

Pareto Improving Social Security Reform when Financial Markets are Incomplete!?^{*}

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Abstract

This paper studies an Overlapping Generations model with stochastic production and incomplete markets to assess whether the introduction of an unfunded social security system can lead to a Pareto improvement. When returns to capital and wages are imperfectly correlated, a social security system that endows retired households with a claim to labor income is an effective tool to share aggregate risk between generations. Our quantitative analysis first shows that, abstracting from the crowding-out effect on the aggregate capital stock, the introduction of social security represents a Pareto improving reform if households are fairly risk-averse *and* fairly willing to intertemporally substitute consumption. Second, the severity of the crowding-out effect in general equilibrium overturns these gains for degrees of risk aversion and intertemporal elasticity of substitution commonly used. Third, our results are robust to the introduction of a low-risk, privately traded bond, as long as the bond return is calibrated to the average empirical long-run return on short-term risk-free debt.

Keywords: Social Security Reform, Intergenerational Risk Sharing, Aggregate Fluctuations, Incomplete Markets

JEL Classification: H55, D91, H31, E62, D58

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1 Introduction

Should the government provide the elderly with a tax-financed, low-risk pension or should each individual be allowed to save for her own retirement consumption in potentially risky privately traded assets? This question was one of the most controversial issues in the 2000 presidential campaign; 2004 appears to be no different.¹ Whereas the proponents of a partial privatization in the Bush camp primarily point to higher *average* returns that can be earned by privately investing in stocks, the opponents on the Democrat side stress the *risk* of low returns to savings for entire generations due to large, unfavorable and persistent aggregate shocks.²

The current US pay-as-you go (PAYG) social security system was introduced in 1935, partially as a response to the great depression, the biggest negative aggregate shock the US economy has experienced so far. In this paper we ask whether the introduction of an unfunded social security system – which re-allocates the impact of aggregate shocks across generations and thus reduces the consumption risk in old age – provides a Pareto improving policy reform (that is, provides a welfare improvement for all generations then alive and for generations to be born into all future states of the world). We will show that the answer to this question depends on the quantitative importance of this positive *intergenerational* risk sharing effect, in comparison to the negative effects from forcing households to implicitly save in a low-return asset and from a declining aggregate capital stock. Choosing the starting point of our thought experiment as the economy without social security (that is, the US just after the great depression) allows us to analyze, from a normative perspective, the risk-return trade-off between social security and private assets, which is at the center of the current reform debate, without having to take a stand on how a potential transition from the current to a partially privatized system has to be financed.³

¹Among many others, see the articles by Michael Barone in USNews on 9/8/03 and by Richard Stevenson in the New York Times on 8/30/03, predicting that George W. Bush will revive his individual account proposal for social security in the 2004 campaign.

²To quote from Al Gore's speech in Kissimmee, FL, Nov. 1, 2000: "Instead of a system where everyone is in together, the Bush plan would turn Social Security into a grab bag where everyone is out for himself. You might call it social insecurity." On the same issue, George Bush "We trust our individual workers [...] we'll allow younger workers at their choice to invest some of their own money in the private markets to get a better return" (speech in St.Charles, MO, Nov. 2, 2000). An academic discussion of this debate is contained in Aaron et al. (2001) or Burtless (2001).

³It is well known that a transition from an unfunded to a funded social security system

How can a social security system lead to enhanced intergenerational risk sharing? As Shiller (1999) and Bohn (1998, 1999) have argued, if returns to capital and wages are imperfectly correlated and subject to aggregate shocks, the consumption variance of all generations can be reduced if private markets or government policies enable them to pool their labor and capital incomes. A social security system that endows retired households with a claim to labor income serves as such an effective tool to share aggregate risk between generations, in the absence of privately traded assets that achieve the same goal. The idea that market failures and missing asset markets might give a normative justification for a PAYG public retirement plan dates back at least to Diamond (1977). He points out that the absence of certain investment opportunities may lead to inefficient risk allocations. Merton (1983) analyzes the economic inefficiencies caused by the non-tradeability of human capital in an overlapping generations model with stochastic production and suggests that the present social security system may help to eliminate these.

While incomplete financial markets can provide a rationale for social security, it is also well known that in a general equilibrium model a PAYG social security system crowds out private savings and thus capital formation, and therefore leads to lower wages for future generations.⁴ These two effects have opposite impacts on agents' welfare and only a careful *quantitative* analysis can reveal which of the two dominates. In this paper we undertake such an analysis.

Our economy is populated by nine overlapping generations that face stochastic, imperfectly correlated wages and returns to capital. Households have a preference for smooth consumption, both over time and across states and can transfer resources across time by purchasing claims to the risky aggregate capital stock. Employing a recursive utility representation as in Kreps and Porteus (1978) and Epstein and Zin (1989) allows us to control risk aversion independently from the willingness to intertemporally substitute consumption. The government administers a pure PAYG social security system by collecting a payroll tax and paying out benefits that vary with stochastic aggregate wages, in order to insure budget balance of the system. With the introduction of such a system, since wages and returns to capital are imperfectly correlated, the government in effect forces households to hold

cannot be Pareto-improving, independent of how outstanding social security benefits have to be honored and financed. See Feldstein and Liebman (2001) and the references cited therein.

⁴See again Feldstein and Liebman (2001) for an elegant survey of the large theoretical and empirical literature studying this crowding-out effect.

a second asset and thus to diversify capital income risk. This risk diversification element is the *only* positive role social security plays in our economy, as we insure (by providing a sufficient theoretical condition that we check computationally in our quantitative exercises) that the equilibrium without social security is not dynamically inefficient in the sense of Samuelson (1958). Therefore social security is *not* beneficial because it cures overaccumulation of capital or leads to better allocation of (average) resources across generations, but *solely* because it enhances *risk sharing* between generations. This beneficial role has to be traded off against its lower average implicit return, compared with other assets, and the crowding-out of physical capital that its introduction induces.

Our quantitative analysis exhibits three main findings. First, abstracting from the crowding-out effect of social security in general equilibrium, the introduction of social security does indeed represent a Pareto improving reform, if (and only if) households are fairly risk-averse *and* fairly willing to intertemporally substitute consumption. This result is obtained even though the return differential between private returns to capital and implicit returns to the social security system amounts to 4.9 percentage points, indicating a strong positive effect of social security on the intergenerational allocation of risk. Second, the severity of the capital crowding-out effect in general equilibrium overturns these gains for degrees of risk aversion and intertemporal elasticity of substitution commonly used in the macroeconomic and public finance literature. However, even in general equilibrium the introduction of social security is a Pareto-improving reform if households are highly risk averse and, in addition, have a very high intertemporal elasticity of substitution or if the excess return of private assets over social security is reduced significantly below that observed in the data. Third, our results are robust to the introduction of a low-risk, privately traded bond with a return that is calibrated to the average empirical return on risk-free (short-term government) debt. For the current reform debate our results imply that, unless American households are thought to be very risk-averse and very willing to accept consumption changes over time, the long-run advantages of higher returns and a higher capital stock from a privatized system more than outweigh risk considerations. In the light of these findings the political discussion should then center around ways to finance the transition and the distribution of the welfare costs associated with this transition.⁵

⁵See Conesa and Krueger (1999) and Fuster et al. (2003) for a discussion of the welfare consequences of different ways to finance this transition.

In the next section we develop a simple, analytically tractable model that aims at formalizing the intuition for the intergenerational risk sharing effect and at providing a back-of-the-envelope calculation of the welfare consequences of social security reform. Section 3 describes the general equilibrium model, relates our paper to the existing theoretical literature and contains the sufficient condition for dynamic efficiency of equilibrium. Section 4 discusses the calibration of the model and Section 5 summarizes our main results, first for a partial equilibrium, then for a general equilibrium version of the model. It also includes sensitivity analysis of our results with respect to crucial preference parameters and the set of privately traded assets. Conclusions are contained in Section 6, with details about theoretical derivations and data used in the paper relegated to the appendix.

2 A Simple Model

2.1 Theory

We now present a simple, two period partial equilibrium model to formalize the intuition from the introduction. Each agent lives for two periods, earns wage w_0 in the first period on which she pays a payroll tax τ . The remainder of her wages is invested into a risky savings technology with stochastic gross return R . In the second period of her life she receives social security payments of $\tau w_0 G$, where G is the stochastic gross return of the social security system. The agent values consumption in the second period of her life, with consumption given by

$$c = (1 - \tau)w_0R + \tau w_0G \quad (1)$$

according to the differentiable utility function $v(c)$. Lifetime utility, as a function of the size of the social security system, is therefore given by

$$U(\tau) = Ev[(1 - \tau)w_0R + \tau w_0G] \quad (2)$$

where $E(\cdot)$ is the expectation with respect to uncertainty realized in the second period of the households' life.

