\}$ to minimize its loss function, subject to the budget constraint (8-2). Also the monetary authority chooses $\{\pi_2\}$ to minimize its loss function, without any regard for the budget constraint (8-2).

4.3.1 The normal case under no confidence crisis (Case III)

First, we consider a situation that confidence crisis does not occur. Under this situation, the government can newly issue an one-period bond (b_{23}). The fiscal and monetary authorities minimize their loss functions in consideration of situation in the third period; whether the government faces a debt default or not. Thus the authorities in the second period have the following loss functions,

$$V_2^{a} = \frac{1}{2}[\alpha_{\pi a}\pi_2^2 + (x_2 - \tilde{x})^2 + \alpha_{gS}(g_2 - \tilde{g})^2] + \beta_S V_3^{aI} \quad (16)$$

where $a = F, M$

V_3^{aI} denotes the value of loss function in Case I in the third period. V_3^{FI} is defined as (11), and V_3^{MI} is obtained by assigning (10) to (7).

The monetary authority minimizes (16) regardless of the government budget constraint, taking policies selected by the fiscal authority and inflation expectation as given. From the first order condition for the choice of $\{\pi_2\}$, we obtain the following condition

$$v(\tilde{x} - x_2) = \alpha_{\pi M}\pi_2 \quad (17\text{-}1)$$

The fiscal authority minimizes its loss function in consideration of situation in the third period; whether the government faces a debt default or not. In the case of $V_3^{FII} > V_3^{FI}$, called Case III, the fiscal authority sets policies to satisfy the following conditions:

$$v(\tilde{x} - x_2) = \alpha_{gS}(\tilde{g} - g_2) = \beta_N^*[\tilde{K} + b_3] \quad (17\text{-}2)$$

where $\beta_N^* \equiv \beta_S(1+\rho)N_F^*/N$

From the above conditions (17-1,2) and the government budget constraint, the following relations are held under the rational expectations formation of the private sector ($\pi^e_2 = \pi_2$)

$$b_{23} = \delta_2[\tilde{K} + b_2 - \beta_N^*\{\tilde{K} + (1+\rho)^3 b_{03} + (1+\rho)^2 b_{13}\}]$$

$$\tau_2 + \frac{\tilde{x}}{v} = \frac{1}{Nv^2}[\tilde{K} + b_2 - b_{23}]$$

$$= \frac{1}{Nv^2}\delta_2\beta_N^*[\tilde{K} + b_2 + \frac{1}{1+\rho}\{\tilde{K} + (1+\rho)^3 b_{03} + (1+\rho)^2 b_{13}\}]$$

$$\pi_2 = \frac{1}{N\alpha_{\pi M}}\delta_2\beta_N^*[\tilde{K} + b_2 + \frac{1}{1+\rho}\{\tilde{K} + (1+\rho)^3 b_{03} + (1+\rho)^2 b_{13}\}]$$

$$\tilde{g} - g_2 = \frac{1}{N\alpha_{gS}}\delta_2\beta_N^*[\tilde{K} + b_2 + \frac{1}{1+\rho}\{\tilde{K} + (1+\rho)^3 b_{03} + (1+\rho)^2 b_{13}\}] \quad (18)$$

where $b_2 \equiv (1+\rho)b_{12} + (1+\rho)^2 b_{02}$, $\delta_2 \equiv \dfrac{1+\rho}{1+\beta_S(1+\rho)^2 N_F^*/N}$

Therefore, we obtain the value of the loss function as follows

$$V_2^{FIII} \equiv \frac{1}{2}\frac{N_F^*}{N^2}[\tilde{K} + b_2 - b_{23}]^2 + \frac{1}{2}\beta_S\frac{N_F^*}{N^2}(\tilde{K} + b_3)^2$$

$$= \frac{1}{2}\frac{N_F^*}{N^2}\delta_2^2\{(\beta_N^*)^2 + \beta_S\}[\tilde{K} + b_2 + \frac{1}{1+\rho}\{\tilde{K} + (1+\rho)^3 b_{03} + (1+\rho)^2 b_{13}\}]^2 \quad (19)$$

4.3.2 The normal case under confidence crisis (Case IV)

Next, we consider a situation that confidence crisis occurs. Under this situation, the government cannot newly issue any bond ($b_{23} = 0$). The fiscal and monetary authorities minimize their loss functions in consideration of situation in the third period; whether the government faces a debt default or not in the third period. Thus the authorities in the second period are written as (16).

The monetary authority minimizes (16) regardless of the government budget constraint, taking policies selected by the fiscal authority and inflation expectation as given. Since this situation is the same as Case III, condition with respect to $\{\pi_2\}$ in this case is (17-1).

The fiscal authority minimizes its loss functions in consideration of situation in the third period. The government decides policies to satisfy the following condition,

$$v(\tilde{x} - x_2) = \alpha_{gS}(\tilde{g} - g_2) \qquad (20)$$

Therefore, we obtain the value of the loss function in this case as follows

$$V_2^{FIV} \equiv \frac{1}{2}\frac{N_F^*}{N^2}[\tilde{K} + b_2]^2 + \frac{1}{2}\beta_S\frac{H_F^*}{H^2}[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}]^2 \qquad (21)$$

4.3.3 The case of default under confidence crisis (Case V)

Also, we discuss the situation that the government in the second period triggers a debt default under confidence crisis. Under this situation, the government cannot newly issue an one-period bond ($b_{23} = 0$).

The fiscal and monetary authorities minimize their loss functions in consideration of situation in the third period. If once the government defaults on payments in the second period, however, the government has no debt in the third period, that is, there is no default in the third period. Thus the authorities in the second period have the following loss functions,

$$V_2^a = \frac{1}{2}[\alpha_{\pi a}\pi_2^2 + (x_2 - \tilde{x})^2 + \alpha_{gS}(g_2 - \tilde{g})^2] + \beta_S V_3^{aI}\,|_{b_3=0} \qquad (16')$$

where $a = F, M$

Also the production in this situation is determined by (1'). The government budget constraint in the second period becomes as follows,

$$\tilde{K} + \frac{(1-z)\tilde{x}}{vz} = (\tau_2 + \frac{\tilde{x}}{vz}) + \kappa\pi_2 + (\tilde{g} - g_2) \qquad 0 < z < 1 \qquad (8\text{-}2')$$

The monetary authority minimizes (16') regardless of the government budget constraint, taking policies selected by the fiscal authority and inflation expectation as given. From the first order condition for the choice of $\{\pi_2\}$, we obtain the following condition

$$vz(\tilde{x} - x_2) = \alpha_{\pi M}\pi_2 \qquad (22\text{-}1)$$

The fiscal authority minimizes its loss function (16'), subject to (8-2'). The authority sets policies to satisfy the following condition:

$$vz(\tilde{x} - x_2) = \alpha_{gS}(\tilde{g} - g_2) \qquad (22\text{-}2)$$

From the above conditions (1'), (22-1, 2), and the government budget constraint (8-2'), the following relations are held under the rational expectations formation of the private sector ($\pi^e_2 = \pi_2$)

$$\pi_2 = \frac{1}{H\alpha_{\pi S}}[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}]$$

$$\tau_2 + \frac{\tilde{x}}{vz} = \frac{1}{Hv^2z^2}[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}]$$

$$\tilde{g} - g_2 = \frac{1}{H\alpha_{gS}}[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}] \qquad (23)$$

Therefore, we obtain the value of the loss function in this case as follows

$$\mathrm{V_3^{FIV}} \equiv \frac{1}{2}\frac{\mathrm{H_F^*}}{\mathrm{H}^2}[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}]^2 + \frac{1}{2}\beta_S\frac{\mathrm{N_F^*}}{\mathrm{N}^2}\tilde{K}^2 \qquad (24)$$

4.3.4 Welfare comparison between Case IV and Case V

Dose the fiscal authority trigger a debt default under no confidence crisis? It depends the value of the loss function of the fiscal authority in each case.

