STABILIZATION, COMPETITIVENESS, AND RISK-SHARING:

A MODEL OF MONETARY INTERDEPENDENCE*

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Abstract

This paper provides a stylized choice-theoretic model to analyze optimal monetary policies among interdependent economies. In response to macroeconomic shocks, policymakers strike a balance between two objectives. The first is to stabilize marginal costs and markups to offset the distortions associated with nominal rigidities, a dimension that entails no international tension. The second is to influence the terms of international trading in contingent assets. Through this channel, policymakers aim at lowering the relative prices of domestic goods, leading to a transfer of world income in favor of the country's residents. This dimension then entails a zero-sum redistribution of welfare across countries. Conducting monetary policy in a coordinated fashion allows policymakers to completely focus on the stabilization objective. While cooperation improves welfare from a worldwide point of view, this need not be true from a national perspective, as the larger country is better off when monetary policy is conducted in a decentralized fashion.

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1 Introduction

Virtually all analyses of the international dimensions of monetary policy, despite significant differences in emphasis and detail, focus implicitly or explicitly on a key policy dilemma: monetary policies that stabilize the global economy in response to macroeconomic shocks need not be optimal from the vantage point of national policymakers, who face the temptation to shift burden and costs of the adjustment onto foreign agents and maximize the net gains accruing to national residents. The argument is often cast in terms of competitiveness gains: when economies are open and interdependent, national policymakers can manipulate international relative prices in a 'beggar-thy-neighbor' fashion. Similar to the argument for 'optimal tariff' protectionism in international trade, consumption interconnections across countries generate a terms of trade externality that national policymakers can exploit in their design of optimal monetary strategies.

A direct implication of these considerations is the existence of gains from international monetary cooperation. Since Bretton Woods, concerns over spirals of 'competitive devaluations' have motivated the design of institutions and rules to prevent countries from adopting exchange rate policies as tools of export promotion.[1] The early game-theoretical analyses of monetary interdependence — as surveyed and systematized e.g. by Canzoneri and Henderson (1991) — emphasized the possibility of significant gains from cooperative responses to macroeconomic shocks. With the experience of the 1970s in the background, it was shown how an uncoordinated reaction to inflationary pressures (say, oil price shocks) could lead to excessive monetary tightening and output contraction in the world economy.

This conclusion has not been left unchallenged, especially on methodological grounds.

[1]The standard reference is Ragnar Nurkse's analysis of the devaluations that took place in the interwar period: "in contemporary discussions much stress was laid on the competitive aspects of currency devaluation. In many quarters devaluation was regarded primarily as a means of improving a country's foreign trade balance and hence its volume of domestic employment — an effective means but one that operated necessarily at the expense of other countries and invited retaliation" (Nurkse (1944), p.129).

As a representative quote, Obstfeld and Rogoff (1996) wrote that: "Virtually all of the literature [on international monetary policy cooperation] is based on obsolete Keynesian models, which lack the micro foundations needed for proper welfare analysis... Because ad-hoc Keynesian analyses of cooperation can yield seriously misleading policy prescriptions, there is a compelling case for basing policy-coordination analysis on choice-theoretic models."[2]

In response to this challenge, there has been a recent resurgence of interest in the analysis of monetary interdependence. Relative to the past, the new literature adopts a choice-theoretic approach in a fully stochastic setting with monopolistic competition, nominal rigidities, and forward-looking price (and/or wage) setters. Such research agenda provides a rigorous micro-founded general equilibrium framework to re-assess the issue of coordination on a firmer footing, and avoid ranking alternative policy regimes on the basis of arbitrary welfare criteria.[3]

Broadly defined, the new literature has come in two strands. The first one has focused on highly stylized models with over-simplified dynamics, sacrificing generality in favor of tractability and intuitive appeal. The benchmark result of this first strand of literature is found in Obstfeld and Rogoff (2002): everything considered, welfare gains from monetary coordination are very small in an environment in which exchange rate pass-through is high and firms' exposure to currency fluctuations is limited. In sharp contrast to earlier contributions, optimizing policymakers maximize global welfare even when they act in a non-coordinated way by stabilizing domestic prices and output gap in response to supply shocks: the ensuing exchange rate movements deliver the optimal cross-country allocation of risk. In related work, Devereux and Engel (2003) and Corsetti and Pesenti (2004) show that gains from coordination are small even when exchange rate pass-through is low (leaving

[2]Obstfeld and Rogoff (1996), pp.656-657.

[3]Early contributions to the so-called New Open Economy Macroeconomics include Obstfeld and Rogoff (1995), Betts and Devereux (2000), Corsetti and Pesenti (2001), Tille (2001), Bacchetta and van Wincoop (2002), Benigno (2002) among others. Lane (2001) provides a survey of early developements.

open the possibility of significant gains for intermediate degrees of pass-through).

Reacting to these conclusions, a second strand of recent contributions have emphasized how they crucially depend on specific limitations of the underlying model, such as the assumption of high correlation between productivities in the traded and non-traded sectors (Canzoneri, Cumby and Diba 2002), the lack of cost-push shocks (Clarida, Gali and Gertler 2003) and, most notably, the parameterization of the basket of consumption goods. The benchmark model rules out preference-shifting shocks to the composition of the consumption basket, and assumes that the elasticity of substitution between home and foreign goods is unity (Cobb-Douglas consumption baskets). As a result of these assumptions, consumers spend a constant share of their income on each good, and exchange rate movements lead to a perfect sharing of risk by equalizing the real exchange rate (i.e. the relative price of a consumption basket across countries) to the ratio of marginal utilities (i.e. the relative need for consumption among the countries' residents) in every state of nature. The Cobb-Douglas assumption is empirically plausible, and it raises significantly the analytical tractability of the model, but of course imposes severe limitations to the generality of the model. Relaxing the unit elasticity assumption — and, more broadly, adopting a more general model — allows to resurrect gains from cooperation, a point reiterated in the work by Benigno and Benigno (2003), Sutherland (2002), Faia and Monacelli (2003), Pappa (2004) and Tchakarov (2004).

This paper falls somewhere between the first and the second strands of literature. Our framework is more general than the benchmark model, as it encompasses both supply and demand shocks, accounts for heterogeneity in the degree of macroeconomic openness, and allows for fluctuations in the cross-country distribution of income by relaxing the assumption that movements in international relative prices are exactly offset by movements in relative quantities. But the model remains sufficiently tractable to allow for a detailed intuitive exposition of the mechanisms at work, an aspect often lost under the analytic complexities of more general frameworks. Second, asset markets play a central role in our analysis. Whereas other contributions also introduce international trade in a complete set of contingent securities, the ensuing risk-sharing allocation is taken as driven by the initial conditions of the economy and unaffected by policy. By contrast, we show that monetary policy does affect the risk-sharing allocation, a property that is key to our results. Finally, we do not restrict our analysis to identical countries, and explore how size asymmetries affect the outcome of the model.

In our analysis, the factors driving optimal monetary policy in open economies reflect two main elements in each country's welfare. The first one reflects a stabilization concern, through which monetary authorities aim at stabilizing firms' markups in order to achieve as low a price level as possible and sustain residents' purchasing power. This dimension is the same for both countries, and is the only one at play under a cooperative design of monetary policy. The second element reflects the influence of monetary policy on the characteristics of the risk-sharing markets. This is because exposure to risk affects the optimal price-setting decisions of the various firms, thus the international competitiveness of a country's industry, a point somewhat overlooked in previous studies.

Each country has an incentive to balance the stabilization concern with achieving a more favorable allocation in risk-sharing. We show that countries then tend to react too little to their own productivity shocks and too much to foreign productivity shocks, compared with the cooperative solution, leading to a suboptimally low volatility of the exchange rate in response to macroeconomic shocks. We show that while the decentralized allocation is suboptimal from a worldwide point of view, it benefits a large country at the expense of a smaller one, making cooperation unlikely to be reached and enforced.

The interaction through risk-sharing is an entirely zero-sum game that is ignored when policy is set in a cooperative manner. The quantitative relevance of the risk-sharing/competitiveness dimension is related to income and substitution effects of price fluctuations: when the latter offset each other, as in the models by Obstfeld and Rogoff (2002) and Corsetti and Pesenti (2004), there are no gains from cooperation in equilibrium.

From a methodological viewpoint, the adoption of non-linear (quadratic) approximations of the solution as in Sutherland (2002) and Tille (2002) makes our analysis the ideal introduction to more complex investigations of optimal monetary policy based on 'perturbation' methods in full-fledged dynamic, stochastic general-equilibrium models.

The paper is organized as follows. Section 2 presents the building blocks of the model, with the solution explored in detail in Section 3.[4] We discuss the design of monetary policy in Section 4, illustrating our results with a simple numerical example. Section 5 concludes.

2 The model

We setup a deliberately minimalist stochastic general-equilibrium model, keeping intertemporal considerations at a bare minimum. The world economy consists of two countries, home and foreign. Foreign variables are indexed with a star. We normalize the world size to unity, and denote the size of the home country with $n \in [0,1]$. In each country there are households, firms, and a government.

Households in each country consume the same basket of goods, both locally produced and imported. We allow for two kinds of demand shocks: a country-specific shock to the marginal utilities of national households, which affects demand for consumption, and a global preference-shifting shock, which alters the composition of the world consumption basket between home-produced goods and foreign-produced goods. Households own, and supply labor to, domestic firms. Households hold real balances. There is international trade across states of nature, and households in both countries have access to a complete set of state-contingent securities.

[4] In the main text we focus on the main points of the model, and leave the detailed solution to the Appendix.

Firms produce a continuum of differentiated tradable goods (varieties) under conditions of monopolistic competition. The technology of production is subject to country-specific productivity shocks, common to all local firms. Prices of differentiated varieties are subject to nominal rigidities, in the sense that they have to be set in the producer's currency before the realization of the shocks. For each traded brand, the law of one price holds internationally.

Governments control the domestic money supply, which is set after the realization of the shocks. Government collect seigniorage revenues that are rebated to households in a lump-sum way.

2.1 The risk-sharing wedge

The representative home household's expected utility is:

$$\mathcal{U} = \sum_k \pi_k \left[f_k U(C_k) + f_k X\left(\frac{M_k}{P_k}\right) - V(L_k) \right] \tag{1}$$

where π_k is the probability of state of nature k. U, X and V measure the contributions to utility of, respectively, consumption C, real balances M/P, and labor effort L. P is the utility-based consumer price index, and f is a demand shocks that affects the utility of consumption relative to effort.[5]

The home household maximizes (1) subject to the following budget constraint in each state k:

$$P_k C_k = W_k L_k + \Pi_k + M_0 - M_k + T_k + P_k \left(B_k - \sum_h q_h B_h \right) \tag{2}$$

where C_k is consumption in state k, $W_k L_k$ is wage income, and Π_k are profits received from home firms. M_0 is the initial stock of money holdings, independent of the realized state, M_k is money holdings in state k, and T_k is a lump-sum nominal net transfer from the government.

[5] We let f affect the utility of real balances as well, so that it does not enter the money demand.

Households trade a complete set of state-contingent securities denominated in units of the consumption basket. Prior to the realization of the shocks, home households purchase or sell an amount B_h of a security paying off 1 unit of consumption in state h. The price of the security in terms of consumption baskets is q_h. Home households then spend $\sum_h q_h B_h$ units of consumption on net purchases of securities, regardless of the realized state of nature, and receive B_k units when state k is realized.

The maximization of (1) subject to (2) with respect to the labor effort, cash holdings and portfolio holdings leads to the following labor supply, money demand, and optimal portfolio allocation:

$$W_k f_k U_k' = P_k V_k', \qquad X_k' = U_k', \qquad q_k = \frac{\pi_k \mu_k}{\sum_k \pi_k \mu_k} \equiv \frac{\pi_k \mu_k}{E(\mu)} \tag{3}$$

where $\mu_k = f_k U_k'$ is the marginal utility of consumption in state k. The optimal portfolio condition states that the cost of purchasing one unit of a security paying off in state k, i.e. q_k, is equal to the probability of this security paying off, i.e. π_k, times the marginal utility of that payoff, i.e. μ_k, normalized with respect to the expected marginal utility across states of nature. E denotes the expectation operator, so that $E(\mu) = \sum_k \pi_k \mu_k$.

Foreign utility is similarly defined. Consumption preferences are identical across countries, so that all households worldwide consume the same baskets of goods. The foreign household faces the following budget constraint for state k:

$$P_k^* C_k^* = W_k^* L_k^* + \Pi_k^* + M_0^* - M_k^* + T_k^* + P_k^* \left(B_k^* - \sum_h q_h B_h^* \right) \tag{4}$$

and optimization leads to the following labor supply, money demand, and optimal portfolio allocation:

$$W_k^* f_k^* U_k^{*\prime} = P_k^* V_k^{*\prime}, \qquad X_k^{*\prime} = U_k^{*\prime}, \qquad q_k = \frac{\pi_k \mu_k^*}{E(\mu^*)} \tag{5}$$

where $\mu_k^* = f_k^* U_k^{*\prime}$.

As there are no trade barriers, the price of each traded good is equalized across borders when expressed in terms of the same currency. The law of one price, coupled with symmetry in consumption baskets across countries, implies that purchasing power parity holds.

Denoting the nominal exchange rate (home currency per unit of foreign currency) with S, we obtain:

$$P_k = S_k P_k^* \tag{6}$$

Combining the optimal portfolio conditions (3) and (5) under purchasing power parity yields:

$$\frac{f_k U_k'}{f_k^* U_k'^*} = 1 + \Gamma = \frac{E\left(fU'\right)}{E\left(f^* U'^*\right)} \tag{7}$$

According to the above equation, the ratio between the marginal utilities of home and foreign consumption is constant across all states of nature k. Risk-sharing however need not involve equalization of the marginal utilities across countries, as the latter can differ by a constant Γ. In what follows we will refer to Γ as the risk-sharing wedge. If $\Gamma > 0$, the portfolio allocation is skewed to the detriment of the home household, in the sense that its need for an additional unit of consumption is greater than the need of the foreign household in any state of nature ($f_k U_k' > f_k^* U_k^{*\prime}$). The risk-sharing wedge Γ is endogenous and reflects the *expected* marginal utilities of income in the two countries. As we discuss below, the risk-sharing wedge reflects structural asymmetries in the global economy, such as different degrees of openness and exposure to foreign competition (measured in terms of the weights of imports in the national consumption baskets). Such fundamental heterogeneity cannot be offset through trading in the asset market.

The link between the risk-sharing wedge and the nominal exchange rate is established by using the money demands (3) and (5), along with purchasing power parity (6) and the risk-sharing relation (7):

$$\frac{f_k X_k'}{f_k^* X_k'^*} = 1 + \Gamma \tag{8}$$

For instance, with symmetric logarithmic utility of real balances,[6] the exchange rate is driven by demand shocks and the relative monetary stance for any level of the risk-sharing

[6]This implies $X_k' = \chi P_k / M_k$ and $X_k'^* = \chi P_k^* / M_k^* = \chi \left(S_k\right)^{-1} P_k / M_k^*$.

wedge:

$$S_k = (1+\Gamma)\,\frac{M_k f_k^*}{M_k^* f_k} \tag{9}$$

A risk-sharing allocation tilted against the home household ($\Gamma > 0$) implies that, other things equal, the price of foreign currency exceeds the ratio of monetary and real fundamentals. Intuitively — albeit rather imprecisely — the home currency is intrinsically 'weak'.

2.2 Optimal price setting

We now turn to the determinants of consumption demand. The consumption index C is a CES aggregate of a basket of home-produced varieties, C_H, and a basket of foreign-produced varieties, C_F. The elasticity of substitution between the two baskets is denoted by λ:

$$C_k = \left[\left(nt_k^{\frac{1}{2}}\right)^{\frac{1}{\lambda}}(C_{Hk})^{\frac{\lambda-1}{\lambda}} + \left((1-n)\,t_k^{-\frac{1}{2}}\right)^{\frac{1}{\lambda}}(C_{Fk})^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}} \tag{10}$$

$$C_k^* = \left[\left(nt_k^{\frac{1}{2}}\right)^{\frac{1}{\lambda}}(C_{Hk}^*)^{\frac{\lambda-1}{\lambda}} + \left((1-n)\,t_k^{-\frac{1}{2}}\right)^{\frac{1}{\lambda}}(C_{Fk}^*)^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}} \tag{11}$$

In the expressions above $t_k > 0$ is a preference-shifting shock determining the composition of the consumption index. When t_k increases above 1, consumption worldwide moves away from foreign-produced goods and toward home-produced goods. Note that the shock t is common to all households, regardless of the country of residence.