We ask when a marginal introduction of a social security system is welfare-improving, that is, seek necessary and sufficient conditions under which $U'(\tau = 0) > 0$. Under the assumption that $v(c) = \ln(c)$ and that G and R are jointly lognormal this condition reduces to (see the appendix)

$$E\left\{\frac{G}{R}\right\} = \frac{E(G)}{E(R)} \cdot \frac{[cv(R)^2 + 1]}{[\rho_{G,R} \cdot cv(G) \cdot cv(R) + 1]} > 1 \quad (3)$$

where $\rho_{G,R} = Cov(G, R)/[Std(G)Std(R)]$ is the correlation coefficient between G and R and $cv(R) = E(R)/Std(R)$ is the coefficient of variation of the risky savings returns, with $cv(G)$ defined accordingly.

From (3) we see that the introduction of a marginal social security system is welfare improving if the implicit expected return to social security, $E(G)$, is sufficiently large relative to the return on the risky saving technology, $E(R)$. Even if the latter is substantially larger than the former, the introduction of social security may still be justified if the stochastic saving returns are sufficiently volatile ($cv(R)$ sufficiently big) or the correlation between private saving returns and returns to social security sufficiently small (or negative). We will calibrate our general equilibrium model exactly to these statistics from the data which this simple model has pointed to as crucial in determining the welfare consequences of social security.

For a general CRRA utility function $v(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ and without any distributional assumptions on (G, R) condition (3) can be generalized to

$$E\left(\frac{G-R}{R^\sigma}\right) = E\left(\frac{G}{R^\sigma}\right) - E\left(R^{1-\sigma}\right) > 0 \quad (4)$$

With appropriate data on private returns to saving R and returns to the social security system G equation (4) can be used to provide a first quantitative assessment whether the introduction of a (small) social security system is justified on the grounds of a better risk allocation, abstracting from intertemporal consumption and general equilibrium effects. It also provides an estimate of the degree of risk aversion required for this argument to work.

2.2 Empirical Implementation

We map the gross returns R into returns to the S&P 500 from Shiller (1989), and the gross return to social security G into the gross growth rate of real total compensation of employees from NIPA, provided by the Bureau of Economic Analysis (BEA). Details are contained in the appendix.

A key question is what time interval in the data corresponds to a model period. The data is available in yearly frequency; since in our simple model agents live for two periods, a model period may more reasonably be interpreted as twenty years. We present results for alternative data frequencies, for annual data, and for data of 18-year frequency.

In Figure 1 we plot, using the data for R and G , condition (4) against the degree of risk aversion σ . We see that for degrees of risk aversion of 2

and higher the introduction of a marginal social security system is beneficial, when the judgement is based on criterion (4).

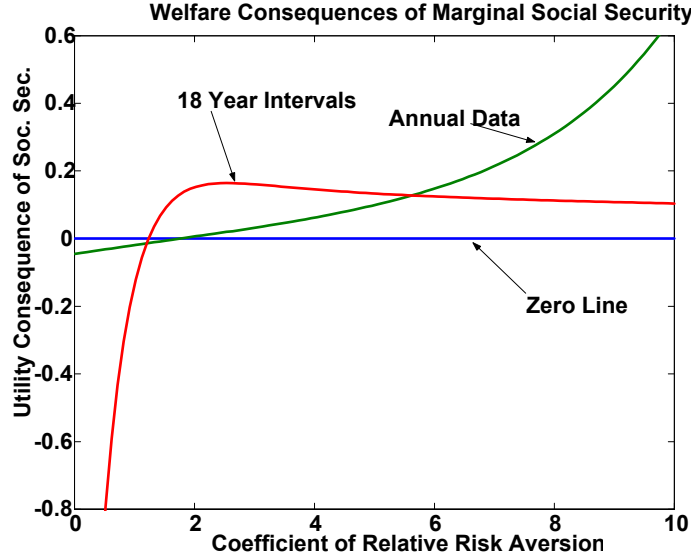


Figure 1: Welfare Consequences of Social Security Reform

The question of whether a better risk allocation induced by the introduction of social security is sufficient to provide a welfare improvement thus becomes a quantitative one. To answer it requires to construct dynamic general equilibrium model, calibrated to data, a task which we turn next to.

3 The General Model

Our model is a straightforward extension of Diamond's (1965) economy to aggregate uncertainty. Time is discrete and extends from $t = 0, \dots, \infty$. Aggregate uncertainty is represented by an event tree. The economy starts with some fixed event z_0 . Each node of the tree is a history of exogenous shocks to the economy $z^t = (z_0, z_1 \dots z_t)$. Let by $\pi_t(z^t)$ denote the probability that the node z^t occurs. We let the notation $z^t \succ z^s$ mean that z^s is a potential successor node of z^s , for $t > s$. The shocks are assumed to follow a Markov chain with finite support \mathcal{Z} and with transition matrix

π . Three commodities are traded, labor, a consumption good and a capital good which can only be used as an input to production.

3.1 Demographics, Endowments and Preferences

The economy is populated by overlapping generations of agents that live for nine periods.⁶ The population growth rate is given by n . In each period t , $L_t = (1 + n)L_{t-1}$ identical new households are born. $L_0 = 1$ denotes the number of newborns in period 0. A household is fully characterized by the node in which she is born (z^t). When there is no ambiguity we index them simply by their date of birth.

An agent born at node z^s has non-negative, deterministic labor endowment over her life-cycle, (l^0, l^1, \dots, l^8) . The price of the consumption good at each date event is normalized to one and at each date event z^t the household supplies her labor endowment inelastically for a market wage $w(z^t)$.

Let by $c^s(z^t)$ denote the consumption of an agent born at time s in period $t \geq s$ and by $U^s(c, z^t)$ the expected continuation utility of an agent born in node z^s from node $z^t \succ z^s$ onwards. An agent born at node z^s therefore has expected lifetime utility from allocation c given by $U^s(c, z^s)$. Individuals have preferences over consumption streams representable by the recursive utility function (see Kreps and Porteus (1978) and Epstein and Zin (1989))

$$U^s(c, z^t) = \left\{ \left[c^s(z^t) \right]^\rho + \beta \left[\sum_{z_{t+1}} \pi(z_{t+1}|z_t) \left(U^s(c, z^{t+1}) \right)^\sigma \right]^{\frac{\rho}{\sigma}} \right\}^{\frac{1}{\rho}} \quad (5)$$

where $\frac{1}{1-\rho}$ is the intertemporal elasticity of substitution and $1 - \sigma$ measures the risk aversion of the consumer with respect to atemporal wealth gambles.⁷

⁶This constitutes a compromise between realism and computational feasibility.

⁷We assume $\sigma < 1$ and $\rho < 1, \rho \neq 0$. Note that if $\rho = \sigma$, then households have standard constant relative risk aversion expected utility, with CRRA of $1 - \sigma$, if the final continuation utility function is given by $U^s(c, z^{s+8}) = c^s(z^{s+8})$, which we assume.