If $V_2^{FV} \geq V_2^{FIV}$, the government does not trigger a default in the second period. On the other hand, in the case of $V_2^{FV} < V_2^{FIV}$, the fiscal authority has incentives to default on payments in the second period.

The details are as follows. In the case of $\tilde{K} > \dfrac{N\tilde{x}vz}{1+z}$ and $D > \tilde{K}^2$, $V_2^{FV} > V_2^{FVI}$ is satisfied under the following conditions

$$b_2 > \sqrt{\beta_S\tilde{K}^2 + (1-\beta_S)D} - \tilde{K}, \qquad (25)$$

These conditions suggest that the government has an incentive to default when the amount of debt outstanding is more than a certain level.

4.3.5 Welfare comparison between Case III and Case IV or Case V

Whether the confidence crisis occurs or not in the second period depends on welfare loss of the fiscal authority in each case. If the government defaults on payments, investors of the government bonds face losses. Thus, they do not buy the bonds at all when they expect that the government trigger a debt default in the second period or the third period.

If $V_2^{FIII} \leq \min\{V_2^{FIV}, V_2^{FV}\}$, the fiscal authority does not have any incentives to default in each period. Hence, investors can purchase the government bonds. We further analyze this situation.

$V_2^{FIII} < V_2^{FIV} < V_2^{FV}$ is satisfied under the following conditions

$$0 < b_{II} < \beta_S D - \tilde{K}^2\{\gamma_2(1+\frac{1}{1+\rho})^2 - 1\} \qquad \text{if } \gamma_2 < \frac{1+\beta_S D}{\left(1+\dfrac{1}{1+\rho}\right)^2} \qquad (26)$$

where $\gamma_2 \equiv \delta_2^2\{(\beta_N^*)^2 + \beta_S\}$,

$$b_{II} \equiv (\gamma_2 - 1)b_2^2 + \gamma_2\{(1+\rho)^2 b_{03} + (1+\rho)b_{13}\}^2 + 2\gamma_2 b_2\{(1+\rho)^2 b_{03} + (1+\rho)b_{13}\} - 2\tilde{K}b_2 + 2\tilde{K}\gamma_2(1+\frac{1}{1+\rho})[b_2 + \{(1+\rho)^2 b_{03} + (1+\rho)b_{13}\}]$$

and

$$b_2 \leq \sqrt{\beta_S\tilde{K}^2 + (1-\beta_S)D} - \tilde{K}, \qquad (25')$$

$V_2^{FIII} < V_2^{FV} < V_2^{FIV}$ is satisfied under (25) and the following conditions

$$0 < b_2 + (1+\rho)^2 b_{03} + (1+\rho)b_{13} < \sqrt{\frac{D + \beta\tilde{K}_2}{\gamma_2}} - \left(1+\frac{1}{1+\rho}\right)\tilde{K}$$

$$\text{if } \gamma_2 < \frac{D + \beta_S\tilde{K}^2}{\left(1+\dfrac{1}{1+\rho}\right)^2\tilde{K}^2} \qquad (27)$$

When $V_2^{FIII} \leq \min\{V_2^{FIV}, V_2^{FV}\}$, that is, conditions (25')and (26) or (25) and (27) are held, investors buy the government bonds in the second period. It means that there is no confidence crisis in the second period under this situation. Otherwise, investors do not buy bonds at all in the second period. Thus confidence crisis occurs in the second period.

4.4 Policy choice in the first period

In the same way, the policy makers choose $\{\pi_1, \tau_1, g_1, b_{12}, b_{13}\}$ to minimize their loss functions in the first period.

4.4.1 The normal case under no confidence crisis (Case VI)

First, we consider a situation that confidence crisis does not occur. Under this situation, the government can newly issue bonds (b_{12} and b_{13}). The fiscal and monetary authorities minimize their loss functions in consideration of situation in the second and third period; whether the government faces a debt default or not. Thus the authorities in the first period have the following loss functions,

$$V_1^a = \frac{1}{2}[\alpha_{\pi a}\pi_1^2 + (x_1 - \tilde{x})^2 + \alpha_{gS}(g_1 - \tilde{g})^2] + \beta_S V_2^{aIII} \tag{28}$$

where $a = F, M$

V_2^{aIII} denotes the value of loss function in Case III in the second period. V_2^{FIII} is defined as (19), and V_2^{MIII} is obtained by assigning (18) to (16).

The monetary authority minimizes (28) regardless of the government budget constraint, taking policies selected by the fiscal authority and inflation expectation as given. From the first order condition for the choice of $\{\pi_1\}$, we obtain the following condition

$$v(\tilde{x} - x_1) = \alpha_{\pi M}\pi_1 \tag{29-1}$$

The fiscal authority minimizes its loss function, subject to (2-1). The fiscal authority sets policies to satisfy the following conditions:

$$v(\tilde{x} - x_1) = \alpha_{gS}(\tilde{g} - g_1) = \beta_N^*\gamma_2[\tilde{K} + b_2 + \frac{1}{1+\rho}\{\tilde{K} + (1+\rho)^3 b_{03} + (1+\rho)^2 b_{13}\}] \tag{29-2}$$