Each basket in turn is defined over a continuum of varieties (home varieties are indexed by $i \in [0,n]$, foreign varieties by $i \in (n,1]$), with elasticity of substitution across varieties equal to $\theta > 1$:

$$C_{Hk} = \left[\left(\tfrac{1}{n}\right)^{\frac{1}{\theta}}\int_0^n (C_{Hk}(i))^{\frac{\theta-1}{\theta}}\,di\right]^{\frac{\theta}{\theta-1}},\ C_{Fk} = \left[\left(\tfrac{1}{1-n}\right)^{\frac{1}{\theta}}\int_n^1 (C_{Fk}(i))^{\frac{\theta-1}{\theta}}\,di\right]^{\frac{\theta}{\theta-1}} \tag{12}$$

We denote with $P_H(i)$ and $P_F(i)$ the home-currency prices of a home- and a foreign-produced variety, respectively. Standard optimality conditions determine the allocation of consumption across varieties:

$$C_{Hk}(i) = t_k^{\frac{1}{2}}\left(\frac{P_{Hk}(i)}{P_{Hk}}\right)^{-\theta}\left(\frac{P_{Hk}}{P_k}\right)^{-\lambda} C_k,\quad C_{Fk}(i) = t_k^{-\frac{1}{2}}\left(\frac{P_{Fk}(i)}{P_{Fk}}\right)^{-\theta}\left(\frac{P_{Fk}}{P_k}\right)^{-\lambda} C_k \tag{13}$$

where the utility-based price indexes P, P_H and P_F are the minimum expenditures required to purchase one unit of the corresponding baskets C, C_H and C_F:

$$P_k = \left[t_k^{\frac{1}{2}} n \left(P_{Hk} \right)^{1-\lambda} + t_k^{-\frac{1}{2}} \left(1-n \right) \left(P_{Fk} \right)^{1-\lambda} \right]^{\frac{1}{1-\lambda}} \tag{14}$$

$$P_{Hk} = \left[\tfrac{1}{n} \int_0^n \left(P_{Hk} \left(i \right) \right)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \qquad P_{Fk} = \left[\tfrac{1}{1-n} \int_n^1 \left(P_{Fk} \left(i \right) \right)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \tag{15}$$

Foreign variables are derived in a similar way. Total demand for home variety i is obtained by aggregating home and foreign households' consumption:

$$nC_{Hk} \left(i \right) + \left(1-n \right) C_{Hk}^* \left(i \right) = t_k^{\frac{1}{2}} \left(\frac{P_{Hk} \left(i \right)}{P_{Hk}} \right)^{-\theta} \left(\frac{P_{Hk}}{P_k} \right)^{-\lambda} \left(nC_k + \left(1-n \right) C_k^* \right) \tag{16}$$

and demand for foreign brands is similarly derived.

On the supply side, firms in each country use a constant return to scale technology with labor as the only input:

$$Y_k \left(i \right) = z_k L_k \left(i \right), \qquad Y_k^* \left(i \right) = z_k^* L_k^* \left(i \right) \tag{17}$$

where $Y_k \left(i \right)$ is production of home variety i, $L_k \left(i \right)$ is labor used in its production, and z_k is a productivity shock that is common to all home firms. Foreign variables are similarly characterized. The resource constraints in the product markets are:

$$Y_k \left(i \right) = nC_{Hk} \left(i \right) + \left(1-n \right) C_{Hk}^* \left(i \right), \qquad Y_k^* \left(i \right) = nC_{Fk} \left(i \right) + \left(1-n \right) C_{Fk}^* \left(i \right) \tag{18}$$

and in the labor markets are:

$$L_k = L_k(i), \qquad L_k^* = L_k^*(i). \tag{19}$$

Due to nominal rigidities, firms set the prices of their varieties before observing supply and demand shocks and the monetary stance. Firms stand ready to meet demand at given prices in any state of nature, so that $P_{Hk} \left(i \right) = P_H \left(i \right)$. As anticipated above, in the absence of market segmentation the law of one price holds: home firms charge the same home-currency price $P_H \left(i \right)$ for both domestic and export sales, while foreign-currency import prices move one-to-one with the exchange rate:[7]

$$P^*_{Hk}(i) = P_H(i)/S_k \tag{20}$$

Foreign variety prices are similarly characterized.

Home firm i's nominal profits are defined as:

$$\Pi_k(i) = nP_H(i)C_{Hk}(i) + (1-n)\,S_k P^*_{Hk}(i)C^*_{Hk}(i) - W_k L_k(i) \tag{21}$$

The objective of a home firm is to maximize its expected real profits, discounted across states by the marginal utility of income of the shareholder, i.e. the representative home household, μ_k:

$$P_H(i) = \arg\max_{p_H(i)} \sum_k \pi_k \frac{\mu_k}{P_k}\left(p_H(i) - \frac{W_k}{z_k}\right) t_k^{\frac{1}{2}} \left(\frac{p_H(i)}{P_{Hk}}\right)^{-\theta} \left(\frac{P_{Hk}}{P_k}\right)^{-\lambda} (nC_k + (1-n)\,C^*_k) \tag{22}$$

Note that, in the above expression, the marginal cost W_k/z_k is equal across firms. The representative firm optimally sets its price as a markup over the expected and appropriately discounted marginal cost. Observing that all home firms charge the same price in equilibrium ($P_H(i) = P_H = P_{Hk}\ \forall k$), we obtain:

$$P_H = \frac{\theta}{\theta-1}\frac{E\left(\mu t^{\frac{1}{2}} P^{\lambda-1}(nC + (1-n)\,C^*)\,W/z\right)}{E\left(\mu t^{\frac{1}{2}} P^{\lambda-1}(nC + (1-n)\,C^*)\right)}. \tag{23}$$

A similar expression holds abroad for P^*_F, while P_F and P^*_H are determined according to the law of one price.

2.3 Equilibrium

Households own local firms, so that:

$$\Pi_k(i) = \Pi_k \tag{24}$$

[7]It is straightforward to show that if a home firm were able to set the home-currency price of its exports at a different level than the price of its domestic sales, it would choose not to in equilibrium. This result need not be robust to changes in model specification: see Bacchetta and van Wincoop (2004) for a general discussion of optimal price setting in an international context

and since all securities are traded in zero net supply, we have:

$$nB_k + (1-n)\,B_k^* = 0. \tag{25}$$

Finally, in our model we abstract from government spending and assume that seigniorage revenue is rebated to a country's residents in a a lump-sum fashion:

$$M_0 - M_k + T_k = 0, \qquad M_0^* - M_k^* + T_k^* = 0 \tag{26}$$

Taking M_0 and M_0^* as given, the monetary authorities in the two countries set M_k and M_k^* after observing the realizations the demand shocks, f_k, f_k^*, and t_k, and the productivity shocks, z_k and z_k^*. Given these exogenous variables, in equilibrium the budget constraints (2) and (4), the money demands, labor supplies, optimal portfolio allocations (3) and (5), the purchasing power parity (6), the risk sharing wedge (7), the consumption demands (13) and their foreign equivalents, the price indexes (14) and (15), the technology (17), the resource constraints (18), the labor resource constraints (19), the law of one price (20), the profits of home firms (21) and their foreign equivalents, the optimal price charged by home firms (23) and its foreign equivalent, the equilibrium in the bond markets (25), and the government budget constraints (26) determine the endogenous variables $C_{Hk}(i)$, $C_{Fk}(i)$,C_{Hk}, C_{Fk}, C_k, $Y_k(i)$, $L_k(i)$, L_k, X_k, B_k, T_k, $P_H(i)$, $P_F(i)$, P_H, P_F, P_k, W_k, Π_k, their foreign analogs, q_k, S_k, and Γ.

2.4 Parametric restrictions and key intuitions

To proceed further, we impose more structure on preferences and assume the following:

$$U(C_k) = \ln C_k, \qquad V(L_k) = \kappa L_k, \qquad X\left(\frac{M_k}{P_k}\right) = \chi \ln \frac{M_k}{P_k} \tag{27}$$

which imply $\mu_k = f_k/C_k$ and, using the properties of the risk-sharing wedge (7), $C_k^* = f_k^*\,(f_k)^{-1}\,(1+\Gamma)\,C_k$. It is easy to show that under our parameterization the expected labor effort in both countries is a simple function of exogenous parameters:

$$E\left(L\right) = \frac{\theta-1}{\theta\kappa}E\left(f\right) \quad , \quad E\left(L^*\right) = \frac{\theta-1}{\theta\kappa}E\left(f^*\right) \tag{28}$$

Accounting for the expressions above, the home equilibrium price (23) can be written as:

$$P_H = \frac{\theta}{\theta - 1} \frac{E(\mathcal{D}W/z)}{E(\mathcal{D})} = \frac{\theta}{\theta - 1} \left(E(W/z) + \frac{Cov(\mathcal{D}, W/z)}{E(\mathcal{D})} \right) \tag{29}$$

The price is equal to expected marginal costs, adjusted by a markup that reflects monopoly power in the product market, and accounting for a premium related to the covariance between marginal costs and the equilibrium stochastic discount rate $\mathcal{D}_k$, equal to:

$$\mathcal{D}_k = P_k^{\lambda-1} t_k^{\frac{1}{2}} \left[n f_k + (1-n) f_k^* (1+\Gamma) \right] \tag{30}$$

This discount factor reflects the 'strength' of the demand faced by the home firm. If prices could be adjusted in response to shocks, the firm would *ex post* bring its price in line with the marginal costs W_k/z_k, that is, would reduce its price when costs are low and vice versa. But as prices cannot be adjusted, the firm has to set P_H *ex ante* by weighing marginal costs across different states of nature with respect to the demand for its product. Intuitively, any misalignment between the price P_H and the ex post marginal cost W_k/z_k is magnified in the states of nature where demand is strong, leading the firm to put more weight on these states. If marginal costs are high when demand is strong (that is, if $Cov(\mathcal{D}, W/z) > 0$), the firm charges a premium over its expected marginal cost in order to reduce markup fluctuations in those states of nature in which these fluctuations matter most.

In more detail, the discount rate $\mathcal{D}_k$ combines three dimensions. The first dimension, captured by the term $P^{\lambda-1}$, is related to the *price competitiveness* of a home firm. Since its price P_H is preset and cannot be modified after observing k, the firm is more competitive in those states in which the consumer price index P is higher (reflecting higher prices of its foreign competitors). This aspect is particularly relevant when the elasticity of substitution between home and foreign goods λ is high. However, higher realizations of P — for a given P_H — reduce the real value of the profit income accruing to shareholders. When the substitution effect prevails over the income effect, the sign of the expression $\lambda - 1$ is positive.

The second dimension, captured by the term t, is the *consumption bias* towards home goods. Trivially, a home firm faces a higher demand for its products when consumers worldwide prefer home-produced goods over foreign-produced ones.

The third dimension, captured by the term in square brackets, is related to the size of *world demand*, with the firm facing a high demand when there is a positive demand shocks in either country (a high f or f^*). Similar considerations hold for the optimal price set by foreign firms.

The previous points can be restated from a different angle. Aggregating (2) across agents and states of nature, we can derive the securitized budget constraint under complete markets:

$$\sum_k q_k C_k = \sum_k q_k \frac{P_H Y_k}{P_k} \tag{31}$$

Intuitively, the home household sells her entire income through issuance of an amount $P_H (P_k)^{-1} Y_k$ of securities paying off in state k, and finance her consumption by purchasing an amount C_k of securities paying off in state k. Using the solution for the securities prices (3), we obtain:

$$E(f) = E\left(t^{\frac{1}{2}} \left(\frac{P}{P_H}\right)^{\lambda-1} [nf + (1-n) f^* (1+\Gamma)]\right) \tag{32}$$

The above expression sheds light on the parallels between the determinants of the risk-sharing wedge Γ and the dimensions underlying the discount rate $\mathcal{D}$. First, when price competitiveness is strong (that is, $(P/P_H)^{\lambda-1}$ is high), consumers worldwide shift their demand towards home firms. When home goods are good substitutes for foreign ones (that is, $\lambda > 1$), such consumption switching is strong enough to raise the sale revenue of home firms, despite their relatively low price. The more competitive home firms are, the higher the real income accruing to the home household relative to its foreign counterpart: this translates into an average consumption differential tilted in favor of the home household (that is, $\Gamma < 0$).

Second, when the consumption bias tends to favor the home country (that is, $E\left(t^{\frac{1}{2}}\right) >$

1), consumers worldwide disproportionately allocate their consumption toward home-produced goods, raising *ceteris paribus* the income and the consumption of the home household. Third, the revenue of the home household is boosted by large exports of home goods if consumption demand is on average higher in the foreign country (that is, $E(f^*) > E(f)$ and, once again, $\Gamma < 0$).[8]

3 A second-order solution of the model

To explore the properties of the model in more detail, we approximate the solution around a non-stochastic equilibrium (baseline) where all firms produce an amount $Y_0 = (\theta - 1)(\theta\kappa)^{-1} f_0 z_0$, corresponding to the amount of consumption by each household $C_0 = Y_0$. Following Sutherland (2002) and Tille (2002), we do not limit ourselves to a first-order linearization of the model, but compute second-order approximations of the various relations. As seen above, the covariance between marginal costs and discount rate is a key determinant of the prices set by optimizing agents. Our methodology allows us to account appropriately for the second-order terms through which macroeconomic fluctuations affect prices, hence average consumption and welfare.

Specifically, a geometric relation between two variables X and Y in a state k is approximated using the following rule:

$$X_k^a Y_k^b = X_0^a Y_0^b \left[1 + a\mathsf{x}_k + b\mathsf{y}_k + \frac{1}{2}(a\mathsf{x}_k + b\mathsf{y}_k)^2\right] \tag{33}$$

where Sans Serif letters denote log deviations ($\mathsf{x}_k = \ln X_k - \ln X_0$). The second-order terms, $(a\mathsf{x}_k + b\mathsf{y}_k)^2$, can in turn be computed based on a first-order approximation of the model.

As regards the stochastic dimension of the model, we assume that the exogenous shocks are mutually independent. We also assume that the expected deviations of the shocks from

[8] This can be easily seen by setting $\lambda = 1$ and $t = 1$ for simplicity. (32) can then written as $1 + \Gamma = E(f)/E(f^*)$, so that $E(f^*) > E(f) \Rightarrow \Gamma < 0$.

the symmetric steady state ($E\mathsf{z}$, $E\mathsf{z}^*$, $E\mathsf{t}$, $E\mathsf{f}$, $E\mathsf{f}^*$, $E\mathsf{g}$, $E\mathsf{g}^*$) and the expected deviations of the monetary stances ($E\mathsf{m}$, $E\mathsf{m}^*$) are all of second order.[9]

3.1 Prices, exchange rate and risk-sharing

Consider first the quadratic approximation of the home CPI in state k:

$$\mathsf{p}_k = \left[n\mathsf{p}_H + (1-n)\left(\mathsf{s}_k + \mathsf{p}_F^*\right) + \frac{1-2n}{2(\lambda-1)}\mathsf{t}_k \right] - \frac{n(1-n)}{2(\lambda-1)}\left(\mathsf{t}_k + (\lambda-1)\,\mathsf{s}_k\right)^2 \tag{34}$$

The above expression shows that the price of a consumption basket depends not only on first-order changes in the components of the CPI, as captured by the term in square brackets, but also on a second-order term reflecting variability in (some of) these components. Intuitively, macroeconomic variability can reduce the utility-based cost of living to the extent that (other things being equal) it enables households to choose a more valuable consumption basket.

Consider for instance the role of exchange rate fluctuations, s_k. As prices are preset in the producer's currency, currency variability affects the price differential between local and imported goods, allowing households to re-allocate their consumption towards the cheaper good. The extent of such expenditure switching depends on the elasticity of substitution between home and foreign goods, λ. However, exchange rate movements also translate into fluctuations in purchasing power, and this income effect leads to a one-to-one loss in utility.[10] If $\lambda > 1$, that is, if the substitution effect prevails over the income effect, exchange

[9]Specifically, we assume that $E(\mathsf{z} - \mathsf{z}^*) = E\mathsf{t} = 0$, $E\mathsf{f} = -0.5E(\mathsf{f})^2$, $E\mathsf{f}^* = -0.5E(\mathsf{f}^*)^2$, so that $E(f) = f_0\left[1 + E\mathsf{f} + \frac{1}{2}E(\mathsf{f})^2\right] = f_0 = E(f^*)$ Our assumptions ensure that expected effort in either country (eq. (28)) does not depend on the volatility of shocks. These assumptions simplify the presentation with no loss of generality.