For the isoelastic utility case $\rho = 0$ preferences are represented recursively by

$$U^s(c, z^t) = \left[c^s(z^t) \left(\sum_{z_{t+1}} \Pi(z_{t+1}|z_t) U^s(c, z^{t+1})^\sigma \right)^{\frac{\sum_{j=1}^{s+8-t} \beta^j}{\sigma}} \right]^{\frac{1}{\sum_{j=0}^{s+8-t} \beta^j}} \quad (6)$$

It can be shown that the limit of the Euler equations for (5), as $\rho \rightarrow 0$, converge to the

Households have access to a storage technology: they can use one unit of the consumption good to obtain one unit of the capital good next period. We denote the investment of household s into this technology by $a^s(z^t)$. All agents are born with zero assets, $a^s(z^{s-1}) = 0$. We do not restrict $a^s(z^t) \geq 0$, thus allowing households to borrow against future labor income. At time t the household sells its capital goods accumulated from last period, $a^s(z^{t-1})$, to the firm for a market price $1 + r(z^t) > 0$. The budget constraint of household s in period $t \geq s$ therefore reads as

$$c^s(z^t) + a^s(z^t) = (1 + r(z^t))a^s(z^{t-1}) + (1 - \tau)l^{t-s}(z_t)w(z^t) + I(s)b(z^t) \quad (7)$$

where τ is the payroll tax to finance social security payments, $b(z^t)$ are the social security benefits received by a retired agent and $I(s)$ is the indicator function, with $I(s) = 1$ for retired agents and $I(s) = 0$ otherwise.⁸ To start off the economy we assume that in period zero there are $L_0/(1+n)^i$ households of ages $i = 0, \dots, 8$ who enter the period with given capital holdings $a_{-1}^0, \dots, a_{-1}^{-8}$, where by assumption $a_{-1}^0 = 0$

3.2 Firms

There is a single representative firm which in each period t uses labor and capital to produce the consumption good according to a constant returns to scale production function $f_t(K, L; z_t)$. Since firms make decisions on how much capital to buy and how much labor to hire after the realization of the shock z_t they face no uncertainty and simply maximize current profits.⁹

In our quantitative work below we will always use the following parametric form for the production function.

$$f_t(K, L; z_t) = \xi(z_t)K^\alpha \left[(1+g)^t L \right]^{1-\alpha} + K(1 - \delta(z_t)) \quad (8)$$

where $\eta(\cdot)$ is the stochastic shock to productivity and where $\delta(\cdot)$ can be interpreted as the (possibly) stochastic depreciation rate.

Euler equations for (6)

⁸Benefits $b(z^t)$ only depend on the aggregate event history, but not on individual income, whereas in the actual U.S. system benefits do depend on individual labor earnings, although in a fairly progressive fashion. There is also a maximum income level beyond which no further social security contributions are levied. Even though our modelling choice may attribute too much intergenerational risk sharing to the social security system, given the progressive nature of the actual system it provides a reasonable first approximation.

⁹We assume that households cannot convert capital goods back into consumption goods at the beginning of the period. This assumption is necessary to prevent households from consuming the capital at the beginning of the period instead of selling it to the firm in states where the net return to capital is negative.

3.3 Government

The only role the government has in our model is to levy payroll taxes to pay for social security benefits. We model social security as a PAYG system that adheres to period by period budget balance, with size characterized by the payroll tax rate τ . Thus taxes and benefits satisfy

$$\tau w(z^t)L(z^t) = b(z^t)L_t^{ret} \quad (9)$$

where $L(z^t)$ is total labor input at node z^t and $L_t^{ret} = L_0 \sum_{s=age_{ret}}^8 (1+n)^{t-s}$ is the total number of retired people in the economy.

3.4 Markets

In this simple economy the only markets are spot markets for consumption, labor and capital, all of which are assumed to be perfectly competitive.

3.5 Equilibrium and Pareto Efficiency

We will present results for two versions of our model. The first is a standard closed general equilibrium economy, in which all capital used in domestic production is owned by domestic agents. In the second economy part of the productive capital stock may be owned by the rest of the world. We assume that in this economy the total supply of capital for the production process is exogenously fixed at \bar{K}_t , which grows at rate $n + g$ per period, $\bar{K}_t = [(1+n)(1+g)]^t \bar{K}$. We refer to this version of the model either as partial equilibrium or small open economy.

For given initial conditions $z_0, (a_{-1}^s)_{s=-8}^0$ a competitive equilibrium for the closed economy is a collection of choices for households $(c^s(z^t), a^s(z^t))_{t=s}^{s+8}$, for the representative firm $\{K(z^t), L(z^t)\}$, a policy $\{\tau, b(z^t)\}$ as well as prices $\{r(z^t), w(z^t)\}$ such that households and the firm maximize the government budget constraint (9) is satisfied and markets clear: for all t, z^t

$$L(z^t) = (1+n)^t \sum_{s=0}^8 \frac{l^s}{(1+n)^s} \quad (10)$$

$$K(z^t) = (1+n)^t \sum_{s=1}^8 \frac{a^{t-s}(z^{t-1})}{(1+n)^s} \quad (11)$$

$$(1+n)^t \sum_{s=0}^8 \frac{c^{t-s}(z^t)}{(1+n)^s} + K(z^{t-1}) = f_t(K(z^t), L(z^t), z_t) \quad (12)$$

By Walras' law market clearing in the labor and capital market imply market clearing in the consumption goods market for the closed economy. For the small open economy the labor market clearing condition (10) remains the same and the capital market clearing condition now reads as¹⁰

$$K(z^t) = \bar{K}_t \quad (13)$$

Finally, an allocation (c, K) is (ex interim) Pareto efficient if it is feasible and there is no other feasible allocation (\hat{c}, \hat{K}) such that $U^s(\hat{c}, z^s) \geq U^s(c, z^s)$ for all z^s and $U^s(\hat{c}, z^s) > U^s(c, z^s)$ for at least one z^s .

In order to solve for the equilibrium numerically using recursive techniques we de-trend the economy by deterministic population growth and technological progress. Denoting growth-adjusted consumption by \tilde{c} , and other variables accordingly¹¹, the Euler equations from the individuals' optimization problem, the recursive version of which our numerical algorithm will operate on, read as

$$\left[E_{z^t} \left(\tilde{U}_{t+1}^s \right)^\sigma \right]^{\frac{\rho}{\sigma}-1} \tilde{\beta} E_{z^t} \left[\frac{\tilde{c}^s(z^{t+1})}{\tilde{c}^s(z^t)} \right]^{\rho-1} \left(\frac{1+r(z^{t+1})}{1+g} \right) \left(\tilde{U}_{t+1}^s \right)^{\sigma-\rho} = 1. \quad (14)$$

Since each agents' optimization problem is finite-dimensional and convex, these Euler equations are necessary and sufficient for optimal household choices.¹²

3.6 The Thought Experiment

We consider the following thought experiment: Suppose that in an equilibrium of the economy with a payroll tax rate $\tau = 0$ at some event z^t (the US in 1935) there is an unanticipated increase of τ . What are the welfare effects for all individuals born at z^t and at all successor nodes? In order to

¹⁰The goods market clearing condition is not any longer part of the equilibrium definition. Foreign investors claim a fraction $(\bar{K}_t - K_{dom}(z^t))F_K$ of output as capital income, with the rest of output being available for investment into the domestically owned capital stock $K_{dom}(z^t)$ and domestic consumption.

¹¹More precisely, define $\tilde{c}^s(z^t) = \frac{c^s(z^t)}{[1+g]^t}$, $\tilde{\beta} = (1+g)^\rho \beta$ and $\tilde{U}_t^s = \frac{U^s(c, z^t)}{(1+g)^t}$.

¹²In order to compute equilibrium allocations numerically we formulate these Euler equations recursively. We then define and compute a Functional Rational Expectations Equilibrium (FREE), following the approach pioneered by Spear (1988) and adapted to stochastic OLG models by Krueger and Kubler (2003). A FREE is a recursive competitive equilibrium where the policy functions are restricted to smooth functions, defined on a compact ergodic set of endogenous and exogenous variables.

determine whether such a reform improves welfare for all future generations, one needs to compare welfare at infinitely many nodes. In our quantitative work below, we report welfare gains and losses for the next 2-3 periods (corresponding to about 20 years in real time) and verify that the qualitative conclusions remain the same over 4-10 periods. We do not have to consider more periods to make conclusive welfare statements, since the welfare consequences of the reform stabilize after at most 4 periods.

3.7 Dynamic Efficiency and Pareto Efficiency

Since financial markets are incomplete in our model, it is well known that equilibrium allocations are generally suboptimal. When discussing possible risk-sharing benefits of a PAYG social security system we focus on a particular policy intervention. We do *not* argue that a social security system guarantees full efficiency and we do *not* attempt to explain why this particular system is in place. We simply examine if it is Pareto-improving to introduce social security.

Ever since Samuelson (1958) and Diamond (1965) it is well known that overlapping generation models can exhibit Pareto suboptimal equilibria for reasons other than inefficient risk allocation. Even in deterministic exchange economies, transfers from young to old agents which do *not* constitute a new asset with different risk characteristics can be Pareto-improving (dynamic inefficiency). In economies with production a reduction in capital accumulation can lead to Pareto improvement through higher aggregate consumption at all future dates (production inefficiency). Demange (2002) generalizes the notion of dynamic efficiency to economies with uncertainty and incomplete markets. An allocation is called dynamically efficient, if there exists no other allocation in the marketed subspace which constitutes a Pareto improvement. An argument similar to Samuelson (1958) can be used to demonstrate that equilibria in our economies may be dynamically inefficient.