From the above conditions (17-1,2) and the government budget constraint, the following relations are held under the rational expectations formation of the private sector ($\pi^e_1 = \pi_1$)

$$b_{12} + b_{13} = \delta_1\frac{1}{1+\rho}[\tilde{K} + (1+\rho)b_{01} - \gamma_2\beta_N^*\{\tilde{K} + (1+\rho)^2 b_{02} + \frac{1}{1+\rho}(\tilde{K} + (1+\rho)^3 b_{03})\}]$$

$$\tau_1 + \frac{\tilde{x}}{v} = \frac{1}{Nv^2}[\tilde{K} + (1+\rho)b_{01} - b_{12} - b_{13}]$$

$$= \frac{1}{Nv^2}\delta_1\gamma_2\beta_N^*[\tilde{K} + (1+\rho)b_{01} + \frac{1}{1+\rho}\{\tilde{K} + (1+\rho)^2 b_{02} + \frac{1}{1+\rho}(\tilde{K} + (1+\rho)^3 b_{03})\}]$$

$$\pi_1 = \frac{1}{N\alpha_{\pi M}}\delta_1\gamma_2\beta_N^*[\tilde{K} + (1+\rho)b_{01} + \frac{1}{1+\rho}\{\tilde{K} + (1+\rho)^2 b_{02} + \frac{1}{1+\rho}(\tilde{K} + (1+\rho)^3 b_{03})\}]$$

$$\tilde{g} - g_1 = \frac{1}{N\alpha_{gS}}\delta_1\gamma_2\beta_N^*[\tilde{K} + (1+\rho)b_{01} + \frac{1}{1+\rho}\{\tilde{K} + (1+\rho)^2 b_{02} + \frac{1}{1+\rho}(\tilde{K} + (1+\rho)^3 b_{03})\}] \tag{30}$$

where $\delta_1 \equiv \dfrac{1+\rho}{1+\gamma_2\beta_S(1+\rho)^2 N_F^*/N}$

Therefore, we obtain the value of the loss function as follows

$$V_1^{FVI} \equiv \frac{1}{2}\frac{N_F^*}{N^2}[\tilde{K} + (1+\rho)b_{01} - b_{12} - b_{13}]^2$$
$$+ \frac{1}{2}\beta_S\frac{N_F^*}{N^2}\gamma_2[\tilde{K} + b_2 - \frac{1}{1+\rho}\{\tilde{K} + (1+\rho)^3 b_{03} + (1+\rho)^2 b_{13}\}]^2$$

$$= \frac{1}{2}\frac{N_F^*}{N^2}\delta_1^2\{(\gamma_2\beta_N^*)^2 + \beta_S\}[\tilde{K}\left(1+\frac{1}{1+\rho}+\frac{1}{(1+\rho)^2}\right) + (1+\rho)(b_{01} + b_{02} + b_{03})]^2 \tag{31}$$

4.4.2 The normal case under confidence crisis (Case VII)

Next, we consider a situation that confidence crisis occurs. Under this situation, the government cannot newly issue any bond. The fiscal and monetary authorities minimize their loss functions in consideration of situation in the second and third period. Thus the authorities in the second period have the following loss functions,

$$V_1^a = \frac{1}{2}[\alpha_{\pi a}\pi_1^2 + (x_1 - \tilde{x})^2 + \alpha_{gS}(g_1 - \tilde{g})^2] + \beta_S[\min\{V_2^{aIV}, V_2^{aV}\}] \tag{32}$$

where $a = F, M$

V_2^{aIV} denotes the value of loss function in Case V in the third period. V_2^{FIV} is defined as (21), and V_2^{MIV} is obtained by assigning (17-1) and (20) to (16). V_2^{aV} is set likewise.

The monetary authority minimizes (32) regardless of the government budget constraint, taking policies selected by the fiscal authority and inflation expectation as given. Since this situation is the same as Case VI, the condition with respect to $\{\pi_1\}$ in this case is (29-1).

The fiscal authority minimizes its loss functions, subject to the following budget constraint.

$$\tilde{K} + (1+\rho)b_{01} = (\tau_1 + \frac{\tilde{x}}{vz}) + \kappa\pi_1 + (\tilde{g} - g_1) \tag{8-1'}$$

The government decides policies to satisfy the following conditions,

$$v(\tilde{x} - x_1) = \alpha_{gS}(\tilde{g} - g_1) \tag{33}$$

Therefore, we obtain the value of the loss function in this case as follows

$$V_1^{FVII} \equiv \frac{1}{2}\frac{N_F^*}{N^2}[\tilde{K} + (1+\rho)b_{01}]^2 + \beta_S[\min\{V_2^{aV}, V_2^{aVI}\}]$$

If $V_2^{aIV} \le V_2^{aV}$,

$$V_1^{FVII(IV)} \equiv \frac{1}{2}\frac{N_F^*}{N^2}[\tilde{K} + (1+\rho)b_{01}]^2 + \frac{1}{2}\frac{N_F^*}{N^2}\beta_S[\tilde{K} + b_2]^2 + \frac{1}{2}\frac{H_F^*}{H^2}\beta_S^2[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}]^2$$

$$= \frac{1}{2}\frac{N_F^*}{N^2}[\{\tilde{K} + (1+\rho)b_{01}\}^2 + \beta_S(\tilde{K} + b_2)^2] + \frac{1}{2}\frac{H_F^*}{H^2}\beta_S^2[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}]^2 \tag{34-1}$$

If $V_2^{aV} < V_2^{aIV}$,

$$V_1^{FVII(V)} \equiv \frac{1}{2}\frac{N_F^*}{N^2}[\tilde{K} + (1+\rho)b_{01}]^2 + \frac{1}{2}\frac{H_F^*}{H^2}\beta_S[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}]^2 + \frac{1}{2}\frac{N_F^*}{N^2}\beta_S^2\tilde{K}^2$$

$$= \frac{1}{2}\frac{N_F^*}{N^2}[\{\tilde{K} + (1+\rho)b_{01}\}^2 + \beta_S^2\tilde{K}^2] + \frac{1}{2}\frac{H_F^*}{H^2}\beta_S[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}] \tag{34-2}$$

4.4.3 The case of default under confidence crisis (Case VIII)

Also, we discuss the situation that the government in the first period triggers a debt default under confidence crisis. Under this situation, the government cannot newly issue an one-period bond.

The fiscal and monetary authorities minimize their loss functions in consideration of situation in the third period. If once the government defaults on payments in the second period, however, the government has no debt in the second and third period, that is, there is no default in the third period. Thus the authorities in the first period have the following loss functions,

$$V_1^a = \frac{1}{2}[\alpha_{\pi a}\pi_2^2 + (x_1 - \tilde{x})^2 + \alpha_{gS}(g_1 - \tilde{g})^2] + \beta_S V_2^{aIII}\ |_{b_2=0, b_3=0} \qquad (35)$$

where $a = F, M$

Also the production in this situation is determined by (1'). The government budget constraint in the second period becomes as follows,

$$\tilde{K} + \frac{(1-z)\tilde{x}}{vz} = (\tau_1 + \frac{\tilde{x}}{vz}) + \kappa\pi_1 + (\tilde{g} - g_1) \qquad 0 < z < 1 \qquad (8\text{-}1'')$$