[10]This effect is best illustrated by setting $\lambda = \mathsf{t} = 0$ in (34) and considering how movements in relative prices increase the cost of living. Consider a case where $n = 0.25$ and $P_H = S_k P_F^* = 1$. From (13) the consumption of each brand is equal to the aggregate consumption C, that we set at 10, and the cost of living is $nP_H C + (1-n)\,S_k P_F^* C = 10$. We then change the prices to $P_H = \exp(0.06)$ and $S_k P_F^* = \exp(-0.02)$. This implies that $n\mathsf{p}_H + (1-n)\left(\mathsf{s}_k + \mathsf{p}_F^*\right) = n \cdot 0.06 - (1-n)\cdot 0.02 = 0$. Purchasing 10 units of consumption however now costs $\left[nP_H + (1-n)\,S_k P_F^*\right] \cdot 10 = 10.00608$.

rate fluctuations increase consumers' utility above the level associated with the purchase of the initial basket at the new prices, thus lowering the (appropriately measured) cost of living from the household's vantage point.[11]

Similar considerations hold in the case of consumption bias shocks. When $\lambda > 1$ movements in t_k prompt households to tilt their consumption in favor of the preferred goods, increasing utility at the margin. When λ is very high, so that world households are virtually indifferent between home and foreign goods, the welfare effects of such changes in consumption are negligible. But when λ is close to 1, these effects can be sizable. Note that when home and foreign goods are hardly substitutable ($\lambda < 1$), fluctuations in t actually *increase* the cost of living.

Turning to the price set by home firms, (29) is written as:

$$\begin{aligned} \mathsf{p}_H &= E\left(\mathsf{m}-\mathsf{z}-\mathsf{f}\right)+\frac{1}{2}E\left(\mathsf{m}-\mathsf{z}-\mathsf{f}\right)^2 \\ &\quad +E\left(\mathsf{m}-\mathsf{z}-\mathsf{f}\right)\left[\left(n\mathsf{f}+\left(1-n\right)\mathsf{f}^*\right)+\left(1-n\right)\left(\left(\lambda-1\right)\mathsf{s}+\mathsf{t}\right)\right] \end{aligned} \tag{35}$$

According to this expression, there are two reasons why home firms set their prices above baseline, i.e. $\mathsf{p}_H > E\left(\mathsf{m}-\mathsf{z}-\mathsf{f}\right)$. The first is that they anticipate fluctuations in their markups stemming from changes in marginal costs, $E\left(\mathsf{m}-\mathsf{z}-\mathsf{f}\right)^2 > 0$. This is the standard channel underlying the case for price stability.[12] The second reason is that firms expect marginal costs to be high in those states of nature in which they face a strong demand (the expression in square brackets). As seen before, this can reflect high price competitiveness due to a weak currency (i.e. $\left(\lambda-1\right)E\left(\mathsf{m}-\mathsf{z}-\mathsf{f}\right)\mathsf{s} > 0$), shifts in world preferences (i.e. $E\left(\mathsf{m}-\mathsf{z}-\mathsf{f}\right)\mathsf{t} > 0$)), or shifts in world demand (i.e. $E\left(\mathsf{m}-\mathsf{z}-\mathsf{f}\right)\left(n\mathsf{f}+\left(1-n\right)\mathsf{f}^*\right) > 0$). Similar considerations hold for foreign firms.

From (9) the exchange rate can be written as a function of the risk-sharing wedge and

[11] Similarly, accounting for purchasing power parity (6) the foreign CPI can be written as $\mathsf{p}_k^* = \mathsf{p}_k - \mathsf{s}_k = n\left(\mathsf{p}_H - \mathsf{s}_k\right) + \left(1-n\right)\mathsf{p}_F^* + \left(1-2n\right)/\left[2\left(\lambda-1\right)\right]\mathsf{t}_k - \left[\left(\lambda-1\right)\mathsf{s}_k+\mathsf{t}_k\right]^2 n\left(1-n\right)/\left[2\left(\lambda-1\right)\right]$

[12] See for instance the analysis by Corsetti and Pesenti (2004).

the relative monetary stance:

$$\mathsf{s}_k = \Gamma + (\mathsf{m}_k - \mathsf{f}_k) - (\mathsf{m}_k^* - \mathsf{f}_k^*) \tag{36}$$

Using (32), (34), (35), (36), we can solve for the risk-sharing wedge as:

$$\Gamma = \frac{\lambda - 1}{\lambda}\frac{1}{2}\left[E\left(\mathsf{m} - \mathsf{z} - \mathsf{f}\right)^2 - E\left(\mathsf{m}^* - \mathsf{z}^* - \mathsf{f}^*\right)^2\right] - \frac{1-2n}{2\lambda}E\left[\left(\lambda - 1\right)\mathsf{s} + \mathsf{t}\right]^2 + \frac{\lambda - 1}{\lambda}E\left[\left(\lambda - 1\right)\mathsf{s} + \mathsf{t}\right]\left[\left(1-n\right)\left(\mathsf{m} - \mathsf{z} - \mathsf{f}\right) + n\left(\mathsf{m}^* - \mathsf{z}^* - \mathsf{f}^*\right)\right] \tag{37}$$

This expression is key to our results. To interpret it, recall that the wedge Γ is defined in relative terms, with a positive value indicating a risk-sharing allocation skewed against the home country. Consider first the case in which $\lambda = 1$. As we have seen before, this is a benchmark parameterization according to which 'income' and 'substitution' effects of price fluctuations offset each other. In this case the expression above boils down to $\Gamma = -0.5\left(1 - 2n\right)E\left(\mathsf{t}\right)^2$. Other things being equal, shifts in consumption preferences between national goods tilt the risk-sharing allocation in favor of the smaller country. This is because the small country is highly exposed to changes in the composition of world demand, whereas the large country is relatively insulated. Worldwide shifts in preferences toward the goods produced by the large country have little economic consequence, but shifts in favor of the goods produced by the small country have a strong impact on income and consumption of the small country's household.

When $\lambda \neq 1$ the analysis becomes considerably more complicated. In what follows we focus on the case $\lambda > 1$, with the understanding that opposite considerations hold when home and foreign goods are weak substitutes in world consumption.

The first element in (37) shows that the country with the most volatile marginal costs is at a disadvantage. Intuitively, this country is fundamentally facing more risk than the other, and its firms set relatively higher prices for their products according to (35) or its foreign analog. As price competitiveness is weak, consumers worldwide shift their demand in favor of the other country. Since $\lambda > 1$, sales revenue, real incomes and consumption all fall in relative terms, despite the comparatively higher price.

Consider now the role of exchange rate fluctuations. The second term in (37) shows that currency volatility has similar implications as variability in preferences: it increases the average income and consumption in the relatively small country.[13] But this is not the end of the story. The final term in (37) captures the impact of co-movements between marginal costs and preference shifters. For instance, the home country is at a disadvantage if it faces high production costs in those states of nature in which world demand is tilted towards its good, either due to a weak home currency or a shift in preferences (that is, $E\left(\left(\lambda-1\right)\mathsf{s}+\mathsf{t}\right)\left(\mathsf{m}-\mathsf{z}-\mathsf{f}\right)>0$). Intuitively, under this scenario home firms produce more in those states in which it is more costly, prompting them to charge a premium over expected marginal costs and suffer the consequences in terms of low competitiveness.

Interestingly, the home country is also adversely affected if world demand is tilted towards home goods when foreign costs are high (that is, $E\left[\left(\lambda-1\right)\mathsf{s}+\mathsf{t}\right]\left(\mathsf{m}^{*}-\mathsf{z}^{*}-\mathsf{f}^{*}\right)>0$). Intuitively, the foreign country is clearly better off if worldwide demand systematically shifts toward home goods when foreign costs are high. Because the risk-sharing wedge is a relative terms reflecting a redistributive, zero-sum dimension, this gain for the foreign country translates into a loss for the home country.

Ultimately, a positive wedge in (37) can be interpreted as a symptom that the home country is fundamentally 'riskier' and less 'competitive' than the foreign country. A key point worth emphasizing is that monetary policy, through its influence on marginal costs and the exchange rate, can affect how risky and uncompetitive each country is *vis-à-vis* the rest of the world. The next sections focus precisely on the analysis of such monetary interdependencies.

[13] Specifically, if the home country is the smaller country, a higher volatility in $\left(\lambda-1\right)\mathsf{s}+\mathsf{t}$ increases $E\left[t^{\frac{1}{2}}\left(P/P_{H}\right)^{\lambda-1}\right]$.

3.2 The welfare objective function

The aim of monetary policy is to maximize domestic households' welfare, as captured by the utility function (1). Following the relevant literature, we abstract from the direct welfare impact of real balances. As equation (28) shows, optimal price setting by firms implies that expected labor effort is a constant, unrelated to monetary policy. Therefore, the welfare functions of the home and foreign households — written in terms of deviations from baseline — are given by:

$$\frac{U - U_0}{f_0} = E\mathsf{m} - E\mathsf{p} + E\left(\mathsf{m} - \mathsf{p}\right)\mathsf{f} \tag{38}$$

$$\frac{U^* - U_0}{f_0} = E\mathsf{m}^* - E\mathsf{p}^* + E\left(\mathsf{m}^* - \mathsf{p}^*\right)\mathsf{f}^* \tag{39}$$

Welfare is higher when the expected consumer price index is low, implying stronger purchasing power, hence higher expected consumption. Welfare is also higher if real balances and consumption tend to be high when positive demand shocks make consumption most valuable.

It is convenient to express the welfare functions in terms of deviations from the flexible price outcome. When firms can adjust their prices in response to monetary and real shocks, they bring them in line with their marginal cost:

$$P_{Hk} = \frac{\theta\kappa}{\theta - 1}\frac{P_k C_k}{z_k f_k} \quad , \quad P^*_{Fk} = \frac{\theta\kappa}{\theta - 1}\frac{P^*_k C^*_k}{z^*_k f^*_k} \tag{40}$$

so that labor effort is constant in all states of nature. Using a second-order approximation of the model, we derive the the welfare functions as:

$$\frac{U_{\text{Flex}} - U_0}{f_0} = -\left(1 - n\right)\Gamma_{\text{Flex}} + \Delta \tag{41}$$

$$\frac{U^*_{\text{Flex}} - U_0}{f_0} = n\Gamma_{\text{Flex}} + \Delta^* \tag{42}$$

$$\frac{U^{World}_{\text{Flex}} - U_0}{f_0} = n\Delta + \left(1 - n\right)\Delta^* \tag{43}$$

where Δ and Δ^* are terms related to non-monetary shocks (they are equal when f and f^* are equally volatile), the '$World$' superscript refers to the global economy as a whole, and the 'Flex' subscript denotes the outcome under flexible prices. The specific solution for Γ_{Flex} is:

$$\Gamma_{\text{Flex}} = -\frac{1-2n}{2\lambda} E\left[(\lambda-1)(\mathsf{z}-\mathsf{z}^*)+\mathsf{t}\right]^2 \tag{44}$$

Equation (44) shows the presence of a risk-sharing wedge even under flexible prices and complete markets. This wedge is similar to the second term in (37), and shows that welfare is tilted in favor of the smaller country when relative prices or relative preferences fluctuate.

Using (34), (35), (36), (37) and (44), we express the welfare of the home and foreign households as deviations from the flexible price outcome as follows:

$$\mathsf{u} = \frac{U_{\text{Sticky}} - U_{\text{Flex}}}{f_0} = -\Omega - (1-n)\left(\Gamma_{\text{Sticky}} - \Gamma_{\text{Flex}}\right) + (1-n)\Psi \tag{45}$$

$$\mathsf{u}^* = -\Omega + n\left(\Gamma_{\text{Sticky}} - \Gamma_{\text{Flex}}\right) - n\Psi \tag{46}$$

where the 'Sticky' subscript denotes the outcome under sticky prices. We have already introduced Γ above. Ω and Ψ are now defined as:

$$\Psi = E\left[n(\mathsf{m}-\mathsf{f}) + (1-n)(\mathsf{m}^*-\mathsf{f}^*)\right](\mathsf{f}-\mathsf{f}^*) \tag{47}$$

and:

$$\begin{aligned}\Omega = {} & \frac{1}{2}\Big[nE(\mathsf{m}-\mathsf{z}-\mathsf{f})^2 + (1-n)E(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*)^2 \\ & + (\lambda-1)n(1-n)E\left[\mathsf{s}-(\mathsf{z}-\mathsf{z}^*)\right]^2\Big]\end{aligned} \tag{48}$$

Similar to the risk-sharing wedge Γ, the term Ψ reflects a redistributive, 'beggar-thy-neighbor' dimension of macroeconomic interdependence (both terms enter the utility functions with opposite signs and their weighted sum is zero in worldwide terms).[14] We will refer to Ψ as the 'demand' wedge. To interpret it, recall the if the home monetary authorities systematically adopt a contractionary stance in response to demand shocks from either country, they contribute to lower home prices according to the optimal pricing rule (35). However, if the home authorities take an expansionary stance following a positive demand

[14]Specifically: $n\left[-(1-n)(\Gamma-\Psi)\right] + (1-n)\left[n(\Gamma-\Psi)\right] = 0$.

shock in their own country, they are able to boost consumption when it is most valuable, as shown by the last term in (38). The second effect prevails in welfare terms in the presence of domestic demand shocks, while only the first effect is relevant if demand shocks originate abroad. As a result, national welfare increases if monetary authorities in both countries respond in an expansionary way to domestic demand shocks, but in a contractionary way to demand shocks abroad.

The term Ω, that is common to both countries, reflects the benefits from economic stabilization worldwide. When the monetary authorities reduce fluctuations in the marginal costs of home and foreign firms, minimizing $E(\mathsf{m}-\mathsf{z}-\mathsf{f})^2$ and $E(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*)^2$, firms worldwide set low prices for their products and consumers' purchasing power rises. In other words, monetary stabilization contributes to undo the distortions of sticky prices, as with constant marginal costs firms have no reason to charge a premium over expected marginal costs.

The stabilization component Ω also also calls for the monetary authorities to generate 'efficient' exchange rate movements. If prices were flexible, relative prices would move in line with the relative productivity shocks. Under sticky prices the relative price of home and foreign goods is affected only by the exchange rate. Generating efficient relative price movements then calls for the exchange rate to track the relative productivity shocks: $s_k = z_k - z_k^*$. To the extent that $\lambda > 1$ and the substitutability between home and foreign goods is relatively high, efficient exchange rate movements reduce the risk that relative price misalignments may lead to substantial inefficient consumption switching between home and foreign goods.

To gain some insight, consider the case where there are no productivity or demand shocks. The welfare components are then:

$$\Gamma_{\text{Sticky}} = \frac{\lambda-1}{2}\left[E(\mathsf{m})^2 - E(\mathsf{m}^*)^2\right] \qquad \Psi = \Delta = \Delta^* = 0 \tag{49}$$

$$\Omega = \frac{1}{2}\left[nE(\mathsf{m})^2 + (1-n)E(\mathsf{m}^*)^2 + (\lambda-1)n(1-n)E(\mathsf{m}-\mathsf{m}^*)^2\right] \tag{50}$$

The risk-sharing wedge provides an incentive to reduce monetary volatility *relative* to the rest of the world, as welfare is tilted against the more volatile country: Γ is positive when the variance of m exceeds the variance of m^*, so that — other things being equal, u falls relative to u^*. The stabilization term, Ω, calls for the authorities to minimize monetary volatility *worldwide*. The monetary authorities in either countries therefore have no incentive to generate monetary volatility for its own sake.