In our quantitative work we focus on inefficient risk sharing across generations (due to *incomplete* markets), rather than on dynamic or production inefficiencies as a source of market failures for the following reason. When returns on available assets are sufficiently high, equilibrium allocations are production efficient and dynamically efficient. When, in addition, markets are sequentially *complete* the equilibrium is Pareto efficient and social security can never be Pareto-improving.¹³ In other words, with high enough

¹³Thus the main arguments and results of our paper do rely on the assumption of incomplete financial markets. This does not mean, however, that, *starting* from a situation

asset returns, the inefficient allocation of risk studied in this paper is the *only* inefficiency that may provide a normative rationale for public transfer programs.

It is a largely empirical question whether one can deduce from asset return that the US economy is dynamically efficient. Abel et al. (1989) provide some supportive evidence. In addition, there is an important theoretical argument for focusing on dynamically efficient economies. An asset that promises to pay a non-negligible fraction of aggregate consumption at each future state of the world (e.g. land, see Demange (2002)) can only have a finite price today if the allocation is dynamically efficient. Therefore equilibria in models with such assets are necessarily dynamically efficient.

However, if markets are incomplete dynamic efficiency does not imply Pareto efficiency. In this paper we therefore want to assess whether in a realistically calibrated economy that is dynamically efficient, the introduction of a social security system is Pareto-improving. For this we need to provide a sufficient and numerically implementable condition on returns to capital that guarantees dynamic efficiency in our model.

For each value of the shock z , define a production function in intensive units $\kappa = \frac{K}{L}$ by

$$\phi(\kappa; z) = \xi(z)F(\kappa, 1) - (1 + \delta(z))\kappa \quad (15)$$

Define a supporting price system $(q(z^t))$ by $q(z_0) = 1$ and

$$E(q(z^t) \frac{\partial \phi(\kappa(z^t), z)}{\partial \kappa} | z^{t-1}) = q(z^{t-1})(1+n)(1+g) \quad (16)$$

Since markets are not sequentially complete, there are several supporting price systems, which we collect in a set \mathcal{Q} . The following proposition (Theorem 1 in Demange (2002)) characterizes dynamically efficient allocations.¹⁴

Proposition 1 *An equilibrium allocation is dynamically efficient if*

$$\lim_{t \rightarrow \infty} \inf_{q \in \mathcal{Q}} E_0 \left(\sum_{s=t}^{t+8} q(z^s) \right) = 0 \quad (17)$$

with incomplete markets and introducing a full set of Arrow securities leads to a Pareto improvement, even though the introduction of social security may lead to a Pareto improvement under the same circumstances. We give examples of this possibility in Krueger and Kubler (2002).

¹⁴Our result extends Demange's (2002) theorem to homothetic economies with population and productivity growth.

The proposition states that it is sufficient for optimality that the *infimum* over all supporting prices tends to zero. Therefore we can verify dynamic efficiency if we find *some* supporting price system that satisfies condition (17).

This condition can obviously not be easily verified since it involves prices ‘at infinity’. Since we focus on equilibria with a compact state space \mathcal{S} , we can give a more useful sufficient condition. For a given time horizon T define the T -period expected discounted present value by

$$R(T) = E_{z_0} \left[\prod_{s=1}^T \frac{(1+n)(1+g)}{1+r(z^s)} \right]. \quad (18)$$

Denote by $\Theta \in \mathcal{S}$ the vector of endogenous state variables (i.e. the aggregate capital stock and the asset holdings of each generation). We have the following

Proposition 2 *A Functional Rational Expectations Equilibrium (FREE) is dynamically efficient if there exists a $T > 0$ such that for all initial conditions $(z, \Theta) \in \mathcal{Z} \times \mathcal{S}$ in the compact state space the resulting equilibrium returns satisfy $R(T) < 1$.*

Proof: By definition of a FREE all $z_T, \Theta(z^T)$ will lie in $\mathcal{Z} \times \mathcal{S}$ themselves and can be viewed as initial conditions as well. Therefore it follows that $R(iT) \rightarrow 0$ as $i \rightarrow \infty$. Defining

$$\tilde{q}(z^{t+1}) = \frac{(1+n)(1+g)\tilde{q}(z^t)}{1+r(z^{t+1})} \quad (19)$$

implies the sufficient condition (17). QED

In the applications below it suffices to consider $T = 1$. With Jensen’s inequality for $T = 1$, our result implies that the allocation is dynamically efficient if the conditional expected returns to capital lies above $(1+g)(1+n)$ for all possible states in the state space.¹⁵

4 Calibration

In order to quantify the welfare effects of introducing an unfunded social security system we first have to parameterize our model. This amounts to

¹⁵Also note that from Zilcha (1990) it follows that, independently of the market structure, allocations are production efficient if (17) holds true.

specifying the aggregate stochastic process governing total factor productivity and stochastic depreciation, population growth and the life-cycle labor income profile, average economic growth, the capital share in the production function and parameters governing preferences.

4.1 Aggregate Growth and Technology

In our model economy agents live for 9 periods. Therefore we interpret one model period to last 6 years. As population growth rate we choose $n = 1.1\%$ per annum, equal to the average population growth rate for the US postwar period. Similarly we choose the average growth rate of wages equal to $g = 1.7\%$, the long-run average for the US. The labor share in the Cobb-Douglas production function is taken to be $\alpha = 0.3$.

We assume that aggregate uncertainty is driven by a four-state Markov chain with state space $Z = \{z_1, z_2, z_3, z_4\}$ and transition matrix $\pi = (\pi_{ij})$. Since we want to model both shocks to total factor productivity and to depreciation, a particular state z_i maps into a combination of low or high TFP and low or high depreciation.

$$\begin{aligned} T(z) &= \begin{cases} 1.0 + \nu & \text{for } z \in \{z_1, z_2\} \\ 1.0 - \nu & \text{for } z \in \{z_3, z_4\} \end{cases} \\ \delta(z) &= \begin{cases} \bar{\delta} - \psi & \text{for } z \in \{z_1, z_3\} \\ \bar{\delta} + \psi & \text{for } z \in \{z_2, z_4\} \end{cases} \end{aligned} \quad (20)$$

We set $\bar{\delta}$, the average depreciation rate, to 0.31, reflecting an average depreciation rate of 6% per year.

The aggregate state z_1 is characterized by a good TFP-shock and a good depreciation shock (low depreciation), whereas z_4 features a bad TFP shock and a bad depreciation shock. To introduce persistence of the process over time we assume that the Markov process is a mixture between an *iid* process and the identity matrix I ,

$$\pi = (1 - w)\Pi + wI \quad (21)$$

where w is a parameter governing the persistence of the process and Π is composed of columns of the form $(\Pi_1, \Pi_2, \Pi_3, \Pi_4)$, and Π_j is the probability of state z_j in the stationary distribution of π . We assume symmetry in that $\Pi_1 = \Pi_4$ and $\Pi_2 = \Pi_3$. Given the restriction $\sum_j \Pi_j = 1$ the matrix π is then uniquely determined by two numbers (Π_1, w) , which, together with (ν, ψ) and possibly \bar{K} completely characterize the production technology.

4.2 Labor Endowments

Labor endowments are deterministic and follow the life cycle pattern documented in Hansen (1993). They are documented in Table 1

Table 1: Labor Endowments

<i>age j</i>	1	2	3	4	5	6	7	8	9
<i>l^j</i>	1	1.35	1.54	1.65	1.67	1.66	1.61	0	0

This profile implies that, absent aggregate shocks, individual labor earnings have a hump-shaped profile, with peak around the age of 48; at that age individuals earn 67% more than at their entry into the labor force in their early 20's. Households of age 63 retire and possibly receive social security benefits.

4.3 Social Security

We consider various sizes of the social security system, with a benchmark of $\tau = 0$ (no social security) and our experiment consisting of the “marginal” introduction of a social security system of size $\tau = 2\%$.

4.4 Preference Parameters

Our recursive preferences are uniquely characterized by the intertemporal elasticity of substitution $\frac{1}{1-\rho}$, the time discount factor β and the risk aversion parameter $1 - \sigma$. Since our results depend crucially on these parameters we report outcomes for different combinations of these parameters.