The monetary authority minimizes (35) regardless of the government budget constraint, taking policies selected by the fiscal authority and inflation expectation as given. From the first order condition for the choice of $\{\pi_1\}$, we obtain the following condition

$$vz(\tilde{x} - x_1) = \alpha_{\pi M}\pi_1 \qquad (36\text{-}1)$$

The fiscal authority minimizes its loss function (35), subject to (8-1"). The authority sets policies to satisfy the following conditions:

$$vz(\tilde{x} - x_1) = \alpha_{gS}(\tilde{g} - g_1) \qquad (36\text{-}2)$$

From the above conditions (1'), (36-1, 2), and the government budget constraint (8-1"), the following relations are held under the rational expectations formation of the private sector ($\pi^e_1 = \pi_1$)

$$\pi_1 = \frac{1}{H\alpha_{\pi S}}[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}]$$

$$\tau_1 + \frac{\tilde{x}}{vz} = \frac{1}{Hv^2z^2}[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}]$$

$$\tilde{g} - g_1 = \frac{1}{H\alpha_{gS}}[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}] \qquad (37)$$

Therefore, we obtain the value of the loss function in this case as follows

$$V_1^{FVIII} \equiv \frac{1}{2}\frac{H_F^*}{H^2}[\tilde{K} + \frac{(1-z)\tilde{x}}{vz}]^2 + \frac{1}{2}\beta_S\frac{N_F^*}{N^2}\gamma_2\left(1 + \frac{1}{1+\rho}\right)^2\tilde{K}^2 \qquad (38)$$

4.4.4 Welfare comparison between Case VII and Case VIII

Dose the fiscal authority trigger a debt default under no confidence crisis? It depends the value of the loss function of the fiscal authority in each case.

If $V_1^{FVIII} \geq \min\{V_1^{FVII(IV)}, V_1^{FVII(V)}\}$, the government does not trigger a default in the first period. On the other hand, in the case of $V_1^{FVIII} < \min\{V_1^{FVII(IV)}, V_1^{FVII(V)}\}$, the fiscal authority has incentives to default on payments in the first period.

The details are as follows. In the case of $\tilde{K} > \frac{N\tilde{x}vz}{1+z}$ and $D > \tilde{K}^2$, $V_2^{FVIII} < V_2^{FVII(V)}$ is satisfied under conditions (25), (27) and the following condition

$$b_{01} > \sqrt{\beta_S\left\{\gamma_2\left(1+\frac{1}{1+\rho}\right)^2 - \beta_S\right\}\tilde{K}^2 + (1-\beta_S)D} - \tilde{K}$$

$$\text{if } \gamma_2 > \frac{(1+\beta_S^2)\tilde{K}^2 - (1-\beta_S)D}{\beta_S\tilde{K}^2\left(1+\frac{1}{1+\rho}\right)^2} \qquad (39)$$

Or $V_2^{FVIII} < V_2^{FVII(IV)}$ is satisfied under conditions (25'), (26) and the following conditions

$$b_0 > (1-\beta_S^2)D - \left[\beta_S\left\{\gamma_2\left(1+\frac{1}{1+\rho}\right)^2 - 1\right\} - 1\right]\tilde{K}^2$$

$$\text{if } \gamma_2 > \frac{(1+\beta_S)\tilde{K}^2 - (1-\beta_S^2)D}{\beta_S\tilde{K}^2\left(1+\frac{1}{1+\rho}\right)^2} \qquad (40)$$

where $b_0 \equiv (1+\rho)^2 b_{01}{}^2 + \beta_S(1+\rho)^4 b_{02}{}^2 + 2\tilde{K}\{(1+\rho)b_{01} + \beta_S(1+\rho)b_{02}\}$

These conditions suggest that the government has an incentive to default when the amount of debt outstanding is more than a certain level.

4.4.5 Welfare comparison between Case VI and Case VII or Case VIII

Whether the confidence crisis occurs or not in the first period depends on welfare loss of the fiscal authority in each case. If the government defaults on payments, investors of the government bonds face losses. Thus, they do not buy the bonds at all when they expect that the government trigger a debt default in any periods.

If $V_2^{FVI} \leq \min\{V_2^{FVII(IV)}, V_2^{FVII(V)}, V_2^{FVIII}\}$, the fiscal authority does not have any incentives to default in each period. Hence, investors can purchase the government bonds. We further analyze this situation.

If $\min\{V_2^{FVII(IV)}, V_2^{FVII(V)}, V_2^{FVIII}\} = V_2^{FVII(V)}$, $V_2^{FVI} < V_2^{FVII(V)}$ is satisfied under conditions (25), (27) and the following conditions

$$0 < b_{01} < \sqrt{\beta_S\left\{\gamma_2\left(1+\frac{1}{1+\rho}\right)^2 - \beta_S\right\}\tilde{K}^2 + (1-\beta_S)D} - \tilde{K}$$

$$\text{if } \gamma_2 > \frac{(1+\beta_S^2)\tilde{K}^2 - (1-\beta_S)D}{\beta_S\tilde{K}^2\left(1+\frac{1}{1+\rho}\right)^2} \qquad (39')$$

and

$$b_{01} > \frac{1}{1-\gamma_1}\left[\gamma_1\left\{\tilde{K}\left(1+\frac{1}{1+\rho}+\frac{1}{(1+\rho)^2}\right) + (1+\rho)(b_{02}+b_{03})\right\} - \tilde{K} - \sqrt{\gamma_1\left\{(1+\rho)(b_{02}+b_{03}) + \tilde{K}\left(\frac{1}{1+\rho}+\frac{1}{(1+\rho)^2}\right)\right\}^2 - (1-\gamma_1)(\beta_S^2\tilde{K}^2 + \beta_S D)}\right]$$

$$\text{if } b_{02}+b_{03} > \frac{1}{1+\rho}\left\{\sqrt{\frac{\beta_S D}{\gamma_1} + \frac{(1+\beta_S^2)\tilde{K}^2}{\gamma_1}} - \left(1+\frac{1}{1+\rho}+\frac{1}{(1+\rho)^2}\right)\tilde{K}\right\}$$

$$\text{and } D > \frac{\tilde{K}^2}{\beta_S\gamma_1}\{1-(1+\beta_S^2)\gamma_1\} \qquad (41)$$

where $\gamma_1 \equiv \delta_1{}^2\{(\gamma_2\beta_N{}^*)^2 + \beta_S\}$.

If $\min\{V_2^{FVII(IV)}, V_2^{FVII(V)}, V_2^{FVIII}\} = V_2^{FVII(IV)}$, $V_2^{FVI} < V_2^{FVII(IV)}$ is satisfied under conditions (25'), (26) and the following conditions

$$b_0 < (1-\beta_S^2)D - \left[\beta_S\left\{\gamma_2\left(1+\frac{1}{1+\rho}\right)^2 - 1\right\} - 1\right]\tilde{K}^2$$

$$\text{if } \gamma_2 > \frac{(1+\beta_S)\tilde{K}^2 - (1-\beta_S^2)D}{\beta_S\tilde{K}^2\left(1+\frac{1}{1+\rho}\right)^2} \qquad (40')$$

and

$$b_0 > \gamma_1\left\{(1+\rho)(b_{01}+b_{02}+b_{03}) + \left(1+\frac{1}{1+\rho}+\frac{1}{(1+\rho)^2}\right)\tilde{K}\right\}^2 - (1+\beta_S)\tilde{K}^2 - \beta_S^2 D$$

$$\text{if } \gamma_1 < \frac{(1+\beta_S)\tilde{K}^2 + \beta_S^2 D}{(1+\rho)(b_{01}+b_{02}+b_{03}) + \left(1+\frac{1}{1+\rho}+\frac{1}{(1+\rho)^2}\right)\tilde{K}^2} \qquad (42)$$