4 Optimal policy responses to macroeconomic shocks

In this section we focus on the optimal monetary stances in the various states, m_k and m_k^*, that maximize households' welfare. We distinguish between cooperative and non-cooperative rules. Under cooperation the monetary stances jointly maximize the weighted average of welfare:

$$\mathsf{u}_{\text{Sticky}}^{World} = n\mathsf{u} + (1-n)\,\mathsf{u}^* = -\Omega \tag{51}$$

The risk-sharing and demand wedges do not enter the global objective function, as it represents a pure 'beggar-thy-neighbor' component. The optimal cooperative monetary policy then calls for an inward-looking rule such that the monetary authority in each country fully stabilizes marginal costs:

$$\mathsf{m}_{k\text{Coop}} = \mathsf{z}_k + \mathsf{f}_k \qquad\qquad \mathsf{m}_{k\text{Coop}}^* = \mathsf{z}_k^* + \mathsf{f}_k^* \tag{52}$$

This policy combination allows the world economy to replicate the flexible price outcome. In particular, the exchange rate mirrors the behavior of relative productivity shocks to a first order ($\mathsf{s}_k = \mathsf{z}_k - \mathsf{z}_k^*$). Worldwide output is fully stabilized and the risk-sharing wedge only reflects asymmetries in country size as described above:

$$\Omega_{\text{Coop}} = 0 \quad , \quad \Gamma_{\text{Coop}} = \Gamma_{\text{Flex}} \quad , \quad \Psi_{\text{Coop}} = 0 \tag{53}$$

In the absence of cooperation, each national policymaker aims at maximizing its country resident's welfare, leading to the following set of self-explanatory first-order conditions:

$$\frac{\partial \mathsf{u}}{\partial \mathsf{m}_{k\mathrm{Nash}}} = -(1-n)\frac{\partial \Gamma_{\mathrm{Sticky}}}{\partial \mathsf{m}_{k\mathrm{Nash}}} + (1-n)\frac{\partial \Psi}{\partial \mathsf{m}_{k\mathrm{Nash}}} - \frac{\partial \Omega}{\partial \mathsf{m}_{k\mathrm{Nash}}} = 0 \tag{54}$$

$$\frac{\partial \mathsf{u}^*}{\partial \mathsf{m}^*_{k\mathrm{Nash}}} = n\frac{\partial \Gamma_{\mathrm{Sticky}}}{\partial \mathsf{m}^*_{k\mathrm{Nash}}} - n\frac{\partial \Psi}{\partial \mathsf{m}^*_{k\mathrm{Nash}}} - \frac{\partial \Omega}{\partial \mathsf{m}^*_{k\mathrm{Nash}}} = 0 \tag{55}$$

This conditions are not met under the cooperative policy (52). While the cooperative allocation is optimal from the point of view of stabilization, the national monetary authority has an incentive to deviate from the cooperative stance by tilting the risk-sharing wedge Γ_{Sticky} and the demand wedge Ψ in favor of the country's residents.

Specifically, the optimal policy stances in a Nash equilibrium are:

$$\mathsf{m}_{k\mathrm{Nash}} = (\mathsf{z}_k + \mathsf{f}_k) - \frac{n(1-n)}{\Theta}\left[(1-n) + n(\lambda-1)\right]\mathsf{T}_k \tag{56}$$

$$\mathsf{m}^*_{k\mathrm{Nash}} = (\mathsf{z}^*_k + \mathsf{f}^*_k) + \frac{n(1-n)}{\Theta}\left[n + (\lambda-1)(1-n)\right]\mathsf{T}_k \tag{57}$$

where:

$$\mathsf{T}_k = \frac{\lambda-1}{\lambda}\left[(\lambda-1)(\mathsf{z}_k - \mathsf{z}^*_k) + \mathsf{t}_k\right] - (\mathsf{f}_k - \mathsf{f}^*_k)$$

$$\Theta = n(1-n) + (1 - n(1-n))(\lambda-1) + 2n(1-n)(\lambda-1)^2 > 0$$

The reaction function (56) implies that, in the absence of coordination, the home monetary authorities under-react to domestic productivity shocks (that is, $\partial \mathsf{m}_{k\mathrm{Nash}}/\partial \mathsf{z}_k < 1$), and over-react to foreign ones (that is, $\partial \mathsf{m}_{k\mathrm{Nash}}/\partial \mathsf{z}^*_k > 0$). They also under-react to preference-shifting shocks (that is, $\partial \mathsf{m}_{k\mathrm{Nash}}/\partial \mathsf{t}_k < 0$). Finally, they over-react to domestic demand shocks (that is, $\partial \mathsf{m}_{k\mathrm{Nash}}/\partial \mathsf{f}_k > 1$) and under-react to foreign ones (that is, $\partial \mathsf{m}_{k\mathrm{Nash}}/\partial \mathsf{f}^*_k < 0$).

The intuition underlying the response of the home authorities to a domestic productivity expansion ($z_k > 0$) is as follows. Optimal stabilization, captured by the term Ω, requires an exact offset to keep marginal costs and markups constant (that is, $m_k = z_k$). Under a Nash equilibrium, however, the home authorities can also choose to influence the risk-sharing wedge (37) in order to make the home country more competitive than the rest of the world. On the one hand, this induces them to reduce the volatility of marginal costs, which is consistent with global stabilization. On the other hand, however, it also prompts them to generate negative co-movements between marginal costs and the exchange rate (that is $E\left(\mathsf{s}\left(\mathsf{m}-\mathsf{z}\right)\right)<0$), so that world demand shifts on average towards the home country when its production costs are low. To achieve this result, the monetary expansion that depreciates the exchange rate must be smaller than the productivity shock, leading to partial stabilization of the marginal cost. The incentive to tilt the risk-sharing wedge in favor of the home country therefore leads to monetary under-reaction to domestic productivity shocks.

In response to foreign productivity expansions ($z_k^* > 0$) which reduce foreign marginal costs, the home authorities have an incentive to tilt the risk-sharing wedge in their favor by switching world demand away from the foreign country, as captured by the last term of (37). Such expenditure switching can be engineered through a weakening of the home currency brought about by a monetary expansion.

Turning to an expansionary demand shock in the home country ($f_k > 0$), markup stabilization requires an exact offset by the home authorities (that is, $m_k = f_k$). Under a Nash outcome, however, the monetary expansion is even larger, as policymakers have an incentive to boost home consumption when it is most valuable to local residents, as captured by the term Ψ. In the case of an expansionary demand shock in the foreign country ($f_k^* > 0$), optimal stabilization requires no action by the home authorities. In the absence of cooperation, however, they adopt a contractionary stance in order to reduce home marginal costs in face of high foreign demand, thereby lowering home prices.

Finally, following a preference-shifting shock that moves world demand towards home goods ($t_k > 0$), optimal stabilization calls for the home monetary authorities to take no action. They however have an incentive to alter the risk sharing wedge by reducing home marginal cost when world demand is tilted towards the home country, as captured by (37), leading to a systematic contractionary response.

The exchange rate correlation with productivity shocks is then dampened under a Nash equilibrium relative to the cooperative outcome. To a first order we can in fact write:

$$\mathsf{s}_{k\text{Nash}} = (\mathsf{z}_k - \mathsf{z}_k^*) - \frac{n(1-n)}{\Theta} \lambda \mathsf{T}_k \tag{58}$$

The above expression (58) shows that the absence of cooperation leads to inefficiently *small* movements in the exchange rate in response to productivity shocks (that is, $\partial \mathsf{s}_{k\text{Nash}} / \partial (\mathsf{z}_k - \mathsf{z}_k^*) < 1$). This occurs because each authority needs to balance two conflicting objectives, stabilizing the economy — which calls for large exchange rate movements — versus enhancing firms' competitiveness at the expense of the rest of the world — which calls for smaller exchange rate movements. By contrast, the absence of cooperation leads to *excessive* movements in the exchange rate in response to preference-shifting shocks (that is, $\partial \mathsf{s}_{k\text{Nash}} / \partial \mathsf{t}_k \neq 0$) and demand shocks (that is, $\partial \mathsf{s}_{k\text{Nash}} / \partial (\mathsf{f}_k - \mathsf{f}_k^*) \neq 0$).

In welfare terms, the non-cooperative allocation is unambiguously suboptimal in terms of stabilization of the world economy, as Ω is positive in a Nash equilibrium. However, the welfare comparison is less straightforward in terms of the risk-sharing and demand wedges. The latter can be written as:

$$\Gamma_{\text{Nash}} - \Psi_{\text{Nash}} = \Phi_1 \Gamma_{\text{Coop}} + \Gamma_{\text{Nash}}^d \tag{59}$$

where:

$$\Gamma_{\text{Nash}}^d = (1 - 2n)(\lambda - 1) \Phi_2 E (\mathsf{f} - \mathsf{f}^*)^2 \tag{60}$$

with $0 \leq \Phi_1 \leq 1$ and $\Phi_2 > 0$.

It is convenient to distinguish between the components of the wedge that reflect productivity and consumption-switching shocks, $\Phi_1 \Gamma_{\text{Coop}}$, and the components reflecting demand shocks, Γ_{Nash}^d. With respect to the former, the sign of the wedge is the same with and without cooperation. Specifically, there is no wedge when the two countries are equally sized ($n = 0.5$) and there is a wedge in favor of the small country otherwise ($n < 0.5 \Rightarrow \Gamma_{\text{Coop}} < 0$).

The magnitude of the wedge is however affected by whether the two countries cooperate or not, with the wedge being smaller in absolute value under a Nash equilibrium. As the wedge translates directly into the cross-country welfare differential, the small country is better off under a cooperative outcome, while the large country prefers the Nash outcome. In terms of the second component there is a gap between the wedge under Nash and under cooperation, reflecting the volatility of relative demands shocks. This gap is again tilted in favor of the large country.

We illustrate our results by looking at a simple numerical example. We set the substitutability between home and foreign goods, λ, to 3. First we contrast the response of the exchange rate under cooperative and Nash policies. Figure 1 illustrates the exchange rate response to a technology shock, $z - z^*$, under a cooperative regime (circled line) and under a Nash equilibrium (thick line), and shows that exchange rate movements are limited in the absence of cooperation. By contrast, the exchange rate moves in response to a switching shock, t, and a relative demand shock, $f - f^*$, under a Nash equilibrium, while no reaction is warranted in a cooperative setup (Figures 2 and 3).

Turning to the welfare effect, we compute the gain associated with moving away from the Nash outcome toward the cooperative outcome for the home country (thin solid line), the foreign country (thin dotted line) and the world (thick solid line). Figures 4 - 6 show these gains for productivity, demand and demand switching shocks respectively.[15] While the levels of the welfare gains of cooperation differ across shocks, the pattern is identical. Specifically, the worldwide gain from cooperation is maximized when the two countries are identical ($n = 0.5$). When the sizes of the countries vary however, the small country achieves large welfare gains by entering a cooperative agreement, while the large country gains much less and actually loses for a broad range of the parameters. This indicates that cooperation

[15] In all three cases shocks are independent across countries. We set $E(\mathsf{z})^2 = E(\mathsf{z}^*)^2 = 1\%$ in figure 4, $E(\mathsf{f})^2 = E(\mathsf{f}^*)^2 = 1\%$ in figure 5, and $E(\mathsf{t})^2 = 1\%$ in figure 6.

is unlikely to be easily implemented between a small and a large country, as the large country tends to be better off under a non-cooperative regime.

5 Conclusion

This paper develops a minimalist choice-theoretic model of international monetary interdependence. We allow for fluctuations in the cross-country distribution of income by relaxing the assumption that movements in international relative prices are exactly offset by movements in relative quantities. Such fluctuations generate a need for risk-sharing, which takes place through international trade in a complete set of state-contingent securities.

We deliberately keep the complexity of the model at a minimum, in order to present a detailed intuitive exposition of the mechanisms at work. Our analysis highlights two conflicting goals for policy makers. Their first aim is to fully compensate for the presence of price rigidities by bringing the economy to the allocation it would reach under flexible prices. This stabilization objective entails no international conflict, and leads policymakers to stabilize firms' marginal costs in order to bring absolute prices as low as possible, thereby maximizing purchasing power and consumption.

The second policymakers' aim is to alter the terms of international risk-sharing in their favor. We emphasize the role of the asset markets, and show how the characteristics of the risk-sharing market are affected by monetary policy, in turn affecting relative prices in the international product markets. Policy makers have an incentive to influence the international competitiveness of domestic firms by reducing the price of domestic goods across all states of nature, relative to the price of foreign goods. With world demand being highly responsive to prices, a reduction in relative prices leads to a shift of world income towards domestic producers.

The influence of monetary policy on the external competitiveness of national firms is a pure zero-sum transfer element that entails no welfare benefit from a worldwide perspective. An optimal coordinated monetary policy then ignores this aspect and focuses solely on bringing the economy around the obstacle of price rigidities. By contrast, policymakers acting in a decentralized fashion balance stabilizing marginal costs against improving international competitiveness. We show that while the decentralized allocation is suboptimal from a worldwide point of view, it benefits a large country at the expense of a smaller one, making cooperative agreements unlikely to be reached and enforced.

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Figure 1: Exchange rate response to a unit productivity shock, z