4.5 Calibration Targets and Benchmark Parameterization

We first present results for the small open economy in which the capital crowding-out effect of social security is absent by construction. The technology parameters $(\Pi_1, w, \nu, \psi, \bar{K})$ are chosen jointly so that the benchmark model competitive equilibrium delivers the following statistics from aggregate data on wages and returns to capital, which we interpret as the S&P 500.¹⁶ These data, and thus the equilibrium of our model, exhibit exactly

¹⁶Note again that our model period lasts for 6 years, and thus the statistics reported below refer to wage and return data over six year periods (see the appendix). Loosely speaking, the parameter \bar{K} determines the average return on capital, the shock to TFP, ν , determines the variability of wages, conditional on ν the shock to depreciation ψ determines the variability of returns to capital, the probability Π_1 determines how correlated returns to capital and labor are and finally w controls the autocorrelation of wages.

the return-risk trade-off on which the current political debate about social security reform centers.

- An average real return on risky capital of 7.7% per annum
- A coefficient of variation for the return of capital of 0.808
- A coefficient of variation of average wages of 0.133
- A correlation coefficient between wages and returns to risky capital of 0.418
- A serial correlation of wages of 0.623

Note that in the small open economy, model-generated statistics for wages and returns are independent of the preference parameters and thus need not be re-calibrated as we perform sensitivity analysis with respect to these parameters. In the closed economy version of our model capital accumulation is endogenous, and therefore the parameter \bar{K} is absent. Consequently we choose one of the preference parameters, namely the time discount factor β , so that the closed economy with $(\Pi_1, w, \nu, \psi, \beta)$ delivers equilibrium observations consistent with the facts above. In anticipation of this we choose as time discount factor for the small open economy $\beta = 0.85$, or a time discount rate of 2.7% per year.

The parameters required for model-generated statistics to coincide with the five empirical observations stated above are summarized in Table 2, together with the other parameters of the model.

Table 2: Benchmark Parameterization

Par.	$n(pa)$	$g(pa)$	α	Π_1	ν	ψ	$\bar{\delta}$	w	$\beta(pa)$	τ
Val.	1.1%	1.7%	0.3	0.29	0.13	0.42	0.31	0.62	0.97	0

Note that a probability $\Pi_1 = \Pi_4 = 0.294 > 0.25$ is required to match the significantly positive correlation of returns to labor and capital for 6 year time periods in the data. For the model to reproduce this observations it has to be sufficiently unlikely that TFP-shocks and depreciation shocks of opposite direction occur simultaneously. The relative magnitude of TFP-shocks and depreciation shocks is explained by the fact that returns to capital are much more volatile in the data than are wages. Since TFP-shocks affect both returns as well as wages directly, the size of these shocks have to be moderate for wages not to be too volatile. Given this, depreciation shocks have to be of large magnitude to generate returns to capital that are sufficiently volatile in the model.

5 Results

5.1 Small Open Economy with Time-Separable Preferences

We first investigate whether the basic results from our simple model in section 2 carry over to a model with nontrivial intertemporal choices. In Figure 2 we plot the welfare consequences of introducing a marginal unfunded social security system $\tau = 2\%$ against the risk aversion of an agent with standard time separable preferences. Note that increasing the agents' risk aversion is automatically associated with reducing her intertemporal elasticity of substitution, since with standard preferences both attitudes towards risk and towards intertemporal substitutability of consumption are controlled by the same parameter.

Since the aggregate capital stock is fixed and wages and returns to capital therefore only vary with the exogenous shock z , the welfare consequences from such a reform for any newborn agent depend only on the current shock. We measure welfare changes in consumption equivalent variation (or “consumption”, for short): we ask what percentage of extra consumption, in each state, an agent would require in the old equilibrium to be as well off as with the introduction of social security. Positive numbers thus indicate welfare gains from an introduction of social security for a newborn agent, negative numbers indicate welfare losses. To better interpret these numbers, note that without aggregate uncertainty a social security reform simply leads to a reduction in the present discounted value of lifetime income worth 1.9% of consumption.

We see that the introduction of social security leads to uniform welfare losses for newborn agents, independent of the state at which it is introduced and the risk aversion of the agent. As risk aversion increases, these losses become more severe, after reaching a minimum at a CRRA of around 5. The magnitude of the losses center around 1% of consumption, significantly less than the 1.9% without uncertainty. Social security *does* have a beneficial role in reducing the variability of retirement consumption: the coefficient of variation of consumption of agents in their last two periods of life (their retirement) declines by 2 to 5 percentage points (depending on the risk aversion), due to the introduction of social security. However, this effect is insufficient quantitatively to overcome the loss in average lifetime consumption stemming from the lower returns of social security, compared to private capital.

The reason for why higher risk aversion does not, as in the simple model

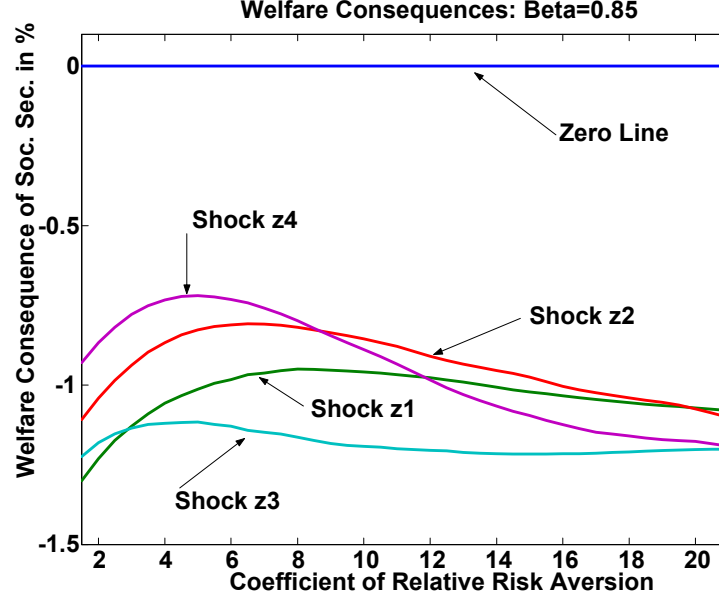


Figure 2: Welfare Consequences of Social Security Reform

of Section 2, lead to welfare gains from better risk allocation via social security, lies in the fact that now households make consumption decisions over time. Increasing risk aversion implies a reduction in a households willingness to intertemporally substitute. In an atemporal model this does not matter, but now it crucially determines our results. In our model with increasing labor income over the life cycle and long run wage growth due to technological progress households borrow at high and risky interest rates when young, and more so with social security which taxes labor income. A reduction in the intertemporal elasticity of substitution makes the desired consumption profile flatter, leads to more borrowing when young and thus lessens the attractiveness of a program that reduces labor income early in life. What drives the results in Figure 2 are therefore not primarily risk considerations, but rather the changes in the IES implied by the changes in attitudes towards risk.¹⁷

¹⁷The welfare consequences of introducing social security improve if one increases the discount factor β , but qualitatively exhibit the same properties as the results summarized in Figure 2.

In order to disentangle these effects it is therefore, for the purpose of this paper, crucial to allow for a utility specification in which the degree of risk aversion and the willingness to intertemporally substitute consumption can be controlled independently. Recursive utility permits exactly this, with minimal deviations from standard von Neumann-Morgenstern utility.

5.2 Small Open Economy with Recursive Preferences

We now repeat our thought experiment of introducing social security for varying degrees of risk aversion, holding the intertemporal elasticity of substitution $\frac{1}{1-\rho}$ constant. There is substantial disagreement about the size of the IES (see Guvenen (2002) for an excellent survey of the literature). Estimates from aggregate consumption data tend to center around 0-0.2, whereas studies that use micro data find higher values of around 0.2-0.8 (see, e.g. Attanasio and Weber, 1993, 1995), and studies using data on stock-holders only conclude that the IES for stock-holders is likely to be above one (see Vissing-Jorgensen and Attanasio, 2003). The macroeconomic studies cited in Guvenen (2002), trying to reconcile observations about interest rates and consumption growth rates, argue for a high IES, which leads him to conclude that “(f)or many macroeconomists economic reasoning constitute a strong, albeit indirect, evidence that the IES is quite high, probably close to unity” (p. 7).

We follow the macroeconomic literature and use a unit elasticity of substitution (the log-case) as our benchmark, but also report results for a lower IES of 0.5, more in line with microeconomic evidence.

From Figure 3 we see that for a fixed IES, the welfare consequences from introducing social security are monotonically increasing in the agents’ risk aversion. In particular, such an introduction is a Pareto-improving reform as long as coefficient of relative risk aversion exceeds 5.5, since newborns in all aggregate states of the world are better off with than without social security.¹⁸

This conclusion depends crucially on the size of the intertemporal elasticity of substitution. Reducing the IES to 0.5 renders the introduction of social security less beneficial. Now such a reform does not constitute a Pareto improvement since agents born into the state with low wages and high returns on capital lose, even for high degree of risk aversion.