If $\min\{V_2^{FVII(IV)}, V_2^{FVII(V)}, V_2^{FVIII}\} = V_2^{FVIII}$, $V_2^{FVI} < V_2^{FVIII}$ is satisfied under conditions {(25), (27), (39)} or {(25'), (26), (40)} and the following conditions

$$0 < b_{01} + b_{02} + b_{03} < \frac{1}{1+\rho}\left\{\sqrt{\frac{D}{\gamma_1} + \frac{\gamma_2\beta\tilde{K}^2}{\gamma_1}\left(1+\frac{1}{1+\rho}\right)^2} - \left(1+\frac{1}{1+\rho}+\frac{1}{(1+\rho)^2}\right)\tilde{K}^2\right\}$$

$$\text{if } \gamma_1 < \frac{\left\{D + \gamma_2\beta_S\tilde{K}^2\left(1+\frac{1}{1+\rho}\right)^2\right\}}{\left(1+\frac{1}{1+\rho}+\frac{1}{(1+\rho)^2}\right)\tilde{K}^2}(1+\rho)^2 \quad (43)$$

4.5 Intuitions of the analysis and policy implications

According to conditions (15), (25) and (39-43), the fiscal authority has an incentive to default when the amount of debt outstanding is more than a certain level. Expecting the debt default, the investors do not buy the public bonds at all. The public bonds, therefore, cannot be sold when the issuance leads the amount of debt outstanding to be more than the certain level. In this respect, the fiscal authority has to take into account the upper limit of stocks of public debt.

This possibility of debt default provides the fiscal authority to issue public bonds strategically in the first period. Suppose that fiscal authority, in the first period, issues public bonds to be paid in the third period. The amount of issuance is, in addition, supposed to set to the extent that fiscal authority has to raise tax rate to finance the government spending and/or cut the government spending itself in the second period because the additional debt issuance is limited due to the possibility of the default in the third period.

This strategic behavior of fiscal authority induces the monetary authority, in the second period, to boost output and raise seigniorage revenues to eliminate the distortion of resource allocation due to the limitation on debt issuance. Therefore, the monetary policy in the second period suffers from an inflation bias from the ax ante point of view. Expecting the future monetary policy, the fiscal authority has an incentive to issue more public bonds strategically in the first period because it will lead the fiscal authority to the advantageous position in the game played in the second period. This strategic behavior of the fiscal authority results in the distortion of the resource allocation.

There are two ways to eliminate this distortion. One of them is to make the monetary authority more conservative than society in the sense that the price stability weight of monetary authority is higher than that of society. If the monetary authority is conservative enough not to raise inflation depending passively on the strategic accumulation of public bonds, the fiscal authority does not engage in the strategic accumulation of debt in the first period. Consequently the central bank should be more conservative to eliminate the distortion due to the strategic behavior of fiscal authority.

The other way of eliminating the distortion of the resource allocation is to design an institutional ceiling on the debt issuance. This institutional framework eliminates directly the distortion stemmed from the strategic behavior of the fiscal authority. This institutional framework does not work effectively if the fiscal authority has not issued public bonds to the extent that the amount of debt outstanding is close to the critical level of debt default. It is therefore certain that the institution might not be necessary for many countries, but it may provide a binding constraint of the public bond issuance for the fiscal authority of Japan because it has accumulated the debt outstanding much more than other countries.

5. Conclusion

If the expansionary trend in Japan's government spending continues at this pace, the fiscal deficit will inflate further and the ability to raise taxes in the future will be politically limited. Investors will lose confidence in Japan's public bonds if they believe that the nation's public finance is bound for long-term crisis. The result is that interest rates will rise and fiscal failure will become a more tangible reality.

This paper has analyzed sustainability issues of Japan's fiscal policy and then discussed the debt management policy based on the maturity structure of government bonds using theoretical models and numerical studies. We also investigated the desirable coordination of fiscal and monetary authorities toward fiscal reconstruction. The partial default attempt by the fiscal authority is not always effective in the long run because the gross rate of return on debt is adjusted to offset changes in the size of default. The size of partial default does not matter once the reform has been anticipated by the private sector. Inflationary taxes by the central bank will not have such an offsetting effect. Thus, the emergency reform by the monetary authority may well be better than the emergency reform by the fiscal authority so as to avoid bankruptcy.

Given the present maturity structure of bonds, Japan's debt policy should be based on the smoothing rule as follows. As the termination date concentrates within a ten-year period, it is preferable not to raise taxes or the inflation rate or to cut expenditures in this period only but to reserve the fiscal resources to a certain extent even if the reliance on government bonds will rise temporarily. At the same time, it is necessary not to cut taxes or increase expenditures much but to reduce reliance on the government bonds gradually when there are relatively a few expiring bonds. Moreover, it is indispensable to restrain the increasing trend of reliance on government bonds.

For a country with large stocks of public debt, the fiscal authority has an incentive to issue public bonds strategically. This strategic behavior distorts the monetary authority to increase inflation. To eliminate this distortion, an institutional ceiling on the debt issuance is one of the effective policy tools to be considered.

Considering the problems that could arise from delays, a fiscal reconstruction program should be implemented as soon as possible, just as reform of the system. Finally, it should be noted that a successful outcome of fiscal reconstruction may increase overall political support for the drastic fiscal reforms.