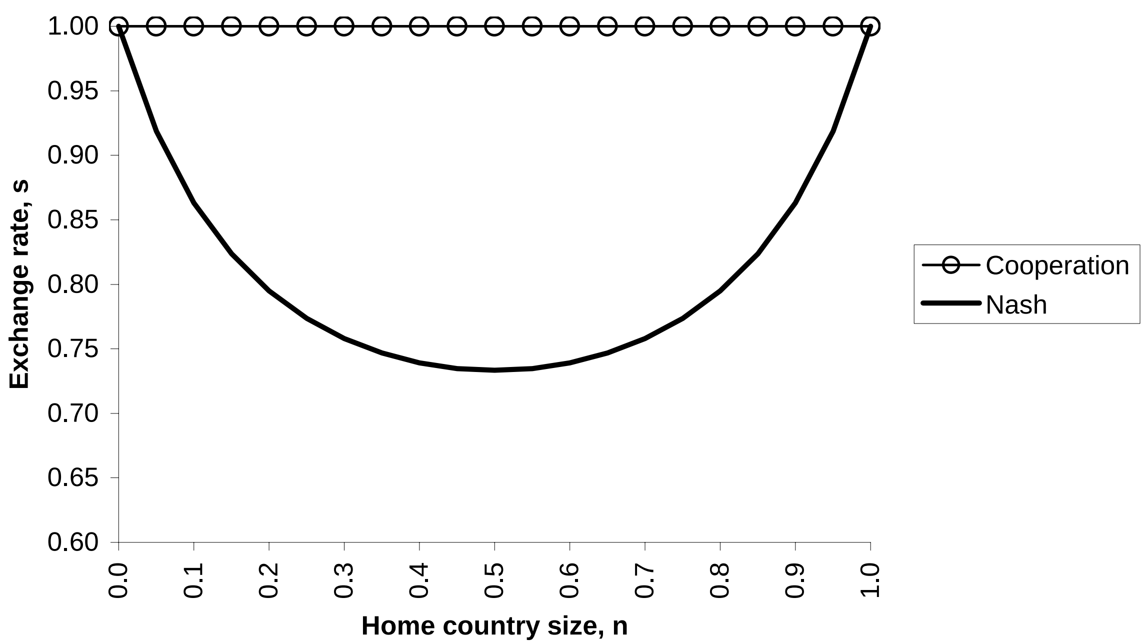


Figure 2: Exchange rate response to a unit relative demand shock, t

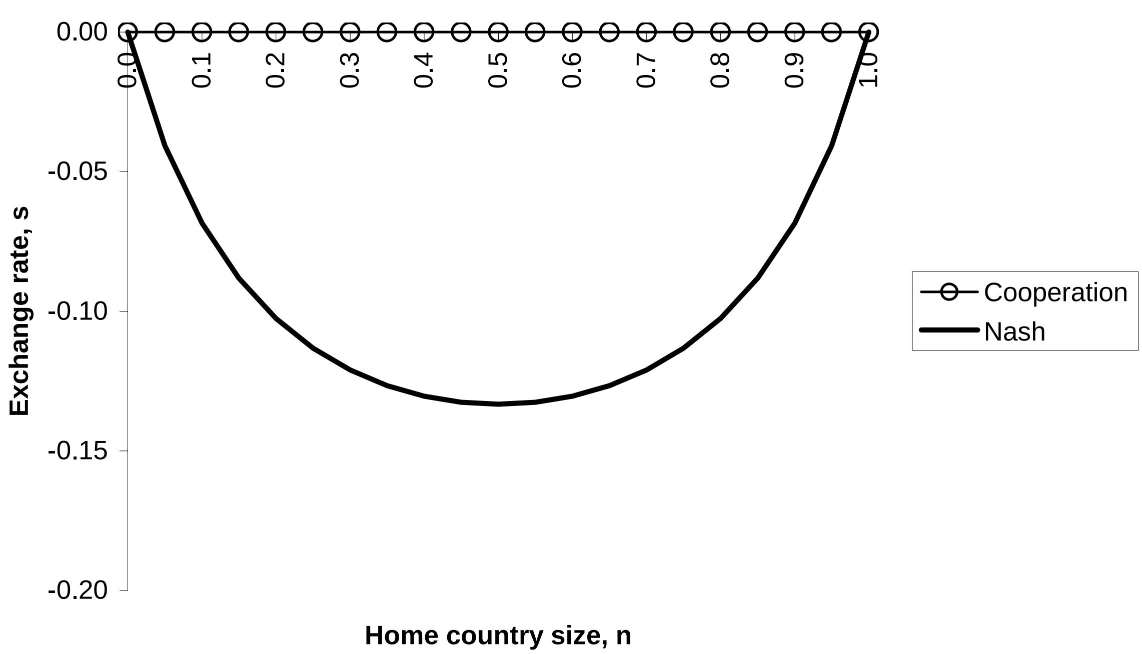


Figure 3: Exchange rate response to a unit demand shock, f

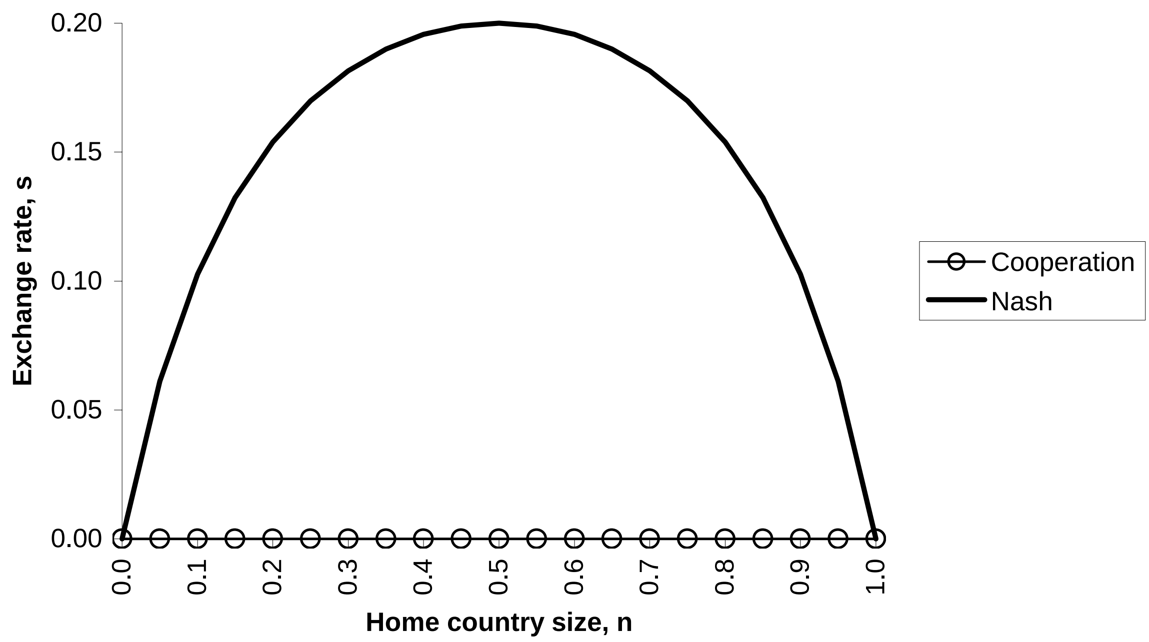


Figure 4: Gains from cooperation, productivity shocks

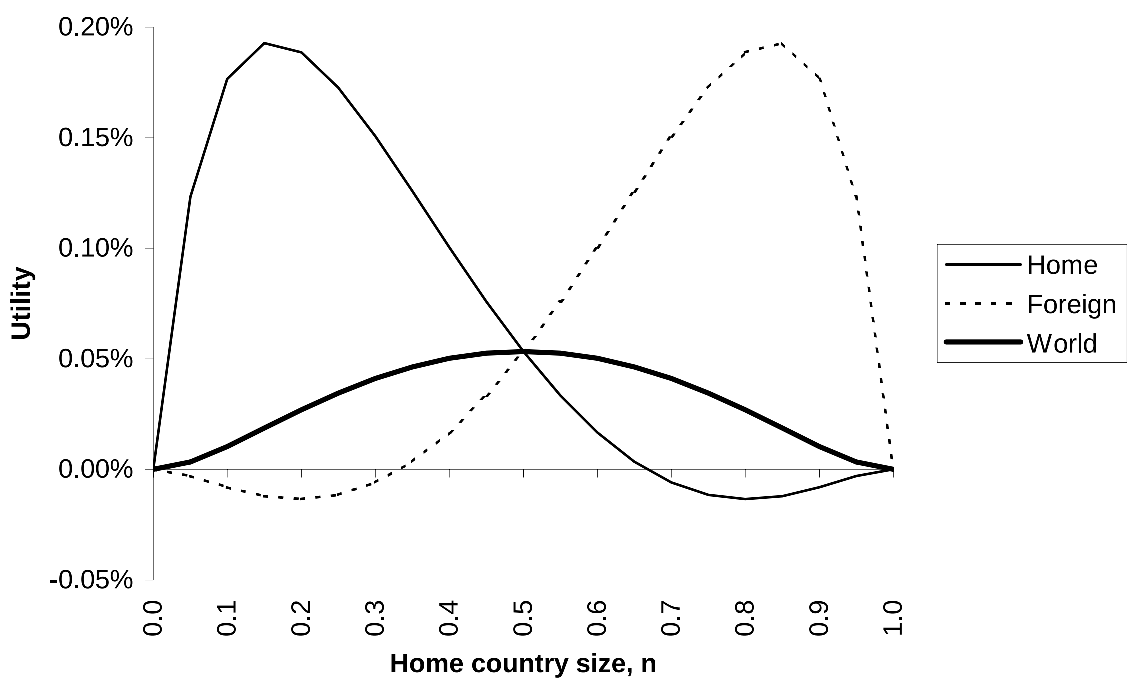


Figure 5: Gains from cooperation, demand shocks

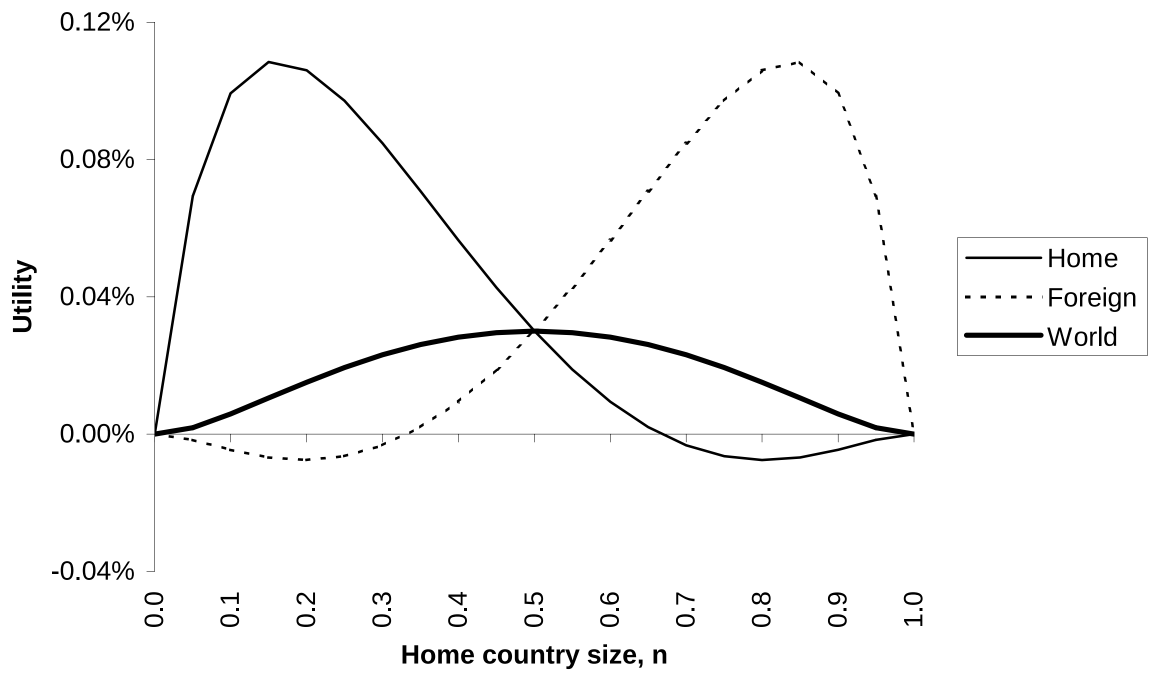
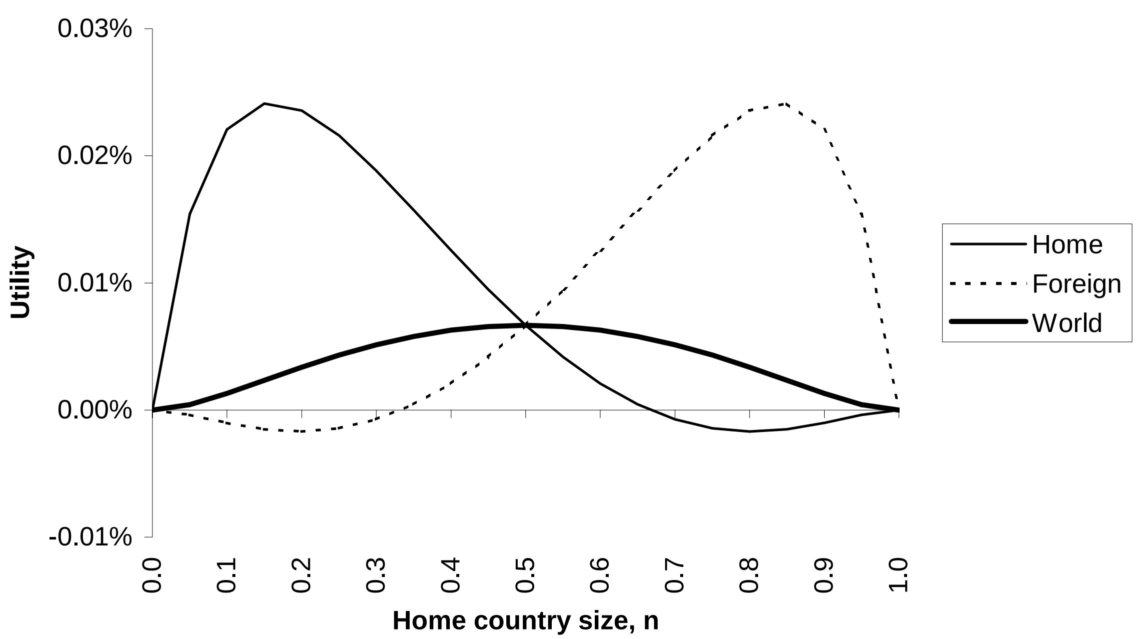


Figure 6: Gains from cooperation, relative demand shocks



Appendix

Household optimization

Money demand, labor supply and portfolio allocation

The optimization problem of the home household is written in a Lagrangian form as:

$$\begin{aligned}\mathcal{L} &= \sum_k \pi_k \left[f_k g_k \ln C_k - \kappa L_k + \chi f_k \ln\left(\frac{M_k}{P_k}\right)\right] \\ &\quad - \sum_k \pi_k \mu_k \left[C_k - \frac{W_k L_k}{P_k} - \frac{\Pi_k}{P_k} - \frac{M_0 - M_k}{P_k} - \frac{T_k}{P_k} - B_k + \sum_h q_h B_h \right]\end{aligned}$$

where g_k is a demand shock that affects the relative weight of consumption and real balances. The first order conditions with respect to C_k, L_k, M_k and B_k lead to:

$$\mu_k = \frac{f_k g_k}{C_k} \qquad \frac{W_k}{P_k} = \frac{\kappa}{\mu_k} \qquad \frac{M_k}{P_k} = \frac{\chi f_k}{\mu_k} \qquad \pi_k \mu_k = q_k E \mu$$

$$\Rightarrow M_k = \frac{\chi}{g_k} P_k C_k \qquad W_k = \frac{\kappa}{\chi f_k} M_k \qquad q_k = \frac{\pi_k (f_k g_k)(C_k)^{-1}}{E(fg)(C)^{-1}} \tag{A.1}$$

Similarly, the optimization problem of the foreign household is written in a Lagrangian form as:

$$\begin{aligned}\mathcal{L}^* &= \sum_k \pi_k \left[f_k^* g_k^* \ln C_k^* - \kappa L_k^* + \chi f_k^* \ln\left(\frac{M_k^*}{P_k^*}\right)\right] \\ &\quad - \sum_k \pi_k \mu_k^* \left[C_k^* - \frac{W_k^* L_k^*}{P_k^*} - \frac{\Pi_k^*}{P_k^*} - \frac{M_0^* - M_k^*}{P_k^*} - \frac{T_k^*}{P_k^*} - B_k^* + \sum_h q_h B_h^* \right]\end{aligned}$$

where S_k is the exchange rate, defined as the amount of home currency required to purchase one unit of foreign currency. The first order conditions with respect to C_k^*, L_k^*, M_k^* and B_k^* lead to:

$$M_k^* = \frac{\chi}{g_k^*} P_k^* C_k^* \qquad W_k^* = \frac{\kappa}{\chi f_k^*} M_k^* \qquad q_k = \frac{\pi_k (f_k^* g_k^*)(C_k^*)^{-1}}{E(f^* g^*)(C^*)^{-1}} \tag{A.2}$$

Combining the optimal portfolio allocation for the home and foreign households (A.1)-(A.2) we obtain the risk sharing wedge:

$$\frac{C_k^*}{C_k} = \frac{f_k^* g_k^*}{f_k g_k} \frac{E(fg)(C)^{-1}}{E(f^* g^*)(C^*)^{-1}} = \frac{f_k^* g_k^*}{f_k g_k}(1 + \Gamma) \tag{A.3}$$

Combining the risk sharing relation (A.3) with the money demands (A.1)-(A.2) we obtain the exchange rate solution:

$$S_k = \frac{M_k}{M_k^*}\frac{f_k^*}{f_k}(1+\Gamma) \tag{A.4}$$

Consumption allocation

The consumption indexes for the home household are:

$$\begin{aligned} C_k &= \left[\left(nt_k^{\frac{1}{2}}\right)^{\frac{1}{\lambda}}(C_{Hk})^{\frac{\lambda-1}{\lambda}} + \left((1-n)\,t_k^{-\frac{1}{2}}\right)^{\frac{1}{\lambda}}(C_{Fk})^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}} \\ C_{Hk} &= \left[\left(\tfrac{1}{n}\right)^{\frac{1}{\theta}}\int_0^n (C_{Hk}(i))^{\frac{\theta-1}{\theta}}\,di\right]^{\frac{\theta}{\theta-1}} \qquad C_{Fk} = \left[\left(\tfrac{1}{1-n}\right)^{\frac{1}{\theta}}\int_n^1 (C_{Fk}(i))^{\frac{\theta-1}{\theta}}\,di\right]^{\frac{\theta}{\theta-1}} \end{aligned}$$

where t_k is a shock on the relative utility of home- and foreign-produced goods. The allocation of consumption is:

$$\begin{aligned} C_{Hk}(i) &= t_k^{\frac{1}{2}}\left[\frac{P_{Hk}(i)}{P_{Hk}}\right]^{-\theta}\left[\frac{P_{Hk}}{P_k}\right]^{-\lambda} C_k \\ C_{Fk}(i) &= t_k^{-\frac{1}{2}}\left[\frac{P_{Fk}(i)}{P_{Fk}}\right]^{-\theta}\left[\frac{P_{Fk}}{P_k}\right]^{-\lambda} C_k \end{aligned} \tag{A.5}$$

where the price indexes are:

$$\begin{aligned} P_k &= \left[t_k^{\frac{1}{2}} n\,[P_{Hk}]^{1-\lambda} + t_k^{-\frac{1}{2}}(1-n)\,[P_{Fk}]^{1-\lambda}\right]^{\frac{1}{1-\lambda}} \\ P_{Hk} &= \left[\tfrac{1}{n}\int_0^n [P_{Hk}(i)]^{1-\theta}\,di\right]^{\frac{1}{1-\theta}} \qquad P_{Fk} = \left[\tfrac{1}{1-n}\int_n^1 [P_{Fk}(i)]^{1-\theta}\,di\right]^{\frac{1}{1-\theta}} \end{aligned} \tag{A.6}$$

Note that when all firms in a given country set identical prices ($P_H(i) = P_H$ and $P_F(i) = P_F$) the relative consumption of home and foreign goods is driven by the relative price and the demand shock t_k:

$$\frac{C_{Hk}(i)}{C_{Fk}(i)} = t_k\left[\frac{P_{Hk}}{P_{Fk}}\right]^{-\lambda}$$