We conclude from this section that introducing an unfunded social se-

¹⁸Not surprisingly, agents already alive at the date of the reform unambiguously gain.

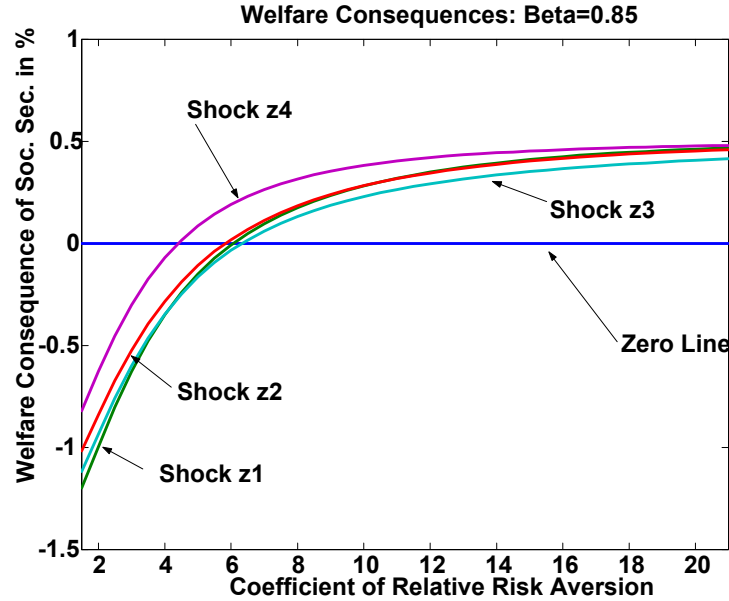


Figure 3: Welfare Consequences of Soc. Sec. Reform: $IES = 1$

curity system may constitute a Pareto improving reform, if agents are sufficiently (but not unreasonably) risk averse and willing to substitute consumption over time. However, if capital accumulation is endogenous, such social security reform will reduce private saving and hence the aggregate capital stock and wages. This adverse effect was, by construction, absent in the previous analysis, but whose quantitative importance we will now examine in the closed economy version of our model.

5.3 The Crowding-Out Effect of Social Security

To assess the importance of the crowding-out effects of social security in this section we only study preference parameterizations for which the small open economy analysis showed that the introduction of social security constitutes a Pareto improvement. This does not mean that we view other parameterizations as empirically unreasonable; it simply reflects the fact that for such parameterizations no additional analysis is needed to arrive at definitive welfare conclusions – the capital crowding out effect of social security

can only make matters worse for the reform.

For the IES we maintain a value of unity. As benchmark we assume a risk aversion parameter of 15. This value, which lies outside the range of values commonly deemed reasonable by macroeconomists, but is not uncommon in the finance literature and has some empirical support from experiments,¹⁹ produces a solid Pareto improvement from social security in the small open economy. We re-calibrate the technology parameters, together with the time discount factor β so that the equilibrium of the closed economy reproduces the empirical wage and return data summarized above.²⁰ The required β equals 0.85, as in the small open economy version of the model.

The introduction of social security, in contrast to the small open economy, now changes aggregate production and interest rates. The average return to capital (in the log run, after the transition has been completed) increases from 7.7% to 7.9% per annum, the average aggregate capital stock declines by 2.6%, output by 1% and consumption by 0.4%.

The crowding-out effect, which sets in immediately after the reform has profound consequences for the welfare implications of the reform, summarized in Figure 4. The number attached to a given node of the event tree represents, in consumption equivalent variation, the welfare gains/losses induced by the reform for an agent born at a particular node of the tree.²¹

Whereas generations already alive still benefit from the reform, households born at or after the reform date now lose, in contrast to the case in which the capital-crowding out effect was absent. Welfare losses increase slightly over time, are fairly uniform across states and amount to roughly 1% of consumption. This result is obtained even though, as in the partial equilibrium case, social security *does* reduce the consumption variance of retired agents significantly (the coefficient of variation of retiree consumption declines by about 2 percentage points).

Increasing the risk aversion further does not qualitatively change the result. It is important to note that the welfare gains from intergenerational risk sharing are bounded (see also Figure 2): any intergenerational retirement transfer program at best can eliminate consumption risk in retirement completely. The ratio of consumption in the best and in the worst state provides an upper bound on the (consumption equivalent) welfare gains,

¹⁹Gollier (2001) summarizes the experimental evidence. For representative papers in the finance literature, see Kandel and Stambaugh (1991) and Cecchetti et al. (1993).

²⁰Since capital accumulation is now endogenous, \bar{K} is no longer a parameter.

²¹The results are fairly independent of the aggregate capital stock and its distribution at the time of the reform.

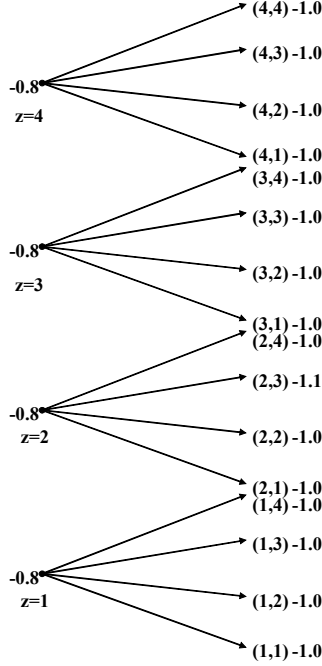


Figure 4: Welfare Cons. with Crowding-Out

independent of the risk aversion. If the loss due to the return dominance of private capital over social security exceeds this upper bound, even with extremely high risk aversion a social security reform does not generate welfare gains. We conclude that even for high risk aversion the crowding-out effect of social security dominates the intergenerational risk sharing effect, and therefore the reform does not provide a Pareto improvement.

5.4 Social Security and Stock Market Returns

The data on returns of the stock market we use in our calibration section stem from the years 1929-2001. A PAYG social security system was in place in the US since the late 1930's. It is therefore possible that high stock market returns in the sample period are partially due to the presence of social security. This possibility is important for our calibration exercise. The main reason why social security has such adverse welfare consequences

in general equilibrium is the return differential between risky capital and an unfunded social security system, before its introduction, of roughly 4.9%.

Suppose we calibrate our economy in such a way that *with* an unfunded social security system our model economy reproduces the empirical targets set forth in the calibration section. Qualitatively, since returns on the risky capital stock in the *absence* of social security now are closer to the potential implicit returns of an unfunded social security system, we expect the welfare consequences of a social security reform to be more favorable. We now ask whether under such a calibration the economy without social security is dynamically inefficient, and if not, if a (marginal) reform now provides a Pareto improvement.

We calibrate to the same observations as above, but now use as social security tax rate the current payroll tax rate of $\tau = 12.4\%$. While this is an extreme assumption (for most of the sample period the tax rate was considerably lower), it provides us with the most stringent robustness check of our previous results. Now a high aversion of agents *is* sufficient to obtain a Pareto improving social security reform. For a risk aversion parameter of 30 all current and all future generations gain from the introduction of an unfunded social security system with payroll tax rate of $\tau = 2\%$. The welfare gains amount to about 0.2% for agents born directly at the date of the reform and 0.1% for agents born farther into the future. The crucial driving force for this result, beyond high risk aversion and fairly high IES of 1 is the reduction of the return differential between capital and social security. Before the introduction of the system the average return to capital now is 6.4% instead of 7.7% as under our previous calibration strategy. The economy is still deep inside the dynamically efficient region, as it passes our sufficient condition easily.

Even though the crowding-out effect is non-negligible (the average capital stock falls by 3.2%, aggregate output by 1% and aggregate consumption by 0.4%, similar to the benchmark calibration), now its welfare implications are dominated by the benefits of better risk allocation. Note that with more moderate degrees of risk aversion (such as a risk aversion parameter of 15 or below) the reform does not constitute a Pareto improvement, although generations born at the time of the reform gains (as opposed to Figure 4).

To us, this example shows that there do exist defensible parameterizations of the closed economy version of the model for which the introduction of a small unfunded social security system provides a Pareto improvement. Given that the average return on the stock market has a sizeable standard error, calibrating to a return of 6.4% is not a priori unreasonable. Reducing

the target return further, to about 5%, generates a Pareto improvement of the reform even for lower degrees of risk aversion than 15 and still leaves the economy dynamically efficient.