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Figure 1

The general account of the central government

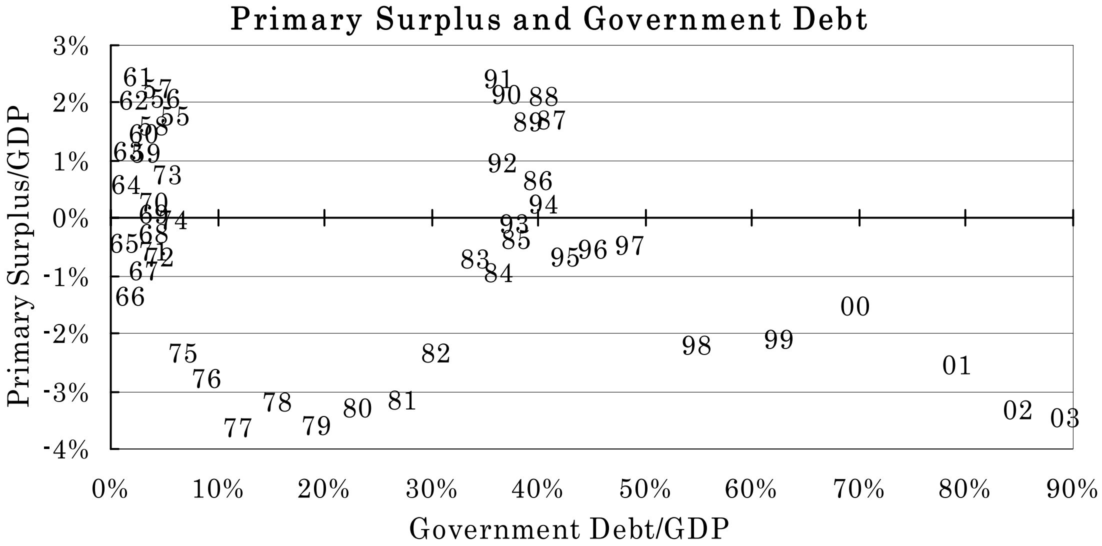


Figure 2

Central and local governments

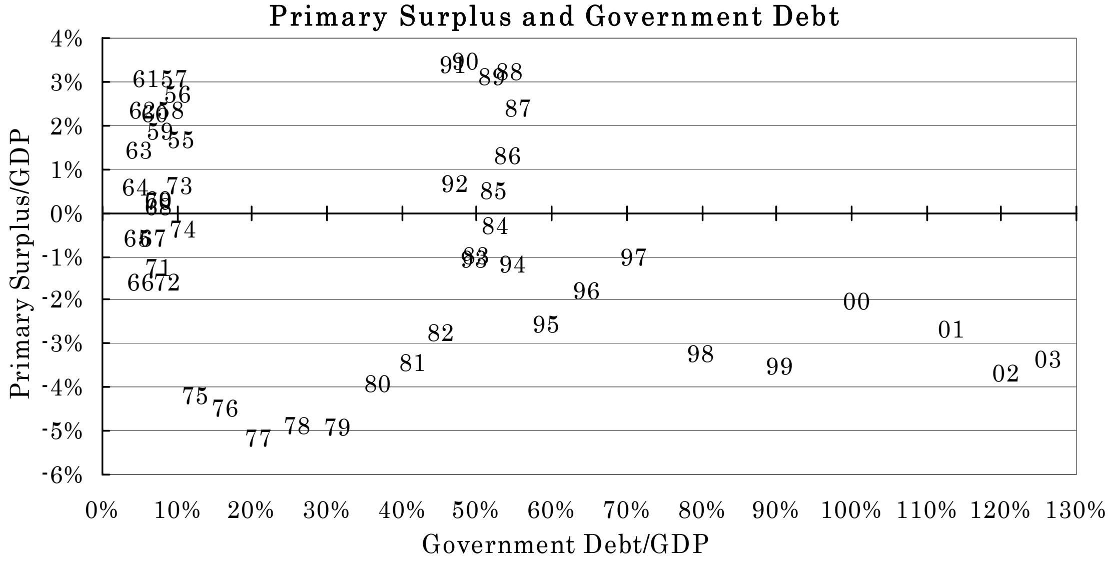


Figure 3

Maturity Structure of Government Bonds

Outstanding Basis

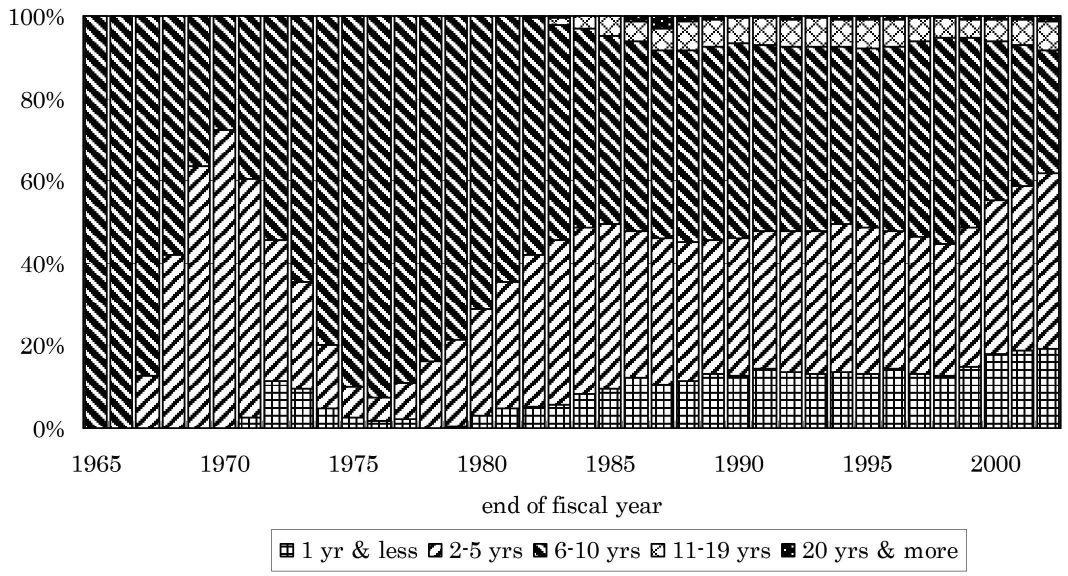
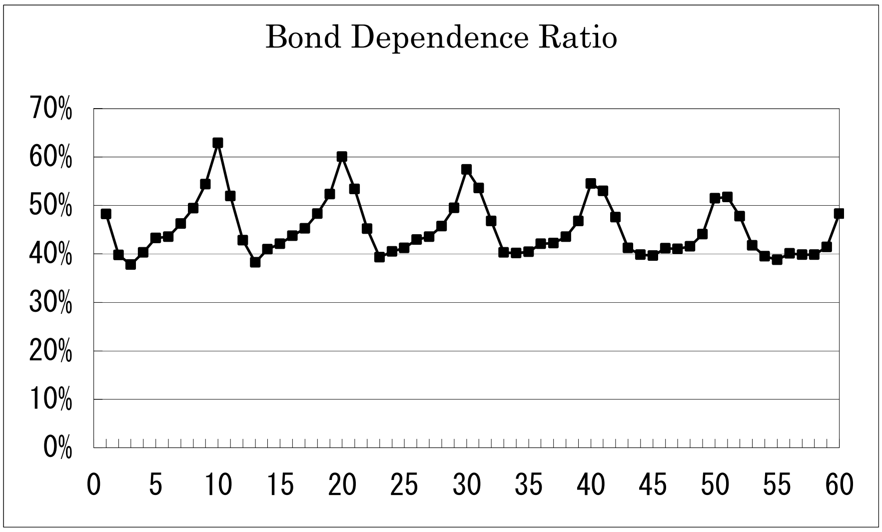