The consumption allocation for the foreign household is similar:

$$\begin{aligned} C_{Hk}^*(i) &= t_k^{\frac{1}{2}}\left[\frac{P_{Hk}^*(i)}{P_{Hk}^*}\right]^{-\theta}\left[\frac{P_{Hk}^*}{P_k^*}\right]^{-\lambda} C_k^* \\ C_{Fk}^*(i) &= t_k^{-\frac{1}{2}}\left[\frac{P_{Fk}^*(i)}{P_{Fk}^*}\right]^{-\theta}\left[\frac{P_{Fk}^*}{P_k^*}\right]^{-\lambda} C_k^* \end{aligned} \tag{A.7}$$

where we assume that the relative demand shock t_k equally affects the consumption pattern of the home and foreign households. Under the assumption of the law of one price, purchasing parity also holds:

$$P_{Hk}(i) = S_k P^*_{Hk}(i) \qquad P_{Fk}(i) = S_k P^*_{Fk}(i) \qquad P_k = S_k P^*_k \tag{A.8}$$

Firms optimization

The demands faced by a typical home and foreign firm are given by aggregating (A.5) and (A.7) across the home and foreign households:

$$\begin{aligned} Y_k(i) &= t_k^{\frac{1}{2}} \left[\frac{P_{Hk}(i)}{P_{Hk}}\right]^{-\theta} \left[\frac{P_{Hk}}{P_k}\right]^{-\lambda} \left(nC_k + (1-n) C^*_k\right) && \text{(A.9)} \\ Y^*_k(i) &= t_k^{-\frac{1}{2}} \left[\frac{P^*_{Fk}(i)}{P^*_{Fk}}\right]^{-\theta} \left[\frac{P^*_{Fk}}{P^*_k}\right]^{-\lambda} \left(nC_k + (1-n) C^*_k\right) && \text{(A.10)} \end{aligned}$$

A home firms sets a price in its own currency, $P_H(i)$, to maximize the expected discounted profits:

$$E\frac{\mu}{P}\left[P_H(i) - W(z)^{-1}\right] t^{\frac{1}{2}} \left[\frac{P_H(i)}{P_H}\right]^{-\theta} \left[\frac{P_H}{P}\right]^{-\lambda} \left(nC + (1-n) C^*\right)$$

where z is the productivity level common to all home firms. Using the money demand, the labor supply, the expression for the marginal utility of income in (A.1), and the risk sharing wedge (A.3), the optimal price is written as:

$$P_H(i) = P_H = \frac{\theta}{\theta-1}\frac{\kappa}{\chi}\frac{Et^{\frac{1}{2}} M (fz)^{-1} [P]^{\lambda-1} \left[nfg + (1-n) f^* g^* (1+\Gamma)\right]}{Et^{\frac{1}{2}} [P]^{\lambda-1} \left[nfg + (1-n) f^* g^* (1+\Gamma)\right]} \tag{A.11}$$

Similarly, a foreign firm sets a price P^*_{Fk} to maximize:

$$E\mu^*\left[P^*_F(i) - W^*(z^*)^{-1}\right] t^{-\frac{1}{2}} \left[\frac{P^*_F(i)}{P^*_F}\right]^{-\theta} \left[\frac{P^*_F}{P^*}\right]^{-\lambda} \left(nC + (1-n) C^*\right)$$

leading to:

$$P^*_F(i) = P^*_F = \frac{\theta}{\theta-1}\frac{\kappa}{\chi}\frac{Et^{-\frac{1}{2}} M^* (f^* z^*)^{-1} [P]^{\lambda-1} [S]^{1-\lambda} \left[nfg\frac{1}{1+\Gamma} + f^* g^* (1-n)\right]}{Et^{-\frac{1}{2}} [P]^{\lambda-1} [S]^{1-\lambda} \left[nfg\frac{1}{1+\Gamma} + f^* g^* (1-n)\right]} \tag{A.12}$$

Risk sharing wedge

Recalling that all home firms are identical in equilibrium, the sales revenue is equal to the sum of the wage bill and profits:

$$W_k L_k + \Pi_k = P_H Y_k$$

The budget constraint of the home household in state k (2) is then written as:

$$C_k = \frac{P_H Y_k}{P_k} + B_k - \sum_h q_h B_h$$

Multiplying both sides by q_k and summing across states k we write:

$$\sum_k q_k C_k = \sum_k q_k \frac{P_H Y_k}{P_k} + \sum_k q_k B_k - \sum_k q_k \sum_h q_h B_h$$

From the optimal portfolio condition in (A.1) we see that:

$$\sum_k q_k = \sum_k \frac{\pi_k \left(f_k g_k\right) \left(C_k\right)^{-1}}{E\left(fg\right)\left(C\right)^{-1}} = \frac{\sum_k \pi_k \left(f_k g_k\right) \left(C_k\right)^{-1}}{E\left(fg\right)\left(C\right)^{-1}} = 1$$

$$\Rightarrow \sum_k q_k B_k - \sum_k q_k \sum_h q_h B_h = \sum_k q_k B_k - \sum_h q_h B_h = 0$$

Using this result we derive:

$$\sum_k q_k C_k = \sum_k q_k \frac{P_H Y_k}{P_k}$$

We use (A.9) to substitute for the output and obtain:

$$\sum_k q_k C_k = \left(P_H\right)^{1-\lambda} \sum_k q_k t_k^{\frac{1}{2}} \left(P_k\right)^{\lambda-1} \left(nC_k + (1-n)\, C_k^*\right)$$

Using the optimal portfolio condition in (A.1) to substitute for q_k this becomes:

$$\sum_k \pi_k f_k g_k = \left(P_H\right)^{1-\lambda} \sum_k \pi_k f_k g_k t_k^{\frac{1}{2}} \left(P_k\right)^{\lambda-1} \left[n + (1-n)\frac{C_k^*}{C_k}\right]$$

Using the risk sharing wedge (A.3) we obtain:

$$Efg = \left(P_H\right)^{1-\lambda} Et^{\frac{1}{2}} \left(P\right)^{\lambda-1} \left[nfg + (1-n)\, f^* g^* \left(1+\Gamma\right)\right] \tag{A.13}$$

A similar relation can be written using the foreign budget constraint, leading to:

$$Ef^*g^* = \left(P_F^*\right)^{1-\lambda} Et^{-\frac{1}{2}} \left(P^*\right)^{\lambda-1} \left[nfg\frac{1}{1+\Gamma} + f^*g^*\left(1-n\right)\right]$$

Expected effort

From the demand faced by a home firm (A.9) we write:

$$\begin{aligned}(P_k)^{\lambda-1} &= (P_H)^{\lambda} Y_k (P_k)^{-1} t_k^{-\frac{1}{2}} (nC_k + (1-n) C_k^*)^{-1} \\ &= (P_H)^{\lambda} Y_k (P_k C_k)^{-1} t_k^{-\frac{1}{2}} fg \left[nfg + (1-n) f^* g^* (1+\Gamma)\right]^{-1}\end{aligned}$$

We use this result to substitute for $(P_k)^{\lambda-1}$ in the numerator and denominators of the optimal price (A.11) and write

$$P_H = \frac{\theta}{\theta-1} \frac{\kappa}{\chi} \frac{EgM(z)^{-1} Y (PC)^{-1}}{EfgY(PC)^{-1}}$$

Using the money demand in (A.1), we obtain:

$$P_H EfgY(PC)^{-1} = \frac{\theta\kappa}{\theta-1} EY(z)^{-1} = \frac{\theta\kappa}{\theta-1} EL$$

Using the output demand faced by a home firm (A.9) we obtain:

$$(P_H)^{1-\lambda} Et^{\frac{1}{2}} (P)^{\lambda-1} \left[nfg + (1-n) f^* g^* (1+\Gamma)\right] = \frac{\theta\kappa}{\theta-1} EL$$

Using the relation between the risk sharing wedge and the prices (A.13), we write:

$$\kappa EL = \frac{\theta-1}{\theta} Efg \tag{A.14}$$

Following similar steps for the foreign expected effort we get:

$$\kappa EL^* = \frac{\theta-1}{\theta} Ef^* g^* \tag{A.15}$$

Second order approximations

Symmetric steady state and method

We expand the model around a symmetric steady-state where the parameters are the same in both countries:

$$f_0 = f_0^* \quad , \quad g_0 = g_0^* \quad , \quad z_0 = z_0^* \quad , \quad t_0 = 1$$

The consumption and price levels are then:

$$C_0 = \frac{\theta - 1}{\theta\kappa} f_0 g_0 z_0 \qquad P_0 = \frac{\theta\kappa}{\theta - 1}\frac{M_0}{\chi f_0 z_0}$$

We write the variables in terms of second order expansions, using the following relation:

$$X_k^a K_k^b = X_0^a K_0^b \left[1 + a\mathsf{x}_k + b\mathsf{k}_k + \frac{1}{2}\left(a\mathsf{x}_k + b\mathsf{k}_k\right)^2\right]$$

where $\mathsf{x}_k = \ln X_k - \ln X_0$. We assume that the shocks are mutually independent, and that the expected deviations of the shocks form the symmetric steady state ($E\mathsf{z}$, $E\mathsf{z}^*$, $E\mathsf{t}$, $E\mathsf{f}$, $E\mathsf{f}^*$, $E\mathsf{g}$, $E\mathsf{g}^*$) are of order 2. Specifically, we consider that $E(\mathsf{z} - \mathsf{z}^*) = E\mathsf{t} = 0$, $E\mathsf{f} = -0.5E(\mathsf{f})^2$, $E\mathsf{f}^* = -0.5E(\mathsf{f}^*)^2$, $E\mathsf{g} = -0.5E(\mathsf{g})^2$, $E\mathsf{g}^* = -0.5E(\mathsf{g}^*)^2$. These assumption simplify the exposition, with no loss of generality.

Consumer prices, exchange rate, and risk sharing

The consumer price index in the home country (A.6) is written as:

$$[P_k]^{1-\lambda} = n t_k^{\frac{1}{2}}[P_{Hk}]^{1-\lambda} + (1-n)\, t_k^{-\frac{1}{2}}[S_k P_{Fk}^*]^{1-\lambda}$$

We take the following expansions:

$$\begin{aligned}
[P_k]^{1-\lambda} &= [P_0]^{1-\lambda}\left[1 + (1-\lambda)\,\mathsf{p}_k + \frac{1}{2}\left[(1-\lambda)\,\mathsf{p}_k\right]^2\right] \\
t_k^{\frac{1}{2}}[P_{Hk}]^{1-\lambda} &= [P_0]^{1-\lambda}\left[1 + (1-\lambda)\,\mathsf{p}_{Hk} + \frac{1}{2}\mathsf{t}_k + \frac{1}{2}\left[(1-\lambda)\,\mathsf{p}_{Hk} + \frac{1}{2}\mathsf{t}_k\right]^2\right] \\
t_k^{-\frac{1}{2}}[S_k P_{Fk}^*]^{1-\lambda} &= [P_0]^{1-\lambda}\left[\begin{array}{c} 1 + (1-\lambda)\,\mathsf{s}_k + (1-\lambda)\,\mathsf{p}_{Fk}^* - \frac{1}{2}\mathsf{t}_k \\ +\frac{1}{2}\left[(1-\lambda)\,\mathsf{s}_k + (1-\lambda)\,\mathsf{p}_{Fk}^* - \frac{1}{2}\mathsf{t}_k\right]^2 \end{array}\right]
\end{aligned}$$

Combining them and re-arranging we get:

$$\begin{aligned}
\mathsf{p}_k &= n\mathsf{p}_{Hk} + (1-n)\left(\mathsf{s}_k + \mathsf{p}_{Fk}^*\right) + \frac{1-2n}{2(\lambda-1)}\mathsf{t}_k \\
&\quad - \frac{n}{2(\lambda-1)}\left[-(\lambda-1)\,\mathsf{p}_{Hk} + \frac{1}{2}\mathsf{t}_k\right]^2 + \frac{1}{2}(\lambda-1)\left[\mathsf{p}_k\right]^2 \qquad \text{(A.16)} \\
&\quad - \frac{1-n}{2(\lambda-1)}\left[-(\lambda-1)\,\mathsf{s}_k + (1-\lambda)\,\mathsf{p}_{Fk}^* - \frac{1}{2}\mathsf{t}_k\right]^2 \\
\mathsf{p}_k^* &= \mathsf{p}_k - \mathsf{s}_k \qquad \text{(A.17)}
\end{aligned}$$

The exchange rate and the money demands in (A.1), (A.2) and (A.4) are exactly linear in logs:

$$\begin{aligned} \mathsf{s}_k &= (\mathsf{m}_k - \mathsf{m}_k^*) - (\mathsf{f}_k - \mathsf{f}_k^*) + \Gamma && \text{(A.18)} \\ \mathsf{m}_k &= \mathsf{p}_k + \mathsf{c}_k - \mathsf{g}_k && \text{(A.19)} \\ \mathsf{m}_k^* &= \mathsf{p}_k^* + \mathsf{c}_k^* - \mathsf{g}_k^* && \text{(A.20)} \end{aligned}$$

Turning to the risk sharing wedge, the relation between the wedge and prices (A.13) is:

$$Efg = nE\left(P_H\right)^{1-\lambda} fgt^{\frac{1}{2}} \left(P\right)^{\lambda-1} + (1-n) E\left(P_H\right)^{1-\lambda} f^* g^* t^{\frac{1}{2}} \left(P\right)^{\lambda-1} (1+\Gamma)$$

We take the following expansions:

$$\begin{aligned} Efg &= f_0 g_0 \\ E\left(P_H\right)^{1-\lambda} fgt^{\frac{1}{2}} \left(P\right)^{\lambda-1} &= f_0 g_0 \left[\begin{array}{c} 1 - \frac{1}{2} E\left(\mathsf{f}+\mathsf{g}\right)^2 + (1-\lambda) E\left(\mathsf{p}_H - \mathsf{p}\right) \\ +\frac{1}{2} E\left[(1-\lambda)\left(\mathsf{p}_H - \mathsf{p}\right) + (\mathsf{f}+\mathsf{g}) + \frac{1}{2}\mathsf{t}\right]^2 \end{array} \right] \\ E\left(P_H\right)^{1-\lambda} f^* g^* t^{\frac{1}{2}} \left(P\right)^{\lambda-1} (1+\Gamma) &= f_0 g_0 \left[\begin{array}{c} 1 - \frac{1}{2} E\left(\mathsf{f}^*+\mathsf{g}^*\right)^2 + (1-\lambda) E\left(\mathsf{p}_H - \mathsf{p}\right) + \Gamma \\ +\frac{1}{2} E\left[(1-\lambda)\left(\mathsf{p}_H - \mathsf{p}\right) + (\mathsf{f}^*+\mathsf{g}^*) + \frac{1}{2}\mathsf{t}\right]^2 \end{array} \right] \end{aligned}$$

Combining we get:

$$\begin{aligned} (\lambda - 1) E\left(\mathsf{p}_H - \mathsf{p}\right) &= (1-n)\Gamma - \frac{1}{2}\left[nE\left(\mathsf{f}+\mathsf{g}\right)^2 + (1-n) E\left(\mathsf{f}^*+\mathsf{g}^*\right)^2\right] \\ &\quad + \frac{n}{2} E\left[(1-\lambda)\left(\mathsf{p}_H - \mathsf{p}\right) + (\mathsf{f}+\mathsf{g}) + \frac{1}{2}\mathsf{t}\right]^2 \qquad \text{(A.21)} \\ &\quad + \frac{1-n}{2} E\left[(1-\lambda)\left(\mathsf{p}_H - \mathsf{p}\right) + (\mathsf{f}^*+\mathsf{g}^*) + \frac{1}{2}\mathsf{t}\right]^2 \end{aligned}$$

Welfare relations

We evaluate the welfares abstracting from the direct impact of real balances. Using (A.14) the welfare of the home household is:

$$U = Efg \ln C - \kappa EL = Efg \ln C - \frac{\theta - 1}{\theta} Efg = Efg \ln C - \frac{\theta - 1}{\theta} f_0 g_0$$

We write the following expansion of the consumption term:

$$\begin{aligned} f_k g_k (\ln C_k) &= f_0 g_0 (\ln C_0) + g_0 (\ln C_0)(f_k - f_0) \\ &\quad + f_0 (\ln C_0)(g_k - g_0) + f_0 g_0 (\ln C_k - \ln C_0) \\ &\quad + \frac{1}{2} \left[\begin{array}{c} 2(\ln C_0)(g_k - g_0)(f_k - f_0) + 2g_0 (\ln C - \ln C_0)(f_k - f_0) \\ +2f_0 (\ln C_k - \ln C_0)(g_k - g_0) \end{array} \right] \\ &= f_0 g_0 (\ln C_0) + f_0 g_0 (\ln C_0) \left[\frac{f_k - f_0}{f_0} + \frac{g_k - g_0}{g_0} \right] \\ &\quad + f_0 g_0 (\ln C_k - \ln C_0) + f_0 g_0 (\ln C_0) \frac{g_k - g_0}{g_0} \frac{f_k - f_0}{f_0} \\ &\quad + f_0 g_0 (\ln C_k - \ln C_0) \frac{f_k - f_0}{f_0} + f_0 g_0 (\ln C_k - \ln C_0) \frac{g_k - g_0}{g_0} \end{aligned}$$