5.5 The Role of the Intertemporal Elasticity of Substitution

It is easy to obtain a Pareto improving reform if one resorts to higher (and thus empirically implausible) values for the intertemporal elasticity of substitution, because a higher IES reduces the magnitude of the crowding-out effect of social security. With our calibrated life cycle profile of labor earnings young agents in the model borrow. As the IES increases, and thus agents value a flat consumption profile less, the incentive to borrow more to offset a payroll tax at young age declines. The weaker the motive of the young to do additional borrowing, the milder is the required increase in interest rates (and consequent fall in the capital stock) to bring about equilibrium in the capital market after the introduction of social security. This point, already discussed in Imrohoroglu et al. (1999), explains why we, with a large IES of 5 find Pareto improvements via social security even for moderate degrees of risk aversion and return differentials between private assets and social security as observed in the data; in these examples the crowding-out effect is virtually absent and the welfare conclusions parallel those of the small open economy discussed previously.

We interpret the findings documented in this section as suggesting that a Pareto improving introduction of social security is a possibility even from a quantitative point of view, but that for parameter values usually deemed reasonable in the macroeconomic literature such a Pareto improvement seems unlikely to occur. For the current reform debate the fairly uniform welfare losses for all generations but the initial old suggest that, from a normative point of view, the long run benefits from higher returns in a (partially) privatized system more than offset risk considerations (which are nevertheless quantitatively important), if US households have preferences that macroeconomists normally attribute to them. Of course these long-run benefits have to be traded off against the transition costs, which is what the political discussion should focus on, from the viewpoint of our results.

5.6 Using Privately Traded Assets to Share Risk

In the model discussed so far the only privately traded asset between generation was risky capital; in the absence of any less risky asset social security

provides a welcome improvement of the risk allocations across generations. It is natural to ask to what extent our results rely on the sparse set of assets traded. We now show that the introduction of a one-period uncontingent bond (as in Mehra and Prescott (1985) and Constantinides et al., 2002) leaves our results qualitatively, and, to a large extent, quantitatively unaffected.

Let $q(z^t)$ denote the price, in node z^t , of a bond that pays off one unit of consumption in *all* nodes $z^{t+1} \succ z^t$ that are potential successors of z^t , and let $d^s(z^t)$ denote the number of bonds generation s buys in node z^t . The budget constraint now reads as

$$c^s(z^t) + a^s(z^t) + q(z^t)d^s(z^t) = (1+r(z^t))a^s(z^{t-1}) + d^s(z^{t-1}) + (1-\tau)l^{t-s}(z_t)w(z^t) + I(s)b(z^t) \quad (22)$$

and the market clearing condition for bonds can be stated as

$$\sum_{s=1}^8 \frac{d^{t-s}(z^{t-1})}{(1+n)^s} = 0,$$

that is, bonds are in zero net supply.

In our calibration, in addition to the previous empirical targets we aim for matching the long run average (real) bond return of 0.82% per annum (see Mehra and Prescott (1985), Constantinides et al., 2002). In partial equilibrium this is easy to achieve by setting the *exogenous* $q(z) = \frac{1}{1.0082}$ for all $z \in Z$. In general equilibrium the process $q(z^t)$ is *endogenous*; we thus check whether the model, calibrated to the same targets as in the benchmark, delivers an average bond return roughly equal to the observed magnitude.

5.6.1 Partial Equilibrium

Abstracting from the capital crowding out effect, the welfare consequences of introducing social security are almost exclusively determined by the relative size of the gross return on bonds, $r_b = \frac{1}{q} - 1$, and the implicit average return on social security $n + g$ (with g the growth rate of labor productivity and n the growth rate of the labor force). If r_b is below $n + g$ even by two to three tenths of one percent, the introduction of social security is Pareto-improving; reversely, if r_b is even slightly higher than $n + g$, the introduction of social security results in welfare losses.²² The welfare consequences are fairly independent of the coefficient of relative risk aversion.

²²This is a computational result, rather than a theorem. With incomplete financial markets it is not true even in partial equilibrium that the introduction of social security

What explains these findings? With the empirically sensible calibration or $r_b = 0.82\%$ the introduction of a bond gives households access to two assets: one with high returns and high risk, one with no risk but returns significantly lower than social security, whose implicit return is roughly $n + g = 2.8\%$. Thus households can offset the reduction in their after-tax income due to the payroll tax by higher borrowing at a rate lower than the implicit return on social security. It is therefore not surprising that the introduction of social security is welfare-improving.²³

5.6.2 General Equilibrium

As in partial equilibrium, our main results from the general equilibrium section of our paper are qualitatively robust to the introduction of the bond. The main *quantitative* change is the size of the capital crowding-out effect of social security. Payroll taxes reduce after-tax wages in exchange for retirement benefits. In order to offset this reduction in current disposable income, households may opt to borrow more. With risky capital as the only asset, households are cautious to do so, thus aggregate saving does not decline that much with the introduction of social security; the capital-crowding out effect remains modest. If households have access to a low-interest, risk-less one period bond, they offset the decline in income to a larger degree by expanding their debt, thus reducing aggregate saving substantially. Therefore the crowding-out effect becomes more severe, which, *ceteris paribus*, leads to a less favorable welfare assessment of the social security reform.

With this intuition in mind, Table 3 documents how our results from section 5.3-5.5 of the paper are altered with the introduction of the bond.²⁴ We retain the same calibration targets; as before we choose parameters to

is Pareto improving if and only if $r_b < n + g$, since the bond has better risk characteristics than social security.

²³Why are these welfare gains fairly uniform with respect to the extent of risk aversion σ ? First, highly risk averse households go long in both risky capital and risk free bonds (in order to balance the risk from capital). The introduction of social security and its associated additional borrowing needs early in life makes it more difficult to be long in bonds. But second, social security is still a less risky asset than physical capital, and thus provides households with welcome risk diversification (at a higher return than bonds). As σ increases, both the negative first and the positive second effect become more important, leaving the overall welfare consequences roughly unchanged. With just risky capital only the second, positive, effect arises, which explains why in our earlier model households value social security the more the higher their risk aversion.

²⁴The average return on capital, without and with social security, is denoted by r_0 and r_∞ , respectively. Similar notation is used for the average bond return, r_{b0} and $r_{b\infty}$.

match the empirical return on capital, but we do *not* choose parameters in order to match the empirical risk-free rate. Whether the model can deliver an empirically plausible risk-free rate and thus an empirically plausible equity premium is then an important quality check of the model.

Table 3: General Equilibrium Results

Calibration	$IES = 1, \sigma = 15$	$IES = 1, \sigma = 30^*$	$IES = 1.5, \sigma = 30^*$	$IES = 4, \sigma = 30$
r_0 p.a.	7.66%	6.64%	7.03%	7.79%
r_∞ p.a.	7.81%	6.81%	7.22%	7.91%
r_{b0} p.a.	2.73%	0.96%	0.02%	0.93%
$r_{b\infty}$ p.a.	2.95%	1.21%	-0.1%	0.96%
ΔK in %	-2.84%	-3.23%	-1.25%	-0.29%
$\Delta \text{Welf.}$ in %	[-1.4%, -1.0%]	[-0.9%, -0.3%]	[0.05%, 0.1%]	[0.1%, 0.3%]

* The economy *with* social security is calibrated to match historical capital returns

If we calibrate the model to reproduce the empirical mean return on capital, as in section 5.3, we again find welfare losses from the reform for all agents born at the time or after the reform. The losses are slightly larger in the model with the bond, since the capital crowding-out effect is now more substantial (2.8% vs. 2.6% in the model with only capital, for $\sigma = 15$). Higher risk aversion does not change the results in a quantitatively significant way.

More important is how robust our results from section 5.4-5.5 are, since in these sections we presented example parameterizations for which the introduction of social security is a Pareto improvement. If we calibrate to an economy with social security, as in section 5.4, reducing the return differential between capital and social security before its introduction to roughly 4%, then for an IES of 1.5 (or higher) and $\sigma = 30$ we find that the introduction of social security is a Pareto improvement, as before. For an IES of 1, as assumed in section 5.4, the reform benefits all households currently alive, but does not constitute a Pareto improvement. Thus, qualitatively the results from the model without the bond are robust, quantitatively a slightly higher IES is needed for the reform to be Pareto-improving. This is due, as mentioned above, to the more powerful capital crowding-out effect in the model with the bond. Not surprisingly, for an IES of 4 or higher (and thus virtually absent crowding-out) the introduction of social security constitutes a Pareto improvement even if we calibrate the economy without social security to historically observed returns, as in section 5.5.