Figure 4

Result of Numerical Analysis

βs	1+ρ	η	κ	απs	αgs
0.985	1.02	0.7	0.36	10	10



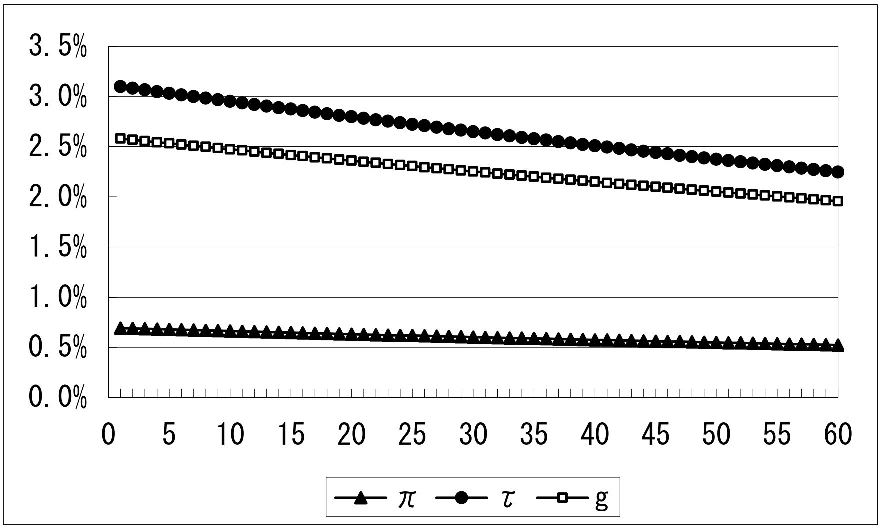
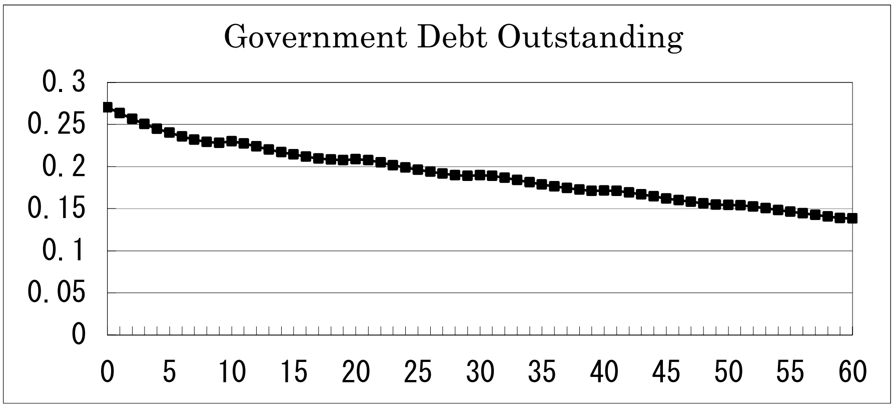


Figure 5
Result of Numerical Analysis (Cont'd)



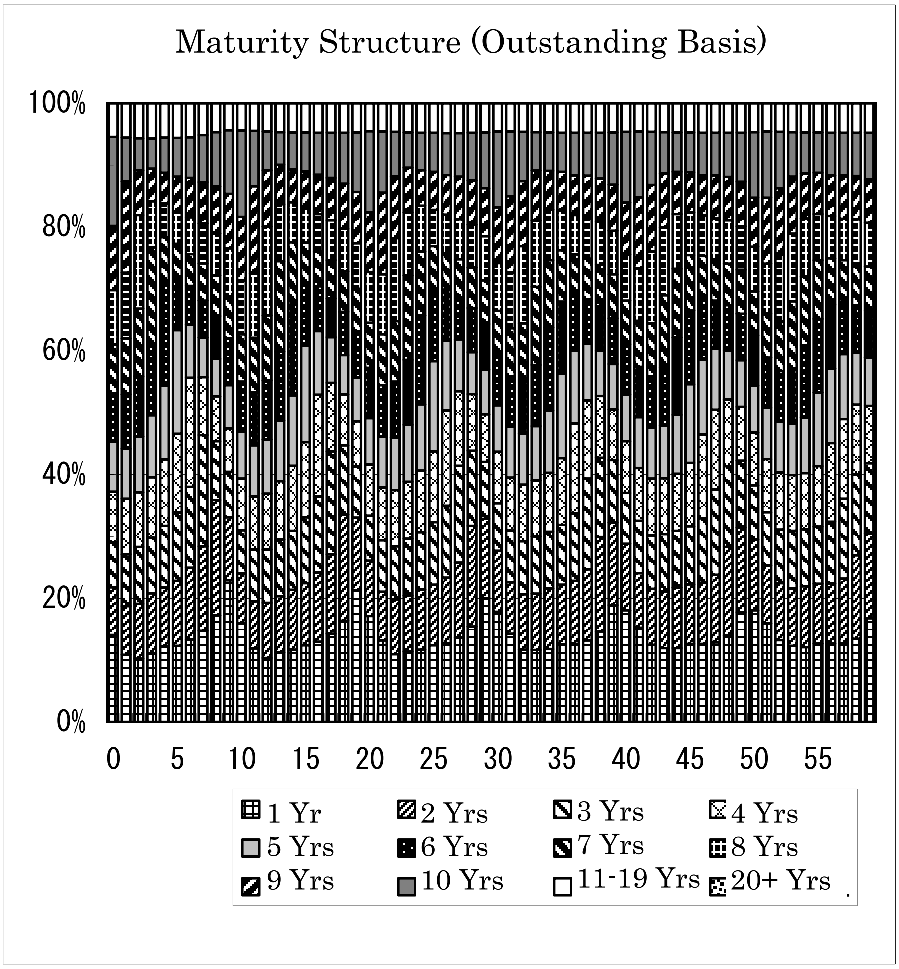


Table 1
Japanese Government Bonds
Classification by Issuance Methods and Maturity

(Billion yen)

	Planned Issuance for FY2003 Initial Budget (a)	Planned Issuance for FY2004 Initial Budget (b)	(b) - (a)
30-year Bonds	1,600.0	2,000.0	400.0
20-year Bonds	4,800.0	6,900.0	2,100.0
15-year Bonds	5,500.0	6,000.0	500.0
10-year Bonds	22,800.0	22,800.0	-
5-year Bonds	22,800.0	22,900.0	100.0
2-year Bonds	20,960.0	19,250.5	-1,709.5
Treasury Bills	34,170.9	34,170.9	-
Inflation-Indexed Bonds	100.0	600.0	500.0
Total Amount in The Market	112,730.9	114,621.4	1,890.5
(excluding Treasury Bills)	(78,560.0)	(80,450.5)	(1,890.5)
Postal Savings	2,100.0	2,300.0	200.0
Bank of Japan	6,441.9	13,219.3	6,777.4
Fiscal Loan Fund	400.0	1,000.0	600.0
Fiscal Loan Fund Special Account Bonds (transitional measures)	18,550.0	29,600.0	11,050.0
Postal Savings	9,960.0	19,700.0	9,740.0
Pension Reserves	5,650.0	7,500.0	1,850.0
Postal Life Insurance	2,940.0	2,400.0	-540.0
Total Amount in Public Sector	27,491.9	46,119.3	18,627.4
JGB for Individual Investors	1,200.0	1,600.0	400.0
TOTAL	141,422.8	162,340.7	20,917.9

Note
1: Figures may not sum up to the total because of rounding.
2: The amount of buy-backs will be approximately 1,000.0 billion yen in FY2003, and approximately 2,000.0 billion yen in FY2004.
3: Figure in "Bank of Japan" in "Planned Issuance for FY2004 Initial Budget" includes 400.0 billion yen of refunding bonds to be issued for the same amount of buy-backs from the Bank of Japan.
4: Figure in "Fiscal Loan Fund" indicates refunding bonds to be issued for the same amount of buy-backs from the Fiscal Loan Fund. At the Fiscal Loan Fund Special Account, funds for the Fiscal Investment and Loan Program are managed.
5: 20-year bonds issuance in FY2003 will be increased by 600.0 billion yen, to meet requests from market participants, which will be transferred within refunding bonds (ie.