Eliminating the terms of order 3 and above, this is written as:

$$f_k g_k (\ln C_k) = f_0 g_0 \left[\begin{array}{c} \ln C_0 + \ln C_0 \left[\mathsf{f}_k + \frac{1}{2} (\mathsf{f}_k)^2 + \mathsf{g}_k + \frac{1}{2} (\mathsf{g}_k)^2 \right] \\ + \mathsf{c}_k + \ln C_0 \mathsf{g}_k \mathsf{f}_k + \mathsf{c}_k \mathsf{f}_k + \mathsf{c}_k \mathsf{g}_k \end{array} \right]$$

Taking expectations we obtain:

$$Efg(\ln C) = f_0 g_0 (\ln C_0) + f_0 g_0 [E\mathsf{c} + E\mathsf{c}(\mathsf{f} + \mathsf{g})]$$

We can now express the welfare in terms of deviations from the steady state where $U_0 = f_0 g_0 \ln C_0 - (\theta - 1)(\theta)^{-1} f_0 g_0$:

$$U - U_0 = f_0 g_0 [E\mathsf{c} + E\mathsf{c}(\mathsf{f} + \mathsf{g})]$$

Using the money demand (A.19) to substitute for consumption, we obtain:

$$\frac{U - U_0}{f_0 g_0} = E(\mathsf{m} + \mathsf{g}) - E\mathsf{p} + E(\mathsf{m} + \mathsf{g} - \mathsf{p})(\mathsf{f} + \mathsf{g}) \tag{A.22}$$

We can derive a similar expression for the foreign welfare:

$$\frac{U^* - U_0}{f_0 g_0} = E(\mathsf{m}^* + \mathsf{g}^*) - E\mathsf{p}^* + E(\mathsf{m}^* + \mathsf{g}^* - \mathsf{p}^*)(\mathsf{f}^* + \mathsf{g}^*) \tag{A.23}$$

The flexible price allocation

We start with the solution under flexible prices. The optimal prices set by home and foreign firms are exactly log linear:

$$\mathsf{p}_{Hk} = \mathsf{m}_k - \mathsf{z}_k - \mathsf{f}_k \qquad \mathsf{p}^*_{Fk} = \mathsf{m}^*_k - \mathsf{z}^*_k - \mathsf{f}^*_k \tag{A.24}$$

Starting with the first order solution, the consumer prices (A.16)-(A.17) are:

$$\begin{aligned} \mathsf{p}_k &= (\mathsf{m}_k - \mathsf{z}_k - \mathsf{f}_k) + (1-n)(\Gamma + \mathsf{z}_k - \mathsf{z}^*_k) + \frac{1-2n}{2(\lambda-1)}\mathsf{t}_k \\ \mathsf{p}^*_k &= \mathsf{p}_k - \mathsf{s}_k \end{aligned}$$

While the risk sharing wedge (A.21) is:

$$\Gamma = \frac{\lambda-1}{1-n} E(\mathsf{p}_H - \mathsf{p}) = (1-\lambda)\Gamma$$

This implies that there to a first order there is no risk sharing wedge: $\Gamma = 0$, therefore to a first order:

$$\begin{aligned} \mathsf{p}_k &= (\mathsf{m}_k - \mathsf{z}_k - \mathsf{f}_k) + (1-n)(\mathsf{z}_k - \mathsf{z}^*_k) + \frac{1-2n}{2(\lambda-1)}\mathsf{t}_k \\ \mathsf{p}^*_k &= \mathsf{p}_k - \mathsf{s}_k \\ \mathsf{s}_k &= (\mathsf{m}_k - \mathsf{m}^*_k) - (\mathsf{f}_k - \mathsf{f}^*_k) \\ \mathsf{c}_k - \mathsf{c}^*_k &= (\mathsf{f}_k - \mathsf{f}^*_k) \\ \mathsf{y}_k - \mathsf{y}^*_k &= \mathsf{t}_k + \lambda(\mathsf{z}_k - \mathsf{z}^*_k) \end{aligned} \tag{A.25}$$

We next turn to the second order solution, using the first order solution (A.25) to evaluate the second order terms. The expected home consumer price index (A.16) solves:

$$\begin{aligned} E\mathsf{p} &= E(\mathsf{m} - \mathsf{z} - \mathsf{f}) + (1-n)\Gamma - \frac{n}{2(\lambda-1)} E\left[-(\lambda-1)(\mathsf{m} - \mathsf{z} - \mathsf{f}) + \frac{1}{2}\mathsf{t}\right]^2 \\ &+ \frac{1}{2}(\lambda-1) E\left[(\mathsf{m} - \mathsf{z} - \mathsf{f}) + (1-n)(\mathsf{z} - \mathsf{z}^*) + \frac{1-2n}{2(\lambda-1)}\mathsf{t}\right]^2 \\ &- \frac{1-n}{2(\lambda-1)} E\left[\begin{array}{c} -(\lambda-1)\left[(\mathsf{m} - \mathsf{m}^*) - (\mathsf{f} - \mathsf{f}^*)\right] \\ +(1-\lambda)(\mathsf{m}^* - \mathsf{z}^* - \mathsf{f}^*) - \frac{1}{2}\mathsf{t} \end{array}\right]^2 \end{aligned}$$

Which is solved by:

$$E\mathsf{p} = E(\mathsf{m} - \mathsf{z} - \mathsf{f}) + (1-n)\Gamma - n(1-n)\frac{1}{2(\lambda-1)}E[(\lambda-1)(\mathsf{z}-\mathsf{z}^*)+\mathsf{t}]^2$$

The relation between prices and the wedge (A.21) is:

$$\Gamma = \frac{\lambda-1}{1-n}E(\mathsf{p}_H - \mathsf{p}) - \frac{1-n}{2}E[(\lambda-1)(\mathsf{z}-\mathsf{z}^*)+\mathsf{t}]^2$$

Combining these expressions, we obtain the risk sharing wedge:

$$\Gamma_{\text{Flex}} = -\frac{1-2n}{2\lambda}E[(\lambda-1)(\mathsf{z}-\mathsf{z}^*)+\mathsf{t}]^2 \tag{A.26}$$

where the Flex subscripts denotes the flexible price allocation. The home and foreign welfare are then given by:

$$\begin{aligned}\frac{U-U_0}{f_0 g_0} &= -(1-n)\Gamma_{\text{Flex}} + \Delta \\ \frac{U^*-U_0}{f_0 g_0} &= n\Gamma_{\text{Flex}} + \Delta^* \\ \frac{U^W-U_0}{f_0 g_0} &= n\Delta + (1-n)\Delta^*\end{aligned} \tag{A.27}$$

where:

$$\Delta = \frac{1}{2}E(\mathsf{f}+\mathsf{g})^2 + \frac{n(1-n)}{2(\lambda-1)}E[(\lambda-1)(\mathsf{z}-\mathsf{z}^*)+\mathsf{t}]^2 \tag{A.28}$$

$$\Delta^* = \frac{1}{2}E(\mathsf{f}^*+\mathsf{g}^*)^2 + \frac{n(1-n)}{2(\lambda-1)}E[(\lambda-1)(\mathsf{z}-\mathsf{z}^*)+\mathsf{t}]^2 \tag{A.29}$$

The sticky price allocation

The price charged by a home firm (A.11) is written as:

$$\begin{aligned}& nEP_H f g t^{\frac{1}{2}}[P]^{\lambda-1} + (1-n)EP_H f^* g^* t^{\frac{1}{2}}[P]^{\lambda-1}(1+\Gamma_{\text{Sticky}}) \\ = \; & \frac{\theta}{\theta-1}\frac{\kappa}{\chi}nEgt^{\frac{1}{2}}M(Z)^{-1}[P]^{\lambda-1} \\ & +\frac{\theta}{\theta-1}\frac{\kappa}{\chi}(1-n)E\frac{f^*g^*}{f}t^{\frac{1}{2}}M(Z)^{-1}[P]^{\lambda-1}(1+\Gamma_{\text{Sticky}})\end{aligned}$$

Take the following approximations:

$$
\begin{aligned}
& EP_H fgt^{\frac{1}{2}} [P]^{\lambda-1} \\
= \quad & f_0 g_0 [P_0]^{\lambda} \left[\begin{array}{c} 1 - \frac{1}{2} E (\mathsf{f} + \mathsf{g})^2 + \mathsf{p}_H + (\lambda - 1) E\mathsf{p} \\ + \frac{1}{2} E \left[(\mathsf{f} + \mathsf{g}) + \frac{1}{2}\mathsf{t} + (\lambda - 1) \mathsf{p} \right]^2 \end{array} \right] \\
& EP_H f^* g^* t^{\frac{1}{2}} [P]^{\lambda-1} (1 + \Gamma_{\text{Sticky}}) \\
= \quad & f_0 g_0 [P_0]^{\lambda} \left[\begin{array}{c} 1 - \frac{1}{2} E (\mathsf{f}^* + \mathsf{g}^*)^2 + \Gamma_{\text{Sticky}} + \mathsf{p}_H + (\lambda - 1) E\mathsf{p} \\ + \frac{1}{2} E \left[(\mathsf{f}^* + \mathsf{g}^*) + \frac{1}{2}\mathsf{t} + (\lambda - 1) \mathsf{p} \right]^2 \end{array} \right] \\
& \frac{\theta}{\theta - 1} \frac{\kappa}{\chi} E fgt^{\frac{1}{2}} M (fz)^{-1} [P]^{\lambda-1} \\
= \quad & f_0 g_0 [P_0]^{\lambda} \left[\begin{array}{c} 1 + E(\mathsf{m} - \mathsf{z} - \mathsf{f}) - \frac{1}{2} E (\mathsf{f} + \mathsf{g})^2 + (\lambda - 1) E\mathsf{p} \\ + \frac{1}{2} E \left[(\mathsf{m} - \mathsf{z} - \mathsf{f}) + (\mathsf{f} + \mathsf{g}) + \frac{1}{2}\mathsf{t} + (\lambda - 1) \mathsf{p} \right]^2 \end{array} \right] \\
& \frac{\theta}{\theta - 1} \frac{\kappa}{\chi} E f^* g^* t^{\frac{1}{2}} M (fz)^{-1} [P]^{\lambda-1} (1 + \Gamma_{\text{Sticky}}) \\
= \quad & f_0 g_0 [P_0]^{\lambda} \left[\begin{array}{c} 1 + E(\mathsf{m} - \mathsf{z} - \mathsf{f}) - \frac{1}{2} E (\mathsf{f}^* + \mathsf{g}^*)^2 + \Gamma_{\text{Sticky}} + (\lambda - 1) E\mathsf{p} \\ + \frac{1}{2} E \left[(\mathsf{m} - \mathsf{z} - \mathsf{f}) + (\mathsf{f}^* + \mathsf{g}^*) + \frac{1}{2}\mathsf{t} + (\lambda - 1) \mathsf{p} \right]^2 \end{array} \right]
\end{aligned}
$$

Combining these results leads to:

$$
\begin{aligned}
\mathsf{p}_H \quad = \quad & E(\mathsf{m} - \mathsf{z} - \mathsf{f}) + \frac{1}{2} E (\mathsf{m} - \mathsf{z} - \mathsf{f})^2 \\
& + E(\mathsf{m} - \mathsf{z} - \mathsf{f}) \left[\frac{1}{2}\mathsf{t} + (\lambda - 1) \mathsf{p} \right] \\
& + E(\mathsf{m} - \mathsf{z} - \mathsf{f}) \left[n (\mathsf{f} + \mathsf{g}) + (1 - n) (\mathsf{f}^* + \mathsf{g}^*) \right]
\end{aligned}
\tag{A.30}
$$

The price charged by a foreign firm (A.12) is written as:

$$
\begin{aligned}
& nEP_F^* fgt^{-\frac{1}{2}} [P]^{\lambda-1} [S]^{1-\lambda} \frac{1}{1 + \Gamma_{\text{Sticky}}} + (1 - n) EP_F^* f^* g^* t^{-\frac{1}{2}} [P]^{\lambda-1} [S]^{1-\lambda} \\
= \quad & \frac{\theta}{\theta - 1} \frac{\kappa}{\chi} nE \frac{fg}{f^*} t^{-\frac{1}{2}} M^* (Z^*)^{-1} [P]^{\lambda-1} [S]^{1-\lambda} \frac{1}{1 + \Gamma_{\text{Sticky}}} \\
& + \frac{\theta}{\theta - 1} \frac{\kappa}{\chi} (1 - n) E g^* t^{-\frac{1}{2}} M^* (Z^*)^{-1} [P]^{\lambda-1} [S]^{1-\lambda}
\end{aligned}
$$

Take the following expansions:

$$
\begin{aligned}
& EP_F^* fgt^{-\frac{1}{2}} \left[P\right]^{\lambda-1} \left[S\right]^{1-\lambda} \frac{1}{1+\Gamma_{\text{Sticky}}} \\
= \quad & f_0 g_0 \left[P_0^*\right]^{\lambda} \left[\begin{array}{c} 1-\frac{1}{2}E\left(\mathsf{f}+\mathsf{g}\right)^2+\mathsf{p}_F^*+\left(\lambda-1\right)E\left(\mathsf{p}-\mathsf{s}\right)-\Gamma_{\text{Sticky}} \\ +\frac{1}{2}E\left[\left(\mathsf{f}+\mathsf{g}\right)-\frac{1}{2}\mathsf{t}+\left(\lambda-1\right)\left(\mathsf{p}-\mathsf{s}\right)\right]^2 \end{array} \right] \\
& EP_F^* f^* g^* t^{-\frac{1}{2}} \left[P\right]^{\lambda-1} \left[S\right]^{1-\lambda} \\
= \quad & f_0 g_0 \left[P_0^*\right]^{\lambda} \left[\begin{array}{c} 1-\frac{1}{2}E\left(\mathsf{f}^*+\mathsf{g}^*\right)^2+\mathsf{p}_F^*+\left(\lambda-1\right)E\left(\mathsf{p}-\mathsf{s}\right) \\ +\frac{1}{2}E\left[\left(\mathsf{f}^*+\mathsf{g}^*\right)-\frac{1}{2}\mathsf{t}+\left(\lambda-1\right)\left(\mathsf{p}-\mathsf{s}\right)\right]^2 \end{array} \right] \\
& \frac{\theta}{\theta-1}\frac{\kappa}{\chi} Efgt^{-\frac{1}{2}} M^* \left(f^* z^*\right)^{-1} \left[P\right]^{\lambda-1} \left[S\right]^{1-\lambda} \frac{1}{1+\Gamma_{\text{Sticky}}} \\
= \quad & f_0 g_0 \left[P_0^*\right]^{\lambda} \left[\begin{array}{c} 1+\left(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*\right)-\frac{1}{2}E\left(\mathsf{f}+\mathsf{g}\right)^2+\left(\lambda-1\right)E\left(\mathsf{p}-\mathsf{s}\right)-\Gamma_{\text{Sticky}} \\ +\frac{1}{2}E\left[\left(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*\right)+\left(\mathsf{f}+\mathsf{g}\right)-\frac{1}{2}\mathsf{t}+\left(\lambda-1\right)\left(\mathsf{p}-\mathsf{s}\right)\right]^2 \end{array} \right] \\
& \frac{\theta}{\theta-1}\frac{\kappa}{\chi} Ef^* g^* t^{-\frac{1}{2}} M^* \left(f^* z^*\right)^{-1} \left[P\right]^{\lambda-1} \left[S\right]^{1-\lambda} \\
= \quad & f_0 g_0 \left[P_0^*\right]^{\lambda} \left[\begin{array}{c} 1+\left(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*\right)-\frac{1}{2}E\left(\mathsf{f}^*+\mathsf{g}^*\right)^2+\left(\lambda-1\right)E\left(\mathsf{p}-\mathsf{s}\right) \\ +\frac{1}{2}E\left[\left(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*\right)+\left(\mathsf{f}^*+\mathsf{g}^*\right)-\frac{1}{2}\mathsf{t}+\left(\lambda-1\right)\left(\mathsf{p}-\mathsf{s}\right)\right]^2 \end{array} \right]
\end{aligned}
$$