In all our general equilibrium experiments the bond return falls between

0–2.7%, roughly consistent with the long-run average real bond return in the data. Our economy, therefore, does a good job of endogenously generating an equity premium of 5–7%, depending on the risk aversion and *IES*. It is crucial for the welfare gains from introducing social security that the private asset available to hedge capital income risk has a significantly lower average return than the implicit return on social security. The low-return bond is not an attractive instrument to hedge risky returns to capital, and therefore the trading-volume in the bond is fairly small. Social security pays decent returns and, in addition, is a good (but not perfect) tool to reduce old-age consumption risk. Therefore, abstracting from crowding-out effects, it is a welcome policy innovation as long as its return is not too much lower than that of capital.

Note however that, in the model with the bond, in addition to providing a better risk allocation social security may be beneficial because the equilibrium without it may be *dynamically inefficient*. In our benchmark model with risky capital we derive and check a sufficient condition for the economy to be dynamically efficient. With the introduction of a risk-free bond with low average returns (as in the data) we cannot assure that this is the case. Thus the welfare gains from social security we find within *that* model may be due to a better allocation of average consumption across generations (as in Samuelson, 1958) rather than better risk allocation across generations from social security. Thus, while our results remain intact with risk-free bonds, their interpretation is less clear than in the benchmark model.

6 Conclusion

Can the introduction of an unfunded social security system provide a Pareto improvement by facilitating intergenerational risk sharing? In this paper we argue that, in the presence of incomplete markets, it potentially can do so in a quantitatively important way. However, in a realistically calibrated economy the intergenerational risk sharing role of unfunded social security is dominated in its importance by the adverse effect on capital accumulation arising from the introduction of such a system, and by the lower average return on social security than on capital. While our results suggest that the current political debate about the return-risk trade-off may be settled in favor of the return-dominance argument, because of the transition cost implied by a reform that reverses the 1935 introduction of a PAYG social security system no clear-cut policy recommendation about the desirability of

a (partial) privatization of social security should be derived from our work.

Future research may extend our work along several important dimensions. First, we abstract from several beneficial roles of an unfunded, redistributive social security system. In the presence of incomplete financial markets social security provides a partial substitute for missing insurance markets against *idiosyncratic* labor income and lifetime uncertainty. On the other hand the distortive effects of payroll taxes on the labor supply decision remain unmodeled. We abstract from these features to more clearly isolate the potential magnitude of the beneficial intergenerational risk sharing role of social security. A complete assessment of its relative quantitative importance, compared to the intragenerational risk sharing and distortion effects would require incorporating these effects explicitly. Allowing for uninsurable idiosyncratic uncertainty would generate intragenerational heterogeneity, a nontrivial wealth distribution within generations and thus induce the same curse of dimensionality that occurs when expanding the number of generations in the model.²⁵

Second, in this paper we are setting a very demanding bar that social security has to pass in order to be judged as welfare improving. Employing the Pareto criterion our normative analysis is silent about the political conflict surrounding the historical adoption or current reform of social security. Extensions of the work of Cooley and Soares (1997) and Boldrin and Rustichini (2000) to our environment with aggregate uncertainty are needed to address the questions why, though not mutually beneficial, the US social security system was introduced when it was introduced and who one would expect the major supporters of this reform (and of its reversal) to be.

²⁵Thus one would have to resort to a different numerical algorithm, e.g. the one developed by Krusell and Smith (1998). In our computational companion paper (Krueger and Kubler, 2003) we argue that for OLG models with sizeable aggregate uncertainty their method, which approximates the aggregate wealth distribution with a small subset of its moments may not work very well.

A Theoretical Appendix

In this appendix we derive equation (3) explicitly. With $v(c) = \ln(c)$ we have

$$U'(\tau = 0) = E \left\{ \frac{G - R}{(1 - 0)R + 0 * G} \right\} > 0 \text{ if and only if } E \left\{ \frac{G}{R} \right\} > 1 \quad (23)$$

We note that

$$E \left(\frac{G}{R} \right) = E \left(e^{\ln(G) - \ln(R)} \right) = E \left(e^{\ln(Z)} \right) \quad (24)$$

where $\ln(Z) := \ln(G) - \ln(R)$, so that $Z = \frac{G}{R}$. Since $(\ln(G), \ln(R))$ are jointly normal, both $\ln(G)$ and $\ln(R)$ are normal random variables, and thus $\ln(Z)$ is normal with mean $\mu_{\ln Z} = \mu_{\ln G} - \mu_{\ln R}$ and variance $\sigma_{\ln Z}^2 = \sigma_{\ln G}^2 + \sigma_{\ln R}^2 - 2\sigma_{\ln G, \ln R}$. Since Z is lognormal we have

$$\begin{aligned} E \left(\frac{G}{R} \right) &= E(Z) = e^{\mu_{\ln Z} + \frac{1}{2}\sigma_{\ln Z}^2} \\ &= e^{\mu_{\ln G} + \frac{1}{2}\sigma_{\ln G}^2} \cdot e^{-(\mu_{\ln R} + \frac{1}{2}\sigma_{\ln R}^2)} \cdot e^{\sigma_{\ln R}^2} \cdot e^{-\sigma_{\ln G, \ln R}} \end{aligned} \quad (25)$$

Since G and R are log-normal we have

$$\begin{aligned} E(G) &= e^{\mu_{\ln G} + \frac{1}{2}\sigma_{\ln G}^2} \text{ and } E(R) = e^{\mu_{\ln R} + \frac{1}{2}\sigma_{\ln R}^2} \\ \text{Var}(R) &= e^{2\mu_{\ln R} + \sigma_{\ln R}^2} \cdot (e^{\sigma_{\ln R}^2} - 1) \\ &= E(R)^2 \cdot (e^{\sigma_{\ln R}^2} - 1) \end{aligned} \quad (26)$$

We thus obtain

$$e^{\mu_{\ln G} + \frac{1}{2}\sigma_{\ln G}^2} = E(G) \quad (27)$$

$$e^{-(\mu_{\ln R} + \frac{1}{2}\sigma_{\ln R}^2)} = \frac{1}{E(R)} \quad (28)$$

$$e^{\sigma_{\ln R}^2} = \frac{\text{Var}(R) + E(R)^2}{E(R)^2} \quad (29)$$

Finally we want to obtain an expression for $e^{-\sigma_{\ln G, \ln R}}$. But

$$\begin{aligned} \text{Cov}(G, R) &= E(GR) - E(G)E(R) = E(e^{\ln(G) + \ln(R)}) - E(G)E(R) \\ &= e^{\mu_{\ln G} - \mu_{\ln R} + \frac{1}{2}\sigma_{\ln G}^2 + \frac{1}{2}\sigma_{\ln R}^2 + \sigma_{\ln G, \ln R}} - E(G)E(R) \\ &= E(G)E(R) (e^{\sigma_{\ln G, \ln R}} - 1) \end{aligned} \quad (30)$$

and thus

$$\begin{aligned} e^{\sigma_{\ln G, \ln R}} &= \frac{Cov(G, R) + E(G)E(R)}{E(G)E(R)} \\ e^{-\sigma_{\ln G, \ln R}} &= \frac{E(G)E(R)}{Cov(G, R) + E(G)E(R)} \end{aligned} \quad (31)$$

Plugging in (27) – (31) into (25) yields

$$\begin{aligned} E\left(\frac{G}{R}\right) &= \frac{E(G)}{E(R)} \cdot \frac{\frac{Var(R) + E(R)^2}{E(R)^2}}{\frac{Cov(G, R) + E(G)E(R)}{E(G)E(R)}} \\ &= \frac{E(G)}{E(R)} \cdot \frac{[cv(R)^2 + 1]}{[\rho_{G, R} \cdot cv(G) \cdot cv(R) + 1]} \end{aligned} \quad (32)$$

as in the main text.

B Data Appendix

We use data for 1929-2001, since reliable wage and stock market data are available only for this period. Our annual data on dividends and stock market prices, in order to compute returns on the S&P 500 are taken from Shiller (1989) and the updates available on his web site. For annual wages we use total compensation of employees from the NIPA, divided by the total number of full-time and (full-time-equivalent) part-time employees. All variables are deflated by the deflator for total consumption expenditures from the NIPA. We remove a constant growth rate of 1.7 per annum from the wage data; the statistics referring to the wage data pertain to the so de-trended data. Where applicable, we aggregate yearly data into 12 six-year intervals to obtain data of frequency comparable to that of our models.

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