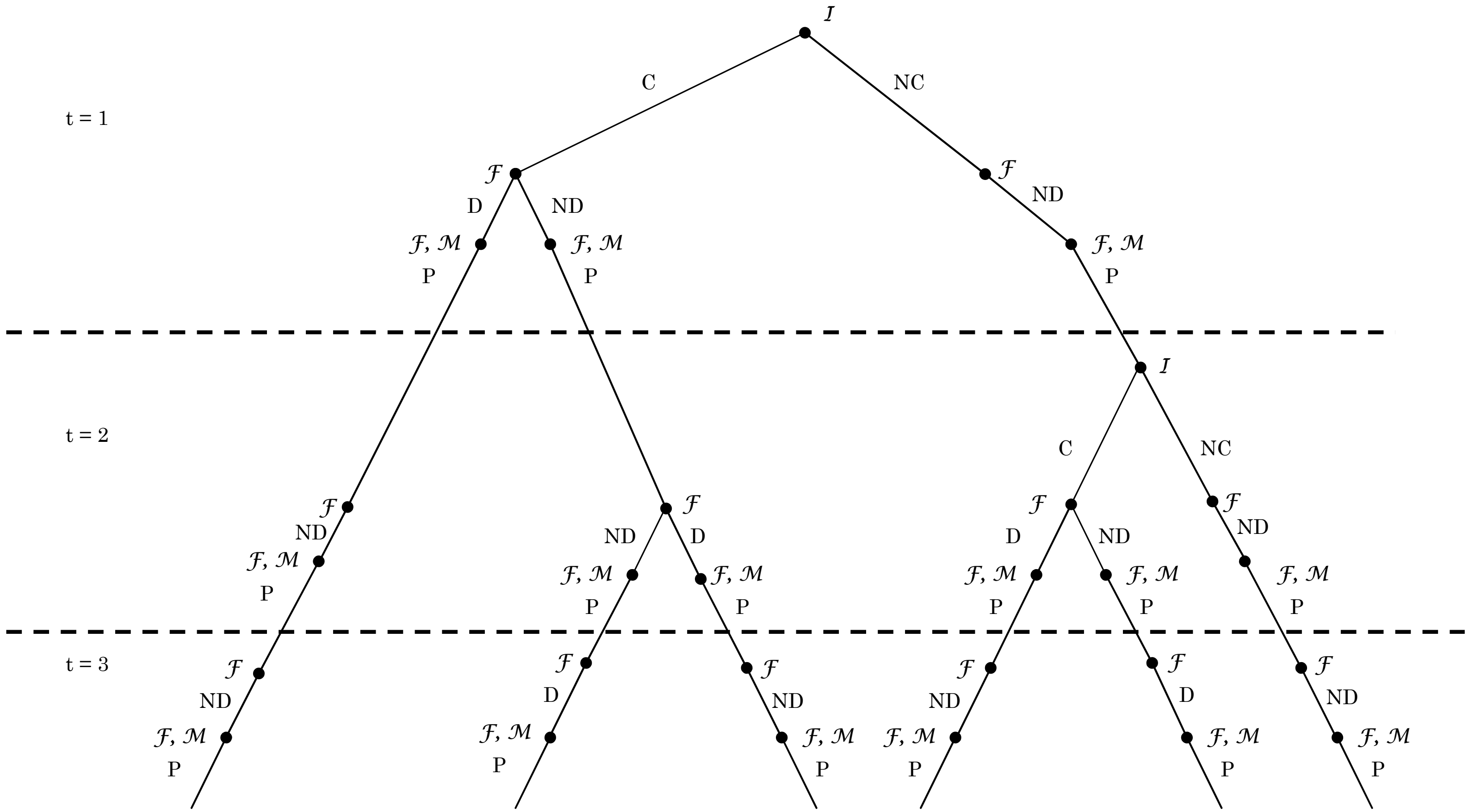
2-year bonds issuance will be reduced by the same amount.).
6: The issue amount of JGB for Individual Investors in FY2003 will exceed its initially planned amount (1,200.0 billion yen in "Planned Issuance for FY2003 Initial Budget").
7: For New Financial Resource Bonds in the FY2003 Supplementary Budget, the Construction Bonds issuance will be increased by 273.0 billion yen and the Special Deficit-Financing Bonds issuance will be reduced by 273.0 billion yen from its initially planned amount shown in "Planned Issuance for FY2003 Initial Budget".

Source: Ministry of Finance “Planned Bond Issuance for FY 2004”

Table 2
Maturity Structure of Japanese Government Bond

	Issuance FY 1998	FY 1987	FY 2000	Outstanding FY 2001
Average Maturity	7.22	10.15	5.02	5.35
Years				
1	24.91%	8.15%	37.00%	18.71%
2	3.55%	5.65%	16.76%	11.58%
3	0%	4.53%	0.37%	6.92%
4	5.71%	1.18%	3.08%	10.87%
5	0.34%	2.53%	12.35%	10.66%
6	5.85%	0%	3.08%	8.14%
7	0%	0%	0%	10.16%
8	0%	0%	0%	5.94%
9	0%	0%	0%	5.58%
10	56.25%	59.46%	20.09%	4.50%
11	0%	0%	0%	0.43%
12	0%	0%	0%	0.27%
13	0%	0%	0%	0.41%
14	0%	0%	0%	1.06%
15	0%	0%	3.45%	1.62%
16	0%	0%	0%	0.58%
17	0%	0%	0%	0.50%
18	0%	0%	0%	0.48%
19	0%	0%	0%	0.74%
20	3.39%	18.50%	2.96%	0.43%
21	0%	0%	0%	0%
22	0%	0%	0%	0%
23	0%	0%	0%	0%
24	0%	0%	0%	0%
25	0%	0%	0%	0%
26	0%	0%	0%	0%
27	0%	0%	0%	0%
28	0%	0%	0%	0.13%
29	0%	0%	0%	0.19%
30	0%	0%	0.86%	0.08%

Figure 6



NC: non confidence crisis, C: confidence crisis, D: default, ND: non default, P: policy choice
$\mathcal{F}$: fiscal authority, $\mathcal{M}$: monetary authority, $\mathcal{I}$: investors