Combining these results we get:

$$
\begin{aligned}
\mathsf{p}_F^* \quad = \quad & E\left(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*\right)+\frac{1}{2}E\left(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*\right)^2 \\
& +E\left(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*\right)\left[-\frac{1}{2}\mathsf{t}+\left(\lambda-1\right)\left(\mathsf{p}-\mathsf{s}\right)\right] \\
& +E\left(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*\right)\left[n\left(\mathsf{f}+\mathsf{g}\right)+\left(1-n\right)\left(\mathsf{f}^*+\mathsf{g}^*\right)\right]
\end{aligned} \tag{A.31}
$$

Starting with the first order solution, there are no deviations of the firms prices (A.30)-(A.31) from the steady state :

$$\mathsf{p}_H = \mathsf{p}_F^* = 0$$

There is no first order risk sharing wedge ($\Gamma_{\text{Sticky}} = 0$), and the consumer price index and the exchange rates are:

$$\begin{aligned}
\mathsf{p}_k &= (1-n)\,\mathsf{s}_k + \frac{1-2n}{2(\lambda-1)}\mathsf{t}_k \\
\mathsf{p}_k^* &= \mathsf{p}_k - \mathsf{s}_k \qquad \text{(A.32)} \\
\mathsf{s}_k &= (\mathsf{m}_k - \mathsf{m}_k^*) - (\mathsf{f}_k - \mathsf{f}_k^*)
\end{aligned}$$

Turning to the second order solution, the prices set by home and foreign firms (A.30)-(A.31) are:

$$\begin{aligned}
\mathsf{p}_H &= E(\mathsf{m}-\mathsf{z}-\mathsf{f}) + \frac{1}{2}E(\mathsf{m}-\mathsf{z}-\mathsf{f})^2 + (1-n)\,E(\mathsf{m}-\mathsf{z}-\mathsf{f})\left[(\lambda-1)\,\mathsf{s}+\mathsf{t}\right] \\
&\quad + E(\mathsf{m}-\mathsf{z}-\mathsf{f})\left[n(\mathsf{f}+\mathsf{g}) + (1-n)(\mathsf{f}^*+\mathsf{g}^*)\right] \qquad \text{(A.33)} \\
\mathsf{p}_F^* &= E(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*) + \frac{1}{2}E(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*)^2 - nE(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*)\left[(\lambda-1)\,\mathsf{s}+\mathsf{t}\right] \\
&\quad + E(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*)\left[n(\mathsf{f}+\mathsf{g}) + (1-n)(\mathsf{f}^*+\mathsf{g}^*)\right] \qquad \text{(A.34)}
\end{aligned}$$

The expected home consumer price index (A.16) solves:

$$\begin{aligned}
E\mathsf{p} &= n\mathsf{p}_H + (1-n)(E\mathsf{s}+\mathsf{p}_F^*) - \frac{1-n}{2(\lambda-1)}E\left[-(\lambda-1)\,\mathsf{s} - \frac{1}{2}\mathsf{t}\right]^2 \\
&\quad - \frac{n}{2(\lambda-1)}E\left[\frac{1}{2}\mathsf{t}\right]^2 + \frac{1}{2}(\lambda-1)\left[(1-n)\,\mathsf{s} + \frac{1-2n}{2(\lambda-1)}\mathsf{t}\right]^2
\end{aligned}$$

which simplifies to:

$$\begin{aligned}
E\mathsf{p} &= (1-n)\,E\left[(\mathsf{m}-\mathsf{f}) - (\mathsf{m}^*-\mathsf{f}^*)\right] \qquad \text{(A.35)} \\
&\quad + n\mathsf{p}_H + (1-n)\,\mathsf{p}_F^* + (1-n)\,\Gamma - \frac{n(1-n)}{2(\lambda-1)}E\left[(\lambda-1)\,\mathsf{s}+\mathsf{t}\right]^2
\end{aligned}$$

From the relation between prices and the wedge (A.21) we obtain:

$$\begin{aligned}
\Gamma_{\text{Sticky}} &= \frac{\lambda-1}{2\lambda}\left[E(\mathsf{m}-\mathsf{z}-\mathsf{f})^2 - E(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*)^2\right] \\
&\quad + \frac{\lambda-1}{\lambda}E\left[(\lambda-1)\,\mathsf{s}+\mathsf{t}\right]\left[\begin{array}{c}(1-n)(\mathsf{m}-\mathsf{z}-\mathsf{f}) \\ +n(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*)\end{array}\right] - \frac{1-2n}{2\lambda}E\left[(\lambda-1)\,\mathsf{s}+\mathsf{t}\right]^2
\end{aligned}$$

which can be re-written as:

$$\begin{aligned}\Gamma_{\text{Sticky}} &= \Gamma_{\text{Flex}} + \frac{\lambda-1}{2\lambda}\left[E\left(\mathsf{m}-\mathsf{z}-\mathsf{f}\right)^2 - E\left(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*\right)^2\right] \\ &\quad + \frac{\lambda-1}{\lambda}E\left[(\lambda-1)\,\mathsf{s}+\mathsf{t}\right]\left[\begin{array}{c}(1-n)\left(\mathsf{m}-\mathsf{z}-\mathsf{f}\right)\\ +n\left(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*\right)\end{array}\right] \\ &\quad - \frac{1-2n}{2\lambda}\left[E\left[(\lambda-1)\,\mathsf{s}+\mathsf{t}\right]^2 - E\left[(\lambda-1)\left(\mathsf{z}-\mathsf{z}^*\right)+\mathsf{t}\right]^2\right]\end{aligned}$$

This allows us to write the home and foreign welfare as:

$$\begin{aligned}\frac{U_{\text{Sticky}}-U_0}{f_0 g_0} &= \Delta - \Omega - (1-n)\,\Gamma_{\text{Sticky}} + (1-n)\,\Psi \\ \frac{U^*_{\text{Sticky}}-U_0}{f_0 g_0} &= \Delta^* - \Omega + n\Gamma_{\text{Sticky}} - n\Psi \\ \frac{U^{World}_{\text{Sticky}}-U_0}{f_0 g_0} &= n\Delta + (1-n)\,\Delta^* - \Omega\end{aligned} \tag{A.36}$$

where:

$$\Omega = \frac{1}{2}\left[\begin{array}{c} nE\left(\mathsf{m}-\mathsf{z}-\mathsf{f}\right)^2 + (1-n)\,E\left(\mathsf{m}^*-\mathsf{z}^*-\mathsf{f}^*\right)^2 \\ +\,(\lambda-1)\,n\,(1-n)\,E\left[\mathsf{s}-\left(\mathsf{z}-\mathsf{z}^*\right)\right]^2\end{array}\right] \geq 0 \tag{A.37}$$

$$\Psi = E\left[n\left(\mathsf{m}-\mathsf{f}\right)+(1-n)\left(\mathsf{m}^*-\mathsf{f}^*\right)\right]\left[\left(\mathsf{f}+\mathsf{g}\right)-\left(\mathsf{f}^*+\mathsf{g}^*\right)\right] \tag{A.38}$$

and Δ and Δ^* are the same as under flexible prices.

Instead of expressing the welfare relations vis-a-vis the symmetric steady state, we can write them in terms of differences from the flexible price allocation:

$$\begin{aligned}\mathsf{u} &= \frac{U_{\text{Sticky}}-U_{\text{Flex}}}{f_0 g_0} = -\Omega - (1-n)\left(\Gamma_{\text{Sticky}}-\Gamma_{\text{Flex}}\right) + (1-n)\,\Psi \\ \mathsf{u}^* &= \frac{U^*_{\text{Sticky}}-U^*_{\text{Flex}}}{f_0 g_0} = -\Omega + n\left(\Gamma_{\text{Sticky}}-\Gamma_{\text{Flex}}\right) - n\Psi \\ \mathsf{u}^{World} &= \frac{U^{World}_{\text{Sticky}}-U^{W}_{\text{Flex}}}{f_0 g_0} = -\Omega\end{aligned}$$

Monetary policy

Components

The welfares under sticky prices are driven by Δ and Δ^*, which are independent of policy, and Γ, Ψ and Ω. The derivatives of the various components (A.28)-(A.29) and

(A.37)-(A.38) with respect to the home monetary stance in state k are written as:

$$\begin{aligned}
\frac{\partial \Omega}{\pi_k \partial \mathsf{m}_k} &= \left[n + (\lambda - 1)\, n\, (1-n)\right] \left(\mathsf{m}_k - \mathsf{z}_k - \mathsf{f}_k\right) \\
&\quad - (\lambda - 1)\, n\, (1-n) \left(\mathsf{m}_k^* - \mathsf{z}_k^* - \mathsf{f}_k^*\right) \\
\frac{\partial \Psi}{\pi_k \partial \mathsf{m}_k} &= n \left[\left(\mathsf{f}_k + \mathsf{g}_k\right) - \left(\mathsf{f}_k^* + \mathsf{g}_k^*\right)\right] \\
\frac{\partial \Gamma_{\text{Sticky}} - \Gamma_{\text{Flex}}}{\pi_k \partial \mathsf{m}_k} &= (\lambda - 1) \left(\mathsf{m}_k - \mathsf{z}_k - \mathsf{f}_k\right) + \frac{\lambda - 1}{\lambda} n \left[(\lambda - 1) \left(\mathsf{z}_k - \mathsf{z}_k^*\right) + \mathsf{t}_k\right]
\end{aligned}$$

Similarly for the derivatives with respect to the foreign monetary stance:

$$\begin{aligned}
\frac{\partial \Omega}{\pi_k \partial \mathsf{m}_k^*} &= \left[(1-n) + (\lambda - 1)\, n\, (1-n)\right] \left(\mathsf{m}_k^* - \mathsf{z}_k^* - \mathsf{f}_k^*\right) \\
&\quad - (\lambda - 1)\, n\, (1-n) \left[\left(\mathsf{m}_k - \mathsf{z}_k - \mathsf{f}_k\right)\right] \\
\frac{\partial \Psi}{\pi_k \partial \mathsf{m}_k^*} &= (1-n) \left[\left(\mathsf{f}_k + \mathsf{g}_k\right) - \left(\mathsf{f}_k^* + \mathsf{g}_k^*\right)\right] \\
\frac{\partial \Gamma_{\text{Sticky}} - \Gamma_{\text{Flex}}}{\pi_k \partial \mathsf{m}_k^*} &= - (\lambda - 1) \left(\mathsf{m}_k^* - \mathsf{z}_k^* - \mathsf{f}_k^*\right) + \frac{\lambda - 1}{\lambda} (1-n) \left[(\lambda - 1) \left(\mathsf{z}_k - \mathsf{z}_k^*\right) + \mathsf{t}_k\right]
\end{aligned}$$

Cooperative policy

Under cooperation, the monetary stances are set to maximize the world welfare U^W, which depends only on Ω. The optimality conditions are therefore:

$$\frac{\partial \Omega}{\pi_k \partial \mathsf{m}_k} = 0 \qquad\qquad \frac{\partial \Omega}{\pi_k \partial \mathsf{m}_k^*} = 0$$

This implies that monetary policy is inward looking:

$$\mathsf{m}_{k\text{Coop}} = \mathsf{z}_k + \mathsf{f}_k \qquad\qquad \mathsf{m}_{k\text{Coop}}^* = \mathsf{z}_k^* + \mathsf{f}_k^* \tag{A.39}$$

This leads to full stabilization, and replicates the flexible price allocation:

$$\Omega_{\text{Coop}} = 0 \qquad \Gamma_{\text{Coop}} = \Gamma_{\text{Flex}} \qquad \Psi_{\text{Coop}} = 0 \tag{A.40}$$

$$\mathsf{s}_{k\text{Coop}} = \left(\mathsf{z}_k - \mathsf{z}_k^*\right) + \Gamma_{\text{Coop}} \tag{A.41}$$

Nash equilibrium

Under a Nash equilibrium the authorities maximize only the welfare of the household living in their own country. The optimality conditions are:

$$\begin{aligned} -\frac{\partial\Omega}{\pi_k\partial\mathsf{m}_k}-(1-n)\frac{\partial\Gamma_{\text{Sticky}}-\Gamma_{\text{Flex}}}{\pi_k\partial\mathsf{m}_k}+(1-n)\frac{\partial\Psi}{\pi_k\partial\mathsf{m}_k} &= 0 \\ -\frac{\partial\Omega}{\pi_k\partial\mathsf{m}_k^*}+n\frac{\partial\Gamma_{\text{Sticky}}-\Gamma_{\text{Flex}}}{\pi_k\partial\mathsf{m}_k^*}-n\frac{\partial\Psi}{\pi_k\partial\mathsf{m}_k^*} &= 0 \end{aligned}$$

These conditions are written as:

$$\begin{aligned} 0 &= [n+(\lambda-1)\,n\,(1-n)+(1-n)\,(\lambda-1)]\,(\mathsf{m}_k-\mathsf{z}_k-\mathsf{f}_k) \\ &\quad -(\lambda-1)\,n\,(1-n)\,(\mathsf{m}_k^*-\mathsf{z}_k^*-\mathsf{f}_k^*)+n\,(1-n)\,\mathsf{T}_k \\ 0 &= [(1-n)+(\lambda-1)\,n\,(1-n)+n\,(\lambda-1)]\,(\mathsf{m}_k^*-\mathsf{z}_k^*-\mathsf{f}_k^*) \\ &\quad -(\lambda-1)\,n\,(1-n)\,(\mathsf{m}_k-\mathsf{z}_k-\mathsf{f}_k)-n\,(1-n)\,\mathsf{T}_k \end{aligned}$$

where:

$$\mathsf{T}_k=\frac{(\lambda-1)^2}{\lambda}\,(\mathsf{z}_k-\mathsf{z}_k^*)+\frac{\lambda-1}{\lambda}\mathsf{t}_k-[(\mathsf{f}_k+\mathsf{g}_k)-(\mathsf{f}_k^*+\mathsf{g}_k^*)]$$

This is solved by:

$$\mathsf{m}_{k\text{Nash}} = (\mathsf{z}_k+\mathsf{f}_k)-\frac{n\,(1-n)}{\Theta}\,[(1-n)+n\,(\lambda-1)]\,\mathsf{T}_k \tag{A.42}$$

$$\mathsf{m}_{k\text{Nash}}^* = (\mathsf{z}_k^*+\mathsf{f}_k^*)+\frac{n\,(1-n)}{\Theta}\,[n+(\lambda-1)\,(1-n)]\,\mathsf{T}_k \tag{A.43}$$

where:

$$\Theta=n\,(1-n)+[1-n\,(1-n)]\,(\lambda-1)+2n\,(1-n)\,(\lambda-1)^2>0$$

This implies the following movements for the exchange rate:

$$\mathsf{s}_{k\text{Nash}}=(\mathsf{z}_k-\mathsf{z}_k^*)-\frac{n\,(1-n)}{\Theta}\lambda\mathsf{T}_k+\Gamma_{\text{Nash}} \tag{A.44}$$

The welfare gap between the two countries is driven by $\Gamma_{\text{Nash}}-\Psi_{\text{Nash}}$, which we write as:

$$\Gamma_{\text{Nash}}-\Psi_{\text{Nash}}=\Phi_1\Gamma_{\text{Coop}}+(1-2n)\,(\lambda-1)\,\Phi_2E\,[(\mathsf{f}+\mathsf{g})-(\mathsf{f}^*+\mathsf{g}^*)]^2$$

where:

$$
\begin{aligned}
\Phi_1 &= 1 - 2\frac{\lambda-1}{\lambda}\left[\frac{(\lambda-1)^2}{\Theta}n(1-n)\right] + \frac{\lambda-2}{\lambda-1}\left[\frac{(\lambda-1)^2}{\Theta}n(1-n)\right]^2 \\
&= 1 - n(1-n)(\lambda-1)^3\begin{pmatrix} 2\left[1-4n(1-n)\right](\lambda-1) \\ +3n(1-n)\lambda^2 \end{pmatrix} \\
&\quad \times \begin{pmatrix} \left[n(1-n)\right]^2 \\ +n(1-n)\left[2-n(1-n)\right](\lambda-1) \\ +\left[1+3\left[n(1-n)\right]^2\right](\lambda-1)^2 \\ +\left[1+2n(1-n)+\left[n(1-n)\right]^2\right](\lambda-1)^3 \\ +4n(1-n)(\lambda-1)^4+4\left[n(1-n)\right]^2(\lambda-1)^5 \end{pmatrix}^{-1} \\
&\in [0,1] \\
\Phi_2 &= \frac{n(1-n)}{2\Theta^2}\left[3n(1-n)\left[1+(\lambda-1)^2\right]+2\left[1-n(1-n)\right](\lambda-1)\right] > 0
\end{aligned}
$$